3

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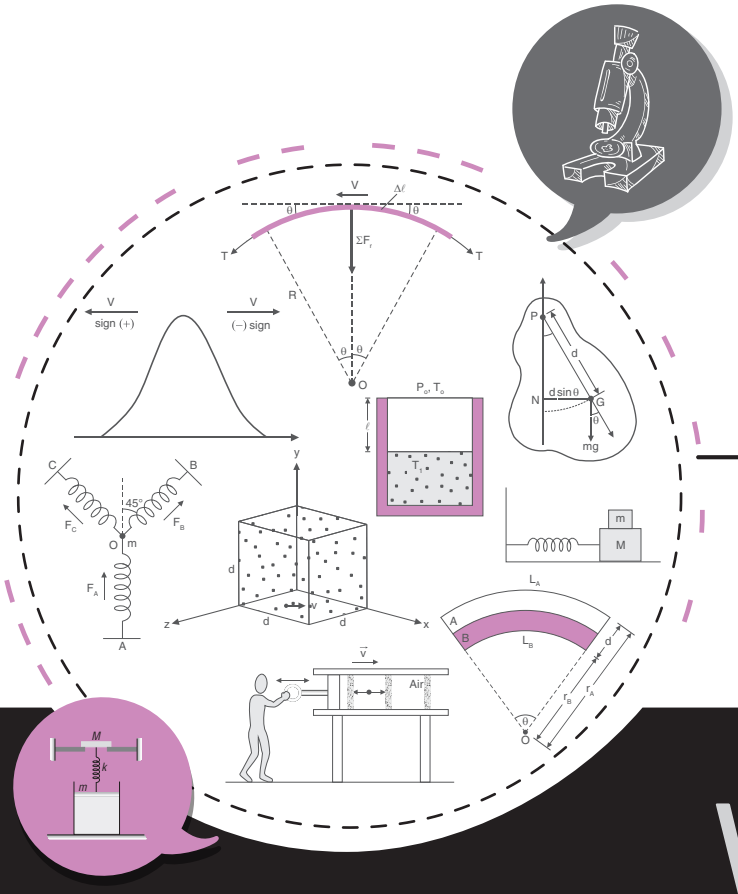

JEE ADVANCED PHYSICS

Waves and Thermodynamics

Rahul Sardana

Pearson

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CONTENTS

Chapter Insight xvi

Preface xxi

About the Author xxii

CHAPTER 1 MECHANICAL PROPERTIES OF MATTER 1.1

Elasticity 1.1

Introduction 1.1

The States of Matter 1.1

Elasticity 1.1

Deforming Force 1.2

Perfectly Elastic Body 1.2

Perfectly Plastic Body 1.2

Cause of Elasticity 1.2

Stress 1.2

Tangential (Shear) Stress and Normal (Tensile) Stress. 1.2

Volumetric Stress or Bulk Stress or Pressure or Normal Stress 1.3

Strain 1.3

Longitudinal Strain 1.3

Volumetric Strain 1.3

Shear Strain 1.4

Relation of Stress to Strain and Elastic Modulus 1.4

Hooke's Law 1.4

Young's Modulus 1.4

Thermal Stress 1.9

Force Constant of a Wire 1.9

Elongation of Rod Under it's Self Weight 1.13

Breaking Stress 1.14

Breaking of a Wire Under its own Weight 1.15

Elastic Potential Energy 1.17

Behaviour of a Wire Under Stress 1.18

Elastic Fatigue 1.19

Behaviour of Rubber Under Stress 1.19

Elastic Hysteresis 1.19

Poisson's Ratio (σ) 1.19

Relation Between Volumetric Strain, Longitudinal Strain and Poisson's Ratio 1.20
Depression of a Beam 1.20
Shear Modulus 1.21
Torsion of a Cylinder 1.22
Bulk Modulus 1.23
Relations Between Elastic Constants 1.23
Density of Compressed Liquids 1.23

Fluid Statics 1.27

Fluids: Introduction and assumptions 1.27
Density 1.27
Relative Density or Specific Gravity. 1.27
Density of a Mixture of Two or More Liquids 1.28
Fluid at Rest 1.28
Pressure 1.29
Atmospheric Pressure (P_0) 1.29
Pressure is Isotropic 1.29
Variation of Pressure With Depth 1.29
The Incompressible Fluid Model 1.30
Absolute Pressure and Gauge Pressure 1.30
Simple Free Body Diagram of a Liquid in a Container 1.32
Hydrostatic Paradox 1.32
Pressure Measuring Device: Manometer 1.34
The Mercury Barometer 1.35
Pascal's Law. 1.36
Hydraulic Lift 1.36
Force and Torque due to Hydrostatic Pressure 1.38
The Compressible Fluid Model 1.41
Liquids in Accelerated Containers 1.43
Horizontally Accelerating U-Tube 1.46
Pressure Difference in Rotating Fluids 1.46
Archimedes' Principle 1.50
Archimedes Principle: A General Proof for an Arbitrary Shaped Body 1.51
Finding the Relative Density (R.D.) or Specific Gravity of a Body 1.52
Finding the Relative Density (R.D.) or Specific Gravity of a Liquid 1.52
Laws of Floatation. 1.52
Apparent Weight of a Body Immersed in Liquid 1.53
Fractional Volume of a Floating Body Inside the Liquid 1.53
Buoyant Force in Accelerating Fluids 1.55
Stability of a Floating Body. 1.56

Viscosity 1.66

Viscosity 1.66
Coefficient of Viscosity (η) 1.66

Similarities Between Viscosity and Solid Friction . . . 1.66
Differences Between Viscosity and Solid Friction . . . 1.67
Some Applications of Viscosity . . . 1.67
Variation of Viscosity . . . 1.69
Stoke's Law . . . 1.69
Terminal Velocity . . . 1.69
Flow of Liquid in Tube: Critical Velocity . . . 1.71
Steady Flow of Liquid Through a Capillary Tube: Poiseuille's Formula . . . 1.72
Combination of Tubes in Series . . . 1.73
Combination of Tubes in Parallel. . . . 1.73

Fluid Dynamics . . . 1.77
Motion of a Fluid . . . 1.77
Flow of Ideal Fluid: Basic Definitions . . . 1.77
Equation of Continuity . . . 1.77
Energies Possessed by a Liquid . . . 1.79
Bernoulli's Equation . . . 1.80
Static, Dynamic and Total Pressure . . . 1.82
Working of an Airplane or Aerodynamic Lift . . . 1.84
Roof's Blowing Off During Wind Storms . . . 1.85
Attraction Between Two Closely Parallel Moving Boats or Trains . . . 1.85
Magnus Effect (Spinning of a Ball or Spinning of a Bullet) . . . 1.85
Action of an Atomiser or Sprayer . . . 1.85
Blood Flow and Heart Attack . . . 1.85
Venturimeter . . . 1.86
Pitot Tube. . . . 1.87
Speed of Efflux . . . 1.88
Force of Reaction due to Ejection of the Liquid From an Orifice . . . 1.89
Speed of Efflux: Torricelli's Theorem . . . 1.90
Range of Liquid Hitting the Ground . . . 1.90
Time to Empty the Beaker to the Orifice . . . 1.91

Surface Tension . . . 1.99
Surface Tension: Introduction . . . 1.99
Explanation to Surface Tension . . . 1.99
Force due to Surface Tension . . . 1.100
Surface Energy . . . 1.102
Relation Between Surface Energy and Surface Tension . . . 1.102
Splitting of Bigger Drop into Small Droplets . . . 1.103
Formation of Bigger Drop From Small Droplets . . . 1.103
Cohesive Forces . . . 1.104
Adhesive Forces . . . 1.104
Water Wets the Glass Surface, but Mercury does not: Explanation . . . 1.105

Angle of Contact . . . 1.105
Shape of Liquid Meniscus in a Glass Tube . . . 1.105
Radius of Curvature of a Curve . . . 1.106
Principal Radii of Curvature of a Surface . . . 1.106
Excess Pressure Inside Liquid Surface with Two Curvatures: Young-Laplace Equation . . . 1.107
Pressure Difference Between the Two Sides Force a Curved Liquid Surface . . . 1.108
Excess Pressure . . . 1.108
Excess Pressure Inside a Liquid Drop and Air Bubble . . . 1.108
Excess Pressure Inside a Soap Bubble . . . 1.109
Capillarity . . . 1.111
Explanation to Capillarity . . . 1.112
Practical Applications of Capillarity . . . 1.113
Rise of Liquid Between Two Parallel Plates . . . 1.113
Liquid Between Two Horizontal Plates . . . 1.115
Rise of Liquid in a Capillary Tube of Insufficient Length . . . 1.115
Solved Problems . . . *1.118*
Practice Exercises . . . *1.132*
Single Correct Choice Type Questions . . . *1.132*
Multiple Correct Choice Type Questions . . . *1.152*
Reasoning Based Questions . . . *1.157*
Linked Comprehension Type Questions . . . *1.159*
Matrix Match/Column Match Type Questions . . . *1.164*
Integer/Numerical Answer Type Questions . . . *1.167*
Archive: JEE Main . . . *1.170*
Archive: JEE Advanced . . . *1.176*
Answer Keys–Test Your Concepts and Practice Exercises . . . *1.183*

CHAPTER 2 HEAT AND THERMODYNAMICS . . . 2.1

Thermometry, Thermal Expansion and Calorimetry . . . 2.1
Heat . . . 2.1
Thermal Equilibrium . . . 2.1
Zeroth Law and Concept of Temperature . . . 2.1
Thermometry and Thermometers . . . 2.2
Temperature Scales . . . 2.2
Temperature Relation Between Scales . . . 2.3
Types of Thermometers (Optional Reading) . . . 2.4
Thermal Expansion . . . 2.5
Effect of Temperature on Pendulum Clocks . . . 2.9
Error in Metal Scale Due to Expansion or Contraction . . . 2.10
Thermal Expansion in Liquids . . . 2.11
Expansion of Gases . . . 2.13
Change in Density of Solids and Liquids with Temperature . . . 2.13

Thermal Stress in a Rod Fixed Between Two Rigid Supports . . . 2.13
Correction For Barometric Reading . . . 2.14
Correction For Expansion of Brass Scale . . . 2.14
Correction For Expansion of Mercury . . . 2.15
Effect of Temperature on Upthrust . . . 2.15
Bimetallic Strip . . . 2.15
Calorimetry: Heat, Work and Mechanical Equivalent of Heat . . . 2.18
Concept of Gram Specific Heat . . . 2.18
Thermal Capacity or Heat Capacity . . . 2.18
Water Equivalent (w) . . . 2.18
Bodies Placed in Contact . . . 2.19
Concept of Latent Heat or the Heat of Transformation . . . 2.20

Kinetic Theory of Gases (KTG) . . . 2.25
Kinetic Theory of Gases . . . 2.25
Molecular Model for the Pressure of an Ideal Gas . . . 2.25
Kinetic Interpretation of Temperature . . . 2.26
Maxwell Boltzmann Law for Distribution of Molecular Speeds . . . 2.27
Average Velocity of Gas Molecules . . . 2.27
Average Speed of Gas Molecules . . . 2.27
Root-Mean-Square Velocity of Gas Molecules . . . 2.28
Most Probable Speed of Gas Molecules . . . 2.28
Mean Free Path . . . 2.28
Gas Laws . . . 2.32
Boyle's Law . . . 2.32
Charle's Law . . . 2.32
Gay-Lussac's Law . . . 2.32
Avogadro's Law . . . 2.33
Dalton's Law of Partial Pressures . . . 2.33
Graham's Law of Diffusion . . . 2.33
Ideal Gas Equation . . . 2.33
Vander Waal's Equation . . . 2.36
Critical Temperature and Pressure (T_c) . . . 2.37
Boyle Temperature (T_b) . . . 2.37
Variation of Pressure With Height or Elevation Above the Sea Level . . . 2.37
Variation of Pressure With Depth Below the Sea Level . . . 2.37
Concept of Internal Energy (U) . . . 2.40
Degrees of Freedom f . . . 2.40
Polyatomic Gases . . . 2.41
Theorem of Equipartition of Energy . . . 2.41
Molar Specific Heat of the Gas (C) . . . 2.42
Specific Heat at Constant Volume (CV) . . . 2.42

C_v for an Ideal Gas . . . 2.42
Specific Heat at Constant Pressure (C_p) . . . 2.42
Internal Energy (U) for an Ideal Gas: Revisited . . . 2.43
Relation Between C_p and C_v . . . 2.43
C_p for an Ideal Gas . . . 2.44
Adiabatic Ratio (γ) for an Ideal Gas . . . 2.44
Relation of C_p and C_v With γ . . . 2.44
Dulong and Petit's Law . . . 2.44

First Law of Thermodynamics and Thermodynamic Processes . . . 2.50
Thermodynamics . . . 2.50
Concept of Work . . . 2.50
Heat Versus Work . . . 2.52
Mechanical Equivalent of Heat . . . 2.52
Thermodynamic Work . . . 2.52
First Law of Thermodynamics . . . 2.53
Misconception Between Heat and Internal Energy . . . 2.53
First Law of Thermodynamics: Revisited . . . 2.53
Isolated System . . . 2.57
Isochoric Process . . . 2.57
Isobaric Process . . . 2.58
Isothermal Process . . . 2.60
Work Done in an Isothermal Process . . . 2.61
Adiabatic Process . . . 2.64
Equation of State for an Adiabatic Process . . . 2.64
Work Done in an Adiabatic Process . . . 2.64
Indicator Diagram for an Adiabatic Process . . . 2.65
Adiabatic Free Expansion . . . 2.67
Polytropic Process . . . 2.69
Molar Specific Heat of a Polytropic Process . . . 2.69
Work Done in a Polytropic Process . . . 2.70
Indicator Diagram for a Polytropic Process . . . 2.70
Cyclic Process . . . 2.74
Efficiency of a Cyclic Process . . . 2.74
Heat Engine . . . 2.74
Refrigerator or Heat Pump . . . 2.76
Relation Between η and β . . . *2.76*
Reversible Process . . . 2.76
Conditions for a Process to be Reversible . . . 2.76
Irreversible Process . . . 2.77
Carnot Engine/Cycle . . . 2.77
Heat Engines in Series . . . 2.78

Limitation of First Law and Introduction to Second Law of Thermodynamics 2.84
The Second Law of Thermodynamics: Revisited 2.85
Entropy 2.85
Calculation of Entropy 2.85

Transfer of Heat 2.88
Modes of Heat Transfer 2.88
Heat Conduction 2.88
Wiedemann Franz Law 2.90
Temperature of Junction. 2.90
Thermometric Conductivity (or Diffusivity) 2.91
Thermal Resistance 2.91
Slabs of Different Materials in Series 2.91
Slabs of Different Materials in Parallel 2.91
Ingen-Hauz Experiment. 2.94
Growth of Ice on Ponds 2.94
Convection 2.98
Radiation 2.99
Prevost's Theory of Heat Exchange 2.100
Perfectly Black Body 2.100
Absorptance, Reflectance and Transmittance 2.100
Emissive Power 2.101
Emissivity 2.101
Energy Density (u) 2.101
Absorptive Power 2.101
Kirchhoff's Law of Heat Radiations 2.102
Applications of Kirchhoff's Law 2.102
Stefan's Law or Stefan's-Boltzmann Law or Stefan's Fourth Power Law 2.103
Newton's Law of Cooling (Special Case of Stefan's Law) 2.106
Solution to Differential Equation Involved in Newton's Law of Cooling 2.106
Solar Constant and Temperature of Sun 2.108
Black Body Radiation Spectrum and Wien's Law (or Wien's Displacement Law) 2.108
Solved Problems *2.111*
Practice Exercises *2.130*
Single Correct Choice Type Questions *2.130*
Multiple Correct Choice Type Questions *2.153*
Reasoning Based Questions *2.159*
Linked Comprehension Type Questions *2.160*
Matrix Match/Column Match Type Questions *2.166*
Integer/Numerical Answer Type Questions *2.168*
Archive: JEE Main *2.171*
Archive: JEE Advanced *2.182*
Answer Keys–Test Your Concepts and Practice Exercises *2.196*

CHAPTER 3 SIMPLE HARMONIC MOTION 3.1

Introduction 3.1
Periodic Motion 3.1
Oscillatory Motion 3.1
Simple Harmonic Motion (SHM). 3.1
Equilibrium Position or Mean Position 3.2
Dynamics of Simple Harmonic Motion 3.2
Phase and Phase Difference 3.3
Equation of SHM and Phase 3.4
Differential Equation for SHM 3.6
Equation of Motion of a Simple Harmonic Motion 3.7
Characteristics of SHM 3.8
Condition for Motion to be SHM. 3.8
Velocity of a Particle in SHM 3.9
Potential Energy of a Particle in SHM 3.10
Kinetic Energy of a Particle in SHM. 3.10
Mechanical Energy of a Particle in SHM 3.10
Understanding SHM for Physical Systems 3.17
Mass-Spring System 3.17
SHM of Free Bodies in Absence of External Forces 3.20
Coupled Spring System 3.21
Rotational Systems or Angular SHM 3.32
Simple Pendulum 3.39
SHM of a Pendulum of Large Length 3.39
Torsional Pendulum 3.41
Physical Pendulum or Compound Pendulum 3.42
Oscillations of a Floating Pole. 3.45
Liquid Oscillating in a U-Tube 3.45
Ball Oscillating in the Neck of an Air Chamber 3.46
Composition of Two SHM of the Same Period Along the Same Line. 3.48
Composition of Two SHM of Same Period at Right Angles to Each Other 3.49
Damped Harmonic Oscillation 3.50
Forced Oscillations and Resonance 3.51
Solved Problems *3.53*
Practice Exercises *3.65*
Single Correct Choice Type Questions *3.65*
Multiple Correct Choice Type Questions. *3.79*
Reasoning Based Questions. *3.82*
Linked Comprehension Type Questions *3.83*
Matrix Match/Column Match Type Questions. *3.89*
Integer/Numerical Answer Type Questions. *3.91*

Archive: JEE Main *3.93*
Archive: JEE Advanced *3.97*
Answer Keys–Test Your Concepts and Practice Exercises *3.103*

CHAPTER 4 MECHANICAL WAVES 4.1

Introduction 4.1
Characteristics of Wave Motion 4.1
Types of Wave Motion 4.2
Wave Motion Parameters 4.2
Relation Between Path Difference (Δx) and Phase Difference ($\Delta\phi$) 4.2
Equation of a Harmonic Wave 4.3
Characteristic Wave Equation 4.4
Particle Velocity, Wave Slope and Particle Acceleration in a Sinusoidal Wave 4.5
Group Velocity 4.7
A Symmetrical Wave Pulse 4.7
Speed of a Transverse Wave in a String 4.10
Energy Density of a Transverse Wave 4.11
Power and Intensity of Wave Travelling Through a Stretched String 4.12
About Amplitude 4.13
Reflection of String Wave 4.13
Wave Properties after Reflection/Refraction (Transmission) 4.14
Partial Reflection and Transmission 4.14
Boundary Conditions 4.14
Relation Between Displacement Wave and Pressure Wave for a Longitudinal Wave 4.18
Speed of a Longitudinal Waves 4.19
Relation Between Pressure Wave and Density Wave 4.20
About Nature of Waves 4.21
Power and Intensity of Wave Travelling Through a Medium 4.21
Intensity of Sound Wave 4.22
Supersonic and Shock Waves 4.23
Intensity Level and Loudness of Sound 4.23
Noise Levels due to Different Sources 4.23
Newton's Formula for Speed of Sound in a Gas 4.24
Laplace's Correction for Speed of Sound in a Gas 4.25
Effect of External Factors on Speed of Sound 4.25
Reflection of Sound Waves 4.27
Refraction 4.28
Echo 4.28
Diffraction 4.29
Superposition of Waves: Introduction 4.29
Superposition Principle 4.29
Interference 4.30

Intensity Factor 4.31
Coherent Sources 4.31
Stationary Waves 4.36
Transverse Waves 4.36
Longitudinal Waves 4.36
Stationary Waves Produced on Reflection From the Free End (Rarer Medium) 4.36
Stationary Waves Produced on Reflection From Fixed End (Denser Medium) 4.38
Comparison of Progressive and Stationary Wave 4.40
Stationary Waves in a String Fixed at Both Ends and Modes of Vibration of a Stretched String 4.40
Stationary Waves in a String Fixed at One End and Modes of Vibration of a Stretched String 4.41
Melde's Experiment 4.41
Sonometer 4.42
Vibrations of a Clamped Rod 4.43
Longitudinal Stationary Waves in Air Columns/Organ Pipes 4.44
Stationary Sound Waves in an Open Organ Pipe 4.45
Closed Pipe 4.45
End Correction in Organ Pipes 4.46
Resonance Tube 4.46
Kundt's Tube 4.46
Beats 4.49
To Find Unknown Frequency 4.50
Pitch (Frequency) 4.52
Quality (Timbre) 4.52
Octave and Interval 4.52
Musical Sound and Musical Scale 4.52
Audible, Infrasonic and Ultrasonic Waves 4.53
Doppler Effect 4.54
Listener and Source Both Stationary 4.55
Source Approaching a Stationary Listener 4.55
Listener Approaching a Stationary Source 4.56
Source and Listener Approaching 4.56
Source Approaching Listener and Both Moving in Same Direction 4.56
Source not Moving Towards Observer 4.60
Effect of Motion of Medium 4.61
Wind Effect 4.61
Principle of Sonar (or Radar) 4.63
Doppler's Effect of Light 4.64
Red Shift and Blue Shift 4.64
Acoustics of Buildings 4.65

Solved Problems 4.67
Practice Exercises 4.82
Single Correct Choice Type Questions 4.82
Multiple Correct Choice Type Questions 4.94
Reasoning Based Questions 4.98
Linked Comprehension Type Questions 4.99
Matrix Match/Column Match Type Questions 4.104
Integer/Numerical Answer Type Questions 4.106
Archive: JEE Main 4.107
Archive: JEE Advanced 4.112
Answer Keys–Test Your Concepts and Practice Exercises 4.121

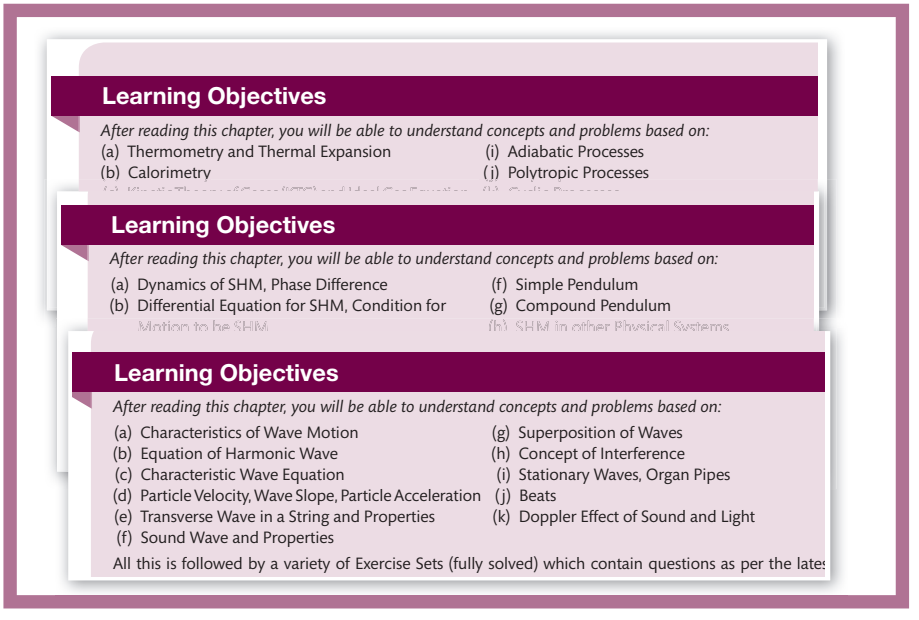
HINTS AND EXPLANATIONS

Chapter 1: Mechanical Properties of Matter H.3
Chapter 2: Heat and Thermodynamics H.75
Chapter 3: Simple Harmonic Motion H.178
Chapter 4: Mechanical Waves H.233

CHAPTER INSIGHT

Learning Objectives

Help the students set an aim to achieve the major take-aways from a particular chapter.



Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Thermometry and Thermal Expansion
(b) Calorimetry
(i) Adiabatic Processes
(j) Polytropic Processes

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Dynamics of SHM, Phase Difference
(b) Differential Equation for SHM, Condition for Motion to be SHM
(f) Simple Pendulum
(g) Compound Pendulum
(h) SHM in other Physical Systems

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Characteristics of Wave Motion
(b) Equation of Harmonic Wave
(c) Characteristic Wave Equation
(d) Particle Velocity, Wave Slope, Particle Acceleration
(e) Transverse Wave in a String and Properties
(f) Sound Wave and Properties
(g) Superposition of Waves
(h) Concept of Interference
(i) Stationary Waves, Organ Pipes
(j) Beats
(k) Doppler Effect of Sound and Light

All this is followed by a variety of Exercise Sets (fully solved) which contain questions as per the lates

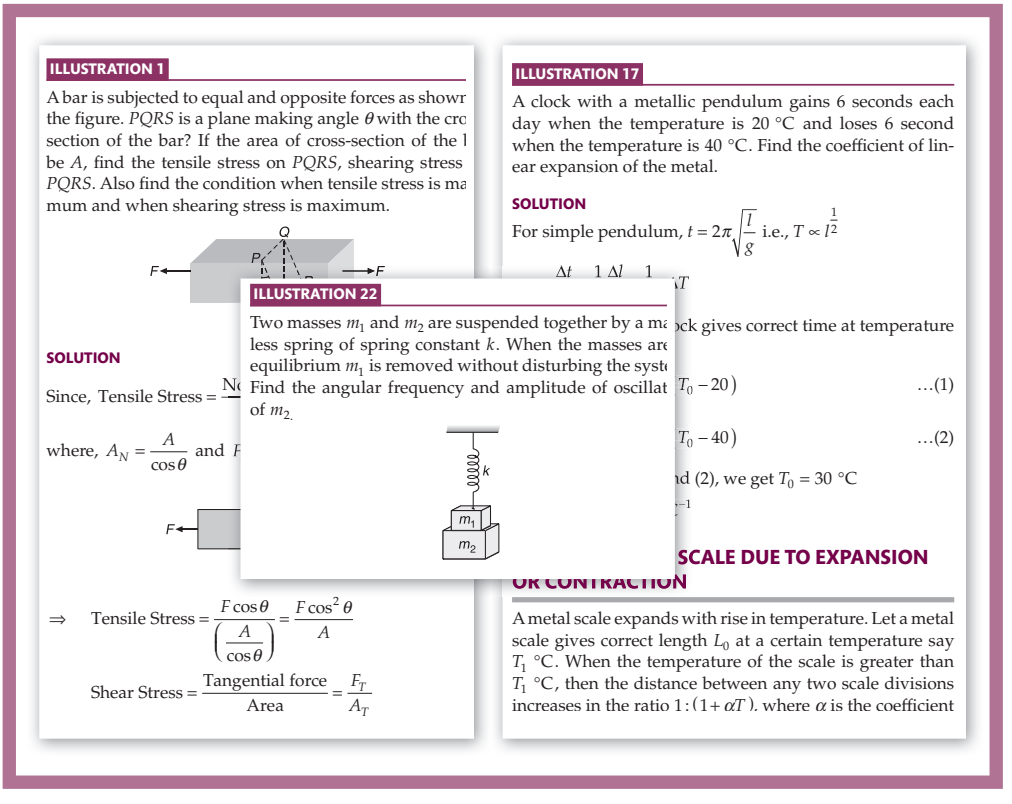


ILLUSTRATION 1

A bar is subjected to equal and opposite forces as showr the figure. *PQRS* is a plane making angle θ with the cro section of the bar? If the area of cross-section of the l be A, find the tensile stress on *PQRS*, shearing stress *PQRS*. Also find the condition when tensile stress is ma mum and when shearing stress is maximum.

SOLUTION

Since, Tensile Stress = N

where, $A_N = \frac{A}{\cos\theta}$ and

$$\Rightarrow \quad \text{Tensile Stress} = \frac{F\cos\theta}{\left(\frac{A}{\cos\theta}\right)} = \frac{F\cos^2\theta}{A}$$

$$\text{Shear Stress} = \frac{\text{Tangential force}}{\text{Area}} = \frac{F_T}{A_T}$$

ILLUSTRATION 22

Two masses m_1 and m_2 are suspended together by a ma less spring of spring constant k. When the masses are equilibrium m_1 is removed without disturbing the syste Find the angular frequency and amplitude of oscillat of m_2.

ILLUSTRATION 17

A clock with a metallic pendulum gains 6 seconds each day when the temperature is 20 °C and loses 6 second when the temperature is 40 °C. Find the coefficient of linear expansion of the metal.

SOLUTION

For simple pendulum, $t = 2\pi\sqrt{\frac{l}{g}}$ i.e., $T \propto l^{\frac{1}{2}}$

ock gives correct time at temperature

$T_0 - 20)$ ...(1)

$T_0 - 40)$ ...(2)

nd (2), we get $T_0 = 30$ °C

SCALE DUE TO EXPANSION OR CONTRACTION

A metal scale expands with rise in temperature. Let a metal scale gives correct length L_0 at a certain temperature say T_1 °C. When the temperature of the scale is greater than T_1 °C, then the distance between any two scale divisions increases in the ratio $1:(1+\alpha T)$. where α is the coefficient

Theory with Illustrations

Elaborative and simple theory helps the students to understand the illustrations supporting the theory. Please note that theory and problem solving techniques are based on simple learning program IF → THEN → ELSE. I would suggest you not to attempt the illustrations without going through the theory of that section.

Test Your Concepts-I

Based on Young's Mo

1. One end of a uniform wire of length L and of W is attached rigidly to a point in the roof and a W_1 is suspended from its lower end. If S is the cross-section of the wire, then find the stress in th at a height $\frac{3L}{4}$ from its lower end.
2. A homogeneous block with a mass m hangs o vertical wires of equal length arranged symme Find the tension of the wires if the middle wi

Test Your Concepts-II

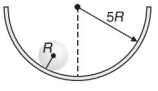
Bas

1. In a container of negligible mass 30 g of ste 100 °C is added to 200 g of water that has a ter ture of 40 °C. If no heat is lost to the surrounding is the final temperature of the system? Also find of water and steam in equilibrium. Take $L_v = 539$ and $c_{water} = 1\,\text{calg}^{-1}(°\text{C})^{-1}$
2. A copper cube of mass 200 g slides down a inclined plane of inclination 37° at a constant Assuming that the loss in mechanical energy go

Test Your Concepts-III

Based

1. A solid sphere of radius R rolls without slippir cylindrical trough of radius $5R$. Find the time pe small oscillations.



2. The disk has a weight of 100 N and rolls witho ping on the horizontal surface as it oscillates ab equilibrium position. If the disk is displaced, b

Test Your Concepts-IV

Base

1. Two harmonic waves are represented in SI un $y_1(x,t) = 0.2\sin(x-3t)$ and
$y_2(x,t) = 0.2\sin(x-3t+\phi)$
Write the expression for the sum $y = y_1 + y_2$ for $\phi =$
If the phase difference ϕ between waves is unl and amplitude of their sum is 0.32 m, calculate ϕ
2. Two waves of equal frequencies have their amp in the ratio of 3:5. They are superimposed or other. Calculate the ratio of minimum and ma intensities of the resultant wave.

Test Your Concepts

These topic based exercise sets are based on simple, single concept classification technique. These are meant for students practice after they study a particular topic and want to practice more on that topic learnt. Finally, in case of any difficulty they can refer to the hints and solutions to these exercise sets given at the end of the book.

Problem Solving Techniques

These techniques ensure that students become capable enough to solve a variety of problems in an easy and quick manner.

Problem Solving Technique(s)

(a) Rise in temperature is same for Celsius and Kelvin
(b) Whenever, ratio of the temperatures has to be do not forget to convert the temperatures to the lute Kelvin scale. Like for example if we are ask take the ratio of the temperatures say $T_1 = 10$ °
$T_2 = 5$ °C, then the ratio $\frac{T_1}{T_2} \neq 2$.
Instead of that the ratio is
$$\frac{T_1}{T_2} = \frac{10+273}{5+273} = \frac{283}{278} = 1.02$$
and this ratio happens to be very close to 1 (n
So please take care while expressing the temper
ratios.
(c) In both the Fahrenheit and Celsius temperature

Problem Solving Technique(s)

In this problem, if friction between block and surface present and coefficient of friction is μ, then following cases arise.
Case (i): $F \leq \mu mg$,
Case (ii): $F > \mu mg$,
For both these cases the answer will be same for elong
i.e., $\Delta\ell = \frac{FL}{2AY}$ for both cases.

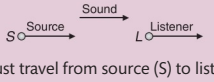
Problem Solving Technique(s)

CALCULATING ANGULAR FREQUENCY OF A PARTICLE IN SHM USING TAYLOR'S METHOD:

If mathematical function is given by $y = f(x)$, ther Taylor's theorem, we have
$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \ldots$$
If x is taken as the displacement of a particle from its n position and the restoring force F acting on the par depends on this x, then this restoring force $F = f(x)$ a ven that displacement x from mean position can be ten by using Taylor's expansion series as
$$F = f(x) = f(0) + xf(0) + \frac{x^2}{2!}f''(0) + \ldots$$

Problem Solving Technique(s)

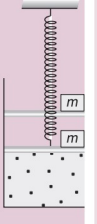
To learn about the general formula and apply it to va situations we proceed by executing the following s instructions.



(a) Sound must travel from source (S) to listener (L).
(b) All velocities along the direction of sound are tak positive.
(c) All velocities opposite to the direction of soun taken as negative.
(d) The apparent frequency is taken as

Problem Solving Technique(s)

(a) Sometimes the piston (which is assumed to be light) is attached to a spring of force constant k and a mass m is placed over the piston. The area of the piston is A. The gas expands. To make the calculation easy we assume that initially the spring was in its natural length. We are required to find the work done by the gas. As the piston is assumed to be light, net force on it at every instant is zero.



Conceptual Note(s)

Modulus of elasticity E (whether it is Y, B or η) is given

$$E = \frac{\text{stress}}{\text{strain}}$$

Following conclusions can be made from the expression:

(a) $E \propto$ stress (for same strain), i.e., if we want the amount of strain in two different materials, t which needs more stress is having more E.

(b) $E \propto \frac{1}{\text{strain}}$ (for same stress). i.e., if the same a

Conceptual Note(s)

A and B both oscillate simple harmonically with sam lar frequency ω but different displacement and amplitudes. Frequency or time period can also be o by energy method as under.

$$E = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 - m_A g x_A + m_B g x_B + \frac{1}{2}k\left(\frac{x_0}{2}\right.$$

$$\Rightarrow \quad E = \frac{1}{2}mv^2 + \frac{1}{2}m\left(\frac{v}{2}\right)^2 - mgx + mg\frac{x}{2} + \frac{1}{2}k\left(\frac{x_0}{2}+\right.$$

where $x_0 = \frac{2mg}{k}$ and x is the displacement (downw A from its mean position x_0. Since E =constant

Conceptual Note(s)

(a) To conclude, with this simplified model of ideal we have arrived at an important result that relate macroscopic quantities of pressure, volume and perature to a microscopic quantity like rms speed we get a key link between the microscopic wor gas molecules and the macroscopic world.

(b) In the derivation of this result, note that we not accounted for collisions between gas molec When these collisions are considered, the results **not alter as collisions do not affect the mom of the particles, with no net effect on the w** Bravo!!! This is consistent with one of our i

Conceptual Note(s)

(a) At nodes displacement of particles is always zer they are permanently at rest. But strain $\frac{\partial y}{\partial x}$ at noc maximum, so pressure and hence energy is maxir at nodes.

(b) At antinodes the displacement is maximum and s $\frac{\partial y}{\partial x}$ is zero, so pressure and hence energy is minir at antinodes.

(c) All particles between two consecutive nodes vibra same phase while the particles on opposite side node vibrate in opposite phase.

Conceptual Notes

The Conceptual Notes, Remarks, Words of Advice, Misconception Removals provide warnings to the students about common errors and help them avoid falling for conceptual pitfalls.

Chapter End Solved Problems

These are based on multiple concept usage in a single problem approach so as to expose a student's brain to the ultimate throttle required to take the JEE examination.

SOLVED PROBLEMS

PROBLEM 1

A steel wire of diameter 0.40 mm is stretched between rigid supports separated by a horizontal distance of 1.8 m

(a) What load suspended from the middle of the wire will produce a stress of 10^9 Nm^{-2}?

(b) Find the stored elastic energy in the wire under this load.

(c) Find the drop in potential energy of the object hung from the centre for this stress. $Y_S = 2\times10^{11}$ Nm^{-2}.

SOLUTION

(a) $2T\cos\theta = Mg$

$$\Rightarrow \quad T = \frac{Mg}{2\cos\theta} \qquad \ldots(1)$$

Further, $\frac{T}{A} = 10^9$ Nm^{-2} $\qquad \ldots(2)$

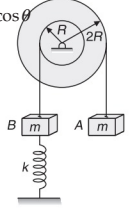
$\Rightarrow T = (10^9)A$

From equations (1) and (2)

$$10^9 = \frac{Mg}{2A\cos\theta}$$

PROBLEM 3

In the figure shown pulley in massless. Initially the bloc are held at a height such that spring is in relaxed positio The block A is released. Find the



(a) amplitude and velocity amplitude of A.

(b) frequency of the oscillation of block B. There is

PROBLEM 14

(b) $U = \frac{(\text{Stress})^2}{2Y}(\text{Volume})$

$$\Rightarrow \quad U = \frac{(10^9)^2}{2(2.0\times10^{11})}(0.125\times10^{-6})(1.8)$$

$$\Rightarrow \quad U = 0.562 \text{ J}$$

(c) $10^9 = \frac{Mg}{2A\cos\theta}$

$$\Rightarrow \quad \cos\theta = \frac{Mg}{(2A)(10^9)} = \frac{25}{2\times0.125\times10^{-6}\times10^9} = 0.1$$

$$\Rightarrow \quad \theta = 84.26°$$

$$\Rightarrow \quad \Delta U = Mgh = (25)(0.9\cot\theta)$$

$$\Rightarrow \quad \Delta U = 2.25 \text{ J}$$

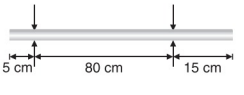
PROBLEM 22

A glass full of water upto a height of 10 cm has a area 10 cm^2, top of area 30 cm^2 and volume 1 l and next higher frequency of natural longitudinal o lation of the rod. Given that Young's modulus of ela ity and density of aluminium are $Y = 1.6\times10^{11}$ Nm^{-2} $\rho = 2500$ kgm^{-3} respectively.

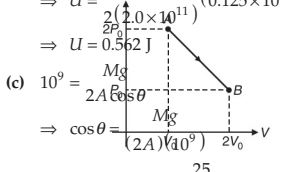
A metal rod of length 1 m is clamped at two point shown in the figure. Distance of the clamp from the ends are 5 cm and 15 cm respectively. Find the minim

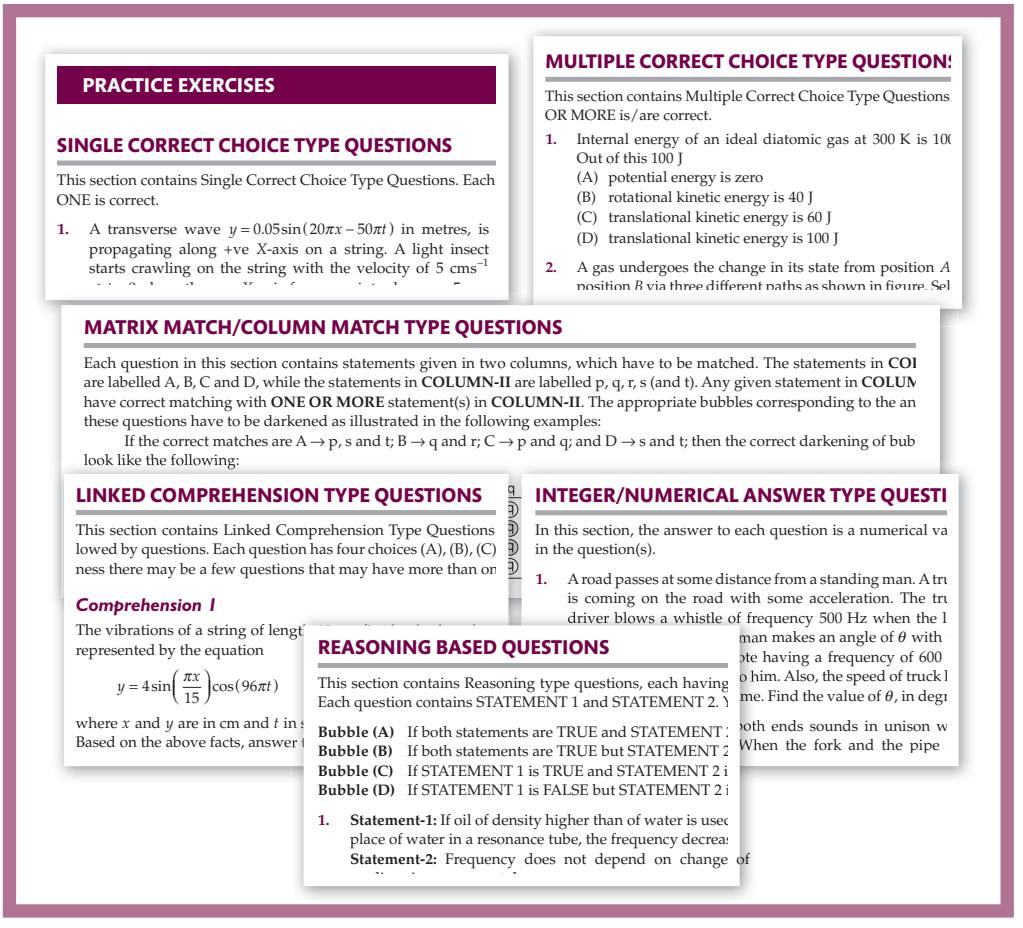
(a) Calculate the force exerted by the water o tom.

(b) Calculate the resultant force exerted by th



SOLUTION





PRACTICE EXERCISES

SINGLE CORRECT CHOICE TYPE QUESTIONS

This section contains Single Correct Choice Type Questions. Each ONE is correct.

1. A transverse wave $y = 0.05\sin(20\pi x - 50\pi t)$ in metres, is propagating along +ve X-axis on a string. A light insect starts crawling on the string with the velocity of 5 cms^{-1}

MULTIPLE CORRECT CHOICE TYPE QUESTIONS

This section contains Multiple Correct Choice Type Questions OR MORE is/are correct.

1. Internal energy of an ideal diatomic gas at 300 K is 10(Out of this 100 J
   (A) potential energy is zero
   (B) rotational kinetic energy is 40 J
   (C) translational kinetic energy is 60 J
   (D) translational kinetic energy is 100 J
2. A gas undergoes the change in its state from position A position B via three different paths as shown in figure. Sel

MATRIX MATCH/COLUMN MATCH TYPE QUESTIONS

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COI** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUM** have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the an these questions have to be darkened as illustrated in the following examples:

If the correct matches are A → p, s and t; B → q and r; C → p and q; and D → s and t; then the correct darkening of bub look like the following:

LINKED COMPREHENSION TYPE QUESTIONS

This section contains Linked Comprehension Type Questions lowed by questions. Each question has four choices (A), (B), (C) ness there may be a few questions that may have more than on

Comprehension I

The vibrations of a string of lengt represented by the equation

$$y = 4\sin\left(\frac{\pi x}{15}\right)\cos(96\pi t)$$

where x and y are in cm and t in Based on the above facts, answer

INTEGER/NUMERICAL ANSWER TYPE QUESTI

In this section, the answer to each question is a numerical va in the question(s).

1. A road passes at some distance from a standing man. A tru is coming on the road with some acceleration. The tru driver blows a whistle of frequency 500 Hz when the l man makes an angle of θ with ote having a frequency of 600 o him. Also, the speed of truck l me. Find the value of θ, in deg

oth ends sounds in unison w When the fork and the pipe

REASONING BASED QUESTIONS

This section contains Reasoning type questions, each having Each question contains STATEMENT 1 and STATEMENT 2. Y

Bubble (A) If both statements are TRUE and STATEMENT
Bubble (B) If both statements are TRUE but STATEMENT 2
Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 i
Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 i

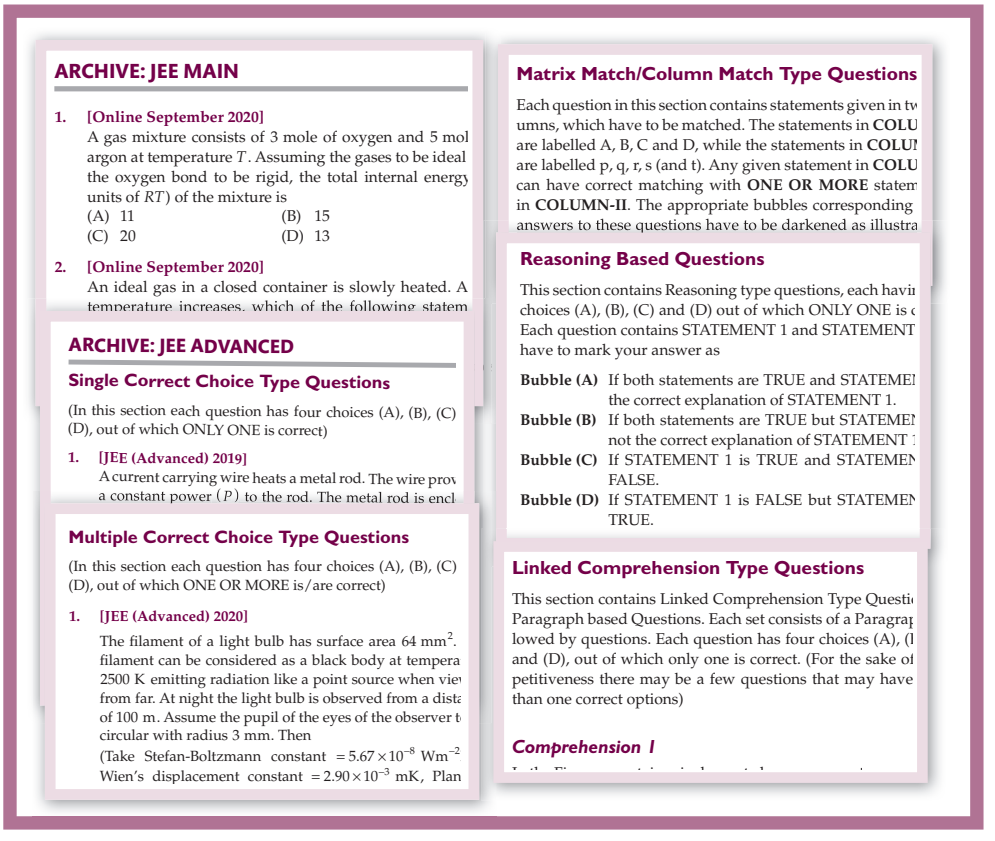
1. **Statement-1:** If oil of density higher than of water is usec place of water in a resonance tube, the frequency decrea
   Statement-2: Frequency does not depend on change of

Practice Exercise

Inclusion of all types of questions asked in JEE Advanced in adequate numbers helps you with enough practice

Archive JEE Main and Advanced

From this fully updated section, students get to know the actual pattern of the problems asked in the past examinations.



ARCHIVE: JEE MAIN

1. **[Online September 2020]**
   A gas mixture consists of 3 mole of oxygen and 5 mol argon at temperature T. Assuming the gases to be ideal the oxygen bond to be rigid, the total internal energy units of RT) of the mixture is
   (A) 11 (B) 15
   (C) 20 (D) 13
2. **[Online September 2020]**
   An ideal gas in a closed container is slowly heated. A temperature increases, which of the following statem

ARCHIVE: JEE ADVANCED

Single Correct Choice Type Questions

(In this section each question has four choices (A), (B), (C) (D), out of which ONLY ONE is correct)

1. **[JEE (Advanced) 2019]**
   A current carrying wire heats a metal rod. The wire prov a constant power (P) to the rod. The metal rod is encl

Multiple Correct Choice Type Questions

(In this section each question has four choices (A), (B), (C) (D), out of which ONE OR MORE is/are correct)

1. **[JEE (Advanced) 2020]**
   The filament of a light bulb has surface area 64 mm^2. filament can be considered as a black body at tempera 2500 K emitting radiation like a point source when vie from far. At night the light bulb is observed from a dist of 100 m. Assume the pupil of the eyes of the observer t circular with radius 3 mm. Then
   (Take Stefan-Boltzmann constant $= 5.67 \times 10^{-8}$ Wm^{-2} Wien's displacement constant $= 2.90 \times 10^{-3}$ mK, Plan

Matrix Match/Column Match Type Questions

Each question in this section contains statements given in tw umns, which have to be matched. The statements in **COLU** are labelled A, B, C and D, while the statements in **COLU** are labelled p, q, r, s (and t). Any given statement in **COLU** can have correct matching with **ONE OR MORE** statem in **COLUMN-II**. The appropriate bubbles corresponding answers to these questions have to be darkened as illustra

Reasoning Based Questions

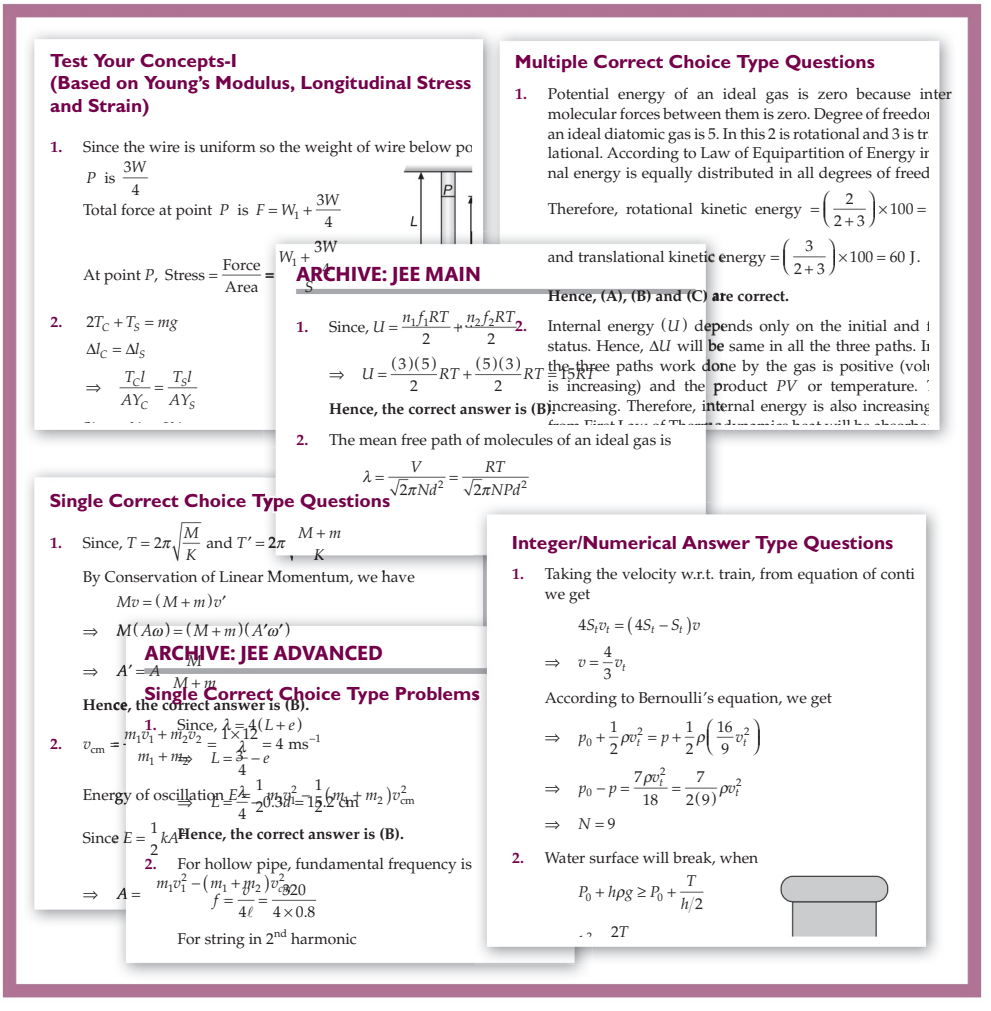
This section contains Reasoning type questions, each havi choices (A), (B), (C) and (D) out of which ONLY ONE is Each question contains STATEMENT 1 and STATEMENT have to mark your answer as

Bubble (A) If both statements are TRUE and STATEME the correct explanation of STATEMENT 1.
Bubble (B) If both statements are TRUE but STATEME not the correct explanation of STATEMENT
Bubble (C) If STATEMENT 1 is TRUE and STATEME FALSE.
Bubble (D) If STATEMENT 1 is FALSE but STATEME TRUE.

Linked Comprehension Type Questions

This section contains Linked Comprehension Type Questi Paragraph based Questions. Each set consists of a Paragra lowed by questions. Each question has four choices (A), (and (D), out of which only one is correct. (For the sake of petitiveness there may be a few questions that may have than one correct options)

Comprehension I



Test Your Concepts-I
(Based on Young's Modulus, Longitudinal Stress and Strain)

1. Since the wire is uniform so the weight of wire below po
P is $\frac{3W}{4}$
Total force at point P is $F = W_1 + \frac{3W}{4}$
At point P, Stress $= \frac{\text{Force}}{\text{Area}} = \frac{W_1 + \frac{3W}{4}}{A}$
2. $2T_C + T_S = mg$
$\Delta l_C = \Delta l_S$
$\Rightarrow \quad \frac{T_C l}{AY_C} = \frac{T_S l}{AY_S}$

ARCHIVE: JEE MAIN

1. Since, $U = \frac{n_1 f_1 RT}{2} + \frac{n_2 f_2 RT}{2}$
$\Rightarrow \quad U = \frac{(3)(5)}{2}RT + \frac{(5)(3)}{2}RT = 15RT$
Hence, the correct answer is (B).
2. The mean free path of molecules of an ideal gas is
$\lambda = \frac{V}{\sqrt{2}\pi N d^2} = \frac{RT}{\sqrt{2}\pi N P d^2}$

Multiple Correct Choice Type Questions

1. Potential energy of an ideal gas is zero because inter molecular forces between them is zero. Degree of freedo an ideal diatomic gas is 5. In this 2 is rotational and 3 is tr lational. According to Law of Equipartition of Energy ir nal energy is equally distributed in all degrees of freed
Therefore, rotational kinetic energy $= \left(\frac{2}{2+3}\right) \times 100 =$
and translational kinetic energy $= \left(\frac{3}{2+3}\right) \times 100 = 60$ J.
Hence, (A), (B) and (C) are correct.
2. Internal energy (U) depends only on the initial and f status. Hence, ΔU will be same in all the three paths. In the three paths work done by the gas is positive (vol is increasing) and the product PV or temperature. increasing. Therefore, internal energy is also increasing

Single Correct Choice Type Questions

1. Since, $T = 2\pi\sqrt{\frac{M}{K}}$ and $T' = 2\pi$ $\frac{M+m}{K}$
By Conservation of Linear Momentum, we have
$Mv = (M+m)v'$
$\Rightarrow \quad M(A\omega) = (M+m)(A'\omega')$
$\Rightarrow \quad A' = A\frac{M}{M+m}$
Hence, the correct answer is (B).
2. $v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{1\times 12}{3} = 4$ ms^{-1}
Energy of oscillation $E = \frac{1}{2}(m_1 + m_2)v_{cm}^2 = \frac{1}{2}(1+2)4^2 = 24$ J
Since $E = \frac{1}{2}kA^2$
$\Rightarrow \quad A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2\times 24}{320}} = 0.3$ m $= 30$ cm

ARCHIVE: JEE ADVANCED

Single Correct Choice Type Problems

1. Since, $\lambda = 4(L+e)$
$\Rightarrow \quad L = \frac{\lambda}{4} - e$
$\Rightarrow \quad L = \frac{1}{4}(0.8) - 0.2 = 0$ cm
Hence, the correct answer is (B).
2. For hollow pipe, fundamental frequency is
$f = \frac{v}{4\ell} = \frac{320}{4\times 0.8}$
For string in 2nd harmonic

Integer/Numerical Answer Type Questions

1. Taking the velocity w.r.t. train, from equation of conti we get
$4S_t v_t = (4S_t - S_t)v$
$\Rightarrow \quad v = \frac{4}{3}v_t$
According to Bernoulli's equation, we get
$\Rightarrow \quad p_0 + \frac{1}{2}\rho v_t^2 = p + \frac{1}{2}\rho\left(\frac{16}{9}v_t^2\right)$
$\Rightarrow \quad p_0 - p = \frac{7\rho v_t^2}{18} = \frac{7}{2(9)}\rho v_t^2$
$\Rightarrow \quad N = 9$
2. Water surface will break, when
$P_0 + h\rho g \geq P_0 + \frac{T}{h/2}$
$\Rightarrow \quad 2T$

Hints and Explanations

Exhaustive solutions with shortcuts (where ever needed), help students enhance their problem-solving skills.

PREFACE

In the past few years, the IIT-JEE has evolved as an examination designed to check a candidate's true scientific skills. The examination pattern needs one to see those little details which others fail to see. These details tell us how much in-depth we should know to explain a concept in the right direction. Keeping the present-day scenario in mind, this series is written for students, to allow them not only to learn the tools but also to see why they work so nicely in explaining the beauty of ideas behind the subject. The central goal of this series is to help the students develop a thorough understanding of Physics as a subject. This series stresses on building a rock-solid technical knowledge based on firm foundation of the fundamental principles followed by a large collection of formulae. The primary philosophy of this book is to guide the aspirants towards detailed groundwork for strong conceptual understanding and development of problem-solving skills like mature and experienced physicists.

This updated Third Edition of the book will help the aspirants prepare for both Advanced and Main levels of JEE conducted for IITs and other elite engineering institutions in India. This book will also be equally useful for the students preparing for Physics Olympiads.

This book is enriched with detailed exhaustive theory that introduces the concepts of Physics in a clear, concise, thorough and easy-to-understand language. A large collection of relevant problems is provided in eight major categories (including updated archive for JEE Advanced and JEE Main), for which the solutions are demonstrated in a logical and stepwise manner.

We have carefully divided the series into seven parts to make the learning of different topics seamless for the students. These parts are

- Mechanics – I
- Mechanics – II
- Waves and Thermodynamics
- Electrostatics and Current Electricity
- Magnetic Effects of Current and Electromagnetic Induction
- Optics
- Modern Physics

Finally, I would like to thank all my teacher friends who had been a guiding source of light throughout the entire journey of writing this book.

To conclude, I apologise in advance for the errors (if any) that may have inadvertently crept in the text. I would be grateful to the readers who bring errors of any kind to my attention. I truly welcome all comments, critiques and suggestions. I hope this book will nourish you with the concepts involved so that you get a great rank at JEE.

PRAYING TO GOD FOR YOUR SUCCESS AT JEE. GOD BLESS YOU!

Rahul Sardana

ABOUT THE AUTHOR

Rahul Sardana, is a Physics instructor and mentor having rich and vast experience of about 20 years in the field of teaching Physics to JEE Advanced, JEE Main and NEET aspirants. Along with teaching, authoring books for engineering and medical aspirants has been his passion. He authored his first book '*MCQs in Physics*' in 2002 and since then he has authored many books exclusively for JEE Advanced, JEE Main and NEET examinations.

He is also a motivational speaker having skills to motivate students and ignite the spark in them for achieving success in all colours of life. Throughout this journey, by the Grace of God, under his guidance and mentorship, many of his students have become successful engineers and doctors.

You can reach him through email at sardanaeducation@gmail.com.

CHAPTER 1

Mechanical Properties of Matter

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Stress, Strain, Hooke's Law, Normal Stress
(b) Longitudinal Strain and Young's Modulus
(c) Elastic Potential Energy and Energy Density
(d) Tangential Stress, Shear Strain and Shear Modulus
(e) Volumetric Stress, Volumetric Strain and Bulk's Modulus
(f) Fluid Properties, Pressure and Pascal's Law
(g) Pressure in Accelerating Fluids
(h) Archimedes' Principle and Buoyancy
(i) Viscosity, Stoke's Law and Terminal Speed
(j) Ideal Fluids, Equation of Continuity, Bernoulli's Theorem and Applications
(k) Surface Tension, Surface Energy, Excess Pressure and Capillarity.

All this is followed by a variety of Exercise Sets (fully solved) which contain questions as per the latest JEE pattern. At the end of Exercise Sets, a collection of problems asked previously in JEE (Main and Advanced) are also given.

ELASTICITY

INTRODUCTION

So far, we have dealt with all the solids that have been modelled as rigid bodies, that is, objects do not change their shape. Real objects, however, deform to some extent when an external force is applied to them. In this chapter we formulate some systematic ways to describe qualitatively the deformation of solids that are subjected to applied forces.

THE STATES OF MATTER

Matter is usually classified into one of three **states** or **phases**: solid, liquid, or gas.
Because they can flow easily, both liquids and gases are called **fluids**.

- A solid has a fixed shape which it tends to retain, whereas fluids have no fixed shape.
- A liquid sinks to the bottom of its container, and a gas expands to fill the available volume.
- The atoms in a solid vibrate about fixed equilibrium positions, whereas the atoms or molecules in a liquid move about relatively freely and collide frequently with each other.
- The atoms in a solid or liquid are quite closely packed, which makes it difficult to reduce their volume, they are almost incompressible.
- On the average, the atoms or molecules in a gas are far apart, typically about ten atomic diameters at room temperature and pressure. They collide much less frequently than those in a liquid. Gases in general are compressible.

ELASTICITY

When external forces are applied on a body, which is not free to move, there is a change in its dimensions. When these forces are removed, the body tends to regain its original shape and size.

The property by virtue of which a body tends to regain its original shape and size when the external deforming forces are removed, is called ***elasticity.***

The property of the material body by virtue of which it does not regain its original configuration when the external force is removed is called plasticity.

DEFORMING FORCE

An external force applied to a body which changes its size or shape or both is called deforming force.

PERFECTLY ELASTIC BODY

A body is said to be perfectly elastic when it completely regains its original configuration on the removal of deforming forces. Since no material can regain completely its original form so the concept of perfectly elastic body is only an ideal concept. A quartz fibre is the nearest approach to the perfectly elastic body.

PERFECTLY PLASTIC BODY

A body is said to be perfectly plastic if it does not regain its original form even slightly when the deforming force is removed. Since every material partially regains its original form on the removal of deforming forces, so the concept of perfectly plastic body is also an ideal concept. Paraffin wax, wet clay are the nearest approach to a perfectly plastic bodies.

CAUSE OF ELASTICITY

In a solid, atoms and molecules are arranged in such a way that each molecule is acted upon by the forces due to the neighbouring molecules. These forces are known as intermolecular forces. When no deforming force is applied on the body, each molecule of the solid (i.e. body) is in its equilibrium position and the inter molecular forces between the molecules of the solid are maximum.

On applying the deforming force on the body, the molecules either come closer or go far apart from each other. As a result of this, the molecules are displaced from their equilibrium position. In other words, intermolecular forces get changed and restoring forces are developed on the molecules. When the deforming force is removed, these restoring forces bring the molecules of the solid to their respective equilibrium positions and hence the solid (or the body) regains its original form.

STRESS

When external deforming forces are applied on a body, the relative positions of the molecules of the body change. This calls into play the internal restoring forces. *The restoring force per unit area is called* ***stress.*** It can also be denoted by

$$\text{Stress} = \frac{\text{Restoring Force}}{\text{Area}}$$

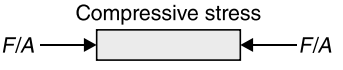
In equilibrium, the restoring force is equal in magnitude to the deforming force. SI unit of stress is Nm^{-2}. Stress is of three types

(a) Longitudinal Stress or Tensile Stress
(b) Tangential Stress or Shearing Stress
(c) Volume Stress or Normal Stress

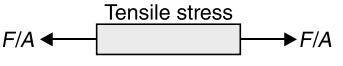
Longitudinal Stress

If the deforming force is applied along some linear dimension of a body, the corresponding stress is called **Longitudinal Stress or Tensile Stress. So, when an object is one dimensional then we use the concept of longitudinal stress.**
It is of two types:

(a) Compressive stress

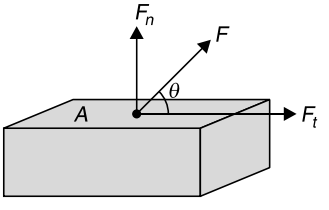


(b) Tensile stress



TANGENTIAL (SHEAR) STRESS AND NORMAL (TENSILE) STRESS

If the force is applied tangentially to one face of a rectangular body, keeping the other face fixed, the stress is called **Tangential or Shearing stress.** So, tangential stress is defined as the restoring force per unit area tangential to the surface of the body.
Consider a block of solid as shown in Figure.



Let a force F be applied to the face which has area A. Resolve $\vec{F}$ into two components $F_n = F\sin\theta$ called normal force and $F_t = F\cos\theta$ called tangential force.
Tangential (shear) stress is given by

$$(\text{Stress})_t = \frac{F_t}{A} = \frac{F\cos\theta}{A}$$

Normal (tensile) stress is given by

$$(\text{Stress})_n = \frac{F_n}{A} = \frac{F\sin\theta}{A}$$

The effect of stress is to produce distortion or a change in size, volume and shape (i.e., configuration of the body).

VOLUMETRIC STRESS OR BULK STRESS OR PRESSURE OR NORMAL STRESS

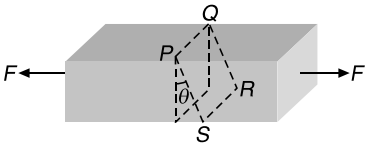
If the force acts all along the surface of the body and normally to its surface, then the stress is called pressure (P) or volumetric Stress because the effect of pressure is to cause a volume change.

The volumetric stress or normal stress or pressure is given by

$$(\text{Stress})_n = \frac{F_n}{A} = P$$

ILLUSTRATION 1

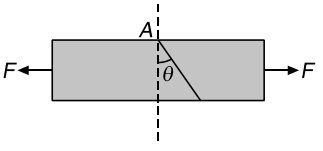
A bar is subjected to equal and opposite forces as shown in the figure. $PQRS$ is a plane making angle θ with the cross-section of the bar? If the area of cross-section of the bar be A, find the tensile stress on $PQRS$, shearing stress on $PQRS$. Also find the condition when tensile stress is maximum and when shearing stress is maximum.



SOLUTION

Since, $\text{Tensile Stress} = \dfrac{\text{Normal force}}{\text{Area}} = \dfrac{F_N}{A_N}$

where, $A_N = \dfrac{A}{\cos\theta}$ and $F_N = F\cos\theta$



$$\Rightarrow \quad \text{Tensile Stress} = \frac{F\cos\theta}{\left(\dfrac{A}{\cos\theta}\right)} = \frac{F\cos^2\theta}{A}$$

$$\text{Shear Stress} = \frac{\text{Tangential force}}{\text{Area}} = \frac{F_T}{A_T}$$

$$\Rightarrow \quad \text{Shear Stress} = \frac{F\sin\theta}{A/\cos\theta} = \frac{F\sin\theta\cos\theta}{A}$$

$$\Rightarrow \quad \text{Shear Stress} = \frac{F\sin\theta\cos\theta}{A} = \frac{F\sin(2\theta)}{2A}$$

Since, $\text{Tensile Stress} = \dfrac{F\cos^2\theta}{A}$

So, for Tensile stress to be maximum,

$$\cos^2\theta = \text{Maximum} = 1$$

$$\Rightarrow \quad \theta = 0°$$

Since, $\text{Shear Stress} = \dfrac{F\sin(2\theta)}{2A}$

So, for Shear stress to be maximum,

$$\sin(2\theta) = \text{Maximum} = 1$$

$$\Rightarrow \quad 2\theta = 90°$$

$$\Rightarrow \quad \theta = 45°$$

STRAIN

When deforming forces are applied on a body, it undergoes a change in shape or size. The fractional (or relative) change in shape or size is called **Strain**. That is

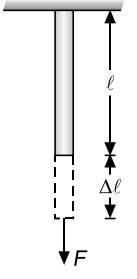
$$\text{Strain} = \frac{\text{Change in Dimension}}{\text{Original Dimension}}$$

Since it is the ratio of two like quantities, so it has no dimensions and units. Strain is of following types

(a) Longitudinal (Linear) Strain
(b) Volume Strain
(c) Shearing Strain

LONGITUDINAL STRAIN

If the deforming force produces a change in length alone, the strain produced in the body is called Longitudinal strain or linear strain or tensile strain. It is the ratio of the change in length (ΔL) to the original length (L).

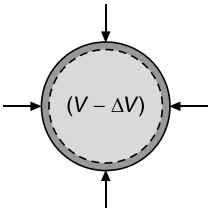


$$\text{Longitudinal Strain} = \frac{\Delta L}{L}$$

Linear strain in the direction of deforming force is called Longitudinal strain and, in a direction, perpendicular to force is called Lateral strain.

VOLUMETRIC STRAIN

When the deforming force produces a change in the volume of the body alone, then the strain produced in the body is called volumetric strain. It is the ratio of the change in volume (ΔV) to the original volume (V).



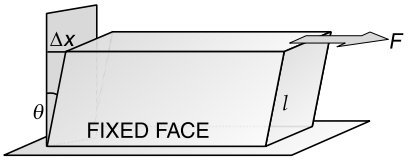
$$\text{Volumetric Strain} = \frac{\Delta V}{V}$$

SHEAR STRAIN

When the deforming force produces a change in the shape of the body without changing its volume, then the strain produced is called shearing strain. It is defined as the angle in radian, through which the face originally perpendicular to the fixed surface of the cubical body gets turned under the effect of tangential force.

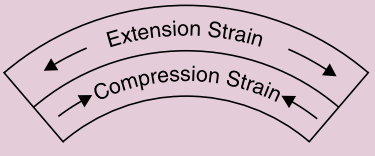
Hence, the angular deformation (θ) in radian is called **shearing strain.** Since θ is generally small, we may write

$$\text{Shearing Strain} = \theta \approx \tan\theta = \frac{\Delta x}{l}$$



Conceptual Note(s)

When a beam is bent both compression strain as well as an extension strain is produced.



RELATION OF STRESS TO STRAIN AND ELASTIC MODULUS

A force applied to an object can change its dimensions and shape. In general, the response of a material to a given type of deforming force is characterized by an **elastic modulus**, which is defined as ratio of stress to strain. So, we have

$$\text{Elastic modulus} = \frac{\text{Stress}}{\text{Strain}}$$

HOOKE'S LAW

This law states that for small deformations (or within elastic limit), stress is directly proportional to strain.

$$\text{Stress} \propto \text{Strain}$$

$$\Rightarrow \quad \text{Stress} = E(\text{Strain})$$

$$\Rightarrow \quad \frac{\text{Stress}}{\text{Strain}} = E$$

where E is a constant called as the modulus of elasticity.

Thus, modulus of elasticity is defined as the ratio of stress to strain. Modulus of elasticity depends on the nature of the material of the body i.e. E is the property of the material of the body and is independent of its shape and dimensions. i.e. length, volume etc. The SI unit of modulus of elasticity is Nm^{-2} or Pascal (Pa), where $1\ \text{Nm}^{-2} = 1\ \text{Pa}$.

Conceptual Note(s)

Modulus of elasticity E (whether it is Y, B or η) is given by

$$E = \frac{\text{stress}}{\text{strain}}$$

Following conclusions can be made from the above expression:

(a) $E \propto$ stress (for same strain), i.e., if we want the equal amount of strain in two different materials, the one which needs more stress is having more E.

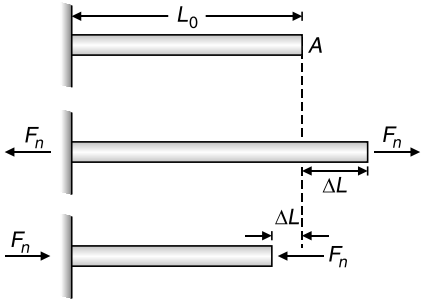
(b) $E \propto \frac{1}{\text{strain}}$ (for same stress), i.e., if the same amount of stress is applied on two different materials, the one having the less strain is having more E. Rather we can say that, the one which offers more resistance to the external forces is having greater value of E. So, we can see that modulus of elasticity of steel is more than that of rubber or

$$E_{\text{steel}} > E_{\text{rubber}}$$

(c) E equals stress for unit strain i.e. when $\frac{\Delta x}{x} = 1$ or $\Delta x = x$. If the length of a wire is 2 metre, then the Young's modulus of elasticity (Y) is the stress applied on the wire to stretch the wire by the same amount of 2 metre.

YOUNG'S MODULUS

Young's modulus is a measure of the resistance of a solid to a change in its length when a force is applied perpendicular to a face. Consider a rod with an unstressed length L_0 and cross-sectional area A, as shown in the Figure.



When it is subjected to equal and opposite forces F_n along it axis and perpendicular to the end faces its length changes by ΔL. These forces tend to stretch the rod. The tensile stress, sometimes denoted by σ, on the rod is defined as

$$\sigma = \frac{F_n}{A}$$

Forces acting in the opposite direction (as shown) would produce a compressive stress. The resulting strain (ε), a dimensionless ratio is given by

$$\varepsilon = \frac{\Delta L}{L_0}$$

Young's modulus Y for the material of the rod is defined as the ratio of tensile stress to tensile strain. So

$$\text{Young's Modulus} = \frac{\text{Tensile stress}}{\text{Tensile strain}}$$

$$\Rightarrow \quad Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\varepsilon} = \frac{\frac{F_n}{A}}{\frac{\Delta L}{L_0}} = \frac{F_n L_0}{A\Delta L}$$

ILLUSTRATION 2

Determine the elongation of the steel bar 1 m long and $1.5\ \text{cm}^2$ cross sectional area when subjected to a pull of 1.5×10^4 N. Take $Y = 2\times10^{11}\ \text{Nm}^{-2}$.

SOLUTION

$$Y = \frac{F/A}{\Delta l/l}$$

$$\Rightarrow \quad \Delta l = \frac{Fl}{AY}$$

Substituting the values,

$$\Delta l = \frac{(1.5\times10^4)(1.0)}{(1.5\times10^{-4})(2\times10^{11})} = 0.5\times10^{-3}\ \text{m}$$

$$\Rightarrow \quad \Delta l = 0.5\ \text{mm}$$

ILLUSTRATION 3

A metal wire 75 cm long and 0.139 cm in diameter stretches 0.035 cm when a load of 8 kg is hung on its end. Find the stress, the strain, and the Young's modulus of the material of the wire.

SOLUTION

$$\text{Stress} = \frac{F}{A} = \frac{(8\ \text{kg})(9.8\ \text{ms}^{-2})}{\pi(6.5\times10^{-4}\ \text{m})^2} = 5.91\times10^7\ \text{Nm}^{-2}$$

$$\text{Strain} = \frac{\Delta L}{L} = \frac{0.035\ \text{cm}}{75\ \text{cm}} = 4.67\times10^{-4}$$

$$\Rightarrow \quad Y = \frac{\text{Stress}}{\text{Strain}} = \frac{5.91\times10^7\ \text{Nm}^{-2}}{4.67\times10^{-4}} = 1.27\times10^{11}\ \text{Nm}^{-2}$$

ILLUSTRATION 4

A circular steel wire 3 m long is to stretch not more than 0.20 cm when a tensile force of 400 N is applied to each end of the wire. What minimum diameter is required for the wire? $Y = 2\times10^{11}\,\text{Pa}$.

SOLUTION

$$A = \frac{Fl}{Y\Delta l}$$

$$\Rightarrow \quad \frac{\pi}{4}d^2 = \frac{Fl}{Y\Delta l}$$

$$\Rightarrow \quad d = \sqrt{\frac{4Fl}{\pi Y\Delta l}} = \sqrt{\frac{4\times400\times3}{\pi\times2\times10^{11}\times0.2\times10^{-2}}}$$

$$\Rightarrow \quad d = 61.8\times10^{-3}\ \text{m} = 61.8\ \text{mm}$$

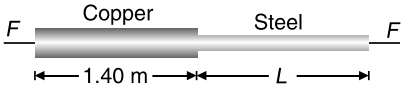
ILLUSTRATION 5

A copper rod with a length of 1.40 m and a cross-section area of $2\ \text{cm}^2$ is fastened to a steel rod with length L and cross-section area $1\ \text{cm}^2$. The compound rod is subjected to equal and opposite pulls of magnitude 6×10^4 N at its ends. Calculate the length L of the steel rod if the elongations of the two rods are equal. Also find the stress and strain in each rod. $Y_{\text{steel}} = 2\times10^{11}$ Pa, $Y_{Cu} = 1.1\times10^{11}$ Pa.

SOLUTION

Given that $\Delta l_{Cu} = \Delta l_{\text{Steel}}$

$$\Rightarrow \quad \frac{Fl_C}{A_C Y_C} = \frac{Fl_S}{A_S Y_S}$$



$$\Rightarrow \quad l_S = \left(\frac{A_S}{A_C}\right)\left(\frac{Y_S}{Y_C}\right)l_C$$

$$\Rightarrow \quad L = \left(\frac{1}{2}\right)\left(\frac{2\times10^{11}}{1.1\times10^{11}}\right)(1.40)$$

$$\Rightarrow \quad L = 1.27\ \text{m}$$

$$\sigma_C = \frac{F}{A_C} = \frac{6\times10^4}{2\times10^{-4}} = 3\times10^8\ \text{Nm}^{-2}$$

and $\sigma_S = \dfrac{F}{A_S} = \dfrac{6\times10^4}{1\times10^{-4}} = 6\times10^8 \text{ Nm}^{-2}$

$$\varepsilon_C = \frac{\sigma_C}{Y_C} = \frac{3\times10^8}{1.1\times10^{11}} = 2.7\times10^{-3}$$

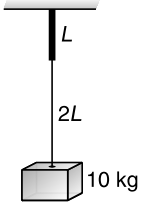
and $\varepsilon_S = \dfrac{\sigma_S}{Y_S} = \dfrac{6\times10^8}{2\times10^{11}} = 3\times10^{-3}$

ILLUSTRATION 6

A length L of copper wire of diameter 1.2 mm is joined to a length $2L$ of steel wire 0.8 mm in diameter, and is hung vertically. When a 10 kg load is suspended from the lower end, the total elongation is 0.65 mm. Find L. $Y_S = 2\times10^{11} \text{ Nm}^{-2}$, $Y_{Cu} = 1.1\times10^{11} \text{ Nm}^{-2}$.

SOLUTION

$$\Delta l = \frac{Fl}{AY} \text{ and } \Delta l = \Delta l_c + \Delta l_s$$



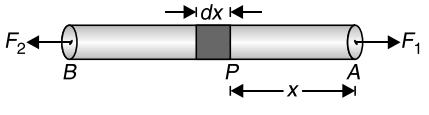
$$\Rightarrow \quad (0.65\times10^{-3}) = \frac{(10\times9.8)L}{\frac{\pi}{4}(1.2\times10^{-3})^2\times1.1\times10^{11}} + \frac{(10\times9.8)(2L)}{\frac{\pi}{4}(0.8\times10^{-3})^2\times2\times10^{11}}$$

Solving this equation, we get

$$L = 23.75 \text{ cm}$$

ILLUSTRATION 7

Consider a metal bar of mass M is pulled by forces F_1 and F_2 applied at its two ends. Consider a small element of length dx at a distance x from end A as shown in Figure.



Calculate the elongation (dL) in small element dx and the total elongation in the bar.

SOLUTION

Acceleration of the bar $= \dfrac{F_1 - F_2}{M}$

Let the tension in bar at distance x from end A is T, then

$$F_1 - T = \frac{Mx}{L}\left(\frac{F_1 - F_2}{M}\right)$$

$$\Rightarrow \quad T = F_1 - \left(\frac{F_1 - F_2}{L}\right)x$$

Stress at this point

$$\frac{T}{A} = \frac{F_1}{A} - \frac{(F_1 - F_2)x}{AL}$$

And $\text{Strain} = \dfrac{\text{Stress}}{Y}$

So, $\dfrac{dL}{dx} = \dfrac{F_1}{AY} - \dfrac{(F_1 - F_2)x}{ALY}$

$$\Rightarrow \quad dL = \left(\frac{F_1}{AY} - \frac{(F_1 - F_2)}{ALY}x\right)dx$$

Total elongation in the bar is

$$\Delta L = \int_0^L dL$$

$$\Rightarrow \quad \Delta L = \int_0^L \left(\frac{F_1}{AY} - \frac{(F_1 - F_2)x}{ALY}\right)dx$$

$$\Rightarrow \quad \Delta L = \frac{F_1}{AY}\int_0^L dx - \left(\frac{F_1 - F_2}{ALY}\right)\int_0^L x\,dx$$

$$\Rightarrow \quad \Delta L = \frac{F_1 L}{AY} - \frac{(F_1 - F_2)L}{2AY}$$

$$\Rightarrow \quad \Delta L = \frac{L}{AY}\left(F_1 - \frac{F_1 - F_2}{2}\right)$$

$$\Rightarrow \quad \Delta L = \frac{(F_1 + F_2)L}{2AY}$$

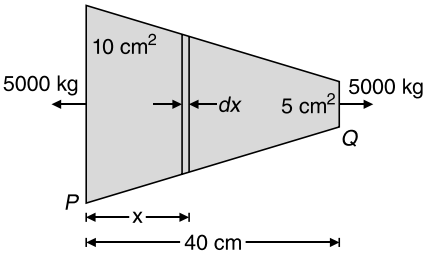
$$\Rightarrow \quad \Delta L = \left(\frac{|F_1| + |F_2|}{2AY}\right)L$$

ILLUSTRATION 8

A steel flat plate PQ tapers uniformly from area 10 cm^2 to 5 cm^2 in length of 40 cm. Calculate the elongation in the plate if an axial tensile force of 5000 kg is acting on it. Take $Y = 2\times10^6 \text{ kgcm}^{-2}$.

SOLUTION

Consider a small element of length dx of the bar at a distance from P as shown in Figure.



Area of cross section at this section is

$$A(x) = 10 - \left(\frac{10-5}{40}\right)x = \left(10 - \frac{x}{8}\right)\ \text{cm}^2$$

Elongation in this elementary length is

$$dl = \frac{F}{A(x)Y}dx = \frac{5000}{\left(10-\frac{x}{8}\right)(2\times10^6)}dx$$

$$\Rightarrow \quad dl = \frac{dx}{400\left(10-\frac{x}{8}\right)}$$

The total extension in the bar is

$$\Delta l = \frac{1}{400}\int_0^{40}\frac{dx}{\left(10-\frac{x}{8}\right)} = \frac{1}{400}(-8)\ln\left(10-\frac{x}{8}\right)\Big|_0^{40}$$

$$\Rightarrow \quad \Delta l = -\frac{8}{400}(\ln 5 - \ln 10) = \frac{1}{50}\ln\left(\frac{10}{5}\right)$$

$$\Rightarrow \quad \Delta l = 0.14\ \text{mm}$$

ILLUSTRATION 9

A light rod of length $2\ \text{m}$ is suspended from the ceiling horizontally by means of two vertical wires of equal length tied to its ends. One of the wires is made of steel and is of cross section $10^{-3}\ \text{m}^2$ and the other is of brass of cross section $2\times10^{-3}\ \text{m}^2$. Locate the position along the rod at which a weight may be hung to produce,

(a) equal stresses in both wires
(b) equal strains on both wires.

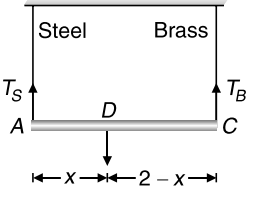
Young's modulus for steel is $2\times10^{11}\ \text{Nm}^{-2}$ and for brass is $10^{11}\ \text{Nm}^{-2}$.

SOLUTION

(a) Given, Stress in steel = Stress in brass

$$\Rightarrow \quad \frac{T_S}{A_S} = \frac{T_B}{A_B}$$

$$\Rightarrow \quad \frac{T_S}{T_B} = \frac{A_S}{A_B} = \frac{10^{-3}}{2\times10^{-3}} = \frac{1}{2} \qquad \ldots(1)$$



As the system is in equilibrium, taking moments about D, we have

$$T_S x = T_B(2-x)$$

$$\Rightarrow \quad \frac{T_S}{T_B} = \frac{2-x}{x} \qquad \ldots(2)$$

From equations (1) and (2), we get

$$x = 1.33\ \text{m}$$

(b) $\text{Strain} = \dfrac{\text{Stress}}{Y}$

Given, Strain in steel = Strain in brass

$$\Rightarrow \quad \frac{\frac{T_S}{A_S}}{Y_S} = \frac{\frac{T_B}{A_B}}{Y_B}$$

$$\Rightarrow \quad \frac{T_S}{T_B} = \frac{A_S Y_S}{A_B Y_B} = \frac{(1\times10^{-3})(2\times10^{11})}{(2\times10^{-3})(10^{11})} = 1 \qquad \ldots(3)$$

From equations (2) and (3), we have

$$x = 1\ \text{m}$$

ILLUSTRATION 10

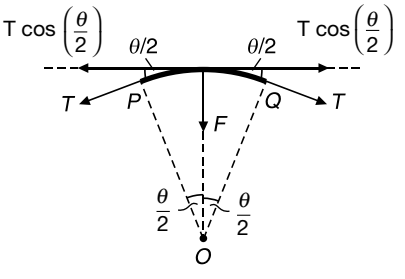
A thin ring of radius R is made of a material of density ρ and Young's modulus Y. If the ring is rotated about its centre in its own plane with angular velocity ω, find the small increase in its radius.

SOLUTION

Consider an element PQ of length dl. Let T be the tension and A the area of cross section of the wire.
Mass of element dm = volume × density $= A(dl)P$
The component of T, towards the centre provides the necessary centripetal force is

$$2T\sin\left(\frac{\theta}{2}\right) = (dm)R\omega^2 \qquad \ldots(1)$$

For small angles $\sin\left(\dfrac{\theta}{2}\right) \approx \dfrac{\theta}{2} = \dfrac{(dl/R)}{2}$



Substituting in equation (1), we get

$$T\frac{dl}{R} = A(dl)\rho R\omega^2$$

$$\Rightarrow \quad T = A\rho\omega^2R^2$$

Let ΔR be the increase in radius,

$$\text{Longitudinal strain} = \frac{\Delta l}{l} = \frac{\Delta(2\pi R)}{2\pi R} = \frac{\Delta R}{R}$$

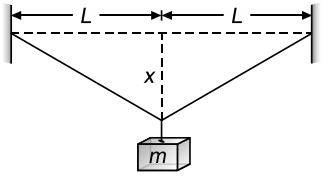
Now, $Y = \dfrac{T/A}{\Delta R/R}$

$$\Rightarrow \quad \Delta R = \frac{TR}{AY} = \frac{(A\rho\omega^2R^2)R}{AY}$$

$$\Rightarrow \quad \Delta R = \frac{\rho\omega^2R^3}{Y}$$

ILLUSTRATION 11

A steel wire of diameter d, area of cross-section A and length $2L$ is clamped firmly at two points A and B which are $2L$ metre apart and in the same plane.



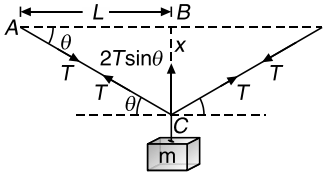
A body of mass m is hung from the middle point of wire such that the middle point sags by x lower from original position. If Young's modulus is Y then find m.

SOLUTION

Let the tension in the string be T, then for the equilibrium of mass m, we have

$$2T\sin\theta = mg$$

$$\Rightarrow \quad T = \frac{mg}{2\sin\theta}$$



Since θ is small, so we have $\sin\theta \approx \frac{x}{L}$ and hence

$$T = \frac{mg}{2\sin\theta} \approx \frac{mgL}{2x}$$

Increment in length of the wire is $\Delta l = AC - AB$

$$\Delta l = \sqrt{L^2 + x^2} - L = \left(L^2 + x^2\right)^{1/2} - L$$

$$\Rightarrow \quad \Delta l = L\left[\left(1 + \frac{x^2}{L^2}\right)^{\frac{1}{2}} - 1\right]$$

Since $(1+x)^n \cong 1 + nx$ for $x \ll 1$, so we have

$$\Delta l = L\left(1 + \frac{1}{2}\frac{x^2}{L^2} - 1\right) = \frac{x^2}{2L}$$

Now, by definition, the Young's modulus is given by

$$Y = \frac{T}{A}\frac{L}{\Delta l}$$

$$\Rightarrow \quad T = \frac{YA\Delta l}{L}$$

Substituting the value of T and Δl in the above equation we get

$$\frac{mgL}{2x} = \left(\frac{YA}{L}\right)\left(\frac{x^2}{2L}\right)$$

$$\Rightarrow \quad m = \frac{YAx^3}{gL^3}$$

ILLUSTRATION 12

The length of an elastic string is a metres when the longitudinal tension is 4 N and b metres when the longitudinal tension is 5 N. Find the length of the string in metres when the longitudinal tension is 9 N.

SOLUTION

Let the original length of elastic string be L and its force constant be k.

When longitudinal tension 4 N is applied on it

$$L + \frac{4}{k} = a \qquad \ldots(1)$$

and when longitudinal tension 5 N is applied on it

$$L + \frac{5}{k} = b \qquad \ldots(2)$$

Solving (1) and (2) we get

$$k = \frac{1}{b-a} \text{ and } L = 5a - 4b$$

When the longitudinal tension of 9 N is applied on elastic string then let its length be c, which is given by

$$c = L + \frac{9}{k} = 5a - 4b + 9(b-a) = 5b - 4a$$

Conceptual Note(s)

To find the actual length of the wire

If a wire has a length L_1 under the tension T_1 and a length L_2 under a tension T_2, then the original length of the wire is

$$L = \frac{L_1T_2 - L_2T_1}{T_2 - T_1}$$

Since, $Y = \frac{F/A}{\Delta L/L} = \frac{T_1/A}{\frac{(L_1 - L_0)}{L_0}} = \frac{T_1L_0}{A(L_1 - L_0)}$

$$\Rightarrow \quad \frac{L_1 - L_0}{T_1} = \frac{L_0}{AY} \qquad \ldots(1)$$

Similarly, $\frac{L_2 - L_0}{T_2} = \frac{L_0}{AY}$ ...(2)

Equating (1) and (2), we get

$$L = \frac{L_1T_2 - L_2T_1}{T_2 - T_1}$$

THERMAL STRESS

If the ends of a rod are rigidly fixed so as to prevent expansion or contraction and the temperature of the rod is changed then, tensile or compressive stresses, called thermal stresses, will be set up in the rod. Hence in the design of many structures which is subject to change in temperature, some provision must be made for expansion to avoid failure of such structures.

Suppose that a rod at a temperature T has its ends rigidly fastened and that while they are thus held, the temperature is reduced to a lower value T_0. The fractional change in length if the rod were free to contract would be

$$\frac{\Delta L}{L} = \alpha\left(T - T_o\right) = \alpha\Delta T$$

Since the rod is not free to contract, the tension must increase to produce some fractional change in length. Let F be the tension produced, then

$$F = AY\frac{\Delta L}{L}$$

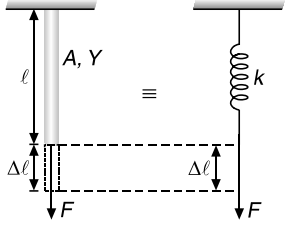
Substituting for $\frac{\Delta L}{L}$, we get,

$$F = AY\alpha\Delta T$$

Hence, the stress in rod is $\frac{F}{A} = Y\alpha\Delta T$

FORCE CONSTANT OF A WIRE

A thin wire or a rod can be imagined to be a series combination of arrays of springs. When we pull the wire by applying a force, then we are actually pulling the springs. Let us take an elastic rod or wire of length l, area of cross-section A, having modulus of elasticity Y and apply a force F to it as shown in Figure.



The force required to produce unit elongation in a wire is called force constant of material of wire, denoted by k. Mathematically

$$k = \frac{F}{\Delta\ell} \qquad \ldots(1)$$

By definition, the Young's modulus of the wire is given by

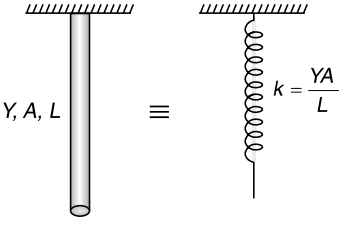
$$Y = \frac{FL}{A\Delta\ell}$$

$$\Rightarrow \quad \frac{F}{\Delta\ell} = \frac{YA}{L} \qquad \ldots(2)$$

From (1) and (2), we get

$$k = \frac{YA}{L}$$

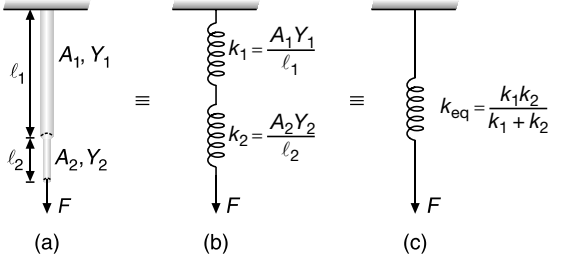
It is clear that the value of force constant depends upon the dimension (length and area of cross section) and material of a substance. So, a wire of length L, Young's Modulus Y and having area of cross section A behaves like a spring of force constant $k = \frac{YA}{L}$. So, all formulae which we use in case of a spring can be applied to a wire also.



For a system of rods joined in series as shown in Figure (a), the replaced spring system is shown in Figure (b) having two springs in series. Figure (c) represents the equivalent spring system having spring constant

$$k_{eq} = \frac{k_1k_2}{k_1 + k_2}$$

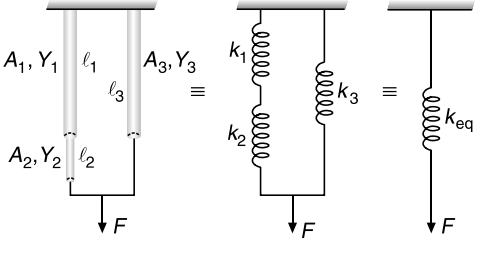
where, $k_1 = \frac{Y_1A_1}{l_1}$, $k_2 = \frac{Y_2A_2}{l_2}$ and $k_3 = \frac{Y_3A_3}{l_3}$



Another combination of rods and its replaced spring system is shown in Figure such that the equivalent spring constant is given by

$$k_{eq} = \frac{k_1k_2}{k_1 + k_2} + k_3$$

where, $k_1 = \frac{Y_1A_1}{l_1}$, $k_2 = \frac{Y_2A_2}{l_2}$ and $k_3 = \frac{Y_3A_3}{l_3}$



Conceptual Note(s)

Since, we have $k_{eq} = \frac{YA}{l}$

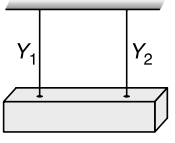
$\Rightarrow \quad k_{eq} l = YA$

This simply means that the product of stiffness constant of an elastic wire and its length is always constant.

So, if we cut a wire in two equal parts, then the stiffness constant of each part will be double that of the original wire. On the same basis, we can say that if a spring is cut into *N* equal parts, each part will have stiffness *N* times the stiffness of the original spring.

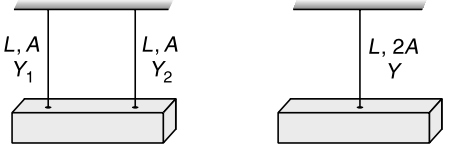
ILLUSTRATION 13

Two wires of equal length and cross-section are suspended as shown. Their Young's moduli are Y_1 and Y_2 respectively. Calculate the equivalent Young's modulus of the arrangement.



SOLUTION

Let the equivalent young's modulus of given combination is Y and the area of cross section is $2A$.



For parallel combination, we have

$$k_1 + k_2 = k_{eq.}$$

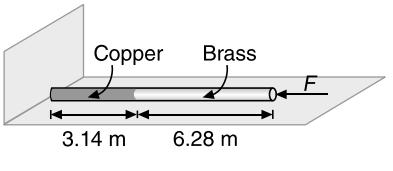
$$\Rightarrow \quad \frac{Y_1 A}{L} + \frac{Y_2 A}{L} = \frac{Y(2A)}{L}$$

$$\Rightarrow \quad Y_1 + Y_2 = 2Y$$

$$\Rightarrow \quad Y = \frac{Y_1 + Y_2}{2}$$

ILLUSTRATION 14

A copper cylinder and a brass cylinder are stacked end to end, as shown in Figure.



Each cylinder has a radius of 0.25 cm. A compressive force of $F = 9000$ N is applied to the right end of the brass cylinder. Find the amount by which the length of the stack decreases if $Y_{copper} = 1.0 \times 10^{11}$ Nm^{-2} and $Y_{brass} = 9.0 \times 10^{10}$ Nm^{-2}.

SOLUTION

METHOD-I

Since both the cylinders experience the same amount of force *F*. Since we know that

$$Y = \frac{F/A}{\Delta L/L}$$

$$\Rightarrow \quad \Delta L = \frac{FL}{AY}$$

Due to the compressive force, we observe that each cylinder decreases in length and hence the total decrease in length is equal to the sum of the decrease in length for each cylinder.

The length of the copper cylinder decreases by

$$\Delta L_{Copper} = \frac{FL_C}{Y_{Copper} A} = \frac{FL_C}{Y_{Copper}(\pi r^2)}$$

Similarly, the length of the brass decreases by

$$\Delta L_{Brass} = \frac{FL_B}{Y_{Brass} A} = \frac{FL_B}{Y_{Brass}(\pi r^2)}$$

Total decrease in length is

$$\Delta L = \Delta L_{Copper} + \Delta L_{Brass}$$

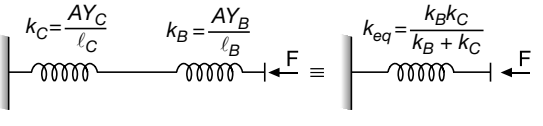
$$\Rightarrow \quad \Delta L = \frac{FL_C}{Y_{Copper} A} + \frac{FL_B}{Y_{Brass} A} = \frac{F}{A}\left(\frac{L_C}{Y_C} + \frac{L_B}{Y_B}\right)$$

$$\Rightarrow \quad \Delta L = \frac{9000}{\pi(0.25 \times 10^{-2})^2}\left(\frac{3.14}{1.0 \times 10^{11}} + \frac{6.28}{9.0 \times 10^{10}}\right)$$

$$\Rightarrow \quad \Delta L = 4.64 \times 10^{-2} \text{ m}$$

METHOD-II

The composite bar can be replaced with a series combination of two springs having spring constants k_C and k_B as shown in Figure.



The equivalent spring constant k_{eq} of the arrangement is

$$\frac{1}{k_{eq}} = \frac{1}{k_C} + \frac{1}{k_B}$$

$$\Rightarrow \quad k_{eq} = \frac{k_B k_C}{k_B + k_C}$$

Deformation in length of the equivalent spring is

$$\Delta L = \frac{F}{k_{eq}} = F\left(\frac{k_B + k_C}{k_B k_C}\right) = F\left(\frac{1}{k_B} + \frac{1}{k_C}\right)$$

where, $k_C = \frac{Y_C A}{L_C}$ and $k_B = \frac{Y_B A}{L_B}$

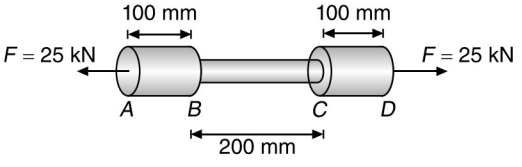
$$\Rightarrow \quad \Delta L = \frac{F}{A}\left(\frac{L_C}{Y_C} + \frac{L_B}{Y_B}\right)$$

$$\Rightarrow \quad \Delta L = \frac{9000}{\pi(0.25\times10^{-2})^2}\left(\frac{3.14}{1.0\times10^{11}} + \frac{6.28}{9.0\times10^{10}}\right)$$

$$\Rightarrow \quad \Delta L = 4.64\times10^{-2} \text{ m}$$

ILLUSTRATION 15

A steel bar $ABCD$ of length 40 cm is made up of three parts AB of diameter 50 mm, BC of diameter 25 mm and CD of diameter 50 mm as shown in Figure.

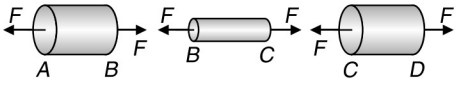


The rod is subjected to a pull of 25 kN. If Young's modulus for steel is 2×10^{11} Nm^{-2}, then calculate stress in each part of the bar and the total extension in the bar.

SOLUTION

METHOD-I: Conventional Method

If F is the stretching force applied on the given steel bar, then the same axial force 25 kN is transmitted longitudinally to each of the three bars. However, stresses induced will be different and hence elongations of each bar will also be different. Figure shows the free-body diagram of the different part of the bar.



Stress in part AB and CD will be equal

$$\sigma_{AB} = \sigma_{CD} = \frac{F}{A_{AB}} = \frac{25000 \text{ N}}{\frac{\pi}{4}(50)^2 \text{ mm}^2}$$

$$\Rightarrow \quad \sigma_{AB} = \frac{40}{\pi} = 12.73 \text{ Nmm}^{-2}$$

Stress in part BC is

$$\sigma_{BC} = \frac{F}{A_{BC}} = \frac{25000}{\frac{\pi}{4}(25)^2} = 50.93 \text{ Nmm}^{-2}$$

Total extension in the rod is

$$\Delta l = \Delta l_{AB} + \Delta l_{BC} + \Delta l_{CD} = 2\Delta l_{AB} + \Delta l_{BC}$$

$$\Rightarrow \quad \Delta l = 2\left(\frac{\sigma_{AB} l_{AB}}{Y}\right) + \left(\frac{\sigma_{BC} l_{BC}}{Y}\right)$$

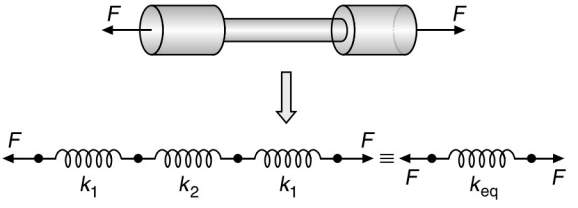
Given that $Y = 2\times10^{11}$ Nm^{-2} $= 2\times10^{5}$ Nmm^{-2}

$$\Rightarrow \quad \Delta l = 2\left(\frac{12.73}{2\times10^5}\times100\right) + \left(\frac{50.93}{2\times10^5}\times200\right)$$

$$\Rightarrow \quad \Delta l = 0.0637 \text{ mm}$$

METHOD-II: Equivalent Spring Method

The steel bar $ABCD$ can be replaced with series combination of three springs.



In series combination of springs, the force in each spring should be equal. Hence, each rod will experience same force F. Hence, stress in rods AB and CD is

$$\sigma_{AB} = \sigma_{CD} = \frac{F}{A_1} = \frac{25\times10^3}{\left(\frac{\pi}{4}\times50^2\right)} = \frac{40}{\pi} \text{ Nmm}^{-2}$$

Stress in rod BC is $\sigma_{BC} = \frac{25\times10^3}{\left(\frac{\pi}{4}\times25^2\right)} = \frac{160}{\pi}$ Nmm^{-2}

Equivalent spring contact can be written as

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_1} = \frac{2}{k_1} + \frac{1}{k_2}$$

where, $k_1 = \frac{YA_1}{l_1}$ and $k_2 = \frac{YA_2}{l_2}$

$$\Rightarrow \quad k_{eq} = \frac{YA_1A_2}{2A_2l_1 + A_1l_2}$$

So, net extension produced in the composite bar is

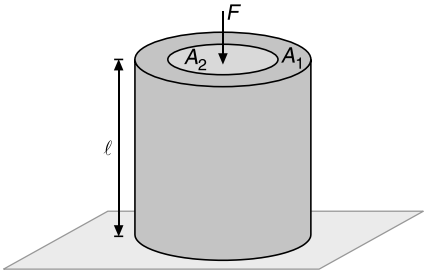
$$x = \frac{F}{k_{eq}}$$

$$\Rightarrow \quad x = F\left(\frac{2A_2l_1 + A_1l_2}{YA_1A_2}\right)$$

$$\Rightarrow \quad x = \frac{1}{5\pi} \text{ mm} = 0.0637 \text{ mm}$$

ILLUSTRATION 16

Consider a cylindrical column (or strut) made up of two different materials as shown in Figure.



The inner section of the column has area A_1, Young's modulus Y_1 and the outer section of the column has area A_2, Young's modulus Y_2. A compressive force F acts on this column. Calculate the load supported by inner and outer section of the column.

SOLUTION

METHOD-I: Conventional Method

Let σ_1 and σ_2 be the stresses induced in the two materials, then sum of restoring forces in the two materials should balance the externally applied compressive force F. Hence, we have

$$\sigma_1 A_1 + \sigma_2 A_2 = F \qquad \ldots(1)$$

Also, the compression in the two portions should be the same. But since their original lengths are same, so strain (ε) will also be the same.

$$\Rightarrow \quad \varepsilon_1 = \varepsilon_2$$

$$\Rightarrow \quad \frac{\sigma_1}{Y_1} = \frac{\sigma_2}{Y_2} \qquad \ldots(2)$$

Solving Equations (1) and (2), we get

$$\sigma_1 = \frac{FY_1}{A_1Y_1 + A_2Y_2}$$

Hence load supported by inner section is

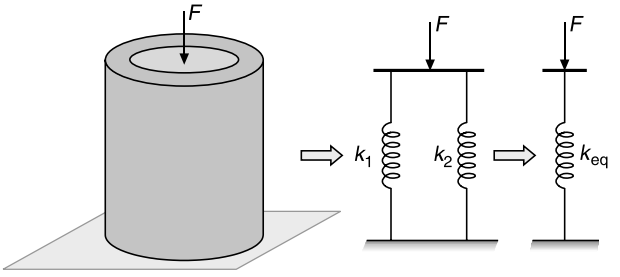
$$F_1 = \sigma_1 A_1 = \frac{FY_1A_1}{A_1Y_1 + A_2Y_2}$$

and load supported by outer section is

$$F_2 = \sigma_2 A_2 = \frac{FY_2A_2}{A_1Y_1 + A_2Y_2}$$

METHOD-II: Equivalent Spring Method

The cylindrical column (or strut) made up of two different materials can be replaced by a combination of two springs in parallel as shown in Figure.



Since the springs are connected in parallel, so the compressions in each spring will be same. Hence

$$x_1 = x_2$$

$$\Rightarrow \quad \frac{F_1}{k_1} = \frac{F_2}{k_2}$$

where, $k_1 = \frac{A_1Y_1}{l_1}$ and $k_2 = \frac{A_2Y_2}{l_2}$

The equivalent force constant of the arrangement is

$$k_{eq} = k_1 + k_2 = \frac{1}{l}(A_1Y_1 + A_2Y_2)$$

Compression in each spring is

$$x_1 = x_2 = x = \frac{F}{k_{eq}} = \frac{Fl}{A_1Y_1 + A_2Y_2}$$

So, load supported by the inner section is

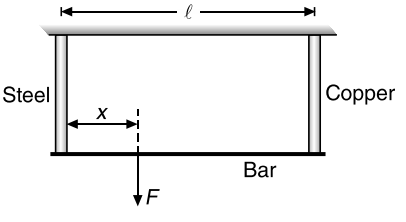
$$F_1 = k_1x_1 = \frac{FY_1A_1}{(A_1Y_1 + A_2Y_2)}$$

and load supported by the outer section is

$$F_2 = k_2x_2 = \frac{FY_2A_2}{A_1Y_1 + A_2Y_2}$$

ILLUSTRATION 17

Two vertical rods of equal lengths, one of steel and the other of copper, are suspended from the ceiling, at a distance l apart and are connected rigidly to a rigid horizontal bar at their lower ends as shown in Figure.



The respective area of cross section and Young's Modulus of steel and copper wires are A_S, Y_S and A_C, Y_C. Calculate the tension in steel and copper wires. Where should a vertical force F be applied to the horizontal bar in order to keep the bar horizontal?

SOLUTION

METHOD-I: Conventional Method

Let the force F be applied at a distance x from the steel bar. Let T_S and T_C be the loads on steel and copper rods, respectively. Then,

$$T_S + T_C = F \qquad \ldots(1)$$

Since the rigid horizontal bar remains horizontal, the extensions produced in the two rods and strains remain the same. That is

$$\frac{T_S}{A_S Y_S} = \frac{T_C}{A_C Y_C} \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$T_S = \frac{FA_S Y_S}{A_S Y_S + A_C Y_C} \text{ and } T_C = \frac{FA_C Y_C}{A_S Y_S + A_C Y_C}$$

Since the steel bar is horizontal and in equilibrium, so, we have

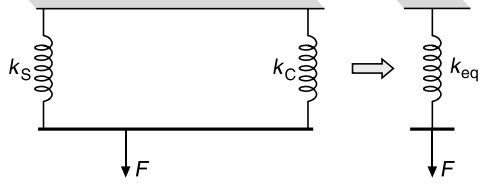
$$\Sigma\tau = 0$$

$$\Rightarrow \quad T_C l = Fx$$

$$\Rightarrow \quad x = \frac{T_C}{F} l = \frac{A_C Y_C l}{A_S Y_S + A_C Y_C}$$

METHOD-II: Equivalent Spring Method

We can treat two rods as two springs, connected in parallel combination



Equivalent spring constant of steel and copper rod

$$k_S = \frac{A_S Y_S}{l} \text{ and } k_C = \frac{A_C Y_C}{l}$$

As the bar always remains horizontal, it means the elongation of the rods should be equal.

$$\Rightarrow \quad \frac{F_S}{k_S} = \frac{F_C}{k_C} = \frac{F}{(k_S + k_C)}$$

$$\Rightarrow \quad F_S = \frac{Fk_s}{k_S + k_C} = \frac{FA_S Y_S}{A_S Y_S + A_C Y_C}$$

and $F_C = \dfrac{Fk_C}{k_S + k_C} = \dfrac{FA_C Y_C}{A_S Y_S + A_C Y_C}$

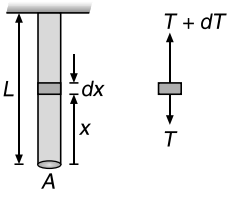
Taking moments about the steel bar, we get

$$T_C l = Fx$$

$$\Rightarrow \quad x = \frac{T_C}{F} l = \frac{A_C Y_C l}{A_S Y_S + A_C Y_C}$$

ELONGATION OF ROD UNDER IT'S SELF WEIGHT

Consider a rope having weight W, area of cross-section A, length L and Young's Modulus Y. Let us consider an element at a distance x from the free end of the rope as shown in Figure.



Consider an element of length dx at distance x from the free end of the rope. The tension in the rope at a distance x from its free end is

$$T = \frac{W}{L} x$$

Elongation dl in this infinitesimal element of length dx of the rope is

$$dl = \frac{Tdx}{AY} = \left(\frac{W}{AY}\right) xdx \qquad \ldots(1)$$

Total elongation ΔL in the rope is obtained by integrating equation (1). So, we get

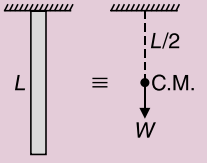
$$\Delta L = \int dl = \int_0^L \frac{Tdx}{AY} = \frac{W}{AY} \int_0^L xdx = \frac{WL}{2AY}$$

If m is mass of the rope, then $W = mg$, so we have

$$\Delta L = \frac{WL}{2AY} = \frac{(mg)L}{2AY}$$

Conceptual Note(s)

One can do this directly by considering that the total weight of wire is acting at CM and using effective original length of the wire as $\frac{L}{2}$.



Since $Y = \dfrac{F/A}{x/L_{eff}} = \dfrac{W/A}{\dfrac{x}{(L/2)}}$

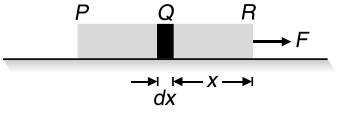
$$\Rightarrow \quad x = \frac{WL}{2AY}$$

ILLUSTRATION 18

A uniform elastic plank moves over a smooth horizontal plane due to a constant force F distributed uniformly over the end face. The surface of the end face is equal to A, and Young's modulus of the material is Y. Find the tensile strain of the plank in the direction of the acting force.

SOLUTION

Let m be the mass of the plank and l its length.



Tension at distance x is

$$T = m_{QR}a$$

$$\Rightarrow \quad T = \left(\frac{m}{l}\right)(l-x)\frac{F}{m}$$

$$\Rightarrow \quad T = F\left(1-\frac{x}{l}\right)$$

Small change in element dx is,

$$dl = \frac{Tdx}{AY} = \frac{F}{AY}\left(1-\frac{x}{l}\right)dx$$

$$\Rightarrow \quad \text{Strain} = \frac{\Delta l}{l} = \frac{F}{2AY}$$

Problem Solving Technique(s)

In this problem, if friction between block and surface is present and coefficient of friction is μ, then following two cases arise.

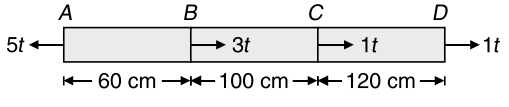
Case (i): $F < \mu mg$

Case (ii): $F > \mu mg$,

For both these cases the answer will be same for elongation i.e., $\Delta\ell = \frac{FL}{2AY}$ for both cases.

ILLUSTRATION 19

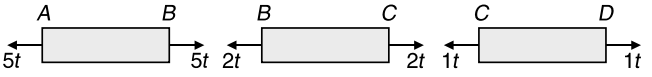
A brass bar, having cross sectional area $10\ \text{cm}^2$ is subjected to axial forces as shown in figure. Find the total elongation of the bar. Take $Y = 8\times10^2\ \text{tcm}^{-2}$.



SOLUTION

Given, $A = 10\ \text{cm}^2$, $Y = 8\times10^2\ \text{tcm}^{-2}$

Let Δl be the total elongation of the bar. For sake of simplicity, the force of $3t$ acting at B may be split into two forces of $5t$ and $2t$ as shown in Figure.



Similarly, the force of $1t$ acting at C may be split into two forces of $2t$ and $1t$.

Since the extension in the bar on which both forces of equal magnitude act in opposite direction is

$$\Delta l = \frac{FL}{AY}$$

So, net extension is given by

$$\Delta l = \frac{1}{AY}\left(F_1l_1 + F_2l_2 + F_3l_3\right)$$

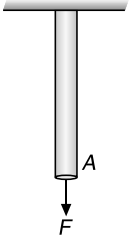
$$\Rightarrow \quad \Delta l = \frac{1}{10\times8\times10^2}\left[(5)(60)+(2)(100)+(1)(120)\right]$$

$$\Rightarrow \quad \Delta l = 0.0775\ \text{cm}$$

BREAKING STRESS

If one end of rod or wire is rigidly fixed and a force is applied at the other end, it will stretch. If the force is small, the extension will also be small and upon the withdrawal of the force, the wire will regain its original state. But if the applied force is gradually increased, after some intermediate states, a state is reached when the wire or rod breaks. The stress corresponding to this breaking point is termed as breaking stress.

When the wire is loaded beyond the elastic limit, then strain increases much more rapidly. The maximum stress after which the wire begins to flow and breaks, is called breaking stress or tensile strength and the force by application of which the wire breaks is called the Breaking Force.



The breaking force depends upon the area of cross-section of the wire i.e.,

$$\text{Breaking Force} \propto A$$

$$\text{Breaking Force} = PA$$

where P is a constant of proportionality known as breaking stress (B.S.) of the wire.

Breaking stress P is a constant for a given material and it does not depend upon the dimension (length or thickness) of wire.

BREAKING OF A WIRE UNDER ITS OWN WEIGHT

Consider a wire of length l, area A, density ρ. Let the breaking stress for the material of the wire be P. Since we know that

$$\begin{pmatrix}\text{Breaking}\\ \text{Force}\end{pmatrix}=\begin{pmatrix}\text{Breaking}\\ \text{Stress}\end{pmatrix}\begin{pmatrix}\text{Area of}\\ \text{cross-section}\end{pmatrix}$$

The weight of wire is given by

$$W = Mg = Al\rho g$$

So, breaking stress is given by

$$P = \frac{W}{A} = l\rho g$$

$$\Rightarrow \quad l = \frac{P}{\rho g}$$

This length of the wire can also be said as the maximum length of the wire which can withstand its own weight before breaking.

Conceptual Note(s)

(a) If a wire can bear maximum force F, then wire of same material but double thickness can bear a maximum force $4F$ because breaking force is proportional to area of cross section and hence r^2.

(b) If a wire of length L is cut into two or more parts, then again, it's each part can hold the same weight, because breaking force is independent of the length of wire.

(c) The working stress is always kept lower than that of a breaking stress.

(d) Safety factor which is the ratio of breaking stress to working stress has a large value as

Breaking Stress $\gg$ Working Stress.

ILLUSTRATION 20

Find the greatest length of steel wire that can hang vertically without breaking. Breaking stress of steel is 8×10^8 Nm^{-2}, density of steel is 8×10^3 kgm^{-3} and $g = 10$ ms^{-2}.

SOLUTION

Let l be the length of the wire that can hang vertically without breaking. Then the stretching force on it is equal to its own weight. If therefore, A is the area of cross section and ρ the density, then breaking stress P is

$$P = \frac{\text{Weight}}{A}$$

$$\Rightarrow \quad P = \frac{(Al\rho)g}{A}$$

$$\Rightarrow \quad l = \frac{P}{\rho g}$$

Substituting the values, we get

$$l = \frac{8\times10^8}{(8\times10^3)(10)} = 10^4 \text{ m}$$

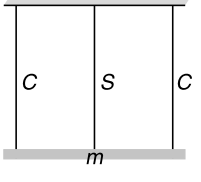
Test Your Concepts-I

Based on Young's Modulus, Longitudinal Stress and Strain

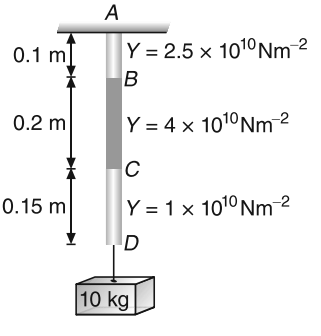
(Solutions on page H.3)

1. One end of a uniform wire of length L and of weight W is attached rigidly to a point in the roof and a weight W_1 is suspended from its lower end. If S is the area of cross-section of the wire, then find the stress in the wire at a height $\frac{3L}{4}$ from its lower end.

2. A homogeneous block with a mass m hangs on three vertical wires of equal length arranged symmetrically. Find the tension of the wires if the middle wire is of steel and the other two are of copper. All the wires have the same cross section. Consider the modulus of elasticity of steel to be double than that of copper.

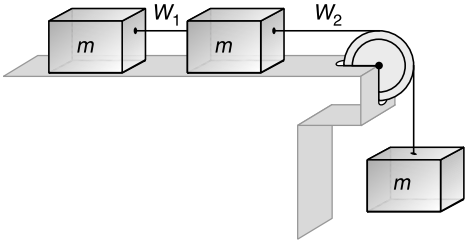


3. A light rod with uniform cross-section of 10^{-7} m^2 is shown in the figure. The rod consists of three different materials whose lengths are 0.1 m, 0.2 m and 0.15 m respectively and whose Young's moduli are 2.5×10^{10} Nm^{-2}, 4×10^{10} Nm^{-2} and 1×10^{10} Nm^{-2} respectively. Calculate the displacement of points *B*, *C* and *D*.

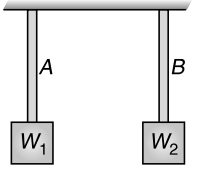


4. Two wires A and B of same dimensions are stretched by same amount of force. Young's modulus of A is twice that of B. Which wire will get more elongation?

5. Three blocks, each of same mass m, are connected with wires W_1 and W_2 of same cross-sectional area a and Young's modulus Y. Neglecting friction, find the strain developed in wire W_2.



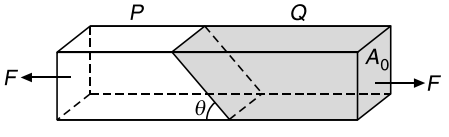
6. Two rods A and B, each of equal length but different materials are suspended from a common support as shown in the figure.



The rods A and B can support a maximum load of $W_1 = 600$ N and $W_2 = 6000$ N, respectively. If their cross-sectional areas are $A_1 = 10\ \text{mm}^2$ and $A_2 = 1000\ \text{mm}^2$, respectively, then identify the stronger material.

7. A wire elongates by 1 mm when a load W is hanged from it. If this wire goes over a pulley and two weights W each are hung at the two ends, find the elongation in the wire.

8. Two bars P and Q are glued together using a strong adhesive. The contact surface of the bars makes an angle θ with its length and the area of cross-section of each bar is A_0. It is known that the adhesive yields if the normal stress at the contact surface exceeds σ_0. Calculate the maximum pulling force F that can be applied to the arrangement without detaching the bars.



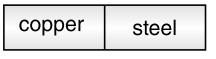
9. The Young's modulus of three materials are in the ratio 2 : 2 : 1. Three wires made of these materials have their cross-sectional areas in the ratio 1 : 2 : 3. For a given stretching force calculate ratio of the elongation's in the three wires.

10. A copper wire has a breaking stress of about $3\times10^8\ \text{Nm}^{-2}$. Calculate the maximum load that can be hung from a copper wire of diameter 0.42 mm. If half this maximum load is hung from the copper wire of same diameter, then calculate the percentage of its length that will be stretched.
Given that $Y_{Cu} = 1.1\times10^{11}\ \text{Nm}^{-2}$.

11. A solid cylindrical steel column is 4 cm long and 9 cm in diameter. What will be its decrease in length when carrying a load of 80000 kg? $Y = 1.9\times10^{11}\ \text{Nm}^{-2}$.

12. A metal rod that is 4.00 m long and 0.50 cm^2 in cross-section area is found to stretch 0.20 cm under a tension of 5000 N. What is the Young's modulus for this metal?

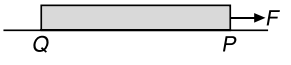
13. A steel wire and a copper wire of equal length and equal cross-sectional area are joined end to end and the combination is subjected to a tension as shown in Figure.



Calculate the ratio of the stresses and the ratio of the strains developed in the two wires.
Given that $Y_{\text{steel}} = 2\times10^{11}\ \text{Nm}^{-2}$ and $Y_{\text{copper}} = 1.3\times10^{11}\ \text{Nm}^{-2}$.

14. A uniform copper bar of density ρ, length L, cross-sectional area S and Young's modulus Y is moving horizontally on a frictionless surface with constant acceleration a_0. Calculate stress at centre of the wire and the total elongation in the wire.

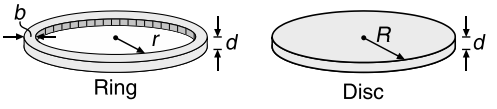
15. A rod PQ of mass m, area of cross section A, length l and Young's modulus of elasticity Y is lying on a smooth table as shown in figure. A force F is applied at P. Find



(a) tension at a distance x from end P,
(b) longitudinal stress at this point,
(c) total change in length and
(d) total strain in the rod.

16. A vertical solid steel post 15 cm in diameter and 3 m long is required to support a load of 8000 kg. The weight of the post can be neglected. What is (a) the stress in the post? (b) the strain in the post? (c) the change in post's length when the load is applied? $Y = 2\times10^{11}$ Pa.

17. A steel ring is to be fitted on a wooden disc of radius R and thickness d. The inner radius of the ring is r which is slightly smaller than R. The outer radius of the ring is $r + b$ and its thickness is d (same as the disc) as shown in Figure.



There is no change in value of b and d after the ring is fitted over the disc, only the inner radius becomes R. If the Young's modulus of steel is Y, calculate the longitudinal stress developed in it. Also calculate the tension force developed in the ring.

ELASTIC POTENTIAL ENERGY

If a force F is applied to a wire of length l and cross-sectional area A, made of material of Young's Modulus Y, and if the wire suffers an extension x then

$$F = \frac{YAx}{l} \qquad \left\{\because Y = \frac{Fl}{Ax}\right\}$$

Work done in extending the wire through Δl is given by

$$W = \int_0^{\Delta l} F\,dx = \frac{YA}{l}\int_0^{\Delta l} x\,dx$$

$$\Rightarrow \quad W = \frac{YA}{l}\frac{(\Delta l)^2}{2} = \frac{1}{2}(Al)\left(\frac{Y\Delta l}{l}\right)\left(\frac{\Delta l}{l}\right)$$

$$\Rightarrow \quad W = \frac{1}{2}\times \text{Volume}\times \text{Stress}\times \text{Strain}$$

This work done is stored in the wire as **elastic potential energy** (U_e) in the wire. Work done per unit volume is called the elastic energy density denoted by (u_e). Hence, we have

$$u_e = \frac{W}{Al} = \frac{U_e}{Al} = \frac{1}{2}(\text{Stress})(\text{Strain})$$

The Work can also be written as

$$W = \frac{1}{2}\left(\frac{YA\Delta l}{l}\right)\Delta l$$

$$\Rightarrow \quad \text{Work} = \frac{1}{2}\times \text{Load}\times \text{Extension}$$

This work done is stored in the wire as **elastic potential energy** (U_e) in the wire. **Energy density** (u_e) is the elastic potential energy per unit volume of the wire.

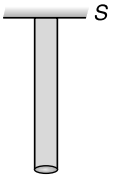
$$\Rightarrow \quad u_e = \frac{W}{Al} = \frac{1}{2}\left(\frac{\text{Load}}{A}\right)\left(\frac{\text{Extension}}{l}\right)$$

$$\Rightarrow \quad u_e = \frac{1}{2}(\text{Stress})(\text{Strain})$$

$$\Rightarrow \quad u_e = \frac{1}{2}Y(\text{Strain})^2 = \frac{(\text{Stress})^2}{2Y}$$

ILLUSTRATION 21

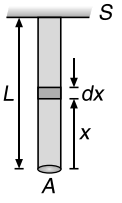
A bar of mass M and length L is hanging from point S as shown in figure.



The Young's modulus of elasticity of the wire is Y and the area of cross-section of the wire is A.

(i) Find the stress at x distance from bottom end.
(ii) Consider a small section dx of the bar at a distance x from lowest point of bar. Find elongation (dL) in that section dx.
(iii) Find total elongation in bar.
(iv) Find energy density at distance x from bottom end.
(v) Find total elastic potential energy stored in bar.

SOLUTION



(i) The weight of x length of the bar is

$$W = \left(\frac{Mg}{L}\right)x$$

So, stress at x distance from bottom

$$\frac{W}{A} = \frac{Mgx}{AL}$$

(ii) $\text{Stress} = Y(\text{Strain})$

$$\frac{Mgx}{AL} = Y\frac{dL}{dx}$$

$$\Rightarrow \quad dL = \frac{Mgxdx}{ALY}$$

(iii) Total elongation in wire

$$\Delta L = \int_0^L dL = \frac{Mg}{ALY}\int_0^L x\,dx$$

$$\Rightarrow \quad \Delta L = \frac{MgL}{2AY}$$

(iv) Elastic energy density at x distance is

$$u_e = \frac{1}{2}(\text{stress})(\text{strain})$$

$$\Rightarrow \quad u_e = \frac{1}{2}\frac{(\text{stress})^2}{Y}$$

$$\Rightarrow \quad u_e = \frac{1}{2Y}\left(\frac{Mgx}{AL}\right)^2 = \frac{M^2g^2x^2}{2YA^2L^2}$$

(v) Energy stored in Adx volume is

$$dU_e = (u_e)Adx$$

$$\Rightarrow \quad dU_e = \frac{M^2g^2x^2}{2YA^2L^2}(Adx)$$

Total energy is given by

$$U_e = \int_0^L \frac{M^2g^2x^2}{2YAL^2}dx = \frac{M^2g^2L}{6AY}$$

ILLUSTRATION 22

A rubber cord has a cross sectional area $1\ \text{mm}^2$ and total unstretched length 10 cm. It is stretched to 12 cm and then released to project a missile of mass 5 g. Taking Young's modulus Y for rubber as $5\times10^8\ \text{Nm}^{-2}$. Calculate the velocity of projection.

SOLUTION

The equivalent force constant of rubber cord is given by

$$k=\frac{YA}{l}=\frac{(5\times10^8)(1\times10^{-6})}{(0.1)}=5\times10^3\ \text{Nm}^{-1}$$

Applying Law of Conservation of Mechanical Energy, we get

$$\begin{pmatrix}\text{Elastic Potential}\\ \text{Energy of Cord}\end{pmatrix}=\begin{pmatrix}\text{Kinetic Energy}\\ \text{of Missile}\end{pmatrix}$$

$$\Rightarrow\quad \frac{1}{2}k(\Delta l)^2=\frac{1}{2}mv^2$$

$$\Rightarrow\quad v=\left(\sqrt{\frac{k}{m}}\right)\Delta l=\left(\sqrt{\frac{5\times10^3}{5\times10^{-3}}}\right)(12-10)\times10^{-2}$$

$$\Rightarrow\quad v=20\ \text{ms}^{-1}$$

Please note that while solving this example following assumptions have been made.

(i) k has been assumed constant, even though it depends on the length (l).

(ii) The entire elastic potential energy is converted into kinetic energy of missile.

ILLUSTRATION 23

The demolition of a building is to be accomplished by swinging a 400 kg steel ball on the end of a 30 m steel wire of diameter 5 mm hanging from a tall crane. The ball is swung through an arc from side to side, the wire making an angle of $60°$ with the vertical at the top of the swing. Calculate the amount by which the wire is stretched at the bottom of the swing and estimate the energy stored in the stretched wire at the bottom of the swing if $Y_S=2\times10^{11}\ \text{Nm}^{-2}$.

SOLUTION

The height to which the ball is raised when it makes an angle of $60°$ with the vertical is given by

$$h=l(1-\cos60°)=\frac{l}{2}=15\ \text{m}$$

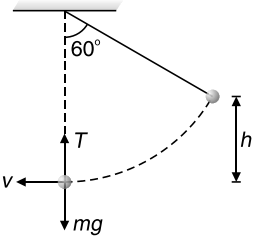
When released from this position, the speed acquired by the ball at the lowest point is obtained by applying the Law of Conservation of Energy, according to which *"loss in gravitational potential energy of the ball equals the gain in kinetic energy of the ball"*, so we get

$$\frac{1}{2}mv^2=mgh$$

$$\Rightarrow\quad v^2=2gh=2\times9.8\times15=294\ \text{m}^2\text{s}^{-2}$$

Also, applying Newton's Second Law to the ball at the lowest point, we get

$$T-mg=\frac{mv^2}{l}$$



$$\Rightarrow\quad T=m\left(g+\frac{v^2}{l}\right)=(400)\left(9.8+\frac{294}{30}\right)$$

$$\Rightarrow\quad T=7840\ \text{N}$$

Since, $\Delta l=\dfrac{Tl}{AY}=\dfrac{(7840)(30)}{\frac{\pi}{4}(5\times10^{-3})^2\times2\times10^{11}}$

$$\Rightarrow\quad \Delta l=0.06\ \text{m}=6\ \text{cm}$$

The elastic energy stored in the wire is

$$U=\frac{1}{2}(Y)(\text{Strain})^2(\text{Volume}),\ \text{where}$$

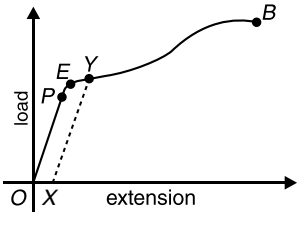
$$\text{Volume}=\left(\frac{\pi}{4}\right)(5\times10^{-3})^2(30)=5.9\times10^{-4}\ \text{m}^3$$

$$\Rightarrow\quad U=\frac{1}{2}(2\times10^{11})\left(\frac{6\times10^{-2}}{30}\right)^2(5.9\times10^{-4})$$

$$\Rightarrow\quad U=236\ \text{J}$$

BEHAVIOUR OF A WIRE UNDER STRESS

A typical load-extension (or stress-strain) curve for a metal wire is shown here.



P	limit of proportionality
E	elastic limit, which may coincide with the limit of proportionality
Y	yield point
B	break point

Up to the point P the curve is straight. This is called **limit of proportionality.** Beyond this point P Hooke's Law is not obeyed.

The point E is called the **elastic limit**. Upto this point the wire returns to its original form when the load is removed. P and E are very close and may not coincide.

Beyond this elastic limit the wire no longer returns to its original size on the removal of load, i.e., a permanent deformation (OX) occurs. At the **yield point** Y, the

material begins to flow, i.e., the length increases with little increase in stress. This continues till the **break point** B is achieved and the wire breaks at some weak point.

Materials which undergo a large increase in length beyond the elastic limit before snapping are called **ductile**. Such materials can be drawn into long wires. Substances which break just after the elastic limit is reached are said to be **brittle.**

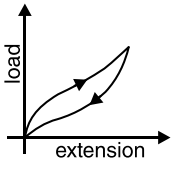
ELASTIC FATIGUE

Consider a wire loaded, within elastic limits, and kept strained for a sufficiently long time. The extension is liable to complete recovery on removal of deforming forces but it is observed that the wire recovers its original configuration slowly on removal of deforming forces, i.e. the recovery lags behind the process of removing one deforming forces.

This phenomenon, by virtue of which a substance exhibits a delay in recovering its original configuration, if it had been subjected to a stress for a longer time, is called **elastic fatigue.**

BEHAVIOUR OF RUBBER UNDER STRESS

For rubber, stress is not proportional to strain at any portion of the curve. However, when the load is removed the specimen recovers its original length – thus it is elastic (but does not obey Hooke's Law). On decreasing the load, the curve is not retraced (as expected) but follows a different path. This property is called **elastic hysteresis.**



ELASTIC HYSTERESIS

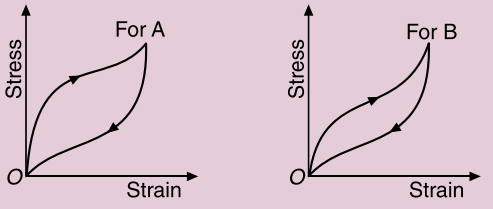
The word Hysteresis actually means "Lagging Behind". When a deforming force is applied on a body then the strain does not change simultaneously with stress rather it lags behind the stress. The lagging of strain behind the stress is called as Elastic Hysteresis. This is the reason due to which the values of strain for same stress are different while increasing the load and while decreasing the load.

Conceptual Note(s)

Significance of Hysteresis Loop

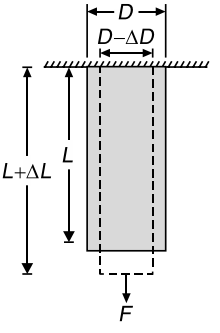
(a) The area of the stress-strain curve is called the hysteresis loop and it is numerically equal to the work done in loading the material and then unloading it.

(b) If we have two tyres of rubber having different hysteresis loop then rubber B should be used for making the car tyres. It is because of the reason that area under the curve i.e. work done in case of rubber B is lesser and hence the car tyre will not get excessively heated and rubber A should be used to absorb vibration of the machinery because of the large area of the curve, a large amount of vibrational energy can be dissipated.



POISSON'S RATIO (σ)

When a long bar is stretched by a force along its length then its length increases and the radius decreases as shown in the Figure.



In that case, we have two types of strain in the wire

(a) Lateral strain: The ratio of change in diameter (ΔD) to the original diameter (D) is called lateral strain i.e. $\frac{\Delta D}{D}$.

(b) Longitudinal strain: The ratio of change in length (ΔL) to the original length (L) is called longitudinal strain i.e. $\frac{\Delta L}{L}$.

Poisson's Ratio (σ) is defined as the ratio of lateral strain to longitudinal strain.

$$\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$\Rightarrow \quad \sigma = -\frac{\Delta D/D}{\Delta L/L}$$

Negative sign indicates that the radius of the bar decreases when it is stretched. Poisson's ratio is a dimensionless and a unit less quantity.

Conceptual Note(s)

(a) The theoretical value of σ lies between –1 and 0.5
(b) The actual value i.e. practical value of σ lies between 0 and 0.5
(c) Poisson's ratio σ is not the modulus of elasticity as it is the ratio of two strains and not of stress to strain.

RELATION BETWEEN VOLUMETRIC STRAIN, LONGITUDINAL STRAIN AND POISSON'S RATIO

Consider a long bar having length L and radius R, then volume of the bar is given by

$$V = \pi R^2 L = \frac{\pi}{4} D^2 L$$

$$\Rightarrow \quad \Delta V = \frac{\pi}{4} \Delta (D^2 L)$$

$$\Rightarrow \quad \Delta V = \frac{\pi}{4} \left[D^2 \Delta L + L \Delta (D^2) \right]$$

Since, we know that $\Delta (D^2) = 2D\Delta D$

$$\Rightarrow \quad \Delta V = \frac{\pi}{4} \left[D^2 \Delta L + L (2D \Delta D) \right]$$

Dividing both the sides of the above equation by volume of the bar i.e. $V = \frac{\pi}{4} D^2 L$, we get

$$\frac{\Delta V}{V} = \frac{D^2 \Delta L}{D^2 L} + \frac{2DL\,\Delta D}{D^2 L}$$

$$\Rightarrow \quad \frac{\Delta V}{V} = \frac{\Delta L}{L} + 2\frac{\Delta D}{D}$$

Since $\frac{\Delta D}{D} = -\sigma \left(\frac{\Delta L}{L} \right)$ $\left\{ \because \ \sigma = -\frac{\Delta D/D}{\Delta L/L} \right\}$

$$\Rightarrow \quad \frac{\Delta V}{V} = (1 - 2\sigma) \frac{\Delta L}{L}$$

Conceptual Note(s)

(a) $\frac{\Delta V}{V} = (1 - 2\sigma) \frac{\Delta L}{L}$
(b) For $\sigma = 0.5$, $\Delta V = 0$
i.e. if value of Poisson's ratio is 0.5, then change in volume for this value is zero.

DEPRESSION OF A BEAM

If a beam of length l is supported horizontally at the ends and is loaded at the middle by a load W, the depression at the centre is given by

$$\delta = \frac{Wl^3}{48YI}$$

where, Y is the Young's Modulus.

For a beam of circular cross-section $I = \frac{1}{4}\pi r^2$, and

For a beam of rectangular cross-section $I = \frac{bd^3}{12}$

where, b being the breadth and d the depth of the beam.

Test Your Concepts-II

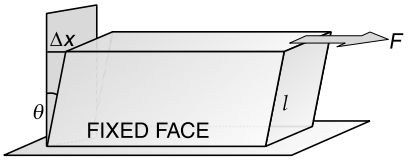
Based on Elastic Energy, Energy Density and Poisson's Ratio

(Solutions on page H.5)

1. Find the elastic deformation energy of a steel rod of mass $m = 3.1\,\text{kg}$ stretched to a tensile strain $\varepsilon = 1 \times 10^{-3}$. Y for steel is $2 \times 10^{11}\ \text{Nm}^{-2}$ and density of steel is $7800\ \text{kgm}^{-3}$.
2. A smooth uniform string of natural length l, cross-sectional area A and Young's modulus Y is pulled along its length by a force F on a horizontal surface. Find the elastic potential energy stored in the string.
3. A wire 4 m long and 0.3 mm in diameter is stretched by a force of 100 N. If extension in the wire is 0.3 mm, calculate the potential energy stored in the wire.
4. A steel wire of length 2 m and $1.2 \times 10^{-7}\ \text{m}^2$ in cross sectional area is stretched by a force of 36 N. Calculate stress, strain, increase in length and work done in stretching the wire $(Y = 1.8 \times 10^{11}\ \text{Nm}^{-2})$
5. Find the work done in stretching a wire of cross section $1\,\text{mm}^2$ and length 2 m through 0.1 mm. Young's modulus for the material of wire is $2 \times 10^{11}\ \text{Nm}^{-2}$.
6. A 45 kg boy whose leg bones are $5\ \text{cm}^2$ in area and 50 cm long falls through a height of 2 m without breaking his leg bones. If the bones can stand a stress of $0.9 \times 10^8\ \text{Nm}^{-2}$, calculate the Young's modulus for the material of the bone. Use $g = 10\ \text{ms}^{-2}$.
7. A catapult consists of two parallel rubber strings, each of lengths 10 cm and cross-sectional area $10\ \text{mm}^{-2}$. When stretched by 5 cm, it can throw a stone of mass 100 g to a vertical height of 25 m. Determine Young's modulus of elasticity of rubber.
8. A steel wire 4.0 m in length is stretched through 2.0 mm. The cross-sectional area of the wire is $2.0\ \text{mm}^2$. If Young's modulus of steel is $2.0 \times 10^{11}\ \text{Nm}^{-2}$. Calculate the energy density of wire and the elastic potential energy stored in the wire.

SHEAR MODULUS

The shear modulus of a solid measures its resistance to a shearing force, which is a force applied tangentially to a surface, as shown in the figure.



Since the bottom of the solid is assumed to be at rest, there is an equal and opposite force on the lower surface. The top surface is displaced by Δx relative to the bottom surface. The shear stress is defined as

$$\text{Shear Stress} = \frac{\text{Tangential force}}{\text{Area}} = \frac{F_t}{A} = \frac{F}{A}$$

where A is the area of the surface. The **shear strain** is denoted by θ and is given by

$$\theta = \frac{\Delta x}{l}$$

where l is the separation between the top and the bottom surfaces.

$$\eta = \frac{\text{Shearing Stress}}{\text{Shearing Strain}}$$

Shear modulus is also called **modulus of rigidity.**

Consider a solid rectangular block whose lower face is held fixed and a tangential force F is applied to the upper face (see figure). This deforms the block into a parallelopiped, turning the vertical end faces through an angle θ. If A is the area of the upper face then

$$\eta = \frac{F}{A\theta} = \frac{F}{A\tan\theta} \qquad \{\text{Using Shearing Strain}\}$$

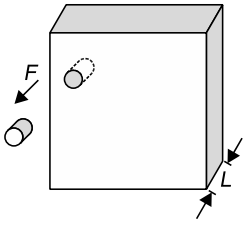
$$\eta = \frac{Fl}{A\Delta x}$$

Conceptual Note(s)

Solids possess all the three moduli of elasticity BUT Liquids and Gases possess only Bulk Modulus.

ILLUSTRATION 24

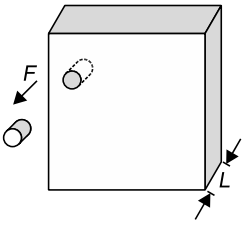
Calculate the force needed to punch a 1.46 cm diameter hole in a steel plate of 1.27 cm thick shown in figure. The ultimate shear stress of steel is 345×10^6 Nm^{-2}.



SOLUTION

According to figure. As in punching, shear elasticity is involved, the hole will be punched if

$$\frac{F}{A} > (\text{Ultimate Shear Stress})$$



$$\Rightarrow \quad F > (\text{Shear Stress})(\text{Area})$$

Since the area $A = 2\pi rL$

$$\Rightarrow \quad A = 2(3.14)(0.73\times10^{-2})(1.27\times10^{-2})$$

$$\Rightarrow \quad A = 5.8\times10^{-4}\ \text{m}^2$$

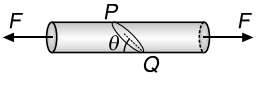
$$\Rightarrow \quad F > (3.45\times10^8)(5.8\times10^{-4}) \approx 200\ \text{kN}$$

$$\Rightarrow \quad F > 200\ \text{kN}$$

Minimum force is required is $F_{\min} = 200$ kN

ILLUSTRATION 25

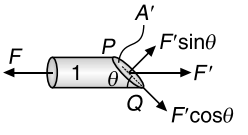
Two equal and opposite forces each of magnitude F act on a rod of uniform cross-sectional area A, as shown in the Figure.



Calculate the shear stress and longitudinal stress on the section PQ.

SOLUTION

Since the net force acting on the rod is zero, so the rod is in equilibrium. Let the tension in the segment PQ be F. Applying Newton's Second Law for the segment 1, we get



$$F_{\text{net}} = F' - F = ma = 0 \qquad \{\because a = 0\}$$

$$\Rightarrow \quad F' = F$$

Resolving the force F' parallel and perpendicular to the given surface PQ having area A', we get the tangential component of the force F_t to be $F_t = F'\cos\theta$ and the normal component of the force to be $F_n = F'\sin\theta$, so tangential stress i.e. shear stress is given by

$$\sigma_t = \frac{F_{\tan}}{A'} = \frac{F'\cos\theta}{A'}$$

Since, $A' = \dfrac{A}{\sin\theta}$ and $F' = F$

$$\Rightarrow \quad \sigma_t = \frac{F\sin\theta\cos\theta}{A}$$

Similarly, longitudinal stress or normal stress is

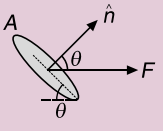
$$\sigma_n = \frac{F_n}{A'} = \frac{F'\sin\theta}{A'}$$

Since, $A' = \dfrac{A}{\sin\theta}$ and $F' = F$

$$\Rightarrow \quad \sigma_n = \frac{F}{A}\sin^2\theta$$

Conceptual Note(s)

To find the normal (or longitudinal) stress, resolve the force perpendicular to the plane of the given surface and then divide it by the area of the surface, i.e.



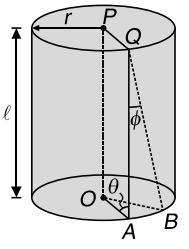
$$\sigma_n = \frac{F_n}{A_{surface}} = \frac{F_\perp}{A_{surface}} = \frac{F\cos\theta}{A_{surface}}$$

To find shearing stress or tangential stress, resolve the force parallel to plane of the given surface and then divide it by the area of the surface, i.e.

$$\sigma_t = \frac{F_t}{A_{surface}} = \frac{F_\parallel}{A_{surface}} = \frac{F\sin\theta}{A_{surface}}$$

TORSION OF A CYLINDER

When one end of a cylinder is clamped and a torque is applied at the other end then the cylinder gets twisted by angle θ due to which a shearing strain ϕ is also produced in the cylinder.



(a) From OAB, we observe that $AB = r\theta$ and from QAB we observe that $AB = l\phi$. So, we have

$$AB = r\theta = l\phi$$

$$\Rightarrow \quad \phi = \frac{r\theta}{l}$$

(b) The restoring torque in the cylinder is given by

$$\tau = \frac{\pi\eta r^4\phi}{2l}$$

where, η is the modulus of rigidity of the material of the cylinder.

(c) The torque per unit twist i.e. the torque required to produce a unit twist in the cylinder (also called as the torsional constant C of the cylinder) is given by

$$C = \frac{\tau}{\phi} = \frac{\pi\eta r^4}{2l}$$

(d) Work done in twisting the cylinder through an angle θ is given by

$$W = \frac{1}{2}C\theta^2 = \frac{\pi\eta r^4\theta^2}{4l}$$

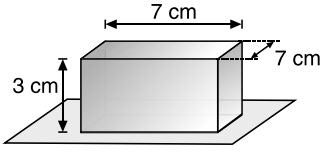
Test Your Concepts-III

Based on Shear Modulus, Tangential Stress, Shear Strain

(Solutions on page H.6)

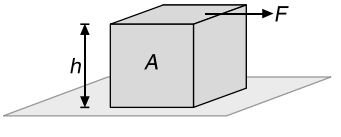
1. A box shaped piece of gelatin dessert has a top area of $15\ cm^2$ and a height of $3\ cm$. When a shearing force of $0.50\ N$ is applied to the upper surface, the upper surface displaces $4\ mm$ relative to the bottom surface. Calculate the shearing stress, the shearing strain, and the shear modulus for the gelatin.

2. A $3\ cm \times 7\ cm \times 7\ cm$ block of soft butter block is resting on a surface as shown in Figure.



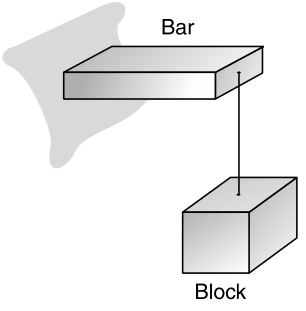
A tangential force of $0.49\ N$ is applied at the top face of the butter block. The top face is observed to move $6\ mm$ relative to the bottom face. Calculate the shear modulus of butter block.

3. A cube of side *h* is fixed to a horizontal surface as shown in Figure.



If a shearing force distorts the cube such that σ is the shear stress, then calculate the work done by the shearing force, assuming deformation to be small.

4. A block of mass 160 kg is hanging from the end of a steel bar as shown in Figure.

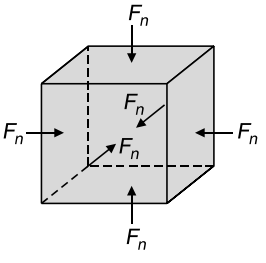


The length of the bar is 0.10 m and its cross-sectional area is 3.2×10^{-4} m^2. Ignoring weight of the bar, calculate shear stress on the bar and vertical deflection of the right end of the bar if $\eta_{steel}=8.0\times10^{10}$ Nm^{-2} and $g=10$ ms^{-2}.

BULK MODULUS

The bulk modulus of a solid or a fluid indicates its resistance to a change in volume. Consider a cube of some material, solid or fluid, as shown in the figure. We assume that all faces experience the same force F_n normal to each face. (One way to accomplish this is to immerse the body in a fluid as long as the change in pressure over the vertical height of the cube is negligible). The **pressure** on the cube is defined as the normal force per unit area

$$p=\frac{F_n}{A}$$



The *SI* unit of pressure is Nm^{-2} and is given the name **pascal (Pa).**

Pressure is a scalar because on any infinitesimal volume, it acts in all directions; it has no unique direction.

When the pressure on a body is increased, its volume decreases. The change in pressure ΔP is called the volume stress and the fractional change in volume $\frac{\Delta V}{V}$ is called the volume strain. The **bulk modulus** B of the material is defined as

$$\text{Bulk modulus}=\frac{\text{Volume stress}}{\text{Volume strain}}$$

$$\Rightarrow \quad B=\frac{-\Delta p}{\frac{\Delta V}{V}}$$

The negative sign is included to make B a positive number since an increase in pressure $(\Delta p>0)$ leads to decrease in volume $(\Delta V<0)$

The inverse of B is called the **compressibility**,

$$k=\frac{1}{B}$$

State	Shear Modulus	Bulk Modulus
Solid	Large	Large
Liquid	Zero	Large
Gas	Zero	Small

RELATIONS BETWEEN ELASTIC CONSTANTS

The elastic constants i.e. Y, K, η and σ are found to depend on each other through the relations given by

(a) $Y=3B(1-2\sigma)$ (b) $Y=2\eta(1+\sigma)$

(c) $\sigma=\frac{3B-2\eta}{6B+2\eta}$ (d) $Y=\frac{9\eta B}{3B+\eta}$

DENSITY OF COMPRESSED LIQUIDS

When a pressure ΔP is applied on a substance its density is changed. If a liquid of density ρ, volume V and bulk modulus K is compressed, then its mass remains same, volume decreases and hence density increases. Since density is

$$\rho=\frac{M}{V} \qquad \ldots(1)$$

When compressed, we have the density ρ' given by

$$\rho'=\frac{M}{V-\Delta V}=\frac{M}{V\left(1-\frac{\Delta V}{V}\right)} \qquad \ldots(2)$$

From (1) and (2), we get

$$\rho'=\frac{\rho}{\left(1-\frac{\Delta V}{V}\right)}$$

Since, $B = -\dfrac{\Delta P}{\left(\dfrac{\Delta V}{V}\right)}$

$\Rightarrow \quad -\dfrac{\Delta V}{V} = \dfrac{\Delta P}{B}$

$\Rightarrow \quad \rho' = \dfrac{\rho}{1-\dfrac{\Delta P}{B}}$

From this expression we can see that ρ' increases as pressure is increased (ΔP is positive) and vice-versa.

ILLUSTRATION 26

Sea water has a Bulk modulus of $23\times10^{10}\ \text{Nm}^{-2}$. Calculate the density of sea water at a depth where the pressure is 800 atm if the density at the surface is $1024\ \text{kgm}^{-3}$.

SOLUTION

The changed density is given by

$$\rho' = \frac{\rho}{1-\dfrac{\Delta P}{B}}$$

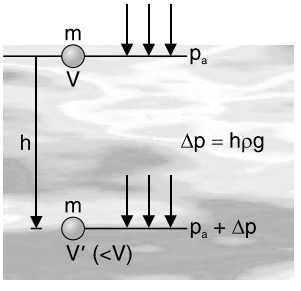
Substituting the value, we get

$$\rho' = \frac{1024}{1-\dfrac{799\times1.01\times10^5}{23\times10^{10}}} = 1024.4\ \text{kgm}^{-3}$$

ILLUSTRATION 27

Find the depth of lake at which density of water is 1% greater than at the surface. Given compressibility $50\times10^{-6}\ \text{atm}^{-1}$.

SOLUTION



$$B = \frac{\Delta p}{-\dfrac{\Delta V}{V}}$$

$\Rightarrow \quad \dfrac{\Delta V}{V} = -\dfrac{\Delta p}{B}$ …(1)

Since, $p = p_{\text{atm}} + h\rho g$

$m = \rho V = \text{constant}$

$\Rightarrow \quad V d\rho + \rho dV = 0$

$\Rightarrow \quad \dfrac{d\rho}{\rho} = -\dfrac{dV}{V}$

$\Rightarrow \quad \dfrac{\Delta\rho}{\rho} = \dfrac{\Delta p}{B}$

Since, $\dfrac{\Delta\rho}{\rho} = \dfrac{1}{100}$

$\Rightarrow \quad \dfrac{1}{100} = \dfrac{h\rho g}{B}$

Assuming ρ to be constant, we get

$$h\rho g = \frac{B}{100} = \frac{1}{100k}$$

$\Rightarrow \quad h\rho g = \dfrac{1\times1\times10^5}{100\times50\times10^{-6}}$

$\Rightarrow \quad h = \dfrac{10^5}{5000\times10^{-6}\times1000\times10}$

$\Rightarrow \quad h = \dfrac{100\times10^3}{50} = 2\ \text{km}$

ILLUSTRATION 28

The pressure P of the gas varies with volume V according to the relation $P = P_0 e^{\alpha V}$. Find the bulk modulus of the gas.

SOLUTION

Since $P = P_o e^{\alpha V}$

$\Rightarrow \quad \dfrac{dP}{dV} = \dfrac{d}{dV}\left(P_o e^{\alpha V}\right) = P_o e^{\alpha V}\alpha = P\alpha \qquad \{\because P = P_o e^{\alpha V}\}$

Since by definition of Bulk's Modulus, we have

$$B = -\frac{dP}{dV/V} = -V\left(\frac{dP}{dV}\right)$$

$$B = \left|\frac{dP}{dV}\right| V = \alpha P V$$

Conceptual Note(s)

(a) A solid will have all the three modulus of elasticity Y, B and η. But in case of a liquid or a gas only B can be defined as a liquid or a gas cannot be framed into a wire or no shear force can be applied on them.

(b) For a liquid or a gas,

$$B = \left(\frac{-dP}{dV/V}\right)$$

So instead of P we are more interested in change in pressure dP.

(c) In case of a gases, the Bulk's modulus is given by

$B = XP$

In the process $PV^x = \text{constant}$

For example, for $x = 1$, or $PV = \text{constant}$ (isothermal process) $B = P$.

i.e., isothermal bulk modulus of a gas (denoted by $B_{\text{isothermal}}$) is equal to the pressure of the gas at that instant of time or

$$B_{\text{isothermal}} = P$$

Similarly, for $x = \gamma = \dfrac{C_P}{C_V}$ or $PV^\gamma = \text{constant}$ (adiabatic process) $B = \gamma P$.

i.e., adiabatic bulk modulus of a gas (denoted by $B_{\text{adiabatic}}$) is equal to γ times the pressure of the gas at that instant of time or

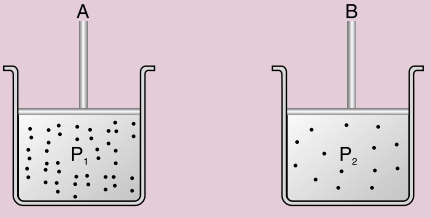
$$B_{\text{adiabatic}} = \gamma P$$

(d) For a gas $B \propto P$

whether it is an isothermal process or an adiabatic process. Physically this can be understood as under.

Suppose we have two containers A and B. Some gas is filled in both the containers. But the pressure in A is more than the pressure in B, i.e.,

$$P_1 > P_2$$



So, bulk modulus of A should be more than the bulk modulus of B, or

$$B_1 > B_2$$

and this is quiet obvious, because it is more difficult to compress the gas in chamber A, i.e., it provides more resistance to the external forces. And as we have said in point number 1 (ii) the modulus of elasticity is greater for a substance which offers more resistance to external forces.

(e) For liquids, the energy density is given by

$$U = \frac{(\text{Stress})^2}{2B} = \frac{(h\rho g)^2}{2B}$$

ILLUSTRATION 29

In a vertical cylindrical vessel of base area $A = 80\ \text{cm}^2$ water is filled to a height $h = 30\ \text{cm}$. If density and the Bulk modulus of water be $\rho = 1000\ \text{kgm}^{-3}$ and $B = 2\times10^9\ \text{Nm}^{-2}$. Calculate elastic deformation energy of water in the vessel. $\left(g = 10\ \text{ms}^{-2}\right)$.

SOLUTION

At a depth h below the free surface of water volume density of elastic deformation energy,

$$U = \frac{(\rho g h)^2}{2B}$$

So, total deformation energy E of water in the vessel is given by

$$E = \int_0^h u\,dV$$

$$\Rightarrow \quad E = \int_0^h \frac{(\rho g h)^2}{2B} A\,dh = \frac{A(\rho g)^2 h^3}{6B}$$

Substituting the values, we have

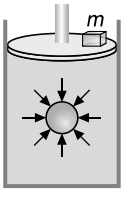
$$E = \frac{(80\times10^{-4})(10^3\times10)^2(0.3)^3}{6\times2\times10^9} = 1.8\times10^{-6}\ \text{J}$$

Test Your Concepts-IV

Based on Bulk's Modulus, Normal Stress and Volumetric Strain

(Solutions on page H.6)

1. A solid sphere of radius R made of a material of bulk modulus B is surrounded by a liquid in a cylindrical container. A heavy piston of mass m and area A floats on the surface of the liquid as shown. Calculate the fractional change in the radius of the sphere.



2. A specimen of oil having an initial volume of $800\ \text{cm}^3$ subjected to a pressure increase of $1.8\times10^6\ \text{Pa}$, and the volume is found to decrease by $0.30\ \text{cm}^3$. Calculate the bulk modulus and compressibility of the material.

3. In taking a solid ball of rubber from the surface to the bottom of a lake of $180\ \text{m}$ depth, reduction in the volume of the ball is 0.1%. The density of water of the lake is $1\times10^3\ \text{kgm}^{-3}$. Determine the value of the bulk modulus of elasticity of rubber. $\left(g = 9.8\ \text{ms}^{-2}\right)$.

4. What is the percentage change in the volume of a glass ball if it is placed in vacuum than in air. Bulk modulus of glass $B = 4 \times 10^{10}\ \text{Nm}^{-2}$.

5. Bulk modulus of water is $2.3 \times 10^{9}\ \text{Nm}^{-2}$. Taking average density of water $\rho = 10^{3}\ \text{kgm}^{-3}$, find increase in density at a depth of $1\,\text{km}$. Take $g = 10\ \text{ms}^{-2}$.

6. Calculate the density of lead under a pressure of $2 \times 10^{8}\ \text{Nm}^{-2}$, if the bulk modulus of lead is $8 \times 10^{9}\ \text{Nm}^{-2}$ and initially the density of lead is $11.4\ \text{gcm}^{-3}$.

7. The bulk modulus of water is $2.3 \times 10^{9}\ \text{Nm}^{-2}$.
(a) Find its compressibility in the units atm^{-1}.
(b) How much pressure in atmospheres is needed to compress a sample of water by 0.1%?

8. Calculate the volume density of the elastic deformation energy in fresh water at the depth of $h = 1000\ \text{m}$. Compressibility of water $k = 4.9 \times 10^{-10}\ \text{m}^2\text{N}^{-1}$.

9. The pressure of a medium is changed from $1.01 \times 10^{5}\ \text{Pa}$ to $1.165 \times 10^{5}\ \text{Pa}$ and change in volume is 10% keeping temperature constant. Calculate the Bulk's modulus of the medium.

FLUID STATICS

FLUIDS: INTRODUCTION AND ASSUMPTIONS

A fluid is a substance that can flow and does not have a shape of its own. All liquids and gases are fluids. However, in this chapter our main study is applicable for

(a) Incompressible Liquids, which have their density to be independent of the variations in pressure and is assumed to be constant.

(b) Non-viscous Liquids, in which the parts of the liquid in contact do not exert any tangential force on each other. The force acted by one part of the liquid on the other part is perpendicular to the surface of contact due to which no friction between the adjacent layers of liquid in contact.

DENSITY

The **density** ρ of a substance is defined as the mass per unit volume of a sample of the substance.
If a small mass element Δm occupies a volume ΔV, the density is given by

$$\rho = \frac{\Delta m}{\Delta V}$$

In general, the density of an object depends on position, so that

$$\rho = f(x, y, z)$$

If the object is homogeneous, its physical parameters do not change with position throughout its volume. Thus, for a homogeneous object of mass M and volume V, the density is defined as

$$\rho = \frac{M}{V}$$

The SI units of density are kgm^{-3}.
In a fluid, at a point, density ρ is defined as

$$\rho = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V} = \frac{dm}{dV}$$

Conceptual Note(s)

(a) In case of homogenous isotropic substance, density has no directional properties, so is a scalar. It has dimensional formula ML^{-3}, SI unit kgm^{-3} and cgs unit gcc^{-1} where

$$1\,\text{gcc}^{-1} = 10^3\ \text{kgm}^{-3}$$

(b) When immiscible liquids of different densities are poured in a beaker, the liquid having the highest density will be at the bottom of the beaker while the liquid having the lowest density will be at the top of the beaker and interfaces separating the liquids will be plane surfaces.

(c) With the rise in temperature (ΔT) due to thermal expansion of a given body, the volume of the body increases whereas the mass of the body remains unchanged. So, the density of the body decreases according to the relation

$$\frac{\rho}{\rho_0} = \frac{m/V}{m/V_0} = \frac{V_0}{V}$$

Since, we know that $V = V_0(1+\gamma\Delta T)$

$$\Rightarrow\ \rho = \frac{\rho_0}{1+\gamma\Delta T}$$

where, γ is the coefficient of volume expansion of the liquid. (Detailed discussion done in Thermal Expansion)

(d) With increase in pressure (ΔP) the volume of the liquid decreases (ΔV), so the density will increase according to the relation

$$\rho = \frac{\rho_0}{1-\frac{\Delta V}{V}} = \frac{\rho_0}{1-\frac{\Delta P}{B}}$$

where, B is the Bulk's Modulus of the liquid. (Detailed discussion done in Elasticity)

RELATIVE DENSITY OR SPECIFIC GRAVITY

Sometimes instead of density we use the term relative density (RD) or specific gravity (SG) which is defined as:

$$\text{RD} = \frac{\text{Density of body}}{\text{Density of water at 4 °C}}$$

Relative density is a pure ratio and has no units. Density of water at $4\ ^\circ\text{C}$ is $1\ \text{gcc}^{-1}$. So, in cgs system of units, the relative density is equal to the density.
For example, the relative density of mercury is 13.6, so its density will be $13.6\times10^3\ \text{gcc}^{-1}$.

ILLUSTRATION 30

Find the density and specific gravity of gasoline if $51\ \text{g}$ occupies $75\ \text{cm}^3$?

SOLUTION

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{0.051\ \text{kg}}{75\times10^{-6}\ \text{m}^3} = 680\ \text{kgm}^{-3}$$

$$\Rightarrow \quad \text{Sp. gravity} = \frac{\text{density of gasoline}}{\text{density of water}}$$

$$\Rightarrow \quad \text{Sp. gravity} = \frac{680 \text{ kgm}^{-3}}{1000 \text{ kgm}^{-3}} = 0.68$$

$$\Rightarrow \quad \text{Sp. gravity} = \frac{\text{mass of 75 cm}^3 \text{ gasoline}}{\text{mass of 75 cm}^3 \text{ water}}$$

$$\Rightarrow \quad \text{Sp. gravity} = \frac{51 \text{ g}}{75 \text{ g}} = 0.68$$

DENSITY OF A MIXTURE OF TWO OR MORE LIQUIDS

If a mass M_1 of liquid of density ρ_1, volume V_1 is mixed with a liquid of mass M_2, density ρ_2, volume V_2 and so on, then

$$\rho_{\text{mixture}} = \frac{M_{\text{mixture}}}{V_{\text{mixture}}} = \frac{M_1 + M_2 + \ldots}{V_1 + V_2 + \ldots}$$

CASE-1

If equal masses of the liquids are mixed, then

$$M_{\text{mixture}} = M + M = 2M \text{ and } V_{\text{mixture}} = V_1 + V_2$$

$$\Rightarrow \quad \rho = \frac{2M}{(M/\rho_1) + (M/\rho_2)}$$

$$\Rightarrow \quad \rho_{\text{mixture}} = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2} = \text{Harmonic Mean}$$

CASE-2

If equal volumes of the liquids are mixed, then

$$M_{\text{mixture}} = M_1 + M_2 \text{ and } V_{\text{mixture}} = 2V$$

$$\Rightarrow \quad \rho = \frac{\rho_1 V + \rho_2 V}{2V} = \frac{\rho_1 + \rho_2}{2}$$

$$\Rightarrow \quad \rho_{\text{mixture}} = \frac{\rho_1 + \rho_2}{2} = \text{Arithmetic Mean}$$

Problem Solving Technique(s)

(a) If n liquids of equal masses are mixed, then

$$\frac{1}{\rho_{\text{mixture}}} = \frac{\frac{1}{\rho_1} + \frac{1}{\rho_2} + \frac{1}{\rho_3} + \ldots}{n}$$

(b) If n liquids of equal volumes are mixed, then

$$\rho_{\text{mixture}} = \frac{\rho_1 + \rho_2 + \rho_3}{n}$$

ILLUSTRATION 31

Two substances of densities ρ_1 and ρ_2 are mixed in equal volume and the relative density of mixture is 4. When they are mixed in equal masses, the relative density of the mixture is 3. Find ρ_1 and ρ_2.

SOLUTION

When substances are mixed in equal volume then density of the mixture is

$$\rho_{\text{mix}} = \frac{\rho_1 + \rho_2}{2} = 4$$

$$\Rightarrow \quad \rho_1 + \rho_2 = 8 \qquad \ldots(1)$$

When substances are mixed in equal masses then density is

$$\frac{2\rho_1\rho_2}{\rho_1 + \rho_2} = 3$$

$$\Rightarrow \quad 2\rho_1\rho_2 = 3(\rho_1 + \rho_2)$$

$$\Rightarrow \quad 2\rho_1\rho_2 = 3 \times 8 = 24$$

$$\Rightarrow \quad \rho_1\rho_2 = 12 \qquad \ldots(2)$$

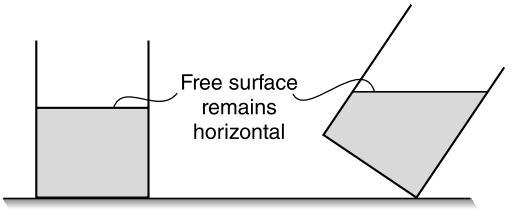
Solving equations (1) and (2), we get

$$\rho_1 = 6 \text{ and } \rho_2 = 2.$$

FLUID AT REST

It has been observed that a fluid at rest cannot sustain a tangential force. If such a force is applied to a fluid, the different layers simply slide over one another. Hence, for a fluid at rest, the forces acting on it have to be normal to the surface.

An important consequence of this property is that the free surface of a liquid at rest, under gravity, in a container, is always horizontal.



Since, the force of gravity acts vertically downwards so, in equilibrium, the liquid surface has to be perpendicular to it and hence is horizontal.

Problem Solving Technique(s)

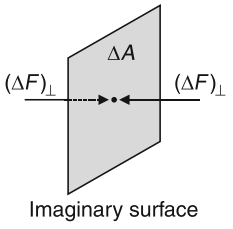
(a) A fluid exerts force on any surface in contact with it.
(b) This force always acts at right angles (normally) to the surface and is also called **Thrust.**

PRESSURE

The normal force or the thrust per unit area is called **Fluid Pressure.** Its S.I. unit is Nm^{-2} or pascal (Pa). Mathematically, we have

$$P = \frac{(\Delta F)_\perp}{\Delta A}$$

where $(\Delta F)_\perp$ is the normal force acting on surface of area A. If we consider an imaginary surface within the liquid, then the fluid on both the sides of the surface exerts equal and opposite force $(\Delta F_\perp)$ on each surface as shown. If this is not the case then the liquid surface will accelerate and the fluid will not remain at rest.



If the pressure is same at all points of a finite plane surface of area A, then

$$P = \frac{F_\perp}{A}$$

where $F_\perp$ is the normal force acting on one side of the imaginary surface. The **average pressure** in the fluid at the position of the element is given by

$$P_{av} = \frac{\Delta F}{\Delta A}$$

When $\Delta A \to 0$, the element reduces to a point, and thus, pressure at a point is defined as

$$P = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} = \frac{dF}{dA}$$

When the force is constant over the surface of area A, then the above equation reduces to

$$P = \frac{F}{A}$$

The SI unit of pressure is Nm^{-2} and is also called **pascal (Pa)**. where, $1\ \mathrm{Pa} = 1\ \mathrm{Nm}^{-2}$

The other common units of pressure are the **atmosphere** and **bar**.

$$1\ \mathrm{atm} = 1.01325 \times 10^5\ \mathrm{Pa}$$

$$1\ \mathrm{bar} = 1.00000 \times 10^5\ \mathrm{Pa}$$

ATMOSPHERIC PRESSURE (P_0)

It is the pressure due to the earth's atmosphere. It changes with weather conditions and elevation. The normal average atmospheric pressure at the sea level is 1.01×10^5 Pa also called as 1 atmosphere i.e.

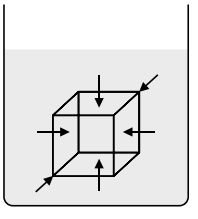
$$1\ \mathrm{atm} = 1.01 \times 10^5\ \mathrm{Pa}$$

and $1\ \mathrm{atm} = 1.01 \times 10^5\ \mathrm{Pa} = 1.01\ \mathrm{bar} = 760\ \mathrm{torr}$

The atmospheric pressure is maximum at the surface of earth and goes on decreasing as we move up into the earth's atmosphere.

PRESSURE IS ISOTROPIC

Imagine a static fluid and consider a small cubic element of it deep within the fluid as shown in Figure. Since this fluid element is in equilibrium, therefore, forces acting on each lateral face of this element must also be equal in magnitude. Because the areas of each face are equal, therefore, the pressure on each of the lateral faces must also be the same. In the limit as the cube element reduces to a point, the forces on the top and bottom surfaces also become equal. Thus, the pressure exerted by a fluid at a point is the same in all directions – **the pressure is isotropic**.



Since the fluid cannot support a shear stress, the force exerted by a fluid pressure must also be perpendicular to the surface of the container that holds it.

VARIATION OF PRESSURE WITH DEPTH

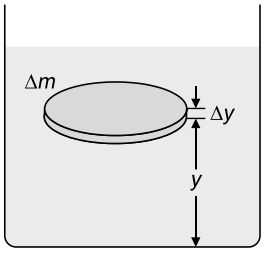
Consider a small cylindrical element of mass Δm at a height y from the base of the container as shown in Figure. Let A be the area of the top and bottom surface of this cylinder.

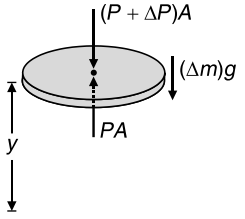
We know that the fluid forces on the opposite vertical faces of the cylinder are equal in magnitude and opposite in direction, and therefore, cancel. The other forces acting on the fluid contained within the imaginary boundary is the gravitational force

$$\Delta W = (\Delta m)\, g$$

and the forces due to fluid pressure.

The pressure force acting on the lower face of the element is pA and that on the upper face is $(p + \Delta p)A$. The free body diagram of the element is shown in Figure.





Applying the condition of equilibrium, we get

$$PA-(P+\Delta P)A-(\Delta m)g=0 \qquad \ldots(1)$$

If ρ be the density of the fluid at the position of the element, then

$$\Delta m=\rho A(\Delta y)$$

$$\Rightarrow \quad PA-(P+\Delta P)A-\rho gA(\Delta y)=0 \qquad \{\text{from (1)}\}$$

$$\Rightarrow \quad \frac{\Delta P}{\Delta y}=-\rho g$$

In the limit Δy approaching to zero, $\frac{\Delta P}{\Delta y}$ becomes

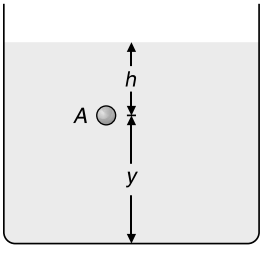
$$\frac{dP}{dy}=-\rho g$$

The above equation indicates that the slope of P versus y is negative. That is, the pressure P decreases with height y from the bottom of the fluid.
In other words, the pressure p increases with depth h, i.e.

$$\frac{dP}{dh}=\rho g$$

THE INCOMPRESSIBLE FLUID MODEL

For an incompressible fluid, the density ρ of the fluid remains constant throughout its volume. It is a good assumption for liquids. To find pressure at the point A in a fluid column as shown in Figure is obtained by integrating.



$$dP=\rho g\,dh$$

$$\Rightarrow \quad \int_{P_0}^{P} dP=\rho g\int_{0}^{h} dh$$

$$\Rightarrow \quad P-P_0=\rho gh$$

$$\Rightarrow \quad P=P_0+\rho gh$$

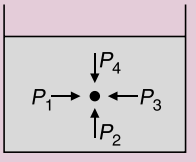
where ρ is the density of the fluid, and ρ_0 is the atmospheric pressure at the free surface of the liquid.

Conceptual Note(s)

(a) Thus, pressure is same at all points at the same depth. The force due to pressure acts normally on any area, irrespective of the orientation of the area.

(b) At same point inside a fluid, the pressure is same in all directions. In the Figure,

$$P_1=P_2=P_3=P_4$$



ABSOLUTE PRESSURE AND GAUGE PRESSURE

Absolute pressure is the total pressure at a point while gauge pressure is relative to the local atmospheric pressure. Gauge pressure may be positive or negative depending upon the fact whether the pressure is more or less than the atmospheric pressure.

$$P_{\text{gauge}}=P_{\text{absolute}}-P_{\text{atm}}$$

ILLUSTRATION 32

If pressure at half the depth of a lake is two third the pressure at the bottom of the lake then find the depth of the lake.

SOLUTION

Let P_0 be the atmospheric, then pressure at bottom of the lake is

$$P=P_0+h\rho g$$

Pressure at half the depth of a lake is

$$P'=P_0+\frac{h}{2}\rho g$$

According to given condition, we have

$$P'=\frac{2}{3}P$$

$$\Rightarrow \quad P_0+\frac{1}{2}h\rho g=\frac{2}{3}(P_0+h\rho g)$$

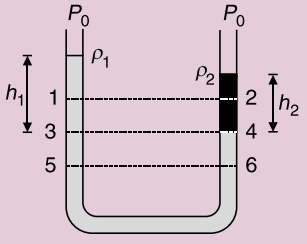
$$\Rightarrow \quad \frac{1}{3}P_0=\frac{1}{6}h\rho g$$

$$\Rightarrow \quad h=\frac{2P_0}{\rho g}=\frac{2\times10^5}{10^3\times10}=20\text{ m}$$

Conceptual Note(s)

(a) Forces acting on a fluid in equilibrium have to be perpendicular to its surface. Because it cannot sustain the shear stress.

(b) In the same liquid pressure will be same at all points at the same level.



For example, in the Figure, we observe that

$$P_1 \neq P_2,\ P_3 = P_4,\ P_5 = P_6$$

For $P_3 = P_4$, we get

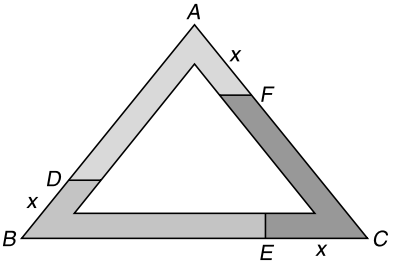
$$P_0 + \rho_1 g h_1 = P_0 + \rho_2 g h_2$$

$$\Rightarrow\ \rho_1 h_1 = \rho_2 h_2$$

$$\Rightarrow\ h \propto \frac{1}{\rho}$$

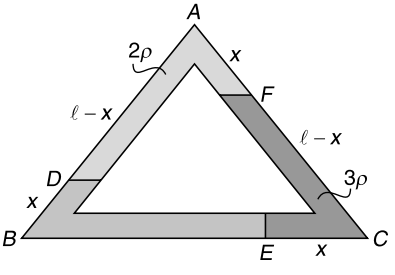
ILLUSTRATION 33

A closed tube in the form of an equilateral triangle of side l contains equal volumes of three liquids which do not mix and is placed vertically with its lowest side horizontal. Find the value of x in the Figure, if the densities of liquids are in arithmetic progression.



SOLUTION

Pressure at E due to the left limb AB must be equal to the pressure at E due to the right limb AC, so using Pascal's equation, we get



$$P_{AB} = (l-x)\sin 60^\circ (2\rho) g + x \sin 60^\circ (3\rho) g \text{ and}$$

$$P_{AC} = (l-x)\sin 60^\circ (3\rho) g + (x \sin 60^\circ)\rho g$$

Equating, these two, we get

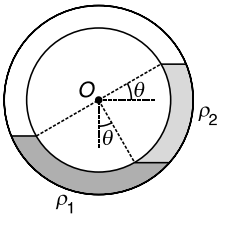
$$2(l-x) + 3x - 3(l-x) - x = 0$$

$$\Rightarrow\ 3x = l$$

$$\Rightarrow\ x = \frac{l}{3}$$

ILLUSTRATION 34

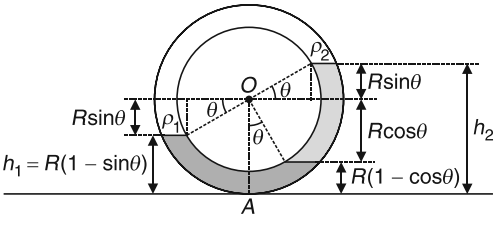
A circular tube of uniform cross section is filled with two liquids of densities ρ_1 and ρ_2 such that half of each liquid occupies a quarter of volume of the tube as shown in Figure. If the line joining the free surfaces of the liquid makes an angle θ with horizontal, then calculate θ.



SOLUTION

Since pressure at A due to the left column and the right column of the liquid should be the same, so we have

$$(P_A)_{\text{due to left column}} = (P_A)_{\text{due to right column}} \quad \ldots(1)$$



Due to left column

Pressure at A is given by

$$(P_A)_{\text{left}} = R(1-\sin\theta)\rho_1 g$$

Due to right column

Pressure at A is given by

$$(P_A)_{\text{right}} = R(\sin\theta + \cos\theta)\rho_2 g + (1-\cos\theta)\rho_1 g$$

Equating both these equations, we get

$$\rho_1(1-\cos\theta) + \rho_2(\sin\theta + \cos\theta) = \rho_1(1-\sin\theta)$$

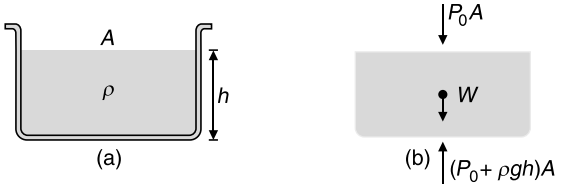
$$\Rightarrow\ \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \frac{\rho_1}{\rho_2}$$

$$\Rightarrow\ \tan\theta = \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}$$

$$\Rightarrow\ \theta = \tan^{-1}\left(\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}\right)$$

SIMPLE FREE BODY DIAGRAM OF A LIQUID IN A CONTAINER

The free body diagram of the liquid (showing the vertical forces only) is shown in Figure (b). For the equilibrium of liquid, the Net downward force must equal the net upward force.

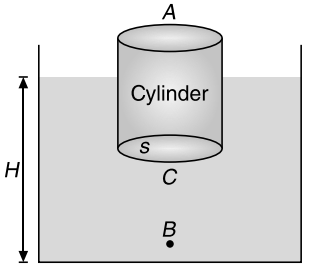


$\Rightarrow \quad P_0A + W = (P_0 + \rho gh)A$

$\Rightarrow \quad W = \rho ghA$ {which is true}

ILLUSTRATION 35

A liquid of density ρ is filled in a beaker of cross-section S to a height H and then a cylinder of mass m and cross-section s is made to float in it as shown in Figure.



If the atmospheric pressure is p_0, find the pressure at the top face A of the cylinder, at the bottom face C of the cylinder and at the base B of the beaker. Can you think of a condition for which these three pressures are equal?

SOLUTION

Above the cross-section A there is external pressure due to atmosphere only. So

$$P_A = \text{Atmospheric pressure} = P_0$$

At the point C the pressure will be due to atmosphere and also due to the weight of the cylinder, so, we have

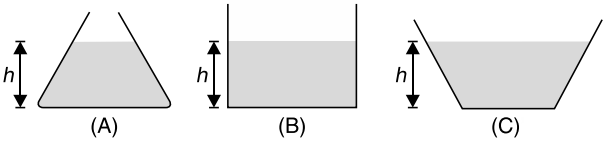
$$P_C = P_0 + \frac{mg}{s}$$

If the system is in free fall (as in a satellite), then $g_{eff} = 0$

$\Rightarrow \quad P_A = P_B = P_C = P_0$

HYDROSTATIC PARADOX

Hydrostatic pressure depends on the depth of the point below the surface (h), nature of liquid (ρ) and acceleration due to gravity (g) while it is independent of the amount of liquid, shape of the container or cross-sectional area considered. So, if a given liquid is filled in vessels of different shapes to same height, the pressure (P) at the base of each vessel's will be the same, though the volume or weight of the liquid in different vessels will be different. This situation is called **Hydrostatic Paradox**.



$$P_A = P_B = P_C = P_0 + h\rho g \text{ but } W_A < W_B < W_C$$

In a liquid at same level, the pressure will be same at all points, if not, due to pressure difference the liquid cannot be at rest. This is why the height of liquid is same in vessels of different shapes containing different amounts of the same liquid at rest when they are in communication with each other.

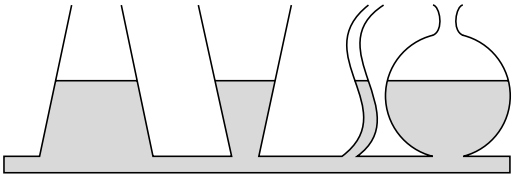
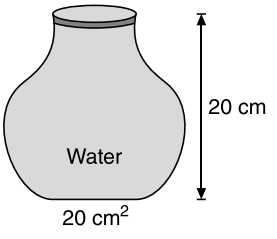


ILLUSTRATION 36

A half-litre vessel of height $20\ \text{cm}$ is full of water. The vessel base has an area of $20\ \text{cm}^2$ and its mouth area is also $20\ \text{cm}^2$ as shown in Figure.



Calculate the force exerted by the water on the base of the vessel. Assuming the water to be in equilibrium calculate the resultant force exerted by the curved surface of the vessel on water if the atmospheric pressure is $P_0 = 1.0 \times 10^5\ \text{Nm}^{-2}$, density of water is $\rho = 1000\ \text{kgm}^{-3}$ and $g = 10\ \text{ms}^{-2}$.

SOLUTION

Pressure of water at the base of vessel is

$$P = P_0 + h\rho g$$

$\Rightarrow \quad P = 1 \times 10^5 + (0.2)(1000)(10)$

$\Rightarrow \quad P = 1.02 \times 10^5\ \text{Nm}^{-2}$

Force on the base of vessel is

$$F = PA_{\text{base}} = (1.02 \times 10^5)(20 \times 10^{-4})$$

$\Rightarrow \quad F = 204\ \text{N}$

So, upward force on water by the base of vessel is

$$F_{\text{upward}} = 204 \text{ N}$$

Weight of water in the vessel is

$$W = (0.5\times10^{-3})(1000)(10) = 5 \text{ N}$$

Downward force on water by atmosphere is

$$F_{\text{downward}} = P_0 A = (1\times10^5)(20\times10^{-4}) = 200 \text{ N}$$

Assuming the force applied by the curved surface of vessel on water to be F_s, then for equilibrium, we have upwards, for equilibrium of water we have

$$\Sigma F = 0$$

$$\Rightarrow \quad F_{\text{up}} + F_{\text{down}} + W + F_s = 0$$

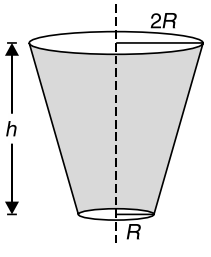
Taking upward direction as positive, we get

$$204 - 200 - 5 + F_s = 0$$

$$\Rightarrow \quad F_s = 1 \text{ N} \text{ (upwards)}$$

ILLUSTRATION 37

Consider a frustum of a cone h having radius $2R$ and R. A liquid of density ρ is filled in it. Calculate the force exerted by the walls of the container on the liquid.

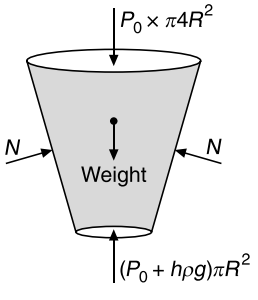


SOLUTION

For vertical equilibrium of the fluid

$$P_0\left[\pi(2R)^2\right] + \text{Weight} = (P_0 + h\rho g)\pi R^2 + N_y$$

Here N_y is the net vertical component of the normal reaction by the wall



Weight of liquid is

$$\rho\left(\frac{7}{3}\pi r^2 h\right)g = \frac{1}{3}\pi h\left[R^2 + (2R)^2 + 2R^2\right]\rho g$$

$$\Rightarrow \quad 4P_0\pi R^2 + \frac{7}{3}\pi R^2 h\rho g = P_0\pi R^2 + h\rho g\pi R^2 + N_y$$

$$\Rightarrow \quad 3P_0\pi R^2 + \frac{4}{3}\pi R^2 h\rho g = N_y$$

For horizontal equilibrium net force by the wall on the liquid in horizontal direction should be zero.

Thus, total force $F = \left(3P_0\pi R^2 + \frac{4}{3}\pi R^2 h\rho g\right)$

$$\Rightarrow \quad F = \pi R^2\left(3P_0 + \frac{4}{3}h\rho g\right)$$

Conceptual Note(s)

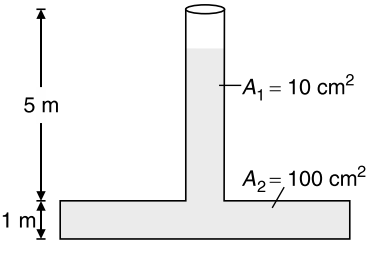
$$V_{\text{frustum}} = \frac{1}{3}\pi h(R^2 + r^2 + Rr)$$

$$\Rightarrow \quad V_{\text{frustum}} = \frac{1}{3}\pi R^2 h\left(1 + \frac{r^2}{R^2} + \frac{r}{R}\right)$$

$$\Rightarrow \quad V = \frac{1}{3}\pi R^2 h\left(\left(\frac{r}{R}\right)^0 + \left(\frac{r}{R}\right)^1 + \left(\frac{r}{R}\right)^2\right)$$

ILLUSTRATION 38

In the Figure shown, calculate the total force acting at the bottom of the tank due to the water pressure, the total weight of water. Can you explain the difference between the two readings?



SOLUTION

Pressure at the base due to water is

$$P = \rho_w g(5+1) = (10)^3(10)(6) = 6\times10^4 \text{ Nm}^{-2}$$

So, force F due to pressure is given by

$$\Rightarrow \quad F = PA_2 = (6\times10^4)(100\times10^{-4}) = 600 \text{ N}$$

Weight of water W is given by

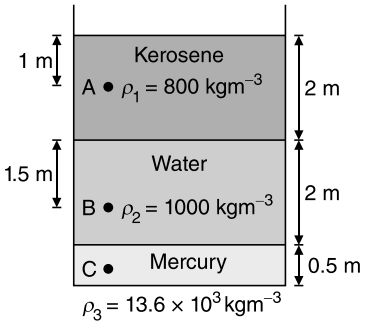
$$W = \rho_w g(5A_1 + A_2)$$

$$\Rightarrow \quad W = 10^4\left[5(10\times10^{-4}) + 100\times10^{-4}\right] = 150 \text{ N}$$

This problem exhibits that a liquid may exert thrust which is more than the weight of the liquid.

ILLUSTRATION 39

Find the absolute pressure and gauge pressure at points A, B and C as shown in Figure.
Take $1 \text{ atm} = 10^5 \text{ Pa}$.



SOLUTION

Given that

$$P_{\text{atm}} = P_0 = 10^5 \text{ Pa} = 100 \text{ kPa}$$

At *A*

Gauge Pressure is measuring the pressure without taking the atmospheric pressure into account, so we have

$$\Delta P_A = \rho_1 g h_A = (800)(10)1$$

$$\Rightarrow \quad \Delta P_A = 8 \text{ kPa}$$

Absolute pressure is given by

$$P_A = P_0 + \Delta P_A$$

$$\Rightarrow \quad P_A = 10^5 \text{ Pa} + 8000 \text{ Pa} = 108 \text{ kPa}$$

At *B*

Gauge pressure is given by

$$\Delta P_B = \rho_1 g(2) + \rho_2 g(1.5)$$

$$\Rightarrow \quad \Delta P_B = (800)(10)(2) + (10)^3(10)(1.5)$$

$$\Rightarrow \quad \Delta P_B = 31 \text{ kPa}$$

Absolute pressure is given by

$$P_B = P_0 + \Delta P_B$$

$$\Rightarrow \quad P_A = 100 \text{ kPa} + 31 \text{ kPa} = 131 \text{ kPa}$$

At *C*

Gauge pressure is given by

$$\Delta P_C = \rho_1 g(2) + \rho_2 g(2) + \rho_3 g(0.5)$$

$$\Rightarrow \quad \Delta P_C = g\left[2\rho_1 + 2\rho_2 + \frac{\rho_3}{2}\right]$$

$$\Rightarrow \quad \Delta P_C = 10\left(1600 + 2000 + \frac{13.6 \times 10^3}{2}\right)$$

$$\Rightarrow \quad \Delta P_C = 104 \text{ kPa}$$

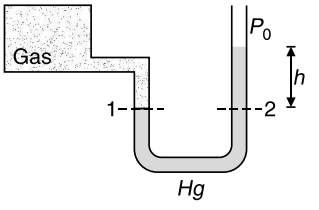
Absolute pressure is given by

$$P_C = P_0 + \Delta P_C$$

$$\Rightarrow \quad P_A = 100 \text{ kPa} + 104 \text{ kPa} = 204 \text{ kPa}$$

PRESSURE MEASURING DEVICE: MANOMETER

It is a device used to measure the pressure of a gas inside a container. A **manometer** is a tube open at both the ends and bent into the shape of a U and partially filled with mercury. When one end of the tube is subjected to an unknown pressure p, the mercury level drops on that side of the tube and rises on the other so that the difference in mercury level is h as shown in Figure.



According to Pascal's Law, when we move down in a fluid pressure increases with depth and when we move up the pressure decreases with depth. When we move horizontally in the same fluid, the fluid pressure remains constant. At equilibrium, we have

$$P_1 = P_2$$

where P_1 is the pressure of the gas in the container and P_2 is given by

$$P_2 = P_{\text{gas}} = P_0 + h\rho g$$

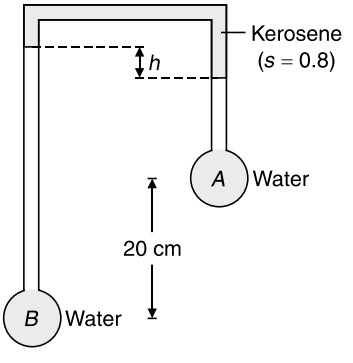
So, gauge pressure is given by

$$P - P_0 = h\rho g$$

where, ρ is the density of the liquid used in U-tube and P_0 is the atmospheric pressure. So, by measuring h we can find absolute (or gauge) pressure in the vessel.

ILLUSTRATION 40

For the arrangement shown in Figure, calculate h if the pressure difference between the vessels A and B is 3 kNm^{-2}.

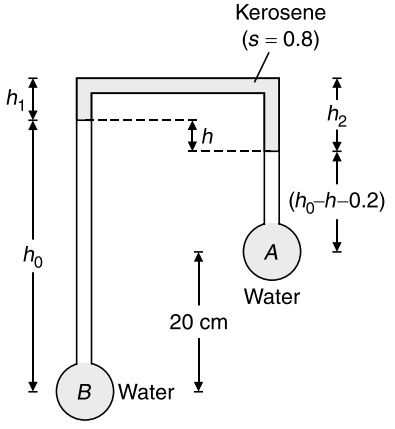


SOLUTION

Let pressure in the horizontal tube be P, then in the left vertical tube, we have

$$P + \rho_k g h_1 + \rho_w g h_0 = P_B$$

$$\Rightarrow \quad P + \rho_k g h_2 + \rho_w g (h_0 - h - 0.2) = P_A$$



Given that, $P_B - P_A = 3 \times 10^3 \text{ Nm}^{-2}$, $\rho_w = 10^3 \text{ kgm}^{-3}$

$$\Rightarrow \quad \rho_k = 800 \text{ kgm}^{-3}$$

$$\Rightarrow \quad h = 0.5 \text{ m} = 50 \text{ cm}$$

ILLUSTRATION 41

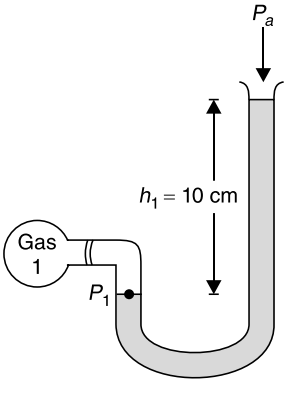
A manometer tube contains a liquid of density $3 \times 10^3 \text{ kg m}^{-3}$. When connected to a vessel containing a gas, the liquid level in the other arm of the tube is higher by 10 cm. When connected to another sample of enclosed gas, the liquid level in the other arm of the manometer tube falls 7 cm below the liquid level in the first arm. Which of the two samples exerts more pressure and by what amount?

SOLUTION

For Sample Gas 1:

$$h_1 = 10 \text{ cm} = 0.1 \text{ m}$$

$$P_1 = P_a + \rho g h_1 \qquad \ldots(1)$$



For Sample Gas 2:

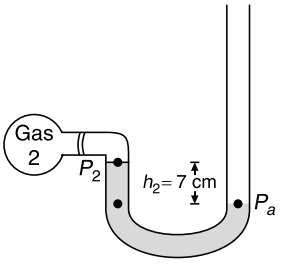
In this case, level of the liquid in the left arm is higher than that in the right arm by 7 cm.

So, atmospheric pressure P_a is greater than the pressure exerted by the sample

$$\Rightarrow \quad P_a = P_2 + \rho g h_2$$

$$\Rightarrow \quad P_2 = P_a - \rho g h_2 \qquad \ldots(2)$$

Comparing equations (1) and (2), we observe that $P_1 > P_2$. Therefore the gas in sample 1 exerts greater pressure than that in sample 2.



The difference in the two pressures is given by

$$\Delta P = P_1 - P_2 = (P_a + \rho g h_1) - (P_a - \rho g h_2)$$

$$\Rightarrow \quad \Delta P = \rho g (h_1 + h_2)$$

where $h_1 + h_2 = 17 \text{ cm} = 0.17 \text{ m}$

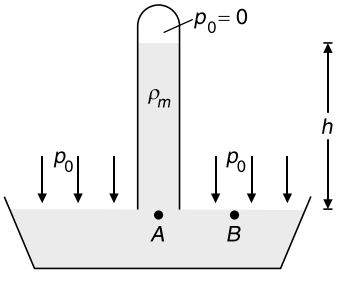
$$\Rightarrow \quad \Delta P = (3 \times 10^3 \text{ kg m}^{-3}) \times (9.8 \text{ ms}^{-2})(0.17 \text{ m})$$

$$\Rightarrow \quad \Delta P = 4.99 \times 10^3 \text{ Pa}$$

$$\Rightarrow \quad \Delta P \approx 5 \text{ kPa}$$

THE MERCURY BAROMETER

It is a straight glass tube (closed at one end) completely filled with mercury and inserted into a dish which is also filled with mercury as shown in Figure. Atmospheric pressure supports the column of mercury in the tube to a height h. The pressure between the closed end of the tube and the column of mercury is zero, $P = 0$.



Therefore, pressure at points A and B are equal, so

$$P_{\text{atm}} = P_0 = 0 + h \rho_m g$$

At the sea level, p_0 can support a column of mercury about 76 cm in height. Hence

$$P_0 = (13.6 \times 10^3)(9.81)(0.76) = 1.01 \times 10^5 \text{ Nm}^{-2}$$

ILLUSTRATION 42

What must be the length of a barometer tube used to measure atmospheric pressure if we are to use water instead of mercury.

SOLUTION

We know that

$$P_0 = \rho_m g h_m = \rho_w g h_w$$

where ρ_w and h_w are the density and height of the water column supporting the atmospheric pressure P_0.

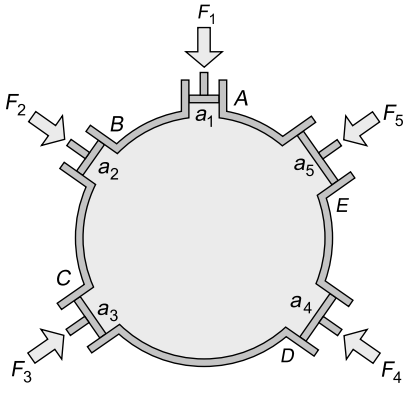
$$\Rightarrow \quad h_w = \left(\frac{\rho_m}{\rho_w}\right) h_m$$

Since, $\frac{\rho_m}{\rho_w} = 13.6$ and $h_m = 0.76$ m

$$\Rightarrow \quad h_w = (13.6)(0.76) = 10.33 \text{ m}$$

PASCAL'S LAW

Liquids, being incompressible are capable of transmitting the pressure applied at one point to any other point. A principle regarding this was formulated by Blaise Pascal (1623-1662), a French mathematician, physicist and philosopher. The formulation is called **Pascal's Law.**



The excess pressure, applied anywhere in a mass of a confined incompressible fluid (or liquid), is transmitted by the fluid in all directions and acts undiminished at every point of the fluid and at right angles to the surfaces exposed to the fluid.

Consider a vessel full of water and filled with air tight pistons in different position as shown. Let the piston at A be pushed down with a force F_1 (shown in Figure). Pressure P on the piston is

$$P = \frac{F_1}{a_1}$$

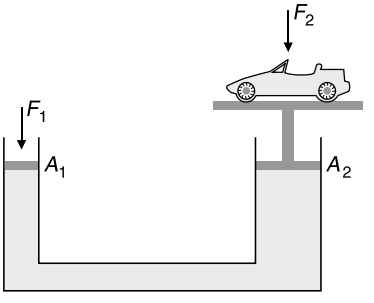
where a_1 is the area of cross-section of piston at A. It will be observed that to hold pistons at B, C, D and E we have to apply forces F_2, F_3, F_4 and F_5 on them such that

$$\frac{F_1}{a_1} = \frac{F_2}{a_2} = \frac{F_3}{a_3} = \frac{F_4}{a_4} = \frac{F_5}{a_5}$$

where a_2, a_3, a_4 and a_5 are the area of cross-section of piston at B, C, D and E respectively. This indicates that pressure is transmitted equally in all directions as stated by Pascal's law.

HYDRAULIC LIFT

Hydraulic lift is used to support or lift heavy objects and works on the principle of Pascal's Law. This principle is used in a hydraulic jack or lift, as shown in Figure.



The pressure due to a small force F_1 applied to a piston of area A_1 is transmitted to the larger piston of area A_2. The pressure at the two pistons is the same because they are at the same level.

$$P = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$\Rightarrow \quad F_2 = \left(\frac{A_2}{A_1}\right) F_1$$

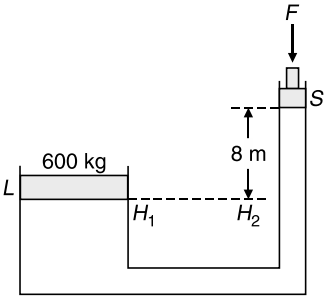
Consequently, the force on the larger piston is large.

Thus, a small force F_1 acting on small area A_1 results in a larger force F_2 acting on larger area A_2. This ratio of the larger force to the smaller force is called the Mechanical Advantage (MA). It is always greater than 1. So,

$$\text{MA} = \frac{F_{\text{larger}}}{F_{\text{smaller}}} = \frac{A_{\text{larger}}}{A_{\text{smaller}}}$$

ILLUSTRATION 43

For the system shown in Figure, the cylinder on the left, at L, has a mass of 600 kg and a cross-sectional area of $800\ \text{cm}^2$. The piston on the right, at S, has cross-sectional area $25\ \text{cm}^2$ and negligible weight. If the apparatus is filled with oil $\left(\rho = 0.78\ \text{gcm}^3\right)$, find the force F required to hold the system in equilibrium as shown in Figure.



SOLUTION

The pressures at point H_1 and H_2 are equal because they are at the same level in the single connected fluid. Therefore,

$$\text{Pressure at } H_1 = \text{Pressure at } H_2$$

$$\Rightarrow \begin{pmatrix}\text{Pressure}\\ \text{due to}\\ \text{left piston}\end{pmatrix} = \begin{pmatrix}\text{Pressure due}\\ \text{to } F \text{ and}\\ \text{right piston}\end{pmatrix} + \begin{pmatrix}\text{Pressure}\\ \text{due to}\\ 8 \text{ m of oil}\end{pmatrix}$$

$$\Rightarrow \frac{(600)(9.8)}{0.08} = \frac{F}{25\times10^{-4}} + (8)(780)(9.8)$$

$$\Rightarrow F = 31 \text{ N}$$

ILLUSTRATION 44

Two pistons of a hydraulic machine have diameters of 30 cm and 2.5 cm. What is the force exerted on the larger piston when 40 kg wt is placed on the smaller piston? If the smaller piston moves in through 6 cm, how much does the other piston move out?

SOLUTION

For smaller piston, Area $a_1 = \pi\times(1.25)^2$ cm^2

For larger piston, Area $a_2 = \pi\times(15)^2$ cm^2

Mechanical advantage at the larger piston is $\frac{a_2}{a_1}$

$$\Rightarrow F_2 = \frac{a_2}{a_1}\times F_1 = \frac{\pi(15)^2}{\pi(1.25)^2}\times 40 \text{ kg wt}$$

$$\Rightarrow F_2 = \frac{225}{1.25\times1.25}\times40\times9.8 \text{ N}$$

$$\Rightarrow F_2 = 56,448 \text{ N}$$

This is the force exerted on the larger piston.

The liquids are considered incompressible. Therefore, volume covered by the movement of smaller piston inwards is equal to that moved outwards by larger piston.

$$\Rightarrow l_1a_1 = l_2a_2$$

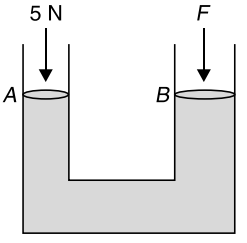
$$\Rightarrow l_2 = \frac{a_1}{a_2}l_1 = \frac{(1.25)^2}{(15)^2}\times6 \text{ cm}$$

$$\Rightarrow l_2 = 0.042 \text{ cm}$$

So, the distance moved out by the larger piston is 0.042 cm.

ILLUSTRATION 45

The area of cross section of the two arms of a hydraulic pressure are 5 cm^2 and 15 cm^2 respectively (shown in Figure). A force of 5 N is applied on water in the thicker arm so that the water may remain in equilibrium?



SOLUTION

In equilibrium, the pressures at the two surfaces should be equal as they lie in the same horizontal level. If atmospheric pressure is P_0 and force F is applied to maintain the equilibrium then

$$P_A = P_B \qquad \ldots(1)$$

The pressure at point A, $P_A = P_0 + \frac{5 \text{ N}}{5\times10^{-4} \text{ m}^2}$

Pressure at point B, $P_B = P_0 + \frac{F}{15\times10^{-4} \text{ m}^2}$

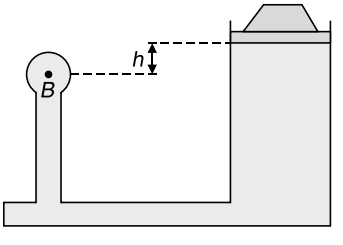
From equation (1), i.e. $P_A = P_B$, we get

$$\Rightarrow P_0 + \frac{5 \text{ N}}{5\times10^{-4} \text{ m}^2} = P_0 + \frac{F}{15\times10^{-4} \text{ m}^2}$$

$$\Rightarrow F = \frac{5}{5\times10^{-4}}\times15\times10^{-4} = 15 \text{ N}$$

ILLUSTRATION 46

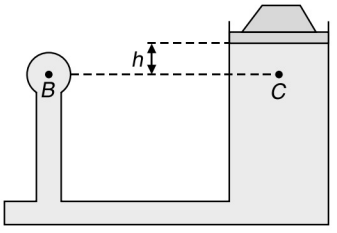
A weighted piston confines a fluid of density ρ in a closed container, as shown in Figure. The combined weight of piston and weight is $W = 200$ N, and the cross-sectional area of the piston is $A = 8$ cm^2. Find the total pressure at point B if the fluid is mercury and $h = 25$ cm $\left(\rho_m = 13600 \text{ kgm}^{-3}\right)$. What would an ordinary pressure gauge read at B?



SOLUTION

Since we know that the pressure at points which are at same level is the same, so we have

$$P_B = P_C$$



where, $P_C = P_0 + \frac{W}{A} + h\rho g$

$\Rightarrow \quad P_B = P_C = P_0 + \frac{W}{A} + h\rho g$, where

Atmospheric pressure is $P_0 = 1\times10^5$ Pa
Pressure due to piston and weight is

$$\frac{W}{A} = \frac{200\text{ N}}{8\times10^{-4}\text{ m}^2} = 2.5\times10^5\text{ Pa}$$

Pressure due to height h of fluid is

$$h\rho g = 0.33\times10^5\text{ Pa}$$

$\Rightarrow \quad P_B = P_0 + \frac{W}{A} + h\rho g$

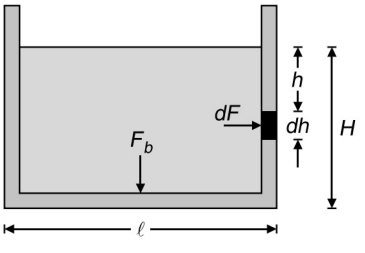
$\Rightarrow \quad P_B = 3.8\times10^5\text{ Pa} = 380\text{ kPa}$

Since the gauge pressure does not include atmospheric pressure, so we have gauge pressure at

$$\Delta P_B = P_B - P_0 = 280\text{ kPa}$$

FORCE AND TORQUE DUE TO HYDROSTATIC PRESSURE

Whenever a fluid comes in contact with solid boundaries it exerts a force on it. Consider a rectangular vessel of base size $l\times b$ filled with water to a height H as shown in Figure.



The force F_b acting at the base of the container is given by

$$F_b = P_{\text{base}}\,(\text{Area of the base})$$

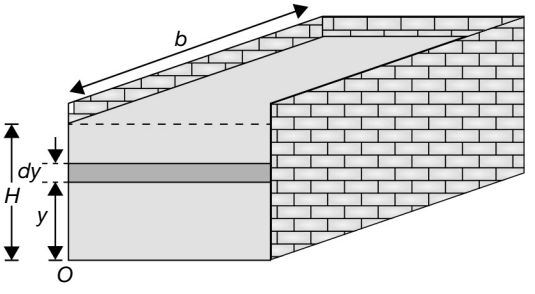
because pressure is same everywhere at the base and is equal to $P_{\text{base}} = \rho gH$.
Therefore, $F_b = \rho gH\,(lb) = \rho glbH$
In this particular case, we see that

$$F_b = \rho gV = \begin{pmatrix}\text{weight of the liquid}\\ \text{inside the vessel}\end{pmatrix}$$

where $V = lbH$ is the volume of the liquid inside the beaker.

Unlike the base, the pressure on the vertical wall of the vessel is not uniform but increases linearly with depth from the free surface. Therefore, we have to perform the integration to calculate the total force on the wall.

Suppose water stands at a depth H behind the vertical face of a dam. It exerts a certain resultant force on the dam tending to slide it and a certain torque tending to overturn the dam's wall. Let's assume width of dam is b as shown in Figure.



The pressure at height y is

$$P = \rho g\,(H - y)$$

Atmospheric pressure can be omitted since it acts against the other face of the dam also. The force against shaded strip is

$$dF = PdA = \rho g\,(H - y)\,b\,dy$$

and the total force is

$$F = \int_0^H \rho gb\,(H - y)\,dy = \frac{\rho gbH^2}{2}$$

The **total force** acting per unit width of the vertical wall is

$$\frac{F}{b} = \frac{1}{2}\rho gH^2$$

The moment of the force dF about an axis through O is

$$d\tau = ydF$$

$\Rightarrow \quad d\tau = \rho gby\,(H - y)\,dy$

The total torque about O is

$$\tau = \int d\tau = \int_0^H \rho gby\,(H - y)\,dy = \frac{1}{6}\rho gbH^3$$

The torque due to hydrostatic forces per unit width of the wall is

$$\frac{\tau}{b} = \frac{\rho gH^3}{6}$$

If H' is the height above O at which the total force F would have to act to produce this torque

$$H' = \frac{1}{3}H$$

Hence, the line of action of the resultant force is at the depth of $\frac{2H}{3}$.

Alternatively, we can keep in mind that the point of application (the centre of force) of the total force from the free surface is given by

$$h_c = \frac{1}{F}\int_0^H h\,dF$$

where $\int_0^H hdF$ is the **moment of force** about the free surface. Here,

$$\int_0^H hdF = \int_0^H h(\rho gbhdh) = \rho gb\int_0^H h^2dh = \frac{1}{3}\rho gbH^3$$

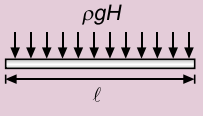
Since, $F = \frac{1}{2}\rho gbH^2$, therefore,

$$h_c = \frac{2}{3}H$$

The **net resultant force** acts at a depth $2H/3$ from the free surface.

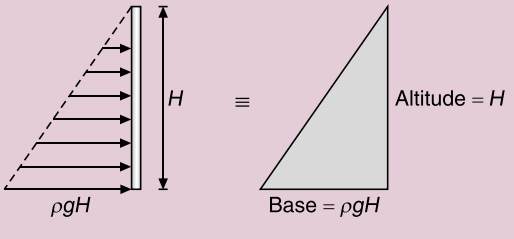
Conceptual Note(s)

(a) The force acting at the base per unit width $\left(\frac{F}{b}\right)$ is equal to the area of the pressure diagram as shown in Figure. That is,



$$\frac{F}{b} = P\ell = \rho gH\ell$$

The force acting per unit width is equal to the area of the pressure diagram shown in Figure.



$$\frac{F}{b} = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(\rho gH)(H) = \frac{1}{2}\rho gH^2$$

Hence, force per unit width of an immersed surface is equal to the area of the pressure diagram on the surface.

(b) Please note that, for the Figures shown, torque due to hydrostatic force about point *O*, the centre of a semi-cylindrical (or hemispherical) gate is zero as the hydrostatic force at all points passes through point *O*.

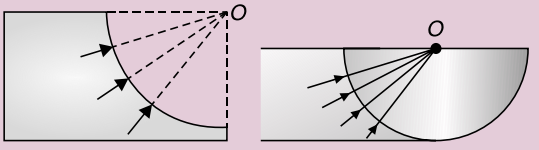
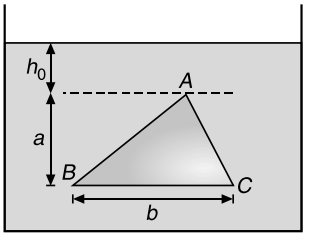


ILLUSTRATION 47

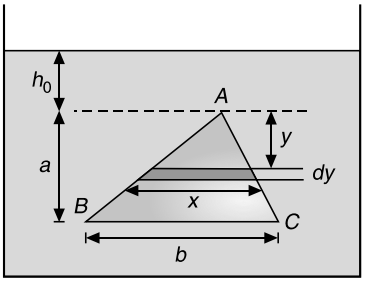
A triangular plate is submerged completely in a liquid of density ρ as shown in Figure. Calculate the hydrostatic force acting on one face of the plate.



SOLUTION

METHOD I:

Consider an infinitesimal strip of width dy, length x at a distance y from the edge A of the triangular plate as shown in Figure.



Pressure at the position of this infinitesimal strip is

$$P = (h_0 + y)\rho g$$

Force on this elemental strip is

$$dF = PdA = (h_0 + y)\rho g(xdy)$$

$$F = \int dF = \int_0^a \rho g(h_0 + y)xdy \qquad \ldots(1)$$

Using similarity of triangles, we get

$$\frac{x}{b} = \frac{y}{a}$$

$$\Rightarrow \quad x = \left(\frac{b}{a}\right) y \qquad \ldots(2)$$

Substituting (2) in (1) and integrating, we get

$$F = \int dF = \int_0^a \rho g (h_0 + y)\left(\frac{b}{a} y\right) dy$$

$$\Rightarrow \quad F = \frac{\rho g b}{a}\left[\int_0^a \left(h_0 y dy + y^2 dy\right)\right]$$

$$\Rightarrow \quad F = \frac{\rho g b}{a}\left(\frac{h_0 a^2}{2} + \frac{a^3}{3}\right)$$

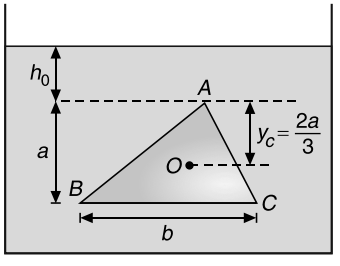
$$\Rightarrow \quad F = \rho g a b\left(\frac{a}{3} + \frac{h_0}{2}\right)$$

METHOD II:

The horizontal force F_H on the plate due to water is

$$F_H = \begin{pmatrix}\text{Pressure at}\\ \text{Centroid of Plate}\end{pmatrix}\begin{pmatrix}\text{Area}\\ \text{of Plate}\end{pmatrix}$$

$$\Rightarrow \quad F_H = P_O A$$



The centroid of the plate is at a distance $\frac{2a}{3}$ from the edge A of the plate. So, pressure at the centroid of the plate is given by

$$P_O = \rho g\left(h_0 + \frac{2a}{3}\right)$$

Area A of the plate is given by

$$A = \frac{1}{2} ab$$

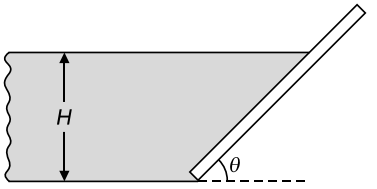
Hence the horizontal force on the plate is

$$\Rightarrow \quad F_H = P_O A = \rho g\left(\frac{2a}{3} + h_0\right)\left(\frac{ab}{2}\right)$$

$$\Rightarrow \quad F_H = \rho g a b\left(\frac{a}{3} + \frac{h_0}{2}\right)$$

ILLUSTRATION 48

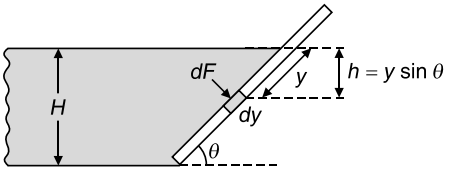
Find the force acting per unit width on a plane wall inclined at an angle θ with the horizontal as shown in Figure.



SOLUTION

METHOD-I:

Consider a small element of thickness dy at a distance y measured along the wall from the free surface as shown in Figure.



The pressure at the position of the element is

$$P = \rho g h = \rho g (y \sin\theta)$$

The force is given by

$$dF = PdA = p(bdy) = \rho g b (ydy)\sin\theta$$

The **total force** per unit width b is given by

$$\frac{F}{b} = \rho g \sin\theta \int_0^{\frac{H}{\sin\theta}} y dy = \rho g \sin\theta\left(\frac{y^2}{2}\right)\Big|_0^{\frac{H}{\sin\theta}}$$

$$\Rightarrow \quad \frac{F}{b} = \frac{1}{2}\left(\frac{\rho g H^2}{\sin\theta}\right)$$

Note that the above formula reduces to $\frac{1}{2}\rho g H^2$ for a vertical wall $(\theta = 90°)$ as discussed earlier.

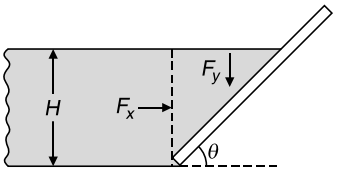
METHOD-II:

Alternatively, the force on the inclined wall may be obtained in two parts viz. horizontal and vertical.

The **horizontal force** F_x acts on the vertical projection of the incline wall, i.e., $F_x = \frac{1}{2}\rho g b H^2$

The **vertical force** F_y acts due to weight of the liquid supported by the wall, i.e.,

$$F_y = \frac{1}{2}\rho g b (H)(H\cot\theta) = \frac{1}{2}\rho g b H^2 \cot\theta$$



The **magnitude** of the resultant force is given by

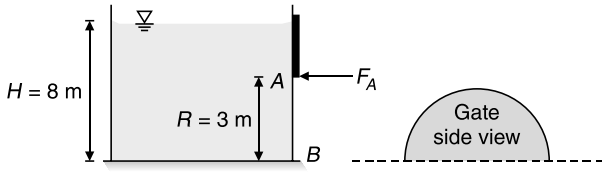
$$F = \sqrt{F_x^2 + F_y^2} = \frac{1}{2}\rho g b H^2 \text{cosec}\theta$$

$$\Rightarrow \quad F = \frac{1}{2}\rho g \frac{bH^2}{\sin\theta}$$

$$\Rightarrow \quad \frac{F}{b} = \frac{1}{2}\left(\frac{\rho g H^2}{\sin\theta}\right)$$

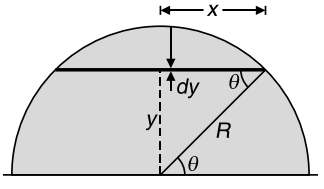
ILLUSTRATION 49

Semi-circular plane gate AB is hinged along B and held by horizontal force F_A applied at A. The liquid to the left of the gate is water, calculate the force F_A required for equilibrium. Take $g = 10\ \text{ms}^{-2}$ and density of water is $10^3\ \text{kgm}^3$.



SOLUTION

For equilibrium of the gate, net clockwise torque due to hydrostatic force is balanced by the anticlockwise torque due to force F_A.



Torque due to hydrostatic force

$$y = R\sin\theta$$

$$\Rightarrow \quad dy = R\cos\theta d\theta$$

$$x = R\cos\theta$$

$$\Rightarrow \quad dA = 2xdy = (2R\cos\theta)(R\cos\theta)d\theta$$

$$\Rightarrow \quad dA = (2R^2\cos^2\theta)d\theta$$

$$\Rightarrow \quad dF = \rho g(8 - y)dA$$

$$\Rightarrow \quad dF = \rho g(8 - R\sin\theta)(2R^2\cos^2\theta)d\theta$$

$$\Rightarrow \quad dF = 2\rho g R^2(8 - R\sin\theta)(\cos^2\theta)d\theta$$

$$\Rightarrow \quad d\tau = ydF = 2\rho g R^3(8 - R\sin\theta)(\sin\theta\cos^2\theta)d\theta$$

$$\Rightarrow \quad \tau_C = \int_0^{\pi/2} d\tau$$

$$\Rightarrow \quad \tau_C = 2\rho g R^3 \int_0^{\pi/2} (8 - R\sin\theta)(\sin\theta\cos^2\theta)d\theta$$

Substituting the values, $\rho = 10^3\ \text{kgm}^{-3}$, $g = 10\ \text{ms}^{-2}$ and $R = 3\ \text{m}$, we get

$$\tau_C = 2\times10^3\times10\times(3)^3\int_0^{\pi/2}(8 - 3\sin\theta)(\sin\theta\cos^2\theta)d\theta$$

$$\Rightarrow \quad \tau_C = 5.4\times10^5\left(\frac{8}{3} - \frac{3\pi}{16}\right)\text{Nm}$$

$$\Rightarrow \quad \tau_C = 1.12\times10^6\ \text{Nm} \qquad \ldots(1)$$

Anticlockwise torque due to F_A

$$\tau_A = F_A r_\perp = 3F_A \qquad \ldots(2)$$

Equating equations (1) and (2), we get

$$F_A = 3.73\times10^5\ \text{N} \approx 373\ \text{kN}$$

THE COMPRESSIBLE FLUID MODEL

For gases, the constant density assumed in the compressible model is often not adequate. However, an alternative simplifying assumption can be made that the density is proportional to the pressure, i.e.,

$$\rho = kP$$

Let ρ_0 be the density of air at the earth's surface where the pressure is atmospheric p_0, then

$$\rho_0 = kP_0$$

After eliminating k, we get

$$\rho = \left(\frac{P}{P_0}\right)\rho_0$$

Substituting the value of ρ in equation $dP = -\rho g dy$, we get

$$dP = -\left(\frac{P}{P_0}\right)\rho_0 g dy$$

$$\Rightarrow \quad \int_{p_o}^{p}\frac{dP}{P} = -\frac{\rho_0 g}{P_0}\int_0^h dy$$

where P is the pressure at a height $y = h$ above the earth's surface. After integrating, we get

$$\ln\left|\frac{P}{P_0}\right| = -\left(\frac{\rho_0 g}{P_0}\right)h$$

$$\Rightarrow \quad P = P_0 e^{-\left(\frac{\rho_0 g}{P_o}\right)h}$$

Note that instead of a linear decrease in pressure with increasing height as in the case of an incompressible fluid, in this case pressure decreases exponentially.

ILLUSTRATION 50

The density of air in atmosphere decreases with height and can be expressed by the relation

$$\rho = \rho_0 e^{-Ah}$$

where ρ_0 is the density at sea-level, A is a constant and h is the height. Calculate the atmospheric pressure at sea-level. Assume g to be constant. $g = 9.8\ \text{ms}^{-2}$, $\rho_0 = 1.3\ \text{kgm}^{-3}$ and $A = 1.2\times10^{-4}\ \text{m}^{-1}$.

SOLUTION

Considering an atmospheric layer at a height y of width dy, the pressure due to this layer is

$$dP = \rho g dy$$

$$\Rightarrow \quad dP = \rho_0 e^{-Ay} g dy$$

$$\Rightarrow \quad P = \int dP = \rho_0 g \int_0^\infty e^{-Ay} dy$$

$$\Rightarrow \quad P = \rho_0 g \left(-\frac{1}{A} e^{-Ay} \right)\Big|_0^\infty = -\frac{\rho_0 g}{A}\left(e^{-\infty} - e^{0}\right)$$

$$\Rightarrow \quad P = \frac{\rho_0 g}{A} = \frac{1.3\times9.8}{1.2\times10^{-4}}$$

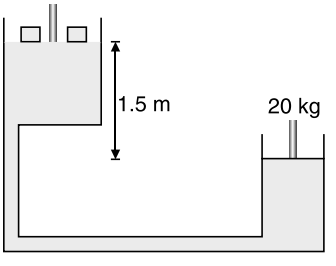
$$\Rightarrow \quad P = 1.0616\times10^{5}\ \text{Nm}^{-2}$$

Test Your Concepts-V

Based on Fluid Properties, Pressure and Pascal's Law

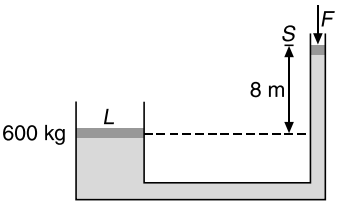
(Solutions on page H.7)

1. Relative density of an oil is 0.8. Find the absolute density of oil in CGS and SI unit.
2. The mass of a litre of milk is $1.032\ \text{kg}$. The butterfat that it contains has a density of $865\ \text{kgm}^{-3}$ when pure, and it constitutes 4 percent of the milk by volume. Calculate the density of the fat-free skimmed milk?
3. Atmospheric pressure is about $1.01\times10^5\ \text{Pa}$. How large a force does the atmosphere exert on a $2\ \text{cm}^2$ area patch on the top of your head assuming it to be flat?
4. Two vessels have the same base area but different shapes. The first vessel takes twice the volume of water that the second vessel requires to fill up to a particular common height. Is the force exerted by water on the base of the vessel the same in the two cases? If so, why do the vessels filled with water to that same height give different readings on a weighing scale?
5. A beaker of circular cross-section of radius $4\ \text{cm}$ is filled with mercury up to a height of $10\ \text{cm}$. Find the force exerted by the mercury at the bottom of beaker. The atmospheric pressure is $10^5\ \text{Nm}^{-2}$, specific gravity of mercury is 13.6 and $g = 10\ \text{ms}^{-2}$.
6. At a depth of $500\ \text{m}$ in an ocean, what is the absolute pressure? Given that the density of sea water is $1.03\times10^3\ \text{kgm}^{-3}$ and $g = 10\ \text{ms}^{-2}$.
7. An inverted bell lying at the bottom of a lake $47.6\ \text{m}$ deep has $50\ \text{cm}^3$ of air trapped in it. The bell is brought gradually to the surface of the lake. Find the volume of the trapped air if atmospheric pressure is taken to be $70\ \text{cm}$ of Hg and density of $Hg = 13.6\ \text{gcm}^{-3}$.
8. To what height should a cylindrical vessel of radius R be filled with a homogeneous liquid such that the force exerted by the liquid on the curved surface of the vessel is equal to the force exerted by the liquid at the bottom of the vessel?
9. The height of a mercury barometer is $75\ \text{cm}$ at sea level and $50\ \text{cm}$ at the top of a hill. If the ratio of density of mercury to that of air is 10^4, calculate the height of this hill.
10. A hydraulic press has a larger piston of diameter 35 cm at a height of 1.5 m relative to the smaller piston of diameter 10 cm as shown in Figure. The mass on the smaller piston is 20 kg. Calculate the force exerted on the load by the larger piston if the density of oil in the press is $750\ \text{kgm}^{-3}$ and $g = 9.8\ \text{ms}^{-2}$.



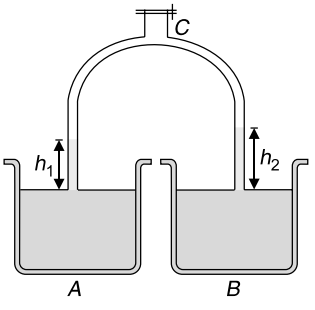
11. Find the pressure in the air column at which the piston remains in equilibrium. Assume the piston to be massless and frictionless.
12. An open U-tube of uniform cross-section contains mercury. When $27.2\ \text{cm}$ of water is poured into one limb of the tube, how high does the mercury rise in the other limb from its initial level? Also calculate the difference in levels of liquids of the two sides if specific gravity of water is 1 and that of mercury is 13.6.
13. The upper edge of a gate in a dam runs along water surface. The gate is $2\ \text{m}$ high and $3\ \text{m}$ wide and is hinged along a horizontal line through its center. Calculate the torque about hinge.

14. For the system shown in Figure, the cylinder on the left at L has a mass of $600\ \text{kg}$ and a cross sectional area of $800\ \text{cm}^2$.



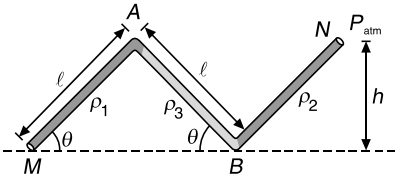
The piston on the right, at S, has cross sectional area $25\ \text{cm}^2$ and negligible weight. If the apparatus is filled with oil $(\rho = 0.75\ \text{gcm}^{-3})$. Find the force F required to hold the system in equilibrium. Take $g = 10\ \text{ms}^{-2}$.

15. The limbs of a glass U-tube are lowered into vessels A and B (Figure).

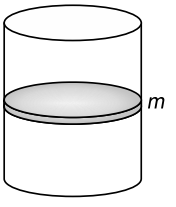


Some air is pumped out through the top of the tube C. The liquid in the left-hand limb then rises to a height h_1 and in the right-hand one to a height h_2. Determine the density of the liquid in limb B if water is present in limb A, $h_1 = 10\ \text{cm}$ and $h_2 = 12\ \text{cm}$.

16. A zig-zag tube open at N, having liquids of densities ρ_1, ρ_2 and ρ_3, is placed in a vertical plane as shown in Figure. (The pressure at M is equal to atmospheric pressure). Calculate the pressures at points A and B. Also find the angle θ.

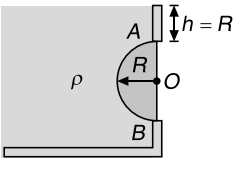


17. A cylindrical vessel containing a liquid is closed by a smooth piston of mass m as shown in Figure.



The area of cross-section of the piston is A. If the atmospheric pressure is P_0, find the pressure of the liquid just below the piston.

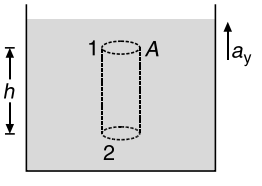
18. A semi-cylindrical massless gate of length ℓ, radius R pivoted at the point O is holding a stationary liquid of density ρ as shown in Figure. Calculate, the horizontal force exerted by the liquid on the gate.



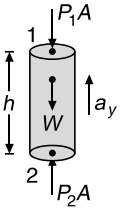
LIQUIDS IN ACCELERATED CONTAINERS

Container Having Vertical Acceleration

Consider a container shown in Figure. The container is accelerating upwards with an acceleration a_y.



Now to write pressure at point P at depth h. Lets consider a hypothetical cylinder of liquid with area of cross section A and length h. If m be the mass of the cylinder, then $m = Ah\rho$.



This hypothetical cylinder of liquid is moving up with an acceleration a_y, so on applying Newton's Second Law, we get

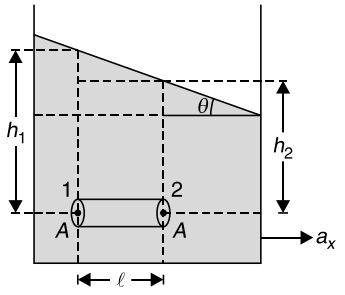
$$P_2A - (P_1A + W) = ma_y$$

$$\Rightarrow \quad P_2A - Ah\rho g - P_1A = (Ah\rho)a_y$$

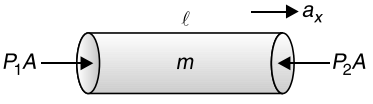
$$\Rightarrow \quad P_2 - P_1 = h\rho(g + a_y)$$

Container Having Horizontal Acceleration

If container has acceleration a_x in the horizontal direction, then its free surface gets tilted as shown in Figure.



Let us apply Newton's Second Law on an imaginary cylinder of liquid of length l and area of cross-section A as shown in Figure.



$$\Rightarrow \quad (P_1 - P_2)A = \rho(Al)a_x$$

$$\Rightarrow \quad P_1 - P_2 = l\rho a_x$$

Now it can be seen very clearly that pressure on a horizontal level is not same throughout. So, we have

$$P_1 = h_1\rho g \text{ and } P_2 = h_2\rho g$$

$$\Rightarrow \quad P_1 - P_2 = (h_1 - h_2)\rho g = l\rho a_x$$

Also, from the Figure, we see that

$$\tan\theta = \frac{h_1 - h_2}{l}$$

$$\Rightarrow \quad \tan\theta = \frac{a_x}{g}$$

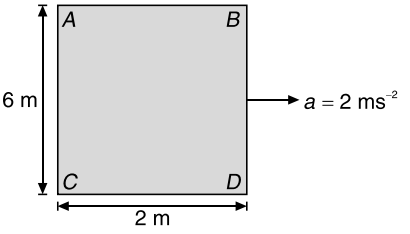
Problem Solving Technique(s)

If the beaker is accelerating both along x and y directions with respective accelerations a_x and a_y, then the liquid surface makes an angle θ with the horizontal given by

$$\tan\theta = \frac{a_x}{g_{\text{eff}}} = \frac{a_x}{g + a_y}$$

ILLUSTRATION 51

A closed container is filled with water as shown in figure. This container is accelerated in horizontal direction with an acceleration, $a = 2\ \text{ms}^{-2}$. Taking the density of water to be $1000\ \text{kgm}^{-3}$, calculate $P_C - P_D$ and $P_A - P_D$.



SOLUTION

Since the container is closed. So, we do not need to take the atmospheric pressure inside the container.
Along the horizontal direction, pressure decreases in the direction of acceleration, so $P_C > P_D$.

$$\Rightarrow \quad P_C - P_D = +\rho ax$$

Substituting the values, we get

$$P_C - P_D = (10^3)(2)(2)$$

$$\Rightarrow \quad P_C - P_D = 4.0 \times 10^3\ \text{Nm}^{-2}$$

In vertical direction, pressure increases with depth, so $P_C > P_A$.

$$\Rightarrow \quad P_A - P_C = -\rho gh = -(10^3)(10)(6)$$

$$\Rightarrow \quad P_A - P_C = -60 \times 10^3\ \text{Nm}^{-2}$$

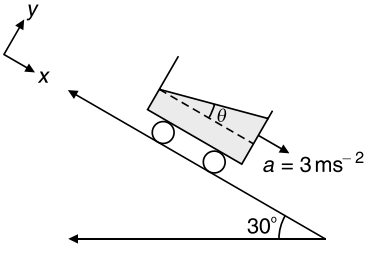
Now, $P_A - P_D = (P_A - P_C) + (P_C - P_D)$

$$\Rightarrow \quad P_A - P_D = (-60 \times 10^3) + (4.0 \times 10^3)$$

$$\Rightarrow \quad P_A - P_D = -56 \times 10^3\ \text{Nm}^{-2}$$

ILLUSTRATION 52

A rectangular container of water undergoes constant acceleration down an incline as shown. Determine the slope $\tan\theta$ of the free surface using the coordinate system shown. Take $g = 10\ \text{ms}^{-2}$.



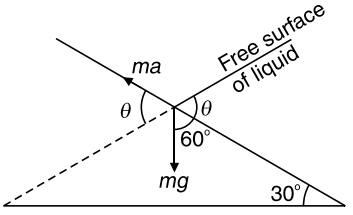
SOLUTION

No force acts along the fluid surface, so net force on a fluid particle of mass m at the surface of the liquid should be perpendicular to its surface when seen from accelerating frame of reference. Two forces are acting on the fluid particle are

(i) weight (mg), acting vertically downwards.

(ii) pseudo force (ma) along negative x-direction.

Since we know that the resultant of these two should be perpendicular to the free surface i.e. along the free surface the components of these two forces should cancel each other.



$$\Rightarrow \quad ma\cos\theta = mg\cos(60^\circ+\theta)$$

$$\Rightarrow \quad 3\cos\theta = 10(\cos 60^\circ\cos\theta - \sin 60^\circ\sin\theta)$$

$$\Rightarrow \quad 3\cos\theta = 5\cos\theta - 5\sqrt{3}\sin\theta$$

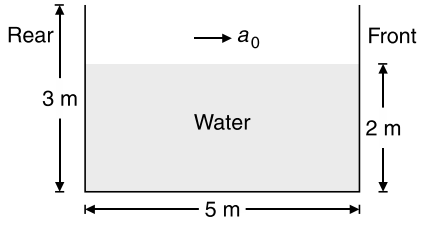
$$\Rightarrow \quad 5\sqrt{3}\sin\theta = 2\cos\theta$$

$$\Rightarrow \quad \tan\theta = \frac{2}{5\sqrt{3}} = 0.23$$

Therefore, the desired slope is 0.23.

ILLUSTRATION 53

An open rectangular tank $5\text{ m}\times 4\text{ m}\times 3\text{ m}$ high containing water up to a height of 2 m is accelerated horizontally along the longer side as shown in Figure.



Calculate the maximum acceleration that can be given without spilling the water. If this acceleration is increased by 20%, calculate the percentage of water spilt over.
If initially, the tank is closed at the top and is accelerated horizontally by 9 ms^{-2}, find the gauge pressure at the bottom of the front and rear walls of the tank. (Take $g = 10\text{ ms}^{-2}$)

SOLUTION

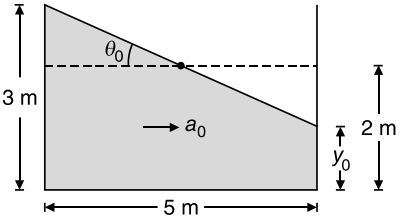
Volume of water inside the tank remains constant

$$\left(\frac{3+y_0}{2}\right)5\times 4 = 5\times 2\times 4$$

$$\Rightarrow \quad y_0 = 1\text{ m}$$

$$\Rightarrow \quad \tan\theta_0 = \frac{3-1}{5} = 0.4$$

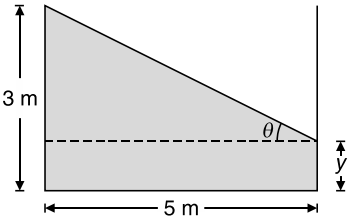
Since, $\tan\theta_0 = \frac{a_0}{g}$, therefore $a_0 = 0.4g = 4\text{ ms}^{-2}$



When acceleration is increased by 20%, then

$$a = 1.2a_0 = 0.48g$$

$$\Rightarrow \quad \tan\theta = \frac{a}{g} = 0.48$$



Now, $y = 2-5\tan\theta = 3-5(0.48) = 0.6\text{ m}$

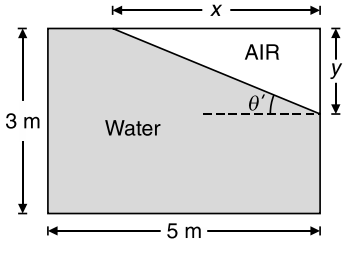
Fraction (f) of water spilt over is

$$f = \frac{(4)(2)(5)-\left(\frac{3+0.6}{2}\right)(5)(4)}{(2)(5)(4)} = 0.1$$

So, percentage of water spilt over is 10%

If $a' = 0.9g$, then we have

$$\tan\theta' = \frac{a'}{g} = 0.9 \qquad \ldots(1)$$



Volume of air remains constant

$$4\times\frac{1}{2}yx = (5)(1)\times 4$$

$$\Rightarrow \quad xy = 10 \qquad \ldots(2)$$

Since $y = x\tan\theta'$

$$\Rightarrow \quad xy = x^2\tan\theta'$$

So, from equation (2), we get

$$\Rightarrow \quad x^2\tan\theta' = 10$$

Using equations (1) and (2), we get

$$x = 3.33 \text{ m} \quad \text{and} \quad y = 3.0 \text{ m}$$

Hence, gauge pressure at the bottom of the

(i) Front wall is

$$p_f = \text{zero}$$

(ii) Rear wall is

$$p_r = (5\tan\theta')\rho_w g = 5(0.9)(10^3)(10)$$

$$\Rightarrow \quad p_r = 4.5\times10^4 \text{ Pa}$$

HORIZONTALLY ACCELERATING *U*-TUBE

When the U tube accelerates horizontally, difference of levels of liquid satisfies the relation,

$$\tan\theta = \frac{a}{g} = \frac{h}{L}$$

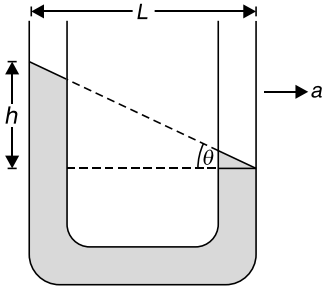
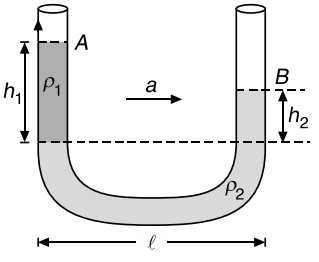


ILLUSTRATION 54

A U-tube containing two liquids of densities ρ_1 and ρ_2. The tube is given an acceleration a in the horizontal direction as shown in Figure.



Calculate the ratio of the densities of liquids.

SOLUTION

Let A and B be the two free liquid levels in the two limbs. Before we solve the problem, we must keep in mind that

1. As we go away from the free surface inside the liquid, pressure increases.
2. As we go towards the free surface inside the liquid, pressure decreases.
3. As we move in the direction of acceleration, the pressure decreases inside the liquid.

Let us start from point A and go to the point B through the tube (i.e. inside the liquid). If P_0 be the atmosphere pressure, then we have

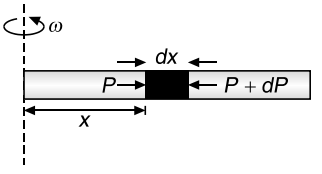
$$P_0 + h_1\rho_1 g - l\rho_2 a - h_2\rho_2 g = P_0$$

$$\Rightarrow \quad h_1\rho_1 g = l\rho_2 a + h_2\rho_2 g = \rho_2(la + h_2 g)$$

$$\Rightarrow \quad \frac{\rho_1}{\rho_2} = \frac{la + h_2 g}{h_1 g}$$

PRESSURE DIFFERENCE IN ROTATING FLUIDS

Consider a liquid of density ρ to be kept inside a tube of area of cross-section A. Let the tube be rotating with an angular velocity ω as shown in Figure.



Consider a small element of length dx at a distance x from the axis of rotation. Mass of this element is,

$$dm = \rho dV = \rho(Adx)$$

This element is rotating in a circle of radius x. So, this element must be accelerated towards centre with centripetal acceleration given by

$$a_c = x\omega^2 \qquad \{\because\ a_c = R\omega^2\}$$

For providing this acceleration, pressure on right hand side of the element should be more than that on the left-hand side, so we have

$$(P + dP)A - PA = (dm)a = (\rho Adx)(x\omega^2)$$

$$\Rightarrow \quad dP = (\rho x\omega^2)dx$$

$$\Rightarrow \quad \Delta P = \int_0^x (\rho\omega^2)x\,dx$$

$$\Rightarrow \quad \Delta P = \frac{\rho\omega^2 x^2}{2}$$

The pressure difference between two points at distances x_1 and $x_2\,(>x_1)$ will be

$$\Delta P = \frac{1}{2}\rho\omega^2(x_2^2 - x_1^2)$$

Conceptual Note(s)

For a rotating fluid (also accelerating) pressure increases in moving away from the rotational axis. At a distance x from the rotational axis, pressure difference is given by

$$\Delta P = \pm\frac{\rho\omega^2 x^2}{2}$$

When moving away from the axis of rotation, the pressure increases, so we take

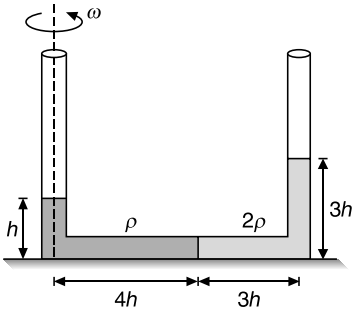
$$\Delta P = \frac{\rho\omega^2 x^2}{2}$$

When moving towards the axis of rotation, the pressure decreases, so we take

$$\Delta P = -\frac{\rho\omega^2 x^2}{2}$$

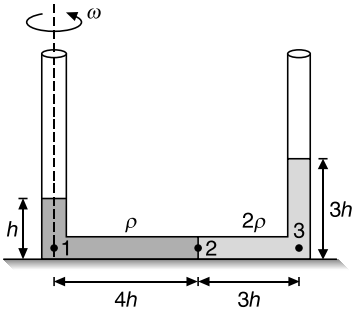
ILLUSTRATION 55

A vertical U-tube having two liquids of densities ρ and 2ρ is rotating with an angular speed ω about a vertical axis passing through its left limb such that the liquid columns in the limbs have heights h and $3h$ as shown in Figure. Calculate ω.



SOLUTION

Let P_1, P_2 and P_3 be the pressures at points 1, 2 and 3. The point 2 lies on the interface separating the two liquids as shown in Figure.



Since $\Delta P = \frac{\rho\omega^2}{2}\left(x_1^2 - x_2^2\right)$, so we have

$$P_2 - P_1 = \frac{1}{2}\rho\omega^2 (4h)^2 = 8\rho\omega^2 h^2 \quad \ldots(1)$$

Similarly,

$$P_3 - P_2 = \frac{1}{2}(2\rho)\omega^2\left[(7h)^2 - (4h)^2\right]$$

$$\Rightarrow \quad P_3 - P_2 = 33\rho\omega^2 h^2 \quad \ldots(2)$$

Also $P_1 = h\rho g$ $\ldots(3)$

and $P_3 = (3h)(2\rho)g = 6h\rho g$ $\ldots(4)$

Adding equations (1) and (2), we get

$$P_3 - P_1 = 41\rho\omega^2 h^2 \quad \ldots(5)$$

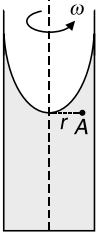
Substituting in equation (5) the value of P_1 from equation (3) and P_3 from equation (4), we get

$$5\rho g h = 41\rho\omega^2 h^2$$

$$\Rightarrow \quad \omega = \sqrt{\frac{5g}{41h}}$$

ILLUSTRATION 56

A liquid of density ρ is in a bucket that spins with angular velocity ω. Calculate the pressure at a point A at radial distance r from the axis as shown in Figure, if p_0 is the atmospheric pressure.

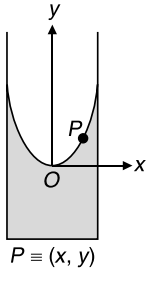


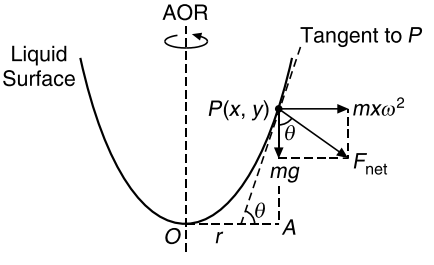
SOLUTION

Consider a fluid particle of mass m at the point $P(x, y)$. From a non-inertial rotating frame of reference two forces are acting on it,

(i) pseudo force $(mx\omega^2)$, radially outwards

(ii) weight (mg), vertically downwards





Net force F_{net} on the particle should be perpendicular to the free surface (in equilibrium). Hence,

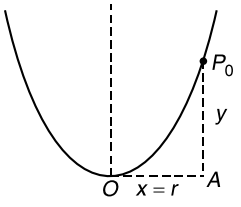
$$\tan\theta = \frac{mx\omega^2}{mg} = \frac{x\omega^2}{g}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x\omega^2}{g}$$

$$\Rightarrow \quad \int_0^y dy = \int_0^x \frac{x\omega^2}{g}\,dx$$

$$\Rightarrow \quad y = \frac{x^2\omega^2}{2g}$$

This is the equation of the free surface of the liquid, which is a parabola



At radial distance r, we have $x = r$

$$\Rightarrow \quad y = \frac{r^2\omega^2}{2g}$$

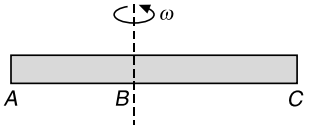
If P_A be the pressure in the liquid at the point A, then we have

$$\Rightarrow \quad P_A = P_0 + \rho g y$$

$$\Rightarrow \quad P_A = P_0 + \frac{\rho\omega^2 r^2}{2}$$

ILLUSTRATION 57

A closed tube of length 6 m filled with water $\left(\rho = 10^3\text{ kgm}^{-3}\right)$ is rotating with an angular velocity $\omega = 2\text{ rads}^{-1}$ about an axis perpendicular to the tube at 2 m from its one end as shown in Figure.



Calculate the pressure difference between the points A and C i.e. $P_A - P_C$.

SOLUTION

Since pressure decreases in moving towards the axis of rotation and increases in moving away from the axis i.e. $\Delta P = \pm\frac{\rho\omega^2 x^2}{2}$, so we must have $P_A > P_B$ and $P_B < P_C$.

Now, $P_A - P_C = (P_A - P_B) + (P_B - P_C)$

$$\Rightarrow \quad P_A = \left(+\frac{\rho\omega^2 x_1^2}{2}\right) + \left(\frac{-\rho\omega^2 x_2^2}{2}\right)$$

$$\Rightarrow \quad P_A = \frac{\rho\omega^2}{2}\left(x_1^2 - x_2^2\right)$$

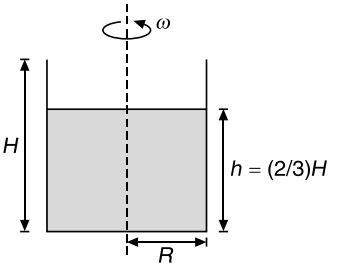
where, $x_1 = AB = 2\text{ m}$ and $x_2 = BC = 4\text{ m}$
Substituting the values, we get

$$P_A - P_C = \frac{(10^3)(2)^2}{2}\left[(2)^2 - (4)^2\right]$$

$$\Rightarrow \quad P_A - P_C = -2.4\times10^3\text{ Nm}^{-2}$$

ILLUSTRATION 58

A cylinder of radius $R = 1\text{ m}$ and height $H = 3\text{ m}$, is filled with water to two thirds its height. It is rotated about its vertical axis, as shown in Figure.



Calculate the angular speed of rotation for which the water just starts spilling over the rim. Also calculate the angular speed of rotation for which the centre of base of the cylinder is just exposed.

SOLUTION

According to the problem, we have

$$y_{\max} = H = 3\text{ m}$$

Also, $h = \frac{2}{3}H = 2\text{ m}$, $R = 1\text{ m}$ and $g = 10\text{ ms}^{-2}$

Since, the maximum height available to the liquid just before spilling is

$$y = H - h = \frac{r^2\omega^2}{2g}$$

$$3 - 2 = \frac{(1)^2\omega^2}{2(10)}$$

$$\Rightarrow \quad \omega = \sqrt{20}\text{ rads}^{-1}$$

For the centre of the base of the cylinder to be just exposed, we have

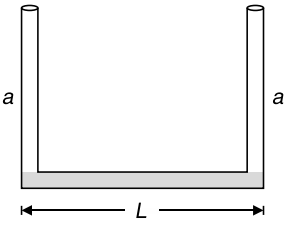
$$y = H = 3\text{ m}$$

$$\Rightarrow \quad y = H = \frac{r^2\omega^2}{2g}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{2gH}{R^2}} = \sqrt{\frac{2(10)(3)}{1^2}} = \sqrt{60}\text{ rads}^{-1}$$

ILLUSTRATION 59

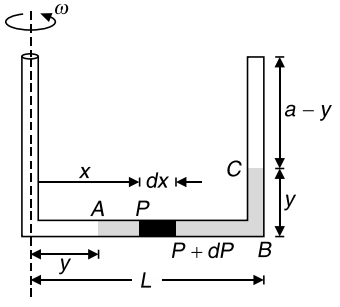
The length of horizontal arm of a U-tube is L and its each vertical arm has a length a. Both the vertical arm ends are open to atmospheric pressure P_0. A liquid of density ρ is poured in the tube such that liquid just fills the horizontal part of the tube as shown in Figure.



Now one of the open ends is sealed and the tube is rotated about the vertical axis passing through the other vertical arm with an angular speed ω_0. If liquid rises to a height y in the sealed end, then calculate pressure in the sealed tube during rotation.

SOLUTION

When tube is rotated, liquid starts to flow radially outward and air in sealed arm is compressed. Let the shift of liquid be y as shown in Figure.



Let the cross-sectional area of tube be S. The pressure difference between points A and B is obtained by integrating the pressure difference dP across an infinitesimal element of width dx, which is given as

$$dP = \rho\omega^2 x dx$$

Integrating from A to B, we get

$$P_B - P_A = \int_y^L \rho\omega^2 x dx$$

$$\Rightarrow \quad P_B - P_A = \frac{\rho\omega^2}{2}\left(L^2 - y^2\right) \qquad \ldots(1)$$

Since at point A, the pressure is atmospheric, so we have $P_A = P_{atm} = P_0$. So, from equation (1), we get

$$P_B = P_A + \frac{\rho\omega^2}{2}\left(L^2 - y^2\right) \qquad \ldots(2)$$

Also, pressure at point C is given by

$$P_C = P_B - y\rho g \qquad \ldots(3)$$

Substituting the value of P_B from equation (2) in equation (3), we get

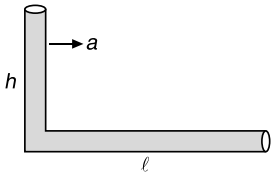
$$P_C = \frac{\rho\omega^2}{2}\left(L^2 - y^2\right) + \rho_0 - y\rho g$$

Test Your Concepts-VI

Based on Pressure in Accelerating Fluids

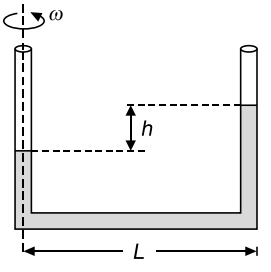
(Solutions on page H.10)

1. An L-shaped tube is filled with a liquid of density ρ as shown in Figure. Calculate the horizontal acceleration a to which the tube must be subjected towards right so that no liquid falls out of the tube.



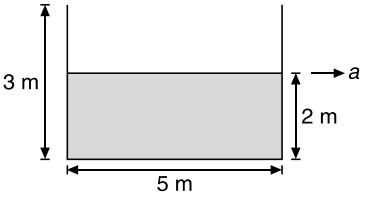
2. A cylindrical tank of radius 20 cm and height 50 cm has water up to 30 cm of height. Calculate the rise in level of liquid at the periphery if the cylinder if it is rotating with an angular velocity of 10 rad s^{-1} about the axis of the cylinder. Also find the frequency of rotation for which the water just starts spilling over sides of the vessel.

3. A U-tube of length L contains a liquid. It is rotated with angular velocity ω about an axis which is co-axial to one of its vertical limbs as shown in Figure.

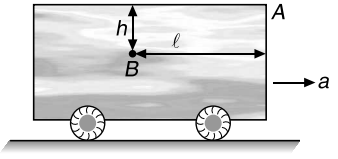


Calculate the difference in heights between the liquid columns in two vertical arms.

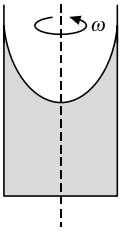
4. A container is accelerated horizontally with acceleration a as shown in Figure. Calculate the minimum value of a for which liquid just starts spilling out. Take $g = 10$ ms^{-2}



5. A closed tank filled with water is mounted on a cart. The cart moves with an acceleration a on a plane road. Calculate the difference in pressure between points B and A shown in Figure.



6. A liquid of density ρ is in a bucket that spins with angular velocity ω as shown in figure. Prove that the free surface of the liquid has a parabolic shape and also find its equation.



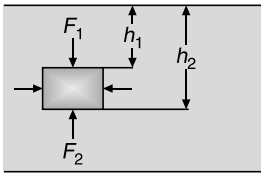
ARCHIMEDES' PRINCIPLE

Archimedes discovered that when a body is immersed partly or completely in a fluid, then the body experiences an upward force that equals the weight of fluid displaced by the immersed part of body. This principle is called Archimedes principle and is a necessary consequence of the Laws of Fluid Statics.

So, according to Archimedes', "*whenever a body is immersed partially or completely in a liquid (or fluid), then it experiences an upward force (called upthrust or buoyant force), which is equal to the weight of the liquid (or fluid) displaced by the immersed part of the body and this upward force or upthrust or buoyant force is equal to the loss in weight of the body*".

Upthrust acts at a point called as the **Centre of Buoyancy,** which is **actually the centre of gravity of the immersed part of the body.**

To determine the magnitude and direction of this force consider a body having volume V, density ρ to be immersed in a fluid of density σ as shown in Figure.



The forces on the vertical sides of the body will cancel each other. The top surface of the body which is at a depth h_1 below the free surface of the liquid will experience a downward force F_1 given by

$$F_1 = P_1 A = (h_1 \sigma g + P_0) A \qquad \{\because P_1 = h_1 \sigma g + P_0\}$$

The lower face of the body will experience an upward force F_2 given by

$$F_2 = P_2 A = (h_2 \sigma g + P_0) A \qquad \{\because P_2 = h_2 \sigma g + P_0\}$$

Since $h_2 > h_1$, so $F_2 > F_1$ and hence the body will experience a net upward force $(F_2 - F_1)$ vertically upwards and this force is called upthrust (U) or the buoyant force (F_B) acting on the body vertically upwards (opposite to the weight of the body).

$$U = F_B = F_2 - F_1 = A\sigma g(h_2 - h_1)$$

If h be the vertical height of the body, then

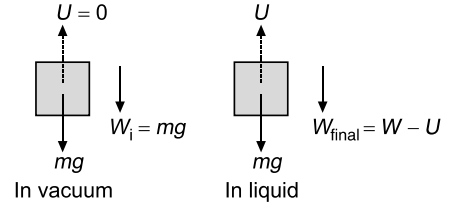
$$h = h_2 - h_1$$

Hence, upthrust is given by

$$U = F_B = (Ah)\sigma g = V\sigma g$$

where, $V = Ah$ is the volume of the liquid displaced by the immersed part of the body, $V\sigma$ is the mass of the liquid displaced and $V\sigma g$ is the weight of the liquid displaced by the immersed part of the body.

Also, we observe that if W_i be the initial weight of body and W_f be the final weight of the body, then



$$W_f = W_i - U$$

$$\Rightarrow \quad U = W_i - W_f = (\text{Loss in Weight of the Body})$$

$$\Rightarrow \quad U = F_B = (V_{\text{immersed}})(\rho_{\text{liquid}})g = W_{\text{initial}} - W_{\text{final}}$$

Please keep in mind that U (or F_B) acts through the centre of gravity of displaced fluid also called as Centre of Buoyancy.

Conceptual Note(s)

(a) Though we have derived this result for a body fully immersed in a fluid, however, this also holds good for bodies partially immersed in a liquid or a body immersed in more than one liquid.

(b) Upthrust is independent of all factors of the body such as its mass, size, density etc. except the volume of the body inside the fluid.

(c) Upthrust depends upon the nature of displaced fluid. This is why upthrust on a fully submerged body is more in sea water than in fresh water because density of sea water is more than fresh water.

(d) **Finding the Volume (V) of the Body:** If a body of volume V is immersed in a liquid of density σ then its weight reduces. If W_1 be the weight of the body in air and W_2 be the weight of the body in liquid, then loss in weight of the body is

$$W_1 - W_2 = V\sigma g$$

$$\Rightarrow V = \frac{W_1 - W_2}{\sigma g}$$

ARCHIMEDES PRINCIPLE: A GENERAL PROOF FOR AN ARBITRARY SHAPED BODY

A body immersed in a fluid experiences an upward buoyant force equivalent to the weight of the fluid displaced by it. To prove this, let us consider a body of any arbitrary shape completely immersed in a liquid of density ρ as shown in Figure (a). A body is being acted upon by the forces from all directions. Let us consider a vertical element of height h and cross-sectional area dA as shown in Figure (b).

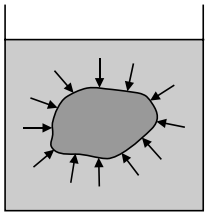


Figure (a)

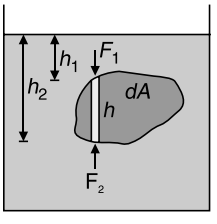


Figure (b)

The force acting on the upper surface of the element is F_1 (downward) and that on the lower surface is F_2 (upward). Since $F_2 > F_1$, therefore, the net upward force acting on the element is

$$dF = F_2 - F_1$$

It can be easily seen from Figure (b), that

$$F_1 = (\rho g h)\,dA \text{ and } F_2 = (\rho g h_2)\,dA$$

so, $dF = \rho g(h)\,dA$

Also, $h_2 - h_1 = h$ and $h(dA) = dV$

The net upward force is

$$F = \int \rho g\,dV = V\rho g$$

Hence, for the entire body, **the buoyant force is the weight of the volume of the fluid displaced.**

The buoyant force acts through the centroid of the displaced fluid.

Conceptual Note(s)

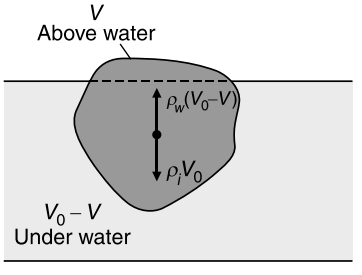
The buoyant force arises because the pressure in the fluid is not uniform because it increases with depth.

An object floats on water if it can displace a volume of water whose weight is greater than that of the object. If the density of the material is less than that of the liquid, it will float even if the material is a uniform solid, such as a block of wood floats on water surface.

If the density of the material is greater than that of water, such as iron, the object can be made to float provided it is not a uniform solid. An iron hulled ship is an example to this case.

ILLUSTRATION 60

An iceberg with a density of $920\ \text{kgm}^{-3}$ floats on an ocean of density $1025\ \text{kgm}^{-3}$. What fraction of the iceberg is visible?



SOLUTION

Let V be the volume of the iceberg above the water surface, then volume under water will be $V_0 - V$.

Under floating conditions, the weight $(\rho_i V_0 g)$ of the iceberg is balanced by the buoyant force $\rho_w(V_0 - V)g$. Thus,

$$\rho_i V_0 g = \rho_w (V_0 - V) g$$

$$\Rightarrow \rho_w V = (\rho_w - \rho_i) V_0$$

$$\Rightarrow \rho_w V = (\rho_w - \rho_i) V_0$$

$$\Rightarrow \quad \frac{V}{V_o} = \left(\frac{\rho_w - \rho_i}{\rho_w}\right)$$

Since, $\rho_w = 1025 \text{ kgm}^{-3}$ and $\rho_i = 920 \text{ kgm}^{-3}$

Therefore, $\frac{V}{V_0} = \frac{1025 - 920}{1025} = 0.10$

Hence 10% of the total volume is visible.

FINDING THE RELATIVE DENSITY (R.D.) OR SPECIFIC GRAVITY OF A BODY

Let a body of volume V and density $\rho(=\rho_b)$ be completely immersed in water of density $\sigma(=\rho_\ell)$, then upthrust acting on the body is equal to the loss in weight $(W_i - W_f)$ of the body. So,

$$U = W_i - W_f = V\sigma g = V\rho_w g \qquad \ldots(1)$$

Now, the weight of the body (W) is given by

$$W = V\rho g = V\rho_b g \qquad \ldots(2)$$

Dividing (2) by (1), we get

$$\frac{W}{U} = \frac{V\rho_b g}{V\rho_w g} = \frac{\rho_b}{\rho_w} = R.D.$$

So, relative density $(R.D.)$ is the ratio of the weight of the body in air (or vacuum) to the loss in weight of the body completely immersed in water.

$$(R.D.)_{\text{body}} = \frac{\text{Weight of Body in Air}}{\left(\begin{matrix}\text{Loss in Weight of Body}\\ \text{Fully Immersed in Water}\end{matrix}\right)}$$

So, if W_1 the weight of body in air and W_2 be the weight of body completely immersed in water, then

$$R.D. = \frac{W_1}{W_1 - W_2}$$

FINDING THE RELATIVE DENSITY (R.D.) OR SPECIFIC GRAVITY OF A LIQUID

If the loss in weight of a body completely immersed in water is "a" while the loss in weight of body completely immersed in a liquid is "b". If W is the weight of body in air, W_ℓ be the weight of body in liquid and W_w be the weight of body in water, then

$$W - W_\ell = a = V\rho_\ell g \qquad \ldots(1)$$

$$W - W_w = b = V\rho_w g \qquad \ldots(2)$$

Dividing (1) by (2), we get

$$\frac{W - W_\ell}{W - W_w} = \frac{a}{b} = \frac{\rho_\ell}{\rho_w} = (R.D.)_{\text{liquid}}$$

$$(R.D.)_{\text{liquid}} = \frac{\left(\begin{matrix}\text{Loss in Weight of Body}\\ \text{Fully Immersed in Liquid}\end{matrix}\right)}{\left(\begin{matrix}\text{Loss in Weight of Body}\\ \text{Fully Immersed in Water}\end{matrix}\right)}$$

ILLUSTRATION 61

When a 2.5 kg crown is immersed in water, it has an apparent weight of 22 N. What is the density of the crown?

SOLUTION

Let $W =$ actual weight of the crown
$W' =$ apparent weight of the crown
$\rho =$ density of crown
$\rho_0 =$ density of water

The buoyant force is given by

$$U = W - W'$$

$$\Rightarrow \quad \rho_0 V_g = W - W'$$

Since, $W = \rho V g$, therefore, $V = \frac{W}{\rho g}$

Eliminating V from the above two equations, we get

$$\rho = \frac{\rho_0 W}{W - W'}$$

Here $W = 25$ N; $W' = 22$ N; $\rho_0 = 10^3 \text{ kgm}^{-3}$

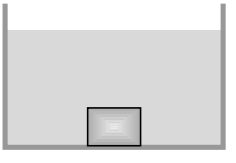
$$\Rightarrow \quad \rho = \frac{(10^3)(25)}{25 - 22} = 8.3 \times 10^3 \text{ kgm}^{-3}$$

LAWS OF FLOATATION

For the different situations that may arise we have summarised the Laws of Floatation in the following Cases.

CASE-1

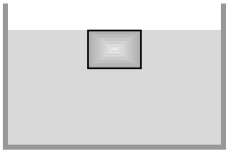
When $\rho_{\text{body}} > \rho_{\text{liquid}}$ i.e. $\rho > \sigma$, then $W_{\text{app}} > 0$ (or $W > U$) and hence the body will sink to the bottom of the beaker as shown in Figure.



CASE-2

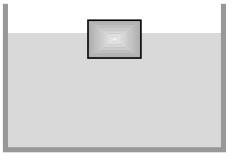
If, $\rho_{\text{body}} = \rho_{\text{liquid}}$ i.e. $\rho = \sigma$, then $W_{\text{app}} = 0$ OR $W = U$

So, the weight of the body will be equal to the upthrust. Hence the body will float fully submerged in neutral equilibrium anywhere in the liquid i.e. the body will just float or remain hanging at whatever height it is left inside the liquid. So, a small downward push to the body in this case will make it sink to the bottom of the beaker.



CASE-3

If, $\rho_{body} < \rho_{liquid}$, i.e. $\rho < \sigma$, then $W < U$ and so the Weight will be less than upthrust so the body will move upwards till the state of equilibrium is attained i.e. The body will rise partly out of the free surface of the liquid until the upward thrust acting on the body due to its immersed part in the liquid becomes equal to the total weight of the body W **(Condition for Floating)**. The body will then float with a fraction of its volume inside (or outside).



APPARENT WEIGHT OF A BODY IMMERSED IN LIQUID

For a body of true weight W, immersed in a liquid, the apparent weight (W' or W_{app}) or the effective weight of the is given by

$$W_{app} = W' = W - U$$

CASE-1: For a Floating Body

Since a body floats when the upthrust acting on the body balances the weight of the body. So, for a floating body, $W = U$ and hence its apparent weight is zero.

$$\begin{pmatrix}\text{Apparent Weight}\\ \text{of a Floating Body}\end{pmatrix} = \text{ZERO}$$

CASE-2: For a Body That Sinks In The Liquid

The body appears to have a lesser weight when immersed in a fluid. So apparent weight of the body is given by

$$W_{app} = W' = W - U$$

Suppose a body of volume V and density ρ is fully immersed in a liquid of density σ. Then

True weight of the body is $W = V\rho g$

Weight of the liquid displaced is $U = V\sigma g$

So, net downward force or apparent weight is

$$W_{app} = W' = (\rho - \sigma)gV$$

$$\Rightarrow \quad W_{app} = W' = \rho g V\left(1 - \frac{\sigma}{\rho}\right)$$

$$\Rightarrow \quad W_{app} = W' = W\left(1 - \frac{\sigma}{\rho}\right)$$

PRINCIPLE OF FLOATATION

For a body to float in a liquid, the weight of the liquid displaced by the immersed portion of the body (i.e. the upthrust) must be equal to its own weight.

FRACTIONAL VOLUME OF A FLOATING BODY INSIDE THE LIQUID

Suppose a body of volume V and density ρ_b floats in a liquid of density ρ_l and let V_{imm} be the volume of the body immersed in the liquid. Then, for the body to be in equilibrium,

$$\begin{pmatrix}\text{Upthrust acting due to}\\ \text{immersed part of the body}\end{pmatrix} = \begin{pmatrix}\text{Weight of}\\ \text{the body}\end{pmatrix}$$

$$\Rightarrow \quad V_{imm}\rho_l g = V\rho_b g$$

$$\Rightarrow \quad \frac{V_{imm}}{V} = \frac{\rho_b}{\rho_l}$$

If the liquid is water, then

$$\frac{V_{imm}}{V} = \text{Relative Density of the Body}$$

So, fraction of volume of body immersed in a liquid is

$$f_{immersed} = \frac{V_{imm}}{V} = \frac{\rho_b}{\rho_l}$$

So, fraction of volume of body immersed in a liquid is independent in the variation of g.

Further, fraction of volume of body outside liquid surface is

$$f_{outside} = 1 - f_{immersed} = 1 - \frac{\rho_b}{\rho_l}$$

Conceptual Note(s)

(a) Translatory Equilibrium of a Floating Body: When a body of density ρ and volume V is immersed in a liquid of density σ, the forces acting on the body are

(i) Weight of body $W = mg = V\rho g$, acting vertically downwards through centre of gravity of the body.

(ii) Upthrust force $= V\sigma g$ acting vertically upwards through the centre of gravity of the displaced liquid i.e., centre of buoyancy.

(b) The body will float in liquid only if

$$U \geq W \text{ OR } \sigma \geq \rho$$

(c) For a floating body

$$V\rho_{body} g = V_{imm}\rho_{liquid} g$$

(d) So, the equilibrium of a floating body is simply not affected by the variation(s) in the acceleration due to gravity g, though both thrust and weight depend on g. If a platform of mass M and cross-section A is floating in a liquid of density σ with its height h inside the liquid

$$Mg = hA\sigma g \quad ...(1)$$

Now if a body of mass m is placed on it and the platform sinks by y then

$$(M+m)g = (y+h)A\sigma g \quad ...(2)$$

Subtracting equation (1) and (2), we get

$$mg = A\sigma yg$$

$$\Rightarrow W \propto y \quad ...(3)$$

So, we can determine the weight of a body by placing it on a floating platform and noting the depression by which the platform further sinks in the liquid.

ILLUSTRATION 62

A block of wood floats in water with one third of its volume submerged. In oil, the block floats with two third of its volume submerged. Find the density of wood and oil if the density of water is 10^3 kgm^{-3}.

SOLUTION

For the case of a floating bod, we have

$$W = U$$

$$\Rightarrow \quad V\rho = V_{\text{in}}\sigma$$

$$\Rightarrow \quad V\rho = \frac{1}{3}V\sigma_w \qquad \left\{\because V_{\text{in}} = \frac{V}{3}\right\}$$

$$\Rightarrow \quad \rho = \frac{1}{3}\sigma_w = \frac{1}{3}\times 10^3 = 333 \text{ kgm}^{-3}$$

So, density of wood is $\rho_{\text{wood}} = 333$ kgm^{-3}

For oil, we have

$$V\rho = \frac{2}{3}V\sigma_{\text{oil}}$$

$$\Rightarrow \quad \frac{2}{3}V\sigma_{\text{oil}} = \frac{1}{3}V\sigma_w$$

$$\Rightarrow \quad \sigma_{\text{oil}} = \frac{1}{2}\sigma_w = \frac{10^3}{2} = 500 \text{ kgm}^{-3}$$

$$\Rightarrow \quad \sigma_{\text{oil}} = 500 \text{ kgm}^{-3}$$

ILLUSTRATION 63

A cubical block of iron 5 cm on each side is floating on mercury in a vessel. Calculate the height of the block above mercury level. If water is poured in the vessel until it just covers the iron block, then calculate the height of water column assuming that $\rho_{Hg} = 13.6$ gcm^{-3} and $\rho_{Fe} = 7.2$ kgm^{-3}.

SOLUTION

CASE-1:

Suppose h be the height of cubical block of iron above mercury.

Volume of iron block is $V_{Fe} = 5\times 5\times 5 = 125$ cm^3

Mass of iron block is $m_{Fe} = 125\times 7.2 = 900$ g

Volume of mercury displaced by the block is

$$V_{Hg} = 5\times 5\times (5-h) \text{ cm}^3$$

Mass of mercury displaced by the block is

$$m_{Hg} = 5\times 5(5-h)\times 13.6 \text{ gram}$$

Since upthrust U equals the weight of liquid displaced by the immersed part of the body, so we have

$$U = (m_{Hg})g$$

According to the Laws of Floatation, this upthrust balance the weight of the iron block, so we have

$$U = W_{Fe}$$

$$\Rightarrow \quad 5\times 5(5-h)\times 13.6 = 900$$

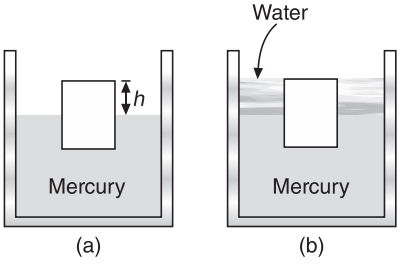
$$\Rightarrow \quad (5-h) = \frac{900}{25\times 13.6} = 2.65$$

$$\Rightarrow \quad h = 5 - 2.65 = 2.35 \text{ cm}$$

CASE-2:

Suppose in this case the height of iron block in water be x, so the height of iron block in mercury will be $(5-x)$ cm.

Mass of the water displaced is



$$m_{\text{water}} = 5\times 5\times (x)\times 1$$

Mass of mercury displaced is

$$m_{Hg} = 5\times 5\times (5-x)\times 13.6$$

For the iron block to float, we must have

$$U_{\text{water}} + U_{Hg} = W_{Fe}$$

$$\Rightarrow \quad (m_{\text{water}})g + (m_{Hg})g = (m_{Fe})g$$

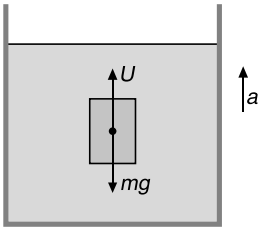
$$\Rightarrow \quad (1)(5)^2 x + (13.6)(5)^2 (5-x) = 900$$

$$\Rightarrow \quad x + (5-x)\times 13.6 = 36$$

$$\Rightarrow \quad x = 2.54 \text{ cm}$$

BUOYANT FORCE IN ACCELERATING FLUIDS

Consider a body to be submerged in a liquid of density ρ. Let the liquid be accelerating up with an acceleration a as shown in Figure.



For the block, we have

$$U - mg = ma$$

$$\Rightarrow \quad U = m(g + a)$$

where m is the mass of the displaced liquid, i.e.

$$m = V\rho$$

$$\Rightarrow \quad U = \rho V(g + a)$$

Conceptual Note(s)

(a) While writing the expression for the buoyant force for an accelerated fluid, instead of taking gravity g we take the effective value of gravity i.e. $g_{\text{eff}} = g \pm a$. So, we write

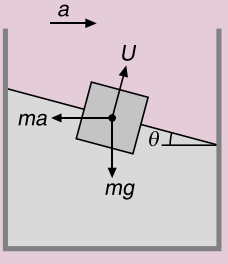
$$F_{\text{buoyant}} = U = V_{\text{immersed}}\rho_{\text{liquid}}g_{\text{eff}}$$

where, $V_{\text{immersed}} = V_{\text{displaced}}$

(b) When the floating object is accelerating in vertical direction, then $g_{eff} = g + a$, if the arrangement is accelerating up or retarding down.

(c) When the floating object is accelerating in vertical direction, then $g_{eff} = g - a$, if the arrangement is accelerating down or retarding up.

(d) When an object is floating in liquid filled in a container and the arrangement is accelerating in horizontal direction, then the liquid surface in the container will be inclined at angle θ with horizontal where, $\tan\theta = \frac{a}{g}$. Since upthrust or the buoyant force acts perpendicular to equi-pressure surface hence, in this case buoyant force will not be vertical but it will act normal to free surface of the liquid as shown in Figure.



The upthrust or the buoyant force is given by

$$U = V_{\text{imm}}\rho_{\text{liq}}g_{\text{eff}} = V_{\text{imm}}\rho_{\text{liq}}\sqrt{g^2 + a^2}$$

ILLUSTRATION 64

A cube of length L, density σ is floating in a beaker filled with water having density ρ_w. Calculate the change in submerged length of cube in water if the arrangement is accelerating in the horizontal direction with an acceleration a.

SOLUTION

Let l be the submerged length of the cube in water when the beaker is at rest, then by applying the Laws of Floatation, we get

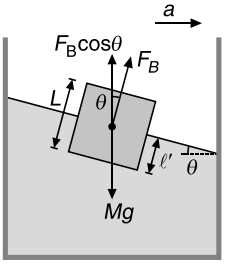
$$Mg = V_{\text{immmersed}}\rho_w g \qquad \ldots(1)$$

where, $M = (AL)\sigma$ and $V_{\text{immersed}} = Al$
So, from equation (1), we get

$$AL\sigma = Al\rho_w$$

$$\Rightarrow \quad l = L\left(\frac{\sigma}{\rho_w}\right)$$

When the beaker is accelerating, then let the new length of the cube immersed in the liquid be l'.



When the arrangement accelerates, then the free surface of the liquid in the beaker makes an angle θ

$$\tan\theta = \frac{a}{g}$$

$$\Rightarrow \quad \cos\theta = \frac{g}{\sqrt{g^2 + a^2}}$$

Since the upthrust $F_B = U$ acts normally to the free surface of the liquid, so we have

$$F_B\cos\theta = Mg$$

$$(Al')\rho_w g_{\text{eff}}\cos\theta = (AL\sigma)g$$

$$\Rightarrow \quad Al'\rho_w \cancel{\sqrt{g^2 + a^2}}\left(\frac{g}{\cancel{\sqrt{g^2 + a^2}}}\right) = AL\sigma g$$

$$\Rightarrow \quad l'\rho_w = L\sigma$$

$$\Rightarrow \quad l' = L\left(\frac{\sigma}{\rho_w}\right) = l$$

So, there is no change in the submerged length of the cube in water when the arrangement is accelerated horizontally.

STABILITY OF A FLOATING BODY

The stability of a floating body depends on the effective point of application of the buoyant force. The weight of the body acts at its centre of gravity. *The buoyant force acts at the centre of gravity of the displaced liquid. This is called the* ***centre of buoyancy***. Under equilibrium condition, the centre of gravity G and the centre of buoyancy B lie along the vertical axis of the body as shown in Figure (a).

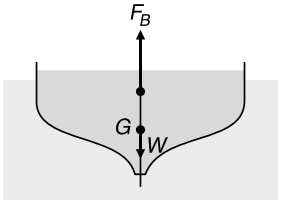


Figure (a)

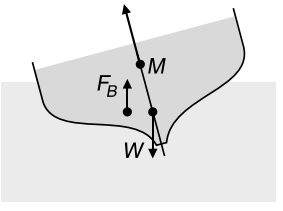


Figure (b)

When the body tilts to one side, the centre of buoyancy shifts relative to the centre of gravity as shown in Figure (b). The two forces act along different vertical lines. As a result, the buoyant force exerts a torque about the centre of gravity. The line of action of the buoyant force crosses the axis of the body at the point M, called the **metacentre**. If G is below M, the torque will tend to restore the body to its equilibrium position.

If G is above M, the torque will tend to rotate the body away from its equilibrium position and the body will be unstable.

ILLUSTRATION 65

A rubber ball of mass m and radius r is submerged in water to a depth h and released. Calculate the height to which the ball will jump above the surface of the water. Neglect resistance of water and air and assume density of water to be ρ.

SOLUTION

Let the ball rise to a height H above water. According to Modified Work Energy Theorem, we have

$$W_{ext} = \Delta U + \Delta K$$

For both the initial and the final positions of the ball, kinetic energy is zero, so $\Delta K = 0$. Also,

$$W_{ext} = Uh\cos 0° = Uh = \frac{4}{3}\pi r^3 \rho g h$$

Assuming Zero Potential Energy Level (ZPEL) at the free surface of water, then

$$\Delta U = mgH - (-mgh) = mg(H + h)$$

$$\Rightarrow \quad \left(\frac{4}{3}\pi r^3 \rho g\right)h = mg(H + h)$$

$$\Rightarrow \quad H = \left(\frac{\frac{4}{3}\pi r^3 \rho - m}{m}\right)h$$

ILLUSTRATION 66

A cube of wood supporting a $200\ \text{g}$ mass just floats in water. When the mass is removed, the cube rises by $2\ \text{cm}$. Calculate the side length of the cube.

SOLUTION

Initially, we have

$$(m_{\text{cube}} + 200)g = U_{\text{initial}} \qquad \ldots(1)$$

Since the cube of wood supporting a $200\ \text{g}$ mass just floats in water, so we have

$$U_{\text{initial}} = l^3 \rho_{\text{water}} g \qquad \ldots(2)$$

From equations (1) and (2), we get

$$m_{\text{cube}} + 200 = l^3 \rho_{\text{water}}$$

$$\Rightarrow \quad l^3 \rho_{\text{wood}} + 200 = l^3 \rho_{\text{water}} \qquad \ldots(3)$$

On removing the block, cube moves up by $2\ \text{cm}$, so we have

$$W = U_{\text{final}}$$

$$\Rightarrow \quad (l^3 \rho_{\text{wood}})g = l^2(l-2)\rho_{\text{water}} g$$

$$\Rightarrow \quad l^3 \rho_{\text{wood}} = l^2(l-2)\rho_{\text{water}} \qquad \ldots(4)$$

Substituting (4) in (3), we get

$$l^2(l-2)\rho_{\text{water}} + 200 = l^3 \rho_{\text{water}}$$

Since, $\rho_{\text{water}} = 1\ \text{gcc}^{-1}$

$$\Rightarrow \quad l^2(l-2) + 200 = l^3$$

$$\Rightarrow \quad l^3 - 2l^2 + 200 = l^3$$

$$\Rightarrow \quad -2l^2 + 200 = 0$$

$$\Rightarrow \quad 2l^2 = 200$$

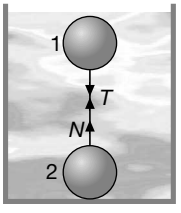
$$\Rightarrow \quad l = 10\ \text{cm}$$

ILLUSTRATION 67

Two solid uniform spheres each of radius $5\ \text{cm}$ are connected by a light string and totally immersed in a tank of water. If the specific gravities of the spheres are 0.5 and 2, calculate the tension in the string and the contact force between the bottom of the tank and the heavier sphere.

SOLUTION

Let the volume of each sphere be V m^3 and density of water be ρ_w kgm^{-3}. The situation given in the problem is shown in Figure.



Upward thrust acting on heavier sphere 2 is

$$U_2 = V\rho g$$

Weight of the heavier sphere 2 is

$$W_2 = V(2\rho_w)g$$

For the heavier sphere 2, we have

$$T + N + V\rho_w g = V(2\rho_w)g \quad \ldots(1)$$

where, N is the contact force between the bottom of the tank and the heavier sphere.

Similarly, for lighter sphere 1, we have

$$T + W_1 = U_1$$

$$\Rightarrow \quad T + V(0.5\rho_w)\rho g = V\rho_w g \quad \ldots(2)$$

$$\Rightarrow \quad T = 0.5V\rho_w g$$

$$\Rightarrow \quad T = (0.5)\left(\frac{4}{3}\times 3.14\times 5^3\times 10^{-6}\right)(1000)(9.8)$$

$$\Rightarrow \quad T = 2.56 \text{ newton}$$

Also, from equation (1), we get

$$0.5V\rho_w g + N = V\rho_w g$$

$$\Rightarrow \quad N = 0.5V\rho_w g$$

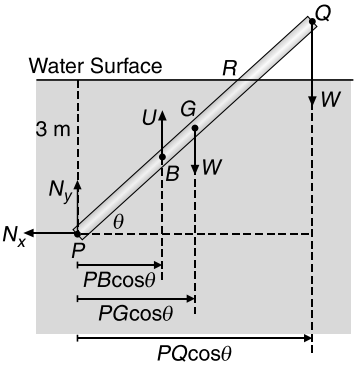
$$\Rightarrow \quad N = 2.56 \text{ newton}$$

ILLUSTRATION 68

A rod of length 6 m has a mass of 12 kg. If it is hinged at one end at a distance of 3 m below a water surface, calculate the weight which must be attached to other end of the rod so that 5 m of the rod is submerged. Also calculate the magnitude and direction of the force exerted by the hinge on the rod. The specific gravity of the material of the rod is 0.5.

SOLUTION

Let PR be the submerged part of the rod PQ hinged at P as shown in Figure.



If G is the centre of gravity of the rod and B is the centre of buoyancy through which force of buoyancy $F_B = U$ acts vertically upwards. Since the rod is uniform, so the mass of the immersed part PR of the rod will be

$$m_{PR} = \frac{5}{6}\times 12 = 10 \text{ kg}$$

The buoyant force on rod at B is

$$F_B = U = V_{\text{imm}}\rho_{\text{liq}}g = \left(\frac{10}{0.5\rho_w}\right)\rho_w g = 20 \text{ kgwt}$$

Let w be the weight attached at the end Q of the rod, then for the rotational equilibrium of the rod, the torques about the point P due to various forces are balanced. So, we have

$$\Sigma\tau_{\text{about } P} = 0$$

$$\Rightarrow \quad W(PG\cos\theta) + w(PQ\cos\theta) = U(PB\cos\theta)$$

$$\Rightarrow \quad W(PG) + w(PQ) = U(PB), \text{ where}$$

$$PG = 3 \text{ m}, \; PQ = 6 \text{ m} \text{ and } PB = \frac{PR}{2} = 2.5 \text{ m}$$

$$\Rightarrow \quad 12\times 3 + w\times 6 = 20\times(2.5)$$

$$\Rightarrow \quad w = 2.33 \text{ kgwt}$$

Let N_x and N_y be the x and y components of the force exerted by hinge on the rod. Since the rod is in equilibrium, so force acting on it is zero. Hence, we have $\Sigma F_x = 0$ and $\Sigma F_y = 0$. As no force acts along the x direction, so we get

$$N_x = 0$$

Also, we have

$$W + w = U + N_y$$

$$\Rightarrow \quad N_y = W + w - U$$

$$\Rightarrow \quad N_y = 12 + 2.33 - 20$$

$$\Rightarrow \quad N_y = -5.67 \text{ kgwt}$$

The negative indicates that the reaction at the hinge is acting in the downward direction and has a magnitude of 5.67 kgwt.

ILLUSTRATION 69

A piece of ice is floating in a glass vessel filled with water. How will the level of water in the vessel change when the ice melts?

SOLUTION

Let m be the mass of ice piece floating in water. In equilibrium (i.e. initially)

$$(\text{Weight of Ice + Stone}) = \text{Upthrust}$$

$$\Rightarrow \quad mg = V_i \rho_w g$$

$$\Rightarrow \quad V_i = \frac{m}{\rho_w} \qquad \ldots(1)$$

where, V_i is the volume of ice piece immersed in water or the volume of water displaced by the immersed part of ice in water.
When the ice melts (i.e. finally), let V be the volume of water formed by m mass of ice. Then

$$V = \frac{m}{\rho_w} \qquad \ldots(2)$$

From equations (1) and (2), we get

$$V_i = V$$

Hence, the level of water in the vessel will not change.

ILLUSTRATION 70

A piece of ice having a stone frozen in it floats in a glass vessel filled with water. How will the level of water in the vessel change when the ice melts?

SOLUTION

Let, m_1 be the mass of ice, m_2 be the mass of stone, ρ_S be the density of stone and ρ_w be the density of water.
In equilibrium (i.e. initially), when the piece of ice having stone in it floats in water, then we have

$$(\text{Weight of Ice + Stone}) = \text{Upthrust}$$

$$\Rightarrow \quad (m_1 + m_2) g = V_i \rho_w g$$

$$\Rightarrow \quad V_i = \frac{m_1}{\rho_w} + \frac{m_2}{\rho_w} \qquad \ldots(1)$$

where, V_i is the volume of ice immersed in water or the volume of water initially displaced by ice stone combination while floating.
When the ice melts (i.e. finally), the mass m_1 of ice converts into water and stone of mass m_2 get sunk completely in water.
The volume of water formed by m_1 mass of ice is

$$V_1 = \frac{m_1}{\rho_w}$$

The volume of stone is also equal to the volume of water displaced by the stone when it sinks in water. So, we have

$$V_2 = \frac{m_2}{\rho_S}$$

Since, $\rho_S > \rho_w$

$$\Rightarrow \quad V_1 + V_2 < V_i$$

Hence the level of water will decrease.

ILLUSTRATION 71

A solid frozen body is floating in a liquid of different material contained in a beaker. Carry out an analysis to find whether the level of liquid in the beaker will rise or fall when the solid melts.

SOLUTION

Let M be the mass of the solid frozen body floating in the liquid, ρ_1 be the density of liquid formed by the melting of the solid, ρ_2 be the density of the liquid in which the solid is floating.
The mass of liquid displaced by the solid is M and hence the volume of liquid displaced by the body is $\frac{M}{\rho_2}$. When the solid melts, the volume occupied by it is $\frac{M}{\rho_1}$.

So, level of liquid in container will rise, when

$$\frac{M}{\rho_1} > \frac{M}{\rho_2} \text{ i.e., } \rho_1 < \rho_2$$

Level of liquid in container will fall, when

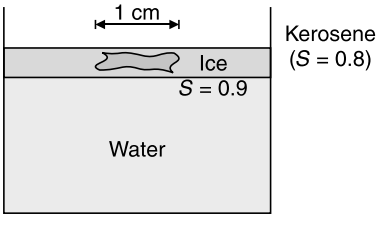
$$\frac{M}{\rho_2} > \frac{M}{\rho_1} \text{ i.e., } \rho_1 > \rho_2$$

Level of liquid in container will remain same, when

$$\frac{M}{\rho_2} = \frac{M}{\rho_1} \text{ i.e., } \rho_1 = \rho_2$$

ILLUSTRATION 72

An ice cube of side $1\ \text{cm}$ is floating at the interface of kerosene and water in a beaker of base area $10\ \text{cm}^2$. The level of kerosene is just covering the top surface of the ice cube. Calculate the depth of submergence in the kerosene and that in the water. Also calculate change in total level of the liquid when the whole ice melts into water.



SOLUTION

Condition of floating

$$0.8\ \rho_w g h_k + \rho_w g h_w = 0.9\ \rho_w g h$$

$$\Rightarrow \quad 0.8 h_k + h_w = (0.9) h \qquad \ldots(1)$$

where h_k and h_w be the submerged depth of the ice in the kerosene and water, respectively.

Also, $h_k + h_w = h$ $\ldots(2)$

Solving equations (1) and (2), we get

$$h_k = 0.5 \text{ cm}, \ h_w = 0.5 \text{ cm}$$

Also, when $1\ \text{cm}^3$ of ice melts, then we get $0.9\ \text{cm}^3$ of water i.e.

$$1\ \underset{(\text{Ice})}{\text{cm}^3} \xrightarrow{\text{melts}} 0.9\ \underset{(\text{Water})}{\text{cm}^3}$$

Fall in the level of kerosene is

$$\Delta h_k = \frac{0.5}{A}$$

Rise in the level of water is

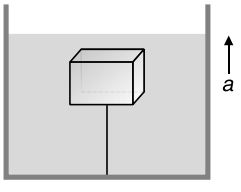
$$\Delta h_w = \frac{0.9 - 0.5}{A} = \frac{0.4}{A}$$

So, net fall in the overall level is

$$\Delta h = \frac{0.1}{A} = \frac{0.1}{10} = 0.01 \text{ cm} = 0.1 \text{ mm}$$

ILLUSTRATION 73

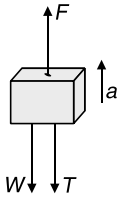
A block of mass 1 kg and density $0.8\ \text{gcm}^{-3}$ is held stationary with the help of a string as shown in Figure.



The tank is accelerating vertically upwards with an acceleration $a = 1.0\ \text{ms}^{-2}$. Calculate the tension in the string. Now, if the string is cut, then calculate acceleration of the block if $g = 10\ \text{ms}^{-2}$ and density of water is $10^3\ \text{kgm}^{-3}$.

SOLUTION

The free body diagram of the block is shown in Figure.



If F_B is the upthrust force, then we have

$$F_B = V \rho_w (g + a)$$

$$\Rightarrow \quad F_B = \left(\frac{\text{mass of block}}{\text{density of block}} \right) \rho_\omega (g + a)$$

$$\Rightarrow \quad F_B = \left(\frac{1}{800} \right) (1000)(10 + 1) = 13.75 \text{ N}$$

Also, the weight of the block is

$$W = mg = 10 \text{ N}$$

So, equation of motion of the block is,

$$F_B - T - W = ma$$

$$\Rightarrow \quad 13.75 - T - 10 = 1 \times 1$$

$$\Rightarrow \quad T = 2.75 \text{ N}$$

When the string is cut, then we have

$$T = 0$$

$$\Rightarrow \quad a = \frac{F_B - W}{m}$$

$$\Rightarrow \quad a = \frac{13.75 - 10}{1}$$

$$\Rightarrow \quad a = 3.75\ \text{ms}^{-2}$$

ILLUSTRATION 74

An iron casting containing a number of cavities weighs 6000 N in air and 4000 N in water. Calculate the volume of the cavities in the casting if the density of iron is $7.87\ \text{gcm}^{-3}$, $g = 9.8\ \text{ms}^{-2}$ and the density of water is $10^3\ \text{kgm}^{-3}$.

SOLUTION

If v be the volume of cavities and V the volume of solid iron, then we have

$$V = \frac{\text{Mass}}{\text{Density}} = \left(\frac{6000/9.8}{7.87 \times 10^3} \right) = 0.078\ \text{m}^3$$

Further, we know that the loss in weight of the body equals the upthrust acting on the body, so we have

$$(6000 - 4000) = (V + v) \rho_w g$$

$$\Rightarrow \quad 2000 = (0.078 + v) \times 10^3 \times 9.8$$

$$\Rightarrow \quad 0.078 + v \approx 0.2$$

$$\Rightarrow \quad v = 0.12\ \text{m}^3$$

ILLUSTRATION 75

An ornament weighing 50 g in air weighs only 46 g is water. Assuming that some copper is mixed with gold to prepare the ornament. Find the amount of copper in it. Specific gravity of gold is 20 and that of copper is 10.

SOLUTION

Let m be the mass of the copper in ornament. Then mass of gold in it is $(50 - m)$. Since,

$$\text{Volume} = \frac{\text{Mass}}{\text{Density}}$$

So, volume of copper is $V_1 = \frac{m}{10}$

and volume of gold is $V_2 = \frac{50-m}{20}$

When immersed in water $\left(\rho_w = 1\ \text{gcm}^{-3}\right)$, we know that the loss in weight of the body equals the upthrust acting on it, so we have

$$(50-46)g = (V_1+V_2)\rho_w g$$

$$\Rightarrow \quad 4 = \frac{m}{10}+\frac{50-m}{20}$$

$$\Rightarrow \quad 80 = 2m+50-m$$

$$\Rightarrow \quad m = 30\ \text{g}$$

ILLUSTRATION 76

A hollow sphere of inner radius $9\ \text{cm}$ and outer radius $10\ \text{cm}$ floats half-submerged in a liquid of specific gravity 0.8. Calculate the density of the material of which the sphere is made. What would be the density of a liquid in which the hollow sphere would just float completely submerged?

SOLUTION

For equilibrium of sphere, the upthrust acting on the body due to the immersed part must balance the weight of the body, so we have

$$\frac{1}{2}\left[\frac{4}{3}\pi(10)^3\right](0.8) = \frac{4}{3}\pi\left((10)^3-(9)^3\right)\rho$$

$$\Rightarrow \quad \rho = \frac{(0.4)(1000)}{1000-729} = \frac{400}{271} = 1.476\ \text{gcm}^{-3}$$

If ρ_L be the density of liquid in which sphere will just float then

$$\frac{4}{3}\pi(10)^3\rho_L = \frac{4}{3}\pi(10)^3\times\frac{1}{2}\times 0.8$$

$$\Rightarrow \quad \rho_L = 0.4\ \text{gcm}^{-3}$$

ILLUSTRATION 77

A piece of gold sample weighs $36\ \text{g}$ in air and $34\ \text{g}$ in water. If this sample has some copper mixed in it, then calculate the amount of copper in the sample if specific gravity of gold is 19.3 and that of copper is 8.9. Now assume that the same mass of sample is made of pure gold but with air cavities inside it and it weighs the same in water, then calculate the volume of the air cavities in the sample.

SOLUTION

Let the sample contain $x\ \text{g}$ of copper, then the amount of gold in this sample is $(36-x)\ \text{g}$. Since we know that the loss in weight of the sample equals the upthrust acting on it, so we have

$$36-34 = \left(\frac{x}{\rho_{Cu}}+\frac{36-x}{\rho_{\text{gold}}}\right)\rho_\omega$$

$$\Rightarrow \quad \frac{x}{8.9}+\frac{36-x}{19.3} = 2$$

$$\Rightarrow \quad 19.3x+8.9\times 36-8.9x = 2\times 8.9\times 19.3$$

$$\Rightarrow \quad 10.4x = 343.54-320.4 = 23.14$$

$$\Rightarrow \quad x = 2.225\ \text{g}$$

If V_c is volume of air cavities inside $36\ \text{g}$ of pure gold sample, then

$$2 = \left(V_c+\frac{36}{\rho_{\text{gold}}}\right)\rho_w$$

$$\Rightarrow \quad V_c = \frac{2}{\rho_w}-\frac{36}{\rho_{\text{gold}}}$$

$$\Rightarrow \quad V_c = 2-\frac{36}{19.3} = 0.135\ \text{cm}^3$$

ILLUSTRATION 78

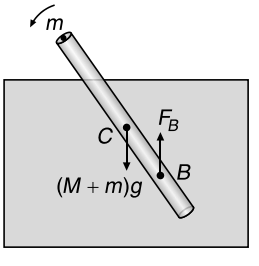
A wooden stick of length L, radius R and density ρ has a small metal piece of mass m (of negligible volume) attached to its one end. Find the minimum value for the mass m (in terms of given parameters) that would make the stick float vertically in equilibrium in a liquid of density $\sigma(>\rho)$.

SOLUTION

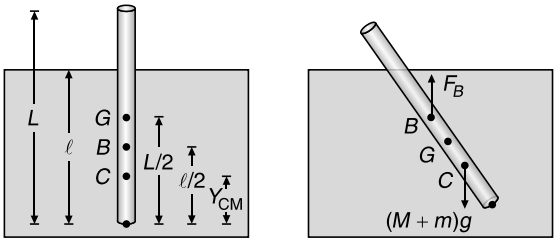
Let M be the mass of the stick, then we have

$$M = \pi R^2\rho L$$

If ℓ be the immersed length of the rod, G be the centre of mass (CM) of rod, B be the centre of buoyant force (F_B), C be the centre of mass of rod plus metal piece system and Y_{CM} be the distance of C from bottom of the rod.
If the metal piece is attached to the top end of the rod, then the centre of buoyancy B will be below the centre of mass C of the rod and combined centre of mass C of rod plus metal piece will be above the centre of mass G of the rod as shown in Figure.



If this is the case, then the torque of the couple of two equal and opposite forces F and $(M+m)g$ will be counter clockwise on displacing the rod leftwards and hence the rod cannot be in rotational equilibrium when m is attached to the upper end of the rod.



When the mass m is attached to the lower end of the rod, then for translational equilibrium of the system, we have

$$F_B = Mg + mg$$

$$\Rightarrow \quad \left(\pi R^2 l\right)\sigma g = \left(\pi R^2 L\right)\rho g + mg$$

$$\Rightarrow \quad l = \left(\frac{\pi R^2 L\rho + m}{\pi R^2 \sigma}\right) \qquad \ldots(1)$$

The position of CM of rod plus metal piece from the bottom of the rod is given by

$$Y_{CM} = \frac{M\left(\frac{L}{2}\right)}{M+m} = \frac{\left(\pi r^2 L\rho\right)\left(\frac{L}{2}\right)}{\left(\pi R^2 L\rho\right)+m} \qquad \ldots(2)$$

Since the centre of buoyancy (B) is at a height $\frac{l}{2}$ from the bottom of the rod, so for rotational equilibrium of the rod, B should either lie above C or at the same level of C. Therefore, we have

$$\frac{l}{2} \geq Y_{COM}$$

$$\Rightarrow \quad \frac{\pi R^2 L\rho + m}{2\pi R^2 \sigma} \geq \frac{\left(\pi R^2 L\rho\right)\frac{L}{2}}{\left(\pi R^2 L\rho\right)+m}$$

$$\Rightarrow \quad m + \pi R^2 L\rho \geq \pi R^2 L\sqrt{\rho\sigma}$$

$$\Rightarrow \quad m \geq \pi R^2 L\left(\sqrt{\rho\sigma} - \rho\right)$$

So, minimum value of m is $\pi R^2 L\left(\sqrt{\rho\sigma} - \rho\right)$.

ILLUSTRATION 79

A flat bottomed thin-walled glass tube has a diameter of 4 cm and it weighs 30 g. The centre of gravity of the empty tube is 10 cm above the bottom. Calculate the amount of water that must be poured into the tube so that when it is floating vertically in a tank of water, the centre of mass of the tube and its contents lies at the midpoint of the immersed length of the tube.

SOLUTION

Assuming that water is filled to a height h and l is length of tube submerged in water, then upthrust acting on the system balances the weight of the system. So, we have

$$\left(l\pi r^2\right)\rho_w g = 30g + \left(\pi r^2 h\right)\rho_w g$$

$$\Rightarrow \quad \left(l\pi r^2\right)\rho_w = 30 + \left(\pi r^2 h\right)\rho_w$$

$$\Rightarrow \quad l = \frac{30 + \left(\pi r^2 h\right)\rho_w}{\left(\pi r^2\right)\rho_w} \qquad \ldots(1)$$

Also, it is given that the centre of mass of the tube and its contents lies at the midpoint of the immersed length of the tube. So, we have

$$\frac{l}{2} = \frac{(30)(10) + \left(\pi r^2 h\rho_w\right)\left(\frac{h}{2}\right)}{30 + \pi r^2 h\rho_w}$$

$$\Rightarrow \quad \left(30 + \pi r^2 h\rho_w\right)\left(\frac{30 + \pi r^2 h\rho_w}{2\pi r^2 \rho_w}\right) = 300 + \frac{\pi r^2 h^2 \rho_w}{2}$$

$$\Rightarrow \quad \frac{\left(30 + \pi r^2 h\rho_w\right)^2}{2\pi r^2 \rho_w} = 300 + \frac{\pi r^2 h^2 \rho_w}{2}$$

$$\Rightarrow \quad \frac{900 + \left(\pi r^2 h\rho_w\right)^2 + 60\left(\pi r^2 h\rho_w\right)}{2\pi r^2 \rho_w} = 300 + \frac{\left(\pi r^2 \rho_w h\right)^2}{2}$$

$$\Rightarrow \quad 450 + 30\left(\pi r^2 h\rho_w\right) = 300\pi r^2 \rho_w$$

$$\Rightarrow \quad h = \frac{300\pi r^2 \rho_w - 450}{30\pi r^2 \rho_w}$$

$$\Rightarrow \quad h = 10 - \frac{450}{30 \times 3.14 \times (2)^2 \times 1}$$

$$\Rightarrow \quad h = 8.8 \text{ cm}$$

Mass of water is given by

$$m = \pi r^2 h\rho_w = 3.14 \times (2)^2 \times 8.8 \times 1$$

$$\Rightarrow \quad m = 110.53 \text{ g}$$

ILLUSTRATION 80

A cylinder of area $300\ \text{cm}^2$ and length 10 cm made of material of specific gravity 0.8 is floated in water with its axis vertical. It is then pushed slowly downwards, so as to be just immersed. Calculate the work done by the external force to push the cylinder completely in water. Take $g = 10\ \text{ms}^{-2}$.

SOLUTION

Weight of the cylinder is

$$mg = \left(300 \times 10^{-4}\right)\left(10 \times 10^{-2}\right)(800) \text{ kgf}$$

$$\Rightarrow \quad mg = 2.4 \text{ kgf} = 24 \text{ N}$$

If l be the length of the cylinder inside the water, then by Law of Floatation, we have

$$2.4g = \left(300 \times 10^{-4}\right) l (1000) g$$

$$\Rightarrow \quad l = 0.08 \text{ m} = 8 \text{ cm}$$

When completely immersed in water, the buoyant force is given by

$$U = F_B = (300\times10^{-4})(0.1)(1000)g$$

$$\Rightarrow \quad U = 30 \text{ N}$$

So, to immerse the cylinder inside water, the external agent has to push it against the upthrust force by

$$h = 0.1 - 0.08 = 0.02 \text{ m} = 2 \text{ cm}$$

Increase in upthrust is

$$\Delta U = 3g - 2.4g = 0.6g = 6 \text{ N}$$

Since this increase in upthrust takes place linearly from zero to 6 N, so the average upthrust against which work has to be done is

$$\langle U \rangle = \frac{0+6}{2} = 3 \text{ N}$$

Hence the work done by the external force is

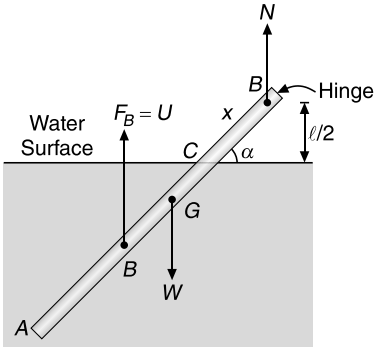
$$W = F_{ext}h = |\langle U \rangle| h = (3)(0.02) = 0.06 \text{ J}$$

ILLUSTRATION 81

A thin uniform rod of length $2l$ and specific gravity $\frac{3}{4}$ is hinged at one end to a point height $\frac{l}{2}$ above the surface of water, with the other end immersed. If the rod is in equilibrium, then calculate the angle of inclination of rod with the water surface.

SOLUTION

Let the length of rod outside water be x and its cross-sectional area be A. The situation is shown in Figure.



If W be the weight of rod and d be the density of water, then weight of the rod is

$$W = A(2l)\left(\frac{3}{4}d\right)g \qquad \ldots(1)$$

The buoyant force $F_B = U$ acting on the rod at the centre of buoyancy B is

$$U = F_B = A(2l-x)(d)g \qquad \ldots(2)$$

Let N be the upward force on rod by the hinge, then for equilibrium of rod we have

$$N + U = W$$

$$\Rightarrow \quad N = W - U$$

$$\Rightarrow \quad N = (Al)\left(\frac{3d}{2}\right)g - A(2l-x)(d)g$$

$$\Rightarrow \quad N = A\left(\frac{3}{2}l - 2l + x\right)(d)g$$

$$\Rightarrow \quad N = A\left(x - \frac{l}{2}\right)(d)g \qquad \ldots(3)$$

Also, for the rotational equilibrium of the rod, we take the net torque about point A to be zero i.e.

$$U\left(\frac{2l-x}{2}\right)\cos\alpha - Wl\cos\alpha + N(2l\cos\alpha) = 0$$

$$\Rightarrow \quad \left[F_B\left(\frac{2l-x}{2}\right) - Wl + 2Nl\right]\cos\alpha = 0$$

$$\Rightarrow \quad F_B\left(\frac{2l-x}{2}\right) - Wl + 2Nl = 0 \qquad \ldots(4)$$

Substituting the values of W, F_B and N from equations (1), (2) and (3) respectively in equation (4), we get

$$\left[\frac{(2l-x)^2}{2} - \frac{3}{2}l^2 + 2l\left(x - \frac{l}{2}\right)\right]A(d)g = 0$$

$$\Rightarrow \quad (2l-x)^2 - 3l^2 + 4l\left(x - \frac{l}{2}\right) = 0$$

$$\Rightarrow \quad 4l^2 + x^2 - 4lx - 3l^2 + 4lx - 2l^2 = 0$$

$$\Rightarrow \quad x^2 - l^2 = 0$$

$$\Rightarrow \quad x^2 = l^2$$

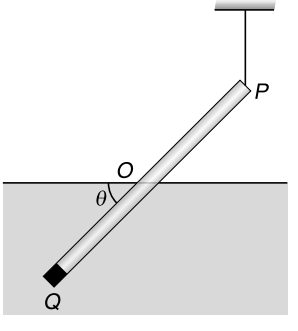
$$\Rightarrow \quad x = l$$

$$\Rightarrow \quad \sin\alpha = \frac{l/2}{l} = \frac{1}{2}$$

$$\Rightarrow \quad \alpha = 30^\circ$$

ILLUSTRATION 82

A uniform rod PQ, 4 m long and weighing 12 kg, is supported at end P, with a 6 kg lead weight attached at Q. The rod floats with one half of its length submerged as shown in Figure.

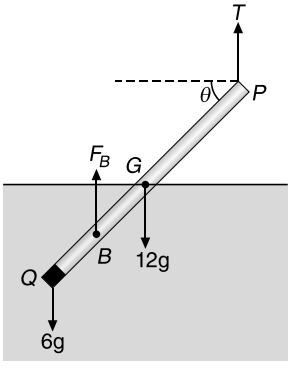


Assuming the lead mass to be of negligible volume, calculate tension in the cord and the total volume of the rod. Take $g = 10\ \text{ms}^{-2}$.

SOLUTION

If A be the area of cross section of the rod, then the buoyant force F_B on submerged part of rod is

$$U = F_B = \left(\frac{4}{2}\right)A\rho_w g = 2A\rho_w g \qquad \ldots(1)$$



For translational equilibrium of rod, we have

$$F_B + T = 18g \qquad \ldots(2)$$

Please note that, since the lead weight has negligible volume, so the buoyant force on it is negligible and hence is not to be taken into account.

For rotational equilibrium of the rod, taking the torque due to all forces about point P to be zero, we get

$$(12g)(2\cos\theta) + (6g)(4\cos\theta) = F_B(3\cos\theta)$$

$$\Rightarrow \quad F_B = 16g = 160\ \text{N} \qquad \ldots(3)$$

$$\Rightarrow \quad T = 18g - F_B = 2g = 20\ \text{N}$$

From equation (1), we get

$$2A(10^3)(10) = 160$$

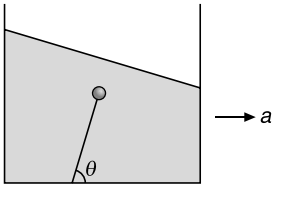
$$\Rightarrow \quad A = 80\times10^{-4}\ \text{m}^2$$

So, volume of the rod is

$$V = Al = (80\times10^{-4})(4) = 3.2\times10^{-2}\ \text{m}^3$$

ILLUSTRATION 83

A container partially filled with water is moved horizontally with acceleration $a = \frac{g}{\sqrt{3}}$. A small wooden ball of mass m is tied to the bottom of the container using a string. The ball remains inside water with the string inclined at an angle θ to the horizontal as shown in Figure. Assuming that the density of ball is half the density of water, calculate the tension in the string.



SOLUTION

As the container is accelerating the liquid surface will be inclined with horizontal such that

$$\tan\theta = \frac{a}{g} = \frac{g/\sqrt{3}}{g} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \sin\theta = \frac{1}{2} \text{ and } \cos\theta = \frac{\sqrt{3}}{2}$$

In this case the buoyant force on the ball will be normal to free surface of the liquid. The magnitude of buoyant force F_B is

$$F_B = V_{\text{imm}}\rho_w g_{\text{eff}}$$

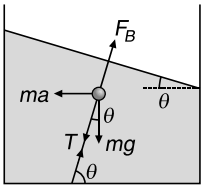
Please note that the immersed volume is equal to the volume of liquid displaced by the immersed part of the body i.e.

$$V_{\text{imm}} = V_{\text{displaced}} = \frac{m}{\rho_w}$$

$$\Rightarrow \quad F_B = \left(\frac{m}{\rho_w}\right)\rho_w\sqrt{g^2 + a^2} = 2m\sqrt{g^2 + \frac{g^2}{3}}$$

$$\Rightarrow \quad F_B = \frac{4mg}{\sqrt{3}}$$

The force acting on the ball are shown in FBD (w.r.t container)



Since the ball is in equilibrium with respect to container, so we have

$$F_B = T + mg\cos\theta + ma\sin\theta$$

$$\Rightarrow \quad \frac{4mg}{\sqrt{3}} = T + mg\frac{\sqrt{3}}{2} + m\left(\frac{g}{\sqrt{3}}\right)\frac{1}{2}$$

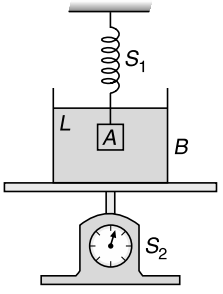
$$\Rightarrow \quad \frac{4mg}{\sqrt{3}} = T + \frac{mg}{2}\left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)$$

$$\Rightarrow \quad \frac{4mg}{\sqrt{3}} = T + \frac{2mg}{\sqrt{3}}$$

$$\Rightarrow \quad T = \frac{2mg}{\sqrt{3}}$$

ILLUSTRATION 84

A block A is hanging from spring balance S_1 and immersed in liquid L which is contained in beaker B as shown in Figure. The mass of beaker B is $1\ \text{kg}$ and mass of liquid L is $1.5\ \text{kg}$. The spring balances S_1 and S_2 read $2.5\ \text{kg}$ and $7.5\ \text{kg}$, respectively. Calculate the readings of spring balances S_1 and S_2 when block A is pulled up out of the liquid.



SOLUTION

When body is immersed in liquid, then it experiences an upthrust U, which makes spring balance S_1 to measure the apparent weight of the body. So, the reading R_{S_1} of spring balance S_1 is equal to the apparent weight W_{app} and is given by

$$R_{S_1} = W_{\text{app}} = W - U$$

$$\Rightarrow \quad R_{S_1} = W_{\text{app}} = W_A - U = 2.5 \text{ kgwt}$$

If W_L be the weight f the liquid, then the reading R_{S_2} of spring balance S_2 is

$$R_{S_2} = W_B + W_L + U$$

$$\Rightarrow \quad W_B + W_L + U = 7.5 \text{ kgwt}$$

Since, $W_B = 1$ kgwt, $W_L = 1.5$ kgwt

$$\Rightarrow \quad 1 + 1.5 + U = 7.5 \text{ kgwt}$$

$$\Rightarrow \quad U = 5 \text{ kgwt}$$

So, $W_A = U + 2.5 = 7.5$ kgwt

Hence the spring balance S_1 reads 7.5 kg and S_2 reads 2.5 kg.

Test Your Concepts-VII

Based on Archimedes' Principle and Buoyancy

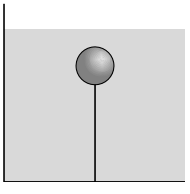
(Solutions on page H.10)

1. A piece of ice is floating in water. Calculate the fraction of volume of the piece of ice outside the water. Density of ice is 900 kgm^{-3} and density of water is 1000 kgm^{-3}.

2. A block is found to weigh W in air when suspended from a spring scale. When completely immersed in water while being attached to the spring scale, it weighs W'. Calculate density ρ of the block in terms of the scale reading and the density of water i.e. ρ_w.

3. A piece of an alloy of mass 96 g is composed of two metals whose specific gravities are 11.4 and 7.4. If the alloy weighs 86 g in water, calculate mass of each metal in the alloy.

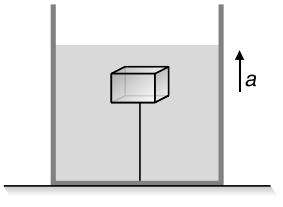
4. A solid sphere of mass 2 kg and specific gravity 0.5 is held stationary relative to a tank filled with water as shown. The tank is accelerating vertically upward with an acceleration of 2 ms^{-2}.



Calculate tension in the thread connecting sphere to the bottom of the tank. If the thread snaps, then calculate acceleration of the sphere with respect to the tank. Assume density of water to be is 1000 kgm^{-3} and $g = 10$ ms^{-2}.

5. A rubber ball of mass 10 g and volume 15 cm^3 is dipped in water to a depth of 10 m. If the ball is released from rest, calculate the acceleration of the ball and the time taken by it to reach the surface. Assume density of water to be uniform throughout the depth and $g = 10$ ms^{-2}.

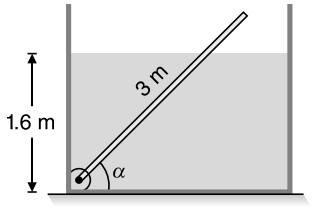
6. A solid block is held below the surface of a liquid (of density greater than that of solid block) with the help of a light string as shown in Figure.



The tension in the string is T_0 when the system is at rest. If the system accelerates upwards with an acceleration a, then calculate the tension in the string.

7. A rough surfaced metal cube of size 4 cm and mass 100 g is placed in an empty vessel. Now water is filled in the vessel so that the cube is just immersed in the water. Calculate the average pressure at the bottom surface of vessel which is in contact with the cube. Take $g = 10$ ms^{-2}.

8. A wooden rod weighing 25 N having length 3 m, uniform cross sectional area of 9.5×10^{-4} m^2 is mounted on a hinge 1.6 m below the free surface of water as shown.

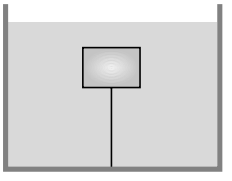


Initially the rod is in the vertical position and when pushed gently it comes to rest making an angle α with the base of the beaker, then calculate α. Also calculate the reaction on the hinge in this position if $\rho_{water} = 1000\ kgm^{-3}$ and $g = 9.8\ ms^{-2}$.

9. A piece of copper having an internal cavity weighs $264\ g$ in air and $221\ g$ in water. Calculate volume of the cavity if density of copper is $8.8\ gcm^{-3}$.

10. A cubical block of length L is floating vertically in a beaker filled with water. Calculate the change in submerged length of block in water if the beaker starts accelerating upwards with an acceleration a.

11. A solid block of volume $1000\ cm^3$ and density $0.8\ gcm^{-3}$ dipped in liquid and tied to the bottom of a container (with the help of a light string) filled with liquid of density $1.2\ gcm^{-3}$ as shown in Figure.

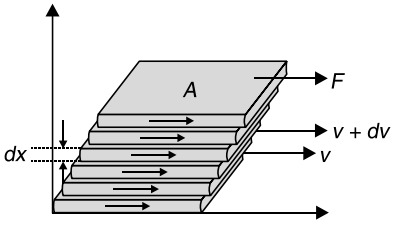


Calculate the tension in the string when the container is stationary and when it is moving up with an acceleration of $5.2\ ms^{-2}$. Take $g = 9.8\ ms^{-2}$.

VISCOSITY

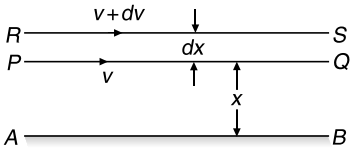
VISCOSITY

For a liquid flowing steadily, it is observed that a layer of liquid slips or tends to slip on adjacent layers in contact. Due to this relative motion between the adjacent layers of liquid, the two layers exert a tangential force on each other which tries to destroy the relative motion between them. The property of a fluid due to which it opposes the relative motion between its different layers is called viscosity (or fluid friction) and the force between the layers opposing the relative motion is called viscous force or viscous drag.



The internal friction of the fluid, which tends to oppose relative motion between different layers of the fluid is called **viscosity.**

Suppose a fluid flows over a fixed surface AB. The layer of the fluid in contact with AB remains at rest and the uppermost layer moves with maximum speed. This shows that every layer opposes the flow of the adjacent upper layer by a tangential viscous force along the surface of the layer.



Consider two layers PQ and RS at distances x and $x+dx$, respectively from AB. Also v and $v+dv$ be the speeds of fluid at PQ and RS respectively. It is found that the viscous force per unit area is proportional to the speed gradient, that is

$$\frac{F}{A} \propto \frac{dv}{dx}$$

$$\Rightarrow \quad F = -\eta A \frac{dv}{dx}$$

where the constant η is called the coefficient of viscosity or simply viscosity of the fluid. The negative sign indicates the opposing nature of the viscous force.

Examples of Viscosity

(i) A stirred liquid, when left, comes to rest on account of viscosity. Thicker liquids like honey, coal tar, glycerine, etc. have a larger viscosity than thinner ones like water. If we pour coal tar and water on a table, the coal tar will stop soon while the water will flow upto quite a large distance.

(ii) If we pour water and honey in separate funnels, water comes out readily from the hole in the funnel while honey takes enough time to do so. This is because honey is much more viscous than water. As honey tends to flow down under gravity, the relative motion between its layers is opposed strongly.

(iii) We can walk fast in air, but not in water. The reason is again viscosity which is very small for air but comparatively much larger for water.

(iv) The cloud particles fall down very slowly because of the viscosity of air and hence appear floating in the sky.

(v) Viscosity comes into play only when there is relative motion between the layers of the same material. This is why it does not act in solids.

COEFFICIENT OF VISCOSITY (η)

It is defined as the tangential force per unit area offered by a fluid layer to create a unit speed gradient. It is measured in $\text{kgm}^{-1}\text{s}^{-1}$ called decapoise in SI system of units. The cgs unit is $\text{gcm}^{-1}\text{s}^{-1}$ and is called **poise**.

$$1 \text{ decapoise } = 10 \text{ poise}$$

$$\text{OR} \quad 1 \text{ poise} = \frac{1}{10} \text{ decapoise}$$

The dimensional formula for coefficient of viscosity is $ML^{-1}T^{-1}$.

Conceptual Note(s)

(a) Viscosity comes into play only when there is a relative motion between the layers of the same material. This is why it does not act in solids.

(b) Viscosity of liquid is much greater (about 100 times more) than that of gases i.e. $\eta_L > \eta_G$

EXAMPLE: Viscosity of water is 0.01 poise while that of air is 200 μpoise.

(c) The viscosity of thick liquids like honey, glycerine, coal tar etc. is more than that of thin liquids like water.

SIMILARITIES BETWEEN VISCOSITY AND SOLID FRICTION

Viscosity and solid friction are similar in the following manner.

(a) Both oppose relative motion. Viscosity opposes the relative motion between two adjacent liquid layers,

solid friction opposes the relative motion between two solid layers.

(b) Both come into play, whenever there is relative motion between layers of liquid or solid surfaces as the case may be.

(c) Both are due to molecular attractions.

DIFFERENCES BETWEEN VISCOSITY AND SOLID FRICTION

Viscosity and solid friction are different in the following manner.

(a) Viscosity (or viscous drag) between layers of liquid is directly proportional to the area of the liquid layers, whereas friction between two solids is independent of the area of solid surfaces in contact.

(b) Viscous drag is proportional to the relative velocity between two layers of liquid, whereas friction is independent of the relative velocity between two surfaces.

(c) Viscous drag is independent of normal reaction between two layers of liquid, whereas friction is directly proportional to the normal reaction between two surfaces in contact.

SOME APPLICATIONS OF VISCOSITY

Knowledge of viscosity of various liquids and gases have been put to use in daily life. Some applications of its knowledge are discussed as under:

(a) As the viscosity of liquids vary with temperature, proper choice of lubricant is made depending upon season.

(b) Liquids of high viscosity are used in shock absorbers and buffers at railway stations.

(c) The phenomenon of viscosity of air and liquid is used to damp the motion of some instruments.

(d) The knowledge of the coefficient of viscosity of organic liquids is used in determining the molecular weight and shape of the organic molecules.

(e) It finds an important use in the circulation of blood through arteries and veins of human body.

ILLUSTRATION 85

A square plate of area $4\ \text{m}^2$ is made to move horizontally with a speed of $4\ \text{ms}^{-1}$, by applying a horizontal tangential force over the free surface of liquid. The depth of liquid is $2\ \text{m}$ and the liquid in contact with the bed is stationary. If coefficient of viscosity of liquid is 0.02 poise, then calculate the tangential force required to move the plate.

SOLUTION

Velocity gradient is $\dfrac{\Delta v}{\Delta x} = \dfrac{4-0}{2-0} = 2\ \text{s}^{-1}$

From Newton's Law, viscous force is given by

$$|F| = \eta A \frac{\Delta v}{\Delta x}$$

$$\Rightarrow \quad F = \left(0.02\times10^{-1}\right)\times 4\times(2)$$

$$\Rightarrow \quad F = 0.16\times10^{-1}$$

$$\Rightarrow \quad F = 16\times10^{-3}\ \text{N}$$

So, to keep the plate moving, a force of $16\times10^{-3}\ \text{N}$ must be applied.

ILLUSTRATION 86

A man is rowing a boat with a constant velocity v_0 in a river the contact area of boat is A and coefficient of viscosity is η. If depth of river is D, then calculate the force required to row the boat.

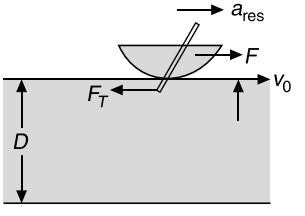
SOLUTION

According to Newton's Second Law, we have

$$F - F_T = ma$$

Since boat moves with constant velocity, so $a = 0$.

$$\Rightarrow \quad F = F_T$$



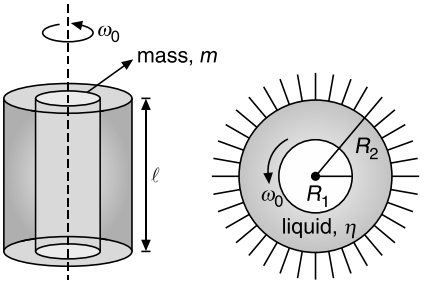
Also, we know that $F_T = \eta A \dfrac{dv}{dx}$

where, $\dfrac{dv}{dx} = \dfrac{v_0 - 0}{D} = \dfrac{v_0}{D}$

$$\Rightarrow \quad F = F_T = \frac{\eta A v_0}{D}$$

ILLUSTRATION 87

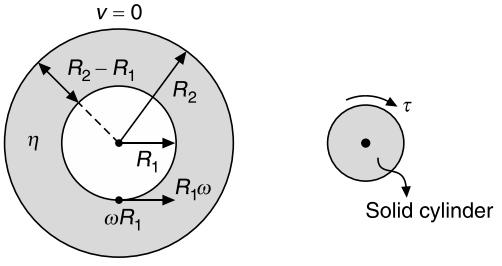
A liquid of viscosity η is filled between an outer fixed cylinder and an inner cylinder as shown in Figure.



The central solid cylinder starts rotating with an initial angular velocity ω_0. Calculate the time after which the angular velocity of the central cylinder becomes half its initial velocity.

SOLUTION

At any instant, the speed of the liquid layer in contact with outer fixed cylinder is zero, whereas the speed of the liquid layer attached with the inner rotating cylinder will be $R_1\omega$, where $\omega < \omega_0$.



The viscous force developed is given by

$$|F| = \eta A\frac{dv}{dx}, \text{ where } \frac{dv}{dx} = \frac{\omega R_1 - 0}{R_2 - R_1}$$

$$\Rightarrow \quad F = \eta(2\pi R_1 l)\left(\frac{\omega R_1}{R_2 - R_1}\right)$$

The torque due to this viscous force about axis is

$$\tau = FR_1 = \frac{2\pi\eta R_1^3 l\omega}{R_2 - R_1}$$

Since $\tau = I\alpha$, where $\alpha = -\frac{d\omega}{dt}$ and $I = \frac{1}{2}mR_1^2$

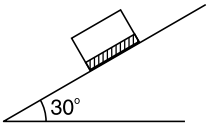
$$\Rightarrow \quad I\alpha = \frac{mR_1^2}{2}\left(-\frac{d\omega}{dt}\right) = \frac{2\pi\eta R_1^3 l\omega}{R_2 - R_1}$$

$$\Rightarrow \quad -\int_{\omega_0}^{\frac{\omega_0}{2}}\frac{d\omega}{\omega} = \frac{4\pi\eta R_1 l}{m(R_2 - R_1)}\int_0^t dt$$

$$\Rightarrow \quad t = \frac{m(R_2 - R_1)\ln 2}{4\pi\eta R_1 l}$$

ILLUSTRATION 88

A cubical block of side 2 m having mass 20 kg slides with constant velocity of $10\ \text{ms}^{-1}$ on an inclined plane lubricated with the oil of viscosity $\eta = 10^{-1}$ poise. Taking $g = 10\ \text{ms}^{-2}$, calculate the thickness of layer of liquid.

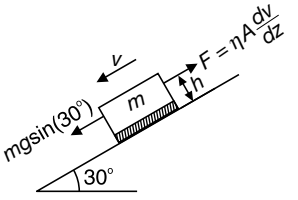


SOLUTION

Since, we have $F = \eta A\frac{dv}{dx}$

$$\Rightarrow \quad F = \eta A\frac{dv}{dx} = mg\sin\theta, \text{ where } \frac{dv}{dx} = \frac{v}{h}$$

$$\Rightarrow \quad (20)(10)(\sin 30°) = \eta(4)\left(\frac{10}{h}\right)$$



Since 1 poise $= \frac{1}{10}$ decapoise

$$\Rightarrow \quad \eta = 10^{-1} \text{ poise} = 10^{-2} \text{ decapoise}$$

$$\Rightarrow \quad h = \frac{40\times 10^{-2}}{100}$$

$$\Rightarrow \quad h = 4\times 10^{-3} \text{ m} = 4 \text{ mm}$$

ILLUSTRATION 89

A plate of area $100\ \text{cm}^2$ is placed on the upper surface of castor oil, 2 mm thick. Taking the coefficient of viscosity to be 15.5 poise, calculate the horizontal force necessary to move the plate with a velocity $3\ \text{cms}^{-1}$.

SOLUTION

The (horizontal tangential) viscous force is given by

$$F = -\eta A\frac{dv}{dx}$$

Given that, $\eta = 15.5$ poise, $A = 100\ \text{cm}^2$ and

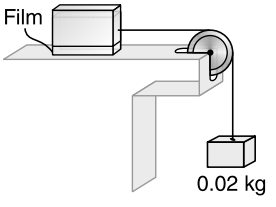
$$\frac{dv}{dx} = \frac{v_2 - v_1}{x_2 - x_1} = \frac{(0-3)\ \text{cms}^{-1}}{(2-0)\ \text{mm}} = -\frac{3}{0.2}\ \text{s}^{-1} = -15\ \text{s}^{-1}$$

$$\Rightarrow \quad F = -15.5\times 100\times(-15) = 2.235\times 10^4 \text{ dyne}$$

$$\Rightarrow \quad F = 0.2325 \text{ N}$$

ILLUSTRATION 90

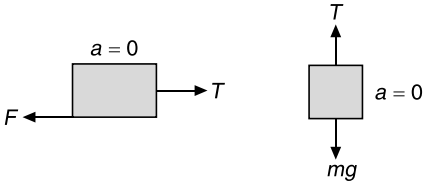
A metal block of area $0.10\ \text{m}^2$ is connected to a 0.02 kg mass through a light. The string passes over an ideal pulley (considered massless and frictionless) as shown in Figure.



A liquid film of thickness 0.15 mm is placed between the metal block and the table. When released the block moves to the right with a constant speed of $0.075\ \text{ms}^{-1}$. Calculate the coefficient of viscosity of the liquid if $g = 10\ \text{ms}^{-2}$.

SOLUTION

Since the block is moving with a constant speed, so tension developed in the string is equal to the viscous force F. The free body diagram of the block and the mass is shown in Figure.



From the FBD of ck and the mass, we have

$$T = F = mg$$

where, $F = \eta A\left(\frac{\Delta v}{\Delta x}\right)$

$$\Rightarrow \quad mg = \eta A\left(\frac{\Delta v}{\Delta x}\right)$$

Given that, area of the film is $A = 0.10\ \text{m}^2$, thickness of the film is $\Delta x = 0.15\ \text{mm} = 0.15\times10^{-3}\ \text{m}$ and the relative velocity of plate $\Delta v = 0.075\ \text{ms}^{-1}$.

$$\Rightarrow \quad (0.02)(10) = \eta(0.10)\left(\frac{0.075}{0.15\times10^{-3}}\right)$$

$$\Rightarrow \quad \eta = \frac{0.2\times0.15\times10^{-3}}{0.10\times0.075}$$

$$\Rightarrow \quad \eta = 4\times10^{-3}\ \text{Pas}$$

$$\Rightarrow \quad \eta = 0.004\ \text{Pas}$$

VARIATION OF VISCOSITY

With Temperature

The viscosity of liquids is mainly due to the cohesive forces between the molecules of the neighbouring layers. When the temperature increases, the kinetic energy of the molecules increases, resulting in decrease of cohesive forces. Thus, the viscosity of liquids decreases (or fluidity increases) with rise in temperature.

The viscosity of gases is mainly due to the diffusion of molecules from one moving layer to the neighbouring layers. With increase of temperature, the rate of diffusion increases. Hence the viscosity of gases increases with increase of temperature.

Problem Solving Technique(s)

(a) For liquids, the viscosity decreases with rise in temperature.
(b) For gases, the viscosity increases with rise in temperature.

With Pressure

As pressure increases, the molecules come marginally closer, resulting in an increase in viscosity of liquids. However, in gases, viscosity is found to be independent of pressure, provided the pressure is not too small.

Conceptual Note(s)

(a) For liquids, as pressure increases viscosity increases.
(b) For gases, as pressure increases or decreases, viscosity remains same provided pressure is not too small.
(c) With increase in pressure, the viscosity of liquids (except water) increases while that of gases is practically independent of pressure. The viscosity of water decreases with increase in pressure.
(d) For temperatures above +32 °C, water behaves like other liquids. Its viscosity increases with increasing pressure.
For temperatures below +32 °C and under pressures of up to 20 MPa, the water's viscosity decreases with increasing pressure.
The reason is that the structure of the three-dimensional network of hydrogen bridges is destroyed. This network is rather stronger than the structures of other low-molecular liquids.

STOKE'S LAW

When a body moves through a liquid, then the liquid layer in immediate contact with the body is dragged with it. This establishes relative motion between consecutive liquid layers near the body, due to which a viscous force starts acting on the body. The liquid exerts a viscous force on the body to oppose its motion. The magnitude of the viscous force depends on the shape and size of the body, its speed and the viscosity of the liquid.

Stoke's law governs the motion of a small sphere through a liquid column of infinite length. When a body moves through a fluid, it experience a viscous drag. This viscous drag experienced by a small sphere of radius r, moving with a speed v through a fluid of viscosity η is given by

$$F = 6\pi\eta r v$$

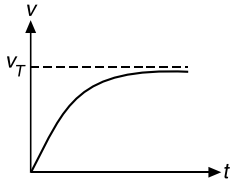
This is called **Stoke's Law.**

TERMINAL VELOCITY

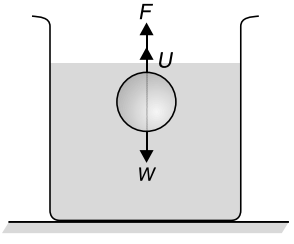
It has been observed that when a spherical body starts falling in a liquid column of infinite length, then initially the viscous drag on the body is zero as initial velocity of the body is zero. However, as the velocity of the body increases, then the viscous drag on the body also increases in accordance with the Stoke's Law. At a certain maximum

velocity called as Terminal velocity (v_T) it is observed that the weight (W) of the body is balanced by the combined effect of Upthrust (U) and the viscous drag (F) acting on the body.

For a body falling through a liquid column of infinite length, the variation in the velocity of a body with time is shown in Figure.



When a sphere is falling under gravity in a liquid of infinite column, the forces acting on it are its weight W (downwards), the upthrust U (upwards) and the viscous drag F (also upwards), as shown in Figure.



The net downward force when $v < v_T$ is

$$F_{net} = W - (U + F)$$

However, when $v = v_T$, the net force on the sphere becomes zero, so

$$F + U = W \text{, where}$$

$$F = 6\pi\eta r v_T,\ U = \left(\frac{4}{3}\pi r^3\right)\sigma g \text{ and } W = \left(\frac{4}{3}\pi r^3\right)\rho g$$

So, we have

$$6\pi\eta r v_T = W - U$$

$$\Rightarrow \quad 6\pi\eta r v_T = \frac{4}{3}\pi r^3(\rho - \sigma)g$$

where, ρ is the density of the sphere and
σ is the density of the liquid.

$$\Rightarrow \quad v_T = \frac{2}{9}\frac{(\rho - \sigma)r^2 g}{\eta} \qquad \{\text{Terminal Velocity}\}$$

If $\sigma > \rho$, the sphere will move upwards with a constant speed. So, we observe that,

(a) Terminal velocity depends on the radius of the sphere, so if radius of the spherical body is made n times, then the terminal velocity will become n^2 times.

(b) Increasing the density of body (ρ), will increase the terminal velocity.

(c) Increasing the density of the liquid (σ) and its viscosity (η) will decrease the terminal velocity.

(d) If $\rho > \sigma$, then terminal velocity will be positive and hence the spherical body will attain constant velocity in downward direction.

(e) If $\rho < \sigma$, then terminal velocity will be negative and hence the spherical body will attain constant velocity in upward direction.

EXAMPLE: Air bubble in a liquid and clouds in sky.

ILLUSTRATION 91

Eight spherical raindrops of equal size are falling vertically through air with a terminal velocity of $0.5\ \text{ms}^{-1}$. What would be the terminal speed of the bigger spherical drop formed if these drops coalesce to form the bigger drop?

SOLUTION

If r be the radius of small rain drop, then the terminal speed of the small drop is

$$v_T \propto r^2 \qquad \ldots(1)$$

If R be the radius of large drop, then equating the initial and the final volumes, we get

$$\frac{4}{3}\pi R^3 = 8\left(\frac{4}{3}\pi r^3\right)$$

$$\Rightarrow \quad R = 2r$$

$$\Rightarrow \quad \frac{v_T'}{v_T} = \left(\frac{R}{r}\right)^2 = 4$$

$$\Rightarrow \quad v_T' = 4v_T = (4)(0.5) = 2\ \text{ms}^{-1}$$

ILLUSTRATION 92

Calculate the terminal speed with which an air bubble 0.8 mm in diameter will rise in a liquid of coefficient of viscosity $0.15\ \text{Nsm}^{-2}$ and specific gravity 0.9, if the density of air is $1.293\ \text{kgm}^{-3}$.

SOLUTION

The terminal speed of bubble is

$$v_T = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$$

where, $r = 0.4 \times 10^{-3}$ m, $\sigma = 0.9 \times 10^3\ \text{kgm}^{-3}$

$\rho = 1.293\ \text{kgm}^{-3}$, $\eta = 0.15\ \text{Nsm}^{-2}$

and $g = 9.8\ \text{ms}^{-2}$

$$\Rightarrow \quad v_T = \frac{2}{9} \times \frac{(0.4 \times 10^{-3})^2 (1.293 - 0.9 \times 10^3) \times 9.8}{0.15}$$

$$\Rightarrow \quad v_T = -0.0021\ \text{ms}^{-1}$$

$$\Rightarrow \quad v_T = -0.21\ \text{cms}^{-1}$$

Please note that the negative sign means that the bubble will rise up.

ILLUSTRATION 93

A spherical ball of radius 3×10^{-4} m and density 10^4 kgm^{-3} falls freely under gravity through a distance h before entering a tank of water. If after entering the water the velocity of the ball does not change, calculate h, if the coefficient of viscosity of water is 9.8×10^{-6} Nsm^{-2}.

SOLUTION

Before entering the water, the velocity of ball is $\sqrt{2gh}$. If after entering the water this velocity does not change then this value should be equal to the terminal velocity. Therefore,

$$\sqrt{2gh}=\frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$$

$$\Rightarrow\quad h=\frac{1}{2g}\left[\frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}\right]^2$$

$$\Rightarrow\quad h=\frac{2}{81}\times\frac{r^4(\rho-\sigma)^2g}{\eta^2}$$

$$\Rightarrow\quad h=\frac{2}{81}\times\frac{(3\times10^{-4})^4(10^4-10^3)^2\times9.8}{(9.8\times10^{-6})^2}$$

$$\Rightarrow\quad h=1.65\times10^3\text{ m}$$

ILLUSTRATION 94

A spherical ball is moving with terminal velocity inside a liquid. Determine the relationship of rate of heat loss with the radius of ball.

SOLUTION

Rate of heat loss is power dissipated (P)

$$\Rightarrow\quad P=F\times v=(6\pi\eta rv)v=6\pi\eta rv^2$$

$$\Rightarrow\quad P=6p\eta r\left[\frac{2}{9}\frac{r^2}{\eta}(\rho_0-\rho_l)g\right]^2$$

Rate of heat loss $\propto r^5$

ILLUSTRATION 95

A sphere is dropped under influence of gravity through a fluid of coefficient of viscosity η. If the average acceleration of the sphere is half of the initial acceleration, then show that time taken by the sphere to attain terminal speed is independent of density of fluid.

SOLUTION

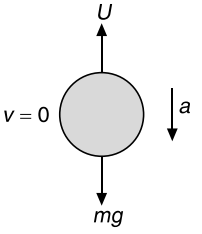
Let r be the radius of the sphere and ρ be its density, then mass (m) of the sphere is

$$m=\frac{4}{3}\pi r^3\rho$$

If σ be the density of the fluid, then mass of fluid displaced by the sphere is

$$m'=\frac{4}{3}\pi r^3\sigma$$

Initially, since $v=0$, so the viscous force acting on the sphere is also zero and hence the free body diagram of the sphere when it is just dropped is shown in Figure.



If a be the initial acceleration of the sphere, then we have

$$mg-U=ma$$

$$\Rightarrow\quad \frac{4}{3}\pi r^3\rho g-\frac{4}{3}\pi r^3\sigma g=\left(\frac{4}{3}\pi r^3\rho\right)a$$

$$\Rightarrow\quad a=\left(\frac{\rho-\sigma}{\rho}\right)g$$

The terminal velocity of the ball is

$$v=\frac{2r^2(\rho-\sigma)g}{9\eta}$$

If the average acceleration of the sphere during the fall (till the drop acquires terminal velocity) is a', then according to the problem, we have

$$a'=\frac{a}{2}=\frac{(\rho-\sigma)g}{2\rho}$$

Also, the average acceleration is given by

$$a_{av}=a'=\frac{\Delta v}{\Delta t}$$

$$\Rightarrow\quad \Delta v=a'\Delta t$$

$$\Rightarrow\quad \frac{2r^2(\rho-\sigma)g}{9\eta}-0=a'(t-0)=a't$$

$$\Rightarrow\quad \frac{2r^2(\rho-\sigma)g}{9\eta}=\left(\frac{\rho-\sigma}{2\rho}\right)gt$$

$$\Rightarrow\quad t=\frac{4r^2\rho}{9\eta}$$

Hence, the time taken to acquire the terminal velocity is independent of the fluid density.

FLOW OF LIQUID IN TUBE: CRITICAL VELOCITY

When a liquid flow in a tube, the viscous forces oppose the flow of the liquid. Hence a pressure difference is applied between the ends of the tube which maintains the flow of the liquid. If all particles of the liquid passing through a

particular point in the tube move along the same path, the flow of the liquid is called 'stream-line flow'. This occurs only when the velocity of flow of the liquid is below a certain limiting value called 'critical velocity'.

When the velocity of flow exceeds the critical velocity, the flow is no longer stream-line but becomes turbulent. In this type of flow, the motion of the liquid becomes zig-zag.

It is found that the flow is streamlined only when the rate of flow is small. Above a certain speed, called the critical speed, the flow becomes turbulent.

Reynold showed that there is a particular combination of four factors, namely density (ρ), velocity (v), tube diameter (d) and viscosity (η), which determines the nature of flow of a viscous fluid through the tube. This combination is called Reynolds Number and is given by

$$R = \frac{\rho v d}{\eta}$$

The Reynolds Number R is a **dimensionless variable**. It is found empirically that when, approximately

(a) $R < 2000$: The flow is Laminar.

(b) $R > 3000$: The flow is turbulent.

(c) $2000 < R < 3000$: The flow is unstable and may be changing from one type to another.

ILLUSTRATION 96

Water is flowing in a pipe of radius 1.5 cm with an average velocity 15 cms^{-1}. What is the nature of flow? Given coefficient of viscosity of water is 10^{-3} kgm^{-1}s^{-1} and its density is 10^3 kgm^{-3}.

SOLUTION

Reynolds number for the given situation is given as

$$R = \frac{\rho v d}{\eta}$$

where, $\rho = 10^3$ kg m^{-3}, coefficient of viscosity of the liquid is $\eta = 10^{-3}$ kgm^{-1}s^{-1}, average velocity of water is $v = 15$ cms^{-1} $= 0.15$ ms^{-1} and diameter of the pipe, is $d = 2 \times 1.5$ cm $= 3$ cm $= 0.03$ m.

$$\Rightarrow \quad R = \frac{10^3 \times 0.15 \times 0.03}{10^{-3}}$$

$$\Rightarrow \quad R = 10^6 \times 0.0045$$

$$\Rightarrow \quad R = 4500$$

Since, $R > 2000$, hence the flow is turbulent.

ILLUSTRATION 97

Calculate the maximum average velocity of water in a tube of diameter 2 cm, so that the flow is laminar. Assume coefficient of viscosity of water to be 10^{-3} Nsm^{-2}.

SOLUTION

For laminar flow through the tube, the maximum value for Reynolds number R is 2000. If v is the maximum average velocity of water, then

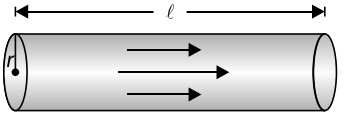
$$R_e = \frac{\rho v d}{\eta}$$

where, $d = 2$ cm $= 0.02$ m, $\eta = 10^{-3}$ Nsm^{-2} and $\rho = 10^3$ kgm^{-3}.

$$\Rightarrow \quad v = \frac{\eta R}{\rho d} = \frac{(10^{-3})(2000)}{(10^3)(0.02\text{ m})} = 0.1 \text{ ms}^{-1}$$

STEADY FLOW OF LIQUID THROUGH A CAPILLARY TUBE: POISEUILLE'S FORMULA

Poiseuille formula is designed for a streamlined flow of a liquid through a horizontal tube. If a viscous liquid flow in a tube, the velocity is greatest at the centre of the tube and decreases to zero at the wall. If the velocity is small, the flow is streamlined (also called the steady or laminar flow).



Poiseuille studied the streamline flow of liquid through horizontal capillary or horizontal tube. He found that if a pressure difference (P) is maintained across the two ends of a capillary tube of length l having radius r, then the volume of liquid flowing per second through the tube i.e. the rate of flow of volume of liquid (V) through the capillary is

(a) directly proportional to the pressure difference $(\Delta P = p)$.

(b) directly proportional to the fourth power of radius (r) of the capillary tube.

(c) inversely proportional to the coefficient of viscosity (η) of the liquid.

(d) inversely proportional to the length (l) of the capillary tube.

Let l be the length of the tube and r be its radius. If a fluid of viscosity η flows steadily under a pressure difference p, then the rate of flow of liquid Q i.e. volume V of fluid flowing per unit time is given by

$$Q = \frac{V}{t} = \frac{\pi r^4 p}{8\eta l} \qquad \{\text{Poiseuille's Formula}\}$$

The quantity $\frac{\Delta P}{l} = \frac{p}{l}$ is also called the pressure gradient.

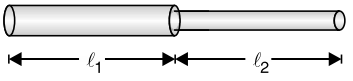
The rate of flow of liquid is due to the pressure difference just like rate of flow of charge (i.e. current) which flows due to the potential difference.

So liquid resistance R is simply defined as the pressure difference (P) per unit rate of flow of liquid (Q).

$$R = \frac{\text{Pressure Difference}}{\text{Rate of Flow of Liquid}} = \frac{\Delta P}{Q} = \frac{8\eta l}{\pi r^4}$$

COMBINATION OF TUBES IN SERIES

When two tubes of length l_1 and l_2 having radii r_1 and r_2 are connected in series across a pressure difference P, then



$$P = P_1 + P_2 \qquad \ldots(1)$$

where P_1 and P_2 are the pressure difference across the first and second tube respectively.

Since the volume of liquid flowing through both the tubes *i.e.* rate of flow of liquid is same.

Therefore $V = V_1 = V_2$

$$\Rightarrow \quad V = \frac{\pi P_1 r_1^4}{8\eta l_1} = \frac{\pi P_2 r_2^4}{8\eta l_2} \qquad \ldots(2)$$

Substituting the value of P_1 and P_2 from equations (1) and (2), we get

$$P = P_1 + P_2 = V\left(\frac{8\eta l_1}{\pi r_1^4} + \frac{8\eta l_2}{\pi r_2^4}\right)$$

$$\Rightarrow \quad V = \frac{P}{\left(\frac{8\eta l_1}{\pi r_1^4} + \frac{8\eta l_2}{\pi r_2^4}\right)} = \frac{P}{R_1 + R_2} = \frac{P}{R_{\text{eff}}}$$

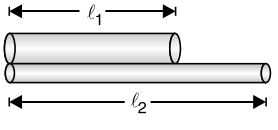
where R_1 and R_2 are the liquid resistances offered to the flow of liquid by the tubes.

So, effective liquid resistance for series combination of the tubes is

$$R_{\text{eff}} = R_1 + R_2$$

COMBINATION OF TUBES IN PARALLEL

When two tubes of length l_1 and l_2 having radii r_1 and r_2 are connected in series across same pressure difference P, then



$$P = P_1 = P_2$$

The total rate of flow of volume (V) of the liquid will be the sum of volume of liquid flowing per second through tube 1 and volume of liquid flowing per second through tube 2, i.e.

$$V = V_1 + V_2$$

$$V = \frac{P\pi r_1^4}{8\eta l_1} + \frac{P\pi r_2^4}{8\eta l_2} = P\left(\frac{\pi r_1^4}{8\eta l_1} + \frac{\pi r_2^4}{8\eta l_2}\right)$$

$$\Rightarrow \quad V = P\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{P}{R_{\text{eff}}}$$

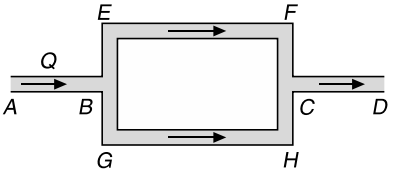
So, the effective liquid resistance in parallel combination

$$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\Rightarrow \quad R_{eff} = \frac{R_1 R_2}{R_1 + R_2}$$

ILLUSTRATION 98

A liquid is flowing through horizontal pipes as shown in Figure.



Length of different pipes have the ratio

$$L_{AB} = L_{CD} = \frac{L_{EF}}{2} = \frac{L_{GH}}{2}$$

Similarly, radii of different pipes have the ratio,

$$r_{AB} = r_{EF} = r_{CD} = \frac{r_{GH}}{2}$$

If pressure at A is $2P_0$ and pressure at D is P_0 and the rate of flow of volume of the liquid through the pipe AB is Q, then calculate the rate of flow of volume of liquid through the pipes EF and GH. Also calculate the pressure at E and F.

SOLUTION

The liquid resistance R is given by

$$R = \frac{\text{Pressure Difference}}{\text{Rate of Flow of Liquid}} = \frac{\Delta P}{Q} = \frac{8\eta l}{\pi r^4}$$

$$\Rightarrow \quad R \propto \frac{l}{r^4}$$

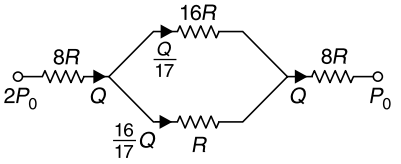
$$\Rightarrow \quad R_{AB} : R_{CD} : R_{EF} : R_{GH} = \frac{\frac{1}{2}}{\left(\frac{1}{2}\right)^4} : \frac{\left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right)^4} : \frac{(1)}{\left(\frac{1}{2}\right)^4} : \frac{(1)}{(1)^4}$$

$$\Rightarrow \quad R_{AB} : R_{CD} : R_{EF} : R_{GH} = 8 : 8 : 16 : 1$$

If $R_{GH} = R$, then

$$R_{AB} = 8R, \; R_{CD} = 8R \text{ and } R_{EF} = 16R$$

The equivalent liquid resistance circuit for the arrangement is shown in Figure.



As the current is distributed in the inverse ratio of the resistance (in parallel). The Q will be distributed in the inverse ratio of R, so the volume flow rate through EF will be $\frac{Q}{17}$ and that from GH will be $\frac{16}{17}Q$. The equivalent liquid resistance offered by the arrangement is

$$R_{net} = 8R + \frac{(16R)(R)}{(16R)+(R)} + 8R = \frac{288}{17}R$$

Since rate of flow of volume is

$$Q = \frac{\Delta P}{R_{net}}$$

$$\Rightarrow \quad Q = \frac{(2P_0 - P_0)}{\frac{288}{17}R} = \frac{17P_0}{288R}$$

Now, let P_1 be the pressure at E, then

$$2P_0 - P_1 = 8QR = 8\left(\frac{17P_0}{288R}\right)R = \left(\frac{8\times 17}{288}\right)P_0$$

$$\Rightarrow \quad P_1 = \left(2 - \frac{8\times 17}{288}\right)P_0 = 1.53P_0$$

Similarly, if P_2 be the pressure at F, then

$$P_2 - P_0 = 8QR$$

$$\Rightarrow \quad P_2 = P_0 + \frac{8\times 17}{288}P_0$$

$$\Rightarrow \quad P_2 = 1.47P_0$$

ILLUSTRATION 99

Water flows through a capillary tube of radius r and length l at a rate of $40\ \text{mls}^{-1}$, when connected to a pressure difference of h cm of water. Another tube of the same length but radius $\frac{r}{2}$ is connected in series with this tube and the combination is connected to the same pressure head. Calculate the pressure difference across each tube and the rate of flow of water through the combination.

SOLUTION

The volume (quantity) of liquid flowing per second through a capillary tube maintained across a pressure difference $P = h\rho g$ is

$$Q = \frac{\pi r^4 P}{8\eta\ell} = \frac{\pi(h\rho g)r^4}{8\eta\ell} = 40\ \text{mls}^{-1} \qquad \ldots(1)$$

where l is length of tube, r is radius of tube and η is coefficient of viscosity of liquid.

When tubes are connected in series, the amount of liquid flowing through them is equal, i.e.,

$$Q = \frac{\pi P_1 r^4}{8\eta l} = \frac{\pi P_2\left(\frac{r}{2}\right)^4}{8\eta l} \qquad \ldots(2)$$

and $P_1 + P_2 = P = \rho gh$ $\ldots(3)$

From equation (2), we get

$$P_1 = \frac{P_2}{16}$$

$$\Rightarrow \quad P_2 = 16P_1$$

Substituting for P_2 in equation (3), we get

$$P_1 + 16P_1 = \rho gh$$

$$\Rightarrow \quad P_1 = \frac{\rho gh}{17} \text{ and } P_2 = \frac{16\rho gh}{17}$$

Substituting P_1 or P_2 in equation (2) and using equation (1), we get

$$Q = \frac{\pi(\rho gh)r^4}{17\times 8\eta\ell} = \frac{1}{17}\left[\frac{\pi(\rho gh)r^4}{8\eta\ell}\right]$$

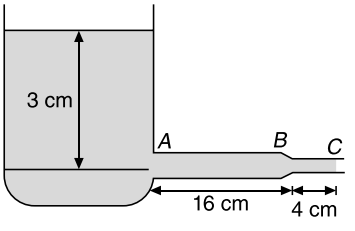
$$\Rightarrow \quad Q = \frac{40}{17}\ \text{mLs}^{-1}$$

ILLUSTRATION 100

Two capillary tubes AB and BC are joined end to end at B. Tube AB is 16 cm long, has a diameter 4 mm and tube BC is 4 cm long, has a diameter 2 mm. The composite tube is held horizontally with A connected to a vessel of water giving a constant pressure head of 3 cm and C is open to the air. Calculate the pressure difference between B and C.

SOLUTION

According to the problem, AB and BC are two capillary tubes which are joined end to end at B as shown in Figure.



Let the atmospheric pressure be equal to h cm of water column. So, pressure at A is $P_A = (h+3)$ and pressure at C is $P_C = h$. Further, if h' be the pressure at B, then pressure difference across AB is

$$P_{AB} = (h+3) - h'$$

and pressure difference across BC is

$$P_{BC} = h' - h$$

Since both tubes are in series, so rate of flow of liquid through both the tubes is same, i.e.

$$Q = \text{constant}$$

$$\Rightarrow \quad \frac{\pi P_1 r_1^4}{8\eta l_1} = \frac{\pi P_2 r_2^4}{8\eta l_2}$$

$$\Rightarrow \quad \frac{P_1 r_1^4}{l_1} = \frac{P_2 r_2^4}{l_2}$$

$$\Rightarrow \quad \frac{P_1}{P_2} = \frac{l_1}{l_2}\left(\frac{r_2}{r_1}\right)^4$$

Since, $l_1 = 16$ cm , $l_2 = 4$ cm

$r_1 = 0.2$ cm and $r_2 = 0.1$ cm

$$\Rightarrow \quad \frac{(h+3-h')}{(h'-h)} = \frac{16}{4}\times\left(\frac{0.1}{0.2}\right)^4$$

$$\Rightarrow \quad \frac{h+3-h'}{h'-h} = \frac{16}{4}\times\frac{1}{16} = \frac{1}{4}$$

$$\Rightarrow \quad h'-h = 4h+12-4h'$$

$$\Rightarrow \quad 5h' = 5h+12$$

$$\Rightarrow \quad h' = h+24$$

So, pressure difference across BC is

$$P_{BC} = h'-h = 2.4 \text{ cm of water column.}$$

ILLUSTRATION 101

A cylindrical vessel of area of cross-section A and filled with liquid to a height of h_1 has a capillary tube of length l and radius r protruding horizontally at its bottom. If the viscosity of liquid is η, density ρ and $g = 9.8\ \text{ms}^{-2}$, find the time in which the level of water in vessel falls to h_2.

SOLUTION

Let h be the height of water level in the vessel at instant t which decrease by dh in time dt.

So, rate of flow of water through capillary tube,

$$Q = -A\left(\frac{dh}{dt}\right) \qquad \ldots(1)$$

The negative sign indicates that as t increases, h decreases. The rate of flow of liquid through a horizontal tube is given by Poiseuille formula, i.e.

$$Q = \frac{\pi p r^4}{8\eta l} \qquad \ldots(2)$$

From (1) and (2), we get

$$-A\frac{dh}{dt} = \frac{\pi\rho g h r^4}{8\eta l} \qquad \{\because\ p = \rho g h\}$$

$$\Rightarrow \quad dt = -\frac{8\eta l A}{\pi\rho g r^4}\frac{dh}{h}$$

So, the required time is obtained by integrating this expression within suitable limits.

$$\Rightarrow \quad t = -\frac{8\eta l A}{\pi\rho g r^4}\int_{h_1}^{h_2}\frac{dh}{h} = -\frac{8\eta l A}{\pi\rho g r^4}\log_e h\Big|_{h_1}^{h_2}$$

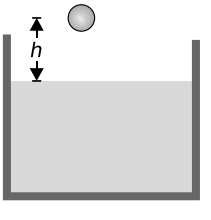
$$\Rightarrow \quad t = -\frac{8\eta l A}{\pi\rho g r^4}\log_e\frac{h_2}{h_1} = \frac{8\eta l A}{\pi\rho g r^4}\log_e\frac{h_1}{h_2}$$

Test Your Concepts-VIII

Based on Viscosity and Terminal Speed

(Solutions on page H.12)

1. The velocity of water in a river is $18\ \text{kmhr}^{-1}$ near the surface. If the river is 5 m deep, find the shear stress between the horizontal layers of water. The coefficient of viscosity of water is 10^{-2} poise.
2. Twenty seven identical spherical raindrops are falling vertically through air with a terminal velocity of $1\ \text{ms}^{-1}$. Calculate the terminal speed of the bigger spherical drop formed if these drops coalesce.
3. A solid rubber ball of density d and radius R falls vertically through an air column of very large length. Assume that the air resistance acting on the ball is $F = KRv$, where K is constant and v is its velocity. Calculate the terminal velocity of the ball.
4. A plate of area $2\ \text{m}^2$ is made to move horizontally with a speed of $2\ \text{ms}^{-1}$ by applying a horizontal tangential force over the free surface of a liquid. If the depth of the liquid is 1 m and the liquid in contact with the bed is stationary. Coefficient of viscosity of liquid is 0.01 poise. Find the tangential force needed to move the plate.
5. A powder comprising particles of various sizes is stirred up in a vessel filled to a height of 10 cm with water. If the particles are assumed to be spherical, then calculate the size of the largest particle that will remain in suspension after 1 hr if density of powder is $4\ \text{gcm}^{-3}$ and coefficient of viscosity of water is 0.01 poise.
6. A ball of radius r and density ρ falls freely under gravity through a distance h before entering water.



Velocity of ball does not change even on entering water. If viscosity of water is η, Find h.

7. Two square metal plates, each of side 10 cm are immersed in water. One plate moves parallel to the other with a speed of 5 cms^{-1}. If the viscous force is 150 dyne, calculate their distance of separation if the coefficient of viscosity of water is $\eta_{\text{water}} = 0.001\text{ Pl}$.

8. Calculate the Reynolds number for blood flowing at 30 cms^{-1} through an aorta of radius 1.0 cm. Assume that the blood has a viscosity of 4 mPa s and a density of 1060 kgm^{-3}. What is the nature of flow?

9. Capillaries of length l, $2l$ and $\frac{l}{2}$ are connected in series. Their radii are r, $\frac{r}{2}$ and $\frac{r}{3}$ respectively. If stream line flow is maintained and pressure across the first capillary is P_1 and across second and third being P_2 and P_3 respectively. Calculate the ratio of P_2 and P_3.

10. An engineer wants to have the same flow rate of water and light machine oil from the pipes of the same length and with the same pressure gradient. Calculate the ratio of the radii of the two pipes, if the coefficient of viscosity of water is 0.01 poise and that of light machine oil is 12.96 poise.

FLUID DYNAMICS

MOTION OF A FLUID

In order to describe the motion of a fluid, in principle one might apply Newton's laws to a particle (a small volume element of fluid) and follow its progress in time. This is a difficult approach. Instead, we consider the properties of the fluid, such as velocity and pressure, at fixed points in space.

In order to simplify the discussion, we make several assumptions:

(i) The flow is steady: The velocity and pressure at each point are constant in time

(ii) The fluid is incompressible: The density of fluid is constant throughout.

(iii) The flow is irrotational: A tiny paddle wheel placed in the liquid will not rotate.
In rotational flow, for example, in eddies, the fluid has net angular momentum about a given point.

(iv) The fluid is non viscous: There is no dissipation of energy due to internal friction between adjacent layer in the fluid.

Such type of fluid is called an **Ideal fluid** also called **SIIN fluid**.

FLOW OF IDEAL FLUID: BASIC DEFINITIONS

Ideal Fluid

An incompressible, streamline, irrotational, non-viscous fluid is called an **ideal fluid (SIIN fluid)**. So, an ideal fluid (i.e. a **S**teady, **I**rrotational, **I**ncompressible and **N**on-viscous liquid is an SIIN fluid.

Steady Flow

If the fluid velocity at any point is constant in time, then the flow is said to be **steady**.

Non-Steady Flow

If the fluid velocity at any point varies with time, then the flow is said to be **non-steady**.

Streamline

A **streamline** is a curve, the tangent to which at a point gives the direction of fluid velocity at that point.

- ✓ It is analogous to a line of force in an electric or magnetic field.
- ✓ In steady flow, the pattern of streamlines is stationary with time and therefore, also called a **streamline** flow.
- ✓ No two streamlines can ever cross one another, for if they did, a fluid particle arriving at that point would have possessed two directions (or two velocities) and hence the flow would never be steady.

Tube of Flow

A tubular region of fluid enclosed in by a boundary consisting of streamlines is called a **tube of flow**.

No fluid can cross the boundaries of a tube of flow and, therefore, a tube of flow behaves like a pipe of the same shape.

Rotational Flow

The flow of liquid is said to be **rotational** if the angular velocity is non zero.

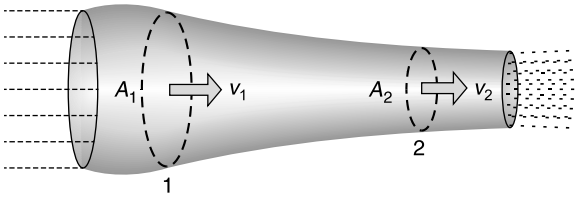
Irrotational Flow

The flow of the fluid is said to be **irrotational** if the element of the fluid at each point has no net angular velocity.

EQUATION OF CONTINUITY

Consider a steady, irrotational flow of an ideal fluid through a tube of varying cross-section having no source or sink between the entry and the exit.

If, A_1 and A_2 are the cross-sectional areas at points 1 and 2 respectively, v_1 and v_2 are the respective velocities of the liquid entering at 1 and leaving at 2 as shown in Figure.



If $\frac{dm}{dt}$ is the mass of liquid entering per second, then we have

$$\frac{dm}{dt} = \frac{A(dx)\rho}{dt} = A\left(\frac{dx}{dt}\right)\rho = Av\rho$$

As there is no source and sink between the entry and the exit, so we have

$$\begin{pmatrix}\text{Mass entering}\\ \text{per second at 1}\end{pmatrix} = \begin{pmatrix}\text{Mass leaving}\\ \text{per second at 2}\end{pmatrix}$$

$$\Rightarrow \quad A_1 v_1 \rho = A_2 v_2 \rho$$

$$\Rightarrow \quad A_1 v_1 = A_2 v_2$$

$$\Rightarrow \quad Av = \text{constant}$$

This is called the **Equation of Continuity** and follows from the Law of Conservation of Mass to the flow of the ideal fluids.

The quantity Av is called the flow rate or volume flux or Volume of liquid flowing per second and is also denoted by Q.

The above equation of continuity states that "for steady incompressible flow the speed of fluid varies inversely with the cross-sectional area".

Conceptual Note(s)

(a) Mathematically, the physical quantity Av is also called **Velocity Flux.** Vectorially,

$$\text{Velocity Flux} = \vec{v} \cdot \vec{A}$$

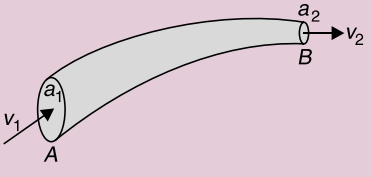
Velocity flux is the measure of the volume of liquid flowing in or out per second for a surface, the surface held normally to the liquid velocity (or liquid flow).

(b) The more the value of velocity flux, the more the number of streamlines cross the surface.

(c) We may also interpret the streamline picture as follows. In a narrow part of the tube the streamlines get closer together than in a wide part. Thus, as the distance between the streamlines decreases, the speed of the fluid increases.

(d) Widely spaced streamlines indicate regions of low speed, whereas closely spaced streamlines indicate regions of high speed.

(e) If a non-viscous compressible liquid in streamline flow passes through a tube AB of varying cross section having cross sectional areas a_1 and a_2 at points A and B. Let the liquid enter at A with normal velocity v_1 and leave at B with velocity v_2 and let ρ_1 and ρ_2 be the densities of the liquid at points A and B respectively, then Equation of Continuity is modified as $a_1v_1\rho_1 = a_2v_2\rho_2$

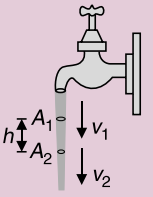


(f) Mathematically av is actually the rate of flow of volume of the liquid (or volume of liquid flowing per second) through a cross section of a tube.

$$av = \frac{dV}{dt} = Q = \begin{pmatrix}\text{Rate of Flow of}\\ \text{Liquid per second}\end{pmatrix}$$

(g) In hilly region, where the river is narrow and shallow (i.e., small cross-section) the water current will be faster, while in plains where the river is wide and deep (i.e., large cross-section) the current will be slower, and so deep water will appear to be still.

(h) When water falls from a tap, the velocity of falling water under the action of gravity will increase with distance from the tap, so $v_2 > v_1$. So, in accordance with continuity equation the cross section of the water stream will decrease and so, $A_2 < A_1$ due to which the falling stream of water becomes narrower.



Consider two such points on the water stream having a separation h, areas A_1, A_2 and respective velocities v_1 and v_2, then from Equation of Continuity, we have

$$A_1v_1 = A_2v_2$$

Now, since we have

$$v_2^2 = v_1^2 + 2gh$$

$$\Rightarrow A_2 = \frac{A_1v_1}{\sqrt{v_1^2 + 2gh}} \qquad (A_2 < A_1)$$

(i) If liquid entering a tube leaves the tube at two other points, and assuming that the tube has no source and sink, then we have

$$a_1v_1 = a_2v_2 + a_3v_3$$

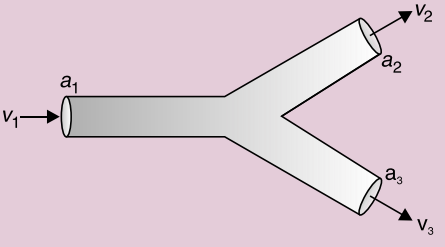


ILLUSTRATION 102

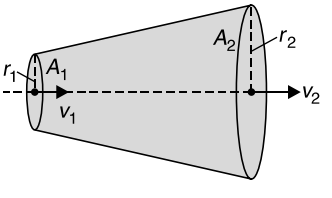
Water is flowing through a horizontal tube of non-uniform cross section. At a place the radius of the tube is 1 cm and the velocity of water is $2\ \text{ms}^{-1}$. What will be the velocity of water where the radius of the pipe is 2 cm?

SOLUTION

Using equation of continuity, $A_1v_1 = A_2v_2$

$$v_2 = \left(\frac{A_1}{A_2}\right)v_1$$

$$\Rightarrow \quad v_2 = \left(\frac{\pi r_1^2}{\pi r_2^2}\right)v_1 = \left(\frac{r_1}{r_2}\right)^2 v_1$$



Substituting the values, we get

$$v_2 = \left(\frac{1.0\times10^{-2}}{2.0\times10^{-2}}\right)^2 (2)$$

$$\Rightarrow \quad v_2 = 0.5\ \text{ms}^{-1}$$

ILLUSTRATION 103

Water is flowing in a circular pipe of varying cross-sectional area, and at all points the water completely fills the pipe.

(a) At one point in the pipe the radius is 0.2 m. What is the magnitude of the water velocity at this point if the volume flow rate in the pipe is $1.20\ \text{m}^3\text{s}^{-1}$?

(b) At a second point in the pipe the water velocity has a magnitude of $3.80\ \text{ms}^{-1}$. What is the radius of the pipe at this point?

SOLUTION

(a) Q = volume flow rate $= Av$

$$v = \frac{Q}{A} = \frac{Q}{\pi r^2} = \frac{1.2}{\pi (0.2)^2} = 9.55\ \text{ms}^{-1}$$

(b) From continuity equation

$$A_1 v_1 = A_2 v_2$$

$$\Rightarrow\ r_1^2 v_1 = r_2^2 v_2$$

$$\Rightarrow\ r_2 = \left(\sqrt{\frac{v_1}{v_2}}\right) r_1 = \left(\sqrt{\frac{9.55}{3.8}}\right)(0.2)$$

$$\Rightarrow\ r_2 = 0.317\ \text{m}$$

ENERGIES POSSESSED BY A LIQUID

A liquid possesses three kinds of energies.

Kinetic Energy (K.E.)

It is the energy possessed by a liquid due to its velocity. So, kinetic energy possessed by a liquid of mass m moving with a speed v is

$$\text{K.E.} = \frac{1}{2}mv^2$$

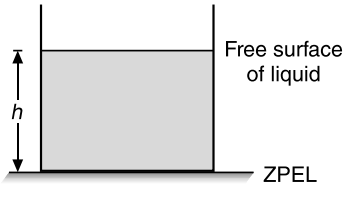
For liquids it is customary to talk about energy per unit mass or energy per unit volume, so we have

$$\frac{\text{K.E.}}{\text{Mass}} = \frac{1}{2}v^2$$

$$\Rightarrow\ \frac{\text{K.E.}}{\text{Volume}} = \frac{1}{2}\rho v^2$$

Potential Energy (P.E.)

It is the energy possessed by a liquid due to its position. Consider a beaker filled to a height h with liquid of density ρ. The beaker is placed on a horizontal surface as shown in Figure.



Assuming zero potential energy level (ZPEL) to be assigned to the horizontal surface, the potential energy of the free surface of the liquid is

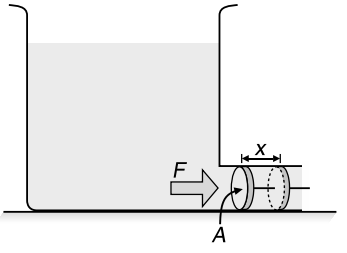
$$\text{P.E.} = mgh$$

$$\Rightarrow\ \frac{\text{Potential Energy}}{\text{Mass}} = gh$$

$$\Rightarrow\ \frac{\text{Potential Energy}}{\text{Volume}} = \rho gh$$

Pressure Energy (Pr.E.)

It is the energy possessed by a liquid due to its pressure. Consider a beaker having a frictionless piston attached to its base and filled with liquid of density ρ as shown in Figure.



The piston will move backwards because of the force due to pressure given by $F = PA$. The area of the piston is assumed to be small, so that there is no variation of pressure on the cross-section of the piston. Work done (W) by this force to displace the piston backwards by x is

$$W = Fx = (PA)x = P(Ax) = PV$$

This work done is store as pressure energy in the liquid, so we have

$$\text{Pressure Energy} = PV$$

$$\Rightarrow\ \frac{\text{Pressure Energy}}{\text{Mass}} = \frac{P}{\rho}$$

$$\Rightarrow\ \frac{\text{Pressure Energy}}{\text{Volume}} = P$$

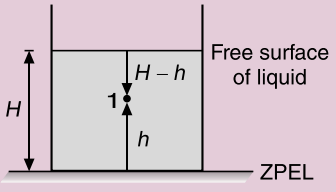
So, from above we see that pressure energy per unit volume is just equal to the pressure of the liquid.

Conceptual Note(s)

To us it may appear that potential energy per unit volume and the pressure energy per unit volume are the same, but let me explain it properly to you, that both are different.

To measure the potential energy per unit volume at a point inside the liquid, we shall be taking the height of the point above the zero potential energy level (ZPEL) and to measure the pressure energy per unit volume (i.e. pressure) at a point inside the liquid, we shall be taking the depth of the point below the free surface of the liquid.

Consider a beaker filled to a height H with a liquid of density ρ. Let us find the potential energy per unit volume and the pressure energy per unit volume at a point 1 inside the liquid as shown in Figure.



Assuming ZPEL to be at the surface on which the beaker is kept, we see that the potential energy per unit volume is

$$\frac{\text{P.E.}}{\text{Volume}} = h\rho g$$

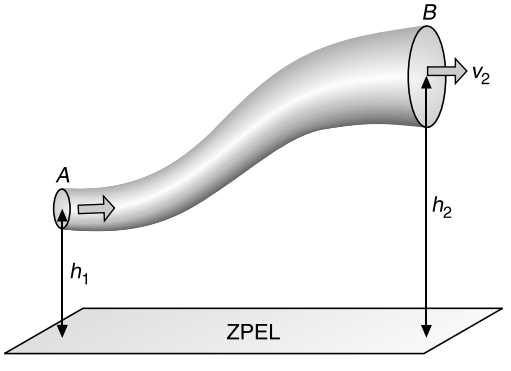
and pressure energy per unit volume is

$$\frac{\text{Pr.E.}}{\text{Volume}} = (H-h)\rho g$$

BERNOULLI'S EQUATION

This theorem is Law of Conservation of Energy for an ideal fluid flowing through a tube of variable cross-section having no source or sink in between.

Consider two cross-sections A and B of area A_1 and A_2 of a tube of flow at heights h_1 and h_2 respectively. Let v_1 and v_2 be the respective fluid velocities at these cross-sections, and let P_1 and P_2 be the respective fluid pressure. Let ρ be the density of the liquid.



Work done at the input A in time t is

$$W_1 = F_1 x_1 \cos 0$$

$$\Rightarrow \quad W_1 = P_1 A_1 v_1 t$$

Work done at the exit point B in time t is

$$W_2 = F_2 x_2 \cos 180$$

$$\Rightarrow \quad W_2 = -P_2 A_2 v_2 t$$

Hence total work done is

$$W = W_1 + W_2 = P_1 A_1 v_1 t - P_2 A_2 v_2 t \qquad \ldots(1)$$

Further by Equation of Continuity we have

$$A_1 v_1 = A_2 v_2 = \frac{\left(\begin{matrix}\text{Volume of liquid}\\ \text{flowing in or out}\end{matrix}\right)}{\text{Time}}$$

$$\Rightarrow \quad A_1 v_1 = A_2 v_2 = \frac{m}{\rho t} \qquad \left\{\because \text{Volume} = \frac{m}{\rho}\right\}$$

$$\Rightarrow \quad A_1 v_1 t = A_2 v_2 t = \frac{m}{\rho} \qquad \ldots(2)$$

Substituting (2) in (1), we get

$$W = (P_1 - P_2)\frac{m}{\rho} \qquad \ldots(3)$$

Further by Work Energy Theorem, we have

$$W = \Delta(KE) + \Delta(PE)$$

$$\Rightarrow \quad (P_1 - P_2)\frac{m}{\rho} = \frac{1}{2}m\left(v_2^2 - v_1^2\right) + mg(h_2 - h_1)$$

$$\Rightarrow \quad P_1 + \frac{1}{2}\rho v_1^2 + h_1\rho g = P_2 + \frac{1}{2}\rho v_2^2 + h_2\rho g$$

$$\Rightarrow \quad P + \frac{1}{2}\rho v^2 + h\rho g = \text{constant}$$

This is called **Bernoulli's Equation** for the steady flow of an ideal fluid according to which the sum total of pressure energy per unit volume, kinetic energy per unit volume and potential energy per unit volume is a constant. It is a consequence of the Law of Conservation of Energy.

Further, we have

$$\frac{P}{\rho} + \frac{1}{2}v^2 + gh = \text{constant}$$

i.e. sum total of pressure energy per unit mass, kinetic energy per unit mass and potential energy per unit mass for an SIIN fluid flowing through a tube of variable cross-section is a constant.

Also, we can write

$$\frac{P}{\rho g} + \frac{v^2}{2g} + h = \text{constant}$$

i.e. sum total of pressure head $\left(\frac{P}{\rho g}\right)$, velocity head $\left(\frac{v^2}{2g}\right)$ and gravitational head (h) for an SIIN fluid flowing through a tube of variable cross-section is a constant.

Problem Solving Technique(s)

If the height of the ends of the tube is same throughout, then the equation reduces to

$$P + \frac{1}{2}\rho v^2 = \text{constant}$$

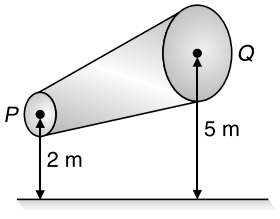
This equation tells us that when pressure is less, then velocity is more and vice-versa.

This pressure arises due to the motion of the fluid and hence is also called as Dynamic Pressure.

There are lot of applications of Bernoulli's theorem in common life. Some of those applications are discussed in detail in the upcoming section but before those examples, it is important for us to discuss the importance of Static, Dynamic and Total Pressure.

ILLUSTRATION 104

A non-viscous liquid of constant density $1000\ \text{kgm}^{-3}$ flows in a streamline motion along a tube of variable cross-section. The tube is kept inclined in the vertical plane as shown in Figure. The area of cross-section of the tube at the points P and Q at heights of 2 metre and 5 metre are respectively $4\times10^{-3}\ \text{m}^2$ and $8\times10^{-3}\ \text{m}^2$. The velocity of the liquid at point P is $1\ \text{ms}^{-1}$. Find the work done per unit volume by the pressure and the gravity forces as the fluid flows from point P to Q. Take $g = 9.8\ \text{ms}^{-2}$.

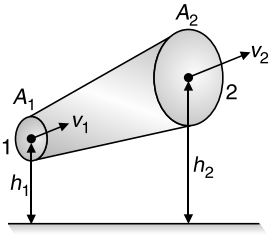


SOLUTION

According to the problem, we have $A_1 = 4\times10^{-3}\ \text{m}^2$, $A_2 = 8\times10^{-3}\ \text{m}^2$, $h_1 = 2\ \text{m}$, $h_2 = 5\ \text{m}$, $v_1 = 1\ \text{ms}^{-1}$ and $\rho = 10^3\ \text{kgm}^{-3}$

From continuity equation, we have

$$A_1v_1 = A_2v_2$$



$$\Rightarrow \quad v_2 = \left(\frac{A_1}{A_2}\right)v_1$$

$$\Rightarrow \quad v_2 = \left(\frac{4\times10^{-3}}{8\times10^{-3}}\right)(1\ \text{ms}^{-1})$$

$$\Rightarrow \quad v_2 = \frac{1}{2}\ \text{ms}^{-1}$$

Applying Bernoulli's equation at section 1 and 2,

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$\Rightarrow \quad P_1 - P_2 = \rho g(h_2 - h_1) + \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) \qquad \ldots(1)$$

Work done for unit volume by the pressure as the fluid flows from P to Q is

$$W_1 = P_1 - P_2$$

$$\Rightarrow \quad W_1 = \rho g(h_2 - h_1) + \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) \qquad \{\text{from (1)}\}$$

$$\Rightarrow \quad W_1 = \left[(10^3)(9.8)(5-2) + \frac{1}{2}(10^3)\left(\frac{1}{4} - 1\right)\right]\ \text{Jm}^{-3}$$

$$\Rightarrow \quad W_1 = (29400 - 375)\ \text{Jm}^{-3}$$

$$\Rightarrow \quad W_1 = 29025\ \text{Jm}^{-3}$$

Work done per unit volume by the gravity as fluid flows from P to Q is

$$W_2 = -\rho g(h_2 - h_1) = -(10^3)(9.8)(5-2)\ \text{Jm}^{-3}$$

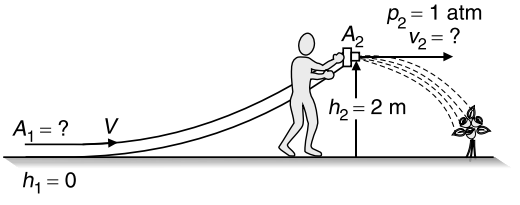
$$\Rightarrow \quad W_2 = -29400\ \text{Jm}^{-3}$$

ILLUSTRATION 105

A garden hose has an inside cross-sectional area of $3\ \text{cm}^2$, and the opening in the nozzle is $0.5\ \text{cm}^2$. If the water velocity is $100\ \text{cms}^{-1}$ in a segment of the hose that lies on the ground, calculate the speed with which water comes out from the nozzle when it is held $2\ \text{m}$ above the ground. Also calculate the water pressure in the hose lying on the ground. Take $g = 10\ \text{ms}^{-2}$ and $P_{\text{atm}} = 10^5\ \text{Nm}^{-2}$.

SOLUTION

The situation given in the problem is shown in Figure.



On applying the Equation of Continuity to find the speed of the fluid at the nozzle, we get

$$v_2 = \frac{A_1}{A_2}v_1 = \left(\frac{3\ \text{cm}^2}{0.5\ \text{cm}^2}\right)(100\ \text{cms}^{-1})$$

$$\Rightarrow \quad v_2 = 600\ \text{cms}^{-1} = 6\ \text{ms}^{-1}$$

According to Bernoulli's Theorem, we have

$$P_1 = P_2 + \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) + \rho g(h_2 - h_1)$$

where, $h_1 = 0$, $h_2 = 2\ \text{m}$, the pressure at the nozzle is atmospheric pressure, i.e. $P_2 = 10^5\ \text{Pa}$ and the density of water is $\rho = 10^3\ \text{kgm}^{-3}$.

$$\Rightarrow \quad P_1 = (10^5) + \frac{1}{2}(10^3)(36-1) + (10^3)(10)(2)$$

$$\Rightarrow \quad P_1 = 137500\ \text{Nm}^{-2}$$

STATIC, DYNAMIC AND TOTAL PRESSURE

Since we know that Bernoulli's Theorem is fundamental to the dynamics of incompressible (ideal or SIIN) liquids. In many fluid flow situations where the tube ends are at the same level or changes in elevation can be ignored, we can simply re-write Bernoulli's equation as

$$P+\frac{1}{2}\rho v^2 = \text{constant} \qquad \ldots(1)$$

where, P is called the **Static Pressure** (sometimes denoted by P_s), $\frac{1}{2}\rho v^2$ is called the **Dynamic Pressure** (denoted by P_d) and $P+\frac{1}{2}\rho v^2$ collectively is called the **Stagnation Pressure or Total Pressure** (denoted by P_0). Interestingly, we see from equation (1) that the Stagnation Pressure is constant. So, we have

$$P+\frac{1}{2}\rho v^2 = P_0 = \text{constant}$$

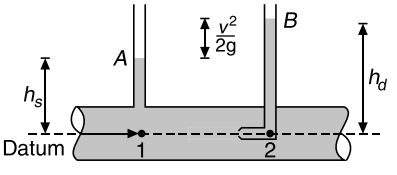
Hence, we conclude that total pressure is constant along any streamline, though static pressure and dynamic pressure may vary.

In simplified form, Bernoulli's Theorem can be summarised as

$$\begin{pmatrix}\text{Static}\\ \text{Pressure}\end{pmatrix}+\begin{pmatrix}\text{Dynamic}\\ \text{Pressure}\end{pmatrix}=\begin{pmatrix}\text{Stagnation}\\ \text{Pressure}\end{pmatrix}$$

The height h_s to which the liquid rises in it with respect to the reference level called Datum or Zero Potential Energy Level (ZPEL) is a measure of the static pressure in the fluid at a point on the axis.

The static pressure at a point in a fluid flow is calculated by connecting a single-tube manometer perpendicular to the direction of the fluid flow as shown by tube A in Figure.



So, we have

$$P_s = \rho g h_s$$

$$\Rightarrow \quad \frac{P_s}{\rho g} = h_s$$

The height h_s is also called the static pressure head.

Now, let a tube B (which is bent at right angle) be inserted in the fluid flow such that its open-end faces against the direction of fluid flow as shown in Figure. Just inside the open end of tube, the velocity of the fluid suddenly reduces to zero and the kinetic energy gets converted into pressure energy. Also, as discussed, the Stagnation Pressure should remain the same on a streamline, so we have

$$P_1+\frac{1}{2}\rho v^2 = P_2+\frac{1}{2}\rho(0)^2$$

$$\Rightarrow \quad P_2-P_1=\frac{1}{2}\rho v^2$$

$$\Rightarrow \quad (P_0+h_d\rho g)-(P_0+h_s\rho g)=\frac{1}{2}\rho v^2$$

$$\Rightarrow \quad h_d-h_s=\frac{v^2}{2g}$$

So, the liquid in tube B rises to a higher level than that in tube A.

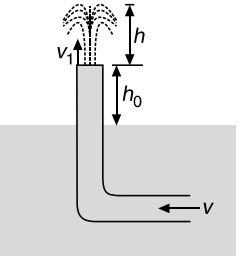
Conceptual Note(s)

The difference in the level of liquid is a measure of velocity of the fluid. Tube B measures the dynamic pressure and the height h_d of the liquid in the tube is called the dynamic head, which is equal to the sum total of the static pressure head and velocity head, i.e.

$$h_d = h_s+\frac{v^2}{2g}$$

ILLUSTRATION 106

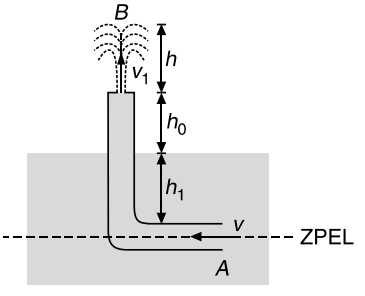
A bent tube is lowered into a water stream as shown in Figure.



The velocity of the stream relative to the tube is equal to $v = 2.5\ \text{ms}^{-1}$. The closed upper end of the tube located at the height $h_0 = 12\ \text{cm}$ has a small orifice. Calculate the height h to which the water jet will spurt.

SOLUTION

Let us consider the zero potential energy level (ZPEL) at the lowest portion of the tube as shown in Figure.



Applying Bernoulli's Theorem at A and B, we get

$$P_A + \rho g h_A + \frac{1}{2}\rho v_A^2 = P_B + \rho g h_B + \frac{1}{2}\rho v_B^2 \qquad \ldots(1)$$

where,

$$P_A = P_0 + h_1\rho g\,,\ h_A = 0\,,\ v_A = v \text{ and}$$

$$P_B = P_0\,,\ h_B = h_1 + h_0 + h\,,\ v_B = 0$$

Substituting these values in equation (1) we get

$$\left(P_0 + h_1\rho g\right) + \frac{1}{2}\rho v^2 = P_0 + \rho g\left(h_1 + h_0 + h\right)$$

$$\Rightarrow \quad \frac{1}{2}\rho v^2 = \rho g\left(h_0 + h\right)$$

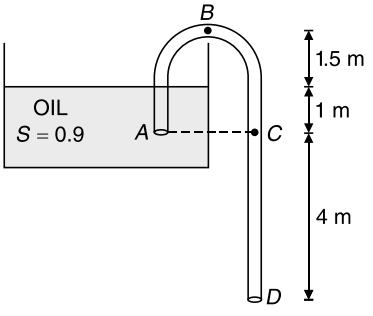
$$\Rightarrow \quad \frac{1}{2}\rho v^2 = \rho g\left(h_0 + h\right)$$

$$\Rightarrow \quad h = \frac{v^2}{2g} - h_0$$

$$\Rightarrow \quad h = \frac{(2.5)^2}{2\times 9.8} - 0.12 = 0.20 \text{ m}$$

ILLUSTRATION 107

A siphon tube is discharging a liquid of specific gravity 0.9 from a reservoir as shown in Figure.



Calculate the velocity of the liquid through the siphon and the pressure at highest point B, at A (out-side the tube) and at C. Also, give correct answer with explanation for each of the following statements.

Statement-1

Would the rate of flow be more, less or the same if the liquid were water?

Statement-2

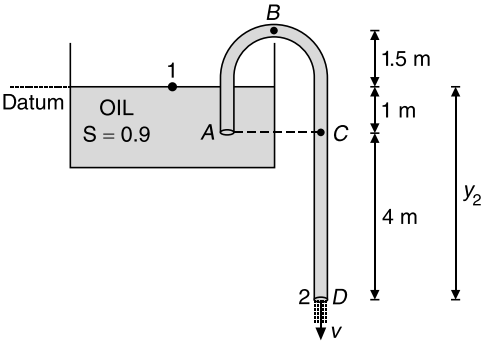
Is there a limit on the maximum height of B above the liquid level in the reservoir?

Statement-3

Is there a limit on the vertical depth of the right limb of the siphon?

SOLUTION

Assume datum at the free surface of the liquid. Applying Bernoulli's equation on point 1 and 2, as shown in Figure.



$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + y_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + y_2$$

where, $P_1 = P_2 = P_0 = 10^5 \text{ Nm}^2$; $y_1 = 0,\ y_2 = -5$ m

Since area of the tube is very small as compared to that of the reservoir, so $v_1 \ll v_2$, hence $\frac{v_1^2}{2g} \approx 0$.

$$\Rightarrow \quad v_2 = \sqrt{2g\left(y_1 - y_2\right)} = \sqrt{2(10)(5)} = 10 \text{ ms}^{-1}$$

Applying Bernoulli's equation at 1 and B.

$$\frac{P_B}{\rho g} + \frac{v_B^2}{2g} + y_B = \frac{P_1}{\rho g} + \frac{v_1^2}{2g} + y_1$$

where, $P_1 = 10^5 \text{ Nm}^{-2}$, $\frac{v_1^2}{2g} \approx 0$,

$$y_1 = 0,\ v_B = v_2 = 10 \text{ ms}^{-1},\ y_B = 1.5 \text{ m}$$

$$\Rightarrow \quad P_B = P_1 - \frac{1}{2}\rho v_2^2 - \rho g y_B$$

$$\Rightarrow \quad P_B = 10^5 - \frac{1}{2}(900)(10)^2 = (900)(10)(1.5)$$

$$\Rightarrow \quad P_B = 41.5 \text{ kNm}^{-2}$$

Applying Bernoulli's equation at 1 and A

$$P_A = P_1 + \rho g\left(y_1 - y_A\right)$$

$$\Rightarrow \quad P_A = 10^5 + (900)(10)(1) = 109 \text{ kNm}^{-2}.$$

Applying Bernoulli's equation at 1 and C,

$$P_C = P_1 - \frac{1}{2}\rho v_C^2 - \rho g y_C$$

$$\Rightarrow \quad P_C = 10^5 - (900) - (10)^2(900)(10)(-1)$$

$$\Rightarrow \quad P_C = 10^5 - 45000 + 9000 = 64 \text{ kNm}^{-2}$$

Statement-1 (Answer and Explanation)

The velocity of flow is independent of the density of the liquid, therefore, the discharge would remain the same.

Statement-2 (Answer and Explanation)

Since the pressure at B is less than atmospheric, the liquid, therefore, has a tendency to get vaporised if the

pressure becomes equal to the vapour pressure of it. Thus, $P_B > P_{vapour}$.

Statement-3 (Answer and Explanation)

The velocity of flow depends on the depth of the point D, below the free surface

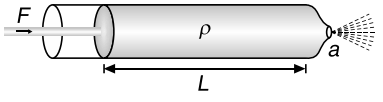
$$\frac{v^2}{2g} = y_1 - y_2 = H$$

$$\Rightarrow \quad P_B = P_1 - \frac{1}{2}\rho v^2 - \rho g y_B = P_1 - \rho g H - \rho g y_B$$

For working of siphon, $H \neq 0$, and H should not be high enough so that P_B may not reduce to vapour pressure.

ILLUSTRATION 108

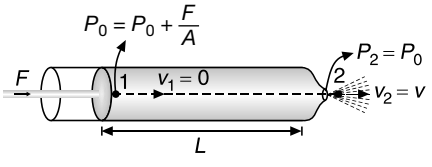
Calculate the work that should be done in order to squeeze all water in time t from a horizontally located cylinder by means of a constant force acting on the piston as shown in Figure. Assume that the volume of water in the cylinder is equal to V, the cross sectional area of the orifice is a, with a being considerably less than the piston area. The friction and viscosity are negligibly small.



SOLUTION

Work done to squeeze all the water from the container in time duration t is

$$W = FL$$



If v be the velocity of the liquid at point 2, then applying Bernoulli's theorem at points 1 and 2, we get

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2 \qquad \ldots(1)$$

where, $P_1 = P_0 + \frac{F}{A}$ and $P_2 = P_0$

Also, according to the problem, $A \gg a$, so v_1 is very much smaller than v_2 and hence v_1 can be neglected. So, from equation (1), we get

$$P_0 + \frac{F}{A} = P_0 + \frac{1}{2}\rho v^2 \qquad \ldots(2)$$

$$\Rightarrow \quad \frac{F}{A} = \frac{1}{2}\rho v^2$$

$$\Rightarrow \quad F = \left(\frac{1}{2}\rho v^2\right)A \qquad \ldots(3)$$

Volume of liquid coming out per second through orifice is

$$Q_2 = av$$

Volume of liquid coming out through the orifice in time duration t is

$$V = Q_2 t = (av)t$$

So, velocity of liquid coming out through orifice is

$$v = \frac{V}{at}$$

From Equation (3), we get

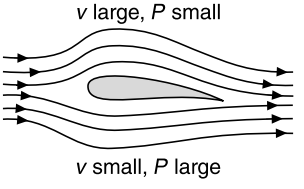
$$F = \frac{1}{2}\rho\left(\frac{V}{at}\right)^2 A$$

$$\Rightarrow \quad W = FL = \frac{1}{2}\rho\left(\frac{V}{at}\right)^2 AL = \frac{1}{2}\rho\left(\frac{V}{at}\right)^2 V$$

$$\Rightarrow \quad W = \frac{1}{2}\rho\left(\frac{V^3}{a^2t^2}\right)$$

WORKING OF AN AIRPLANE OR AERODYNAMIC LIFT

Because of the specific shape of wings of an airplane (as shown), when the airplane runs fast on the runway then air passes at higher speed over the upper surface of the wing as compared to its lower surface as shown in Figure.



This difference of air speeds above and below the wings, in accordance with Bernoulli's principle, creates a pressure difference, due to which an upward force called **dynamic lift** (= pressure difference × area of wing) acts on the plane and when this force becomes greater than the weight of the plane, the plane starts rising up.

ILLUSTRATION 109

An airplane wing is designed so that the speed of the air across the top of the wing is $251\ ms^{-1}$ when the speed of the air below the wing is $225\ ms^{-1}$. The density of the air is $1.29\ kgm^{-3}$. What is the lifting force on a wing of area $24.0\ m^2$?

SOLUTION

According to Bernoulli's theorem, we have

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$\Rightarrow \quad P_2 - P_1 = \Delta P = \frac{1}{2}\rho\left(v_1^2 - v_2^2\right)$$

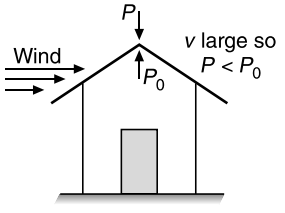
$$\Rightarrow \quad F = A\Delta P = \frac{1}{2}\rho A\left(v_1^2 - v_2^2\right)$$

$$\Rightarrow \quad F = \frac{1}{2}\times 1.29\times 24\left[(251)^2 - (225)^2\right]$$

$$\Rightarrow \quad F = 1.92\times 10^5 \text{ N}$$

ROOF'S BLOWING OFF DURING WIND STORMS

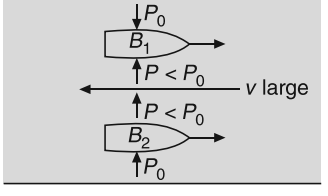
When a high-speed wind blows over a straw or tin roof, it creates a low pressure (P) in accordance with Bernoulli's principle.



However, the pressure below the roof (i.e., inside the room) is still atmospheric $(= P_0)$. So due to this difference of pressure the roof experiences an upward force and hence is then blown off by the wind.

ATTRACTION BETWEEN TWO CLOSELY PARALLEL MOVING BOATS OR TRAINS

It is observed that, when two boats move side by side in the same direction, the water in the region between them moves faster than that on the remote sides.

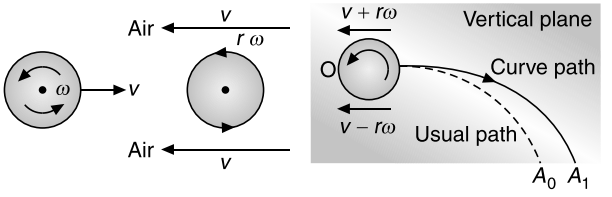


Consequently, in accordance with Bernoulli's principle the pressure between them is reduced and hence due to pressure difference they are pulled towards each other.

MAGNUS EFFECT (SPINNING OF A BALL OR SPINNING OF A BULLET)

When a ball is thrown with a spin, then the ball deviates from its usual path in flight. This effect is called Magnus effect and plays as important role in games like tennis, cricket, soccer, etc. where the player throws the ball by applying appropriate spin so that the ball can be made to follow a curved track in the desired direction.

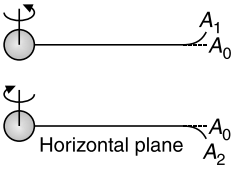
Consider a ball moving from left to right and also spinning about a horizontal axis perpendicular to the direction of motion as shown in Figure.



We observe that air will be moving from right to left relative to the ball. The resultant velocity of air above the ball will be $(v + r\omega)$ while below it $(v - r\omega)$. So, in accordance with Bernoulli's principle pressure above the ball will be less than below it. Due to this difference of pressure an upward force will act on the ball and hence the ball will deviate from its usual path OA_0 and will hit the ground at A_1 following the path OA_1 i.e., if a ball is thrown with back-spin, the pitch will curve less sharply prolonging the flight.

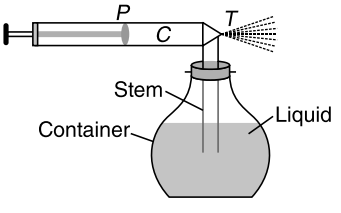
Similarly, if the spin is clockwise i.e., the ball is thrown with top-spin, the force due to pressure difference will act in the direction of gravity and so the pitch will curve more sharply shortening the flight.

Furthermore, if the ball is made to spin about a vertical axis, the curving will be sideways as shown in producing the so called out swing or in swing.



ACTION OF AN ATOMISER OR SPRAYER

The action of gun, scent-spray or insect-sprayer is based on Bernoulli's principle. In all these, by means of motion of a piston P in a cylinder C, high speed air is passed over a tube T dipped in liquid L to be sprayed. High speed air creates low pressure over the tube due to which the liquid (petrol, paint, scent, insecticide) rises in it and is then blown off as tiny droplets along with the expelled air.



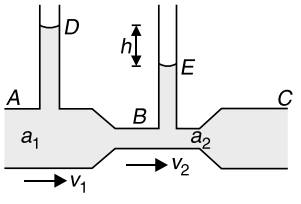
BLOOD FLOW AND HEART ATTACK

The occurrence of Heart Attack can be explained on the basis of Bernoulli's theorem. When the inner walls of a blood artery get constricted due to the deposition of plaque, the heart is required to exert excess pressure so as to derive the blood through this constriction. Blood flow speed increases through it as in accordance with the

equation of continuity. The increased blood flow reduces the pressure at the constriction. This leads to the collapse of artery due to external pressure. The heart has to push harder to push the blood through this artery. As the blood rushes through the opening, the pressure inside the artery drops once again. The artery collapses again and this repeated collapse may lead to a heart attack.

VENTURIMETER

Venturimeter is a device used to measure the speed of an incompressible liquid flowing through a pipe. It can also measure the rate of flow of liquid i.e. volume of liquid flowing per second, through a pipe.



It consists of two identical coaxial tubes A and C connected by a narrow co-axial tube B. Two vertical tubes D and E are mounted on the tubes A and B to measure the pressure of the following liquid.

When liquid of density ρ, flows through the tube ABC, the velocity of flow of liquid will be larger in tube B than in the tube A or C (as B has lesser area compared to A and C). So, the pressure in part B will be less than that in tube A or C. By measuring the pressure difference between A and B, the rate of flow of the liquid in the tube can be calculated.

Let a_1, a_2 be the area of cross section of tube A and B respectively and v_1, v_2 be the velocity of flow of liquid through tube A and B respectively. Let P_1, P_2 be the liquid pressure at A and B respectively, then we have

$$P_1 - P_2 = h\rho g \qquad \ldots(1)$$

Applying Bernoulli's theorem for horizontal flow of liquid, we have

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) \qquad \ldots(2)$$

From (1) and (2), we get

$$h\rho g = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) \qquad \ldots(3)$$

According to Equation of Continuity, we have

$$a_1 v_1 = a_2 v_2$$

$$\Rightarrow \quad v_2 = \frac{a_1 v_1}{a_2}$$

So, from equation (3), we get

$$h\rho g = \frac{1}{2}\rho v_1^2\left(\frac{a_1^2}{a_2^2} - 1\right)$$

$$\Rightarrow \quad v_1 = \sqrt{2gh}\sqrt{\frac{a_2^2}{a_1^2 - a_2^2}}$$

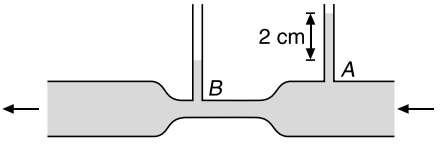
If Q be the volume of liquid flowing per second, then from Equation of continuity, we have

$$Q = a_1 v_1 = a_2 v_2$$

$$\Rightarrow \quad Q = a_1 a_2\sqrt{\frac{2gh}{a_1^2 - a_2^2}}$$

ILLUSTRATION 110

Water flows through a horizontal tube as shown in Figure.



If the difference of heights of water column in the vertical tubes is 2 cm and the areas of cross-section at A and B are 4 cm^2 and 2 cm^2 respectively, calculate the rate of flow of water across any section. Take $g = 10\ ms^{-2}$.

SOLUTION

Using Bernoulli's theorem for venturi tube, flow velocity is

$$v_A = \sqrt{2gh}\sqrt{\frac{a_A^2}{a_A^2 - a_B^2}}$$

$$\Rightarrow \quad v_A = \sqrt{2(10)(0.02)}\sqrt{\frac{(2)^2}{(4)^2 - (2)^2}}$$

$$\Rightarrow \quad v_A = \sqrt{\frac{0.4}{3}} = 0.365\ ms^{-1}$$

Rate of flow of liquid is

$$Q = a_A v_A = \left(4 \times 10^{-4}\right)(0.365)$$

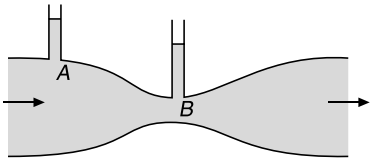
$$\Rightarrow \quad Q = 1.46 \times 10^{-4}\ m^3 s^{-1}$$

Since $1\ m^3 = 10^6\ cm^3$

$$\Rightarrow \quad Q = 146\ cm^3 s^{-1}$$

ILLUSTRATION 111

Water is flowing through a horizontal tube as shown in Figure. If the difference of heights of water column in the vertical tubes is 2 cm and the areas of cross-section at A and B are 4 cm^2 and 2 cm^2 respectively, then calculate the rate of flow of water across any section of the tube.



SOLUTION

Applying Bernoulli's equation at A and B, we get

$$p_A + \frac{1}{2}\rho v_A^2 + \rho g h_A = p_B + \frac{1}{2}\rho v_B^2 + \rho g h_B$$

$$\Rightarrow \quad \frac{1}{2}\rho v_B^2 - \frac{1}{2}\rho v_A^2 = \rho g\left(h_A - h_B\right)$$

$$\Rightarrow \quad v_B^2 - v_A^2 = 2g\left(h_A - h_B\right) = 2(10)(0.02)$$

$$\Rightarrow \quad v_B^2 - v_A^2 = 0.4 \text{ m}^2\text{s}^{-2} \qquad \ldots(1)$$

Applying Equation of Continuity at A and B, we get

$$v_A a_A = v_B a_B$$

$$\Rightarrow \quad 4v_A = 2v_B$$

$$\Rightarrow \quad v_B = 2v_A \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$v_A = 0.363 \text{ ms}^{-1}$$

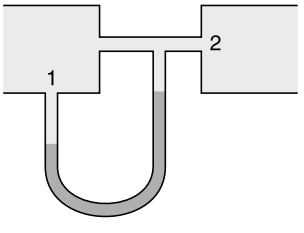
Volume flow rate Q is given by

$$Q = v_A A_A = (0.365)\left(4\times10^{-4}\right)$$

$$\Rightarrow \quad Q = 1.46\times10^{-4} \text{ m}^3\text{s}^{-1} = 146 \text{ cm}^3\text{s}^{-1}$$

ILLUSTRATION 112

Water flows through the tube as shown in Figure. The areas of cross-section of the wide and the narrow portions of the tube are 5 cm^2 and 2 cm^2 respectively. The rate of flow of water through the tube is $500 \text{ cm}^3\text{s}^{-1}$. Find the difference of mercury levels in the U-tube.



SOLUTION

Applying Equation of Continuity at 1 and 2, we get

$$A_1 v_1 = A_2 v_2 = Q \text{ i.e., rate of flow of water}$$

$$\Rightarrow \quad 5v_1 = 2v_2 = 500 \text{ cm}^3\text{s}^{-1}$$

$$\Rightarrow \quad v_1 = 100 \text{ cm}^{-1}\text{s}^{-1} = 1.0 \text{ ms}^{-1}$$

and $\quad v_2 = 250 \text{ cm}^{-1}\text{s}^{-1} = 2.5 \text{ ms}^{-1}$

Applying Bernoulli's equation at 1 and 2, we get

$$p_1 + \frac{1}{2}\rho_w v_1^2 + \rho_w g h_1 = p_2 + \frac{1}{2}\rho_w v_2^2 + \rho_w g h_2$$

Since $h_1 = h_2$, so we get

$$\frac{1}{2}\rho_w\left(v_2^2 - v_1^2\right) = p_1 - p_2 = \rho_{Hg} g h_{Hg}$$

$$\Rightarrow \quad \frac{1}{2}\rho_w\left(v_2^2 - v_1^2\right) = \rho_{Hg} g h_{Hg}$$

$$\Rightarrow \quad h_{Hg} = \frac{\rho_w\left(v_2^2 - v_1^2\right)}{2\rho_{Hg} g}$$

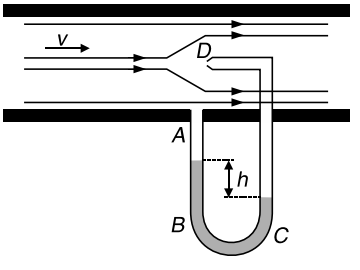
$$\Rightarrow \quad h_{Hg} = \frac{10^3\,(6.25-1)}{2\times13.6\times10^3\times9.8} = 0.0196 \text{ m}$$

$$\Rightarrow \quad h_{Hg} = 1.96 \text{ cm}$$

PITOT TUBE

It is based on Bernoulli's principle. It is a device used to measure the velocity of flow of liquid and hence the rate of flow at any depth in a flowing liquid. It is mounted on an aeroplane wing to measure the speed of the plane.

The arrangement consists of bent tube $ABCD$ with small apertures at A and D as shown in Figure. This tube is inserted in a pipe P through which the fluid (air in aeroplanes) is flowing.



Let ρ be the density of the fluid/air. The plane of aperture A is parallel to the direction of the air flow as such the velocity v_1 of air at A is same as its velocity in pipe which is equal to the velocity of the plane relative to air. Let P_1 is the pressure at A. However, plane of aperture at D is normal to the direction of air flow in the pipe. Therefore the velocity v_2 of the air flow rapidly falls to zero at D and pressure increases to P_2.

Applying Bernoulli's theorem to the horizontal flow at A and D, we get

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

where, $v_1 = v$ and $v_2 = 0$

$$\Rightarrow \quad P_1 + \frac{1}{2}\rho v^2 = P_2$$

$$\Rightarrow \quad v = \sqrt{\frac{2\left(P_2 - P_1\right)}{\rho}}$$

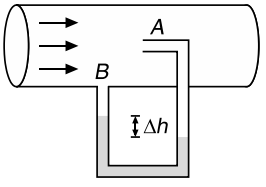
If σ is the density of liquid in tube $ABCD$, then

$$P_2 - P_1 = h\sigma g$$

$$\Rightarrow \quad v = \sqrt{\frac{2h\sigma g}{\rho}}$$

ILLUSTRATION 113

A Pitot tube is mounted along the axis of a gas pipeline whose cross-sectional area is equal to S as shown in Figure.



Assuming the viscosity to be negligible, find the volume of the gas flowing cross the section of the pipe per unit time, if the difference in the liquid columns is equal to Δh and the densities of the liquid and the gas are ρ_0 and ρ respectively.

SOLUTION

Using Bernoulli's theorem at points A and B, we get

$$P_A + \frac{1}{2}\rho v_A^2 = P_B + 0 \qquad \{\because v_B = 0\}$$

$$\Rightarrow \quad \frac{1}{2}\rho v_A^2 = P_B - P_A = \rho_0 g \Delta h$$

$$v_A = \sqrt{\frac{2\rho_0 g \Delta h}{\rho}}$$

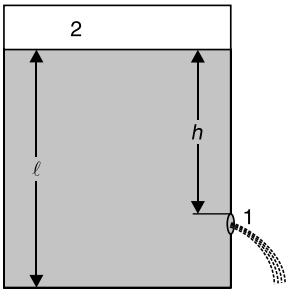
Volume of the gas flowing per unit time is

$$Q = Sv_A$$

$$\Rightarrow \quad Q = S\sqrt{\frac{2\rho_0 g \Delta h}{\rho}}$$

SPEED OF EFFLUX

The word 'efflux' means the outflow of the fluid. Let us find an expression for the velocity of efflux for a fluid, from a small hole of its container. Consider a closed vessel filled with a liquid up to height l. A small hole (or orifice) is made in its side at a depth h below the surface of the liquid s shown in Figure.



Taking the liquid to be incompressible and its flow through the hole as streamlined, we can apply the equation of continuity at points 1 and 2

$$A_2 v_2 = A_1 v_1$$

$$\Rightarrow \quad v_2 = \frac{A_1}{A_2} v_1$$

Applying Bernoulli's equation at the two points, we have

$$P_2 + \frac{1}{2}\rho v_2^2 + \rho g l = P_1 + \frac{1}{2}\rho v_1^2 + \rho g (l - h)$$

Conceptual Note(s)

While writing the Bernoulli's equation, we have

(a) considered ZPEL to be at the base of the vessel and hence potential energy per unit volume at 1 and 2 are $\rho g(l-h)$ and $\rho g l$ respectively.

(b) not taken atmospheric pressure into account because the vessel is closed.

If the cross-sectional area of the vessel A_2 is much larger than that of the orifice i.e., A_1 then,

$$v_2 \approx 0$$

$$\Rightarrow \quad P_2 + \rho g l = P_1 + \frac{1}{2}\rho v_1^2 + \rho g (l - h)$$

$$\Rightarrow \quad \frac{1}{2}\rho v_1^2 = (P_2 - P_1) + \rho g h$$

$$\Rightarrow \quad v_1 = \sqrt{\frac{2}{\rho}(P_2 - P_1) + 2gh}$$

Since the hole is open to atmosphere so, the pressure P_1 is the same as the atmospheric pressure P_a.

$$\Rightarrow \quad v_1 = \sqrt{\frac{2}{\rho}(P_2 - P_a) + 2gh} \qquad \ldots(1)$$

This expression gives the velocity of efflux.

CASE-1:

Torricelli's Law: In case the vessel containing the fluid is open i.e., not covered, the pressure P_2 at the top of the liquid surface is same as the atmospheric pressure P_a. Thus, the velocity of efflux given above in equation (1) becomes

$$v_{\text{efflux}} = v = \sqrt{2gh} \qquad \ldots(2)$$

This is same as the speed acquired by a body after falling freely through a height h. The expression (2) is known as Torricelli's law.

CASE-2:

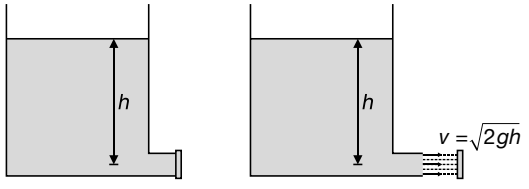
In case pressure $P_2 \gg P_a$, we can ignore the value $2gh$ in equation (1), to get the speed of efflux as

$$v_{\text{efflux}} = \sqrt{\frac{2}{\rho}(P_2 - P_a)}$$

The speed of efflux is determined by the container pressure. This occurs during rocket propulsion.

FORCE OF REACTION DUE TO EJECTION OF THE LIQUID FROM AN ORIFICE

Consider a cylindrical vessel that has an opening of cross-sectional area a near its bottom at a depth h below free surface of liquid. Let a disc be held against the opening to prevent liquid of density ρ from coming out of the opening.



The disc experiences a hydrostatic pressure from the liquid inside the vessel. Pressure at the level of the disc is

$$P_1 = P_{\text{atm}} + \rho gh$$

The air pressure on the outside the disc is $P_2 = P_{\text{atm}}$.
The net outward/rightward force acting on disc is

$$F_{\text{rightwards}} = (P_2 - P_1)a = \rho gha$$

Now, let the disc be moved a short distance away in horizontal direction. Due to this, the liquid comes out and strikes the disc. Assuming the collision of the liquid with the disc to be perfectly inelastic such that the liquid stops after hitting the disc, then the liquid in this case will exert an impulsive (impact) force on the disc.

When the disc is moved away, the liquid moves out with speed $v = \sqrt{2gh}$. The (mass per second) i.e., the rate of mass coming out of the opening is given by

$$\frac{dm}{dt} = \rho av = \rho a\sqrt{2gh}\ .$$

The thrust force acting on the liquid will be leftwards (opposite to v) and is given by

$$F_{\text{leftwards}} = v\frac{dm}{dt} = v(\rho av) = av^2\rho = 2h\rho ga$$

Taking the rightward direction as positive, the net force on the liquid is towards the left and its reaction on the disc is towards the right. Also, we note that the force due to atmospheric pressure is cancelled out from both the sides.

Conceptual Note(s)

Dear Student's, always keep in mind that the thrust force due to the liquid/gas coming out of the orifice is opposite to the velocity of a liquid (or gas) coming out through the orifice. If the liquid having density ρ is coming out of an orifice of cross-sectional area a with a velocity v, then thrust force due to liquid coming out the orifice is given by

$$F = \rho av^2$$

The direction of the thrust force will be just opposite to the direction of velocity as shown in examples taken in the Figure.

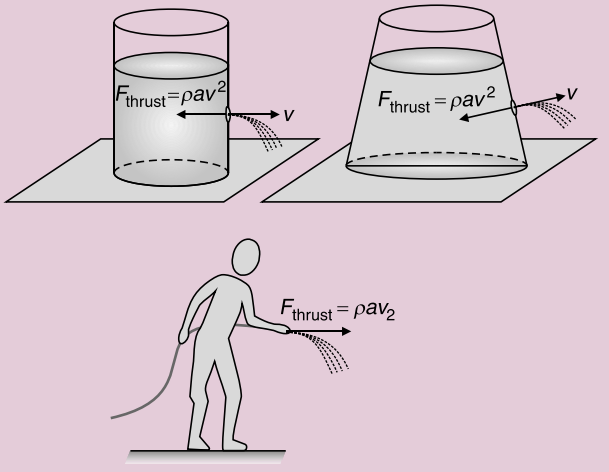


ILLUSTRATION 114

Find the velocity of efflux of water from an orifice near the bottom of a tank in which pressure is $500\ \text{gfcm}^{-2}$ above atmosphere.

SOLUTION

Pressure at orifice, $P = 500\ \text{gfcm}^{-2}$

$$\Rightarrow \quad P = 500 \times 9.8 \times \frac{(100)^2}{1000\ \text{Nm}^{-2}}$$

$$\Rightarrow \quad P = 500 \times 98\ \text{Nm}^{-2}$$

Let h be the depth of orifice below the surface
As $P = h\rho g$

$$\Rightarrow \quad h = \frac{P}{\rho g} = \frac{(500)(98)}{(10^3)(9.8)} = 5\ \text{m}$$

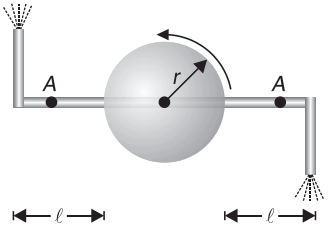
So, the velocity of efflux is

$$v = \sqrt{2gh} = \sqrt{(2 \times 9.8 \times 5)} = 9.893\ \text{ms}^{-1}$$

ILLUSTRATION 115

Figure shows the top view of a cylindrical vessel mounted on a turntable. The vessel is filled with water. At a depth h below the water surface are two horizontal tubes of length

l and cross-sectional area a, with right-angle bends at their ends. Show that, as the water jet just emerges from the tubes, there is a torque τ exerted on the system given by the expression $\tau = 4\rho gh(r+l)a$, where ρ is the density of the water.



SOLUTION

Just when the water jet leaves the tube, the velocity of efflux of water coming out of each tube is

$$v = \sqrt{2gh}$$

Thrust due to the emerging water jet stream on each tube is

$$F = av^2\rho = a(2gh)\rho$$

These forces acting opposite to each other have different line of action and hence they produce a torque given by

$$\tau = 2(Fr_{\perp})$$

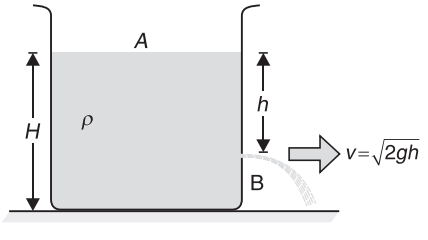
$$\Rightarrow \quad \tau = 2\left[(\rho av^2)\times(r+l)\right]$$

$$\Rightarrow \quad \tau = 2\left[(\rho a)\left(\sqrt{2gh}\right)^2\times(r+l)\right]$$

$$\Rightarrow \quad \tau = 4\rho gha(r+l)$$

SPEED OF EFFLUX: TORRICELLI'S THEOREM

Consider an ideal liquid of density ρ contained in a tank having a small orifice at a point B in its wall. If the tank is large, the velocity of liquid at any point A on its free surface can be assumed to be zero. Atmospheric pressure P acts at both A and B. If v is the velocity with which the liquid flows out of B, then according to Bernoulli's Equation, we have



$$\begin{pmatrix}\text{Total energy per}\\ \text{unit volume at}\\ \text{the orifice}\end{pmatrix} = \begin{pmatrix}\text{Total energy per}\\ \text{unit volume at}\\ \text{the surface}\end{pmatrix}$$

$$\Rightarrow \quad P_a + (H-h)\rho g + \frac{1}{2}\rho v^2 = P_a + H\rho g$$

$$\Rightarrow \quad \frac{1}{2}\rho v^2 = h\rho g$$

$$\Rightarrow \quad v = \sqrt{2gh}$$

So, according to Torricelli's Theorem , the speed of efflux through an orifice at a depth h below the free surface the liquid equals the speed that is acquired by a body falling freely from a height h.

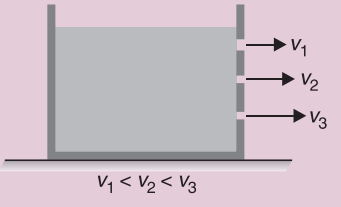
Please note that this result for the speed of efflux is applicable only when the area of the orifice a is very small compared to the area of the beaker A. However, if this is not the case i.e. the area of the orifice a is comparable to the area of the beaker A, then the speed of efflux is given by

$$v = \sqrt{2gh}\sqrt{\frac{A^2}{A^2-a^2}}$$

Problem Solving Technique(s)

(a) The velocity of efflux is independent of the nature of liquid, quantity of liquid in the vessel and the area of orifice.

(b) The greater is the depth of the orifice below the free surface the greater will be the velocity of efflux i.e., $v \propto \sqrt{h}$



RANGE OF LIQUID HITTING THE GROUND

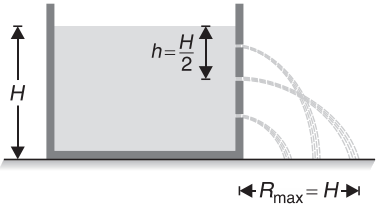
Just when the liquid leaves the orifice, the vertical velocity of liquid at the orifice is zero and since the orifice is at a height $(H-h)$ from the bottom, so the time taken by the liquid to reach the level of the base is

$$t = \sqrt{\frac{2(H-h)}{g}}$$

The range of the liquid is thus given by

$$R = vt = \sqrt{2gh}\sqrt{\frac{2(H-h)}{g}}$$

$$\Rightarrow \quad R = 2\sqrt{h(H-h)}$$



For R to be maximum i.e. for R^2 to be maximum, we have

$$\frac{d}{dh}(R^2)=0$$

$$\Rightarrow \quad \frac{d}{dh}[h(H-h)]=0$$

$$\Rightarrow \quad \frac{d}{dh}(Hh-h^2)=0$$

$$\Rightarrow \quad H-2h=0$$

$$\Rightarrow \quad h=\frac{H}{2}$$

So, range will be maximum when

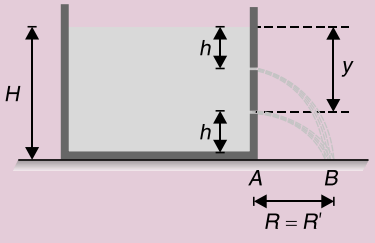
$$h=\frac{H}{2} \text{ and } R_{\max}=H$$

Conceptual Note(s)

INTERESTING OBSERVATION

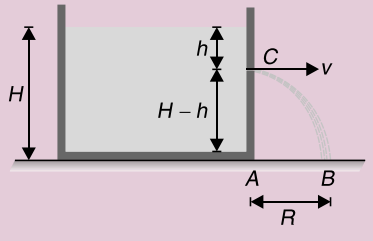
If h is replaced by $H-h$ in the above formula, then range R remains the same.

So, if the level of free surface in a container is at height H from the base and there are two holes one at depth h below the free surface and at the other at height h above the base, then



$R=2\sqrt{h(H-h)}$ and $R'=2\sqrt{(H-h)h}$

So, the range will be same if the orifice is at a depth h or $(H-h)$ below the free surface. Now as the distance $(H-h)$ from top means $H-(H-h)=h$ from the bottom, so the range is same for liquid coming out of holes at same distance below the top and above the bottom.



TIME TO EMPTY THE BEAKER TO THE ORIFICE

If a be the area of orifice and the depth of the orifice below the free surface of the liquid be y, then speed of efflux (v) of liquid coming out of the orifice is

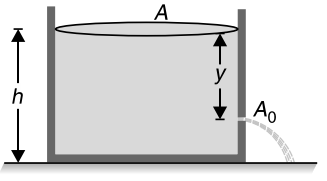
$$v=\sqrt{2gy}$$

If A is the area of the mouth of container, then volume of liquid coming out of the orifice per second will be

$$Q=-\frac{dV}{dt}=av=a\sqrt{2gy} \qquad \ldots(1)$$

Due to the liquid coming out of the orifice, the level of liquid in the container will decrease and so if the level of liquid in the container above the hole changes from y to $y-dy$ in time t to $t+dt$, then we have

$$dV=Ady$$



Substituting value of dV in equation (1), we get

$$-A\frac{dy}{dt}=a\sqrt{2gy}$$

$$\Rightarrow \quad \int_0^t dt=-\frac{A}{a}\frac{1}{\sqrt{2g}}\int_{h_1}^{h_2} y^{-\frac{1}{2}}dy$$

So, the time taken for the level to fall from h to h' is

$$t=-\frac{A}{a}\frac{1}{\sqrt{2g}}\left(\frac{y^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\Bigg|_h^{h'}\right)$$

$$\Rightarrow \quad t=\frac{A}{a}\sqrt{\frac{2}{g}}\left(\sqrt{h}-\sqrt{h'}\right)$$

ILLUSTRATION 116

A tank is filled with a liquid up to a height H. A small hole is made at the bottom of this tank. Let t_1 be the time taken to empty first half of the tank and t_2 the time taken to empty rest half of the tank. Then find $\frac{t_1}{t_2}$.

SOLUTION

Substituting the proper limits in equation (1), derived in the theory we have,

$$\int_0^{t_1} dt = -\frac{A}{a\sqrt{2g}} \int_H^{H/2} y^{-1/2} dy$$

$$\Rightarrow \quad t_1 = \frac{2A}{a\sqrt{2g}} \left[\sqrt{y}\right]_{H/2}^{H}$$

$$\Rightarrow \quad t_1 = \frac{2A}{a\sqrt{2g}} \left[\sqrt{H} - \sqrt{\frac{H}{2}}\right]$$

$$\Rightarrow \quad t_1 = \frac{A}{a}\sqrt{\frac{H}{g}}\left(\sqrt{2}-1\right) \qquad \ldots(1)$$

Similarly, $\int_0^{t_2} dt = -\frac{A}{a\sqrt{2g}} \int_{H/2}^{0} y^{-1/2} dy$

$$\Rightarrow \quad t_2 = \frac{A}{a}\sqrt{\frac{H}{g}} \qquad \ldots(2)$$

From equations (1) and (2), we get

$$\frac{t_1}{t_2} = \sqrt{2}-1$$

$$\Rightarrow \quad \frac{t_1}{t_2} = 0.414$$

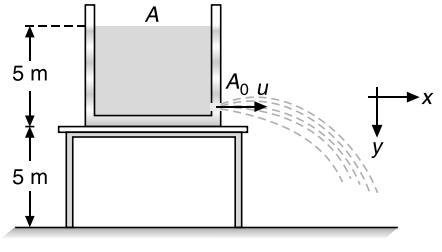
From here we see that $t_1 < t_2$. This is because initially the pressure is high and the liquid comes out with greater speed.

ILLUSTRATION 117

A cylindrical tank $1\ \text{m}$ in radius rests on a platform $5\ \text{m}$ high. Initially the tank is filled with water up to a height of $5\ \text{m}$. A plug having an area of $10^{-4}\ \text{m}^2$ is removed from the orifice at the bottom of the tank. Calculate the speed with which the water leaves the orifice and the speed with which it hits the ground. Also calculate the time taken to empty the tank to half its original volume. Does this time depend on the height of platform above ground?

SOLUTION

The situation given in the problem is shown in Figure.



The speed with which water leaves the orifice is equal to the speed of efflux given by

$$u_x = u = \sqrt{2gh} = \sqrt{2\times10\times5} = 10\ \text{ms}^{-1}$$

Since initial vertical velocity of water is zero, so its vertical velocity when it hits the ground is

$$v_y = \sqrt{2gh} = \sqrt{2\times10\times5} = 10\ \text{ms}^{-1}$$

Also, motion is uniform along the horizontal direction, so

$$v_x = u_x = u$$

Hence the speed (v) with which water strikes the ground is

$$v = \sqrt{v_x^2 + v_y^2} = 10\sqrt{2} = 14.1\ \text{ms}^{-1}$$

When height of water level above the orifice is y, then velocity of efflux will be $v = \sqrt{2gy}$ and so rate of flow of water is

$$\frac{dV}{dt} = A_0 v = A_0\sqrt{2gy}$$

$$\Rightarrow \quad -Ady = \left(\sqrt{2gy}\right)A_0 dt \qquad \{\because dV = -Ady\}$$

$$\Rightarrow \quad \int_H^0 \frac{Ady}{\sqrt{2gy}} = \int_0^t A_0 dt$$

$$\Rightarrow \quad t = \frac{A}{A_0}\sqrt{\frac{2}{g}}\left(\sqrt{H} - \sqrt{H'}\right)$$

$$\Rightarrow \quad t = \left(\frac{\pi\times1^2}{10^{-4}}\right)\sqrt{\frac{2}{10}}\left[\sqrt{5} - \sqrt{\left(\frac{5}{2}\right)}\right]$$

$$\Rightarrow \quad t = 9.2\times10^3\ \text{s} \approx 2.5\ \text{h}$$

The expression of t is independent of height of platform on which beaker is kept.

ILLUSTRATION 118

At what velocity does the water emerge from a small orifice in an open tank if the gauge pressure at the orifice is $2\times10^5\ \text{Nm}^{-2}$ before the flow starts?

SOLUTION

Since the tank is open, velocity of efflux can be directly given by Torricelli's law.

$$v = \sqrt{2gh} \qquad \ldots(1)$$

If the gauge pressure of $2\times10^5\ \text{Nm}^{-2}$ corresponds to a height h, then we have

$$h\rho_w g = \Delta P = 2\times10^5\ \text{Nm}^{-2}$$

$$\Rightarrow \quad gh = \frac{2\times10^5}{\rho_w}$$

$$\Rightarrow \quad gh = \frac{2\times10^5}{10^3}$$

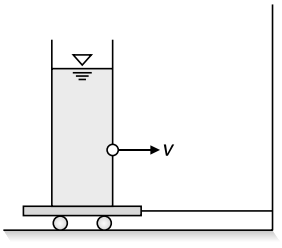
Substituting this value of gh in equation (1), we get

$$v = \sqrt{\frac{2\times2\times10^5}{10^3}}$$

$$\Rightarrow \quad v = 20\ \text{ms}^{-1}$$

ILLUSTRATION 119

A large tank of height $h = 1$ m and diameter $D = 0.6$ m is affixed to a cart as shown. Water issues from a tank through a nozzle of diameter $d = 10$ mm. The speed of the liquid leaving the tank is approximately $v = \sqrt{2gy}$ where y is the height from the nozzle to the free surface. Determine the tension in the wire when $y = 0.8$ m. Plot the tension in the wire as a function of water depth $0 \le y \le 0.8$ m. Density of water is 10^3 kgm^{-2} and $g = 9.8$ ms^{-2}.



SOLUTION

Tension in the thread is equal to the thrust force due to liquid on the beaker, so we have

$$T = \rho a v^2$$

where, $a = \left(\frac{\pi}{4}\right)(d^2)$ and $v^2 = 2gy$

$$\Rightarrow \quad T = \rho\left(\frac{\pi}{4}\right)(d^2)(2gy) \qquad \ldots(1)$$

$$\Rightarrow \quad T = (10^3)\left(\frac{\pi}{4}\right)(10^{-2})^2(2)(9.8)(0.8)$$

$$\Rightarrow \quad T = 1.23 \text{ N}$$

From equation (1) $T \propto y$

i.e. T-y graph is a straight line passing through origin as shown in Figure.

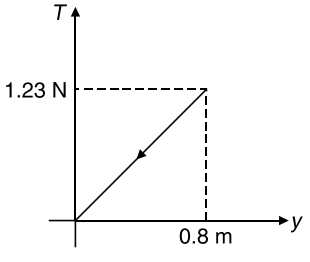
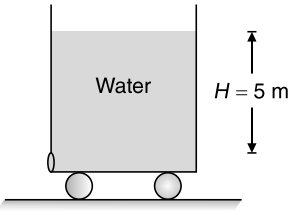


ILLUSTRATION 120

A tank, initially at rest, is filled with water to a height $H = 5$ m. A small orifice is made at the bottom of the wall. Find the velocity attained by the tank when it becomes completely empty. Assume mass of the tank to be negligible. Friction is negligible. Take $g = 10$ ms^{-2}.



SOLUTION

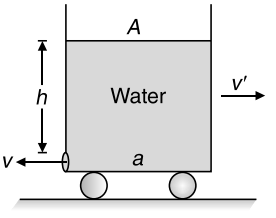
Let v' be the instantaneous velocity of the tank, and v be the instantaneous velocity of efflux with respect to the tank. **Thrust** exerted on the tank is

$$F = \rho a v^2$$

where a is the cross-sectional area of the orifice.

$$v = \sqrt{2gh}$$

where h is the depth of the orifice below the free surface of water in the tank.



If A be the cross-sectional area of the tank, then mass of the tank at any time t is

$$m = \rho A h$$

According to Newton's Second Law, we have

$$F = m\frac{dv'}{dt} = \rho A h\frac{dv'}{dt}$$

$$\Rightarrow \quad \rho A h\frac{dv'}{dt} = \rho a v^2 = 2\rho g a h$$

$$\Rightarrow \quad \frac{dv'}{dt} = 2g\left(\frac{a}{A}\right) \qquad \ldots(1)$$

In a time dt, if the water level falls by dh, then by Law of conservation of mass applied to liquids, we have

$$-\rho A dh = \rho a v dt$$

$$\Rightarrow \quad \frac{dh}{dt} = -\frac{av}{A}$$

Equation (1) can be re-written as

$$\left(\frac{dv'}{dh}\right)\left(\frac{dh}{dt}\right) = 2g\left(\frac{a}{A}\right)$$

$$\Rightarrow \quad \frac{dv'}{dh}\left(-\frac{av}{A}\right) = 2g\left(\frac{a}{A}\right)$$

$$\Rightarrow \quad \frac{dv'}{dh} = -\frac{2g}{v} = -\frac{2g}{\sqrt{2gh}} = -\sqrt{\frac{2g}{h}}$$

On integrating, we get

$$\int_0^{v'} dv' = -\sqrt{2g}\int_H^0 \frac{dh}{\sqrt{h}}$$

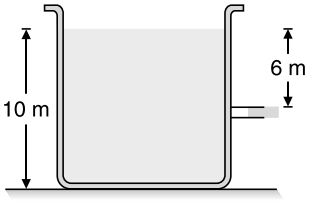
$$\Rightarrow \quad v = 2\sqrt{2gH}$$

Since $H = 5$ m,

$$\Rightarrow \quad v = 2\sqrt{2(10)(5)} = 20 \text{ ms}^{-1}$$

ILLUSTRATION 121

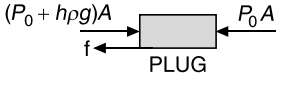
A fresh water on a reservoir is 10 m deep. A horizontal pipe 4 cm in diameter passes through the reservoir 6 m below the water surface as shown in Figure. A plug secures the pipe opening.



Calculate the frictional force between the plug and pipe wall. If the plug is removed, then calculate the volume of water flows out of the pipe in 1 h. Assume area of reservoir to be too large.

SOLUTION

The force of friction between the plug and the inner surface of the pipe is equal to the force due to pressure at the depth of 6 m. The free body diagram for the plug is shown in Figure.



For equilibrium, we have

$$f + P_0A = (P_0 + h\rho g)A$$

$$\Rightarrow \quad f = (h\rho g)A$$

$$\Rightarrow \quad f = (10^3)(9.8)(6.0)(\pi)(2\times10^{-2})^2$$

$$\Rightarrow \quad f = 73.9 \text{ N}$$

Assuming the area of the reservoir to be too large, the velocity of efflux will be

$$v = \sqrt{2gh} \approx \text{constant}$$

$$\Rightarrow \quad v = \sqrt{2\times9.8\times6} = 10.84 \text{ ms}^{-1}$$

Volume of water coming out per second is

$$Q = Av = \pi(2\times10^{-2})^2(10.84)$$

$$\Rightarrow \quad Q = 1.36\times10^{-2} \text{ m}^3\text{s}^{-1}$$

So, the volume of water flowing through the pipe in $t = 1$ hr $= 3600$ s is

$$\Rightarrow \quad V = Qt = \left(\frac{dV}{dt}\right)t = (1.36\times10^{-2})(3600)$$

$$\Rightarrow \quad V = 48.96 \text{ m}^3$$

ILLUSTRATION 122

A cylindrical tank of base area A has a small hole of area a at the bottom. At time $t = 0$, a tap starts to supply water into the tank at a constant rate α m^{3}s^{-1}. Calculate the maximum level of water h_{max} in the tank. Also calculate the time when level of water becomes $h(< h_{max})$.

SOLUTION

Level will be maximum when

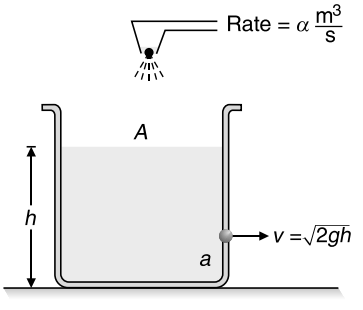
rate of inflow of water = rate of outflow of water

i.e., $\alpha = av$

$$\Rightarrow \quad \alpha = a\sqrt{2gh_{max}}$$

$$\Rightarrow \quad h_{max} = \frac{\alpha^2}{2ga^2}$$

Let at time t, the level of water be h. Then



$$A\left(\frac{dh}{dt}\right) = \alpha - a\sqrt{2gh}$$

$$\Rightarrow \quad \int_0^h \frac{dh}{\alpha - a\sqrt{2gh}} = \int_0^t \frac{dt}{A}$$

On solving this equation, we get

$$t = \frac{A}{ag}\left[\frac{\alpha}{a}\ln\left(\frac{\alpha - a\sqrt{2gh}}{\alpha}\right) - \sqrt{2gh}\right]$$

ILLUSTRATION 123

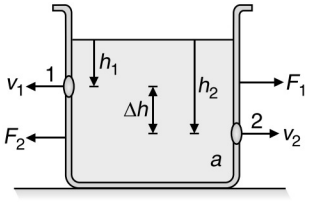
On the opposite sides of a wide vertical vessel filled with water, two identical holes each having cross-sectional area A are opened. The height difference between the holes is equal to Δh. Calculate the force of reaction due to the water flowing out of vessel.

SOLUTION

Force of reaction due to ejection of fluid through a hole is

$$F = Av^2\rho$$

This force is opposite to the velocity of efflux of the liquid coming out from the orifice.



Due to two holes, net force on vessel is

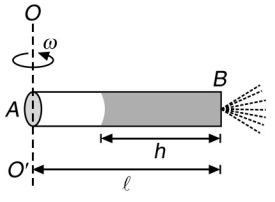
$$F = F_2 - F_1 = A\left(v_1^2 - v_2^2\right)\rho$$

$$\Rightarrow \quad F = A\left[\left(\sqrt{2g(h+\Delta h)}\right)^2 - \left(\sqrt{2gh}\right)^2\right]\rho$$

$$\Rightarrow \quad F = 2\rho g A\Delta h$$

ILLUSTRATION 124

A horizontal oriented tube AB of length l rotates with a constant angular velocity ω about a stationary vertical axis OO' passing through the end A as shown in Figure.



The tube is filled with an ideal fluid. The end A of the tube is open, the closed end B has a very small orifice. Find the velocity of the fluid relative to the tube as a function of the column height h.

SOLUTION

Using Bernoulli's theorem, we get

$$P_B = P_{atm} + \frac{1}{2}\rho v^2 \qquad \ldots(1)$$

Also, we know that

$$P_B - P_{atm} = \int_{l-h}^{l} \rho\omega^2 x\,dx$$

$$\Rightarrow \quad P_B - P_{atm} = \frac{\rho\omega^2}{2}\left[l^2 - (l-h)^2\right]$$

$$\Rightarrow \quad P_B - P_{atm} = \frac{1}{2}\rho\omega^2\left(2lh - h^2\right)$$

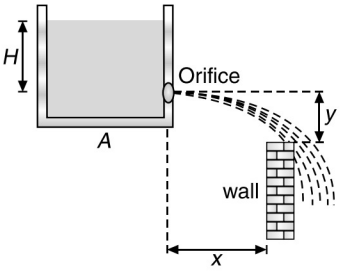
Using equation (1), we get

$$\frac{1}{2}\rho\omega^2\left(2lh - h^2\right) = \frac{1}{2}\rho v^2$$

$$\Rightarrow \quad v = \omega h\sqrt{\frac{2l}{h} - 1}$$

ILLUSTRATION 125

For the arrangement shown in Figure, calculate the time interval after which the water jet ceases to cross the wall. Assume area of the tank to be A and area of orifice to be $a \ll A$.



SOLUTION

At any instant, when liquid level is at a height h, then the speed of ejection of water is

$$v = \sqrt{2gh} \qquad \{\because a \ll A\}$$

The water jet will be crossing the wall when

$$x = vt = v\sqrt{\frac{2y}{g}}$$

$$\Rightarrow \quad x = \sqrt{2gh}\sqrt{\frac{2y}{g}}$$

$$\Rightarrow \quad h = \frac{x^2}{4y}$$

According to Equation of Continuity, we have

$$-A\frac{dh}{dt} = a\sqrt{2gh}$$

$$\Rightarrow \quad \int_H^h \frac{dh}{\sqrt{h}} = -\frac{a}{A}\sqrt{2g}\int_0^t dt$$

$$\Rightarrow \quad 2\left(\sqrt{H} - \sqrt{h}\right) = \frac{a}{A}\sqrt{2gt}$$

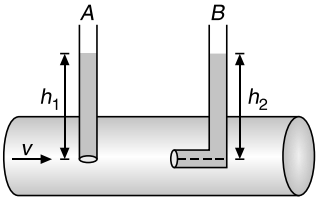
$$\Rightarrow \quad t = \frac{A}{a}\sqrt{\frac{2}{g}}\left(\sqrt{H} - \sqrt{\frac{x^2}{4y}}\right)$$

Test Your Concepts-IX

Based on Equation of Continuity, Bernoulli's Theorem and Applications

(Solutions on page H.14)

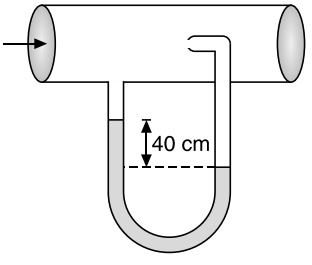
1. A liquid is flowing in a horizontal pipe of uniform cross section with velocity v. Two tubes A and B of small cross-sectional area are inserted into the pipe as shown.



Assuming the flow to remain streamline inside the pipe, calculate the difference in height of the liquid in two tubes

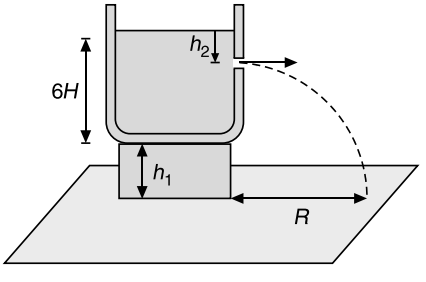
2. A cylindrical bucket, open at the top, is $0.2\ \text{m}$ high and $0.1\ \text{m}$ in diameter. A circular hole with cross-section area $1\ \text{cm}^2$ is cut in the centre of the bottom of the bucket. Water flows into the bucket from a tube above it at the rate of $1.3\times10^{-4}\ \text{m}^3\text{s}^{-1}$. How high will the water in the bucket rise?

3. A pitot tube is mounted on an aeroplane wing to measure the speed of the plane. The tube contains alcohol and shows a level difference of $40\ \text{cm}$ as shown in Figure.



Calculate the speed of the plane relative to air if relative density of alcohol is 0.8 and density of air is $1\ \text{kgm}^{-3}$.

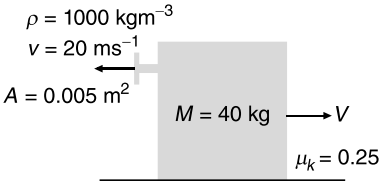
4. A beaker containing liquid to a height $6H$ is having an orifice at a depth h_2 below the free surface of the liquid. The beaker is placed on a horizontal platform of height h_1 as shown in Figure.



Calculate the value of h_2 for range R to be maximum, if $h_1 = 4H$ and $h_1 = 8H$. Also find the value of this maximum range in both cases.

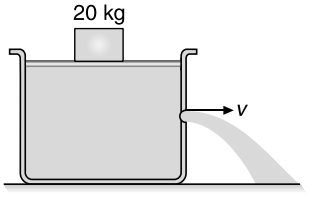
5. On the opposite side of a wide vertical vessel filled with water two identical holes are opened, each having the cross-sectional area $0.50\ \text{cm}^2$. The height difference between them is equal to $51\ \text{cm}$. Calculate the resultant force of reaction of the water flowing out of the vessel.

6. A slider assembly of mass $m = 40\ \text{kg}$ moves under the influence of a liquid jet. At any instant, the slider is moving to the right with a speed $V = 10\ \text{ms}^{-1}$ by emitting a water of density $1000\ \text{kgm}^{-3}$ from an area of $A = 0.005\ \text{m}^2$ with a velocity of $v = 20\ \text{ms}^{-1}$ to the left as shown in Figure.

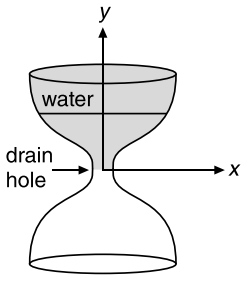


If the coefficient of kinetic friction between the slider and the surface is $\mu_k = 0.25$, then calculate the acceleration of the slider at that instant. Also calculate terminal speed of the slider.

7. A long cylindrical tank of cross-section area $0.5\ \text{m}^2$ is filled with water. It has a hole of cross-section $1\times10^{-4}\ \text{m}^2$ at a height $50\ \text{cm}$ from the bottom. A movable piston of cross-sectional area almost equal to $0.5\ \text{m}^2$ is fitted on the top of the tank such that it can slide in the tank freely. A load of $20\ \text{kg}$ is applied on the top of the water by piston, as shown in the figure. Calculate the speed of the water jet with which it hits the surface when piston is $1\ \text{m}$ above the base. Ignore the mass of the piston and take $g = 10\ \text{ms}^{-2}$.



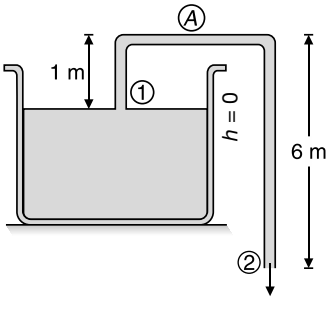
8. The shape of an ancient water clock jug is such that water level descends at a constant rate at all times. If the water level falls by 4 cm every hour, determine the shape of the jar, i.e., specify x as a function of y. Assume that the radius of drain hole is 2 mm which is very small compared to x.



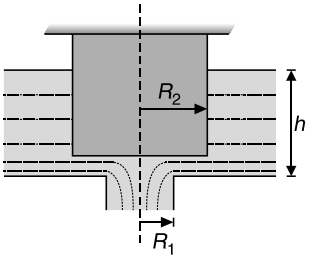
9. Calculate the rate of flow of glycerine of density 1.25×10^3 kgm^{-3} through the conical section of a pipe, if the radii of its ends are 0.1 m and 0.04 m and the pressure drop across its length is 10 Nm^{-2}.

10. A cylindrical vessel is filled with water upto a height of 1 m. The cross-sectional area of the orifice at the bottom is $\left(\frac{1}{400}\right)$ that of the vessel. Calculate the time required to empty the tank through the orifice at the bottom. Also calculate the time required for the same amount of water to flow out, if the water level in tank is maintained always at a level of 1 m above the orifice.

11. The U-tube acts as a water siphon. The bend in the tube is 1 m above the water surface. The tube outlet is 5 m below the water surface. The water issues from the bottom of the siphon as a free jet at atmospheric pressure. Determine the speed of the free jet and the minimum absolute pressure of the water in the bend. Given atmospheric pressure to be 10^5 Nm^{-2}, density of water to be 10^3 kgm^{-3} and $g = 9.8$ ms^{-2}.



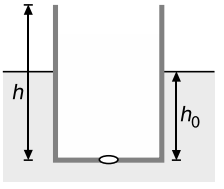
12. The horizontal bottom of a wide vessel with an ideal fluid has a round orifice of radius R_1 over which a round closed cylinder is mounted, whose radius $R_2 > R_1$ as shown in Figure.



The clearance between the cylinder and the bottom of the vessel is very small, the fluid density is ρ. Assuming the atmospheric pressure to be P_0 calculate the static pressure of the fluid in the clearance as a function of the distance r from the axis of the orifice (and the cylinder) if the height of the fluid is equal to h. Assume that $R_1 < r < R_2$.

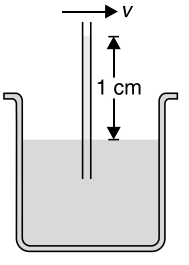
13. A large tank of area A, filled with water to a height H is to be emptied to half the height through a small hole of area a at the bottom. Calculate the ratio of times taken for the level of water to fall from H to $\frac{H}{2}$ and from $\frac{H}{2}$ to zero.

14. A cylindrical can of height h and base area A is immersed in water to a depth h_0 as shown in Figure.

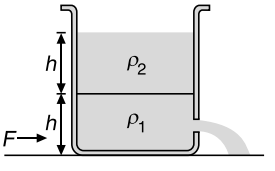


If there is a small hole of area a in the base of the can, then calculate the time taken by the can to sink.

15. When air of density 1.3 kgm^{-3} flows across the top of the tube shown in Figure, water rises in the tube to a height of 1 cm. Calculate the speed of air.



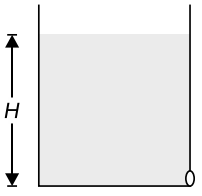
16. A cylindrical tank having cross-sectional area $A = 0.5$ m^2 is filled with two liquids of density $\rho_1 = 900$ kgm^{-3} and $\rho_2 = 600$ kgm^{-3} to a height $h = 60$ cm each as shown in Figure.



A small hole having area $a = 5$ cm^2 is made in right vertical wall at a height $y = 20$ cm from the bottom. Calculate the velocity of efflux and the horizontal force F required to keep the cylinder in static equilibrium, if it is placed on a smooth horizontal plane. If coefficient of friction between the cylinder and the surface on which it is kept is $\mu = 0.01$, then calculate the maximum

and minimum values of F to keep the cylinder in equilibrium. Take $g = 10\ \text{ms}^{-2}$.

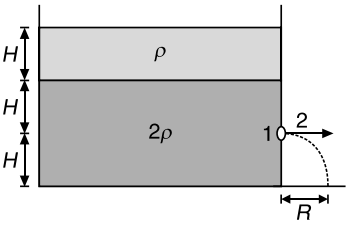
17. Find the time to empty the container filled with liquid up to height H initially. Area of hole at bottom is a and area of cross-section of the base of container is A.



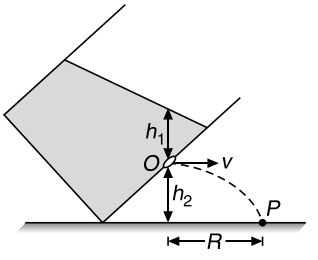
18. The side wall of side vertical cylindrical vessel of height $h = 75\ \text{cm}$ has a narrow vertical slit running all the way down to the bottom of the vessel. The length of the slit is $l = 50\ \text{cm}$ and the width $b = 1.0\ \text{mm}$. With the slit closed, the vessel is filled with water. Calculate the resultant force of reaction of the water flowing out of the vessel immediately after the slit is opened.

19. A water line with an internal radius of 6.5×10^{-3} m is connected to a shower head that has 12 holes. The speed of the water in the line is $1.2\ \text{ms}^{-1}$. Calculate the volume flow rate in the line. Also calculate the speed with which the water leaves one of the holes in the head assuming the effective hole radius to be 4.6×10^{-4} m.

20. A cylindrical tank is filled with two liquids of density ρ and 2ρ as shown in Figure. Calculate the range R of the liquid coming out from the orifice.



21. A beaker is kept such that the orifice O is located as shown in Figure. Calculate the speed v with which liquid comes out of the orifice, the time taken by the liquid to hit the ground and the range R.



22. A wide vessel with a small hole in the bottom is filled with water and kerosene. Neglecting the viscosity, find the velocity of the water flow, if the thickness of the water layer is equal to $h_1 = 30\ \text{cm}$ and that of the kerosene layer to $h_2 = 20\ \text{cm}$. Density of kerosene is $600\ \text{kgm}^{-3}$.

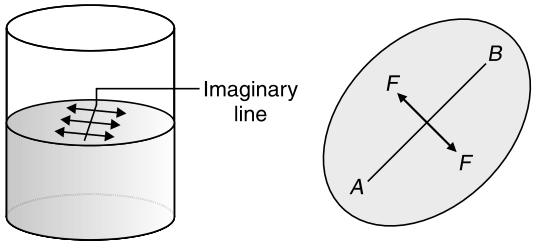
23. Liquid is filled in a container upto a height of H. A small hole is made at the bottom of the tank. Time take to empty from H to $\frac{H}{3}$ is t_0. Find the time taken to empty the tank from $\frac{H}{3}$ to zero.

SURFACE TENSION

SURFACE TENSION: INTRODUCTION

A liquid has the property that its free surface tends to attain minimum possible area and is therefore, in a state of tension, somewhat like a stretched membrane. It is a molecular phenomenon based on electromagnetic interaction between the molecules. The force of contraction at right angles to an imaginary line of unit length, tangential to the surface of liquid, is called its **surface tension.**

Surface tension of a liquid is measured by the force acting per unit length on either side of an imaginary line drawn on the free surface of liquid as shown in Figure.



The direction of this force being perpendicular to the imaginary line and tangential to the free surface of liquid. So if F is the force acting on one side of imaginary line AB of length l, then mathematically surface tension is

$$T = \frac{F}{l}$$

The S.I. unit of surface tension is newton per metre (Nm^{-1}) and dynecm^{-1}. The Dimensional formula of surface tension is MT^{-2} which is same as that of force constant or spring constant.

Surface Tension depends only on the nature of liquid and is independent of the area of surface or length of line considered. It is a scalar as it has a unique direction which is not to be specified.

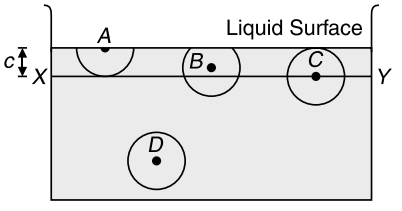
Small liquid drops tend to be spherical due to surface tension, because for a given volume, sphere has the minimum surface area. It follows that in order to increase the surface area of liquid, work has to be done against this force of contraction. This work is stored in the surface as its potential energy.

EXPLANATION TO SURFACE TENSION

Laplace explained the phenomenon of surface-tension on the basis of inter-molecular forces. We know that if the distance between two molecules is less than the molecular range $c(\approx 10^{-9}\ \text{meter})$ then they attract each other, but if the distance is more than this range, then attraction becomes negligible.

Therefore, if we draw a sphere of radius c with a molecule as center, then only those molecules which are enclosed within this sphere can attract, or be attracted by, the molecule at the center of the sphere. This is called 'sphere of molecular activity'.

In order to understand the tension existing in the free surface of a liquid, let us consider four liquid molecules like A, B, C and D along with their spheres of molecular activity as shown in Figure.



The molecule D is well inside the liquid and so it is attracted equally in all directions. Hence the resultant force acting on molecule D is zero.

The sphere of molecule C is just below the liquid surface and the resultant force on it is also zero.

The molecule B which is a little below the liquid surface has its sphere of molecular activity partly outside the liquid. Thus, the number of liquid molecules in upper half (attracting in downward). Hence the molecule B experience a resultant downward force.

The molecule A is at the surface of the liquid, so that its sphere of molecular activity is half outside the liquid and half inside. So it experiences a maximum downward force.

Hence all the molecules situated between the surface and a plane XY, distance c below the surface, experience a resultant downward cohesive force.

When the surface area of liquid is increased, molecules from the interior of the liquid rise to the surface. As these molecules reach near the surface, work is done against the (downward) cohesive force. This work is stored in the molecule in the form of potential energy. Thus, the potential energy of the molecules lying at the surface is greater than that of the molecules in the interior of the liquid.

Since we know that a system is in stable equilibrium when it has minimum potential energy. So, in order to have minimum potential energy, the liquid surface tends to have minimum number of molecules in it (which is possible if the free surface tries to attain the minimum surface area). In other words, the surface tends to contract to a minimum possible area. *This tendency of the free surface of the liquid is exhibited as surface tension.*

Conceptual Note(s)

It must be noted that surface tension is a property of a liquid and it does not depend on the area of the free surface of the liquid. For example, both containers have the same liquid under same conditions. $A_2 > A_1$ but surface tension will be same in both cases.

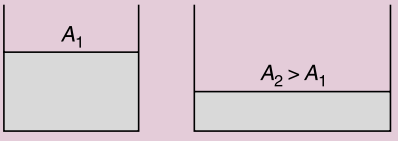


ILLUSTRATION 126

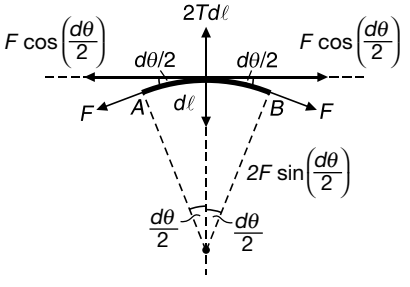
Consider a horizontal film of soap solution, on which a light thread soaked in soap solution is placed in the form of a loop. The film is then pierced inside the loop and the thread becomes a circular loop of radius R. If the surface tension of the soap solution be T, then calculate the tension in the thread.

SOLUTION

Let the tension in the thread be F. Consider a small arc AB of length dl of the circular loop. Let this arc subtend an angle $d\theta$ at the centre of the circular loop of radius R. On this small arc, we observe the following two forces to be acting.

(a) Force due to surface tension $2T\Delta l$, acting outwards.

(b) Component of tension force $2F\sin\left(\frac{d\theta}{2}\right)$, acting inwards.



At equilibrium, we have

$$2F\sin\left(\frac{d\theta}{2}\right) = 2Tdl \qquad \ldots(1)$$

Since $d\theta$ is small, so

$$\sin\left(\frac{d\theta}{2}\right) \approx \frac{d\theta}{2}, \text{ where } d\theta = \frac{dl}{R}$$

So, from (1), we get

$$2F\left(\frac{d\theta}{2}\right) = Fd\theta = 2T(Rd\theta)$$

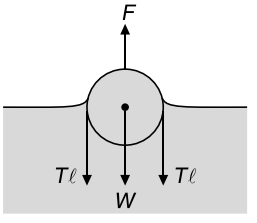
$$\Rightarrow \quad F = 2TR$$

FORCE DUE TO SURFACE TENSION

If a body of weight W is placed on the liquid surface, whose surface tension is T. Let us calculate the value of minimum force F required to pull it away from the water for different body shapes.

CASE-1: Needle on water surface

If a needle of length l is placed on the liquid surface whose cross-sectional view is shown in Figure. as shown in Figure.

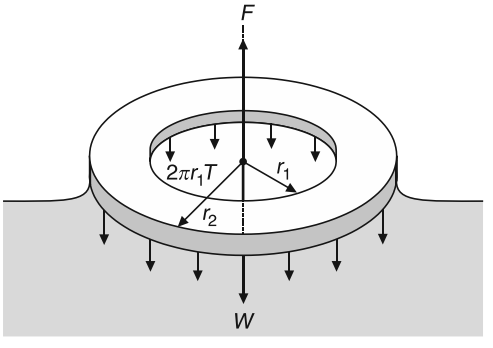


Minimum force required to lift up the needle is

$$F = 2lT + W$$

CASE-2: Annular disc on water surface

If an annular disc of inner radius r_1 and outer radius r_2 is placed on the liquid surface and we try to pull it up with a minimum force F, then forces acting on the disc are shown in Figure.



The weight W acts vertically downwards, the force due to surface tension acting on outer perimeter of the disc is $2\pi r_2 T$ (downwards) and that acting on inner perimeter of the disc is $2\pi r_1 T$ (downwards). The applied minimum force should be equal to the sum of these forces, so we have

$$F = 2\pi(r_1 + r_2)T + W$$

Special Case(s)

(a) **For a disc** of radius r, we can think $r_1 = 0$ and $r_2 = r$, so the minimum force F is

$$F = 2\pi rT$$

(b) **For a ring** of radius r, we can think $r_1 \approx r_2 \approx r$, so the minimum force F is

$$F \approx 2\pi(r + r)T = 4\pi rT$$

(c) For a square plate of side l, the minimum force required to pick up the plate is

$$F = 4lT + W$$

(d) For a square frame of side l, the minimum force required to pick up the frame is

$$F = (4lT + 4lT) + W = 8lT + W$$

ILLUSTRATION 127

A ring is cut form a platinum tube of 8.5 cm internal and 8.7 cm external diameter. It is supported horizontally from a pan of a balance so that it comes in contact with the water in a glass vessel. Calculate the surface tension of water if an extra 3.97 gwt is required to pull it away from water. Take $g = 980\ \text{cms}^{-2}$.

SOLUTION

The ring is in contact with water along its inner and outer circumference. When pulled out, the total force on it due to surface tension will be

$$F = T(2\pi r_1 + 2\pi r_2) + mg$$

$$\Rightarrow \quad T = \frac{F - mg}{2\pi(r_1 + r_2)}$$

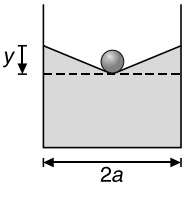
Given that an extra 3.97 gwt is required to pull it away from water so we have

$$F - mg = 3.97 \text{ gwt} = (3.97)(980) \text{ dyne}$$

$$\Rightarrow \quad T = \frac{3.97 \times 980}{3.14 \times (8.5 + 8.7)} = 72.13 \text{ dynecm}^{-1}$$

ILLUSTRATION 128

A container of width $2a$ is filled with a liquid. A thin wire of linear mass density λ is gently placed over the liquid surface in the middle of surface as shown. As a result, the liquid surface is depressed by a distance $y\,(y \ll a)$. Determine the surface tension of the liquid.

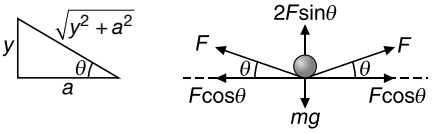


SOLUTION

If l be the length of wire and λ is mass per unit length of wire, then weight of wire is

$mg = (\lambda x)g$, acting vertically downwards

The force due to surface tension acting on each side of the wire is $F = Tl$. The free body diagram showing forces acting on the wire is shown in Figure.



Vertical force acting upwards due to surface tension balances the weight of the wire, so we have

$$2Tl\sin\theta = mg = (\lambda l)g$$

$$\Rightarrow \quad T = \frac{\lambda g}{2\sin\theta} \qquad \ldots(1)$$

Since $y \ll a$, so θ is small and hence $\sin\theta \approx \theta \approx \frac{y}{a}$

So, from equation (1), we get

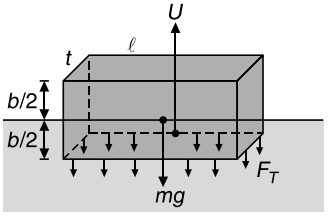
$$T = \frac{\lambda g}{2(y/a)} = \frac{\lambda a g}{2y}$$

ILLUSTRATION 129

A glass plate of length 40 cm, height 20 cm and thickness 10 cm weighs 5 kg in air. If the plate is held vertically with long side horizontal and half its height immersed in a liquid, then calculate the apparent weight of the plate assuming the surface tension of the liquid to be $50 \times 10^{-2}\ \text{Nm}^{-1}$.

SOLUTION

The glass plate is held inside liquid as shown in Figure.



According to the problem, we have $l = 40$ cm, $b = 20$ cm, $t = 10$ cm and $T = 50 \times 10^{-3}\ \text{Nm}^{-1}$. The glass plate is under the action of following forces

(a) weight mg, acting vertically downwards

(b) force (F_T) due to surface tension, acting vertically downwards such that

$$F_T = 2(l + t)T = 2\left(\frac{40}{100} + \frac{10}{100}\right)(0.5) = 0.5 \text{ N}$$

(c) upthrust (U), acting vertically upwards, experienced due to the immersed part of the plate such that

$$U = V_{\text{immersed}}\rho_{\text{liquid}}g = \left(\frac{ltb}{2}\right)\rho_w g$$

$$\Rightarrow \quad U = \left[\left(\frac{40 \times 10 \times 20}{2}\right) \times 10^{-6}\right](10^3)(10)$$

$$\Rightarrow \quad U = 40 \text{ N}$$

So, apparent weight of the glass plate is

$$W' = mg + F_T - U$$

$$\Rightarrow \quad W' = 50 + 0.5 - 40 = 10.5 \text{ N}$$

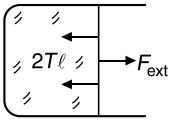
SURFACE ENERGY

A liquid molecule completely inside the liquid is surrounded by similar molecules on all sides and hence experiences no resultant force on it, whereas a molecule at the free surface of liquid is surrounded by similar liquid molecules on one side of the free surface (while on the other side it may be surrounded by air molecules or the molecules of the vapour of the liquid etc). Since air or liquid vapours have negligible density compared to liquid, so they exert only a small force on the molecules at the free surface. Hence, a resultant inward force acts on molecule lying at the surface. This inward force tries to pull the molecule into the liquid due to which the free surface layer remains in microscopic turbulence in which the molecules are pulled back from the free surface layer to the liquid bulk and hence new molecules from the liquid bulk come to the surface in an attempt to fill the empty space.

When a molecule is taken from inside the liquid to the free surface, then work has to be done against the inward resultant force for moving the molecule up to the free surface. The potential energy is increased due to this work. A molecule at the free surface has greater potential energy than a molecule completely inside the liquid. *This extra energy possessed by the free surface layer is called the surface energy.*

RELATION BETWEEN SURFACE ENERGY AND SURFACE TENSION

To find the relation between surface energy and surface tension, let us consider a U-shaped frame that guides a sliding wire on its arm. Let the arrangement be dipped in a soap solution, then taken out and placed in a horizontal position as shown in Figure.



We may think that the soap film formed is extremely thin, however at the molecular scale its thickness is not ignorable as it may possess several hundred thousand molecular layers. So, we say that the soap film has two free surfaces both of which are in contact with the sliding wire and hence exert forces of surface tension on the wire.

If T be the surface tension of the soap solution and l be the length of the sliding wire, then each surface will pull the wire parallel to itself with a force Tl and hence the net force of pull F on the wire due to both the surfaces is

$$F = Tl + Tl = 2Tl$$

In order to keep the wire in equilibrium, we have to apply a constant external force F_{ext} equal and opposite to F.

Now, let the wire be slowly pulled out (so that change in kinetic energy is zero) with the help of external force through a distance x so that the area of the frame is increased by $\Delta A = lx$. Since the film has two free surfaces of the soap solution, hence total change in surface area of the film is $2\Delta A = 2lx$.

The work done by the external force in moving the wire through x is

$$W_{ext} = F_{ext}x = (2Tl)x = T(2lx) = 2T\Delta A$$

Since there is no change in kinetic energy, so the work done by external force is stored as the change in potential energy ΔU of the surface, so we have

$$\Delta U = W_{ext} = 2T\Delta A$$

$$\Rightarrow \quad T = \frac{\Delta U}{2\Delta A} = \frac{W_{ext}}{\Delta A_{total}} \qquad \ldots(1)$$

where, $\Delta A = lx$ is the change in surface area of each free surface.

So, we observe that the surface tension of a liquid is equal to the surface energy per unit change in surface area.

So, from equation (1), we observe that the work done by an external force to increase the surface area of the film by ΔA_{total} is

$$W_{ext} = T\Delta A_{total} = 2T\Delta A_{\text{each surface}}$$

However, for the case where we may have only one free surface, then $\Delta A_{total} = \Delta A$ and hence

$$W_{ext} = T\Delta A_{total} = T\Delta A$$

The SI unit of surface tension is also written as Jm^{-2} and it can also be verified that $1\ \text{Nm}^{-1} = 1\ \text{Jm}^{-2}$.

Conceptual Note(s)

Please note that the work done to change the surface area of

(a) a soap film or a liquid film will be $W = 2T\Delta A$, because a soap film or a liquid film has two free surfaces.

(b) a soap bubble will be $W = 2T\Delta A$, because a soap bubble has two free surfaces.

(c) an air bubble or a liquid drop will be $W = T\Delta A$, because an air bubble or a liquid drop has only one free surface.

(d) In case of the liquid drop, if the initial radius of liquid drop (having surface tension T) is f_1 and final radius is f_2, then work done in changing the radius from f_1 to f_2 is

$$W = T\Delta A = 4\pi T\left(r_2^2 - r_1^2\right)$$

(e) In case of soap bubble, which has two free surfaces, if the initial radius of soap bubble (having surface tension T) is f_1 and final radius is f_2, then then work done in changing the radius from f_1 to f_2 is

$$W = 8\pi T\left(r_2^2 - r_1^2\right)$$

ILLUSTRATION 130

A U-shaped wire loop having a slider of negligible mass having a length of 0.1 m is dipped in a soap solution and then removed. Due to this, a thin soap film is formed between the wire and the light slider. It is observed that the film can support a weight of 0.006 N, before it can break. Calculate the surface tension of the film.

SOLUTION

Because of the phenomenon of surface tension, the soap film tries to occupy minimum surface area. Since a soap film has two free surfaces, so the total force acting on the slider due to surface tension is

$$F = 2(Tl)$$

Since the film can support a weight of 0.006 N, before it can break, so we have

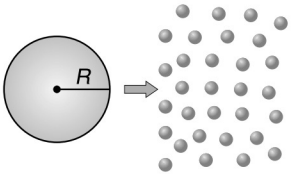
$$2(Tl) = W = mg$$

where, $W = mg = 0.006\text{ N}$ and $l = 0.1\text{ m}$

$$\Rightarrow \quad T = \frac{W}{2l} = \frac{0.006}{2\times 0.1} = 0.03\text{ Nm}^{-1}$$

SPLITTING OF BIGGER DROP INTO SMALL DROPLETS

When a drop of radius R splits into n smaller drops, (each of radius r) then surface area of liquid increases.



Hence the work is to be done against surface tension. Since the volume of liquid remains constant therefore

$$\frac{4}{3}\pi R^3 = n\left(\frac{4}{3}\pi r^3\right)$$

$$\Rightarrow \quad R^3 = nr^3$$

Work done is given by

$$W = T\Delta A = T\left[n(4\pi r^2) - 4\pi R^2\right]$$

$$\Rightarrow \quad W = 4\pi T\left(nr^2 - R^2\right)$$

$$\Rightarrow \quad W = 4\pi R^2 T\left(n^{1/3} - 1\right)$$

$$\Rightarrow \quad W = 4\pi T r^2 n^{2/3}\left(n^{1/3} - 1\right)$$

$$\Rightarrow \quad W = 4\pi TR^3\left(\frac{1}{r} - \frac{1}{R}\right)$$

Conceptual Note(s)

If the work is not done by an external source then internal energy of liquid decreases, subsequently temperature decreases. This is the reason why spraying causes cooling. By Law of Conservation of energy, loss in thermal energy equals the work done against surface tension. So

$$W = JQ$$

where, $Q = mc\Delta\theta$, c is the gram specific heat of the liquid, $\Delta\theta$ is the decrease in temperature an J is the Joule's Mechanical Equivalent of heat.

$$J(mc\Delta\theta) = 4\pi TR^3\left(\frac{1}{r} - \frac{1}{R}\right)$$

Since $m = \left(\frac{4}{3}\pi r^3\right)\rho$

$$\Rightarrow \quad \left(\frac{4}{3}\pi R^3\right)\rho c\,\Delta\theta = \frac{4\pi R^3 T}{J}\left(\frac{1}{r} - \frac{1}{R}\right)$$

So, decrease in temperature is

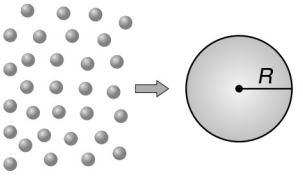
$$\Delta\theta = \frac{3T}{Jc\rho}\left(\frac{1}{r} - \frac{1}{R}\right)$$

In cgs system and for water, we have $s = 1\,\text{calg}^{-1}(^\circ\text{C})^{-1}$ and $\rho = 1\,\text{gcc}^{-1}$, so we get

$$\Delta\theta = \frac{3T}{J}\left(\frac{1}{r} - \frac{1}{R}\right)$$

FORMATION OF BIGGER DROP FROM SMALL DROPLETS

If n small drops of radius r coalesce to form a big drop of radius R then surface area of the liquid decreases.



Energy released is

$$\Delta E = T\left(A_{\text{initial}} - A_{\text{final}}\right)$$

$$\Rightarrow \quad \Delta E = n(4\pi r^2)T - 4\pi R^2 T$$

$$\Rightarrow \quad \Delta E = 4\pi T\left(nr^2 - R^2\right)$$

$$\Rightarrow \quad \Delta E = 4\pi R^2 T\left(n^{1/3} - 1\right)$$

$$\Rightarrow \quad \Delta E = 4\pi T r^2 n^{2/3}\left(n^{1/3} - 1\right)$$

$$\Rightarrow \quad \Delta E = 4\pi TR^3\left(\frac{1}{r} - \frac{1}{R}\right)$$

Conceptual Note(s)

(a) If this released energy is absorbed by a big drop, its temperature increases and rise in temperature can be given by

$$\Delta\theta = \frac{3T}{J\rho c}\left(\frac{1}{r}-\frac{1}{R}\right)$$

In cgs system and for water, we have $c = 1\ \text{calg}^{-1}(^\circ\text{C})^{-1}$ and $\rho = 1\ \text{gcc}^{-1}$, so we have

$$\Delta\theta = \frac{3T}{J}\left(\frac{1}{r}-\frac{1}{R}\right)$$

(b) If this released energy is converted into kinetic energy of a big drop without dissipation, then by the Law of Conservation of Energy, we get

$$\frac{1}{2}mv^2 = 4\pi R^3 T\left(\frac{1}{r}-\frac{1}{R}\right)$$

$$\Rightarrow \frac{1}{2}\left(\frac{4}{3}\pi R^3\right)\rho v^2 = 4\pi R^3 T\left[\frac{1}{r}-\frac{1}{R}\right]$$

$$\Rightarrow v^2 = \frac{6T}{\rho}\left(\frac{1}{r}-\frac{1}{R}\right)$$

$$\Rightarrow v = \sqrt{\frac{6T}{d}\left(\frac{1}{r}-\frac{1}{R}\right)}$$

ILLUSTRATION 131

How much work will be done in increasing the diameter of a soap bubble from 2 cm to 5 cm. Surface tension of soap solution is $3\times10^{-2}\ \text{Nm}^{-1}$.

SOLUTION

Soap bubble has two surfaces. Hence,

$$W = T\Delta A$$

where, $\Delta A = 2\left[4\pi\left\{(2.5\times10^{-2})^2-(1\times10^{-2})^2\right\}\right]$

$$\Rightarrow \Delta A = 1.32\times10^{-2}\ \text{m}^2$$

$$\Rightarrow W = (3\times10^{-2})(1.32\times10^{-2})\ \text{J}$$

$$\Rightarrow W = 3.96\times10^{-4}\ \text{J}$$

ILLUSTRATION 132

Calculate the energy released when 1000 small water drops each of same radius 10^{-7} m coalesce to form one large drop. The surface tension of water is $7\times10^{-2}\ \text{Nm}^{-1}$

SOLUTION

Let r be the radius of smaller drops and R of bigger one. Equating the initial and final volumes, we have,

$$\frac{4}{3}\pi R^3 = (1000)\left(\frac{4}{3}\pi r^3\right)$$

$$\Rightarrow R = 10r = (10)(10^{-7})\ \text{m}$$

$$\Rightarrow R = 10^{-6}\ \text{m}$$

Further, the water drops have only one free surface. Therefore,

$$\Delta A = 4\pi R^2 - (1000)(4\pi r^2)$$

$$\Rightarrow \Delta A = 4\pi\left[(10^{-6})^2-(10^3)(10^{-7})^2\right]$$

$$\Rightarrow \Delta A = -36\pi(10^{-12})\ \text{m}^2$$

Here, negative sign implies that surface area is decreasing. Hence, energy released in the process.

$$\Delta U = T|\Delta A| = (7\times10^{-2})(36\pi\times10^{-12})\ \text{J}$$

$$\Rightarrow \Delta U = 7.9\times10^{-12}\ \text{J}$$

COHESIVE FORCES

The force of attraction between the molecules of the same substance is called force of cohesion or cohesive force. In case of solids, the force of cohesion is very large and due to this, solids have definite shape and size. On the other hand, the force of cohesion in case of liquids is weaker than that of solids. Hence liquids do not have definite shape but have definite volume. The force of cohesion is negligible in case of gases. Because of this fact, gases have neither fixed shape nor volume.

Examples

(a) Two drops of a liquid coalesce into one when brought in mutual contact because of the cohesive force.
(b) It is difficult to separate two sticky plates of glass wetted with water because a large force has to be applied against the cohesive force between the molecules of water.
(c) It is very difficult to break a drop of mercury into small droplets because of large cohesive force between mercury molecules.

ADHESIVE FORCES

The force of attraction between molecules of different substances is called adhesion.

Examples

(a) Adhesive force enables us to write on the black board with a chalk.
(b) Adhesive force helps us to write on the paper with ink.
(c) Large force of adhesion between cement and bricks helps us in construction work.
(d) Due to force of adhesive, water wets the glass plate.
(e) Fevicol and gum are used in gluing two surfaces together because of adhesive force.

WATER WETS THE GLASS SURFACE, BUT MERCURY DOES NOT: EXPLANATION

The adhesive force between water molecules and glass molecules is greater than the cohesive force between the molecules of water. Hence when water is poured on glass, the water molecules cling with the glass molecules and the glass surface is wetted.

However, the adhesive force between mercury-glass molecules is less than the cohesive force between mercury-mercury molecules. So, mercury molecules do not cling with glass molecules i.e. mercury does not wet the glass.

If, however, the glass surface is greasy, then water also does not wet the glass because the adhesive force between water-grease molecules is less than the cohesive force between water-water molecules.

The adhesive force between oil and water is less than the cohesive force of water, but greater than the cohesive force of oil. Therefore, a water drop poured on the surface of oil contracts to take the form of a globule, while a drop of oil poured on the surface of water spreads to a large area in the form of a thin film.

The adhesive force between ink and paper is greater than the cohesive force of ink. That is why ink sticks on paper. Writing on blackboard by chalk is also possible due to adhesive force.

ANGLE OF CONTACT

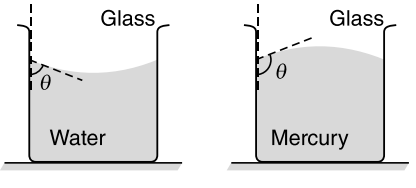
When the free surface of a liquid comes in contact of a solid, it becomes curved near the place of contact.

The angle of contact between a liquid and a solid is defined as the angle inside the liquid between the tangent to the solid surface and the tangent to the liquid surface at the point of contact of the liquid with the solid.

Liquids which wet the surface have acute angle of contact. Angle of contact is zero for pure water and clean glass. For ordinary water and glass angle of contact is about $8°$.

Liquids which do not wet the surface have obtuse angle of contact. For mercury and glass the angle of contact is $135°$.

The angles of contact θ for water-glass and mercury-glass are shown in Figure.



The angle of contact for water and silver is $90°$. Hence in a silver vessel the surface of water at the edges also remains horizontal.

Conceptual Note(s)

(a) Angle of contact θ lies between $0°$ and $180°$.
(b) It does not depend upon the inclination of the solid in the liquid.
(c) If $\theta < 90°$, the liquid wets the surface of solid and if $\theta > 90°$, the liquid does not wet the surface of solid.
(d) The angle of contact depends on
  (i) the nature of the liquid.
  (ii) the nature of the solid.
  (iii) the cleanliness of the surfaces.
  (iv) the medium above the liquid surface.
  (v) temperature. (on increasing the temperature, angle of contact decreases).
(e) Soluble impurities increase the angle of contact.
(f) Partially soluble impurities decrease the angle of contact.

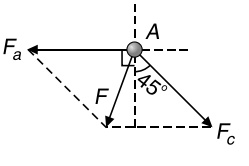
SHAPE OF LIQUID MENISCUS IN A GLASS TUBE

When a liquid is brought in contact with a solid surface, the surface of the liquid becomes curved near the place of contact. The nature of the curvature (concave or convex) depends upon the relative magnitudes of the cohesive force between the liquid molecules and the adhesive force between the molecules of the liquid and those of the solid. A liquid takes the shape of the vessel in which it is contained, so it cannot permanently oppose any force that tries to change its shape.

The free surface of liquid at rest always adjusts itself at right angles to the resultant force such that the component of force tangential to the liquid is zero.

When a capillary tube is dipped in a liquid, the liquid surface becomes curved at the point of contact. This surface is curved due to forces of cohesion and forces of adhesion. This curved surface of the liquid is called the Meniscus of the Liquid or Liquid Meniscus. Consider a liquid molecule A in contact with solid (i.e. wall of capillary tube), then forces acting on molecule A are

(a) Weight of the molecule A which acts vertically downwards along the wall of the tube and is negligible compared to forces of adhesion and cohesion.
(b) Force of adhesion F_a (acts outwards at right angle to the wall of the tube).
(c) Force of cohesion F_c (acts at an angle $45°$ to the vertical) i.e., F_c makes an angle of $135°$ with F_a as shown in Figure (in which weight of the particle is not shown as it is negligible in comparison to F_a and F_c).



The resultant force F depends on the values of F_a and F_c. If the resultant force F makes an angle α with F_a, then we have

$$\tan\alpha = \frac{F_c \sin(135°)}{F_a + F_c \cos(135°)} = \frac{F_c}{\sqrt{2}F_a - F_c} \quad \ldots(1)$$

By knowing the direction of resultant force, we can find out the shape of meniscus because the free surface of the liquid adjusts itself at right angle to this resultant force.

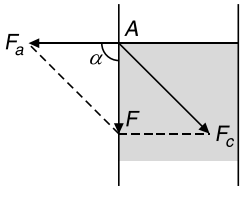
CASE-1: For Horizontal Meniscus

From equation (1) we see that, when $F_c = \sqrt{2}F_a$, then

$\tan\alpha \to \infty$

$\Rightarrow \quad \alpha \to 90°$

So, the resultant force F acts vertically downwards and hence the liquid meniscus must be horizontal.



EXAMPLE: Pure water in silver coated capillary tube.

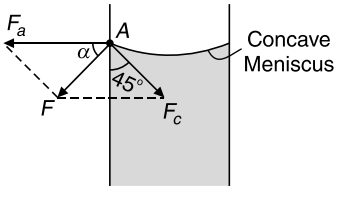
CASE-2: For Concave Meniscus

From equation (1), we see that, if $F_c < \sqrt{2}F_a$, then

$\tan\alpha$ = positive

$\Rightarrow \quad \alpha$ is acute

So, the resultant force is directed outside the liquid and hence the liquid meniscus must be concave upward.



EXAMPLE: Water in glass capillary tube.

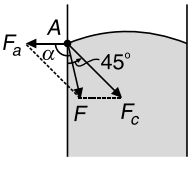
CASE-3: For Convex Meniscus

From equation (1), we see that, if $F_c > \sqrt{2}F_a$, then

$\tan\alpha$ = negative

$\Rightarrow \quad \alpha$ is obtuse

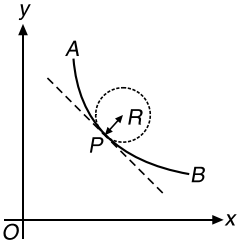
So, the resultant force is directed inside the liquid and hence the liquid meniscus must be convex upward.



EXAMPLE: Mercury in glass capillary tube.

RADIUS OF CURVATURE OF A CURVE

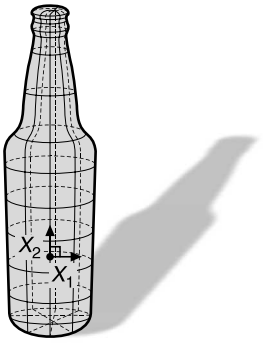
Shape of a curved surface or interface can be described if we know the radius of curvature of the curved surface at some point. Consider a point P on the curve AB as shown in Figure.



The radius of curvature R of the curve at the point P is actually the radius of the circle which is tangent to the curve at the point P.

PRINCIPAL RADII OF CURVATURE OF A SURFACE

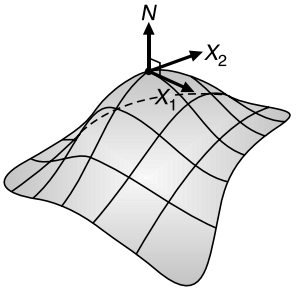
We must understand the fact that all known shapes are made of surfaces which have some curvature. The spherical and cylindrical surfaces are rather simple cases for mathematical treatment. Consider a bottle shown in Figure.



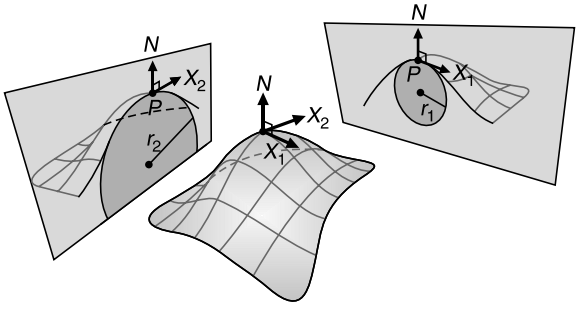
Along one direction X_1, the bottle quickly curves around in a circle but along another direction X_2 it is completely flat and travels along a straight line.

This way of looking at curvature in terms of curves traveling along the surface is often how we treat curvature in general. Also note that the curvature of a curve κ is inversely proportional to the radius of curvature.

In many other cases however, the shapes are more complicated. Let us now consider a curved surface shown in Figure.



At each point on a given surface, two radii of curvature (which are denoted by r_1 and r_2) are required to describe the shape. If we want to determine these radii at any point (say P), a normal to the surface at this point is drawn and a plane containing the normal is constructed through the surface. This plane will intersect the surface in a curve. The radius of curvature of this curve at point P is denoted by r_1 as shown in Figure.



An infinite number of such planes can be constructed each of which intersects the surface at P. For each of these planes, a radius of curvature can be obtained. If we construct a second plane through the surface, containing the normal and perpendicular to the first plane, the second curve of intersection and hence the second radius of curvature at point P (i.e., r_2) is obtained. These two radii define the curvature at P completely.

It can be shown that the mean curvature i.e. $\frac{1}{2}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$ of the surface is constant, which is independent of the choice of the planes. An infinite set of such pairs of radii is possible.

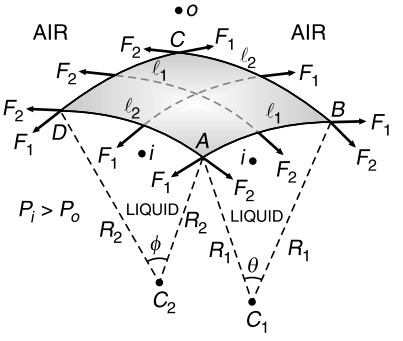
For standardization, the first plane is rotated around the normal until the radius of curvature in that plane reaches maximum and hence the other radius of curvature becomes minimum. These are called the principal radii of curvature and are denoted by R_1 and R_2.

EXCESS PRESSURE INSIDE LIQUID SURFACE WITH TWO CURVATURES: YOUNG-LAPLACE EQUATION

There exists a difference in pressure across a curved liquid surface which is a consequence of surface tension. The pressure is greater on the concave side. The Laplace equation relates the pressure difference to the shape of the Young surface. This difference in pressure is given by

$$\Delta P = T\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

where R_1 and R_2 re the principal radii of curvature of the surface. To derive this equation, consider a small section of liquid surface having two curvatures R_1 and R_2 as shown in Figure 1.



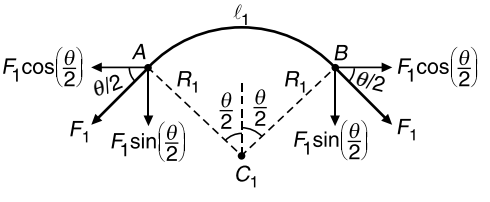
Since the surface is small, the angle subtended at the respective centres are also small. Net outward force on the liquid surface is

$$F_{\text{out}} = (P_i - P_o)A_{\text{surface}} = (P_i - P_o)(R_1\theta)(R_2\phi) \qquad \ldots(1)$$

Let us now calculate the force due to surface tension on each surface. Let us draw the force acting on the portion AB due to surface tension. Interestingly, we see that the force F_1 is acting along the length $l_2 = R_2\phi$ and the force F_2 is acting on the length $l_1 = R_1\theta$ as shown in Figure 1. So, we have

$$F_1 = Tl_2 = TR_2\phi \text{ and } F_2 = Tl_1 = TR_1\theta$$

The force $F_1 = Tl_2 = T(R_2\phi)$, acting along the length AD has components $F_1\cos\left(\frac{\theta}{2}\right)$ and $F_1\sin\left(\frac{\theta}{2}\right)$. Similarly, this same value of force is acting along the length BC has components $F_1\cos\left(\frac{\theta}{2}\right)$ and $F_1\sin\left(\frac{\theta}{2}\right)$ with cross sectional view for the curve AB shown in Figure 2.



The components $F_1\cos\left(\frac{\theta}{2}\right)$ and $F_1\cos\left(\frac{\theta}{2}\right)$ cancel, so the inward force acting due to surface tension on the curves AD and BC is $2F_1\sin\left(\frac{\theta}{2}\right)$.

Similarly, the inward force acting due to surface tension on the curves AB and CD is $2F_2\sin\left(\frac{\phi}{2}\right)$.

If F_{in} is the net inward force, then

$$F_{\text{in}} = 2F_1\sin\left(\frac{\theta}{2}\right)+2F_2\sin\left(\frac{\phi}{2}\right)$$

Since, θ and ϕ are small, so we have

$$\sin\left(\frac{\theta}{2}\right)\approx\frac{\theta}{2} \text{ and } \sin\left(\frac{\phi}{2}\right)\approx\frac{\phi}{2}$$

Also, $F_1 = Tl_2 = TR_2\phi$ and $F_2 = Tl_1 = TR_1\theta$

$$\Rightarrow \quad F_{in} = F_1\theta + F_2\phi = (TR_2\phi)\theta + (TR_1\theta)\phi$$

$$\Rightarrow \quad F_{in} = T\theta\phi(R_1 + R_2) \qquad \ldots(2)$$

For equilibrium of the surface, the net outward force must be balanced by the net inward force, so we have

$$F_{out} = F_{in}$$

$$\Rightarrow \quad (P_i - P_o)(R_1\theta)(R_2\phi) = T\theta\phi(R_1 + R_2)$$

$$\Rightarrow \quad P_i - P_o = \Delta P = T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

However, for the case of a film, we have

$$\Delta P = 2T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

because a film has two free surfaces.

Conceptual Note(s)

The simplified forms of spherical, cylindrical and planar surfaces are given below

For a spherical surface, we have

$R_1 = R_2 = R$, therefore $\Delta P = \frac{2T}{R}$

For a cylindrical surface, we have

$R_1 \to \infty$ and $R_2 = R$, therefore $\Delta P = \frac{T}{R}$

For a planar surface, we have

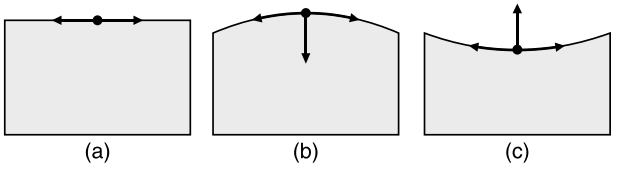
$R_1 \to \infty$ and $R_2 \to \infty$, therefore $\Delta P = 0$

PRESSURE DIFFERENCE BETWEEN THE TWO SIDES FORCE A CURVED LIQUID SURFACE

A molecule lying in the surface of a liquid is attracted by other molecules on the surface in all directions. If the surface is plane, then the molecule is attracted equally in all directions. Hence the resultant force on the molecule due to surface tension is zero.

If the surface is convex, then a resultant component of all the forces of attraction acting on every molecule acts normal to the surface is directed inward.

Similarly, if the surface is concave, then every molecule experiences a resultant force due to surface tension acting normally outward.



Obviously, for the equilibrium of a curved surface, there must be a difference of pressure between its two sides so that the excess pressure force may balance the resultant force due to surface tension. Hence the pressure on the concave side must be greater than the pressure on the convex side. This difference of pressure is equal to $\frac{2T}{R}$, where T is the surface tension and R is radius of curvature of the surface.

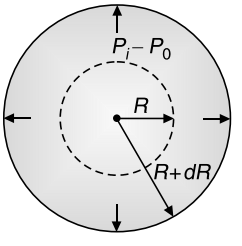
EXCESS PRESSURE

Due to the property of surface tension a drop or bubble tries to contract and so compresses the matter enclosed. This in turn increases the internal pressure which prevents further contraction and equilibrium is achieved. So, in equilibrium the pressure inside a bubble or drop is greater than outside and the difference of pressure between two sides of the liquid surface is called excess pressure.

In case of a drop, excess pressure is provided by hydrostatic pressure of the liquid within the drop while in case of bubble the gauge pressure of the gas confined in the bubble provides it.

EXCESS PRESSURE INSIDE A LIQUID DROP AND AIR BUBBLE

Let P_i be the pressure inside the liquid drop and P_0 be the pressure outside it. Then $(P_i - P_0)$ is the excess pressure inside the liquid drop.



Let T be the surface tension of drop. Let the radius of bubble increase from R to $(R+dR)$ under the influence of excess pressure $(P_i - P_0)$ acting only on the inner surface of area $4\pi R^2$.

So, work done by the force due to pressure is

$$W = (P_i - P_0)4\pi R^2 dR \qquad \ldots(1)$$

Also, work done to change surface area of the drop is

$$W = T\left[4\pi(R+dR)^2 - 4\pi R^2\right]$$

$$\Rightarrow \quad W = 4\pi TR^2\left[\left(1 + \frac{dR}{R}\right)^2 - 1\right]$$

$$\Rightarrow \quad W \approx 4\pi TR^2\left[1 + 2\frac{dR}{R} - 1\right]$$

$$\Rightarrow \quad W \approx 8\pi TR^2 dR \qquad \ldots(2)$$

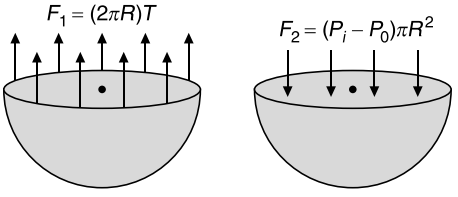
Equating (1) and (2), we get

$$P_i - P_0 = \frac{2T}{R}$$

For an air bubble also, the excess pressure is

$$P_i - P_0 = \frac{2T}{R} \qquad \{\because \text{ Both have a single free surface}\}$$

We can also do this quickly, by equating the force due to surface tension with the force due to excess pressure (as done earlier too) as shown in Figure.



The force due to surface tension is

$$F_1 = (2\pi R)T \qquad \ldots(3)$$

The force due to excess pressure is

$$F_2 = (P_i - P_0)\pi R^2 \qquad \ldots(4)$$

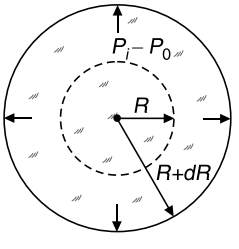
Equating (3) and (4), we get

$$P_i - P_0 = \frac{2T}{R}$$

EXCESS PRESSURE INSIDE A SOAP BUBBLE

Let P_i be the pressure inside the bubble and P_0 be the pressure outside it. Then $(P_i - P_0)$ is the excess pressure inside the bubble.

Let T be the surface tension of the soap bubble. Let the radius of bubble increase from R to $(R + dR)$ under the influence of excess pressure $(P_i - P_0)$ acting only on the inner surface of area $4\pi R^2$.



Work done by the force due to pressure is

$$W = (P_i - P_0)4\pi R^2 dR \qquad \ldots(1)$$

Since a soap bubble has two free surfaces, so work done to change surface area of the soap bubble is

$$W = T\left[4\pi(R + dR)^2 - 4\pi R^2\right] \times 2$$

$$\Rightarrow \quad W = 8\pi TR^2\left[\left(1 + \frac{dR}{R}\right)^2 - 1\right]$$

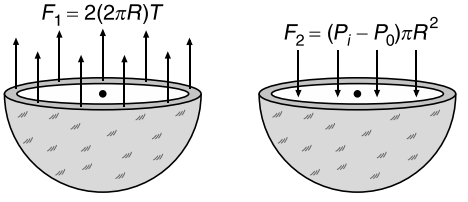
$$\Rightarrow \quad W \approx 8\pi TR^2\left[\cancel{1} + \frac{2dR}{R} - \cancel{1}\right]$$

$$\Rightarrow \quad W \approx 16\pi TR^2 dR \qquad \ldots(2)$$

Equating (1) and (2), we get

$$P_i - P_0 = \frac{4T}{R}$$

We can also do this quickly, by equating the force due to surface tension with the force due to excess pressure (as done earlier too) as shown in Figure.



The force due to surface tension is

$$F_1 = 2[(2\pi R)T] \qquad \ldots(3)$$

The force due to excess pressure is

$$F_2 = (P_i - P_0)\pi R^2 \qquad \ldots(4)$$

Equating (3) and (4), we get

$$P_i - P_o = \frac{4T}{R}$$

Conceptual Note(s)

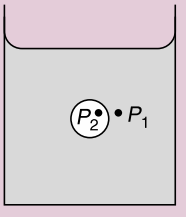
(a) Thus, the pressure difference is greater for smaller drops and bubbles than for larger ones.

(b) Pressure difference is inversely proportional to the radius.

(c) The pressure inside the surface is greater than the pressure outside the surface.

(d) The pressure on the concave side is greater than the pressure on the convex side.

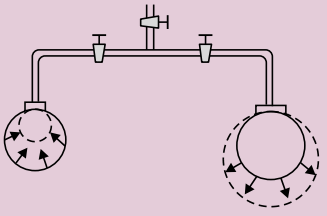
(e) Consider an air bubble inside the liquid as shown in the figure.



We observe that a single surface is formed with air on the concave side and liquid on the convex side. The pressure at the concave side is greater than the pressure at the convex side, by an amount $\frac{2T}{R}$. So, we have

$$P_2 - P_1 = \frac{2T}{R}$$

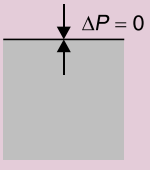
(f) Excess pressure is inversely proportional to the radius of bubble (or drop), i.e., pressure inside a smaller bubble (or drop) is higher than inside a larger bubble (or drop).



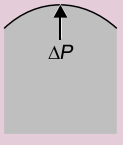
This is why when two bubbles of different sizes are connected to each other, the air will rush from smaller bubble to larger bubble. Due to this the smaller will shrink while the larger will expand till the smaller bubble reduces to droplet.

(g) Excess Pressure in Different Cases

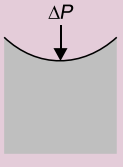
(i) For a plane surface, $\Delta P = 0$



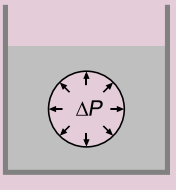
(ii) For a convex surface, $\Delta P = \frac{2T}{R}$



(iii) For a concave surface, $\Delta P = \frac{2T}{R}$



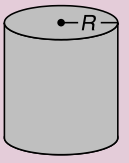
(iv) For an air bubble in liquid, $\Delta P = \frac{2T}{R}$



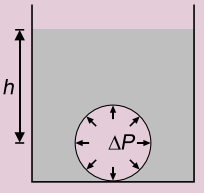
(v) For a liquid Drop, $\Delta P = \frac{2T}{R}$

(vi) For a soap bubble $\Delta P = \frac{4T}{R}$

(vii) For a cylindrical liquid surface, $\Delta P = \frac{T}{R}$

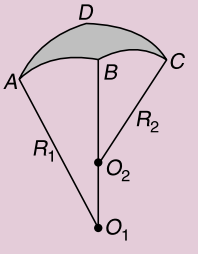


(viii) For an air bubble at depth h below the free surface of liquid of density d, $\Delta P = \frac{2T}{R} + hdg$



(ix) For a liquid surface of unequal radii,

$$\Delta P = T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$



(x) For a liquid film of unequal radii,

$$\Delta P = 2T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

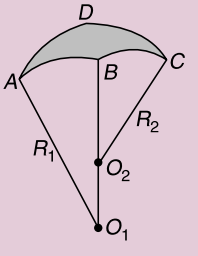


ILLUSTRATION 133

Calculate the pressure inside a small air bubble of 0.1 mm radius situated just below the water surface. Assume the surface tension of water to be 7.2×10^{-2} Nm^{-1} and atmospheric pressure to be 1.013×10^{5} Nm^{-2}.

SOLUTION

Surface tension of water $T = 7.2\times10^{-2}\ \text{Nm}^{-1}$
Radius of air bubble $R = 0.1\ \text{mm} = 10^{-4}\ \text{m}$
The excess pressure inside the air bubble is given by,

$$P_2 - P_1 = \frac{2T}{R}$$

Pressure inside the air bubble is

$$P_2 = P_1 + \frac{2T}{R}$$

Substituting the values, we have

$$P_2 = (1.013\times10^5) + \frac{(2\times7.2\times10^{-2})}{10^{-4}}$$

$$\Rightarrow \quad P_2 = 1.027\times10^5\ \text{Nm}^{-2}$$

ILLUSTRATION 134

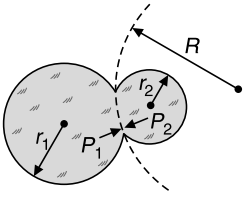
Two separate air bubbles (radii 0.006 m and 0.003 m) formed of the same liquid (surface tension $0.07\ \text{Nm}^{-1}$) come together to form a double bubble. Calculate the radius and the sense of curvature of the internal film surface common to both the bubbles.

SOLUTION

The pressure inside the bubbles is

$$P_1 = P_0 + \frac{4T}{r_1} \text{ and } P_2 = P_0 + \frac{4T}{r_2}$$

Since $r_2 < r_1$, so we get $P_2 > P_1$
The pressure inside the smaller bubble will be more as shown in Figure.



The excess pressure at the interface is

$$P = P_2 - P_1 = 4T\left(\frac{r_1 - r_2}{r_1 r_2}\right) \qquad \ldots(1)$$

This excess pressure acts from concave to convex side, the interface will be concave towards smaller bubble and convex towards larger bubble. Let R be the radius of interface then,

$$P = \frac{4T}{R} \qquad \ldots(2)$$

From equations (1) and (2)

$$R = \frac{r_1 r_2}{r_1 - r_2} = \frac{(0.006)(0.003)}{0.006 - 0.003}$$

$$\Rightarrow \quad R = 0.006\ \text{m}$$

ILLUSTRATION 135

Under isothermal condition two soap bubbles of radii r_1 and r_2 coalesce to form a single bubble of radius r. The external pressure is P_0. Find the surface tension of the soap in terms of the given parameters.

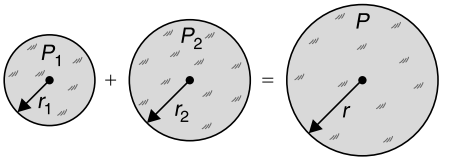
SOLUTION

As mass of the air is conserved,

$$\Rightarrow \quad n_1 + n_2 = n$$

Since, $PV = nRT$

$$\Rightarrow \quad \frac{P_1V_1}{RT_1} + \frac{P_2V_2}{RT_2} = \frac{PV}{RT}$$



As temperature is constant,

$$T_1 = T_2 = T$$

$$\Rightarrow \quad P_1V_1 + P_2V_2 = PV \qquad \ldots(1)$$

where, $V_1 = \frac{4}{3}\pi r_1^3$, $V_2 = \frac{4}{3}\pi r_2^3$ and $V = \frac{4}{3}\pi r^3$

To avoid confusion with temperature, let us denote surface tension by σ. From equation (1), we get

$$\left(P_0 + \frac{4\sigma}{r_1}\right)r_1^3 + \left(P_0 + \frac{4\sigma}{r_2}\right)r_2^3 = \left(P_0 + \frac{4\sigma}{r}\right)r^3$$

$$\Rightarrow \quad \sigma = \frac{P_0\left(r^3 - r_1^3 - r_2^3\right)}{4\left(r_1^2 + r_2^2 - r^2\right)}$$

Special Case

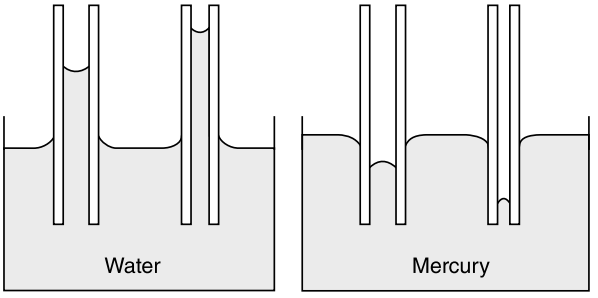
If the bubbles coalesce in vacuum, then $P_0 = 0$ and hence from equation (2), we get

$$r^2 = r_1^2 + r_2^2$$

CAPILLARITY

When a glass capillary tube open at both ends is dipped vertically in water, the water rises up in the tube to a certain height above the water level outside the tube. The narrower the tube, the higher is the rise of water. On the other hand, if the tube is dipped in mercury, the mercury is depressed below the outside level.

The phenomenon of rise or depression of liquids in a capillary tube is called capillarity. The liquids which wet glass (for which the angle of contact is acute) rise up in the capillary tube, while those which do not wet glass (for which the angle of contact is obtuse) are depressed down in the capillary.



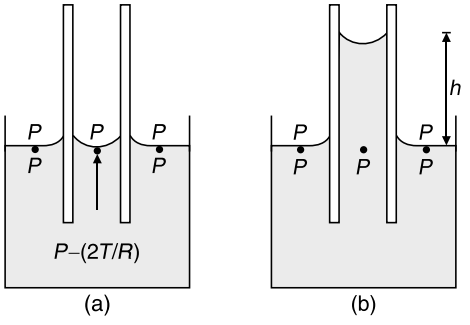
EXPLANATION TO CAPILLARITY

The phenomenon of capillarity arises due to the surface tension of liquids. When a capillary tube is dipped in water, the water meniscus inside the tube is concave. The pressure just below the meniscus is less than the pressure just above it by $\frac{2T}{R}$, where T is the surface tension of water and R is the radius of curvature of meniscus.

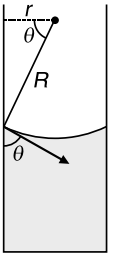
The pressure of the surface of water is atmospheric pressure P. The pressure just below plane surface of water outside the tube is also P, but that just below the meniscus inside the tube is $P-\left(\frac{2T}{R}\right)$. We know that pressure at all points in the same level of water must be the same.

Therefore, to make up the deficiency of pressure, $\frac{2T}{R}$, below the meniscus, water begins to flow from outside into the tube. The rising force water in the capillary stops at a certain height h. In this position the pressure of the water-column of height h becomes equal to $\frac{2T}{R}$, that is,

$$h\rho g=\frac{2T}{R}$$



where, ρ is the density of water and g is the acceleration due to gravity. If r be the radius of the capillary tube and θ the angle of contact of water-glass, then the radius of curvature R of the meniscus is given by $R=\frac{r}{\cos\theta}$.



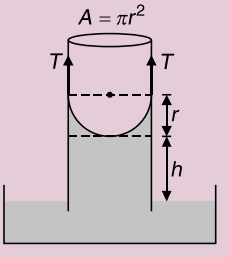
$$\Rightarrow \quad h\rho g=\frac{2T}{\left(\frac{r}{\cos\theta}\right)}$$

$$\Rightarrow \quad h=\frac{2T\cos\theta}{r\rho g}$$

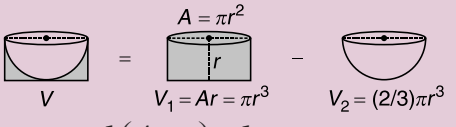
This shows that as r decreases, h increases, that is, narrower the tube, greater is the height to which the liquid rises in the tube.

Conceptual Note(s)

If weight of the liquid in the meniscus is to be considered, then the correction due to the weight of the liquid contained in the meniscus can be made for contact angle of $0°$ for which the meniscus will be hemispherical in shape as shown in Figure.



The volume of the shaded part of liquid inside the meniscus is $V=V_1-V_2$ as shown in Figure.



$$\Rightarrow \quad V=(\pi r^2)r-\frac{1}{2}\left(\frac{4}{3}\pi r^3\right)=\frac{1}{3}\pi r^3$$

Since the force due to surface tension will balance weight of the liquid W in the capillary, so we have

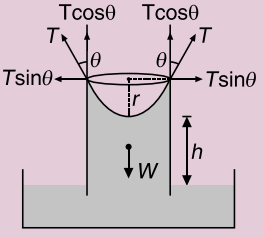
$$T(2\pi r)=W$$

where, $W=\left(\pi r^2h+\frac{1}{3}\pi r^3\right)\rho g$

$$\Rightarrow \quad T(2\pi r)=\left(\pi r^2h+\frac{1}{3}\pi r^3\right)\rho g$$

$$\Rightarrow \quad \left(h+\frac{r}{3}\right)=\frac{2T}{r\rho g}$$

However, if the contact angle is non-zero and acute, then too the meniscus is nearly hemispherical as shown in Figure, then we have



$$(T\cos\theta)(2\pi r)=W$$

where, $W=\left(\pi r^2 h+\frac{1}{3}\pi r^3\right)\rho g$

$$\Rightarrow \quad (T\cos\theta)(2\pi r)=\left(\pi r^2 h+\frac{1}{3}\pi r^3\right)\rho g$$

$$\Rightarrow \quad \left(h+\frac{r}{3}\right)=\frac{2T\cos\theta}{r\rho g}$$

PRACTICAL APPLICATIONS OF CAPILLARITY

(a) The oil in a lamp rises in the wick by capillary action.
(b) The tip of nib of a pen is split up, to make a narrow capillary so that the ink rises upto the tin or nib continuously.
(c) Sap and water rise upto the top of the leaves of the tree by capillary action.
(d) If one end of the towel dips into a bucket of water and the other end hangs over the bucket the towel soon becomes wet throughout due to capillary action.
(e) Ink is absorbed by the blotter due to capillary action.
(f) Sandy soil is more dry than clay. It is because the capillaries between sand particles are not so fine as to draw the water up by capillaries.
(g) The moisture rises in the capillaries of soil to the surface, where it evaporates. To preserve the moisture in the soil, capillaries must be broken up. This is done by ploughing and levelling the fields.
(h) Bricks are porous and behave like capillaries.

Conceptual Note(s)

(a) For liquids which wet the glass tube or capillary tube, angle of contact $\theta<90°$. Hence,

$\cos\theta=$ positive

$\Rightarrow$ $h=$ positive.

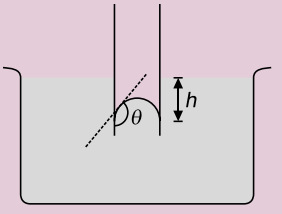
It means that these liquids rise in the capillary tube. So, the liquids which wet capillary tube rise in the capillary tube. For example, water, milk, kerosene oil, petrol etc.

(b) For liquids which do not wet the glass tube or capillary tube, angle of contact $\theta>90°$. Hence,

$\cos\theta=$ negative

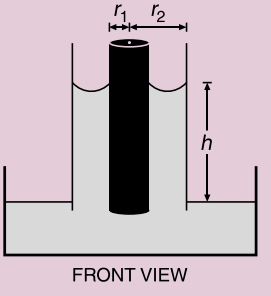
$\Rightarrow$ $h=$ negative.

So, the liquids which do not wet capillary tube are depressed in the capillary tube. For example, mercury.

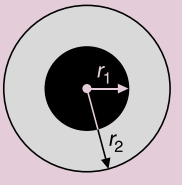


(c) Since, T, θ, ρ and g are constant and hence $h\propto\frac{1}{r}$. Thus, the liquid rises more in a narrow tube and less in a wider tube. **This is called Jurin's Law.**

(d) If two concentric tubes of radii r_1 and r_2 (inner one is solid) are placed in water reservoir, then height to which the liquid rises is obtained by equating the force due to surface tension with the weight of liquid in the tube. So, we get



FRONT VIEW



TOP VIEW

$$T(2\pi r_1+2\pi r_2)=\left(\pi r_2^2 h-\pi r_1^2 h\right)\rho g$$

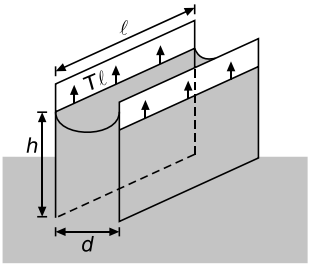
$$\Rightarrow \quad h=\frac{2T}{(r_2-r_1)\rho g}$$

RISE OF LIQUID BETWEEN TWO PARALLEL PLATES

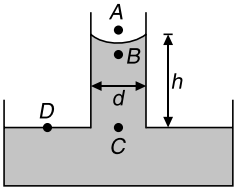
If two parallel plates with the spacing 'd' are placed in water reservoir, then liquid will rise till force due to surface tension balances the weight of liquid, so we have

$$2Tl\cos(0°)=(\rho lhd)g$$

$$\Rightarrow \quad h=\frac{2T}{\rho g d}$$



We can also calculate this height by applying a different approach. Since the liquid meniscus between two plates is cylindrical in shape, so the cross-sectional view of the arrangement is shown in Figure.



The pressure at A is equal to the atmospheric pressure $P_0 = P_{atm}$.

The pressure difference between the point A and the point B (which lies just inside the cylindrical meniscus) is

$$P_A - P_B = \frac{T}{R}, \text{ where } R \approx \frac{d}{2}$$

$$\Rightarrow \quad P_B = P_0 - \frac{T}{R}$$

Also, we see that

$$P_C - P_B = h\rho g$$

$$\Rightarrow \quad P_C = P_B + h\rho g = \left(P_0 - \frac{T}{R}\right) + h\rho g \qquad \ldots(1)$$

Now we must see that the points C and D are at the same level, hence

$$P_C = P_D = P_{atm} = P_0$$

So, from equation (1), we get

$$P_0 = \left(P_0 - \frac{T}{R}\right) + h\rho g$$

$$\Rightarrow \quad h = \frac{T}{\rho g R} = \frac{2T}{\rho g d}$$

ILLUSTRATION 136

Mercury has an angle of contact of $120°$ with glass. A narrow tube of radius 1.0 mm made of this glass is dipped in a trough containing mercury. By what amount does the mercury dip down in the tube relative to the liquid surface outside. Surface tension of mercury at the temperature of the experiment is $0.5\ \text{Nm}^{-1}$ and density of mercury is $13.6\times10^3\ \text{kgm}^{-3}$. Take $g = 9.8\ \text{ms}^{-2}$.

SOLUTION

$$h = \frac{2T\cos\theta}{r\rho g}$$

Substituting the values, we get,

$$h = \frac{2\times0.5\times\cos120°}{10^{-3}\times13.6\times10^3\times9.8}$$

$$\Rightarrow \quad h = -3.75\times10^{-3}\ \text{m}$$

$$\Rightarrow \quad h = -3.75\ \text{mm}$$

Please note that the negative sign indicates that mercury suffers capillary depression.

ILLUSTRATION 137

A glass tube of radius 0.4 mm is dipped vertically in water. Find upto what height the water will rise in the capillary? If the tube in inclined at an angle of $60°$ with the vertical, how much length of the capillary is occupied by water. Surface tension of water is $7\times10^{-2}\ \text{Nm}^{-1}$, density of water $10^3\ \text{kgm}^{-3}$.

SOLUTION

For glass-water, angle of contact $\theta = 0°$

Now, $h = \dfrac{2T\cos\theta}{r\rho g}$

$$\Rightarrow \quad h = \frac{(2)(7\times10^{-2})\cos0°}{(0.4\times10^{-3})(10^3)(9.8)}$$

$$\Rightarrow \quad h = 3.57\times10^{-2}\ \text{m}$$

$$\Rightarrow \quad h = 3.57\ \text{cm}$$

$$\Rightarrow \quad l = \frac{h}{\cos60°} = \frac{3.57}{1/2} = 7.14\ \text{cm}$$

ILLUSTRATION 138

Two narrow bores of radius 3.0 mm and 6.0 mm are joined together to form a U-shaped tube open at both ends. If the U-tube contains water, what is the difference in its levels in the two limbs of the tube. Surface tension of water is $7.3\times10^{-2}\ \text{Nm}^{-1}$. Take the angle of contact to be zero and density of water to be $10^3\ \text{kgm}^{-3}$ and $g = 9.8\ \text{ms}^{-2}$.

SOLUTION

$$h\rho g = \Delta P$$

$$\Rightarrow \quad h\rho g = \frac{2T\cos\theta}{r_1} - \frac{2T\cos\theta}{r_2}$$

$$\Rightarrow \quad h = \frac{2T\cos\theta}{\rho g}\left(\frac{r_2 - r_1}{r_1 r_2}\right)$$

Substituting the values, we have

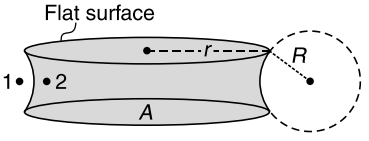
$$h = \frac{2\times7.3\times10^{-2}\times\cos0°}{10^3\times9.8}\left(\frac{6.0-3.0}{6.0\times3.0}\right)\times\frac{1}{10^{-3}}$$

$$\Rightarrow \quad h = 2.48\times10^{-3}\ \text{m}$$

$$\Rightarrow \quad h = 2.48\ \text{mm}$$

LIQUID BETWEEN TWO HORIZONTAL PLATES

When a small drop of water is placed between two glass plates placed face to face, it forms a thin cylindrical film which is concave outward along its boundary.



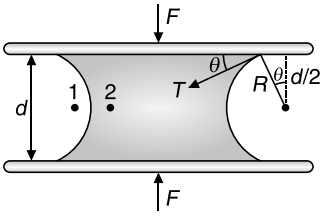
Let R and r be the radii of curvature of the enclosed film in two perpendicular directions.

The pressure difference on two sides of a cylindrical surface is

$$\Delta P = P_1 - P_2 = \frac{T}{R}, \text{ where } P_1 = P_{atm} = P_0$$

$$\Rightarrow \quad P_2 = P_1 - \frac{T}{R} = P_0 - \frac{T}{R} \qquad \ldots(1)$$

If d be the distance between the two plates and θ the angle of contact for water and glass as shown in Figure, then we get



$$\cos\theta = \frac{d/2}{R}$$

$$\Rightarrow \quad \frac{1}{R} = \frac{2\cos\theta}{d}$$

Substituting for $\frac{1}{R}$ in Equation (1), we get

$$P_2 = \left(P_1 - \frac{2T}{d}\cos\theta\right) \approx \left(P_0 - \frac{2T}{d}\cos\theta\right)$$

For water and glass, angle θ can be taken zero, so

$$\cos\theta \approx 1$$

The pressure above the upper pale is equal to the atmospheric pressure, whereas just below the plate inside the liquid, pressure is

$$P_2 = P_0 - \frac{T}{R} \approx P_0 - \frac{2T}{d}$$

So, the upper plate is pressed downwards by a force that corresponds to a pressure of $\frac{2T}{d}$. If $A = \pi r^2$ be the area of plate wetted by the film, then the resultant force F pressing the upper plate downwards is

$$F = A(\Delta P) = \frac{2TA}{d}$$

ILLUSTRATION 139

A drop of water of mass $m = 0.2$ g is placed between two clean glass plates, the distance between which is 0.01 cm. Calculate the force of attraction between the plates. Surface tension of water is $0.07\ \text{Nm}^{-1}$.

SOLUTION

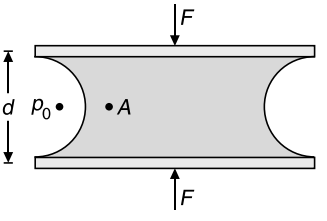
Let R be the radius of the circular layer of water, then

$$m = (\pi R^2 d)\rho \qquad \ldots(1)$$

Since meniscus is cylindrical in shape, so pressure at A is

$$P_A = P_0 - \frac{2T}{d}$$

Thus, pressure between the plates is less than the atmospheric pressure and so the plates are pressed together as though they are attracted towards each other.



If F is the force of attraction, then

$$F = (\Delta P)A = \left(\frac{2T}{d}\right)(\pi R^2) = \left(\frac{2T}{d}\right)\left(\frac{m}{\rho d}\right)$$

$$\Rightarrow \quad F = \frac{2Tm}{d^2\rho} = \frac{2\times 0.2\times 10^{-3}\times 0.07}{0.01^2\times 10^{-4}\times 1000} = 2.8\ \text{N}$$

RISE OF LIQUID IN A CAPILLARY TUBE OF INSUFFICIENT LENGTH

Suppose a liquid of density ρ and surface tension T rises in a capillary tube to a height h. Then

$$h\rho g = \frac{2T}{R}$$

where R is the radius of curvature of the liquid meniscus in the tube. From this we may write

$$hR = \frac{2T}{\rho g} = \text{ constant (for a given liquid)}$$

When the length of the tube is greater than h, the liquid rises in the tube to a height so as to satisfy the above relation. But if the length of the tube is less than h, say h', then the liquid rise up to the top of the tube and then spreads out until its radius of curvature R increases to R', such that

$$h'R' = hR = \frac{2T}{\rho g}$$

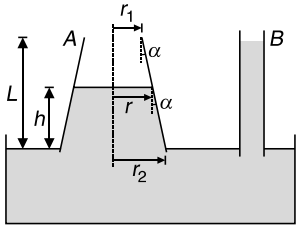
It is clear that liquid cannot emerge in the form of a fountain from the upper end of a short capillary tube.

ILLUSTRATION 140

A conical glass capillary tube A of length $0.1\ \text{m}$ has diameters $10^{-3}\ \text{m}$ and $5\times10^{-4}\ \text{m}$ at the ends. When it is just immersed in a liquid at 0 °C with larger diameter in contact with it, the liquid rises to $8\times10^{-2}\ \text{m}$ in the tube. If another cylindrical glass capillary tube B is immersed in the same liquid at 0 °C, the liquid rises to $6\times10^{-2}\ \text{m}$ height. The rise of liquid in the tube B is only $5.5\times10^{-2}\ \text{m}$ when the liquid is at $50\ °\text{C}$. Find the rate at which the surface tension changes with temperature considering the change to be linear. The density of the liquid is $\left(\frac{1}{14}\right)\times10^4\ \text{kgm}^{-3}$ and angle of contact is zero. Effect of temperature on density of liquid and glass is negligible.

SOLUTION

Let r be radius of meniscus in the conical tube as shown in Figure. Then, we have



$$\tan\alpha=\frac{r-r_1}{L-h}=\frac{r_2-r_1}{L}$$

$$\Rightarrow\quad \frac{r-2.5\times10^{-4}}{0.1-0.08}=\frac{(5.2.5)\times10^{-4}}{0.1}$$

$$\Rightarrow\quad r\times10^4-2.5=0.2\times2.5$$

$$\Rightarrow\quad r=3\times10^{-4}\ \text{m}$$

Since the phenomenon of capillarity is independent of the shape of tube, so at same temperature of $\theta=0$ °C, we have

$$h_Ar_A=h_Br_B=\frac{2T_0}{\rho g}=\text{constant}$$

$$\Rightarrow\quad r_B=\frac{(0.08)\left(3\times10^{-4}\right)}{0.06}=4\times10^{-4}\ \text{m}$$

For a cylindrical tube, we have $h=\dfrac{2T}{r\rho g}$

$$\Rightarrow\quad T_{0\,°\text{C}}=T_0=\frac{h_0\rho gr}{2}$$

$$\Rightarrow\quad T_0=\frac{1}{2}\left[(0.06)\left(\frac{1}{14}\times10^4\right)(9.8)\left(4\times10^{-4}\right)\right]$$

$$\Rightarrow\quad T_0=8.4\times10^{-2}\ \text{Nm}^{-1}$$

For a given tube and liquid, we have $T=\dfrac{h\rho gr}{2}$

$$\Rightarrow\quad T\propto h$$

$$\Rightarrow\quad \frac{T_{50}}{T_0}=\frac{h_{50}}{h_0}\ \text{ so}$$

$$\Rightarrow\quad T_{50\,°\text{C}}=T_{50}=\left(\frac{5.5\times10^{-2}}{6\times10^{-2}}\right)\left(8.4\times10^{-2}\right)$$

$$\Rightarrow\quad T_{50}=7.7\times10^{-2}\ \text{Nm}^{-1}$$

Hence, rate of change of surface tension (T) with temperature (θ) assuming linearity is given by

$$\frac{\Delta T}{\Delta\theta}=\frac{T_{50}-T_0}{50-0}=\frac{(7.7-8.4)\times10^{-2}}{50}$$

$$\Rightarrow\quad \frac{\Delta T}{\Delta\theta}=-1.4\times10^{-2}\ \text{Nm}^{-1}(°\text{C})^{-1}$$

Negative sign shows that with rise in temperature surface tension decreases.

ILLUSTRATION 141

The lower end of a capillary tube of diameter $2\ \text{mm}$ is dipped $8.5\ \text{cm}$ below the surface of water in a beaker. Calculate the pressure required in the tube in order to blow a hemispherical bubble at its end in the water. Assume that the surface tension of water is $7.5\times10^{-2}\ \text{Nm}^{-1}$, density of water is $10^3\ \text{kgm}^{-3}$, $g=10\ \text{ms}^{-2}$ and $1\ \text{atm}=10^5\ \text{Nm}^{-2}$. Also find excess pressure inside the bubble.

SOLUTION

Let P_i be the pressure inside the hemispherical air bubble blown at the lower end of the tube and P_0, just outside it. Then,

$$P_i-P_0=\frac{2T}{r}$$

$$\Rightarrow\quad P_i=P_0+\frac{2T}{r}\qquad\ldots(1)$$

Now, pressure outside hemispherical air bubble is

$$P_0=P_{\text{atm}}+h\rho g\qquad\ldots(2)$$

where, h is the length of capillary tube dipped inside water.

The pressure required to blow the bubble is equal to pressure inside the bubble.

$$\Rightarrow\quad P_i=P_0+\frac{2T}{r}=\left(P_{\text{atm}}+h\rho g\right)+\frac{2T}{r}$$

According to the problem, we have

$$T=7.5\times10^{-2}\ \text{Nm}^{-1},\ \rho=10^3\ \text{kgm}^{-3},$$

$$r=\frac{2\ \text{mm}}{2}=1\ \text{mm}=10^{-3}\ \text{m},\ P_{\text{atm}}=10^5\ \text{Nm}^{-2},$$

$$g=10\ \text{ms}^{-2}\ \text{and}\ h=8.5\ \text{cm}=\frac{8.5}{100}\ \text{m}$$

$$\Rightarrow \quad P_i = 10^5 + \left(\frac{8.5}{100}\right)(10^3)(10) + \frac{2(7.5\times10^{-2})}{10^{-3}}$$

$$\Rightarrow \quad P_i = 10^5 + 850 + 150 = 101000 \text{ Nm}^{-2}$$

$$\Rightarrow \quad P_i = 1.01\times10^5 \text{ Nm}^{-2}$$

The excess pressure inside the bubble is

$$P_i - P_0 = \frac{2T}{r} = \frac{2(7.5\times10^{-2})}{10^{-3}} = 150 \text{ Nm}^{-2}$$

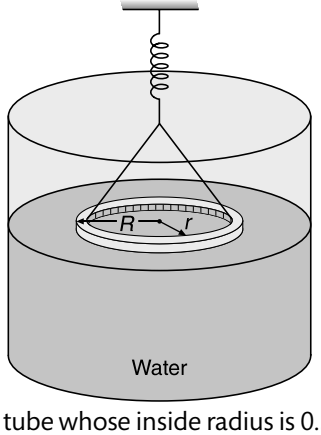
Test Your Concepts-X

Based on Surface Tension, Surface Energy, Excess Pressure and Capillarity

(Solutions on page H.18)

1. A liquid of specific gravity 1.5 is observed to rise 3 cm in a capillary tube of diameter 0.50 mm and the liquid wets the surface of the tube. Calculate the excess pressure inside a spherical bubble of 1 cm diameter blown from the same liquid if angle of contact is 0°.

2. A circular ring has inner and outer radii equal to r and R respectively. Mass of the ring is m. It gently pulled out vertically from a water surface by a sensitive spring. When the spring is stretched 3.4 cm from its equilibrium position, then ring is on verge of being pulled out from the water surface. If spring constant is k. Calculate the surface tension of water.



3. A capillary tube whose inside radius is 0.5 mm is dipped in water having surface tension 7×10^{-2} Nm^{-1}. To what height is the water raised above the normal water level. Angle of contact of water with glass is 0°. Density of water is 10^3 kgm^{-3} and $g = 9.8$ ms^{-2}.

4. A tube of insufficient length is immersed in water (surface tension = 0.07 Nm^{-1}) with 1 cm of it projecting vertically upwards outside the water. What is the radius of the meniscus? Radius of tube is 1 mm.

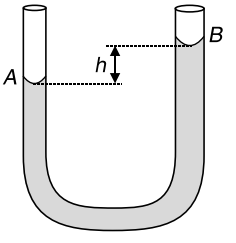
5. A soap bubble of radius r is placed on another soap bubble of radius R. What is the radius of the film separating the two bubbles?

6. Calculate the maximum possible mass of a greased needle that floats on a water surface. Assume that the length of the needle is l and surface tension of water to be T.

7. Water rises in a capillary tube to a height of 2 cm. In another capillary tube whose radius is one third of it, how much the water will rise?

8. Calculate the amount of energy evolved when 8 droplets of water each of radius 0.5 mm combine to form a single drop. Assume surface tension of water to be 72×10^{-3} Nm^{-1}.

9. Calculate difference (h) in the levels of water in two communicating capillary tubes A and B of radii 1 mm and 1.5 mm assuming that the surface tension of water is 0.07 Nm^{-1}.



10. Calculate the work to be done (in millijoule) against surface tension in blowing a soap bubble from a radius of 10 cm to 20 cm, if the surface tension of soap solution in 25×10^{-3} Nm^{-1}.

11. A film of water is formed between two straight parallel wires each 10 cm long and at separation 0.5 cm. Calculate the work required to increase 1 mm distance between the wires if surface tension of water is 72×10^{-3} Nm^{-1}.

12. Two soap bubbles in vacuum having radii 6 cm and 8 cm respectively coalesce under isothermal conditions to form a single bubble. Calculate the radius of the new bubble formed.

13. A small hollow sphere having a small hole in it is immersed in water to a depth 40 cm before any water enters the sphere. Calculate radius of the hole if surface tension of water is 75×10^{-3} Nm^{-1}, density of water is 10^3 kgm^{-3} and $g = 10$ ms^{-2}.

14. There are 1000 droplets of mercury of 1 mm diameter on a glass plate. Subsequently they merge into one big drop. How will the surface energy of the surface layer change? The process is isothermal. Surface tension of mercury is 0.475 Nm^{-1}.

SOLVED PROBLEMS

PROBLEM 1

A steel wire of diameter 0.40 mm is stretched between rigid supports separated by a horizontal distance of 1.8 m.

(a) What load suspended from the middle of the wire will produce a stress of 10^9 Nm^{-2}?
(b) Find the stored elastic energy in the wire under this load.
(c) Find the drop in potential energy of the object hung from the centre for this stress. $Y_S = 2\times10^{11}$ Nm^{-2}.

SOLUTION

(a) $2T\cos\theta = Mg$

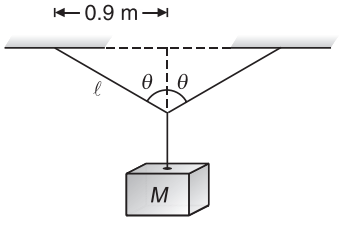
$\Rightarrow T = \dfrac{Mg}{2\cos\theta}$ …(1)

Further, $\dfrac{T}{A} = 10^9$ Nm^{-2} …(2)

$\Rightarrow T = (10^9)A$ …(3)

From equations (1) and (2)

$$10^9 = \frac{Mg}{2A\cos\theta}$$



$\Rightarrow Mg = (2A\cos\theta)\times10^9$

$\Rightarrow l = \dfrac{0.9}{\sin\theta}$

$\Rightarrow \Delta l = 0.9\left(\dfrac{1}{\sin\theta} - 1\right) = \dfrac{0.9T}{AY}$

$\Rightarrow \sin\theta = \dfrac{AY}{AY+T}$

$A = \dfrac{\pi}{4}(d^2) = 1.25\times10^{-7}$ m^2

$\Rightarrow T = (10^9)A = 125$ N

$\Rightarrow \sin\theta = \dfrac{(1.25\times10^{-7})(2\times10^{11})}{(1.25\times10^{-7}\times2\times10^{11})+125} = 0.995$

$\Rightarrow \cos\theta = \sqrt{1-\sin^2\theta} = 0.0996$

$\Rightarrow Mg = (2A\cos\theta)\times10^9$

$\Rightarrow Mg = (2\times1.25\times10^{-7})(0.0996)(10^9) \approx 25$ N

(b) $U = \left(\dfrac{(\text{Stress})^2}{2Y}\right)(\text{Volume})$

$\Rightarrow U = \dfrac{(10^9)^2}{2(2.0\times10^{11})}(0.125\times10^{-6})(1.8)$

$\Rightarrow U = 0.562$ J

(c) $10^9 = \dfrac{Mg}{2A\cos\theta}$

$\Rightarrow \cos\theta = \dfrac{Mg}{(2A)(10^9)}$

$\Rightarrow \cos\theta = \dfrac{25}{2\times0.125\times10^{-6}\times10^9} = 0.1$

$\Rightarrow \theta = 84.26°$

$\Rightarrow \Delta U = Mgh = (25)(0.9\cot\theta)$

$\Rightarrow \Delta U = 2.25$ J

PROBLEM 2

A glass full of water upto a height of 10 cm has a bottom of area 10 cm^2, top of area 30 cm^2 and volume 1 litre.

(a) Calculate the force exerted by the water on the bottom.
(b) Calculate the resultant force exerted by the sides of the glass on the water.
(c) If the glass is covered with a jar and the air inside the jar is completely pumped out, then recalculate the answers to parts (a) and (b).
(d) If a glass of different shape having same height, bottom area and volume is used then again recalculate the answers to parts (a) and (b).

Take $g = 10$ ms^{-2}, density of water to be 10^3 kgm^{-3} and atmospheric pressure to be 1.01×10^5 Nm^{-2}.

SOLUTION

(a) Force exerted by water at the bottom of glass is

$F_1 = (P_0 + \rho gh)A_1$ …(1)

where, $P_0 = P_{atm} = 1.01\times10^5$ Nm^{-2}

$\rho = \rho_w = 10^3$ kgm^{-3}, $g = 10$ ms^{-2}

$h = 10$ cm $= 0.1$ m and

Area of the base is

$A_1 = 10$ cm^2 $= 10^{-3}$ m^2

Substituting these values in equation (1), we get

$F_1 = (1.01\times10^5 + 10^3\times10\times0.1)\times10^{-3}$

$\Rightarrow F_1 = 102$ N (downwards)

(b) Force exerted by atmosphere on water

$$F_2 = P_0 A_2$$

where, A_2 is area of the top given by

$$A_2 = 30 \text{ cm}^2 = 3\times10^{-3} \text{ m}^2$$

$$\Rightarrow F_2 = (1.01\times10^5)(3\times10^{-3})$$

$$\Rightarrow F_2 = 303 \text{ N (downwards)}$$

Force exerted by bottom on the water is

$$F_3 = -F_1$$

$$\Rightarrow F_3 = 102 \text{ N (upwards)}$$

Weight of water in glass is

$$W = V\rho_w g = (10^{-3})(10^3)(10)$$

$$\Rightarrow W = 10 \text{ N (downwards)}$$

Let F be the force exerted by side walls on the water (upwards), then due to the equilibrium of water, we have

$$\Sigma F_{\text{upwards}} = \Sigma F_{\text{downwards}}$$

$$\Rightarrow F + F_3 = F_2 + W$$

$$\Rightarrow F = F_2 + W - F_3 = 303 + 10 - 102$$

$$\Rightarrow F = 211 \text{ N (upwards)}$$

(c) When air inside the jar is completely pumped out, then the atmospheric pressure becomes zero and hence we have

$$F_1 = (\rho g h) A_1$$

$$\Rightarrow F_1 = (10^3)(10)(0.1)(10^{-3})$$

$$\Rightarrow F_1 = 1 \text{ N (downwards)}$$

In this case, $F_2 = 0$ and $F_3 = 1$ N (upwards)

$$\Rightarrow F = F_2 + W - F_3$$

$$\Rightarrow F = 0 + 10 - 1 = 9 \text{ N (upwards)}$$

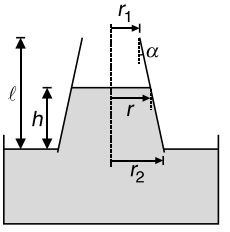
(d) When a glass of different shape having same height, bottom area and volume is used then the answers to (a) and (b) will remain the same, because they depend upon P_0, ρ, g, h, A_1 and A_2.

PROBLEM 3

A tube of conical bore of length 10 cm is just dipped inside the water. The diameters of upper and lower ends are 0.04 cm and 0.06 cm respectively. If the surface tension of water is 70 dyne cm^{-1} and angle of contact 0°, calculate the height to which the liquid rises in the tube.

SOLUTION

Let r_1, r_2 and r be the respective radii of the tube at the upper end, lower end and the meniscus of the water surface. Suppose that water rises to a height h in the tube. The view of the conical tube of length l dipped inside water is shown in Figure.



Now, $h = \dfrac{2T\cos\theta}{r\rho g}$

where, $T = 70$ dynecm^{-1}, $\theta = 0°$, $\rho = 1$ gcm^{-3} and $g = 980$ cms^{-1}

$$\Rightarrow h = \frac{2\times70\times\cos0°}{r\times1\times980} = \frac{1}{7r}$$

$$\Rightarrow r = \frac{h}{7} \quad \ldots(1)$$

If the wall of the conical tube makes an angle α with the vertical, then

$$\tan\alpha = \frac{r_2 - r_1}{l} = \frac{r - r_1}{l - h}$$

where, $r_1 = \dfrac{0.04}{2} = 0.02$ cm,

$$r_2 = \frac{0.06}{2} = 0.03 \text{ cm and } l = 10 \text{ cm}$$

$$\Rightarrow \frac{0.03 - 0.02}{10} = \frac{r - 0.02}{10 - h}$$

$$\Rightarrow (10 - h)\times0.001 = r - 0.02$$

$$\Rightarrow 0.001h + r - 0.03 = 0 \quad \ldots(2)$$

Substituting value of r from equation (1) in equation (2), we get

$$0.001h + \frac{1}{7h} - 0.03 = 0$$

$$\Rightarrow 7h^2 - 210h + 1000 = 0$$

$$\Rightarrow h = \frac{210 \pm \sqrt{(-210)^2 - 4\times7\times1000}}{14}$$

$$\Rightarrow h = \frac{210 \pm 126.9}{14} = 24.06 \text{ cm OR } 5.94 \text{ cm}$$

It can be checked that in a tube of uniform bore $r_1 (< r)$, water can rise up to a height of 7.35 cm only. So, the water cannot rise up to 24.06 cm and hence the water will rise up to a height of 5.94 cm in the conical tube.

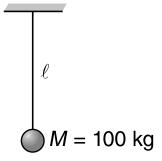
$$\Rightarrow h = 5.94 \text{ cm}$$

PROBLEM 4

A 5 m long cylindrical steel wire with radius 2×10^{-3} m is suspended vertically from a rigid support and carries a bob of mass 100 kg at the other end. If the bob gets snapped, calculate the change in temperature of the

wire ignoring radiation losses. Take $g = 10\ \text{ms}^{-2}$. (For the steel wire: Young's modulus $= 2.1\times10^{11}\ \text{Nm}^{-2}$. Density $= 7860\ \text{kgm}^{-3}$; specific heat $= 420\ \text{Jkg}^{-1}\text{K}$).

SOLUTION



Given, Length of the wire, $l = 5$ m

Radius of the wire, $r = 2\times10^{-3}$ m

Density of wire, $\rho = 7860\ \text{kgm}^{-3}$

Young's modulus, $Y = 2.1\times10^{11}\ \text{Nm}^{-2}$

and specific heat, $c = 420\ \text{Jkg}^{-1}\text{K}$

Mass of wire, $m = (\text{density})(\text{volume})$

$$m = (\rho)(\pi r^2 l)$$

$$\Rightarrow \quad m = (7860)(\pi)(2\times10^{-3})^2(5)\ \text{kg}$$

$$\Rightarrow \quad m = 0.494\ \text{kg}$$

Elastic potential energy stored in the wire,

$$U = \frac{1}{2}(\text{Stress})(\text{Strain})(\text{Volume}) \qquad \left\{\because \frac{\text{Energy}}{\text{Volume}} = \frac{1}{2}\text{stress}\times\text{strain}\right\}$$

$$\Rightarrow \quad U = \frac{1}{2}\left(\frac{Mg}{\pi r^2}\right)\left(\frac{\Delta l}{l}\right)(\pi r^2 l)$$

$$\Rightarrow \quad U = \frac{1}{2}(Mg)\Delta l \qquad \left(\Delta l = \frac{Fl}{AY}\right)$$

$$\Rightarrow \quad U = \frac{1}{2}(Mg)\frac{Mgl}{(\pi r^2)Y} = \frac{1}{2}\frac{M^2g^2l}{\pi r^2 Y}$$

Substituting the values, we get

$$U = \frac{1}{2}\frac{(100)^2(10)^2(5)}{(3.14)(2\times10^{-3})^2(2.1\times10^{11})}\ \text{J}$$

$$\Rightarrow \quad U = 0.9478\ \text{J}$$

When the bob gets snapped, this energy is utilised in raising the temperature of the wire, through $\Delta\theta$, so

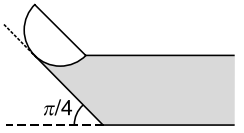
$$U = mc\Delta\theta$$

$$\Rightarrow \quad \Delta\theta = \frac{U}{ms} = \frac{(0.9478)}{(0.494)(420)}\ ^\circ\text{C or K}$$

$$\Rightarrow \quad \Delta\theta = 4.568\times10^{-3}\ ^\circ\text{C}$$

PROBLEM 5

A uniform half cylinder of density ρ_2 is resting in equilibrium on a rough inclined plane (having inclination $\pi/4$) with liquid of density ρ_1 on its right as shown in Figure.



Calculate the minimum coefficient of friction to ensure equilibrium and the ratio $\frac{\rho_1}{\rho_2}$ assuming that the flat surface of the half cylinder is parallel to the inclined plane.

SOLUTION

Let us draw the arrangement in the Figure(s) shown.

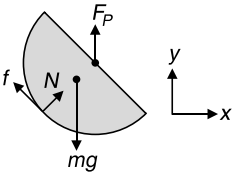


Figure (a)

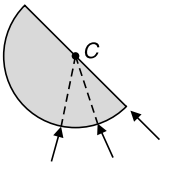


Figure (b)

From Figure (b) following conclusion can be drawn about the pressure force.

(i) Torque of pressure force about C is zero and
(ii) The resultant force acts vertically (by symmetry)

See the force diagram in Figure (a). Cylinder is in equilibrium under the four coplanar forces shown in figure. So, we have

$$\Sigma F_x = 0$$

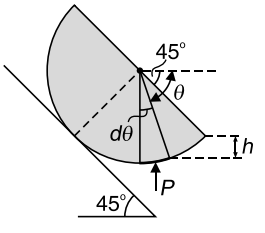
$$\Rightarrow \quad \frac{f}{\sqrt{2}} = \frac{N}{\sqrt{2}}$$

$$\Rightarrow \quad f = N \qquad \ldots(1)$$

Since, we know that $\mu \geq \frac{f}{N}$, so on using equation (1), we get

$$\mu_{\min} = 1$$

Calculation of F_P



Since, $h = R\left(\sin\theta - \sin\frac{\pi}{4}\right)$

$$\Rightarrow \quad F_P = \int_{\pi/4}^{3\pi/4} PdA\sin\theta$$

$$\Rightarrow \quad F_P = \int_{\pi/4}^{3\pi/4} \rho_1 gR\left(\sin\theta - \sin\frac{\pi}{4}\right)(LRd\theta)\sin\theta$$

$$\Rightarrow \quad F_P = \rho_1 g R^2 L\left(\frac{\pi}{4}-\frac{1}{2}\right)$$

Also, $\Sigma F_y = 0$

$$\Rightarrow \quad F_P = mg - \frac{1}{\sqrt{2}}(f+N)$$

$$\Rightarrow \quad \left(\frac{\pi}{4}-\frac{1}{2}\right)LR^2\rho_1 g = mg - \sqrt{2}f \qquad \ldots(2)$$

Since in equilibrium, torque about the point C is also zero, so we have

$$\Sigma\tau_C = 0$$

$$\Rightarrow \quad fR = (mg)\left(\frac{4R}{3\pi}\sin\frac{\pi}{4}\right)$$

$$\Rightarrow \quad f = \frac{4mg}{3\sqrt{2}\pi}$$

Substituting this value of f in equation (2), we get

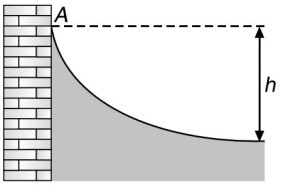
$$\left(\frac{\pi}{4}-\frac{1}{2}\right)LR^2\rho_1 g = mg\left(1-\frac{4}{3\pi}\right)$$

$$\Rightarrow \quad \left(\frac{\pi}{4}-\frac{1}{2}\right)LR^2\rho_1 g = \left(\frac{\pi R^2}{2}\right)(L\rho_2)g\left(\frac{3\pi-4}{3\pi}\right)$$

$$\Rightarrow \quad \frac{\rho_1}{\rho_2} = \frac{2(3\pi-4)}{3(\pi-2)}$$

PROBLEM 6

A liquid having surface tension T and density ρ is in contact with a vertical solid wall due to which the liquid surface gets curved as shown in Figure.



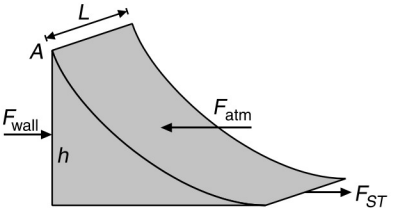
Assuming that at the bottom, the liquid surface is flat and the atmospheric pressure is P_0, Calculate the pressure inside the liquid at the top most point A of the meniscus. Also calculate the difference in height (h) between highest and the lowest points of the meniscus.

SOLUTION

At the lowest level of meniscus liquid surface is flat, so pressure at the lowest level of meniscus is P_0 and hence pressure inside the liquid at A is

$$P_A = P_0 - \rho g h$$

For horizontal equilibrium of liquid in the meniscus, let us consider a depth L of the liquid section perpendicular to the plane as shown in Figure.



Force due to wall on the liquid is

$$F_{\text{wall}} = P_{av}A = P_{av}(hL)$$

$$\Rightarrow \quad F_{\text{wall}} = \left[\frac{P_0 + (P_0 - \rho g h)}{2}\right]hL = \left(P_0 - \frac{\rho g h}{2}\right)hL$$

Force due to the atmospheric pressure is

$$F_{\text{atm}} = P_0 A = P_0(hL)$$

Force due to surface tension is

$$F_{ST} = TL$$

For horizontal equilibrium of liquid in the meniscus, we have

$$F_{\text{wall}} + F_{ST} = F_{\text{atm}}$$

$$\Rightarrow \quad \left(P_0 - \frac{\rho g h}{2}\right)hL + TL = P_0 hL$$

$$\Rightarrow \quad P_0 h - \frac{\rho g h^2}{2} + T = P_0 h$$

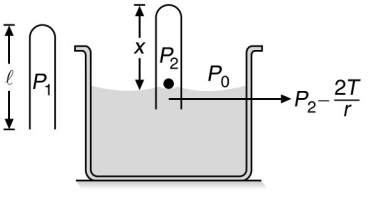
$$\Rightarrow \quad h = \sqrt{\frac{2T}{\rho g}}$$

PROBLEM 7

A glass capillary sealed at the upper end is of length 0.11 m and internal diameter 2×10^{-5} m. The tube is immersed vertically into a liquid of surface tension 5.0×10^{-2} Nm^{-1} and density 10^3 kgm^{-3}. To what length has the capillary to be immersed so that the liquid level inside and outside the capillary becomes the same. What will happen to the level of liquid inside the capillary if the seal is now broken. Assume the temperature to be constant, atmospheric pressure to be $P_0 = 1.01\times10^5$ Nm^{-2} and contact angle to be $\theta = 0°$.

SOLUTION

From the figure shown, we have



$$P_1 = P_0 \qquad \{\text{atmospheric pressure}\}$$

If A is the area of cross section of tube, then we have

$$V_1 = Al$$

Also, we know that

$$P_2 - \frac{2T}{r} = P_0$$

$$\Rightarrow \quad P_2 = \left(P_0 + \frac{2T}{r}\right)$$

and $V_2 = Ax$

Under isothermal (T = constant) conditions, we have

$$P_1V_1 = P_2V_2$$

$$\Rightarrow \quad P_0Al = \left(P_0 + \frac{2T}{r}\right)Ax$$

$$\Rightarrow \quad x = \frac{P_0 l}{P_0 + \frac{2T}{r}}$$

Substituting the values, we get

$$x = \frac{(1.01\times10^5)(0.11)}{(1.01\times10^5) + \frac{2\times5.0\times10^{-2}}{10^{-5}}}$$

$$\Rightarrow \quad x = 0.1 \text{ m}$$

The length of the tube immersed in liquid is

$$l' = l - x = 0.11 - 0.1 = 0.01 \text{ m}$$

If the seal is now broken, then the pressure now becomes P_0 and the rise of liquid will be given by

$$h = \frac{2T}{r\rho g} \qquad (\theta = 0^\circ)$$

$$\Rightarrow \quad h = \frac{2\times5.0\times10^{-2}}{10^{-5}\times10^3\times9.8} = 1.02 \text{ m}$$

However, the length of tube outside the liquid is only 0.1 m, so the liquid will rise only to the top of the tube and will stay there with radius of meniscus

$$r_2 = \frac{h_1 r_1}{h_2}$$

$$\Rightarrow \quad r_2 = \frac{(1.02)(10^{-5})}{0.1} = 1.02\times10^{-4} \text{ m}$$

PROBLEM 8

A thin wire of cross-sectional area a is bent to form a circular ring of radius R. The ring rotates about an axis perpendicular to its plane and through its centre with angular frequency ω. Given density of the wire is ρ, Young's modulus of elasticity is Y. Find the total energy stored in the wire.

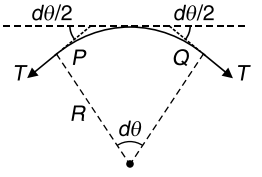
SOLUTION

dm = mass of element PQ of the wire

$$\Rightarrow \quad dm = (Rd\theta)(a\rho)$$

Now, $2T\sin\left(\frac{d\theta}{2}\right) = (dm)R\omega^2$

$$\Rightarrow \quad Td\theta = a\rho R^2\omega^2 d\theta$$



Since, $\sin\frac{d\theta}{2} \approx \frac{d\theta}{2}$ for small angles

$$\Rightarrow \quad T = a\rho R^2\omega^2$$

So, stress in the wire is given by $\sigma = \frac{T}{a} = \rho R^2\omega^2$

Potential energy per unit volume,

$$U = \frac{1}{2}\frac{(\text{stress})^2}{Y} = \frac{1}{2}\frac{\rho^2R^4\omega^4}{Y}$$

Total potential energy is the product of energy density and volume. So,

$$E = u(\text{Volume}) \qquad \ldots(1)$$

$$\Rightarrow \quad E = \frac{1}{2}\frac{\rho^2R^4\omega^4}{Y}(2\pi Ra) = \frac{\pi\rho^2R^5a\omega^4}{Y}$$

Kinetic energy

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}(mR^2)\omega^2$$

$$\Rightarrow \quad K = \frac{1}{2}(2\pi Ra\rho)(R^2\omega^2)$$

$$\Rightarrow \quad K = \pi R^3a\rho\omega^2 \qquad \ldots(2)$$

So, total energy

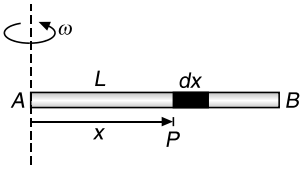
$$E_{\text{total}} = U + K = \pi\omega^2R^3\rho a\left(1 + \frac{\rho R^2\omega^2}{Y}\right)$$

PROBLEM 9

A uniform material rod of length L is rotated in a horizontal plane about a vertical axis through one of its ends. The angular speed of rotation is ω. Calculate the increase in length of the rod if the density and Young's modulus of rod are ρ and Y respectively.

SOLUTION

Consider an element of length dx at a distance x from the axis of rotation of the rod as shown in Figure.



The centripetal force required to move this element in a circle of radius x is

$$dF = (dm)x\omega^2 = (\rho A dx)\omega^2 x$$

Tension at a point P at a distance x from the axis is equal to sum of centripetal forces on all elements lying between P and B. So, at point P, tension is given by

$$T = \rho A\omega^2 \int_x^L x dx = \frac{\rho A\omega^2}{2}(L^2 - x^2)$$

Now assume that dl is extension in an element of length dx located at a distance x from the axis.

$$\text{Strain} = \frac{dl}{dx}$$

$$\text{Stress} = \frac{T}{A} = \frac{1}{2}\rho\omega^2(L^2 - x^2)$$

$$\Rightarrow \quad Y\frac{dl}{dx} = \frac{1}{2}\rho\omega^2(L^2 - x^2)$$

$$\Rightarrow \quad dl = \frac{1}{2}\frac{\rho\omega^2}{Y}(L^2 - x^2)dx$$

So, change in length of the entire rod is given by

$$\Delta l = \int dl = \frac{\rho\omega^2}{2Y}\left(L^2 x - \frac{x^3}{3}\right)\Bigg|_0^L$$

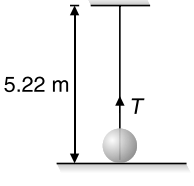
$$\Rightarrow \quad \Delta\ell = \frac{\rho\omega^2}{2Y}\left(L^3 - \frac{L^3}{3}\right) = \frac{1}{3}\frac{\rho\omega^2 L^3}{Y}$$

PROBLEM 10

A sphere of radius 0.1 m and mass 8π kg is attached to the lower end of a steel wire of length 5 m and diameter 10^{-3} m. The wire is suspended from 5.22 m high ceiling of a room. When the sphere is made to swing as a simple pendulum, it just grazes the floor at its lowest point. Calculate the velocity of the sphere at the lowest position. Young's modulus of steel is 1.994×10^{11} Nm^{-2}.

SOLUTION

Let Δl be the extension of wire when the sphere is at mean position. Then, we have



$$l + \Delta l + 2r = 5.22$$

$$\Rightarrow \quad \Delta l = 5.22 - l - 2r$$

$$\Rightarrow \quad \Delta l = 5.22 - 5 - 2\times0.1$$

$$\Rightarrow \quad \Delta l = 0.02 \text{ m}$$

Let T be the tension in the wire at mean position during oscillations, then from Hooke's Law

$$Y = \frac{T/A}{\Delta l/l}$$

$$\Rightarrow \quad T = \frac{YA\Delta l}{l} = \frac{Y\pi r^2\Delta l}{l}$$

Substituting the values, we get

$$T = \frac{(1.994\times10^{11})\times\pi\times(0.5\times10^{-3})^2\times0.02}{5}$$

$$\Rightarrow \quad T = 626.43 \text{ N}$$

The equation of motion at mean position is, given by

$$T - mg = \frac{mv^2}{R} \qquad \ldots(1)$$

where, $R = 5.22 - r = 5.22 - 0.1 = 5.12$ m

and $m = 8\pi$ kg $= 25.13$ kg

Substituting the values in equation (1), we get

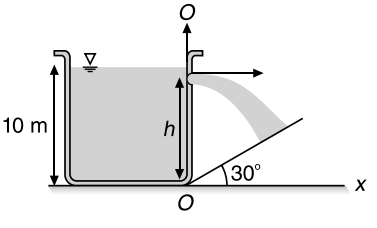
$$(626.43) - (25.13\times9.8) = \frac{(25.13)v^2}{5.12}$$

Solving this equation, we get

$$v = 8.8 \text{ ms}^{-1}$$

PROBLEM 11

A rectangular tank of height 10 m filled with water, is placed near the bottom of a plane inclined at an angle $30°$ with horizontal. At height h from bottom a small hole is made (as shown in figure) such that the stream coming out from hole, strikes the inclined plane normally. Calculate h.



SOLUTION

Speed of the liquid coming out of the orifice is

$$v = \sqrt{2g(10 - h)}$$

Component of its velocity parallel to the plane is $v\cos30°$. Let stream strike the plane after time t at the point (x, y) i.e. when the velocity of the stream is perpendicular to the plane, so we get

$$0 = v\cos30° - g\sin30°t$$

$$\Rightarrow \quad t = \frac{v\cot 30^\circ}{g}$$

Also, we have $\tan(30^\circ) = \frac{y}{x}$

$$\Rightarrow \quad x = vt = \frac{v^2\cot 30^\circ}{g} = \sqrt{3}y$$

$$\Rightarrow \quad \frac{v^2\cos 30^\circ}{g} = \sqrt{3}\left(h - \frac{1}{2}gt^2\right)$$

$$\Rightarrow \quad \frac{\sqrt{3}v^2}{g} = \sqrt{3}\left(h - \frac{g}{2}\frac{v^2\cot^2 30^\circ}{g^2}\right)$$

$$\Rightarrow \quad \frac{v^2}{g} = h - \frac{3}{2}\frac{v^2}{g}$$

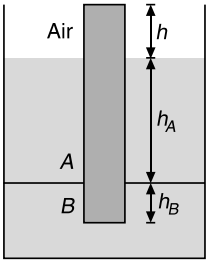
$$\Rightarrow \quad \frac{5}{2}\frac{v^2}{g} = h$$

$$\Rightarrow \quad 5(10-h) = h$$

$$\Rightarrow \quad h = 8.33 \text{ m}$$

PROBLEM 12

A uniform solid cylinder of density $0.8\ \text{gcm}^{-3}$ floats in equilibrium in a combination of two non-mixing liquids A and B with its axis vertical as shown in Figure.



The densities of liquids A and B are $0.7\ \text{gcm}^{-3}$ and $1.2\ \text{gcm}^{-3}$, respectively. The height of liquid A is $h_A = 1.2$ cm. The length of the part of the cylinder with liquid B is $h_B = 0.8$ cm. Calculate the net force exerted by liquid A on the cylinder and the length h of part of the cylinder in air. If the cylinder is depressed in such a way that its top surface is just below the upper surface of liquid A and is then released, then calculate the acceleration of cylinder immediately after being released. Take $g = 10\ \text{ms}^{-2}$.

SOLUTION

The liquid A exerts horizontal forces on all surface area elements from all the directions. Due to symmetry, the resultant of these horizontal forces is zero. So, the net force exerted by liquid A on the cylinder is zero.

Let a be the area of cross-section of the cylinder. Then for equilibrium of the cylinder, weight of the cylinder equals the buoyant force, which equals the loss in weight of the cylinder.

$$\Rightarrow \quad W = U$$

$$\Rightarrow \quad a(h + h_A + h_B)(0.8)g = (ah_A)\rho_A g + (ah_B)\rho_B g$$

$$\Rightarrow \quad (h + h_A + h_B)(0.8) = h_A\rho_A + h_B\rho_B$$

$$\Rightarrow \quad (h + 1.2 + 0.8)(0.8) = (1.2)(0.7) + (0.8)(1.2)$$

$$\Rightarrow \quad 0.8h + 1.6 = 1.8$$

$$\Rightarrow \quad 0.8h = 0.2$$

$$\Rightarrow \quad h = 0.25 \text{ cm}$$

Mass of cylinder is

$$M = a(h + h_A + h_B)(0.8)$$

$$\Rightarrow \quad M = a(0.2 + 1.2 + 0.8)(0.8) = 1.8a$$

When the cylinder is depressed just completely, its height h goes inside the liquid B and hence the extra buoyant force due to liquid B acts on it, due to which the cylinder is accelerated upward.

Additional upthrust is given by

$$F = (ah)\rho_B g$$

$$\Rightarrow \quad F = (0.25a)(1.2)g = 0.3ag$$

Acceleration is given by

$$\text{Acceleration} = \frac{F}{M} = \frac{0.3ag}{1.8a} = \frac{g}{6}$$

$$\Rightarrow \quad \text{Acceleration} = \frac{10}{6} = \frac{5}{3}\ \text{ms}^{-2}$$

PROBLEM 13

A thin uniform metallic rod of length 0.5 m rotates with an angular velocity $400\ \text{rads}^{-1}$ in a horizontal plane about a vertical axis passing through one of its ends. Calculate the elongation of the rod. The density of the material of the rod is $10^4\ \text{kgm}^{-3}$ and the Young's modulus is $2\times10^{11}\ \text{Nm}^{-2}$.

SOLUTION

Let the mass per unit length of rod be $\lambda\left(=\frac{m}{l}\right)$ (say).

Consider an element of length dx at a distance x from centre. Then centripetal force on this element is

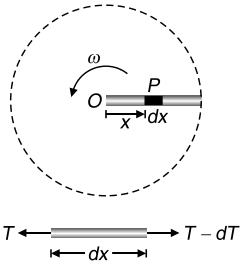
$$dF = (\lambda dx)x\omega^2$$

Tension in the rod is a function of x. The difference in tension at two ends of the section P provides the necessary centripetal force.

$$-dT = \lambda(dx)x\omega^2$$

The negative sign indicates decrease in the value of T and x increases.

$$\Rightarrow \quad -\int_0^T dT = \lambda\omega^2\int_l^x x\,dx$$



Now, we can also write $\lambda = \frac{m}{l} = \rho A = \rho\pi r^2$

$$\Rightarrow \quad T(x) = \frac{\pi r^2 \rho l^2 \omega^2}{2}\left(1 - \frac{x^2}{l^2}\right)$$

Let dl be the extension of the rod in the small element dx at P, then

$$dl = \frac{T(x)}{AY}dx$$

$$\Rightarrow \quad dl = \frac{\pi r^2 \rho l^2 \omega^2}{2AY}\left(1 - \frac{x^2}{l^2}\right)dx$$

So, total extension is given by $\Delta l = \int_0^l dl$

$$\Rightarrow \quad \Delta l = \frac{\pi r^2 \rho l^2 \omega^2}{2AY}\int_0^l \left(1 - \frac{x^2}{l^2}\right)dx$$

$$\Rightarrow \quad \Delta l = \frac{\rho \omega^2 l^3}{3Y} \text{ as } A = \pi r^2$$

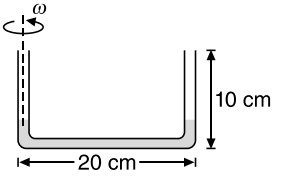
Substituting the values, we get

$$\Delta l = \frac{1}{3}\frac{(10^4)(400)^2(0.5)^3}{2\times 10^{11}}$$

$$\Rightarrow \quad \Delta l = 3.33\times 10^{-4} \text{ m}$$

PROBLEM 14

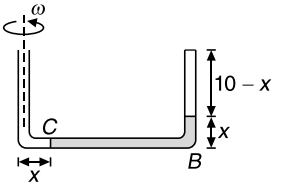
Length of a horizontal arm of a *U*-tube is 20 cm and ends of both the vertical arms are open to a pressure of 1.01×10^3 Nm^{-2}. Water is poured into the tube such that liquid just fills horizontal part of the tube. Now, one of the open ends is sealed and the tube is then rotated about a vertical axis passing through the other vertical arm with angular velocity ω. If length of water in sealed arm is 5 cm, then calculate ω. Assume the density of water to be 10^3 kgm^{-3}, $g = 10$ ms^{-2} and temperature to be constant.



SOLUTION

Let the cross-sectional area of the tube be A. The initial pressure of air in sealed tube is

$$P_i = 1.01\times 10^3 \text{ Nm}^{-2}$$



Initial volume, $V_i = 0.1A$

Final volume, $V_f = (0.1 - x)A$

If final pressure is P_f, then for constant temperature, we have

$$P_i V_i = P_f V_f$$

$$\Rightarrow \quad P_f = \frac{P_i V_i}{V_f} = (1.01\times 10^3)\left(\frac{0.1}{0.1 - x}\right) \qquad \ldots(1)$$

Also, we have

$$P_B = P_f + \rho g x \qquad \ldots(2)$$

and $P_C = 1.01\times 10^3$ Nm^{-2} $\qquad \ldots(3)$

So, pressure difference between B and C is

$$\Delta P = P_B - P_C$$

$$\Rightarrow \quad \Delta P = (P_f + \rho g x) - P_C \qquad \ldots(4)$$

The centripetal force required for circular motion of vertical column is provided by reaction of the tube while the centripetal force required for circular motion of the horizontal part is provided by the excess pressure at B.

$$\Rightarrow \quad F = (\Delta P)A = (m_{CB})r\omega^2 = [(0.2 - x)\rho A]r\omega^2$$

$$\Rightarrow \quad \Delta P = [(0.2 - x)\rho]r\omega^2 \qquad \ldots(5)$$

where, r is the distance of centre of mass of horizontal portion of liquid from the axis of rotation.

$$\Rightarrow \quad r = x + \left(\frac{0.2 - x}{2}\right) = \frac{0.2 + x}{2} \qquad \ldots(6)$$

Substituting the value of r from equation (5) and ΔP from equation (4) in equation (5), we get these values we have,

$$(1.01\times 10^3)\left(\frac{0.1}{0.1 - x}\right) + \rho g x - (1.01\times 10^3) =$$

$$(0.2 - x)\rho\left(\frac{0.2 + x}{2}\right)(\omega)^2$$

Substituting the values given in the problem, i.e.

$$x = 0.05 \text{ m}, \ \rho = 10^3 \text{ kgm}^{-3} \text{ and } g = 10 \text{ ms}^{-2}$$

we get

$$2.02 + 0.5 - 1 = 0.01875\omega^2$$

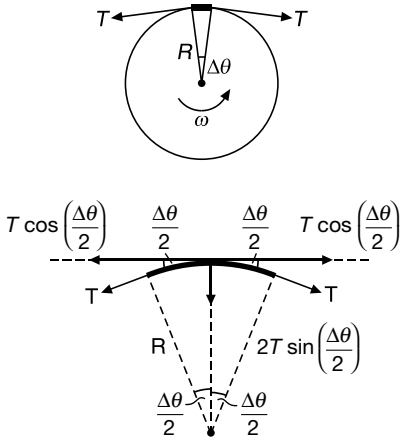
$$\Rightarrow \quad \omega = 9.15 \text{ rads}^{-1}$$

PROBLEM 15

A circular ring of radius R, mass M, made of a uniform wire of cross-sectional area A is rotated about a stationary vertical axis passing through its centre and perpendicular to the plane of the ring. If the breaking stress of the material of the ring is σ_0, then calculate the maximum angular speed ω_{max} at which the ring may be rotated without breaking it. If Young's modulus of the material of ring is Y, then calculate the increment ΔR in the radius of the ring.

SOLUTION

Every element of the ring rotates in a circle of radius R about the axis of rotation. The radial component of tension (T) in the wire provides the centripetal force. The free body diagram of a small element of mass Δm is shown in Figure.



The mass of this element is given by

$$\Delta m = \frac{M\Delta\theta}{2\pi}$$

The net force acting towards the centre is

$$2T\sin\left(\frac{\Delta\theta}{2}\right) \approx 2T\left(\frac{\Delta\theta}{2}\right) \approx T\Delta\theta$$

This force provides the necessary centripetal force to the ring to revolve in a circle of radius R.

$$\Rightarrow \quad T\Delta\theta = \left(\frac{M\Delta\theta}{2\pi}\right)\omega^2 R$$

$$\Rightarrow \quad T = \frac{M\omega^2 R}{2\pi}$$

According to the problem, the breaking stress is σ_0, so the maximum value of tension that the ring can bear is

$$T_{max} = \sigma_0 A$$

$$\Rightarrow \quad \frac{M\omega_{max}^2 R}{2\pi} = \sigma_0 A$$

$$\Rightarrow \quad \omega_{max} = \sqrt{\frac{2\pi\sigma_0 A}{MR}}$$

Stress (σ) in the ring is

$$\sigma = \frac{T}{A} = \frac{MR\omega^2}{2\pi A}$$

If ΔR is the increment in the radius of the ring, then the elongation in the length of the ring is

$$\Delta l = 2\pi(R+\Delta R) - 2\pi R = 2\pi\Delta R$$

The strain (ε) is given by

$$\varepsilon = \frac{\Delta l}{l} = \frac{\Delta l}{2\pi R} = \frac{\Delta R}{R}$$

By definition, we have $Y = \dfrac{\text{Stress}}{\text{Strain}} = \dfrac{\sigma}{\varepsilon}$

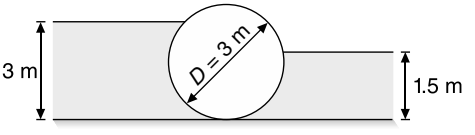
$$\Rightarrow \quad \varepsilon = \frac{\sigma}{Y}$$

$$\Rightarrow \quad \Delta R = \left(\frac{\sigma}{Y}\right)R = \left(\frac{MR\omega^2}{2\pi A}\right)\frac{R}{Y}$$

$$\Rightarrow \quad \Delta R = \frac{MR^2\omega^2}{2\pi AY}$$

PROBLEM 16

A cylindrical weir has a diameter of 3 m and a length of 6 m, whose cross-sectional view is shown in Figure. Calculate the magnitude of the resultant force acting on the weir due to water. Take $g = 10\ \text{ms}^{-2}$ and $\rho_{water} = 1000\ \text{kgm}^{-3}$.

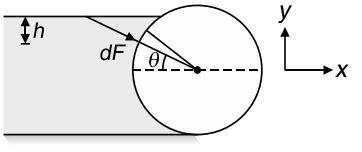


SOLUTION

Let us calculate the force on weir due to water from the left side and the right side.

Hydrostatic force from the left side

Consider an infinitesimal element on the left side of the weir subtending an angle $d\theta$ at the centre of the weir and making an angle θ with the diameter as shown in Figure.



The arc length of the element is $Rd\theta$, so area of the element measured across the length of the weir is

$$dA = (6)Rd\theta$$

Depth of the element below the free surface of water is

$$h = R(1-\sin\theta)$$

Force on this element due to water is

$$dF = (\rho gh)dA = 6\rho gR^2(1-\sin\theta)d\theta$$

The x and y components of this force are

$$dF_x = dF\cos\theta = 6\rho gR^2\cos\theta(1-\sin\theta)d\theta \text{ and}$$

$$dF_y = -dF\sin\theta = -6\rho gR^2(1-\sin\theta)\sin\theta d\theta$$

$$\Rightarrow \quad F_x = \int_{-\pi/2}^{+\pi/2} dF_x = 6\rho gR^2\int_{-\pi/2}^{+\pi/2}\cos\theta(1-\sin\theta)d\theta$$

$$\Rightarrow \quad F_x = 6\rho gR^2\left(\sin\theta+\frac{\cos 2\theta}{4}\right)\Bigg|_{-\pi/2}^{+\pi/2}$$

$$\Rightarrow \quad F_x = 12\rho gR^2 \qquad \ldots(1)$$

Similarly, we have

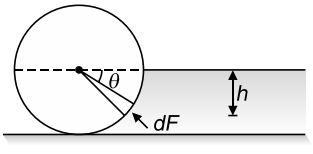
$$F_y = \int_{-\pi/2}^{+\pi/2} dF_y = -6\rho gR^2\int_{-\pi/2}^{+\pi/2}(1-\sin\theta)\sin\theta d\theta$$

$$\Rightarrow \quad F_y = -6\rho gR^2\left(-\cos\theta+\frac{\sin 2\theta}{4}-\frac{\theta}{2}\right)\Bigg|_{-\pi/2}^{+\pi/2}$$

$$\Rightarrow \quad F_y = 3\pi\rho gR^2 \qquad \ldots(2)$$

Hydrostatic force from the right side

Similarly, consider an infinitesimal element on the right side of the weir subtending an angle $d\theta$ at the centre of the weir and making an angle θ with the diameter as shown in Figure.



Area of this element is

$$dA = 6(Rd\theta)$$

Depth of this element below the free surface of the liquid is

$$h = R\sin\theta$$

Force on this element due to water is

$$dF = (\rho gh)dA = 6\rho gR^2\sin\theta d\theta$$

The x and y components of this force are

$$dF_x = -dF\cos\theta = -3\rho gR^2(\sin 2\theta)d\theta \text{ and}$$

$$dF_y = dF\sin\theta = 3\rho gR^2(2\sin^2\theta)d\theta$$

$$\Rightarrow \quad F_x = -3\rho gR^2\int_0^{\pi/2}(\sin 2\theta)d\theta$$

$$\Rightarrow \quad F_x = -3\rho gR^2 \qquad \ldots(3)$$

$$\Rightarrow \quad F_y = 3\rho gR^2\int_0^{\pi/2}(1-\cos 2\theta)d\theta$$

$$\Rightarrow \quad F_y = \frac{3}{2}\pi\rho gR^2 \qquad \ldots(4)$$

From equations (1), (2), (3) and (4), we get

$$(F_x)_{\text{net}} = 12\rho gR^2 - 3\rho gR^2 = 9\rho gR^2 \text{ and}$$

$$(F_y)_{\text{net}} = \frac{9}{2}\pi\rho gR^2$$

$$\Rightarrow \quad F_{\text{net}} = \rho gR^2\sqrt{81+\left(\frac{9\pi}{2}\right)^2} = 16.76\rho gR^2$$

$$\Rightarrow \quad F_{\text{net}} = (16.76)(10^3)(10)(1.5)^2$$

$$\Rightarrow \quad F_{\text{net}} = 377100 \text{ N} \approx 377 \text{ kN}$$

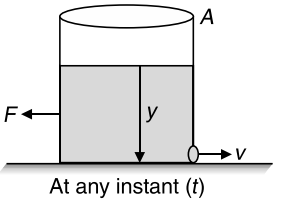
PROBLEM 17

A large open top container of negligible mass and uniform cross-sectional area A has a small hole of cross-sectional area $\frac{A}{100}$ in its side wall near the bottom. The container is kept on a smooth horizontal floor and contains a liquid of density ρ and mass M_0. Assuming that the liquid starts flowing out horizontally through the hole at $t = 0$, calculate

(a) the acceleration of the container and
(b) its velocity when 75% of the liquid has drained out.

SOLUTION

Let y be the depth of the orifice below the free surface of the liquid at any instant t after the start.



(a) The velocity of efflux of the liquid coming out of the orifice is

$$v = \sqrt{2gy}$$

The thrust exerted on the beaker due to the ejection of the liquid is

$$F = av^2\rho, \text{ opposite to } v$$

$$\Rightarrow \quad F = a(2gy)\rho$$

So, acceleration of the container plus liquid system at the instant discussed is

$$\text{Acceleration} = \frac{F}{M_{\text{liquid}} + M_{\text{container}}}$$

Since, the container has negligible mass, so

$$\text{Acceleration} \approx \frac{F}{M_{\text{liquid}}} = \frac{a(2gy)\rho}{Ay\rho}$$

$$\Rightarrow \text{Acceleration} = 2g\left(\frac{a}{A}\right) = 2g\left(\frac{1}{100}\right)$$

$$\Rightarrow \text{Acceleration} = 2g\left(\frac{a}{A}\right) = \frac{g}{50}$$

(b) When 75% of the liquid has drained out, then height y' of the free surface of the liquid is given by

$$Ay'\rho = \frac{25}{100}M_0$$

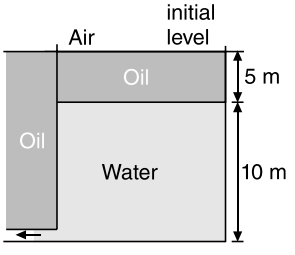
$$\Rightarrow y' = \frac{M_0}{4A\rho}$$

So, the new speed of efflux is

$$v' = \sqrt{2gy'} = \sqrt{2g\left(\frac{M_0}{4A\rho}\right)} = \sqrt{\frac{M_0 g}{2A\rho}}$$

PROBLEM 18

A tank having a small circular hole contains oil on top of water. It is immersed in a large tank of the same oil. Water flows through the hole. What is the velocity of this flow initially? When the flow stops, what would be the position of the oil-water interface in the tank from the bottom. If the ratio of the cross-sectional area of tank to that of hole is 50, determine the time in which the flow stops. The specific gravity of oil is 0.5.



SOLUTION

Since, the difference in pressure is

$$\Delta P = h(\rho_w - \rho_0)g = (10)(1000-500)9.8$$

$$\Rightarrow \Delta P = 49000 \text{ Nm}^{-2}$$

According to Bernoulli's Theorem, we have

$$\Delta P = \frac{1}{2}\rho_w v^2$$

$$\Rightarrow v = \sqrt{\frac{2\Delta P}{\rho_w}} = \sqrt{\frac{2\times 49000}{1000}} = 9.8 \text{ ms}^{-1}$$

When the flow stops, then let h be the height of oil water interface. Also, flow will stop, because the pressure difference will be zero. Hence, we have

$$(10+5)\rho_0 g = 5\rho_0 g + h\rho_w g$$

$$\Rightarrow 10\rho_0 = h\rho_w$$

$$\Rightarrow h = \frac{10\times 500}{1000} = 5 \text{ m}$$

i.e., flow will stop when the water-oil interface is at a height of 5.0 m

When the oil-water interface is at a height of y from ground, then we have

$$\Delta P = 5\rho_0 g + y\rho_w g - 15\rho_0 g$$

$$\Rightarrow \Delta P = y\rho_w g - 10\rho_0 g$$

According to Bernoulli's Theorem, we get

$$\frac{1}{2}\rho_w v^2 = y\rho_w g - 10\rho_0 g$$

$$\Rightarrow v = \sqrt{2g\left(y - \frac{10\rho_0}{\rho_w}\right)} = 4.43\sqrt{y-5}$$

Further, according to the Equation of Continuity, we have

$$av = A\left(-\frac{dy}{dt}\right)$$

$$\Rightarrow dt = \left(\frac{A}{a}\right)\left(-\frac{dy}{v}\right) = 50\left(-\frac{dy}{4.43\sqrt{y-5}}\right)$$

$$\Rightarrow \int_0^t dt = -\frac{50}{4.43}\int_{10}^{5}\frac{dy}{\sqrt{y-5}} = 11.29\int_5^{10}\frac{dy}{\sqrt{y-5}}$$

$$\Rightarrow t = 11.29\left(\frac{\sqrt{y-5}}{1/2}\right)\Bigg|_5^{10} = 22.58(\sqrt{5})$$

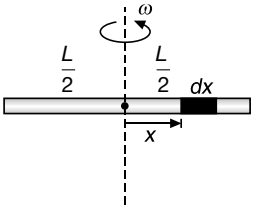
$$\Rightarrow t \approx 50.5 \text{ s}$$

PROBLEM 19

A horizontally oriented copper rod of length l is rotated about a vertical axis passing through its middle. Calculate the angular frequency at which the rod ruptures. Assume that the breaking or rupture strength of copper is σ and density of copper is ρ.

SOLUTION

Since the stress is zero at the free ends and maximum at the axis, hence the rod will rupture at the middle. Let us consider the element of length dx of the rod at a distance x from the axis of rotation of the rod as shown in Figure.



The mass dm of this element is $dm = \rho A dx$

The centripetal force required to move this element in a circle of radius x is

$$dF = (dm)x\omega^2 = (\rho A dx)\omega^2 x$$

Tension at a point P at a distance x from the axis is equal to sum of centripetal forces on all elements lying between P and B. So at point P, tension is given by

$$T = \rho A\omega^2 \int_0^{L/2} x\,dx = \frac{\rho A\omega^2 L^2}{8}$$

So, stress is given by

$$\frac{F}{A} = \frac{\rho\omega^2 l^2}{8} \qquad \ldots(1)$$

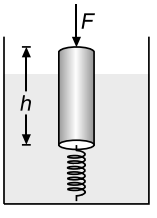
The rod will rupture when the stress calculated in equation (1) equals the breaking stress σ of the rod.

$$\Rightarrow \quad \sigma = \frac{\rho\omega^2 l^2}{8}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{8\sigma}{\rho l^2}}$$

PROBLEM 20

A solid cylinder of radius $r = 10$ cm is floating with its axis vertical in a cylindrical vessel of radius $R = 20$ cm, containing water. A spring of force constant $k = 500\ \text{Nm}^{-1}$ attached at the bottom of the cylinder is initially in its natural position. The cylinder is slowly pushed from top till it gets just immersed in the water and in doing so 25 J of energy was spent. The height of the cylinder is $h = 40$ cm. Calculate the density of cylinder. Neglect viscosity of the liquid and take $g = 10\ \text{ms}^{-2}$.



SOLUTION

For equilibrium, the weight of the cylinder must be balanced by the upthrust acting on the cylinder. If ρ be density of cylinder and h_i be length of cylinder immersed in water, then we have

$$(\pi r^2 h)\rho g = (\pi r^2 h_i)\rho_w g$$

$$\Rightarrow \quad h_i = \left(\frac{\rho}{\rho_w}\right)h = \left(\frac{\rho}{10^3}\right)h \qquad \ldots(1)$$

So, length of cylinder above the surface of water is

$$H = h - h_i$$

$$\Rightarrow \quad H = h - \left(\frac{\rho}{10^3}\right)h$$

$$\Rightarrow \quad H = \left(\frac{10^3 - \rho}{10^3}\right)h \qquad \ldots(2)$$

When the cylinder is pushed downward by a distance y, let the water level rise up through y'.

$$\Rightarrow \quad \pi r^2 y = (\pi R^2 - \pi r^2)y'$$

$$\Rightarrow \quad y' = \left(\frac{r^2}{R^2 - r^2}\right)y = \left(\frac{100}{400 - 100}\right)y = \frac{y}{3}$$

Since $y + y' = H$

$$\Rightarrow \quad \frac{4}{3}y = H$$

$$\Rightarrow \quad y = \frac{3h}{4}\left(\frac{10^3 - \rho}{10^3}\right) = 0.3\left(\frac{10^3 - \rho}{10^3}\right) \qquad \ldots(3)$$

The net force acting on the cylinder is

$$F = ky + \pi r^2 (y + y')\rho_w g = ky + \frac{4}{3}y\pi r^2 \rho_w g$$

The work done by this variable force is

$$W = \int_0^y F\,dy$$

$$\Rightarrow \quad W = \left(k + \frac{4}{3}\pi r^2 \rho_w g\right)\frac{y^2}{2}$$

Substituting the values, we get

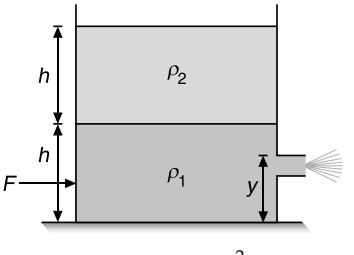
$$25 = \left[\frac{500 + \frac{4\pi}{3}(0.1)^2(10^3)(10)}{2}\right](0.3)^2\left(\frac{10^3 - \rho}{10^3}\right)^2$$

$$\Rightarrow \quad \frac{10^3 - \rho}{10^3} = 0.774$$

$$\Rightarrow \quad \rho = 225.5\ \text{kgm}^{-3}$$

PROBLEM 21

A cylindrical vessel having cross-sectional area $A = 0.5\ \text{m}^2$ is filled with two liquids of density $\rho_1 = 500\ \text{kgm}^{-3}$ and $\rho_2 = 1000\ \text{kgms}^{-3}$, to a height $h = 50$ cm each as shown in Figure.



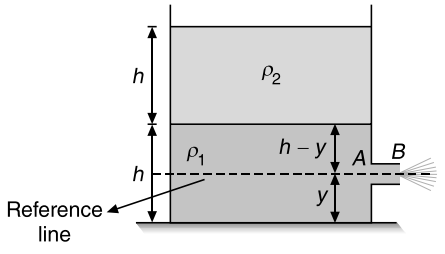
A small hole having area $a = 5\ \text{cm}^2$ is made in right vertical wall at a height $y = 10$ cm from the bottom. Calculate the velocity of efflux of the liquid coming out of the orifice. Calculate the horizontal force F to keep the cylinder in static equilibrium, if it is placed on a smooth horizontal

plane. Also find the minimum and maximum values of F to keep the cylinder in static equilibrium, assuming that the coefficient of friction between the cylinder and the plane is $\mu = 0.02$ and $g = 10\ \text{ms}^{-2}$.

SOLUTION

The area a of hole is very small in comparison to base area A of the cylinder, therefore, velocity of liquid inside the cylinder is negligible. Let velocity of efflux be v and atmospheric pressure P_0.

Consider two points A (inside the cylinder) and B (just outside the hole) on the same horizontal line as shown in Figure.



Pressure at A, $P_A = P_0 + h\rho_2 g + (h-y)\rho_1 g$

Pressure at B, $P_B = P_0$

According to Bernoulli's theorem, we have

$$P_A = P_B + \frac{1}{2}\rho_1 v^2$$

$$\Rightarrow \quad v = 3\ \text{ms}^{-1}$$

When cylinder is on smooth horizontal plane, force F required to keep cylinder stationary equals horizontal thrust exerted by the water jet emerging out from the orifice.

$$F_{\text{thrust}} = v_{\text{rel}}\frac{dm}{dt} = (v-0)(av\rho) = av^2\rho$$

$$\Rightarrow \quad F_{\text{thrust}} = 5\times10^{-4}\times1000\times(3)^2 = 4.5\ \text{N}$$

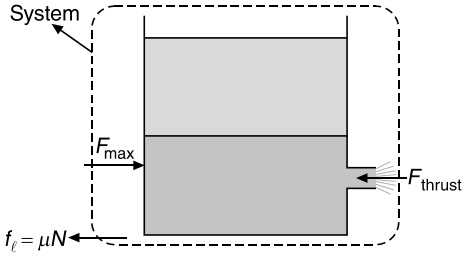
Total mass of the liquid in the cylinder is

$$M = Ah\rho_1 + Ah\rho_2 = 750\ \text{kg}$$

Limiting friction between the cylinder and the surface is

$$f_l = \mu N = \mu Mg = 150\ \text{N}$$

Since $F_{\text{thrust}} = 4.5$ N, which is less than the limiting value of the force of friction, so minimum force required to keep the cylinder in static equilibrium is zero.



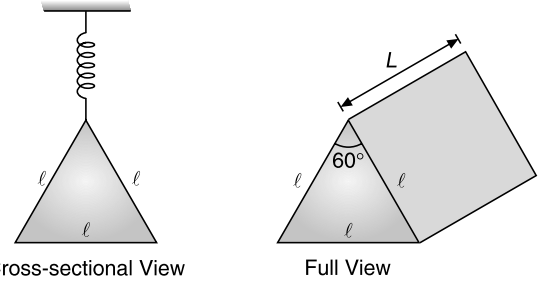
Considering free body diagram of the system for maximum value of force, we get

$$F_{\max} = F_{\text{thrust}} + \mu N$$

$$\Rightarrow \quad F_{\max} = 4.5 + 150 = 154.5\ \text{N}$$

PROBLEM 22

A glass prism of length L has its principal section in the form of an equilateral triangle of side length l whose cross sectional view and full view are shown in the Figure.



The prism, with its base horizontal, is supported by a vertical spring of force constant k such that half the slant surface of the prism is submerged in water. If the surface tension of water is T, contact angle between water and glass is $0°$, density of glass is d and that of water is $\rho(<d)$, then calculate the extension in the spring in this position of equilibrium.

SOLUTION

Volume of prism will be

$$V = AL = \left(\frac{\sqrt{3}}{4}l^2\right)L = \frac{\sqrt{3}}{4}l^2L$$

Since half the slant surface of prism is submerged in water, so the volume of prism immersed in water is

$$V_{\text{imm}} = V' = \frac{\sqrt{3}}{4}l^2L - \frac{\sqrt{3}}{4}\left(\frac{l}{2}\right)^2 L = \frac{3\sqrt{3}}{16}l^2L$$

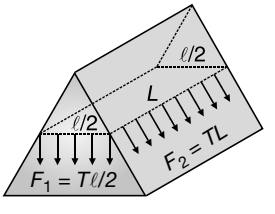
The forces acting on prism are

(i) Weight of the prism, $W = \dfrac{\sqrt{3}}{4}l^2Ldg$

(ii) Spring force, $F_{\text{sp}} = kx$

(iii) Buoyant force, $F_B = \left(\dfrac{3\sqrt{3}}{16}l^2L\right)\rho g$

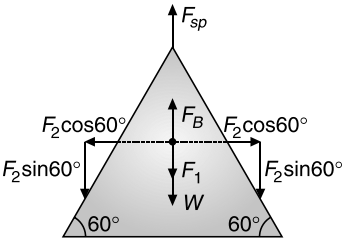
(iv) Surface tension force, F_{ST} made up of two forces $F_1 = T\left(\dfrac{l}{2}\right)$ and $F_2 = TL$ as shown in Figure.



$$F_{ST} = 2(F_1 + F_2) = 2\left(T\frac{l}{2} + TL\sin 60°\right)$$

$$\Rightarrow \quad F_{ST} = T(l + \sqrt{3}L)$$

The Free Body Diagram (FBD) of prism showing all the forces acting on it is shown in Figure.



In equilibrium,

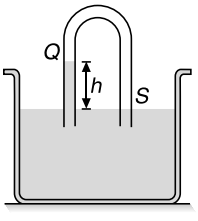
$$F_{sp} + F_B = W + F_{ST}$$

$$\Rightarrow \quad kx + \frac{3\sqrt{3}}{16}l^2L\rho g = \frac{\sqrt{3}}{4}l^2Ldg + T(l + \sqrt{3}L)$$

$$\Rightarrow \quad x = \frac{1}{k}\left[\frac{\sqrt{3}}{4}l^2Ldg - \frac{3\sqrt{3}}{16}l^2L\rho g + \sqrt{3}TL + Tl\right]$$

PROBLEM 23

A glass *U*-tube is inverted with its open ends of diameters $0.5\ \text{mm}$ and $1.0\ \text{mm}$ below the surface of water in a beaker as shown in Figure.



The air pressure in the upper part is increased until the meniscus in one limb is in level with the water outside. If density of water is $10^3\ \text{kgm}^{-3}$, surface tension of water is $7.5\times10^{-2}\ \text{Nm}^{-1}$, contact angle is $\theta = 0°$, then calculate height of water in the other limb.

SOLUTION

Let P be the pressure of air inside the *U*-tube and P_0 be the atmospheric pressure, then pressure inside the liquid below Q is $(P_0 - h\rho g)$

Since, we know that

$$\Delta P = \frac{2T}{R}$$

$$\Rightarrow \quad P - (P_0 - h\rho g) = \frac{2T}{r_1} \qquad \ldots(1)$$

where, r_1 is radius of the left limb.

$$\Rightarrow \quad r_1 = 0.25\times10^{-3}\ \text{m}$$

Pressure just inside the liquid below S is P_0

$$\Rightarrow \quad P - P_0 = \frac{2T}{r_2} \qquad \ldots(2)$$

where, r_2 is radius of the left limb.

$$\Rightarrow \quad r_2 = 0.5\times10^{-3}\ \text{m}$$

From equations (1) and (2), we get

$$h = \frac{2T}{\rho g}\left(\frac{r_2 - r_1}{r_2 r_1}\right)$$

$$\Rightarrow \quad h = \frac{2\times7.5\times10^{-2}}{10^3\times9.8}\left(\frac{0.5 - 0.25}{0.5\times0.25}\right)\frac{1}{10^{-3}}$$

$$\Rightarrow \quad h \approx 3.1\times10^{-2}\ \text{m}$$

PRACTICE EXERCISES

SINGLE CORRECT CHOICE TYPE QUESTIONS

This section contains Single Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. Capillaries of length l and $2l$ are connected in series. Their radii are r and $2r$ respectively. If stream line flow is maintained and pressure across first and second capillaries are P_1 and P_2 respectively. Then the ratio $\frac{P_1}{P_2}$ will be

(A) 4 (B) 16
(C) 8 (D) 32

2. A force of $200\ \text{N}$ is applied at one end of a wire of length $2\ \text{m}$ and having area of cross-section $10^{-2}\ \text{cm}^2$. The other end of the wire is rigidly fixed. If coefficient of linear expansion of the wire $\alpha = 8\times10^{-6}\ ^\circ\text{C}^{-1}$ and Young's modulus $Y = 2.2\times10^{11}\ \text{Nm}^{-2}$ and its temperature is increased by $5\ ^\circ\text{C}$, then the increase in the tension of the wire will be

(A) 4.2 N (B) 4.4 N
(C) 2.4 N (D) 8.8 N

3. Two identical spherical soap bubbles collapses. If V is the consequent change in volume of the contained air, S is the change in the total surface area and T is the surface tension of the soap solution, then (if P is atmospheric pressure and assume temperature to remain same in all the bubbles).

(A) $3PV + 4ST = 0$
(B) $4PV + 3ST = 0$
(C) $PV + 4TS = 0$
(D) $4PV + ST = 0$

4. Two wires of the same material have diameters in the ratio $2:1$ and lengths in the ratio $1:2$. If they are stretched by the same force, their elongations will be in the ratio

(A) $8:1$ (B) $1:8$
(C) $2:1$ (D) $1:4$

5. A ball of density ρ is released from deep inside of a liquid of density 2ρ. It will move up

(A) with an increasing acceleration
(B) with a decreasing acceleration
(C) with a constant acceleration
(D) with zero acceleration

6. A cylinder with a movable piston contains air under a pressure P_1 and a soap bubble of radius r. The pressure P_2 to which the air should be compressed by slowly pushing the piston into the cylinder for the soap bubble to reduce its size by half will be (The surface tension is T and the temperature is maintained constant).

(A) $8P_1 + \frac{24T}{r}$ (B) $4P_1 + \frac{24T}{r}$
(C) $2P_1 + \frac{24T}{r}$ (D) $2P_1 + \frac{24T}{r}$

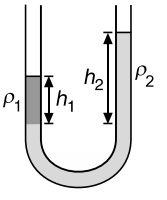
7. A steel rod of length l, area of cross-section A, Young's modulus Y and coefficient of linear expansion α is heated through $t\ ^\circ\text{C}$. The work that can be performed by a rod when heated is

(A) $(YA\alpha t)(l\alpha t)$ (B) $\frac{1}{2}(YA\alpha t)(l\alpha t)$
(C) $\frac{1}{2}(YA\alpha t)\left(\frac{1}{2}l\alpha t\right)$ (D) $2(YA\alpha t)(l\alpha t)$

8. An ice cube of size $a = 10\ \text{cm}$ is floating in a tank (base area $A = 50\ \text{cm}\times50\ \text{cm}$) partially filled with water. The change in gravitational potential energy, when ice melts completely is: (density of ice is $900\ \text{kgm}^{-2}$)

(A) –0.072 J (B) –0.24 J
(C) –0.016 J (D) –0.045 J

9. Two liquids which do not react chemically are placed in a bent tube as shown in Figure.



The heights of the liquids above their surface of separation are

(A) directly proportional to their densities
(B) inversely proportional to their densities
(C) directly proportional to square of their densities
(D) equal

10. A thin metal ring of internal radius $8\ \text{cm}$ and external radius $9\ \text{cm}$ is supported horizontally from the pan of a balance so that it comes in contact with water in a glass vessel, if is found that an extra weight of $7.48\ \text{g}$ is required to pull the ring out of water. The surface tension of water is $\left(g = 10\ \text{ms}^{-2}\right)$

(A) $80\times10^{-3}\ \text{Nm}^{-1}$ (B) $25\times10^{-3}\ \text{Nm}^{-1}$
(C) $45\times10^{-3}\ \text{Nm}^{-1}$ (D) $70\times10^{-3}\ \text{Nm}^{-1}$

11. A boy's catapult is made of rubber cord which is $42\ \text{cm}$ long, with $6\ \text{mm}$ diameter of cross-section and of negligible mass. The boy keeps a stone weighing $0.02\ \text{kg}$ on it and stretches the cord by $20\ \text{cm}$ by applying a constant force. When released, the stone flies off with a velocity of $20\ \text{ms}^{-1}$. Neglect the change in the area of cross-section of the cord while stretched. The Young's modulus of rubber is closest to

(A) $10^4\ \text{Nm}^{-2}$ (B) $10^3\ \text{Nm}^{-2}$

(C) $10^8\ \text{Nm}^{-2}$ (D) $10^6\ \text{Nm}^{-2}$

12. A metallic wire of density ρ floats horizontal in water. The maximum radius of the wire so that the wire may not sink will be (if surface tension of water is T and angle of contact is $0°$)

(A) $\sqrt{\frac{2T}{\pi\rho g}}$ (B) $\sqrt{\frac{4T}{\rho g}}$

(C) $\sqrt{\frac{T}{\pi\rho g}}$ (D) $\sqrt{\frac{T\rho}{\pi g}}$

13. A soap bubble of radius r is placed on another bubble of radius $2r$. The radius of the surface common to both the bubbles is

(A) $\frac{2r}{3}$ (B) $3r$

(C) $2r$ (D) r

14. Young's moduli of two wires A and B are in the ratio $7:4$. Wire A is $2\ \text{m}$ long and has radius R. Wire B is $1.5\ \text{m}$ long and has radius $2\ \text{mm}$. If the two wires stretch by the same length for a given load, then the value of R is close to

(A) 1.3 mm (B) 1.9 mm

(C) 1.5 mm (D) 1.7 mm

15. A cylindrical vessel of area of cross-section A is filled with water (density ρ) to a height H. It has capillary tube of length l and radius r fitted horizontally at its bottom. If the coefficient of viscosity of water is η, then time in which level will fall to a height $\frac{H}{2}$ is

(A) $\frac{\eta l r^2}{4A\rho}\ln(2)$ (B) $\frac{4\eta l r^4}{\pi g A\rho}\ln\left(\frac{1}{2}\right)$

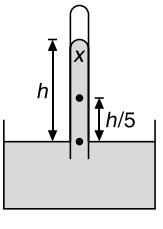
(C) $\frac{8\eta l A}{\rho\pi g r^4}\ln(2)$ (D) $\frac{4H\eta l\rho}{\pi g r^4}\ln(2)$

16. A brass rod of cross-sectional area $1\ \text{cm}^2$ and length $0.2\ \text{m}$ is compressed lengthwise by a weight of $5\ \text{kg}$. If Young's modulus of elasticity of brass is $1\times10^{11}\ \text{Nm}^{-2}$ and $g = 10\ \text{ms}^{-2}$, then increase in the energy of the rod will be

(A) 10^{-5} J (B) 2.5×10^{-5} J

(C) 5×10^{-5} J (D) 2.5×10^{-4} J

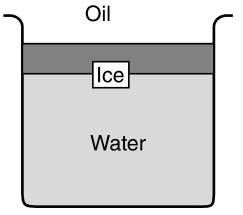
17. The height of mercury barometer is h when the atmospheric pressure is 10^5 Pa. The pressure at x in the shown diagram is



(A) 10^5 Pa (B) 0.8×10^5 Pa

(C) 0.2×10^5 Pa (D) 120×10^5 Pa

18. An ice cube is floating in water above which a layer of lighter oil is poured. As the ice melts completely, the level of interface and the upper most level of oil will respectively



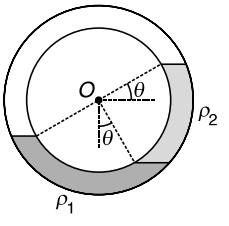
(A) rise and fall

(B) fall and rise

(C) not change and not change

(D) not change and fall

19. A thin uniform circular tube is kept in a vertical plane. Equal volumes of two immiscible liquids whose densities are ρ_1 and ρ_2 fill half of the tube. In equilibrium the radius passing through the interface makes an angle of $\theta = 30°$ with vertical as shown in Figure. The ratio of densities $\left(\frac{\rho_1}{\rho_2}\right)$ is equal to



(A) $\frac{\sqrt{3}-1}{2-\sqrt{3}}$ (B) $\frac{\sqrt{3}+1}{2+\sqrt{3}}$

(C) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ (D) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$

20. The terminal velocity of a ball (of density ρ) in air (of density σ) is v, where acceleration due to gravity is g. Now the same ball is taken in a gravity free space where all other conditions are same. The ball is now given the same velocity v, then for $\rho = 2\sigma$

(A) the terminal velocity of the ball will be $\frac{v}{2}$

(B) the ball will move with a constant velocity

(C) the initial acceleration of the ball is $2g$ in direction opposite to the ball's velocity

(D) the ball will finally stop

21. A massless spring $(k = 800\ \text{Nm}^{-1})$, attached with a mass $(500\ \text{g})$ is completely immersed in $1\ \text{kg}$ of water. The spring is stretched by $2\ \text{cm}$ and release so that it starts vibrating. What would be the order of magnitude of the change in the temperature of water when the vibrations stop completely? (Assume that the water container and spring receive negligible heat and specific heat of mass is $400\ \text{Jkg}^{-1}\text{K}^{-1}$, specific heat of water is $4184\ \text{Jkg}^{-1}\text{K}^{-1}$)

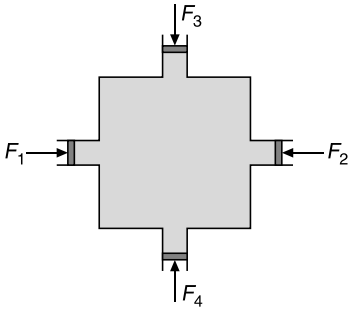
(A) 10^{-5} K (B) 10^{-1} K

(C) 10^{-3} K (D) 10^{-4} K

22. A ball of relative density 0.8 falls into water from a height of 2 m. The depth to which the ball will sink (if viscous forces are neglected) is

(A) 8 m (B) 2 m
(C) 6 m (D) 4 m

23. Water is filled in a symmetrical container as shown in Figure.



Four pistons each of area A are used at the four openings to keep the water in equilibrium. Now an additional force each of magnitude F is applied at each piston. The increase in the pressure at the centre of the container due to this additional force is

(A) $\frac{F}{A}$ (B) $\frac{2F}{A}$
(C) $\frac{4F}{A}$ (D) 0

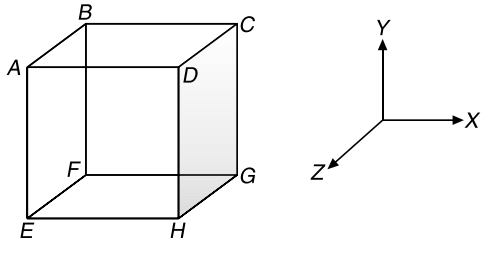
24. Three capillary tubes of same radius 1 cm but of lengths 1 m, 2 m and 3 m are fitted horizontally to the bottom of a long cylinder containing a liquid at constant pressure and flowing through these tubes. What is the length of a single tube which can replace the three capillaries?

(A) $\frac{6}{11}$ m (B) 6 m
(C) 5 m (D) None of these

25. If a million tiny droplets of water of the same radius coalesce into one larger drop the ratio of the surface energy of the large drop to the total surface energy of all the droplets will be

(A) $1:10$ (B) $1:10^2$
(C) $1:10^4$ (D) $1:10^6$

26. The cubical container $ABCDEFGH$ which is completely filled with an ideal (non-viscous and incompressible) fluid, moves in a gravity free space with an acceleration of $a = a_0\left(\hat{i} - \hat{j} + \hat{k}\right)$ where a_0 is a positive constant. Then the only point in the container where pressure can be zero is

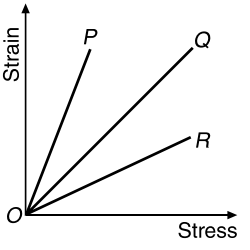


(A) B (B) C
(C) E (D) H

27. A uniform plank of Young's modulus Y is moved over a smooth horizontal surface by a constant horizontal force F. The area of cross section of the plank is A. The strain on the plank in the direction of the force is

(A) $\frac{F}{AY}$ (B) $\frac{2F}{AY}$
(C) $\frac{1}{2}\left(\frac{F}{AY}\right)$ (D) $\frac{3F}{AY}$

28. The strain-stress curves of three wires of different materials are shown in Figure.



P, Q and R are the elastic limits of the wires. The figure shows that

(A) Elasticity of wire P is maximum
(B) Elasticity of wire Q is maximum
(C) Tensile strength of R is maximum
(D) None of the above is true

29. A cylinder of mass m and density ρ hanging from a string is lowered into a vessel of cross-sectional area s containing a liquid of density $\sigma(<\rho)$ until it is fully immersed. The increase in pressure at the bottom of the vessel is

(A) $\frac{m\rho g}{\sigma s}$ (B) $\frac{mg}{s}$
(C) $\frac{m\sigma g}{\rho s}$ (D) ZERO

30. The elastic limit of brass is 379 MPa. What should be the minimum diameter of a brass rod if it is to support a 400 N load without exceeding its elastic limit?

(A) 0.90 mm (B) 1.16 mm
(C) 1.00 mm (D) 1.36 mm

31. The viscous force on a liquid passing through a length L of pipe in laminar flow is given by $F = 4\pi\eta L v_m$ where η is the liquid viscosity and v_m is the maximum velocity of the liquid (i.e., along the central axis of the pipe). If pressure difference at the two ends of the pipe (of radius r) is ΔP, then v_m is equal to

(A) $\frac{4\eta\Delta P}{r^2 L}$ (B) $\frac{(\Delta P)r^2}{4\eta L}$
(C) $\frac{(\Delta P)L}{4\eta r^2}$ (D) $\frac{\Delta P}{4\eta L r^2}$

32. The elastic deformation energy per unit volume in fresh water (of density d) of compressibility β at a depth h below the surface is

(A) $\frac{1}{2}\beta(hdg)^2$ (B) $\frac{(hdg)^2}{2\beta}$
(C) $\frac{1}{2}\beta(hdg)$ (D) $\beta(hdg)^2$

33. A capillary tube of radius r is immersed in water and water rises in it to a height h. The mass of the water in the capillary tube is m. Another capillary of radius $2r$ is immersed in water. The mass of water that will rise in this tube is

(A) $\frac{m}{2}$ (B) m

(C) $2m$ (D) $4m$

34. A material has Poisson's ratio 0.20. If a uniform rod of it suffers a longitudinal strain of 2×10^{-3}, then the percentage change in volume is

(A) +0.12 (B) –0.12

(C) +0.28 (D) –0.28

35. A wire of cross-sectional area $3\ \text{mm}^2$ is first stretched between two fixed points at a temperature of $20\ °\text{C}$. Determine the tension when the temperature falls to $10\ °\text{C}$. Coefficient of linear expansion $\alpha = 10^{-5}\ °\text{C}^{-1}$ and $Y = 2\times10^{11}\ \text{Nm}^{-2}$

(A) 20 N (B) 30 N

(C) 60 N (D) 120 N

36. If ρ is the density of the material of a wire and σ the breaking stress. The greatest length of the wire that can hang freely without breaking is

(A) $\frac{2\sigma}{pg}$ (B) $\frac{\rho}{\sigma g}$

(C) $\frac{\rho g}{2\sigma}$ (D) $\frac{\sigma}{\rho g}$

37. A uniform cylindrical rod of length L and radius r, is made from a material whose Young's modulus of Elasticity equals Y. When this rod is heated by temperature T and simultaneously subjected to a net longitudinal compressional force F, its length remains unchanged. The coefficient of volume expansion, of the material of the rod, is (nearly) equal to

(A) $\frac{9F}{(\pi r^2 YT)}$ (B) $\frac{3F}{(\pi r^2 YT)}$

(C) $\frac{F}{(\pi r^2 YT)}$ (D) $\frac{6F}{(\pi r^2 YT)}$

38. A block of volume V and density ρ is floating in a liquid of density 2ρ filled in a vessel. Now the vessel starts falling freely with acceleration g. Then the volume of block inside the liquid in the following condition is

(A) V (B) $\frac{1}{2}V$

(C) arbitrary (D) zero

39. A cylindrical wooden float having base area A and the height H drifts on the water surface. If density of wood is d and density of water is ρ then calculate the minimum work performed to take the float out of the water (ignore surface tension) is

(A) $\frac{A^2gd}{2\rho}$ (B) $\frac{Agd^2H^2}{\rho}$

(C) $\frac{Agd^2H^2}{2\rho}$ (D) $\frac{2A^3gd^2}{\rho H^2}$

40. A ball of mass m and density ρ is immersed in a liquid of density 3ρ at a depth h and released. To what height will the ball jump up above the surface of liquid? (neglect the resistance of water and air).

(A) h (B) $\frac{h}{2}$

(C) $2h$ (D) $3h$

41. A large block of ice cuboid of edge length l and density $\rho_{\text{ice}} = 0.9\rho_w$, has a large vertical hole along its axis. This block is floating in a lake. The minimum length of the rope required to raise a bucket of water through the hole is

(A) $\frac{l}{2}$ (B) $\frac{l}{4}$

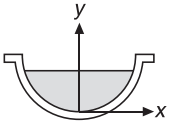
(C) $\frac{l}{8}$ (D) $\frac{l}{10}$

42. The pressure inside two soap bubbles is 1.01 and 1.02 atmosphere. The ratio of their respective volumes is

(A) 8 (B) 4

(C) 16 (D) 2

43. A small hole is made at the bottom of a symmetrical jar as shown in Figure. A liquid is filled into the jar upto a certain height. The rate of descent of liquid is independent of the level of liquid in the jar. Then the surface of jar is a surface of revolution of the curve



(A) $y = kx^4$ (B) $y = kx^2$

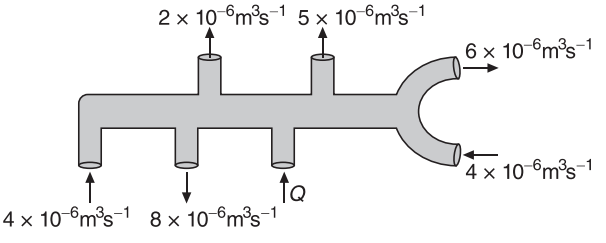
(C) $y = kx^3$ (D) $y = kx^5$

44. A glass rod of radius $1\ \text{m}$ is inserted symmetrically into a glass capillary tube with inside radius 2 mm. Then the whole arrangement is brought in contact of the surface of water. Surface tension of water is $7\times10^{-2}\ \text{Nm}^{-1}$. To what height will the water rise in the capillary: $(\theta = 0°)$

(A) 1.4 cm (B) 4.2 cm

(C) 2.1 cm (D) 6.8 cm

45. The volume flow rate of an ideal liquid in m^3s^{-1} at certain time and its direction at various points of a pipe is shown in Figure.

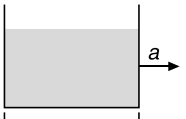


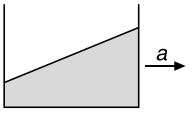
The value of Q in m^3s^{-1} is (Assume steady state and equal area of cross section at each opening)

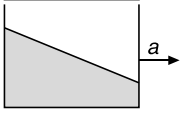
(A) 10×10^{-6} (B) 11×10^{-6}

(C) 13×10^{-6} (D) 18×10^{-6}

46. A vessel containing water is given a constant acceleration a towards the right along a straight horizontal path. Which of the following diagrams represents the surface of the liquid?

(A) 

(B) 

(C) 

(D) None of these

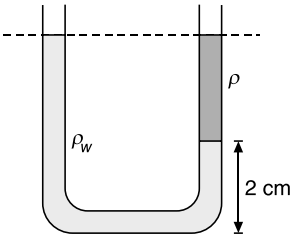
47. A load of mass M kg is suspended from a steel wire of length 2 m and radius 1.0 mm in Searle's apparatus experiment. The increase in length produced in the wire is 4.0 mm. Now the load is fully immersed in a liquid of relative density 2. The relative density of the material of the load is 8.
The new value of increase in length of the steel wire is
(A) 4.0 mm (B) zero
(C) 5.0 mm (D) 3.0 mm

48. A cubical block of wood of specific gravity 0.5 and chunk of concrete of specific gravity 2.5 are fastened together. The ratio of mass of concrete to the mass of wood which makes the combination to float with its entire volume submerged in water is
(A) $\frac{5}{3}$ (B) $\frac{5}{1}$
(C) $\frac{5}{2}$ (D) $\frac{4}{3}$

49. A small steel ball falls through a syrup at a constant speed of $1\ \text{ms}^{-1}$. If the steel ball is pulled upwards with a force equal to twice its effective weight, how fast will it move upward?
(A) $1\ \text{ms}^{-1}$ (B) $2\ \text{ms}^{-1}$
(C) $0.5\ \text{ms}^{-1}$ (D) zero

50. A submarine experiences a pressure of 5.05×10^6 Pa at a depth of d_1 in a sea. When it goes further to a depth of d_2, it experiences a pressure of 8.08×10^6 Pa. Then d_2-d_1 is approximately (density of water $=10^3\ \text{kgm}^{-3}$ and acceleration due to gravity $=10\ \text{ms}^{-2}$)
(A) 600 m (B) 500 m
(C) 300 m (D) 400 m

51. A U-tube of uniform cross-section filled with two immiscible liquids as shown in Figure. One is water with density ρ_w and the other liquid is of density ρ. The liquid interface lies 2 cm above the base. The relation between ρ and ρ_w is



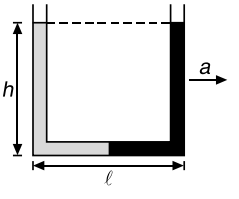
(A) $\rho=\rho_w$ (B) $\rho=1.02\rho_w$
(C) $\rho=1.2\rho_w$ (D) $\rho=\frac{\rho_w}{2}$

52. If viscosity of air is taken into account, then the orbital speed of the satellite moving close to earth
(A) increases till the satellite falls back on the earth
(B) increases till the satellite overcomes earth's gravitational pull
(C) decrease continuously
(D) remains unaffected

53. A spring balance reads 10 kg when a bucket of water is suspended from it. The reading of the balance when an iron piece of mass 7.2 kg suspended by a string is immersed with half its volume inside the water in the bucket (Relative density of iron is 7.2) is
(A) 10 kg (B) 10.5 kg
(C) 13.6 kg (D) 17.2 kg

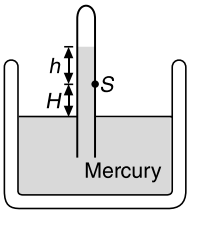
54. A water hose pipe of cross-sectional area $5\ \text{cm}^2$ is used to fill a tank of 120 L. It has been observed that it takes 2 min to fill the tank. Now, a nozzle with an opening of cross-sectional area $1\ \text{cm}^2$ is attached to the hose. The nozzle is held so that water is projected horizontally from a point 1 m above the ground. The horizontal distance over which the water can be projected is (Take $g=10\ \text{ms}^{-2}$)
(A) 3 m (B) 8 m
(C) 4.47 m (D) 8.64 m

55. A U-tube of base length l filled with same volume of two liquids of densities ρ and 2ρ is moving with an acceleration a on the horizontal plane. If the height difference between the two surfaces (open to atmosphere) becomes zero, then the height h is given by



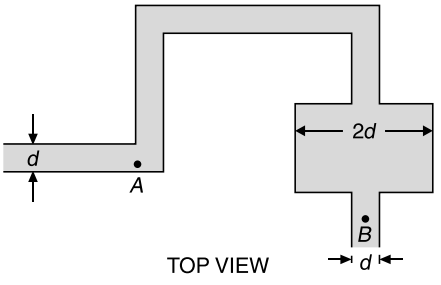
(A) $\left(\frac{a}{2g}\right)l$ (B) $\left(\frac{3a}{2g}\right)l$
(C) $\left(\frac{a}{g}\right)l$ (D) $\left(\frac{2a}{3g}\right)l$

56. Consider the barometer shown in Figure.



A small hole is made at point S as shown. The mercury (of density ρ) comes out from this hole with speed v equal to
(A) $\sqrt{2gh}$ (B) $\sqrt{2gH}$
(C) $\sqrt{2g(H-h)}$ (D) None of these

57. An ideal fluid is flowing through the given tubes of diameter d and $2d$ which is placed on a horizontal surface. If the liquid has velocities v_A and v_B and pressures P_A and P_B at points A and B respectively, then the correct relation is (A and B are at same height from ground level)



(A) $v_A > v_B$, $P_A < P_B$ (B) $v_A < v_B$, $P_A > P_B$

(C) $v_A = v_B$, $P_A = P_B$ (D) $v_A > v_B$, $P_A = P_B$

58. To measure the atmospheric pressure, four different tubes of length 1 m, 2 m, 3 m and 4 m are used. If the height of the mercury column in the tubes is h_1, h_2, h_3, h_4 respectively in the four cases, then $h_1 : h_2 : h_3 : h_4$ is

(A) $1:2:3:4$ (B) $4:3:2:1$

(C) $1:2:2:1$ (D) $1:1:1:1$

59. The amount of work done is forming a soap bubble of radius $r = 1\text{ cm}$ having surface tension $T = 3\times10^{-2}\text{ Nm}^{-1}$ is

(A) $37.38\ \mu\text{J}$ (B) $40.20\ \mu\text{J}$

(C) $75.36\ \mu\text{J}$ (D) $20.10\ \mu\text{J}$

60. One end of a glass capillary tube with a radius $r = 0.005\text{ cm}$ is immersed into water to a depth of $h = 2\text{ cm}$. The pressure is required to blow an air bubble out of the lower end of the tube. If $T = 7\times10^{-2}\text{ Nm}^{-1}$ and $\rho = 10^3\text{ kgm}^{-3}$ is

(A) 480 Nm^{-2} (B) 680 Nm^{-2}

(C) 120 Nm^{-2} (D) 820 Nm^{-2}

61. The length of a rubber cord is L_1 metres when the tension 4 N and L_2 metres when the tension is 5 N. The length in metres when the tension is 9 N is

(A) $5L_1 - 4L_2$ (B) $5L_2 - 4L_1$

(C) $9L_1 - 8L_2$ (D) $9L_2 - 8L_1$

62. Young's modulus of steel is $2\times10^{11}\text{ Nm}^{-2}$. A steel wire has a length of 1 m and area of cross section 1 mm^2. The work required to increase its length by 1 mm is

(A) 0.1 J (B) 1 J

(C) 10 J (D) 100 J

63. Bulk modulus of water is $2\times10^9\text{ Nm}^{-2}$. The change in pressure required to increase the density of water by 0.1% is

(A) $2\times10^9\text{ Nm}^{-2}$ (B) $2\times10^8\text{ Nm}^{-2}$

(C) $2\times10^6\text{ Nm}^{-2}$ (D) $2\times10^4\text{ Nm}^{-2}$

64. A rod of length ℓ and radius r is joined to a rod of length $\frac{\ell}{2}$ and radius $\frac{r}{2}$ of same material. The free end of small rod is fixed to a rigid base and the free end of larger rod is given a twist of $\theta°$, the twist angle at the joint will be

(A) $\frac{\theta}{4}$ (B) $\frac{\theta}{2}$

(C) $\frac{5\theta}{6}$ (D) $\frac{8\theta}{9}$

65. There is a horizontal film of soap solution. On it a thread is placed in the form of a loop. The film is pierced inside the loop and the thread becomes a circular loop of radius R. If the surface tension of the loop be T, then tension in the thread will be

(A) $\pi R^2 T$ (B) $2RT$

(C) RT (D) $\frac{\pi R^2}{T}$

66. A body of density ρ is dropped from rest from a height h into a lake of density $\sigma(\sigma > \rho)$. The maximum depth the body sinks inside the liquid is (neglect viscous effect of liquid)

(A) $\frac{h\rho}{\sigma - \rho}$ (B) $\frac{h\sigma}{\sigma - \rho}$

(C) $\frac{h\rho}{\sigma}$ (D) $\frac{h\sigma}{\rho}$

67. If the volume of the given mass of a gas is increased four times, the temperature is raised from $27\ °\text{C}$ to $127\ °\text{C}$. The elasticity will become

(A) 4 times (B) $\frac{1}{4}$ times

(C) 3 times (D) $\frac{1}{3}$ times

68. A satellite revolves round the earth. Air pressure inside the satellite is maintained at 76 cm of mercury. The height of mercury column in a barometer tube 90 cm long placed in the satellite is

(A) 76 cm (B) 90 cm

(C) ZERO (D) can be any of these

69. A material has Poisson's ratio 0.5. If a uniform rod of it suffers a longitudinal strain of 2×10^{-3}, the percentage increase in its volume is

(A) 1 (B) 1.5

(C) 0.5 (D) 0

70. A metallic rod of Young's modulus $2\times10^{11}\text{ Nm}^{-2}$ undergoes a strain of 0.5%. Then the energy stored per unit volume in the rod will be

(A) $2.5\times10^6\text{ Jm}^{-3}$

(B) $5\times10^8\text{ Jm}^{-3}$

(C) $2.5\times10^8\text{ Jm}^{-3}$

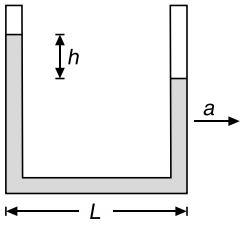
(D) $0.5\times10^{11}\text{ Jm}^{-3}$

71. Two wires A and B have the same length and area of cross section. But Young's modulus of A is two times the Young's modulus of B. Then the ratio of force constant of A to that of B is

(A) 1 (B) 2

(C) $\frac{1}{2}$ (D) $\sqrt{2}$

72. A liquid stands at the plane level in the U-tube when at rest. If areas of cross-section of both the limbs are equal. The difference in heights h of the liquid in the two limbs of U-tube, when the system is given an acceleration a in horizontal direction towards right as shown in Figure is



(A) $\frac{Lg}{a}$ (B) $\frac{La}{g}$

(C) $\frac{Lg^2}{a^2}$ (D) ZERO

73. Water from a pipe is coming at a rate of 100 liters per minute. If the radius of the pipe is 5 cm, the Reynolds number for the flow is of the order of (if density of water is 1000 kgm^{-3}, coefficient of viscosity of water is 1 mPas)

(A) 10^2 (B) 10^4

(C) 10^3 (D) 10^6

74. In case of a liquid

(A) only bulk modulus is defined

(B) only bulk modulus and Young's modulus are defined

(C) only bulk modulus and shear modulus are defined

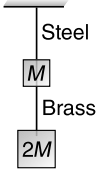
(D) all the three modulii (bulk, Young's and shear) are defined

75. The top of a water tank is open to air and its water level is maintained. It is giving out 0.74 m^3 water per minute through a circular opening of 2 cm radius in its wall. The depth of the centre of the opening from the level of water in the tank is close to

(A) 9.6 m (B) 2.9 m

(C) 4.8 m (D) 6.0 m

76. If the ratio of lengths, radii and Young's modulii of steel and brass wires in the figure are a, b and c respectively. Then the corresponding ratio of increase in their lengths would be



(A) $\frac{2a^2c}{b}$ (B) $\frac{3a}{2b^2c}$

(C) $\frac{2ac}{b^2}$ (D) $\frac{3c}{2ab^2}$

77. A wooden block floating in a bucket of water has $\frac{4}{5}$ of its volume submerged. When certain amount of an oil is poured into the bucket, it is found that the block is just under the oil surface with half of its volume under water and half in oil. The density of oil relative to that of water is

(A) 0.5 (B) 0.8

(C) 0.7 (D) 0.6

78. A wooden cube floating in water supports a mass $m = 0.2$ kg on its top. When the mass is removed the cube rises by 2 cm. The side of the cube is (density of water $= 10^3$ kgm^{-3})

(A) 6 cm (B) 12 cm

(C) 8 cm (D) 10 cm

79. A solid sphere, of radius R acquires a terminal velocity v_1 when falling (due to gravity) through a viscous fluid having a coefficient of viscosity η. The sphere is broken into 27 identical solid spheres. If each of these spheres acquires a terminal velocity, v_2, when falling through the same fluid, the ratio $\left(\frac{v_1}{v_2}\right)$ equals

(A) $\frac{1}{27}$ (B) 9

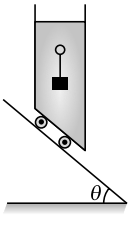
(C) $\frac{1}{9}$ (D) 27

80. Two cylinders of same cross-section and length L but made of two material of densities d_1 and d_2 are cemented together to form a cylinder of length 2L. The combination floats in a liquid of density d with a length $\frac{L}{2}$ above the surface of the liquid. If $d_1 < d_2$ then

(A) $d_1 < \frac{3}{4}d$ (B) $\frac{d}{2} > 4$

(C) $\frac{d}{4} > d$ (D) $d < d$

81. A body floats in completely immersed condition in water as shown in Figure. When the whole system is allowed to slide down freely along the inclined surface, the magnitude of buoyant force



(A) remains unchanged (B) increases

(C) decreases (D) becomes zero

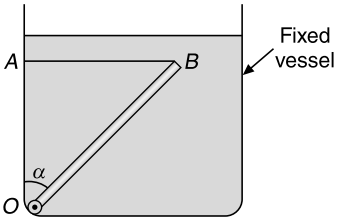
82. A soap bubble of radius r is placed on another bubble of radius 2r. The angles formed between the films (of soap bubbles) at the points of contact are

(A) 60°, 60°, 60° (B) 45°, 60°, 90°

(C) 30°, 60°, 90° (D) 120°, 120°, 120°

83. A uniform rod OB of length 1 m, cross-sectional area 0.012 m^2 and relative density 2.0 is free to rotate about O in vertical plane. The rod is held with a horizontal string AB which can withstand a maximum tension of 45 N. The rod and string system is kept in water as shown in Figure. The

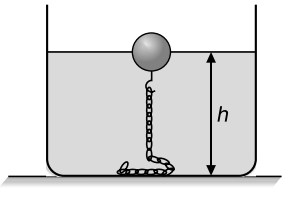
maximum value of angle α which the rod can make with vertical without breaking the string is (Take $g = 10\ \text{ms}^{-2}$)



(A) 45° (B) 37°
(C) 53° (D) 60°

84. A barometer tube reads 76 cm of mercury. If the tube is gradually inclined at an angle of 60° with vertical, keeping the open end immersed in the mercury reservoir, the length of the mercury column will be
(A) 152 cm (B) 76 cm
(C) 38 cm (D) $38\sqrt{3}$ cm

85. One end of a long iron chain of linear mass density λ is fixed to a sphere of mass m and specific density $\frac{1}{3}$ while the other end is free. The sphere along with the chain is immersed in a deep lake. If specific density of iron is 7, the height h above the bed of the lake at which the sphere will float in equilibrium is (Assume that the part of the chain lying on the bottom of the lake exerts negligible force on the upper part of the chain)

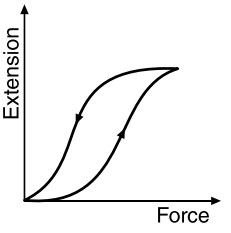


(A) $\frac{5m}{2\lambda}$ (B) $\frac{7m}{3\lambda}$
(C) $\frac{8m}{3\lambda}$ (D) $\frac{16m}{7\lambda}$

86. Two identical cylindrical vessels with their bases at the same level, each contains a liquid of density ρ. The height of the liquid in one vessel is h_1 and that in the other is h_2. The area of either base is A. The work done by gravity in equalizing the levels when the vessels are interconnected is
(A) $A\rho g\left(\frac{h_1 - h_2}{2}\right)$ (B) $A\rho g\left(\frac{h_1 - h_2}{2}\right)^2$
(C) $A\rho g\left(\frac{h_1 - h_2}{4}\right)$ (D) $A\rho g\left(\frac{h_1 - h_2}{4}\right)^2$

87. A rubber cord of length L is suspended vertically. Density of rubber is D and Young's modulus is Y. If the cord extends by a length ℓ under its own weight, then
(A) $l = \frac{L^2 Dg}{Y}$ (B) $l = \frac{L^2 Dg}{2Y}$
(C) $l = \frac{L^2 Dg}{4Y}$ (D) $l = \frac{L^2 Dg}{8Y}$

88. The diagram shows a force-extension graph for a rubber band. Consider the following statements



(I) It will be easier to compress this rubber than expand it
(II) Rubber does not return to its original length after it is stretched
(III) The rubber band will get heated if it is stretched and released
Which of these can be deduced from the graph?
(A) III only (B) II and III
(C) I and III (D) I only

89. A large tank is filled with water having density $\rho_w = 10^3\ \text{kgm}^{-3}$. A small hole is made at a depth 10 m below water surface. The range of water coming out of the hole is R on ground. The extra pressure that must be applied on the water surface so that the range becomes $2R$ (If $1\ \text{atm} = 10^5\ \text{Pa}$ and $g = 10\ \text{ms}^{-2}$) is
(A) 9 atm (B) 4 atm
(C) 5 atm (D) 3 atm

90. If a rubber ball is taken at the depth of 200 m in a pool, its volume decreases by 0.1%. If the density of the water is $1\times10^3\ \text{kgm}^{-3}$ and $g = 10\ \text{ms}^{-2}$, then the volume elasticity in Nm^{-2} will be
(A) 10^8 (B) 2×10^8
(C) 10^9 (D) 2×10^9

91. A weight of 200 kg is suspended by vertical wire of length 600.5 cm. The area of cross-section of wire is $1\ \text{mm}^2$. When the load is removed, the wire contracts by 0.5 cm. The Young's modulus of the material of wire will be
(A) $2.35\times10^{12}\ \text{Nm}^{-2}$ (B) $1.35\times10^{10}\ \text{Nm}^{-2}$
(C) $13.5\times10^{11}\ \text{Nm}^{-2}$ (D) $23.5\times10^{9}\ \text{Nm}^{-2}$

92. When a force is applied on a wire of uniform cross-sectional area $3\times10^{-6}\ \text{m}^2$ and length 4 m, the increase in length is 1 mm. Energy stored in it will be $(Y = 2\times10^{11}\ \text{Nm}^{-2})$
(A) 6250 J (B) 0.177 J
(C) 0.075 J (D) 0.150 J

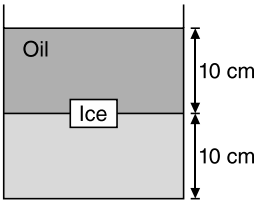
93. The mean density of sea water is ρ, and bulk modulus is B. The change in density of sea water in going from the surface of water to a depth of h is
(A) $\frac{B\rho^2}{gh}$ (B) $B\rho gh$
(C) $\frac{\rho^2 gh}{B}$ (D) $\frac{\rho gh}{B}$

94. A liquid of density ρ is coming out of a hose pipe of radius a with horizontal speed v and hits a mesh. 50% of the liquid passes through the mesh unaffected. 25% looses all of its momentum and 25% comes back with the same speed. The resultant pressure on the mesh will be

(A) $\frac{3}{4}\rho v^2$ (B) $\frac{1}{4}\rho v^2$

(C) $\frac{1}{2}\rho v^2$ (D) ρv^2

95. A cubical block of side $10\ \text{cm}$ floats at the interface of an oil and water as shown in Figure. The density of oil is $0.6\ \text{gcm}^{-3}$ and the lower face of ice cube is $2\ \text{cm}$ below the interface. The pressure above that of the atmosphere at the lower face of the block is



(A) 200 Pa (B) 620 Pa

(C) 900 Pa (D) 800 Pa

96. A cubical block of side $0.5\ \text{m}$ floats on water with 30% of its volume under water. What is the maximum weight that can be put on the block without fully submerging it under water? (Take, density of water $= 10^3\ \text{kgm}^{-3}$)

(A) 30.1 kg (B) 46.3 kg

(C) 87.5 kg (D) 65.4 kg

97. A rubber balloon has $200\ \text{g}$ of water in it. Its weight in water will be (neglect the weight of balloon)

(A) 100 g (B) 200 g

(C) 50 g (D) ZERO

98. A body floats in water with its one-third volume above the surface. The same body floats in a liquid with one-third volume immersed. The density of the liquid is

(A) 9 times more than that of water

(B) 2 times more than that of water

(C) 3 times more than that of water

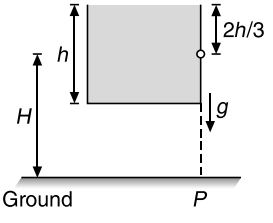
(D) 1.5 times more than that of water

99. A flat plate of area $20\ \text{cm}^2$ is placed on a horizontal surface coated with a layer of glycerine $1\ \text{mm}$ thick. What force must be applied to the plate to keep it moving with a speed of $1\ \text{cms}^{-1}$ over the horizontal surface? (Coefficient of viscosity of glycerine $= 2\ \text{kgms}^{-1}$)

(A) 2×10^{-3} N (B) 2 N

(C) 1×10^{-4} N (D) 4×10^{-2} N

100. An open vessel full of water is falling freely under gravity. There is a small hole in one face of the vessel as shown in Figure. The water which comes out from the hole at the instant when hole is at height H above the ground, strikes the ground at a distance of x from P. Which of the following is correct for the situation described?



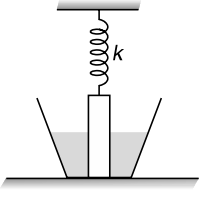
(A) The value of x is $2\sqrt{\frac{2hH}{3}}$.

(B) The value of x is $\sqrt{\frac{4hH}{3}}$.

(C) The value of x can't be computed from information provided.

(D) No water comes out from the hole.

101. A block of mass $2\ \text{kg}$ and specific gravity $\frac{5}{2}$ is attached with a spring of force constant $k = 100\ \text{Nm}^{-1}$ and is half dipped in the water. If extension in the spring is $1\ \text{cm}$. The force exerted by the bottom of the tank on block is (take $g = 10\ \text{ms}^{-2}$)



(A) 20 N (B) 19 N

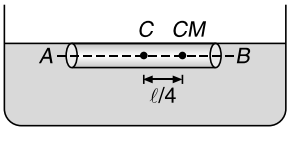
(C) 15 N (D) 16 N

102. A leakage begins in water tank at position P as shown in Figure. The initial gauge pressure (pressure above that of the atmosphere) at P was $5\times10^5\ \text{Nm}^{-2}$. If the density of water is $1000\ \text{kgm}^{-3}$ the initial velocity with which water gushes out is approximately

(A) $3.2\ \text{ms}^{-1}$ (B) $32\ \text{ms}^{-1}$

(C) $28\ \text{ms}^{-1}$ (D) $2.8\ \text{ms}^{-1}$

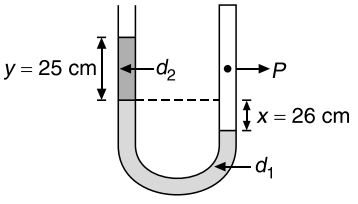
103. A non-uniform cylinder of mass m, length l and radius r is having its centre of mass at a distance $\frac{l}{4}$ from the centre C and lying on the axis of the cylinder. The cylinder is kept in a liquid of uniform density ρ. The moment of inertia of the rod about the centre of mass is I. The angular acceleration of point A relative to point B just after the rod is released from the horizontal position shown in Figure is



(A) $\frac{\pi\rho g l^2 r^2}{I}$ (B) $\frac{\pi\rho g l^2 r^2}{4I}$

(C) $\frac{\pi\rho g l^2 r^2}{2I}$ (D) $\frac{3\pi\rho g l^2 r^2}{4I}$

104. In a given U-tube open at one-end and closed at other end as shown in Figure, the correct relation between P and P_{atm} is (If $d_2 = 2\times13.6\ \text{gcm}^{-3}$ and $d_1 = 13.6\ \text{gcm}^{-3}$)



(A) $P_{atm} = P$ (B) $P_{atm} = 2P$

(C) $P_{atm} = \frac{P}{2}$ (D) $P_{atm} = \frac{P}{4}$

105. A substance breaks down by a stress of $10^6\ \text{Nm}^{-2}$. If the density of the material of the wire is $3\times10^3\ \text{kgm}^{-3}$, then the length of the wire of that substance which will break under its own weight when suspended vertically is

(A) 3.4 m (B) 34 m

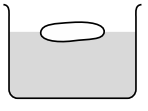
(C) 340 m (D) None of these

106. A small block of wood of specific gravity 0.5 is submerged at a depth of 1.2 m in a vessel filled with water. The vessel is accelerated upwards with an acceleration $a_0 = \frac{g}{2}$. Time taken by the block to reach the surface, if it is released with zero initial velocity is $\left(g = 10\ \text{ms}^{-2}\right)$

(A) 0.6 s (B) 0.4 s

(C) 1.2 s (D) 1 s

107. A body floats in a liquid contained in a beaker. The whole system as shown in Figure falls freely under gravity. The upthrust on the body is



(A) ZERO

(B) equal to the weight of liquid displaced

(C) equal to the weight of the body is air

(D) equal to the weight of the immersed portion of the body

108. A 5 m long aluminium wire $\left(Y = 7\times10^{10}\ \text{Nm}^{-2}\right)$ of diameter 3 mm supports a 40 kg mass. In order to have the same elongation in a copper wire $\left(Y = 12\times10^{10}\ \text{Nm}^{-2}\right)$ of the same length under the same weight, the diameter should now be, in mm.

(A) 1.75 (B) 1.5

(C) 2.5 (D) 5

109. The pressure of a medium is changed from 1.01×10^5 Pa to 1.165×10^5 Pa and change in volume is 10% keeping temperature constant. The Bulk modulus of the medium is

(A) 204.8×10^5 Pa (B) 102.4×10^5 Pa

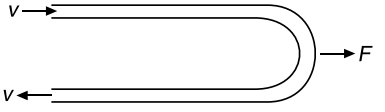
(C) 51.2×10^5 Pa (D) 1.55×10^5 Pa

110. A cube of aluminium of sides 0.1 m is subjected to a shearing force of 100 N. The top face of the cube is displaced through 0.02 cm with respect to the bottom face. The shearing strain would be

(A) 0.02 (B) 0.1

(C) 0.005 (D) 0.002

111. A horizontal tube of uniform cross-sectional area A is bent in the form of U as shown in Figure. If the liquid of density ρ enters and leaves the tube with velocity v, then the external force F required to hold the bend stationary is



(A) $F = 0$ (B) ρAv^2

(C) $2\rho Av^2$ (D) $\frac{1}{2}\rho Av^2$

112. If a bar is made of copper whose coefficient of linear expansion is one and a half times that of iron, the ratio of the force developed in the copper bar to the iron bar of identical lengths and cross-sections, when heated through the same temperature range (Young's modulus for copper may be taken to be equal to that of iron) is

(A) $\frac{3}{2}$ (B) $\frac{2}{3}$

(C) $\frac{9}{4}$ (D) $\frac{4}{9}$

113. Two drops of same radius are falling through air with steady speed v. If the two drops coalesce, the terminal speed of bigger drop is

(A) v (B) $2^{1/3}v$

(C) $3v$ (D) $4^{1/3}v$

114. A vertical capillary is brought in contact with the water surface (surface tension = T). The radius of the capillary is r and the contact angle $\theta = 0°$. The increase in potential energy of the water (density $= \rho$) is

(A) independent of ρ (B) independent of r

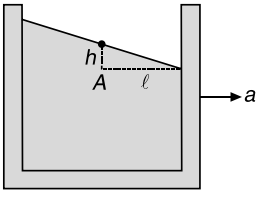
(C) independent of T (D) zero

115. A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth $4y$ from the top. When the tank is completely filled with water, the quantities of water flowing out per second from both holes are the same. Then, R is equal to

(A) $\frac{L}{\sqrt{2\pi}}$ (B) $2\pi L$

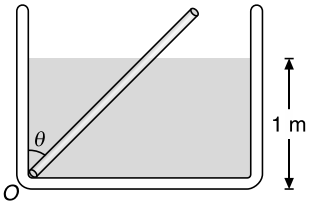
(C) L (D) $\frac{L}{2\pi}$

116. A rectangular container moves with an acceleration a along the positive direction as shown in Figure. The pressure at the point A in excess of the atmospheric pressure p_0 is (take ρ as the density of liquid)



(A) ρgh (B) ρal
(C) $\rho(gh+al)$ (D) Both (A) and (B)

117. A uniform rod of length $2.0\ \text{m}$ specific gravity 0.5 and mass $2\ \text{kg}$ is hinged at one end to the bottom of a tank of water (specific gravity $=1.0$) filled upto a height of $1.0\ \text{m}$ as shown in Figure. Taking the case $\theta \neq 0^o$ the force exerted by the hinge on the rod is $\left(g = 10\ \text{ms}^{-2}\right)$



(A) 10.2 N upwards (B) 4.2 N downwards
(C) 8.3 N downwards (D) 6.2 N upwards

118. If M is the mass of water that rises in a capillary tube of radius r, then mass of water which will rise in a capillary tube of radius $2r$ is

(A) $2M$ (B) M
(C) $4M$ (D) $\frac{M}{2}$

119. A piece of ice is floating in a beaker containing thick sugar solution of water. As the ice melts, the total level of the liquid
(A) increases (B) decreases
(C) remains unchanged (D) insufficient data

120. Mercury is poured in a U-tube. Temperature of one side is $50\ ^\circ\text{C}$ and level of mercury on this side is h_1. Temperature of the other side is $100\ ^\circ\text{C}$ and level of mercury on this side is h_2. Then
(A) $h_1 = h_2$ (B) $h_2 < h_1$
(C) $h_2 > h_1$ (D) $h_2 = 2h_1$

121. An open rectangular tank $1.5\ \text{m}$ wide $2\ \text{m}$ deep and $2\ \text{m}$ long is half filled with water. It is accelerated horizontally at $3.27\ \text{ms}^{-2}$ in the direction of its length. The depth of water at rear end of tank if $g = 9.81\ \text{ms}^{-2}$ is
(A) 0.9 m (B) 1.2 m
(C) 1.5 m (D) 1.7 m

122. The magnitude of the force developed by raising the temperature from $0\ ^\circ\text{C}$ to $100\ ^\circ\text{C}$ of the iron bar $1.00\ \text{m}$ long and $1\ \text{cm}^2$ cross-section when it is held so that it is not permitted to expand or bend is ($\alpha = 10^{-5}\ ^\circ\text{C}^{-1}$ and $Y = 10^{11}\ \text{Nm}^{-2}$)
(A) 10^3 N (B) 10^4 N
(C) 10^5 N (D) 10^9 N

123. A cylindrical vessel contains a liquid of density ρ upto a height h. The liquid is closed by a piston of mass m and area of cross section A. There is a small hole at the bottom of the vessel. The speed v with which the liquid comes out of the hole is

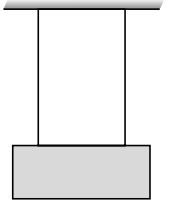
(A) $\sqrt{2gh}$ (B) $\sqrt{2\left(gh+\frac{mg}{\rho A}\right)}$
(C) $\sqrt{2\left(gh+\frac{mg}{A}\right)}$ (D) $\sqrt{2gh+\frac{mg}{A}}$

124. The pressure applied from all directions on a cube is P. How much its temperature should be raised to maintain the original volume? The volume elasticity of the cube is β and the coefficient of volume expansion is α

(A) $\frac{P}{\alpha\beta}$ (B) $\frac{P\alpha}{\beta}$
(C) $\frac{P\beta}{\alpha}$ (D) $\frac{\alpha\beta}{P}$

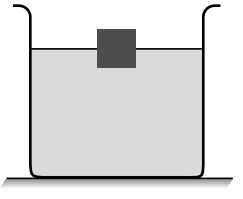
125. The extension produced in a wire by the application of a load is $3.0\ \text{mm}$. The extension in a wire of the same material and length but half the radius, by the same load, is
(A) 0.75 mm (B) 1.5 mm
(C) 6.0 mm (D) 12.0 mm

126. Two wires of equal length and cross-section are suspended as shown, Their Young's modulii are Y_1 and Y_2 respectively. The equivalent Young's modulus will be



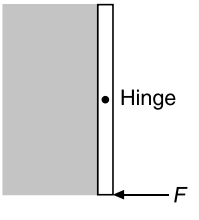
(A) $Y_1 + Y_2$ (B) $\frac{Y_1+Y_2}{2}$
(C) $\frac{Y_1Y_2}{Y_1+Y_2}$ (D) $\sqrt{Y_1Y_2}$

127. A uniform cube of mass M is floating on the surface of a liquid with three fourth of its volume immersed in the liquid of density ρ. The length of the side of the cube is equal to



(A) $\left(\frac{4M}{3\rho}\right)^{\frac{2}{3}}$ (B) $\left(\frac{M}{3\rho}\right)^{\frac{2}{3}}$
(C) $\left(\frac{M}{4\rho}\right)^{\frac{2}{3}}$ (D) $\left(\frac{4M}{3\rho}\right)^{\frac{1}{3}}$

128. A square gate of size $1\ \text{m} \times 1\ \text{m}$ is hinged at its mid point. A fluid of density ρ fills the space to the left of the gate. The force F required to hold the gate stationary is



(A) $\frac{\rho g}{3}$ (B) $\frac{1}{2}\rho g$

(C) $\frac{\rho g}{6}$ (D) None of these

129. Power required to raise 300 litres of water per minute through a height of 6 m using a pipe of diameter 2.4 cm is $\left(g = 10\ \text{ms}^{-2}\right)$

(A) 600 watt (B) 400 watt

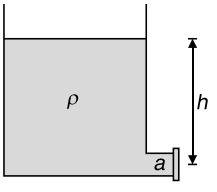
(C) 1000 watt (D) 2000 watt

130. Young's modulus of rubber is $10^4\ \text{Nm}^{-2}$ and area of cross-section is $2\ \text{cm}^2$. If force of 2×10^5 dynes is applied along its length, then its initial length L becomes

(A) $3L$ (B) $4L$

(C) $2L$ (D) L

131. A disk is held against the opening near the bottom of a vessel to keep the liquid, from running out as shown in Figure.



Let the opening be having an area a which is very small compare to the area of the mouth of the vessel. If F_1 be the net force acting on disk by the liquid and air in this case. Now the disk is moved away from the opening through a small distance due to which the liquid comes out of the opening and strikes the disk completely elastically. Let F_2 be the force exerted by the liquid in this condition. Then $\frac{F_1}{F_2}$ is

(A) $\frac{1}{2}$ (B) 1

(C) $\frac{2}{1}$ (D) $\frac{1}{4}$

132. A solid sphere of radius R made of material of Bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area A floats on the surface of the liquid when a mass m is placed on the piston to compress the liquid, the fractional change in the radius of the sphere, $\frac{dR}{R}$ is

(A) $\frac{mg}{4AK}$ (B) $\frac{mg}{3AK}$

(C) $\frac{mg}{2AK}$ (D) $\frac{mg}{AK}$

133. An area of cross-section of rubber string is $2\ \text{cm}^2$. Its length is doubled when stretched with a linear force of 2×10^5 dyne. The Young's modulus of the rubber in dyne cm^{-2} will be

(A) 4×10^5 (B) 1×10^5

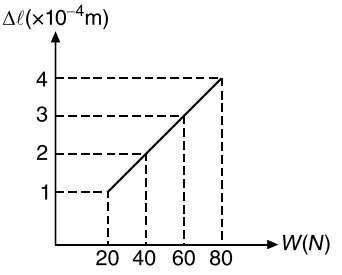
(C) 2×10^5 (D) 1×10^4

134. The diameter of a brass rod is 4 mm and Young's modulus of brass is $9\times10^{10}\ \text{Nm}^{-2}$. The force required to stretch by 0.1% of its length is

(A) 360π N (B) 36 N

(C) $144\pi\times10^3$ N (D) $36\pi\times10^5$ N

135. The graph shows the extension (Δl) of a wire of length l m suspended from the top of a roof at one end and with a load W connected to the other end. If the cross-sectional area of the wire is $10^{-6}\ \text{m}^2$, calculate the Young's modulus of the material of the wire



(A) $2\times10^{11}\ \text{Nm}^{-2}$ (B) $2\times10^{-11}\ \text{Nm}^{-2}$

(C) $3\times10^{12}\ \text{Nm}^{-2}$ (D) $2\times10^{13}\ \text{Nm}^{-2}$

136. A uniform rod of length L has a mass per unit length λ and area of cross section A. The elongation in the rod is ℓ due to its own weight if it is suspended from the ceiling of a room. The Young's modulus of the rod is

(A) $\frac{2\lambda gL^2}{A\ell}$ (B) $\frac{\lambda gL^2}{2A\ell}$

(C) $\frac{2\lambda gL}{A\ell}$ (D) $\frac{\lambda g\ell^2}{AL}$

137. A drop of water of mass m surface tension T and density ρ is placed between two well cleaned glass plates separated by a distance d. The force of attraction between the plates is

(A) $\frac{Tm}{2\rho d^2}$ (B) $\frac{4Tm}{\rho d^2}$

(C) $\frac{2Tm}{\rho d^2}$ (D) $\frac{Tm}{\rho d^2}$

138. The height to which a liquid of surface tension T and density ρ rises between two long parallel plates, a distance d apart is

(A) $\frac{4T}{\rho gd}$ (B) $\frac{2T}{\rho gd}$

(C) $\frac{T}{\rho gd}$ (D) $\frac{T}{2\rho gd}$

139. A plate moves normally with the speed v_1 towards a horizontal jet of water of uniform area of cross-section. The jet discharges water at the rate of volume V per second at a speed of v_2. The density of water is ρ. Assume that water splashes along the surface of the plate at right angles to the original motion. The magnitude of the force acting on the plate due to the jet of water is

(A) $\rho V v_1$ (B) $\rho\left(\frac{V}{v_2}\right)(v_1+v_2)^2$

(C) $\frac{\rho V}{v_1+v_2}(v_1)^2$ (D) $\rho V(v_1+v_2)$

140. Water flows into a large tank with flat bottom at the rate of $10^{-4}\ \text{m}^3\text{s}^{-1}$. Water is also leaking out of a hole of area $1\ \text{cm}^2$ at its bottom. If the height of the water in the tank remains steady, then this height is

(A) 5.1 cm (B) 1.7 cm

(C) 2.9 cm (D) 4 cm

141. The ratio of surface tensions of mercury and water is given to be 7.5 while the ratio of their densities is 13.6. Their contact angles, with glass, are close to 135° and 0°, respectively. It is observed that mercury gets depressed by an amount h in a capillary tube of radius r_1, while water rises by the same amount h in a capillary tube of radius r_2. The ratio, $\left(\frac{r_1}{r_2}\right)$, is then close to

(A) $\frac{4}{5}$ (B) $\frac{2}{3}$

(C) $\frac{3}{5}$ (D) $\frac{2}{5}$

142. A closed compartment containing gas is moving with some acceleration in horizontal direction. Neglect effect of gravity. Then the pressure in the compartment is

(A) same everywhere (B) lower in front side

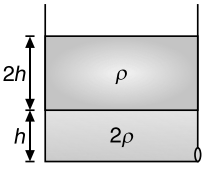
(C) lower in rear side (D) lower in upper side

143. When a tap is closed, the manometer attached to the pipe reads $3.5\times10^5\ \text{Nm}^{-2}$. When the tap is opened, the reading of manometer falls to $3.0\times10^5\ \text{Nm}^{-2}$. The velocity of water in the pipe is

(A) $0.1\ \text{ms}^{-1}$ (B) $1\ \text{ms}^{-1}$

(C) $5\ \text{ms}^{-1}$ (D) $10\ \text{ms}^{-1}$

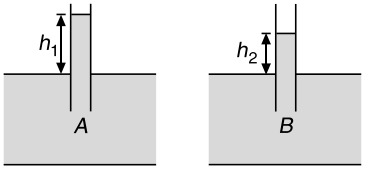
144. The velocity of the liquid coming out of a small hole of a large vessel containing two different liquids of densities 2ρ and ρ as shown in Figure is



(A) $\sqrt{6gh}$ (B) $2\sqrt{gh}$

(C) $2\sqrt{2gh}$ (D) $\sqrt{gh}$

145. Two ideal liquids are flowing through two different pipes A and B of uniform cross section. When a glass tube is inserted in these pipes, liquids rise in the tube to heights h_1 and $h_2(<h_1)$ as shown in Figure.



From this we may conclude that

(A) pressure in A is more than the pressure in B

(B) pressure in B is more than the pressure in A

(C) density of liquid in pipe A is less than the density of liquid in pipe B

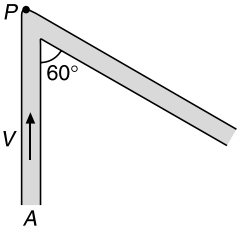
(D) None of these

146. A pump draws water from a reservoir and sends it through a horizontal pipe with speed v. The power of the pump is proportional to

(A) v (B) v^2

(C) v^3 (D) $v^{3/2}$

147. Water having density ρ is flowing through the uniform horizontal tube of cross-sectional area A with a constant speed v as shown in Figure.

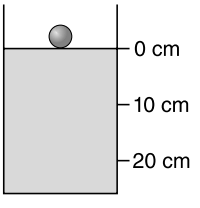


The magnitude of force exerted by the water on the curved corner of the tube (neglect viscous forces) is

(A) $\sqrt{3}\rho A v^2$ (B) $2\rho A v^2$

(C) $\sqrt{2}\rho A v^2$ (D) $\frac{\rho A v^2}{\sqrt{2}}$

148. A spherical ball of density ρ and radius 0.003 m is dropped into a tube containing a viscous fluid up to the 0 cm mark as shown in Figure. Viscosity of the fluid is $1.26\ \text{Nsm}^{-2}$ and its density ρ_L is $\frac{\rho}{2}=1260\ \text{kgm}^{-3}$. Assuming that the ball reaches a terminal speed at 10 cm mark, the time taken by the ball to travel the distance between the 10 cm and 20 cm mark is $\left(g=10\ \text{ms}^{-2}\right)$



(A) 2 s (B) 1 s

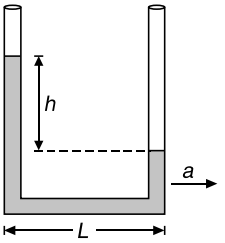
(C) 0.5 s (D) 5 s

149. A material has Poisson's ratio 0.50. If a uniform rod of it suffers a longitudinal strain of 2×10^{-3}, then the percentage change in volume is

(A) 0.6 (B) 0.4

(C) 0.2 (D) ZERO

150. When at rest, a liquid stands at the same level in both limbs of a U-tube. However a height difference h occurs, when the system is given an acceleration a towards the right as shown in Figure, then h equals



(A) $\frac{aL}{2g}$ (B) $\frac{gL}{2a}$

(C) $\frac{gL}{a}$ (D) $\frac{aL}{g}$

151. If a rubber ball is taken down to a $100\ \text{m}$ deep lake, its volume decreases by 0.1%. If $g = 10\ \text{ms}^{-2}$ then the Bulk modulus of elasticity for rubber, in Nm^{-2}, is

(A) 10^8 (B) 10^9

(C) 10^{11} (D) 10^{10}

152. A wooden block is floating in a liquid with 50% of its volume inside the liquid when the vessel is stationary.

Percentage of volume immersed, when the vessel moves upwards with an acceleration $a = \frac{g}{2}$ is

(A) 75% (B) 25%

(C) 50% (D) 33.33%

153. A force of $10^3\ \text{N}$ stretches the length of a hanging wire by $1\ \text{mm}$. The force required to stretch a wire of same material and length but having four times the diameter by $1\ \text{mm}$ is

(A) $4\times10^3\ \text{N}$ (B) $16\times10^3\ \text{N}$

(C) $\frac{1}{4}\times10^3\ \text{N}$ (D) $\frac{1}{16}\times10^3\ \text{N}$

154. The height of a mercury barometer is $75\ \text{cm}$ at sea level and $50\ \text{cm}$ at the top of a hill. Ratio of density of mercury to that of air is 10^4. The height of the hill is

(A) 250 m (B) 2.5 km

(C) 1.25 km (D) 750 km

155. Two rods of different materials having coefficients of linear expansions α_1, α_2 and Young's moduli Y_1, Y_2 respectively are fixed between two rigid massive walls. The rods are heated such that they undergo the same increase in temperature. There is no bending of the rods. If $\alpha_1 : \alpha_2 = 2:3$, the thermal stresses developed in the two rods are equal provided $Y_1 : Y_2$ is equal to

(A) 2 : 3 (B) 1 : 1

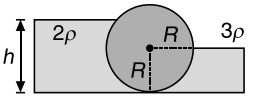
(C) 3 : 2 (D) 4 : 9

156. Two spheres of volume $250\ \text{cc}$ each but of relative densities 0.8 and 1.2 are connected by a string and the combination is immersed in a water. The tension in the string is $\left(g = 10\ \text{ms}^{-2}\right)$

(A) 5.0 N (B) 0.5 N

(C) 1.0 N (D) 2.0 N

157. In the Figure shown, the heavy cylinder (radius R) resting on a smooth surface separates two liquids of densities 2ρ and 3ρ. The height h for the equilibrium of cylinder must be



(A) $\frac{3R}{2}$ (B) $R\sqrt{\frac{3}{2}}$

(C) $R\sqrt{2}$ (D) $R\sqrt{\frac{2}{3}}$

158. Water from a tap emerges vertically downwards with an initial speed of $1\ \text{ms}^{-1}$. The cross-sectional area of tap is $10^{-4}\ \text{m}^2$. Assume that the pressure is constant throughout and that the flow is steady, the cross-sectional area of stream $0.15\ \text{m}$ below the tap is

(A) $5\times10^{-4}\ \text{m}^2$ (B) $1\times10^{-4}\ \text{m}^2$

(C) $5\times10^{-5}\ \text{m}^2$ (D) $2\times10^{-5}\ \text{m}^2$

159. A long cylindrical vessel is half filled with a liquid. When the vessel is rotated about its own vertical axis, the liquid rises up near the wall. If the radius of vessel is $5\ \text{cm}$ and its rotational speed is 2 rotations per second, then the difference in the heights between the centre and the sides, in cm, will be

(A) 1.2 (B) 0.1

(C) 0.4 (D) 2.0

160. A glass tube $80\ \text{cm}$ long and open at both ends is half immersed in mercury. Then the top of the tube is closed and it is taken out of the mercury. A column of mercury $20\ \text{cm}$ long then remains in the tube. The atmospheric pressure (in cm of Hg) is

(A) 75 (B) 30

(C) 60 (D) 90

(Assume temperature to be constant)

161. There is a small hole of area a at the bottom of a fixed container having area A containing a liquid upto height h. The top of the liquid as well as the hole at the bottom are exposed to atmosphere. As the liquid comes out of the hole, the

(A) top surface of the liquid (A) accelerates with acceleration g

(B) accelerates with acceleration $g\frac{a^2}{A^2}$

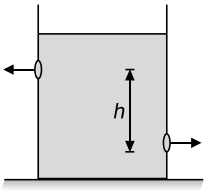
(C) retards with retardation $g\frac{a}{A}$

(D) retards with retardation $\frac{ga^2}{A^2}$

162. A U-tube having horizontal arm of length $20\ \text{cm}$, has uniform cross-sectional area of $1\ \text{cm}^2$. It is filled with $60\ \text{cc}$ water. The volume of a liquid of density $4\ \text{gcc}^{-1}$ that should be poured from one side into the U-tube so that no water is left in the horizontal arm of the tube is

(A) 60 cc (B) 45 cc
(C) 50 cc (D) 35 cc

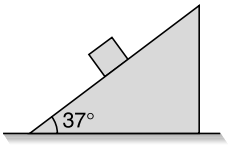
163. There are two identical small holes of area of cross-section a on the opposite sides of a tank (lying on a smooth horizontal surface) containing a liquid of density ρ. The difference in height between the holes is h. The horizontal force which will has to be applied to the tank to keep it in equilibrium is



(A) $ghpa$ (B) $\frac{2gh}{\rho a}$
(C) $2\rho agh$ (D) $\frac{\rho gh}{a}$

164. An open tank 10 m long and 2 m deep is filled up to 1.5 m height of oil of specific gravity 0.8. The tank is uniformly accelerated along its length from rest to a speed of 20 ms^{-1} horizontally. The shortest time in which the speed may be attained without spilling any oil is $(g = 10\text{ ms}^{-2})$
(A) 20 s (B) 18 s
(C) 10 s (D) 5 s

165. A cubical block of side a and density ρ slides over a fixed inclined plane with constant velocity v. There is a thin film of viscous fluid of thickness t between the plane and the block. Then the coefficient of viscosity of the film will be



(A) $\frac{3\rho agt}{5v}$ (B) $\frac{4\rho agt}{5v}$
(C) $\frac{\rho agt}{v}$ (D) $\frac{5\rho agt}{3v}$

166. One end of a uniform wire of length L and of weight W is attached rigidly to a point in the roof and a weight W_1 is suspended from its lower end. If S is the area of cross-section of the wire, the stress in the wire at a height $\frac{3L}{4}$ from its lower end is

(A) $\frac{W_1}{S}$ (B) $\frac{W_1 + \frac{W}{4}}{S}$
(C) $\frac{W_1 + \frac{3W}{4}}{S}$ (D) $\frac{W_1 + W}{S}$

167. For silver, Young's modulus is $7.25 \times 10^{10}\text{ Nm}^{-2}$ and Bulk modulus is $11 \times 10^{10}\text{ Nm}^{-2}$. Its Poisson's ratio will be
(A) –1 (B) 0.5
(C) 0.39 (D) 0.25

168. A highly rigid cubical block A of small mass M and side L is fixed rigidly onto another cubical block B of the same dimensions and of low modulus of rigidity η such that the lower face of B is rigidly held on a horizontal surface. A small force F is applied perpendicular to one of the side faces of A. After the force is withdrawn, block A executes small oscillations the time period of which is given by

(A) $2\pi\sqrt{M\eta L}$ (B) $2\pi\sqrt{\frac{M\eta}{L}}$
(C) $2\pi\sqrt{\frac{ML}{\eta}}$ (D) $2\pi\sqrt{\frac{M}{\eta L}}$

169. Three points A, B and C on a steady flow of a non-viscous and incompressible fluid are observed. The pressure, velocity and height of the points A, B and C are (2, 3, 1), (1, 2, 2) and (4, 1, 2) respectively. If the density of the fluid is 1 kgm^{-3}, $g = 10\text{ ms}^{-2}$ and all other parameters are given in SI units, then which of the following is correct?
(A) Points A and B lie on the same stream line
(B) Points B and C lie on the same stream line
(C) Points C and A lie on the same stream line
(D) None of these

170. The length of a wire is 1 m and the area of cross-section is $1 \times 10^{-2}\text{ cm}^2$. If the work done for increase in length by 0.2 cm is 0.4 joule, then Young's modulus of the material of the wire is
(A) $2 \times 10^{10}\text{ Nm}^{-2}$ (B) $4 \times 10^{10}\text{ Nm}^{-2}$
(C) $2 \times 10^{11}\text{ Nm}^{-2}$ (D) $4 \times 10^{11}\text{ Nm}^{-2}$

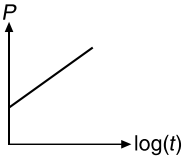
171. Equal volumes of two immiscible liquids of densities ρ and 2ρ are filled in a vessel as shown in Figure. Two small holes are punched at depth $\frac{h}{2}$ and $\frac{3h}{2}$ from the surface of lighter liquid. If v_1 and v_2 are the velocities of efflux at these two holes, then $\frac{v_1}{v_2}$ is
(A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{2}$
(C) $\frac{1}{4}$ (D) $\frac{1}{\sqrt{2}}$

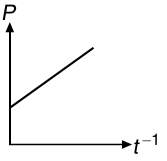
172. One end of a long metallic wire of length L is tied to a ceiling. The other end is tied to a massless spring of spring constant K. A mass M hangs from the free end of the spring. The area of cross-section and the Young's modulus of the wire are A and Y respectively. If the mass is slightly pushed down and released, it will oscillate with a time period T equal to
(A) $2\pi\sqrt{\frac{M}{K}}$ (B) $2\pi\sqrt{\frac{(YA + KL)M}{YAK}}$
(C) $2\pi\sqrt{\frac{MYA}{KL}}$ (D) $2\pi\sqrt{\frac{ML}{AY}}$

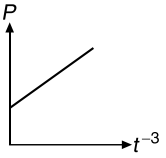
173. A ball floats on the surface of water in a container exposed to the atmosphere. Volume V_1 of its volume is inside the water. If the container is now covered and the air is pumped out. Now let V_2 be the volume immersed in water. Then

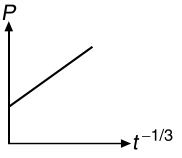
(A) $V_1 = V_2$ (B) $V_1 > V_2$
(C) $V_2 > V_1$ (D) $V_2 = 0$

174. A soap bubble, blown by a mechanical pump at the mouth of a tube, increases in volume, with time, at a constant rate. The graph that correctly depicts the time dependence of pressure inside the bubble is given by

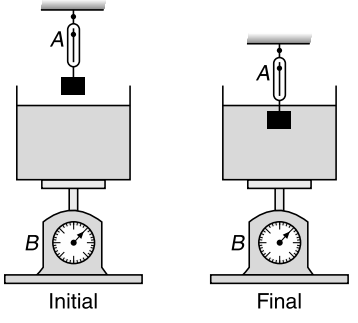
(A) 

(B) 

(C) 

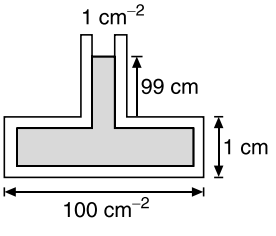
(D) 

175. For the arrangement shown in Figure, initially the balance A and B reads F_1 and F_2 respectively and $F_1 > F_2$. Finally when the block is immersed in the liquid then the readings of balance A and B are f_1 and f_2 respectively. Identify the statement which is not always (where, F is some force) correct statement.



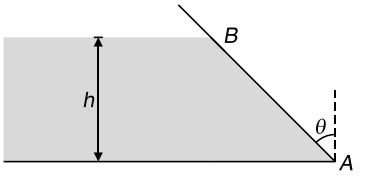
(A) $f_1 > f_2$ (B) $F_1 + F > F_2 + F$
(C) $f_1 + f_2 = F_1 + F_2$ (D) None of these

176. A tube $1\ \text{cm}^2$ in cross section is attached to the top of a vessel $1\ \text{cm}$ high and cross-section $100\ \text{cm}^2$. Water fills the system upto a height of $100\ \text{cm}$ from the bottom of the vessel. The force exerted by the liquid at the bottom of the vessel is $\left(g = 10\ \text{ms}^{-2}\right)$



(A) 1000 N (B) 990 N
(C) 900 N (D) 100 N

177. The height of water in a vessel is h. The vessel wall of width b is at an angle θ to the vertical. The net force exerted by the water on the wall is

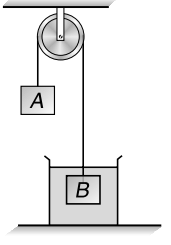


(A) $\frac{1}{3}\rho bh^2 g\cos\theta$ (B) $\frac{1}{2}bh^2\rho g$
(C) $\frac{1}{2}\rho bh^2 g\sec\theta$ (D) ZERO

178. A vessel contains oil (density $= 0.8\ \text{gcm}^{-3}$) over mercury (density $= 13.6\ \text{gcm}^{-3}$). A uniform sphere floats with half its volume immersed in mercury and the other half in oil. The density of the material of sphere in gcm^{-3} is

(A) 3.3 (B) 6.4
(C) 7.2 (D) 12.8

179. In the arrangement shown in Figure $\frac{m_A}{m_B} = \frac{2}{3}$ and the ratio of density of block B and of liquid is $2:1$. The system is released from rest. Then



(A) block B will oscillate but not simple harmonically
(B) block B will oscillate simple harmonically
(C) the system will remain in equilibrium
(D) None of these

180. A circular cylinder of height $h_0 = 10\ \text{cm}$ and radius $r_0 = 2\ \text{cm}$ is opened at the top and filled with liquid. It is rotated about its vertical axis. The speed of rotation if only half the area of the bottom gets exposed is $\left(\text{take } g = 10\ \text{ms}^{-2}\right)$

(A) $25\ \text{rads}^{-1}$ (B) $50\ \text{rads}^{-1}$
(C) $100\ \text{rads}^{-1}$ (D) $200\ \text{rads}^{-1}$

181. Water flows in a streamline manner through a capillary tube of radius a. The pressure difference being P and the rate of flow is Q. If the radius is reduced to $\frac{a}{2}$ and the pressure is increased to $2P$, then the rate of flow becomes

(A) $4Q$ (B) $\frac{Q}{2}$
(C) Q (D) $\frac{Q}{8}$

182. The following four wires are made of the same material. Which of these will have the largest extension when the same tension is applied on all?

(A) length = 50 cm , diameter = 0.5 mm

(B) length = 100 cm , diameter = 1 mm

(C) length = 200 cm , diameter = 2 mm

(D) length = 300 cm , diameter = 3 mm

183. A body of density ρ is dropped from rest from height h (from the surface of water) into a lake of density of water $\sigma(\sigma > \rho)$. Neglecting all dissipative effects, the acceleration of body while it is in the lake is

(A) $g\left(\frac{\sigma}{\rho}-1\right)$, upwards (B) $g\left(\frac{\sigma}{\rho}-1\right)$, downwards

(C) $g\left(\frac{\sigma}{\rho}\right)$, upwards (D) $g\left(\frac{\sigma}{\rho}\right)$, downwards

184. A ball falling in a lake of depth 200 m shows 0.1% decrease in its volume at the bottom. What is the bulk modulus of the material of the ball?

(A) 19.6×10^{8} Nm^{-2} (B) 19.6×10^{-10} Nm^{-2}

(C) 19.6×10^{10} Nm^{-2} (D) 19.6×10^{-8} Nm^{-2}

185. Two rods of different materials having coefficients of thermal expansion α_1, α_2 and Young's modulii Y_1, Y_2 respectively are fixed between two rigid massive walls. The rods are heated such that they undergo the same increase in temperature. There is no bending of the rods. If $\alpha_1 : \alpha_2 = 2:3$, the thermal stresses developed in the two roads are equal provided $Y_1 : Y_2$ is equal to

(A) 2 : 3 (B) 1 : 1

(C) 3 : 2 (D) 4 : 9

186. An air bubble of radius 1 mm is formed inside water at a depth 10 m below free surface (where air pressure is 10^5 Nm^{-2}). The pressure inside the bubble (if surface tension of water is 7×10^{-2} Nm^{-1}) is

(A) 2.28×10^{-5} Nm^{-2} (B) 2.0028×10^{5} Nm^{-2}

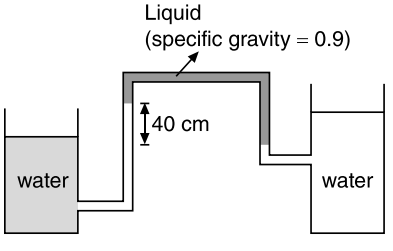
(C) 2.14×10^{-5} Nm^{-2} (D) 2.0014×10^{5} Nm^{-2}

187. A piece of gold $(\rho = 19.3\ \text{gcm}^{-3})$ has a cavity in it. It weights 38.2 g in air and 36.2 g in water. The volume of the cavity in gold is

(A) 0.2 cm^3 (B) 0.04 cm^3

(C) 0.02 cm^3 (D) 0.01 cm^3

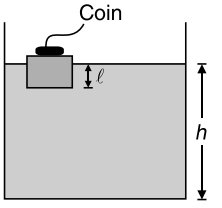
188. The manometer shown below is used to measure the difference in water level between the two tanks. This difference for the conditions shown in Figure is



(A) 2 cm (B) 4 cm

(C) 6 cm (D) 8 cm

189. A wooden block, with a coin placed on its top, floats in water as shown in Figure. The distance l and h are shown there. After sometime the coin falls into the water. Then



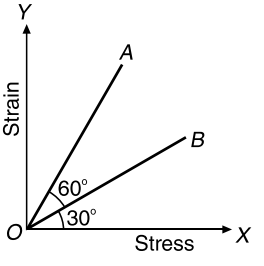
(A) l decrease and h increases

(B) l increase and h decreases

(C) both l and h increase

(D) both l and h

190. The stress versus strain graphs for wires of two materials A and B are as shown in Figure. If Y_A and Y_B are the Young's modulii of the materials, then



(A) $Y_B = 2Y_A$ (B) $Y_A = Y_B$

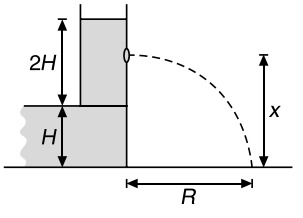
(C) $Y_B = 3Y_A$ (D) $Y_A = 3Y_B$

191. A cube of side 10 cm is subjected to a tangential force of 5×10^5 N at the upper face, keeping lower face fixed and the upper face is displaced by 0.001 rad relative to lower face along the direction of tangential force. Then Shear modulus of the material of the cube is

(A) 5×10^{6} Nm^{-2} (B) 5×10^{8} Nm^{-2}

(C) 5×10^{10} Nm^{-2} (D) 5×10^{11} Nm^{-2}

192. A tank is filled up to a height $2H$ with a liquid and is placed on a platform of height H from the ground. The distance x from the ground where a small hole is punched to get the maximum range R is



(A) H (B) $1.25H$

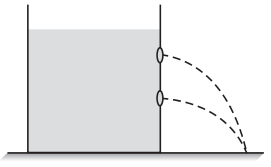
(C) $1.5H$ (D) $2H$

193. A force F is applied on the wire of radius r and length L and change in the length of wire is l. If the same force F is applied on the wire of the same material and radius $2r$ and length $2L$, Then the change in length of the other wire is

(A) l (B) $2l$

(C) $\frac{l}{2}$ (D) $4l$

194. In a cylindrical vessel containing liquid of density ρ, there are two holes in the side walls at heights of h_1 and h_2 respectively such that the range of efflux at the bottom of the vessel is same. The height of a hole for which the range of efflux would be maximum, will be



(A) $h_2 - h_1$ (B) $h_2 + h_1$

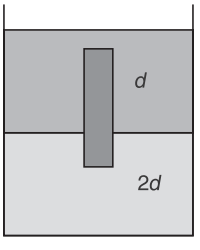
(C) $\frac{h_2 - h_1}{2}$ (D) $\frac{h_2 + h_1}{2}$

195. A cubical block of wood of edge a and density ρ floats in water of density 2ρ. The lower surface of the cube just touches the free end of a massless spring of force constant k fixed at the bottom of the vessel. The weight W put over the block so that it is completely immersed in water without wetting the weight is

(A) $a\left(a^2\rho g + k\right)$ (B) $a\left(a\rho g + 2k\right)$

(C) $a\left(\frac{a\rho g}{2} + 2k\right)$ (D) $a\left(a^2\rho g + \frac{k}{2}\right)$

196. A homogeneous solid cylinder of length $L\left(< \frac{H}{2}\right)$, cross-sectional area $\frac{A}{5}$ is immersed such that it floats with its axis vertical at the liquid interface with length $\frac{L}{4}$ in the denser liquid as shown in Figure. The lower density liquid is open to atmosphere having pressure P_0. Then, density D of solid is given by



(A) $\frac{5}{4}d$ (B) $\frac{4}{5}d$

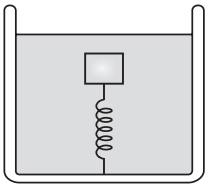
(C) $4d$ (D) $\frac{d}{5}$

197. A block of ice of total area A and thickness 0.5 m is floating in water. In order to just support a man of mass 100 kg, the area A should be (the specific gravity of ice is 0.9)

(A) 2.2 m^2 (B) 1.0 m^2

(C) 0.5 m^2 (D) None of these

198. A block is submerged in a vessel filled with water by a spring attached to the bottom of the vessel. In equilibrium, the spring is compressed. The vessel now moves downwards with an acceleration $a(< g)$. The spring length



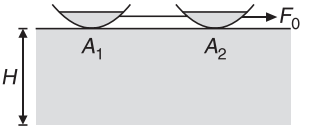
(A) will become zero

(B) may increase, decrease or remain constant

(C) will decrease

(D) will increase

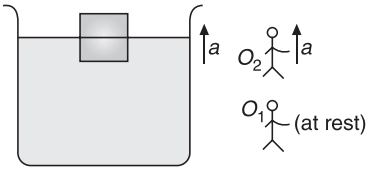
199. Two boats of base areas A_1 and A_2, connected by a string are being pulled by an external force F_0. The viscosity of water is η and depth of the water body is H. When the system attains a constant speed, the tension in the thread will be



(A) $F_0\left(\frac{A_1}{A_2}\right)$ (B) $F_0\frac{A_2}{(A_1 + A_2)}$

(C) $F_0\frac{A_1}{(A_1 + A_2)}$ (D) $F_0\left(\frac{A_2}{A_1}\right)$

200. A block is partially immersed in a liquid and the vessel is accelerating upwards with an acceleration a. The block is observed by two observers O_1 and O_2, one at rest and the other accelerating with an acceleration a upward. The total buoyant force on the block is



(A) same for O_1 and O_2

(B) greater for O_1 than O_2

(C) greater for O_2 than O_1

(D) data is not sufficient

201. The normal density of gold is ρ and its Bulk modulus is K. The increase in density of a piece of gold when a pressure P is applied uniformly from all sides is

(A) $\frac{\rho P}{2K}$ (B) $\frac{\rho K}{2P}$

(C) $\frac{\rho P}{K - P}$ (D) $\frac{\rho K}{K - P}$

202. At the bottom of a vessel filled with mercury of density ρ surface tension T there is a round hole of radius r. The maximum height of the mercury layer for which the liquid will not flow out through this hole is

(A) $\frac{T}{r\rho g}$ (B) $\frac{T}{2r\rho g}$

(C) $\frac{2T}{r\rho g}$ (D) $\frac{4T}{r\rho g}$

203. If the work done in stretching a wire by 1 mm is 2 J, the work necessary for stretching another wire of the same material but double the radius and half the length by 1 mm is

(A) 16 J (B) 8 J

(C) 4 J (D) $\frac{1}{4}\text{ J}$

204. A given quantity of an ideal gas is at pressure P and absolute temperature T. The isothermal bulk modulus of the gas is

(A) $\frac{2}{3}P$ (B) P

(C) $\frac{3}{2}P$ (D) $2P$

205. A solid shell loses half its weight in water. If relative density of shell is 5, the fraction of its volume which happens to be hollow is

(A) $\frac{3}{5}$ (B) $\frac{2}{5}$

(C) $\frac{1}{5}$ (D) $\frac{4}{5}$

206. In a U-tube the radii of two columns are respectively r_1 and r_2. When a liquid of density $\rho(\theta = 0°)$ is filled in it, a level difference of h is observed on two arms, then the surface tension of the liquid is

(A) $\frac{\rho g h r_1 r_2}{2(r_2 - r_1)}$ (B) $h\rho g(r_2 - r_1)$

(C) $\frac{h\rho g(r_2 - r_1)}{2}$ (D) $\frac{h\rho g}{2(r_2 - r_1)}$

207. A copper piece of mass 10 g is suspended by a vertical spring. The spring elongates 1 cm over its natural length to keep the piece in equilibrium. A beaker containing water is now placed below the piece so as to immerse the piece completely in water. If density of copper is 9000 kgm^{-3} and $g = 10\text{ ms}^{-2}$, then the approximate elongation of the spring is

(A) 0.45 cm (B) 0.89 cm

(C) 1.02 cm (D) 1.86 cm

208. A U-tube is partially filled with water. Oil which does not mix with water is next poured into one side, until water rises by 25 cm on the other side. If the density of oil is 0.8 gcm^{-3}, the oil level will stand higher than the water level by

(A) 6.25 cm (B) 12.50 cm

(C) 31.75 cm (D) 25 cm

209. The compressibility of water is 5×10^{-5} per unit atmospheric pressure. The decrease in volume of 100 cm^3 of water under a pressure of 100 atmosphere will be

(A) 0.5 cm^3 (B) $5\times10^{-5}\text{ cm}^3$

(C) 0.025 cm^3 (D) 0.005 cm^3

210. There is a hole of radius r at the bottom of a long cylindrical vessel of glass. The depth to which the vessel can be lowered vertically in a deep water bath (surface tension is T and density is ρ) without any water entering inside is

(A) $\frac{4T}{\rho rg}$ (B) $\frac{3T}{\rho rg}$

(C) $\frac{2T}{\rho rg}$ (D) $\frac{T}{\rho rg}$

211. A rubber cord 10 m long is suspended vertically. How much does it stretch under its own weight (Density of rubber is 1500 kgm^{-3}, $Y = 5\times10^8\text{ Nm}^{-2}$, $g = 10\text{ ms}^{-2}$)

(A) $15\times10^{-4}\text{ m}$ (B) $7.5\times10^{-4}\text{ m}$

(C) $12\times10^{-4}\text{ m}$ (D) $25\times10^{-4}\text{ m}$

212. A wooden cube just floats inside water when a 200 g mass is placed on it. When the mass is removed, the cube is 2 cm above the water level. What is the size of each sides of the cube?

(A) 6 cm (B) 8 cm

(C) 10 cm (D) 12 cm

213 A solid ball of density ρ_1 and radius r falls vertically through a liquid of density ρ_2. Assume that the viscous force acting on the ball is $F = krv$, where k is a constant and v its velocity. The terminal velocity of the ball is

(A) $\frac{4\pi g r^2(\rho_1 - \rho_2)}{3k}$ (B) $\frac{2\pi r(\rho_1 - \rho_2)}{3gk}$

(C) $\frac{2\pi g(\rho_1 + \rho_2)}{3gr^2k}$ (D) None of these

214. A vessel contains oil of density 0.8 gcm^{-3} over mercury of density 13.6 gcm^{-3}. A homogeneous sphere floats with half its volume immersed in mercury and the other half in oil. The density of the material of the sphere in gcm^{-3} is

(A) 3.3 (B) 6.4

(C) 1.05 (D) 1.0

215. A steel wire having a radius of 2.0 mm, carrying a load of 4 kg, is hanging from a ceiling. Given that $g = 3.1\pi\text{ ms}^{-2}$, what will be the tensile stress that would be developed in the wire?

(A) $4.8\times10^6\text{ Nm}^{-2}$ (B) $3.1\times10^6\text{ Nm}^{-2}$

(C) $5.2\times10^6\text{ Nm}^{-2}$ (D) $6.2\times10^6\text{ Nm}^{-2}$

216. Water rises to a height of 10 cm in a certain capillary tube. When another identical tube is dipped in mercury, the level of mercury is depressed by 3.42 cm. If the density of mercury is 13.6 gcc^{-1}, the angle of contact for water in contact with glass is $0°$ and mercury in contact with glass is $135°$, then the ratio of surface tension of water to that of Hg is

(A) 1:3 (B) 1:4

(C) 1:5.5 (D) 1:6.5

217. What is the radius of a steel sphere that will float on water with exactly half the sphere submerged? Density of steel is $7.9\times10^3\text{ kgm}^{-3}$ and surface tension of water is $7\times10^{-2}\text{ N}$

(A) 2.6 cm (B) 4.6 mm

(C) 1.2 mm (D) 6.5 mm

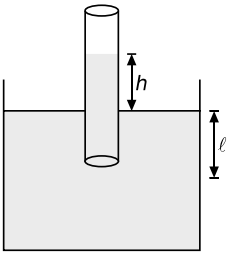
218. A fixed container of height H with large cross-sectional area A is completely filled with water. Two small orifice of cross-sectional area a are made, one at the bottom and

the other on the vertical side of the container at a distance $\frac{H}{2}$ from the top of the container. The time taken by the water level to reach a height of $\frac{H}{2}$ from the bottom of the container is

(A) $\frac{2A}{3a}\sqrt{\frac{2H}{g}}$ (B) $\frac{2A}{3a}(\sqrt{2}-1)\sqrt{\frac{H}{g}}$

(C) $\frac{3A}{2a}(\sqrt{2}-1)\sqrt{\frac{H}{g}}$ (D) $\frac{3A}{2a}\sqrt{\frac{2H}{g}}$

219. A capillary tube is dipped in water to a depth and the water rises to a height $h(<l)$ in the capillary tube. The lower end of the tube is closed in water by putting a thumb over it. The tube is now taken out and the thumb is removed from the lower end and it is kept open. The length of liquid column in the tube will be



(A) l (B) $l+h$

(C) h (D) $2h$

220. A steel wire of length 20 cm and uniform cross-section 1 mm^2 is tied rigidly at both the ends. The temperature of the wire is altered from 40 °C to 20 °C. What is the magnitude of force developed in the wire? (Coefficient of linear expansion for steel, $\alpha = 1.1\times10^{-5}\text{ °C}^{-1}$ and Y for steel is $2.0\times10^{11}\text{ Nm}^{-2}$)

(A) $2.2\times10^{6}\text{ N}$ (B) 16 N

(C) 8 N (D) 44 N

221. There is a horizontal film of soap solution. On it a thread is placed in the form of a loop. The film is punctured inside the loop and the thread becomes a circular loop of radius R. If the surface tension of the soap solution be T, then the tension in the thread will be

(A) $\frac{\pi R^2}{T}$ (B) π^2R^2T

(C) $2\pi RT$ (D) $2RT$

222. When too many water drops coalesce to form a bigger drop

(A) energy is absorbed

(B) energy is liberated

(C) energy is neither liberated nor absorbed

(D) energy may either be liberated or be absorbed depending on the nature of liquid

223. A U -tube of uniform cross-section is partially filled with a liquid 1. Another liquid 2 which does not mix with liquid 1 is poured into one side. It is found that the liquid levels of the two sides of the tube are the same, while the level of liquid 1 has risen by 2 cm. If the specific gravity of liquid 1 is 1.1, the specific gravity of liquid 2 must be

(A) 1.12 (B) 1.1

(C) 1.05 (D) 1.0

224. In an experiment, brass and steel wires of length 1 m each with areas of cross section 1 mm^2 are used. The wires are connected in series and one end of the combined wire is connected to a rigid support and other end is subjected to elongation. The stress required to produce a net elongation of 0.2 mm is, [Given, the Young's Modulus for steel and brass are, respectively, $120\times10^{9}\text{ Nm}^{-2}$ and $60\times10^{9}\text{ Nm}^{-2}$]

(A) $8\times10^{6}\text{ Nm}^{-2}$ (B) $1.2\times10^{6}\text{ Nm}^{-2}$

(C) $4.0\times10^{6}\text{ Nm}^{-2}$ (D) $0.2\times10^{6}\text{ Nm}^{-2}$

225. A capillary tube of radius R is immersed in water and water rises in it to a height h. Mass of water in capillary tube is M. If the radius of the tube is doubled, mass of water that will rise in the capillary tube will be

(A) $2M$ (B) M

(C) $\frac{M}{2}$ (D) $4M$

226. A capillary is dipped in water vessel kept on a freely falling lift, then

(A) water will not rise in the tube

(B) water will rise to the maximum available height of the tube

(C) water will rise to the height observed under normal condition

(D) water will rise to the height below that observed under normal condition

227. A thin metal disc of radius r floats on a liquid surface and bends the surface downwards along the perimeter making an angle θ with vertical edge of the disc. If the disc displaces a weight of liquid w and surface tension of liquid is T, then the weight of metal disc is

(A) $2\pi rT + w$ (B) $2\pi rT\cos\theta - w$

(C) $2\pi rT\cos\theta + w$ (D) $w - 2\pi rT\cos\theta$

228. A spherical steel ball released at the top of a long column of glycerine of length l, falls through a distance $\frac{l}{2}$ with accelerated motion and the remaining distance $\frac{l}{2}$ with uniform velocity. Let t_1 and t_2 denote the times taken to cover the first and second half and W_1 and W_2 the work done against gravity in the two halves, then

(A) $t_1 < t_2,\ W_1 > W_2$ (B) $t_1 > t_2,\ W_1 < W_2$

(C) $t_1 = t_2,\ W_1 = W_2$ (D) $t_1 > t_2,\ W_1 = W_2$

229. The radii of the two columns is U-tube are r_1 and $r_2(>r_1)$. When a liquid of density ρ (angle of contact is $0°$) is filled in it, the level difference of liquid in two arms is h. The surface tension of liquid is

(A) $\frac{\rho ghr_1r_2}{2(r_2-r_1)}$ (B) $\frac{\rho gh(r_2-r_1)}{2r_1r_2}$

(C) $\frac{2(r_2-r_1)}{\rho ghr_1r_2}$ (D) $\frac{\rho gh}{2(r_2-r_1)}$

230. A fixed volume of iron is drawn into a wire of length L. The extension x produced in this wire by a constant force F is proportional to

(A) $\frac{1}{L^2}$ (B) $\frac{1}{L}$

(C) L^2 (D) L

231. A piece of steel has a weight w in air, w_1 when completely immersed in water and w_2 when completely immersed in an unknown liquid. The relative density (specific gravity) of liquid is

(A) $\frac{w-w_1}{w-w_2}$ (B) $\frac{w-w_2}{w-w_1}$

(C) $\frac{w_1-w_2}{w-w_1}$ (D) $\frac{w_1-w_2}{w-w_2}$

232. A metal ball immersed in alcohol weight W_1 at $0\ ^\circ C$ and W_2 at $50\ ^\circ C$. The coefficient of cubical expansion of the metal is less than that of the alcohol. Assuming that the density of the metal is large compared to that of alcohol, it can be shown that

(A) $W_1 > W_2$ (B) $W_1 = W_2$

(C) $W_1 < W_2$ (D) All of the above

233. At $40\ ^\circ C$, a brass wire of $1\ mm$ radius is hung from the ceiling. A small mass, M is hung from the free end of the wire. When the wire is cooled down from $40\ ^\circ C$ to $20\ ^\circ C$ it regains its original length of $0.2\ m$. The value of M is close to (Coefficient of linear expansion and Young's modulus of brass are $10^{-5}\ ^\circ C^{-1}$ and $10^{11}\ Nm^{-2}$, respectively; $g = 10\ ms^{-2}$)

(A) 0.5 kg (B) 0.9 kg

(C) 6.28 kg (D) 9 kg

234. A and B are two soap bubbles. Bubble A is larger than B. If these are now joined by a tube then

(A) the bubble A becomes more large

(B) the bubble B becomes more large

(C) both the bubbles acquire the same size

(D) both the bubbles will get bursted

235. The volume of a liquid flowing per second out of an orifice at the bottom of a tank does not depend upon

(A) the height of the liquid above the orifice

(B) the acceleration due to gravity

(C) the density of the liquid

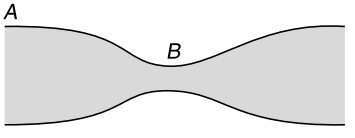
(D) the area of the orifice

236. Two parallel glass plates are dipped partly in the liquid of density ρ keeping them vertical. If the distance between the plates is x, Surface tension for liquid is T and angle of contact is θ then rise of liquid between the plates due to capillary will be

(A) $\frac{T\cos\theta}{x\rho}$ (B) $\frac{2T\cos\theta}{x\rho g}$

(C) $\frac{2T}{x\rho g\cos\theta}$ (D) $\frac{T\cos\theta}{x\rho g}$

237. Water flows in a horizontal tube shown in Figure. The pressure of water changes by $600\ Nm^{-2}$ between A and B where the areas of cross-section are $30\ cm^2$ and $15\ cm^2$ respectively. The rate of flow of water through the tube is



(A) $600\ cm^3s^{-1}$ (B) $1200\ cm^3s^{-1}$

(C) $1800\ cm^3s^{-1}$ (D) $2400\ cm^3s^{-1}$

MULTIPLE CORRECT CHOICE TYPE QUESTIONS

This section contains Multiple Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

1. A large wooden plate of area $10\ m^2$ floating on the surface of a river is made to move horizontally with a speed of $2\ ms^{-1}$ by applying a tangential force. If river is $1\ m$ deep, coefficient of viscosity of water is $10^{-3}\ Nsm^{-2}$ and the water in contact with the bed, then select the correct statement(s).

(A) The speed gradient is $2\ s^{-1}$

(B) The speed gradient is $1\ s^{-1}$

(C) The force required to keep the plate moving with constant speed is $0.02\ N$

(D) The force required to keep the plate moving with constant speed is $0.01\ N$

2. A sample of metal weighs $210\ g$ in air, $180\ g$ in water and $120\ g$ in liquid. Then relative density (RD) of

(A) metal is 3 (B) metal is 7

(C) liquid is 3 (D) liquid is $\frac{1}{3}$

3. An ideal liquid is flowing in streamline motion through a tube with its axis horizontal. Consider two points A and B in the tube at the same horizontal level, then select the correct statement(s).

(A) The pressure at A and B are equal for any shape of the tube.

(B) The pressures are equal if the tube has a uniform cross-section.

(C) The pressure can never be equal.

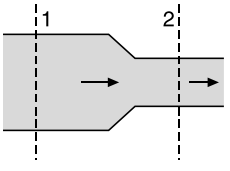
(D) The pressure may be equal even if the tube has a non-uniform cross-section.

4. A light rod of length $2\ m$ is suspended from a ceiling horizontally by means of two vertical wires of equal length tied to its ends. One of the wires is made of steel and is of cross-section $10^{-3}\ m^2$ and the other is of brass of cross-section $2\times10^{-3}\ m^2$. If x is the distance from steel wire end, at which a weight may be hung, $Y_{steel} = 2\times10^{11}\ Pa$ and $Y_{brass} = 10^{11}\ Pa$, then which of the following statement(s) is/are **correct**?

(A) $x = 1.33$ m, if stress is equal in both wires.
(B) $x = 1.42$ m, if stress is equal in both wires.
(C) $x = 1$ m, if strain is equal in both wires.
(D) $x = 1.2$ m, if strain is equal in both wires.

5. A ball of density ρ is dropped on a horizontal solid surface. It bounces elastically from the surface and returns to its original position in time t_1. Another identical ball is released from the same height, but this time it strikes the surface of a liquid of density ρ_L and takes time t_2 to return to its original height. If $\rho < \rho_L < 2\rho$, then select the correct statement(s).
(A) $t_2 < t_1$
(B) $t_2 > t_1$
(C) If $\rho = \rho_L$, then the speed of the ball inside the liquid will be independent of its depth.
(D) The motion of the ball is not simple harmonic.

6. A non-viscous incompressible liquid is flowing through a horizontal pipe of variable cross section as shown in Figure. Select the correct option(s).

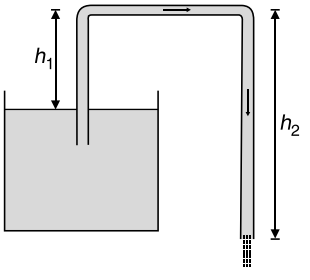


(A) The speed of liquid at section-2 is more.
(B) The volume of liquid flowing per second from section-2 is more.
(C) The mass of liquid flowing per second at both the sections is same.
(D) The pressure at section-2 is less.

7. For two different materials it is given that $Y_1 > Y_2$ and $B_1 < B_2$. Here, Y is Young's modulus of elasticity and B, the Bulk modulus of elasticity. Then can conclude that
(A) 1 is more ductile (B) 2 is more ductile
(C) 1 is more malleable (D) 2 is more malleable

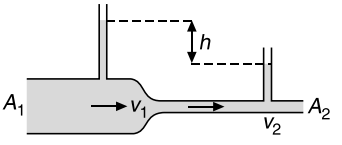
8. Two wires A and B have equal lengths and are made of the same material, but diameter of wire A is twice that of wire B. Then, for a given load attached to the wires, select the correct statement(s).
(A) Extension is same in both the wires.
(B) Extension in B will be four times that in A.
(C) Strain is equal in both the wires.
(D) Strain in B is four times that in A.

9. A siphon is a device used to transfer liquid from a higher level to a lower level. For the siphon to work, the condition of working is



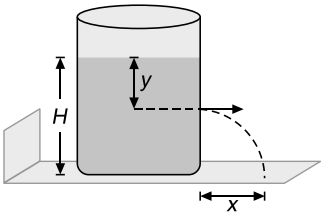
(A) $h_2 > h_1$
(B) $h_1 = 2h_2$
(C) h_1 should be greater than the height of corresponding liquid barometer.
(D) h_1 should be less than the height of corresponding liquid barometer.

10. An ideal liquid flows through a horizontal tube. The velocities of the liquid in the two sections, which have areas of cross-section A_1 and A_2, are v_1 and v_2 respectively. The difference in the levels of the liquid in the two vertical tubes is h



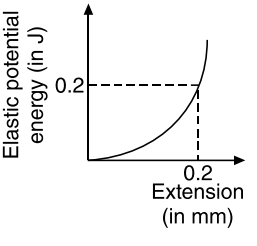
(A) The volume of the liquid flowing through the tube in unit time is $A_1 v_1$
(B) $v_2^2 - v_1^2 = 2gh$
(C) $v_2 - v_1 = \sqrt{2gh}$
(D) The energy per unit mass of the liquid is the same in both sections of the tube

11. A tank open at the top contains a liquid up to a height H as shown in Figure.



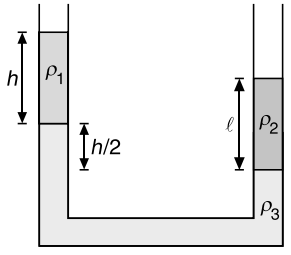
A small orifice is made in the wall of the tank at a distance y below the free surface of the liquid in the tank and the liquid emerging from the orifice hits the ground at a distance x from the tank. Select the correct statement(s).
(A) The maximum value of x will depend on the density of the liquid
(B) x is maximum for $y = \frac{H}{2}$
(C) The maximum value of x is H
(D) If y is increased from zero to H, x first increases and then decreases.

12. The graph of elastic potential energy (U) stored versus extension, for a steel wire of volume 200 cc having $Y_S = 2 \times 10^{11}$ Pa is shown in Figure. If A is the area of cross-section of the wire and its original length is L, then



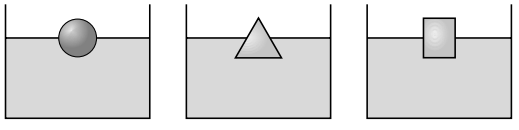
(A) $A = 10^{-3}$ m^2 (B) $A = 10^{-4}$ m^2
(C) $L = 2$ m (D) $L = 1.5$ m

13. Three different liquids of densities ρ_1, ρ_2 and ρ_3 are filled in a U-tube as shown in Figure.



Select the correct conclusion(s) you infer from the Figure.
(A) $\rho_1 > \rho_2$ (B) $\rho_2 > \rho_1$
(C) $\rho_2 = 2\rho_3 - \rho_1$ (D) $\rho_3 = 2(\rho_2 - \rho_1)$

14. A solid sphere, a cone and a cylinder all having same mass, density and radius are floating in water. Let f_1, f_2 and f_3 be the respective fraction of their volumes inside the water and h_1, h_2 and h_3 be their respective depths inside water, then

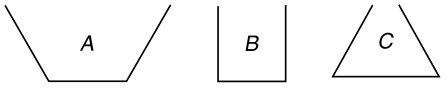


(A) $f_3 > f_2 > f_1$
(B) $f_1 = f_2 = f_3$
(C) $h_3 < h_1$
(D) $h_3 < h_2$

15. A body of mass m is attached to the lower end of a metal wire, whose upper end is fixed. If elongation in the wire is l, then select the correct statement(s).
(A) Elastic potential energy stored in wire is mgl.
(B) Loss in gravitational potential energy of mass is mgl.
(C) Elastic potential energy stored in wire is $\frac{mgl}{2}$.
(D) Heat produced is $\frac{mgl}{2}$.

16. A liquid of density ρ comes out with a velocity v from a horizontal tube of area of cross-section A. The reaction force exerted by the liquid on the tube is F. Then, we have
(A) $F \propto \rho$ (B) $F \propto v^2$
(C) $F \propto A$ (D) $F \propto v$

17. Equal volumes of a liquid are poured in the three vessels A, B, and C. All the vessels have same base area. Select the correct statement(s).

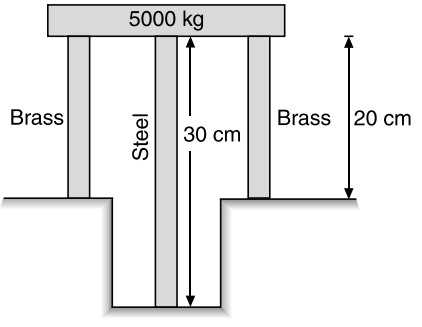


(A) Force on base of vessel A is maximum.
(B) Force on base of vessel C is maximum.
(C) Net force exerted by liquid in all the three vessels is equal.
(D) Net force exerted by liquid in vessel A is maximum

18. Which of the following option(s) is correct?
(A) Viscosity of liquids increases with temperature
(B) Viscosity of gases increases with temperature
(C) Surface tension of liquids decreases with temperature
(D) For angle of contact $\theta = 0°$, liquid neither rises nor falls on capillary

19. Select the correct alternative(s)
(A) elastic forces are always conservative
(B) elastic forces are not always conservative
(C) elastic forces are conservative only when Hooke's law is obeyed
(D) elastic forces may be conservative even when Hooke's law is not obeyed

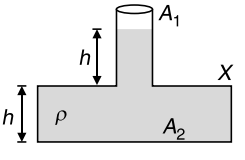
20. A steel rod of cross-sectional area 16 cm^2 and two brass rods each of cross-sectional area 10 cm^2 together support a load of 5000 kg as shown in Figure.



If $Y_{steel} = 2 \times 10^6$ kgcm^{-2} and $Y_{brass} = 10^6$ kgcm^{-2}, then select the **correct** option(s).
(A) Stress in brass rod is 141 kgfcm^{-2}
(B) Stress in steel rod is 141 kgfcm^{-2}
(C) Stress in brass rod is 121 kgfcm^{-2}
(D) Stress in steel rod is 161 kgfcm^{-2}

21. A load W is suspended from a wire of length L and area of cross-section A. If change in length of the wire is say ΔL, then ΔL can be made twice if
(A) W is made two times
(B) L is made two times
(C) A is made half
(D) A is made two times

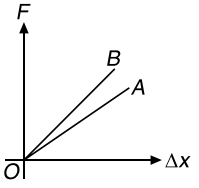
22. The cross-sectional area of the mouth of vessel is A_1 and that of its base is A_2 as shown in Figure.



A liquid of density ρ fills both the sections, each up to a height h. Neglecting atmospheric pressure, select the correct statement(s).
(A) The pressure at the base of the vessel is $2h\rho g$
(B) The force exerted by the liquid on the base of the vessel is $2h\rho g A_2$

(C) The weight of the liquid is $< 2h\rho gA_2$
(D) The walls of the vessel at the level X exert a downward force $h\rho g(A_2 - A_1)$ on the liquid

23. Two wires A and B of same length are made of same material. The load F versus extension Δx graph for the two wires is shown in Figure. Select the correct statement(s).

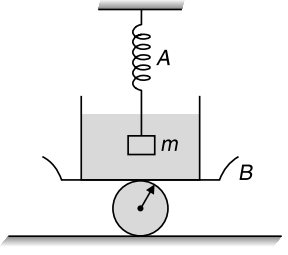


(A) Area of cross section of A is greater than B.
(B) Wire B is more elastic than A.
(C) Area of cross section of B is greater than A.
(D) Wire A is more elastic than B.

24. The viscous force acting on a solid ball of surface area A, volume V moving with terminal velocity v is proportional to

(A) A (B) $A^{1/2}$
(C) $V^{1/3}$ (D) v

25. A spring balance A reads 2 kg with a block of mass m suspended from it and another balance B reads 5 kg when a beaker with liquid is put on the pan of the balance. The two balances are now so arranged that the hanging block is inside the liquid in the beaker as shown in Figure. Then the balance

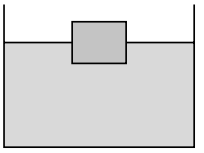


(A) A reads more than 2 kg
(B) B reads more than 5 kg
(C) A reads less than 2 kg
(D) A reads 2 kg and B reads 5 kg

26. An iron casting weighs 27 kg in air and 18 kg in water. If density of iron is 7800 kgm^{-3}, then select the correct option(s).

(A) Outer volume of casting is 9000 cm^3
(B) Outer volume of casting is 6000 cm^3
(C) Volume of cavity inside the casting is 5538 cm^3
(D) Volume of cavity inside the casting is 780 cm^3

27. A block of weight W is floating in a liquid as shown in Figure. Let P be the pressure at bottom of block and U be the upthrust due to liquid acting on the block. If the container starts moving upwards with some positive acceleration, then let the new respective values of weight, pressure and upthrust be W', P' and U', then

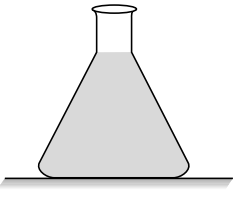


(A) $W' > W$ (B) $P' > P$
(C) $U' > U$ (D) $F' = W'$

28. Two large identical beakers 1 and 2 each are filled with identical liquids to a same height $5h$. Each beaker has a small orifice in its wall, one h below the free surface of liquid and the other h above the horizontal surface on which the beakers are kept. If v_1 and v_2 be the respective speeds of liquid coming out of orifice, t_1 and t_2 be the respective times taken by the liquid to hit the ground, R_1 and R_2 be the respective ranges of the liquid where it hits the horizontal surface, then

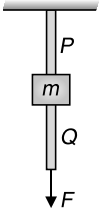
(A) $\frac{v_1}{v_2} = \frac{1}{2}$ (B) $\frac{v_1}{v_2} = \frac{1}{4}$
(C) $\frac{t_1}{t_2} = 2$ (D) $\frac{R_1}{R_2} = 1$

29. A massless conical flask filled with a liquid is kept on a horizontal surface and the arrangement is kept in vacuum. If the force exerted by the liquid on the base of the flask is W_1 and the force exerted by the flask on the table is W_2, then



(A) $W_1 = W_2$
(B) $W_1 > W_2$
(C) $W_1 < W_2$
(D) The force exerted by the liquid on the walls of the flask is $(W_1 - W_2)$

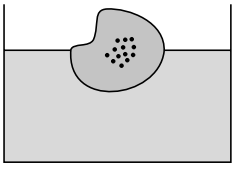
30. Two light wires P and Q are made of same material, have radii r_P and r_Q, respectively have a block of mass m attached between them as shown in Figure.



When a force $F = \frac{mg}{3}$ is applied on the lower end of the wire Q, then one of the wires breaks. Select the **correct** statement(s).

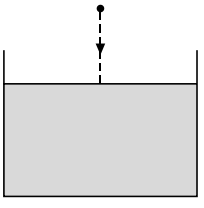
(A) When $r_P = r_Q$, then P breaks.
(B) When $r_P < 2r_Q$, then P breaks.
(C) When $r_P = 2r_Q$, then either P or Q may break.
(D) The lengths of P and Q must be known in order to predict which wire will break.

31. In a block of ice floating in water of density ρ_w in a beaker some pieces of impurity of density ρ are embedded. When the ice melts, the level of water in the beaker will



(A) rise if $\rho > \rho_w$
(B) fall if $\rho < \rho_w$
(C) fall if $\rho > \rho_w$
(D) remain unchanged, if $\rho < \rho_w$

32. A ball of density ρ is dropped from some height on the surface of a non-viscous liquid of density 2ρ. Select the correct statement(s).

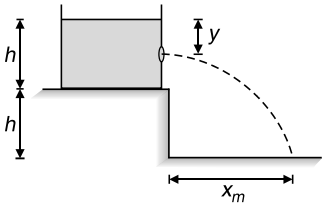


(A) Motion of ball is periodic but not simple harmonic
(B) Acceleration of ball in air and in liquid are equal
(C) Magnitude of upthrust in the liquid is two times the weight of ball
(D) Net force on ball in air and in liquid are equal and opposite

33. A metal wire of length L, area of cross-section A and Young's modulus Y is stretched by a variable force F such that F is always slightly greater than the elastic forces of resistance in the wire. When the elongation in the wire is l, then the

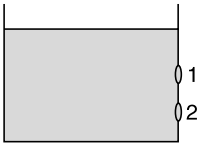
(A) work done by F is $\frac{YAl^2}{2L}$
(B) work done by F is $\frac{YAl^2}{L}$
(C) elastic potential energy stored in wire is $\frac{YAl^2}{2L}$
(D) elastic potential energy stored in wire is $\frac{YAl^2}{4L}$

34. A tank is filled up to a height h with a liquid and is placed on a platform of height h from the ground. To get maximum range x_m, a small hole is punched at a distance of y from the free surface of the liquid. Then



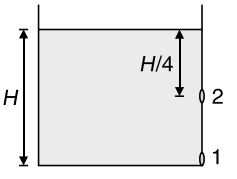
(A) $x_m = 2\,h$ (B) $x_m = 1.5\,h$
(C) $y = h$ (D) $y = 0.75\,h$

35. A vessel of very large height filled with an ideal liquid has two small circular holes 1 and 2 made at depths h and $16h$ below the free surface of the liquid respectively. If radius of hole 1 is twice that of hole 2, then select the correct alternative(s).



(A) Initially more volume of liquid will flow from hole-2 per unit time
(B) Initially equal volumes of liquid will flow from both the holes in unit time
(C) After some time more volume of liquid will flow from hole-1 per unit time
(D) After some time more volume of liquid will flow from hole-2 per unit time

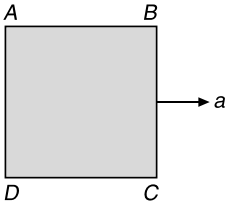
36. Water in a tank is filled to a height H. The tank has two holes 1 and 2. The hole 1 and 2 are at the respective depth H and $H/4$ below the free surface of water. The area of hole 2 is twice that of area of hole 1. If v be the speed of efflux of liquid coming out from the hole and Q is the rate of flow of liquid, then



(A) $v_1 = 4v_2$ (B) $v_1 = 2v_2$
(C) $Q_1 = Q_2$ (D) $Q_1 = 2Q_2$

37. A piece of ice is floating in a liquid kept in a beaker. The level of liquid in the beaker
(A) remains same if liquid is water.
(B) falls if liquid is water.
(C) rises if liquid is denser than water.
(D) falls if liquid is lighter than water.

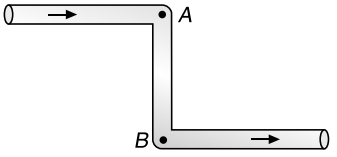
38. Four points A, B, C and D are taken inside an incompressible liquid that completely fills a closed container accelerating toward the right as shown in Figure. If P be the pressure at a point inside the liquid, then select the correct option(s).



(A) $P_C > P_A$ (B) $P_A > P_B$
(C) $P_A > P_C$ (D) $P_D > P_B$

39. Water is being poured in a vessel at a constant rate α m^3s^{-1}. There is a small hole of area a at the bottom of the tank. The maximum level of water in the vessel is proportional to
(A) α (B) α^2
(C) a^{-2} (D) a^{-1}

40. In the figure, an ideal liquid flows through the tube, which is of uniform cross-section. The liquid has velocities v_A, v_B and pressures P_A, P_B at points A and B respectively



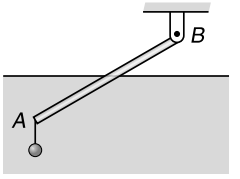
(A) $v_B > v_A$ (B) $v_A = v_B$
(C) $P_B > P_A$ (D) $P_A = P_B$

41. A vessel is filled with mercury to a height of 0.9 m. If the barometric height is 0.7 m mercury, then select the correct statement(s).
(A) The vessel can be completely emptied with the aid of a siphon
(B) The vessel cannot be emptied completely with the aid of a siphon
(C) The vessel can be emptied with at least 0.7 m height of mercury remaining in the vessel
(D) None of the given statements is correct.

42. An oil drop falls through air with a terminal velocity of 5×10^{-4} ms^{-1}. Viscosity of oil is 1.8×10^{-5} Nsm^{-2} and density of oil is 900 kgm^{-3}. Neglecting density of air as compared to that of the oil, select the correct option(s).
(A) radius of the drop is 6.2×10^{-2} m
(B) radius of the drop is 2.14×10^{-6} m
(C) terminal velocity of the drop at half of this radius is 1.25×10^{-4} ms^{-1}
(D) terminal velocity of the drop at half of this radius is 2.5×10^{-4} ms^{-1}

43. An oil drop falls through air with a terminal velocity of 5×10^{-4} ms^{-1}. If viscosity of air is 1.8×10^{-5} Nsm^{-2} and density of oil is 900 kgm^{-3} and density of air be ignored compared to the density of oil, then select the correct option(s).
(A) Radius of the drop is 2.14×10^{-6} m
(B) Radius of the drop is 6.7×10^{-6} m
(C) Terminal velocity of the drop at half of this radius is 1.25×10^{-4} ms^{-1}
(D) Terminal velocity of the drop at half of this radius is 2.5×10^{-4} ms^{-1}

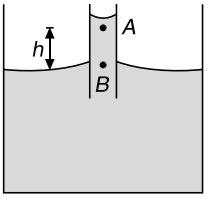
44. A uniform rod AB of length 12 m, mass 24 kg is supported at end B by a flexible light string. A lead weight (of very small size) of 12 kg attached to the end A of the rod as shown in Figure.



If the rod floats in water with one-half of its length submerged, $g = 10$ ms^{-2} and density of water is 1000 kgm^{-3}, then for this situation, select the incorrect statement(s).
(A) The point of application of the buoyant force is passing through C (centre of mass of rod)
(B) The tension in the string is 20 N
(C) The tension in the string is 36 N
(D) The volume of the rod is 6.4×10^{-2} m^3

45. Following are some statements about buoyant force on a body placed in liquid of uniform density. Select the incorrect statement(s).
(A) Buoyant force depends upon orientation of the concerned body inside the liquid.
(B) Buoyant force depends upon the density of the body immersed.
(C) Buoyant force depends on the fact whether the system is on moon or on the earth.
(D) Buoyant force depends upon the depth at which the body (fully immersed in the liquid) is placed inside the liquid.

46. A liquid of density ρ and surface tension σ rises in a capillary tube of inner radius R. The angle of contact between the liquid and the glass is θ. The point A lies just below the meniscus in the tube and the point B lies at the outside level of liquid in the beaker as shown in Figure. If pressure at B is P_B and P_0 is atmospheric pressure, then pressure at A is equal to



(A) $P_B - \rho g h$ (B) $P_B - \dfrac{2\sigma\cos\theta}{R}$
(C) $P_0 - \dfrac{2\sigma\cos\theta}{R}$ (D) $P_B - P_0$

REASONING BASED QUESTIONS

This section contains Reasoning type questions, each having four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Each question contains STATEMENT 1 and STATEMENT 2. You have to mark your answer as

Bubble (A) If both statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1.
Bubble (B) If both statements are TRUE but STATEMENT 2 is not the correct explanation of STATEMENT 1.
Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE.
Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 is TRUE.

1. **Statement-1:** A rain drop after falling through some height attains a constant velocity.
Statement-2: At constant velocity the viscous drag is just equal to its net weight.

2. **Statement-1:** Smaller drops of liquid resist deforming forces better than the larger drops.
Statement-2: Excess pressure inside a drop is directly proportional to its surface area.

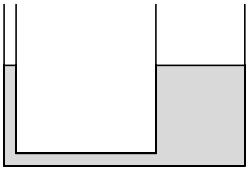
3. **Statement-1:** If barometer is accelerated upwards, the level of mercury in the tube of barometer will decrease.
Statement-2: The effective value of g will increase, so upthrust will increase.

4. **Statement-1:** Aeroplanes are made to run on the runway before take-off, so that they acquire the necessary lift.
Statement-2: This is as per Bernoulli's theorem.

5. **Statement-1:** Upthrust on a solid block of iron when immersed in a lake will be less on the surface than on the bed of the lake.
Statement-2: On the surface of the lake density of water will be less than that at the bed and upthrust depends on the density of liquid.

6. **Statement-1:** Dust particles generally settle down in a closed room.
Statement-2: The terminal velocity is inversely proportional to the square of their radii.

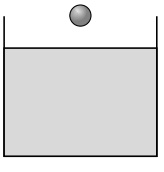
7. **Statement-1:** Water is filled in a U-tube of different cross-sectional area on two sides as shown in Figure. Now equal amount of oil $(RD = 0.5)$ is poured on two sides. Level of water on both sides will remain unchanged.



Statement-2: Same weight of oil poured on two sides will produce different pressures.

8. **Statement-1:** To float, a body must displace liquid whose weight is greater than actual weight of the body.
Statement-2: The body will experience no net downward force in that case.

9. **Statement-1:** A ball is dropped from a certain height above the free surface of an ideal fluid. When the ball enters the liquid, it may accelerate or retard.
Statement-2: Ball accelerates or retards it all depends on the density of ball and the density of liquid.



10. **Statement-1:** Bulk modulus of elasticity B represents incompressibility of the material.
Statement-2: $B = -\dfrac{\Delta P}{\Delta V/V}$, where symbols have their usual meaning.

11. **Statement-1:** An ideal fluid is flowing through a pipe. Speed of fluid particles is more at places where pressure is low.
Statement-2: Bernoulli's theorem can be derived from work-energy theorem.

12. **Statement-1:** A dam for water reservoir is built thicker at bottom than at the top.
Statement-2: Pressure of water is very large at the bottom.

13. **Statement-1:** From the relation $Y = \dfrac{Fl}{A\Delta l}$, we observe that, if length of a wire is doubled, then its Young's modulus (Y) also becomes two times.
Statement-2: Modulus of elasticity is the property of the material.

14. **Statement-1:** Pascal's law is the working principle of hydraulic lift.
Statement-2: Pressure $= \dfrac{\text{thrust}}{\text{area}}$

15. **Statement-1:** A needle placed carefully on the surface of water may float, whereas the ball of the same material will always sink.
Statement-2: The buoyancy of an object depends both on the material and shape of the object.

16. **Statement-1:** Air flows from a small bubble to a large bubble when they are connected to each other by a capillary tube.
Statement-2: The excess pressure because of surface tension inside a spherical bubble decreases as its radius increases.

17. **Statement-1:** A small drop of mercury is spherical but bigger drops are oval in shape.
Statement-2: Surface tension of liquid decreases with increase in temperature.

18. **Statement-1:** Bulk's modulus of elasticity can be defined for all three states of matter, i.e. solids liquids and gases.
Statement-2: Young's modulus is not defined for liquids and gases.

19. **Statement-1:** Steel is more elastic than rubber.
Statement-2: Under given deforming force, steel is deformed less than rubber.

20. **Statement-1:** Force of buoyancy due to atmosphere on a small body is almost zero (or negligible).
Statement-2: If a body is completely submerged in a fluid, then buoyant force is zero.

21. **Statement-1:** Stress is internal restoring force per unit area of a body.
Statement-2: Rubber is more elastic than steel.

22. **Statement-1:** At depth h below the water surface pressure is p. Then at depth $2h$ pressure will be $2p$. (Ignore density variation).
Statement-2: With depth pressure increases linearly.

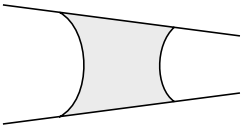
23. **Statement-1:** If length of a rod is doubled the breaking load remains unchanged.
Statement-2: Breaking load is equal to the elastic limit.

24. **Statement-1:** On moon, barometer height will be six times compared to the height on earth.

Statement-2: Value of g on moon's surface is $\frac{1}{6}$ the value of g on earth's surface.

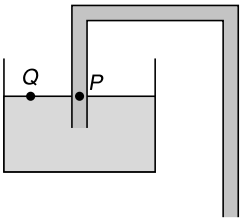
25. **Statement-1:** Lead is more elastic than rubber.
Statement-2: If the same load is attached to lead and rubber wires of the same cross-sectional area, the strain of lead is very much less than that of rubber.

26. **Statement-1:** Two identical conical pipes shown in figure have a drop of water. The water drop tends to move towards tapered end.
Statement-2: Excess pressure is directed towards centre of curvature and inversely proportional to radius of curvature. Net excess pressure is therefore, directed towards tapered end. So, the water drop tends to move towards tapered end.



27. **Statement-1:** Finer the capillary, greater is the height to which the liquid rises in the tube.
Statement-2: This is accordance with ascent formula.

28. **Statement-1:** In the siphon shown in Figure, pressure at P is equal to atmospheric pressure.



Statement-2: Pressure at Q is atmospheric pressure and points P and Q are at same levels.

29. **Statement-1:** Machine parts are jammed in winter.
Statement-2: The viscosity of lubricant used in machine parts increases at low temperature.

LINKED COMPREHENSION TYPE QUESTIONS

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct. (For the sake of competitiveness there may be a few questions that may have more than one correct options)

Comprehension 1

A large open top container of negligible mass and uniform cross-sectional area A has a small hole of cross-sectional area $\frac{A}{100}$ in its side wall near the bottom. The container is kept on a smooth horizontal floor and container a liquid of density ρ and mass m_0. Assuming that the liquid starts flowing out horizontally through the hole at $t = 0$. Based on the above facts, answer the following questions.

1. The velocity of efflux at the hole initially is

(A) $v = \sqrt{\frac{m_0 g}{A\rho}}$ (B) $v = \sqrt{\frac{m_0 g}{2A\rho}}$

(C) $v = \sqrt{\frac{m_0 g}{3A\rho}}$ (D) $v = \sqrt{\frac{2m_0 g}{A\rho}}$

2. The acceleration of the container is a. Then a equals

(A) $\frac{g}{25}$ (B) $\frac{g}{50}$

(C) $\frac{g}{75}$ (D) $\frac{g}{100}$

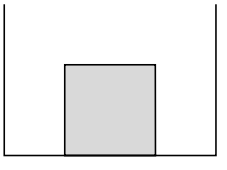
3. The velocity of the liquid when 75% of it has drained out is

(A) $\sqrt{\frac{m_0 g}{A\rho}}$ (B) $\sqrt{\frac{m_0 g}{2A\rho}}$

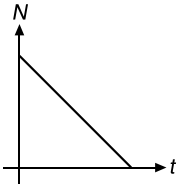
(C) $\sqrt{\frac{m_0 g}{3A\rho}}$ (D) $\sqrt{\frac{2m_0 g}{A\rho}}$

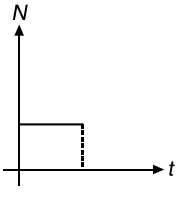
Comprehension 2

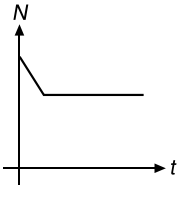
A cube of side $10\ \text{cm}$ having a density of $0.5\ \text{gcm}^{-3}$ is placed in a vessel of base area $20\ \text{cm} \times 20\ \text{cm}$ as shown in Figure. A liquid of density $1\ \text{gcm}^{-3}$ is gradually filled in the vessel at a constant rate $Q = 50\ \text{cm}^3\text{s}^{-1}$. Based on the information given, answer the following questions.

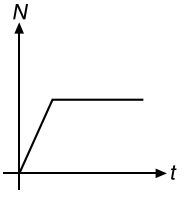


4. Graph between the normal reaction acting on cube by the vessel versus time is

(A) 

(B) 

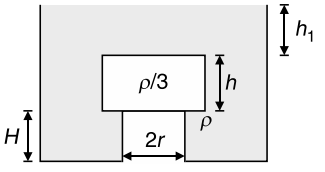
(C) 

(D) 

5. The cube will leave contact with the vessel after
(A) 20 s (B) 30 s
(C) 40 s (D) 60 s

Comprehension 3

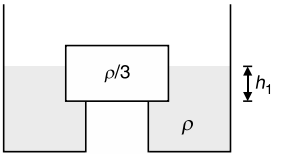
A wooden cylinder of diameter $4r$, height h and density $\frac{\rho}{3}$ is kept on a hole of diameter $2r$ of a tank, filled with water of density ρ as shown in Figure.



The height of the base of the cylinder from the base of the tank is H. Based on the above facts, answer the following questions.

6. If level of the liquid starts decreasing slowly, when the level of liquid is at a height h_1 above the cylinder the block starts moving up. Then, the value of h_1 is
(A) $\frac{4h}{9}$ (B) $\frac{5h}{9}$
(C) $\frac{5h}{3}$ (D) remains same

7. Let the cylinder be prevented from moving up, by applying a force and water level is further decreased. Then height of water level $(h_2$ in figure$)$ for which the cylinder remains in original position without application of force is

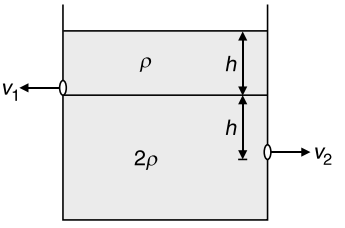


(A) $\frac{4h}{9}$ (B) $\frac{5h}{9}$
(C) remains same (D) $\frac{2h}{3}$

8. If height h_2 of water level is further decreased, then
(A) cylinder will not move up and remains at its original position
(B) for $h_2 = \frac{h}{3}$, cylinder again starts moving up
(C) for $h_2 = \frac{h}{4}$, cylinder again starts moving up
(D) for $h_2 = \frac{h}{5}$ cylinder again starts moving up

Comprehension 4

A liquid of density ρ and height h is filled over another liquid of density 2ρ. Two holes are made at depths h and $2h$ from the top free surface. Speeds of liquids ejecting from two holes are say v_1 and v_2. Based on the information given, answer the following questions.

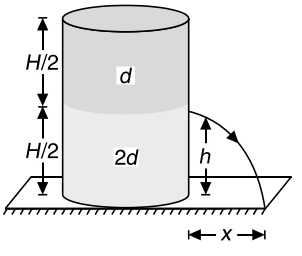


9. The ratio $\frac{v_1}{v_2}$ will be
(A) $\sqrt{\frac{1}{3}}$ (B) $\sqrt{\frac{1}{2}}$
(C) $\sqrt{\frac{2}{3}}$ (D) $\sqrt{\frac{4}{3}}$

10. Assume that the density of lower liquid to be increased, then
(A) v_1 will decrease, v_2 will increase.
(B) v_1 remains same, v_2 will decrease.
(C) v_1 remains same, v_2 will increase.
(D) both v_1 and v_2 decrease.

Comprehension 5

A container of large uniform cross-sectional area A resting on a horizontal surface, holds two immiscible, non-viscous and incompressible liquids of densities d and $2d$, each of height $\frac{H}{2}$ as shown in figure. The lower density liquid is open to the atmosphere having pressure P_0.



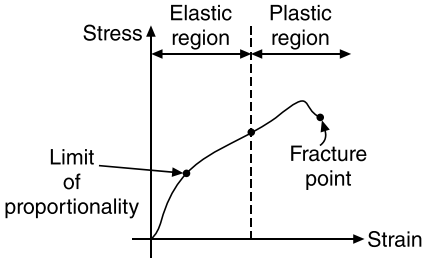
A homogeneous solid cylinder of density D, length $L\left(L < \frac{H}{2}\right)$, cross-sectional area $\frac{A}{5}$ is immersed such that if floats with its axis vertical at the liquid-liquid interface with length $\frac{L}{4}$ in the denser liquid. The total pressure at the bottom of the container is P.
Based on the above facts, answer the following questions.

11. The density of the solid cylinder is
(A) $D = \frac{5}{2}d$ (B) $D = \frac{7}{20}d$
(C) $D = \frac{5}{4}d$ (D) $D = \frac{7}{10}d$

12. The value of P in terms of the known parameters is
(A) $P = P_0 + \frac{gd}{2}(L + 3H)$ (B) $P = P_0 + \frac{gd}{4}(L + 6H)$
(C) $P = P_0 + gd(L + 6H)$ (D) $P = P_0 + \frac{gd}{2}(L + 6H)$

Comprehension 6

The stress-strain relationship for a metal wire subjected to the gradual loading process is shown in Figure.



Within elastic limit, Hooke's Law is obeyed, i.e. ratio of stress to strain is constant and that constant is called the modulus of elasticity. Based on the information given, answer the following questions.

13. Two wires of same material have length and radius (L, r) and $\left(3L, \frac{r}{3}\right)$. The ratio of their Young's moduli is

(A) 1 : 2 (B) 2 : 1
(C) 1 : 1 (D) 2 : 3

14. Just when the yield region is crossed, the material will experience

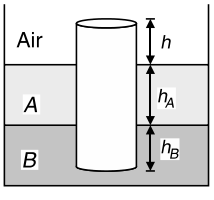
(A) constant stress.
(B) breaking stress that increases.
(C) breaking stress that decreases.
(D) None of these

15. Consider the ratio of stress to strain to be x in elastic region and y in the region of yield, then

(A) $x = y$ (B) $x = 2y$
(C) $x < y$ (D) $x > y$

Comprehension 7

A uniform solid cylinder of density $0.8\ \text{gcm}^{-3}$ floats in equilibrium in a combination of two non-mixing liquids A and B with its axis vertical. The densities of the liquids A and B are $0.7\ \text{gcm}^{-3}$ and $1.2\ \text{gcm}^{-3}$, respectively. The height of liquid A is $h_A = 1.2$ cm. The length of the part of the cylinder immersed in liquid B is $h_B = 0.8$ cm.



Based on the above facts, answer the following questions.

16. Net force applied by the liquid A on the cylinder is

(A) zero
(B) non-zero
(C) non-zero and very large
(D) cannot be determined

17. The height of the portion of cylinder in air is

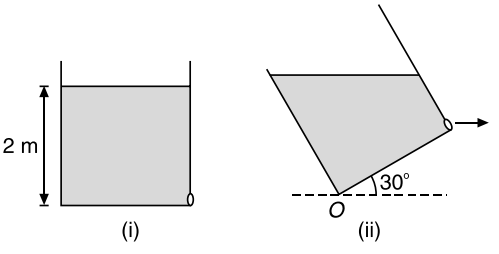
(A) $h = 1$ cm
(B) $h = 0.75$ cm
(C) $h = 0.25$ cm
(D) $h = 0$ cm

18. The cylinder is depressed in such a manner so that its top surface is just below the upper surface of the liquid A and is then released. The acceleration of the cylinder immediately after it is released is

(A) $\frac{g}{12}$ (B) $\frac{g}{6}$
(C) $\frac{g}{3}$ (D) $\frac{g}{2}$

Comprehension 8

Liquid is filled in a vessel having a square base $(2\ \text{m} \times 2\ \text{m})$ up to a height of 2 m. The vessel has an orifice that is sealed tightly such that no liquid could emerge from it as shown in Figure-(i). Now, the vessel is tilted such that it makes an angle of $30°$ with the horizontal and the water in the vessel is just not spilt over the edge of the mouth of vessel as shown in Figure-(ii). Based on the information given, answer the following questions.



19. The speed of efflux of the liquid coming out of the orifice, when the orifice in Figure-2 is opened is nearly

(A) $3\ \text{ms}^{-1}$ (B) $4\ \text{ms}^{-1}$
(C) $5\ \text{ms}^{-1}$ (D) $6\ \text{ms}^{-1}$

20. The time when liquid hits the ground is

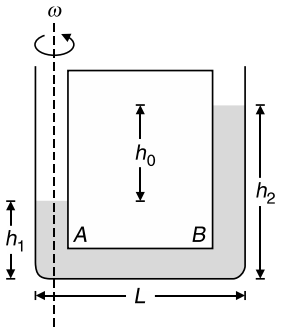
(A) $\frac{1}{\sqrt{2}}$ s (B) $\frac{1}{\sqrt{3}}$ s
(C) $\frac{1}{\sqrt{5}}$ s (D) $\sqrt{2}$ s

21. The distance from point O, where the liquid strikes the ground is

(A) $\sqrt{3}$ m (B) $\sqrt{5}$ m
(C) $(\sqrt{5}+\sqrt{3})$ m (D) $(\sqrt{5}-\sqrt{3})$ m

Comprehension 9

A U-tube of base length L contains a liquid of density ρ in it. The tube is rotated about one of its vertical limbs with angular velocity ω as shown. The diameter of the tube is negligibly small as compared to the length of the tube. Take P_0 as the atmospheric pressure.



Based on the above facts, answer the following questions.

22. What is the force at B due to rotation of the U-tube?

(A) $A\rho\omega^2$ (B) $A\rho\omega^2L^2$

(C) $\frac{A\rho\omega^2L^2}{2}$ (D) $\frac{A\rho\omega^2L^2}{4}$

23. What is the pressure at B from the left side?

(A) $P_0 + h_1\rho g$ (B) $P_0 + \frac{h_1\rho g}{2}$

(C) $2P_0 + h_1\rho g$ (D) $P_0 + h_1\rho g + \frac{\rho\omega^2L^2}{2}$

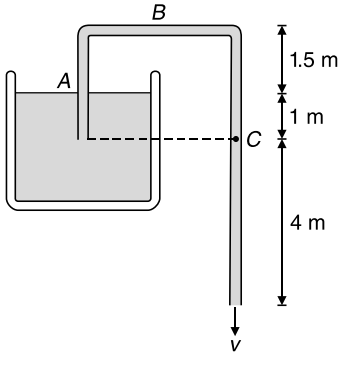
24. The value of h_0 is

(A) $\frac{\omega^2L}{2g}$ (B) $\frac{\omega^2L^2}{2g}$

(C) $\frac{\omega^2L^2}{3g}$ (D) $\frac{\omega^2L^2}{4g}$

Comprehension 10

In the arrangement shown, a siphon tube is discharging a liquid of density $900\ \text{kgm}^{-3}$. Assuming the atmospheric pressure to be $P_0 = 1.01\times10^5\ \text{Nm}^{-2}$, answer the following questions.



25. The speed of liquid through the siphon is

(A) $12\ \text{ms}^{-1}$ (B) $10\ \text{ms}^{-1}$

(C) $8\ \text{ms}^{-1}$ (D) $6\ \text{ms}^{-1}$

26. The pressure at point B is

(A) $2.50\times10^4\ \text{Nm}^{-2}$ (B) $2\times10^9\ \text{Nm}^{-2}$

(C) $4.25\times10^4\ \text{Nm}^{-2}$ (D) $6.25\times10^4\ \text{Nm}^{-2}$

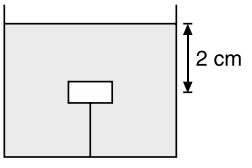
27. The pressure at point C is

(A) $10.5\times10^4\ \text{Nm}^{-2}$ (B) $8\times10^4\ \text{Nm}^{-2}$

(C) $6.5\times10^4\ \text{Nm}^{-2}$ (D) $5.5\times10^4\ \text{Nm}^{-2}$

Comprehension 11

A small block of weight W is kept inside a vessel filled with some liquid with the help of a string attached to the bottom of the vessel as shown in Figure. The tension in the string is $\frac{W}{2}$ and this entire situation is referred to as first case.



In the second case, if the weight of the block is doubled, then it is observed that tension in the string to keep the block in equilibrium becomes x times the tension in the first case. Also, when the string is cut, then time taken by the block to reach the liquid surface becomes y times the value calculated in the first case. Based on the information given, answer the following questions.

28. If the string is cut, then the time taken by the block to reach the surface of the liquid is

(A) $\frac{1}{\sqrt{5}}$ s (B) $\sqrt{5}$ s

(C) $\sqrt{3}$ s (D) $\frac{2}{\sqrt{5}}$ s

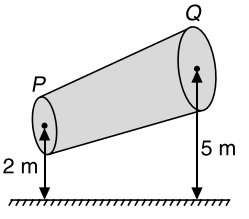
29. The ratio of x to y is

(A) 1 (B) 2

(C) 3 (D) 4

Comprehension 12

A non-viscous liquid of constant density $1000\ \text{kgm}^{-3}$ flows in streamline motion along a tube of variable cross-section. The tube is kept inclined in the vertical plane as shown in the figure. The area of cross-section of the tube at two points P and Q at heights of 2 m and 5 m are respectively $4\times10^{-3}\ \text{m}^2$ and $8\times10^{-3}\ \text{m}^2$. The velocity of the liquid at point P is $1\ \text{ms}^{-1}$.



Based on the above facts, answer the following questions.

30. The velocity of the liquid at the point Q is

(A) $0.5\ \text{ms}^{-1}$ (B) $1\ \text{ms}^{-1}$

(C) $2\ \text{ms}^{-1}$ (D) $4\ \text{ms}^{-1}$

31. Work done per unit volume by the pressure forces as the liquid goes from P to Q is

(A) $25290\ \text{Jm}^{-3}$ (B) $29250\ \text{Jm}^{-3}$

(C) $29025\ \text{Jm}^{-3}$ (D) $25029\ \text{Jm}^{-3}$

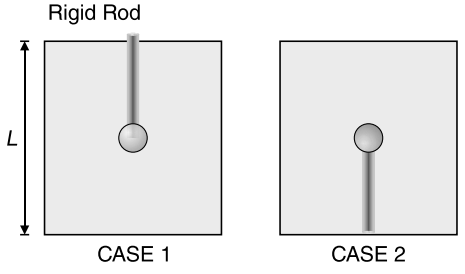
32. Work done per unit volume by the gravitational forces as the liquid flows from P to Q is

(A) 29004 Jm^{-3} (B) 29400 Jm^{-3}

(C) 24900 Jm^{-3} (D) 24009 Jm^{-3}

Comprehension 13

A cylindrical container of length L is full to the brim with a liquid which has mass density ρ. The cylinder is placed on a weighing scale and the scale reading is observed to be W. A light ball of mass m, volume V capable of floating on the liquid (when allowed to do so), is pushed gently down and halfway below the surface of the liquid with a rigid rod of negligible volume as shown in Case-1. If instead of being pushed down by a rod, the ball is held in place by a thin string attached to the bottom of the container as shown in Case-2, then answer the following questions.



33. Mass M of the liquid which overflows while the ball was being pushed into the liquid is

(A) ρV (B) m

(C) $m-\rho V$ (D) $m+\rho V$

34. Reading of the scale when the ball is fully immersed (in Case-1) is

(A) $W-V\rho g$ (B) W

(C) $W+mg-V\rho g$ (D) $W+mg$

35. If instead of being pushed down by a rod, the ball is held in place by a thin string attached to the bottom of the container as shown on the right. What is the tension T in the string?

(A) $(\rho V-m)g$ (B) ρVg

(C) mg (D) None of these

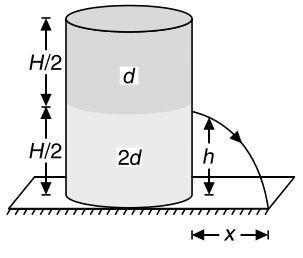
36. Reading of the scale in Case-2 is

(A) $W-V\rho g$ (B) W

(C) $W+mg-V\rho g$ (D) $W+mg$

Comprehension 14

A container of large uniform cross-sectional area A resting on a horizontal surface, holds two immiscible, non-viscous and incompressible liquids of densities d and $2d$, each of height $\frac{H}{2}$ as shown in Figure.



The lower density liquid is open to the atmosphere having pressure P_0. A tiny hole of area $s(s \ll A)$ is punched on the vertical side of the container at a height $h\left(h<\frac{H}{2}\right)$. Neglecting the air resistance in these calculations. Based on the above facts, answer the following questions.

37. The initial speed of efflux of the liquid at the hole is

(A) $v=\sqrt{\left(\frac{3H}{2}-2h\right)g}$ (B) $v=\sqrt{(3H-2h)g}$

(C) $v=\sqrt{\left(2H-\frac{3h}{2}\right)g}$ (D) $v=\sqrt{(2H-3h)g}$

38. The horizontal distance x travelled by the liquid initially is

(A) $x=\sqrt{h(4H-3h)}$

(B) $x=\sqrt{h(3H-4h)}$

(C) $x=\sqrt{3h(H-4h)}$

(D) $x=\sqrt{4h(H-3h)}$

39. The height h_m at which the hole should be punched so that the liquid travels the maximum distance (x_m) initially is

(A) $h_m=\frac{H}{4}$ (B) $h_m=\frac{3H}{8}$

(C) $h_m=\frac{H}{2}$ (D) $h_m=\frac{H}{8}$

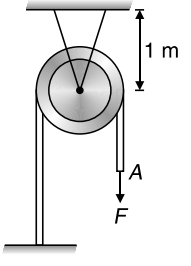
40. The corresponding value of x_m for the condition achieved is

(A) $x_m=\frac{H}{4}$ (B) $x_m=\frac{H}{2}$

(C) $x_m=\frac{3H}{4}$ (D) $x_m=H$

Comprehension 15

The axle of a pulley of mass 1 kg is attached to the end of an elastic string of length 1 m, cross-sectional area 10^{-3} m^2 and Young's modulus 2×10^5 Nm^{-2}, whose other end is fixed to the ceiling. A rope of negligible mass is placed on the pulley such that its left end is fixed to the ground and its right end is hanging freely, from the pulley which is at rest in equilibrium. Now, the free end A of the rope is subjected to pulling with constant force $F=10$ N. Friction between the rope and the pulley can be neglected. (Given, $g=10$ ms^{-2})



41. The elongation of the string before applying force is

(A) 50 cm (B) 5 cm

(C) 0.5 cm (D) 0.05 cm

42. The greatest elongation of the string is
(A) 35 cm (B) 30 cm
(C) 25 cm (D) 20 cm

43. The maximum displacement of point A after applying F is
(A) 70 cm (B) 60 cm
(C) 40 cm (D) 50 cm

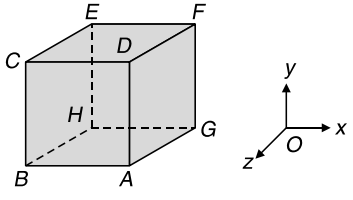
MATRIX MATCH/COLUMN MATCH TYPE QUESTIONS

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

If the correct matches are A → p, s and t; B → q and r; C → p and q; and D → s and t; then the correct darkening of bubbles will look like the following:

	p	q	r	s	t
A	●	●	○	●	●
B	●	●	●	○	○
C	●	●	○	○	○
D	○	○	○	●	●

1. The cubical container completely filled with water is given an acceleration $\vec{a} = a_0\hat{i} + a_0\hat{j} + a_0\hat{k}$ in a gravity free space. Compare the situations in **COLUMN-I** to the respective conclusions in **COLUMN-II**.



COLUMN-I	COLUMN-II
(A) Pressure at point A is	(p) less than pressure at point G
(B) Pressure at point D is	(q) less than pressure at point F
(C) Pressure at point E is	(r) greater than pressure at point C
(D) Pressure at point H is	(s) greater than pressure at point B
	(t) equal to pressure at point F

2. An ideal liquid is flowing through a pipe of non-uniform cross-section. Match the quantities in **COLUMN-I** to their respective match in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Rate of flow of liquid	(p) more
(B) Speed of liquid at the narrow cross-section of pipe	(q) less
(C) Speed of liquid at the broader cross-section of pipe	(r) positive
(D) ressure difference between the narrow and broad cross-section of the pipe	(s) negative
	(t) constant

(*Continued*)

3. Match the relations and laws related to fluids given in **COLUMN-I**, to the physical reasons behind them given in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Stoke's law	(p) behaviour of free surface of liquid like a stretched elastic membrane.
(B) Equation of continuity	(q) force of viscosity on a spherical body.
(C) Bernoulli's theorem	(r) conservation of mass
(D) Surface tension	(s) conservation of energy

4. Match the states of matter in **COLUMN-I** with the respective characteristics in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Steel	(p) Young's modulus of elasticity
(B) Water	(q) Bulk modulus of elasticity
(C) Hydrogen gas filled in a chamber	(r) Shear modulus of elasticity
(D) Oil	(s) Surface Tension

5. Consider a wire having length L, cross-sectional area A and Young's modulus Y. Match the details in **COLUMN-I** with the respective information given in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Let us suspend the wire vertically from a rigid supported and attach a mass m at its lower end. If the mass is slightly pulled down and released, it executes S.H.M. The time period will depend on	(p) Young's Modulus (Y)
(B) If the given wire is fixed between two rigid supports and its temperature is increased thermal stress that develops in the rod will depend on	(q) Extension (x)
(C) Work done in stretching the wire to a length $L + x$ depends on	(r) Length (L)
(D) If the wire is pulled at its ends by equal and opposite forces of magnitude F so that it undergoes an elongation x, according to Hooke's law, $F = kx$, where k is the force constant. Force constant (k)of the wire will depend on	(s) Area of cross-section (A)
	(t) Independent of elongation (x)

6. **COLUMN-I** contains different arrangements and **COLUMN-II** contains capillary rise of water in those arrangements. If the surface tension of water is T and density is ρ, match the following.

COLUMN-I	COLUMN-II
(A) Silver tube of radius r immersed in water	(p) $\frac{2T}{r\rho g}$
(B) Glass tube of radius r immersed in water	(q) $\frac{4T}{r\rho g}$
(C) Hollow co-axial cylinder made of glass, having inner and outer radius r and $2r$ respectively immersed in water	(r) $\frac{3T}{2r\rho g}$
(D) Two parallel glass plate separated by distance r immersed in water	(s) ZERO

7. Match the relations in **COLUMN-I** to their respective SI units given in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) $\frac{(\text{Stress})^2}{Y}$	(p) J
(B) $\frac{YA}{l}$	(q) Nm^{-1}
(C) Yl^3	(r) Jm^{-3}
(D) $\frac{Fl}{AY}$	(s) m

8. Match the phenomenon and equations in **COLUMN-I** with their respective match in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Excess pressure inside a drop of radius R	(p) decreases with increase in temperature
(B) Excess pressure inside a soap bubble of radius R	(q) $\frac{2T}{R}$
(C) Excess pressure inside an air cavity of radius R	(r) $\frac{4T}{R}$
(D) Surface tension	(s) decreases with increase in radius

9. Match the physical quantities in **COLUMN-I** with the respective dimensional formulae in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Pressure per unit speed gradient Coefficient of viscosity	(p) $ML^{-1}T^{-2}$
(B) Surface tension	(q) $M^0L^0T^0$
(C) Modulus of elasticity	(r) $ML^{-1}T^{-1}$
(D) Energy per unit volume of a fluid	(s) MLT^{-2}
(E) Reynold's Number	(t) MT^{-2}

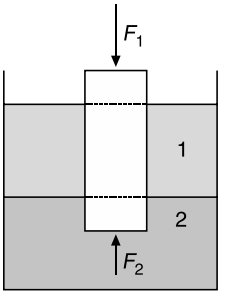
10. Match the contents of **COLUMN-I** to the contents given in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Equation of continuity	(p) Non-viscous flow
(B) Bernoulli's equation	(q) Steady flow

(*Continued*)

COLUMN-I	COLUMN-II
(C) Velocity at a given point is constant with	(r) Incompressible liquid
(D) Ideal flow	(s) Irrotational flow

11. A cylinder of weight W is floating in two liquids. A force F_1 acts at the top of cylinder and a force F_2 acts at the bottom of the cylinder as shown in Figure. Match the following two columns.

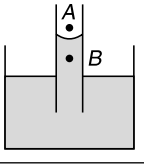


COLUMN-I	COLUMN-II
(A) Net force on cylinder from liquid-1	(p) Zero
(B) $F_2 - F_1$	(q) W
(C) Net force on cylinder from liquid-2	(r) Net upthrust
(D) Net force on cylinder from atmosphere	(s) None of these

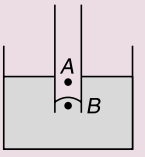
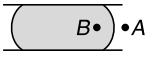
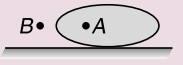
12. For an incompressible liquid match the contents of **COLUMN-I** with their respective equations in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Equation of continuity	(p) $\Delta P = h\rho g$
(B) Bernoulli's equation	(q) $v = \sqrt{2gh}$
(C) Torricelli's theorem	(r) $A_1v_1 = A_2v_2$
(D) Hydrostatic pressure	(s) $P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$

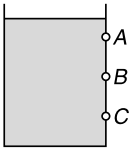
13. Match the arguments in **COLUMN-I** to the capillary rise and shape of droplets on a plate due to surface tension shown in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Adhesive force is greater than cohesive force	(p) 

(*Continued*)

COLUMN-I	COLUMN-II
(B) Cohesive force is greater than adhesive force	(q) 
(C) Pressure at A greater pressure at B	(r) A mercury drop is pressed between two parallel plates of glass 
(D) Pressure at B greater pressure at A	(s) 

14. Three holes A, B and C are made at depths 1 m, 2 m and 5 m as shown. Total height of liquid in the container is 9 m. Let v is the speed with which liquid comes out of the hole and R the range on ground. Match the following two columns.

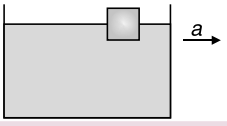
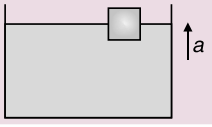
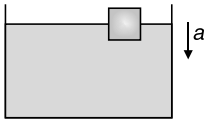
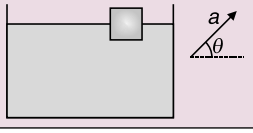


COLUMN-I	COLUMN-II
(A) v is maximum for	(p) hole A
(B) v is minimum for	(q) hole B
(C) R is maximum for	(r) hole C
(D) R is minimum for	(s) will depend on density of liquid

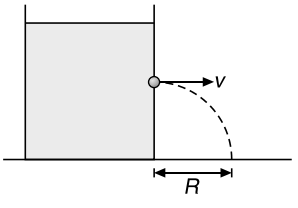
15. Match the contents of **COLUMN-I** to the respective contents in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Mercury Barometer	(p) Measures difference of absolute pressure and atmospheric pressure
(B) Pascal's law	(q) Measures absolute pressure
(C) Pressure gauge	(r) Measures atmospheric pressure
(D) Manometer	(s) The change in pressure is transferred to the entire liquid without being diminished in magnitude

16. A block of wood is floating on the surface of water inside a container at rest. Now the container is given some acceleration as shown in **COLUMN-I**. Some observation are shown in **COLUMN-II**. Match the columns.

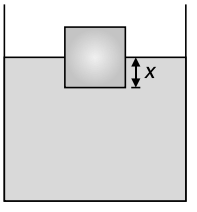
COLUMN-I	COLUMN-II
(A) 	(p) Buoyant force on the block increases
(B) 	(q) Buoyant force on the block decreases
(C) 	(r) Pressure at the bottom of the container is non-uniform
(D) 	(s) Pressure at the bottom of the container is uniform

17. In the arrangement shown, the speed of liquid coming out from the orifice is v, time taken by the liquid to hit the ground is t and distance at which the liquid hits the ground is R. If the vessel is taken to a mountain, match the following, (consider all cases which might possible)



COLUMN-I	COLUMN-II
(A) v	(p) will increase
(B) t	(q) will decrease
(C) R	(r) will remain same
(D) R_{max}	(s) when the orifice is at the middle of the beaker
	(t) when the orifice is at the middle of the liquid level that fills the beaker.

18. A uniform solid cube is floating in a liquid as shown in Figure, with part x inside the liquid. Some changes in parameters are mentioned in **COLUMN-I**. Assuming no other changes, match the following

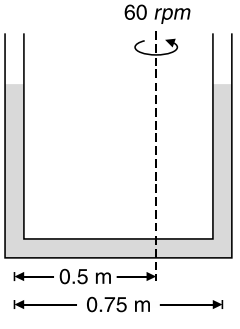


COLUMN-I	COLUMN-II
(A) If density of the liquid is increased, x will	(p) Increase
(B) If height of the cube is increased keeping base area and density same, x will	(q) Decreases
(C) if the whole system is accelerated upward, then x will	(r) Remain same
(D) If the cube is replaced by another cube of same size but lesser density, x will	(s) May increase or decrease

INTEGER/NUMERICAL ANSWER TYPE QUESTIONS

In this section, the answer to each question is a numerical value obtained after doing series of calculations based on the data given in the question(s).

1. A vertical U-tube with the two limbs 0.74 m apart is filled with water and rotated about a vertical axis 0.5 m from the left limb, as shown in Figure. Determine the difference in elevation of the water levels in the two limbs in centimetre when the speed of rotation is 60 rpm.

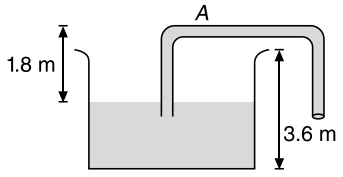


2. The pressure increases by 1.0×10^{4} Nm^{-2} for every meter of depth beneath the surface of the ocean. At what depth (in metre) does the volume of a Pyrex glass cube, 1.0×10^{-2} m on an edge at the ocean's surface, decrease by 1.0×10^{-10} m^{3}? Assume the Bulk's modulus to be 2.6×10^{10} Nm^{-2}.

3. Calculate the increment in the length (in micrometre) of a steel wire of length 5 m and radius 6 mm under its own weight. Assume density of steel to be 8000 kgm^{-3}, $g = 9.8$ ms^{-2} and Young's modulus of steel to be 2×10^{11} Nm^{-2}. Also calculate energy stored in the wire in microjoule.

4. When the load on a wire is increased slowly from 3 kgwt to 5 kgwt, the elongation increases from 0.61 mm to 1.02 mm. Calculate the work done (in millijoule) during the extension of the wire.

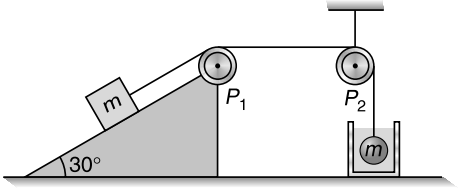
5. A siphon has a uniform circular base of diameter $\frac{8}{\sqrt{\pi}}$ cm with its crest A lying at 1.8 m above the water level as shown in Figure.



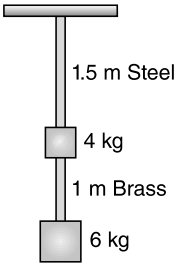
Calculate the velocity of flow in ms^{-1}, discharge rate of the flow in cm^3s^{-1} and the absolute pressure at the crest level A in kNm^{-2} if atmospheric pressure is $100\ kNm^{-2}$ and $g = 10\ ms^{-2}$.

6. A glass tube of 1 mm bore is dipped vertically into a container of mercury, with its lower end 2 cm below the mercury surface. The gauge pressure of air in the tube required to blow a hemispherical bubble at its lower end is x mm of Hg. Assuming that the density of mercury is $13600\ kgm^{-3}$ and its surface tension is $0.465\ Nm^{-1}$, calculate x.

7. A block of mass m is kept over a fixed smooth wedge. Block is attached to a sphere of same mass through fixed massless pullies P_1 and P_2. Sphere is dipped inside the water as shown. If specific gravity of material of sphere is 2. Calculate the acceleration of sphere.



8. Two wires of diameter 0.25 cm, one made of steel and the other made of brass are loaded as shown in Figure.



The unloaded length of steel wire is 1.5 m and that of brass wire is 1.0 m. Calculate the elongations of the steel and the brass wires in mm. (Young's modulus for brass is $9\times10^{10}\ Nm^{-2}$ and Young's modulus for steel is $2\times10^{11}\ Nm^{-2}$)

9. Air is streaming past a horizontal aeroplane wing such that its speed is $120\ ms^{-1}$ over the upper surface and $90\ ms^{-1}$ at the lower surface. If the density of air is $1.3\ kgm^{-3}$, the pressure difference between the top and bottom of the wing is x kPa. Calculate x. If the wing is 10 m long and has an average width 2 m, calculate the lift force acting on the wing in kN.

10. Between each pair of vertebrae in the spinal column is a cylindrical disc of cartilage. Typically, this disc has a radius of about 3.0×10^{-2} m and a thickness of about 7.0×10^{-3} m. The shear modulus of cartilage is $1.2\times10^{7}\ Nm^{-2}$. Suppose that a shearing force of magnitude 11 N is applied parallel to the top surface of the disc while the bottom surface remains fixed in place. How far does the top surface move (in micrometre) relative of the bottom surface?

11. A cylindrical steel wire of 3 m length is to stretch no more than 0.2 cm when a tensile force of 400 N is applied to each end of the wire. Calculate the minimum diameter of the wire (in mm) required for the purpose. Given $Y_{steel} = 2.1\times10^{11}\ Nm^{-2}$.

12. Calculate the density of water, in kgm^{-3}, at a depth where the pressure is 501 atm. Given that the density of water at the surface is $1.0\times10^{3}\ kgm^{-3}$, Bulk modulus of water is 2.0×10^{9} Pa and the atmospheric pressure is $1\ atm = 1.01\times10^{5}$ Pa.

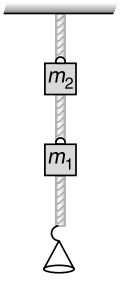
13. A cylindrical vessel filled with water up to a height of 2 m stands on a horizontal plane. The side wall of the vessel has a plugged circular hole touching the bottom. Find the minimum diameter of the hole (in centimetre) so that the vessel begins to move on the floor when the plug is removed. The coefficient of friction between the bottom of the vessel and the plane is 0.4 and total mass of water plus vessel is 100 kg.

14. A wire having a length L and cross-sectional area A is suspended at one of its ends from a ceiling. Density and Young's modulus of material of the wire are ρ and Y, respectively. Its strain energy due to its own weight is $\frac{\rho^2 g^2 AL^3}{pY}$. Calculate the value of p.

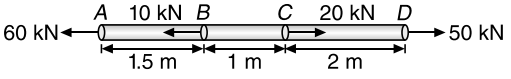
15. A rod 100 cm long and of $2\ cm\times2\ cm$ cross-section is subjected to a pull of 1000 kg force. If the modulus of elasticity of the material is $2.0\times10^{6}\ kgcm^{-2}$, determine the elongation of the rod in mm.

16. Spherical particles of pollen are shaken up in water and allowed to settle. The depth of water is 2×10^{-2} m. Calculate the diameter of the largest particle, in μm remaining in suspension one hour later. Density of pollen $= 1.8\times10^{3}\ kgm^{-3}$, viscosity of water is 1×10^{-2} poise and density of water is $1\times10^{-5}\ kgm^{-3}$.

17. Two wires shown in figure are made of the same material which has a breaking stress of $8\times10^{8}\ Nm^{-2}$. The area of cross-section of the upper wire is $0.006\ cm^2$ and that of the lower wire is $0.003\ cm^2$. The mass $m_1 = 10$ kg, $m_2 = 20$ kg and the hanger is light. Calculate the maximum load (in kg) that can be put on the hanger without breaking a wire. Which wire will break first if the load is increased? (Take $g = 10\ ms^{-2}$).



18. A steel rod of cross-sectional area $1\ m^2$ is acted upon by forces shown in Figure. Determine the total elongation of the bar in micrometre. Take $Y = 2.0 \times 10^{11}\ Nm^{-2}$.



19. If the elastic limit of copper is $1.5 \times 10^8\ Nm^{-2}$, determine the minimum diameter a copper wire (in mm) can have under a load of $10.0\ kg$, if its elastic limit is not to be exceeded.

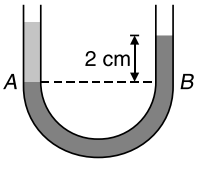
20. Water is flowing through a horizontal tube of non-uniform cross section. At a place the radius of the tube is 1.0 cm and velocity of water is $2.0\ ms^{-1}$. What will be the velocity of water, in cms^{-1}, where the radius of the pipe is 2.0 cm?

21. A water pipe with an internal diameter of $2.5\ cm$ carries water at ground floor of a house with velocity $3\ ms^{-1}$ and at pressure $317.6\ kPa$. Another pipe of internal diameter $1.25\ cm$ is connected to it and takes water to the first floor $25\ m$ above ground. Calculate the velocity in ms^{-1} and water pressure at first floor in Nm^{-2}.

22. The elastic limit and ultimate strength for steel is $2.48 \times 10^8\ Pa$ and $4.89 \times 10^8\ Pa$ respectively. A steel wire of length $10\ m$ and cross-sectional diameter $2\ mm$ is subjected to longitudinal tensile stress. If Young's modulus of steel is $Y = 2 \times 10^{11}\ Pa$, then calculate the maximum elongation (in cm) that can be produced in the wire without permanently deforming it and the force (in newton) required to produce this extension. Also calculate the maximum stretching force (in kilonewton) that can be applied without breaking the wire.

23. Calculate the approximate change in density, in kgm^{-3}, of water in a lake at a depth of 400 m below the surface. The density of water at the surface is $1030\ kgm^{-3}$ and bulk modulus of water is $2 \times 10^9\ Nm^{-2}$.

24. The liquids shown in figure in the two limbs of the tube are mercury (specific gravity $= 13.6$) and water.



If difference of heights of the mercury column in the two limbs of the tube is $2\ cm$, calculate the height h of the water column in cm.

25. A cube of mass $m = 800\ g$ floats on the surface of water such that water wets it completely. The cube is $10\ cm$ on each edge. Calculate the additional distance in μm through which it buoyed up or down due to the surface tension of water. Assume that the surface tension of water is $0.07\ Nm^{-1}$ and $g = 10\ ms^{-2}$

26. A 0.1 kg mass is suspended from a wire of negligible mass. The length of the wire is $1\ m$ and its cross-sectional area is $4.9 \times 10^{-7}\ m^2$. If the mass is pulled a little in the vertically downward direction and released, it performs SHM with angular frequency $140\ rads^{-1}$. If the Young's modulus of the material of the wire is $x \times 10^9\ Nm^{-2}$, find the value of x.

27. Two stretched cables both experience the same stress. The first cable has a radius of $3.5 \times 10^{-3}\ m$ and is subject to a stretching force of $270\ N$. The radius of the second cable is $5.1 \times 10^{-3}\ m$. Determine the stretching force (in newton) acting on the second cable.

28. A metallic sphere of radius $1\ mm$ and density $10^4\ kgm^{-3}$ enters a tank of water, after a free fall through distance of h in the earth's gravitational field. If its velocity remains unchanged after entering water, calculate the value of h in metre, if coefficient of viscosity of water is $1.0 \times 10^{-3}\ Ns^{-1}m^2$, $g = 10\ ms^{-2}$ and density of water is $10^3\ kgm^{-3}$.

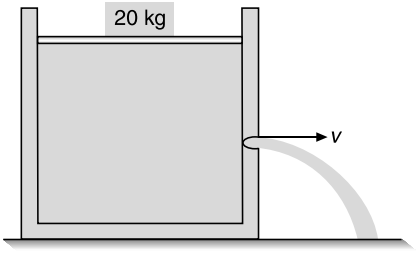
29. A cast iron column has internal diameter of 200 mm. Calculate the minimum external diameter of the column (in mm) so that it may carry a load of $1.6\ MN$ without the stress exceeding $90\ Nmm^{-2}$.

30. A river of width $12\ m$ flowing at $20\ kmh^{-1}$ mixes with another river of width $8\ m$ flowing at $16\ kmh^{-1}$ and spreads in another stream of width $16\ m$. Calculate the flow velocity in kmh^{-1} of water after mixing, assuming the depth of river are same.

31. A sphere of radius $10\ cm$ and mass $25\ kg$ is attached to the lower end of a steel wire of length $5\ m$ and diameter $4\ mm$ which is suspended from the ceiling of a room. The point of support is $521\ cm$ above the floor. When the sphere is set swinging as a simple pendulum, its lowest point just grazes the floor. Calculate the velocity of the ball, rounded off to the nearest integer in ms^{-1}, at its lowest position $\left(Y_{steel} = 2 \times 10^{11}\ Nm^{-2}\right)$.

32. The limiting stress of a typical human bone is $0.9 \times 10^8\ Nm^{-2}$ while Young's modulus is $1.4 \times 10^{10}\ Nm^{-2}$. Calculate the energy (in joule) that can be absorbed by two leg bones (without breaking) if each has a typical length of $50\ cm$ and an average cross-sectional area of $5\ cm^2$.

33. A long cylindrical tank of cross-sectional area $0.5\ m^2$ is filled with water. It has a small hole at a height $50\ cm$ from the bottom. A movable piston of cross-sectional area almost equal to $0.5\ m^2$ is fitted on the top of the tank such that it can slide in the tank freely. A load of $20\ kg$ is applied on the top of the water by piston, as shown in Figure. Calculate the speed of the water jet (in ms^{-1}) with which it hits the surface when piston is $1\ m$ above the bottom. (Ignore the mass of the piston).



34. In a horizontal pipe line of uniform area of cross section, the pressure falls by $8\ \text{Nm}^{-2}$ between two points separated by a distance of $1\ \text{km}$. Calculate the change in kinetic energy per unit mass (in millijoule per kg) of the oil flowing at these points if density of oil is $800\ \text{kgm}^{-3}$.

35. A $6\ \text{kg}$ weight is fastened to the end of a steel wire of unstretched length $60\ \text{cm}$. It is whirled in a vertical circle and has an angular velocity of $2\ \text{revs}^{-1}$ at the bottom of the circle. The area of cross-section of the wire is $0.05\ \text{cm}^2$. Calculate the elongation of the wire (in mm) when the weight is at the lowest point of the path. Young's modulus of steel $= 2\times10^{11}\ \text{Nm}^{-2}$.

36. A mild steel wire of length $1.0\ \text{m}$ and cross-sectional area $0.50\times10^{-2}\ \text{cm}^2$ is stretched well within its elastic limit horizontally between two pillars. A mass of $100\ \text{g}$ is suspended from the midpoint of the wire. Calculate the depression at the midpoint in metre. $(Y_{\text{steel}} = 200\ \text{GPa})$

37. The elastic limit of a steel cable is $3\times10^{8}\ \text{Nm}^{-2}$ and the cross-section area is $4\ \text{cm}^2$. Calculate the maximum upward acceleration, in ms^{-2}, that can be given to a $900\ \text{kg}$ elevator supported by the cable if the stress is not to exceed one third of the elastic limit.

38. In an experimental model of the Venturimeter, the diameter of the pipe is $4\ \text{cm}$ and that of constriction is $3\ \text{cm}$. With water filling the pipe and flowing at a certain rate, the height of the liquids in the pressure tube is $20\ \text{cm}$ at the pipe and $15\ \text{cm}$ at the constrictions. Calculate the discharge rate of the liquid, in cm^3s^{-1} rounded off to the nearest integer.

39. A wire of length $3\ \text{m}$, diameter $0.4\ \text{mm}$ and Young's modulus $8\times10^{10}\ \text{Nm}^{-2}$ is suspended from a point and supports a heavy cylinder of volume $10^{-3}\ \text{m}^3$ at its lower end. Calculate the decrease in length (in mm) when the metal cylinder is immersed in a liquid of density $800\ \text{kgm}^{-3}$.

40. In a hospital, a patient receives a $500\ \text{cm}^3$ blood transfusion through a needle with a length of $5\ \text{cm}$ and inner radius of $0.03\ \text{cm}$. If the blood bag is kept $85\ \text{cm}$ above the needle, calculate the time in minute for which the transfusion takes place. Given that the viscosity of blood is 0.017 poise, density of blood is $1.02\ \text{gcm}^{-3}$ and $g = 9.8\ \text{ms}^{-2}$.

41. A uniform ring of radius R and made up of a wire of cross-sectional radius r is rotated about its axis with a frequency f. If density of the wire is ρ and Young's modulus is Y. The fractional change in radius of the ring is $a\left(\dfrac{f^2R^2\rho}{Y}\right)$. Take $\pi^2 \approx 10$.

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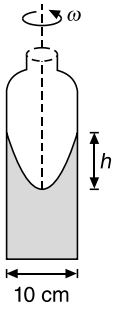
1. **[Online September 2020]**
A cylindrical vessel containing a liquid is rotated about its axis so that the liquid rises at its sides as shown in Figure. The radius of vessel is $5\ \text{cm}$ and the angular speed of rotation is $\omega\ \text{rads}^{-1}$. The difference in the height, h (in cm) of liquid at the center of vessel and at the side will be



(A) $\dfrac{25\omega^2}{2g}$ (B) $\dfrac{2\omega^2}{5g}$

(C) $\dfrac{5\omega^2}{2g}$ (D) $\dfrac{2\omega^2}{25g}$

2. **[Online September 2020]**
A capillary tube made of glass of radius $0.15\ \text{mm}$ is dipped vertically in a beaker filed with methylene iodide (surface tension $= 0.05\ \text{Nm}^{-1}$, density $= 667\ \text{kgm}^{-3}$) which rises to height h in the tube. It is observed that the two tangents drawn from liquid-glass interfaces (from opp. Sides of the capillary) make an angle of $60°$ with one another. Then h is close to $(g = 10\ \text{ms}^{-2})$

(A) $0.137\ \text{m}$ (B) $0.172\ \text{m}$
(C) $0.087\ \text{m}$ (D) $0.049\ \text{m}$

3. **[Online September 2020]**
Pressure inside two soap bubbles are 1.01 and 1.02 atmosphere, respectively. The ratio of their volumes is

(A) $8:1$ (B) $0.8:1$
(C) $2:1$ (D) $4:1$

4. **[Online September 2020]**
A air bubble of radius $1\ \text{cm}$ in water has an upward acceleration $9.8\ \text{cms}^{-2}$. The density of water is $1\ \text{gcm}^{-3}$ and water offers negligible drag force on the bubble. The mass of the bubble is $(g = 980\ \text{cms}^{-2})$

(A) $3.15\ \text{g}$ (B) $4.51\ \text{g}$
(C) $4.15\ \text{g}$ (D) $1.52\ \text{g}$

5. [Online September 2020]
A cube of metal is subjected to a hydrostatic pressure of 4 GPa. The percentage change in the length of the side of the cube is close to (Given bulk modulus of metal, $B = 8\times10^{10}$ Pa)
(A) 0.6 (B) 1.67
(C) 5 (D) 20

6. [Online September 2020]
A hollow spherical shell at outer radius R floats just submerged under the water surface. The inner radius of the shell is r. If the specific gravity of the shell material is $\frac{27}{8}$ w.r.t. water, the value of r is
(A) $\frac{4}{9}R$ (B) $\frac{8}{9}R$
(C) $\frac{1}{3}R$ (D) $\frac{2}{3}R$

7. [Online September 2020]
In an experiment to verify stokes law, a small spherical ball of radius r and density ρ falls under gravity through a distance h in air before entering a tank of water. If the terminal velocity of the ball inside water is same as its velocity just before entering the water surface, then the value of h is proportional to (ignore viscosity of air)
(A) r (B) r^4
(C) r^3 (D) r^2

8. [Online September 2020]
A fluid is flowing through a horizontal pipe of varying cross-section, with speed v ms^{-1} at a point where the pressure is P Pascal. P at another point where pressure is $\frac{P}{2}$ Pascal its speed is V ms^{-1}. If the density of the fluid is ρ kgm^{-3} and the flow is streamline, then V is equal to
(A) $\sqrt{\frac{P}{2\rho}+v^2}$ (B) $\sqrt{\frac{P}{\rho}+v^2}$
(C) $\sqrt{\frac{2P}{\rho}+v^2}$ (D) $\sqrt{\frac{P}{\rho}+v}$

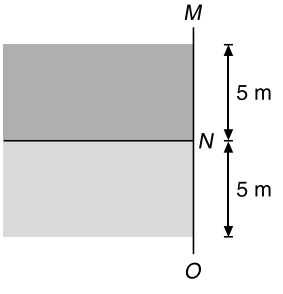
9. [Online September 2020]
When a long glass capillary tube of radius 0.015 cm is dipped in a liquid, the liquid rises to a height of 15 cm within it. If the contact angle between the liquid and glass to close to 0°, the surface tension of the liquid, in millinewton per metre, is $\left[\rho_{(\text{liquid})} = 900 \text{ kgm}^{-3},\ g = 10 \text{ ms}^{-2}\right]$ (Give answer in closest integer) ……

10. [Online January 2020]
An ideal fluid flows (laminar flow) through a pipe of non-uniform diameter. The maximum and minimum diameters of the pipes are 6.4 cm and 4.8 cm, respectively. The ratio of the minimum and the maximum velocities of fluid in this pipe is
(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{3}{4}$
(C) $\frac{81}{256}$ (D) $\frac{9}{16}$

11. [Online January 2020]
Consider a solid sphere of radius R and mass density $\rho(r) = \rho_0\left(1-\frac{r^2}{R^2}\right)$, $0 < r \le R$. The minimum density of a liquid in which it will float is
(A) $\frac{\rho_0}{5}$ (B) $\frac{\rho_0}{3}$
(C) $\frac{2\rho_0}{3}$ (D) $\frac{2\rho_0}{5}$

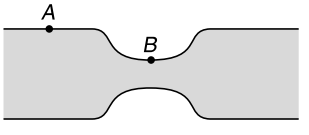
12. [Online January 2020]
A leak proof cylinder of length 1 m, made of a metal which has very low coefficient of expansion is floating vertically in water at 0 °C such that its height above the water surface is 20 cm. When the temperature of water is increased to 4 °C, the height of the cylinder above the water surface becomes 21 cm. The density of water at $T = 4$ °C, relative to the density at $T = 0$ °C is close to
(A) 1.01 (B) 1.04
(C) 1.03 (D) 1.26

13. [Online January 2020]
Two liquids of densities ρ_1 an $\rho_2(\rho_2 = 2\rho_1)$ are filled up behind a square wall of side 10 m as shown in Figure. Each liquid has a height of 5 m. The ratio of the forces due to these liquids exerted on upper part MN to that at the lower part NO is (Assume that the liquids are not mixing)



(A) $\frac{1}{4}$ (B) $\frac{2}{3}$
(C) $\frac{1}{3}$ (D) $\frac{1}{2}$

14. [Online January 2020]
Water flows in a horizontal tube (see figure). The pressure of water changes by 700 Nm^{-2} between A and B where the area of cross section are 40 cm^2 and 20 cm^2, respectively. Find the rate of flow of water through the tube. (density of water = 1000 kgm^{-3})



(A) 1810 cm^{3}s^{-1}
(B) 3020 cm^{3}s^{-1}
(C) 2720 cm^{3}s^{-1}
(D) 2420 cm^{3}s^{-1}

15. [Online January 2020]
A small spherical droplet of density d is floating exactly half immersed in a liquid of density ρ and surface tension T. The radius of the droplet is (take not that the surface tension applies an upward force on the droplet)

(A) $r=\sqrt{\frac{2T}{3(d+\rho)g}}$ (B) $r=\sqrt{\frac{3T}{(2d-\rho)g}}$

(C) $r=\sqrt{\frac{T}{(d-\rho)g}}$ (D) $r=\sqrt{\frac{T}{(d+\rho)g}}$

16. [Online January 2020]
Two steel wires having same length are suspended from a ceiling under the same load. If the ratio of their energy stored per unit volume is $1:4$, the ratio of their diameters is

(A) $1:\sqrt{2}$ (B) $1:2$

(C) $2:1$ (D) $\sqrt{2}:1$

17. [Online January 2020]
A body of mass $m=10$ kg is attached to one end of a wire of length 0.3 m. The maximum angular speed (in rads^{-1}) with which it can be rotated about its other end in space station is (Breaking stress of wire $=4.8\times10^7$ Nm^{-2} and area of cross-section of the wire $=10^{-2}$ cm^2) is ……

18. [Online April 2019]
Water from a pipe is coming at a rate of 100 liters per minute. If the radius of the pipe is 5 cm, the Reynolds number for the flow is of the order of (density of water 1000 kgm^{-3}, coefficient of viscosity of water 1 mPas)

(A) 10^2 (B) 10^4

(C) 10^3 (D) 10^6

19. [Online April 2019]
If M is the mass of water that rises in a capillary tube of radius r, then mass of water which will rise in a capillary tube of radius $2r$ is

(A) $2M$ (B) M

(C) $4M$ (D) $\frac{M}{2}$

20. [Online April 2019]
A wooden block floating in a bucket of water has $\frac{4}{5}$ of its volume submerged. When certain amount of an oil is poured into the bucket, it is found that the block is just under the oil surface with half of its volume under water and half in oil. The density of oil relative to that of water is

(A) 0.5 (B) 0.8

(C) 0.7 (D) 0.6

21. [Online April 2019]
The ratio of surface tensions of mercury and water is given to be 7.5 while the ratio of their densities is 13.6. Their contact angles, with glass, are close to 135° and 0°, respectively. It is observed that mercury gets depressed by an amount h in a capillary tube of radius r_1, while water rises by the same amount h in a capillary tube of radius r_2. The ratio, $\left(\frac{r_1}{r_2}\right)$, is then close to

(A) $\frac{4}{5}$ (B) $\frac{2}{3}$

(C) $\frac{3}{5}$ (D) $\frac{2}{5}$

22. [Online April 2019]
A cubical block of side 0.5 m floats on water with 30% of its volume under water. What is the maximum weight that can be put on the block without fully submerging it under water? [Take, density of water $=10^3$ kgm^{-3}]

(A) 30.1 kg (B) 46.3 kg

(C) 87.5 kg (D) 65.4 kg

23. [Online April 2019]
Water from a tap emerges vertically downwards with an initial speed of 1.0 ms^{-1}. The cross-sectional area of the tap is 10^{-4} m^2. Assume that the pressure is constant throughout the stream of water and that the flow is streamlined. The cross-sectional area of stream, 0.15 m below the tap would be (Take $g=10$ ms^{-2})

(A) 1×10^{-5} m^2 (B) 5×10^{-5} m^2

(C) 5×10^{-4} m^2 (D) 2×10^{-5} m^2

24. [Online April 2019]
A submarine experiences a pressure of 5.05×10^6 Pa at a depth of d_1 in a sea. When it goes further to a depth of d_2, it experiences a pressure of 8.08×10^6 Pa. Then d_2-d_1 is approximately (density of water $=10^3$ kgm^{-3} and acceleration due to gravity $=10$ ms^{-2})

(A) 600 m (B) 500 m

(C) 300 m (D) 400 m

25. [Online April 2019]
A solid sphere, of radius R acquires a terminal velocity v_1 when falling (due to gravity) through a viscous fluid having a coefficient of viscosity η. The sphere is broken into 27 identical solid spheres. If each of these spheres acquires a terminal velocity, v_2, when falling through the same fluid, the ratio $\left(\frac{v_1}{v_2}\right)$ equals

(A) $\frac{1}{27}$ (B) 9

(C) $\frac{1}{9}$ (D) 27

26. [Online January 2019]
The top of a water tank is open to air and its water level is maintained. It is giving out 0.74 m^3 water per minute through a circular opening of 2 cm radius in its wall. The depth of the centre of the opening from the level of water in the tank is close to

(A) 9.6 m (B) 2.9 m

(C) 4.8 m (D) 6.0 m

27. [Online January 2019]
Water flows into a large tank with flat bottom at the rate of 10^{-4} m^3s^{-1}. Water is also leaking out of a hole of area 1 cm^2

at its bottom. If the height of the water in the tank remains steady, then this height is

(A) 5.1 cm (B) 1.7 cm

(C) 2.9 cm (D) 4 cm

28. [Online January 2019]

A liquid of density ρ is coming out of a hose pipe of radius a with horizontal speed v and hits a mesh. 50% of the liquid passes through the mesh unaffected. 25% looses all of its momentum and 25% comes back with the same speed. The resultant pressure on the mesh will be

(A) $\frac{3}{4}\rho v^2$ (B) $\frac{1}{4}\rho v^2$

(C) $\frac{1}{2}\rho v^2$ (D) ρv^2

29. [Online January 2019]

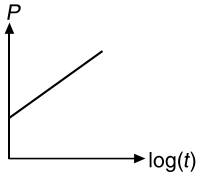
A load of mass M kg is suspended from a steel wire of length 2 m and radius 1.0 mm in Searle's apparatus experiment. The increase in length produced in the wire is 4.0 mm. Now the load is fully immersed in a liquid of relative density 2. The relative density of the material of the load is 8. The new value of increase in length of the steel wire is

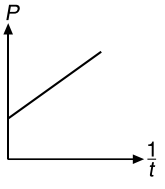
(A) 4.0 mm (B) zero

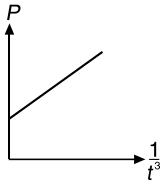
(C) 5.0 mm (D) 3.0 mm

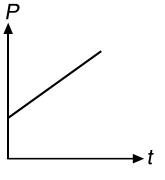
30. [Online January 2019]

A soap bubble, blown by a mechanical pump at the mouth of a tube, increases in volume, with time, at a constant rate. The graph that correctly depicts the time dependence of pressure inside the bubble is given by

(A) 

(B) 

(C) 

(D) 

31. [Online January 2019]

A long cylindrical vessel is half filled with a liquid. When the vessel is rotated about its own vertical axis, the liquid rises up near the wall. If the radius of vessel is 5 cm and its rotational speed is 2 rotations per second, then the difference in the heights between the centre and the sides, in cm, will be

(A) 1.2 (B) 0.1

(C) 0.4 (D) 2.0

32. [2018]

A solid sphere of radius r made of a soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area a floats on the surface of the liquid, covering entire cross section of cylindrical container. When a mass m is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere, $\left(\frac{dr}{r}\right)$, is

(A) $\frac{Ka}{mg}$ (B) $\frac{Ka}{3mg}$

(C) $\frac{mg}{3Ka}$ (D) $\frac{mg}{Ka}$

33. [Online 2018]

When an air bubble of radius r rises from the bottom to the surface of a lake, its radius becomes $\frac{5r}{4}$. Taking the atmospheric pressure to be equal to 10 m height of water column, the depth of the lake would approximately be (ignore the surface tension and the effect of temperature)

(A) 11.2 m (B) 10.5 m

(C) 9.5 m (D) 8.7 m

34. [Online 2018]

As shown in the figure, forces of 10^5 N each are applied in opposite directions, on the upper and lower faces of a cube of side 10 cm, shifting the upper face parallel to itself by 0.5 cm, if the side of another cube of the same material is 20 cm, then under similar conditions as above, the displacement will be

(A) 0.25 cm (B) 0.37 cm

(C) 1.00 cm (D) 0.75 cm

35. [Online 2018]

A small soap bubble of radius 4 cm is trapped inside another bubble of radius 6 cm without any contact. Let P_2 be the pressure inside the inner bubble and P_0, the pressure outside the outer bubble. Radius of another bubble with pressure difference $P_2 - P_0$ between its inside and outside would be

(A) 12 cm (B) 4.8 cm

(C) 2.4 cm (D) 6 cm

36. [2017]

A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that his density remains same, the stress in the leg will change by a factor of

(A) 9 (B) $\frac{1}{9}$

(C) 81 (D) $\frac{1}{81}$

37. [Online 2017]

Two tubes of radii r_1 and r_2 and lengths l_1 and l_2, respectively, are connected in series and a liquid flows through each of them in streamline conditions. P_1 and P_2 are pressure differences across the two tubes. If P_2 is $4P_1$ and l_2 is $\frac{l_1}{4}$, then the radius r_2 will be equal to

(A) $2r_1$ (B) $\frac{r_1}{2}$

(C) $4r_1$ (D) r_1

38. [Online 2016]

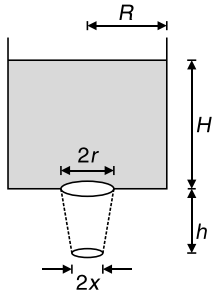
A uniformly tapering conical wire is made from a material of Young's modulus Y and has a normal, unextended length L. The radii, at the upper and lower ends of this conical wire, have values R and $3R$, respectively. The upper end of the wire is fixed to a rigid support and a mass M is suspended from its lower end. The equilibrium extended length, of this wire, would equal

(A) $L\left(1+\frac{2}{9}\frac{Mg}{\pi YR^2}\right)$ (B) $L\left(1+\frac{1}{9}\frac{Mg}{\pi YR^2}\right)$

(C) $L\left(1+\frac{1}{3}\frac{Mg}{\pi YR^2}\right)$ (D) $L\left(1+\frac{2}{3}\frac{Mg}{\pi YR^2}\right)$

39. [Online 2016]

Consider a water jar of radius R that has water filled up to height H and is kept on a stand of height h (see figure). Through a hole of radius $r(r \ll R)$ at its bottom, the water leaks out and the stream of water coming down towards the ground has a shape like a funnel as shown in the figure. If the radius of the cross-section of water stream when it hits the ground is x. Then

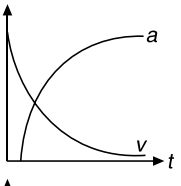


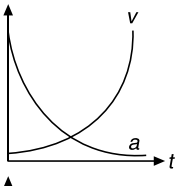
(A) $x = r\left(\frac{H}{H+h}\right)^{\frac{1}{4}}$ (B) $x = r\left(\frac{H}{H+h}\right)$

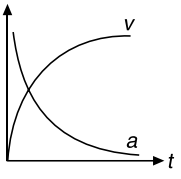
(C) $x = r\left(\frac{H}{H+h}\right)^{2}$ (D) $x = r\left(\frac{H}{H+h}\right)^{\frac{1}{2}}$

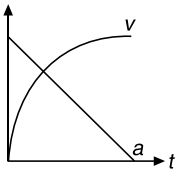
40. [Online 2016]

Which of the following option correctly describes the variation of the speed v and acceleration a of a point mass falling vertically in a viscous medium that applies a force $F = -kv$, where k is a constant, on the body? (Graphs are schematic and not drawn to scale)

(A)


(B)


(C)


(D)


41. [Online 2016]

A bottle has an opening of radius a and length b. A cork of length b and radius $(a+\Delta a)$ where $(\Delta a \ll a)$ is compressed to fit into the opening completely (see figure). If the bulk modulus of cork is B and frictional coefficient between the bottle and cork is μ then the force needed to push the cork into the bottle is

(A) $(\pi\mu Bb)a$ (B) $(2\pi\mu Bb)\Delta a$

(C) $(\pi\mu Bb)\Delta a$ (D) $(4\pi\mu Bb)\Delta a$

42. [Online 2015]

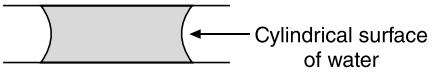
If it takes 5 minutes to fill a 15 litre bucket from a water tap of diameter $\frac{2}{\sqrt{\pi}}$ cm then the Reynolds number for the flow is (density of water is 10^3 kgm^{-3} and viscosity of water is 10^{-3} Pas) close to

(A) 5500 (B) 11000

(C) 550 (D) 1100

43. [Online 2015]

If two glass plates have water between them and are separated by very small distance (see figure), it is very difficult to pull them apart. It is because the water in between forms cylindrical surface on the side that gives rise to lower pressure in the water in comparison to atmosphere. If the radius of the cylindrical surface is R and surface tension of water is T then the pressure in water between the plates is lower by



(A) $\frac{T}{4R}$ (B) $\frac{T}{2R}$

(C) $\frac{4T}{R}$ (D) $\frac{2T}{R}$

44. [2014]

An open glass tube is immersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open end of the tube is then closed and sealed and the tube is raised vertically up by additional 46 cm. What will be length of the air column above mercury in the tube now? (Atmospheric pressure = 76 cm of Hg)

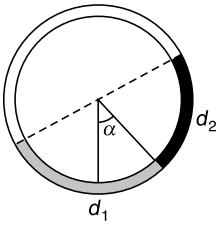
(A) 16 cm (B) 22 cm

(C) 38 cm (D) 6 cm

45. [2014]

There is a circular tube in a vertical plane. Two liquids which do not mix and of densities d_1 and d_2 are filled in the tube.

Each liquid subtends 90° angle at centre. Radius joining their interface makes an angle α with vertical. Ratio $\frac{d_1}{d_2}$ is

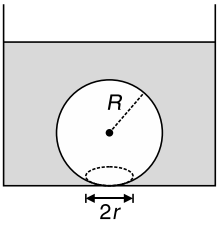


(A) $\frac{1+\sin\alpha}{1-\sin\alpha}$ (B) $\frac{1+\cos\alpha}{1-\cos\alpha}$

(C) $\frac{1+\tan\alpha}{1-\tan\alpha}$ (D) $\frac{1+\sin\alpha}{1-\cos\alpha}$

46. [2014]

On heating water, bubbles being formed at the bottom of the vessel detach and rise. Take the bubbles to be spheres of radius R and making a circular contact of radius r with the bottom of the vessel. If $x \ll R$ and the surface tension of water is T, value of r just before bubbles detach is (density of water is ρ_w)



(A) $R^2\sqrt{\frac{2\rho_w g}{3T}}$ (B) $R^2\sqrt{\frac{\rho_w g}{6T}}$

(C) $R^2\sqrt{\frac{\rho_w g}{T}}$ (D) $R^2\sqrt{\frac{3\rho_w g}{T}}$

47. [2013]

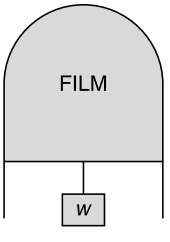
A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical, from a fixed point by a massless spring, such that it is half submerged in a liquid of density σ at equilibrium position. The extension x_0 of the spring when it is in equilibrium is

(A) $\frac{Mg}{k}$ (B) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{M}\right)$

(C) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{2M}\right)$ (D) $\frac{Mg}{k}\left(1+\frac{LA\sigma}{M}\right)$

48. [2012]

A thin liquid film formed between a U-shaped wire and a light slider supports a weight of 1.5×10^{-2} N (see figure). The length of the slider is 30 cm and its weight negligible. The surface tension of the liquid film is



(A) 0.1 Nm^{-1} (B) 0.05 Nm^{-1}

(C) 0.025 Nm^{-1} (D) 0.0125 Nm^{-1}

49. [2011]

Water is flowing continuously from a tap having an internal diameter 8×10^{-3} m. The water velocity as it leaves the tap is 0.4 ms^{-1}. The diameter of the water stream at a distance 2×10^{-1} m below the tap is close to

(A) 5.0×10^{-3} m (B) 7.5×10^{-3} m

(C) 9.6×10^{-3} m (D) 3.6×10^{-3} m

50. [2011]

Work done in increasing the size of a soap bubble from a radius of 3 cm to 5 cm is nearly (Surface tension of soap solution = 0.03 Nm^{-1})

(A) 4π mJ (B) 0.2π mJ

(C) 2π mJ (D) 0.4π mJ

51. [2010]

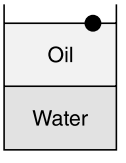
The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x)=\frac{a}{x^{12}}-\frac{b}{x^6}$, where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is $D=\left[U(x=\infty)-U_{\text{at equilibrium}}\right]$, D is

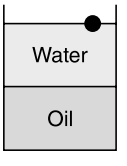
(A) $\frac{b^2}{6a}$ (B) $\frac{b^2}{2a}$

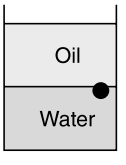
(C) $\frac{b^2}{12a}$ (D) $\frac{b^2}{4a}$

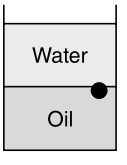
52. [2010]

A ball is made of a material of density ρ where $\rho_{oil}<\rho<\rho_{water}$ with ρ_{oil} and ρ_{water} representing the densities of oil and water, respectively. The oil and water are immiscible. If the above ball is in equilibrium in a mixture of this oil and water, which of the following pictures represents its equilibrium position?

(A)


(B)


(C)


(D)


53. [2009]

Two wires are made of the same material and have the same volume. However, wire 1 has cross-sectional area A and wire 2 has cross-sectional area $3A$. If the length of wire 1 increases by Δx on applying force F, how much force is needed to stretch wire 2 by the same amount?

(A) F (B) $4F$

(C) $6F$ (D) $9F$

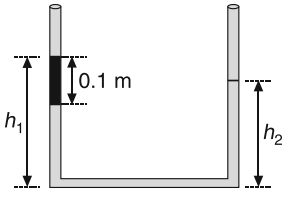
ARCHIVE: JEE ADVANCED

Single Correct Choice Type Problems

(In this section each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct)

1. [JEE (Advanced) 2020]

An open-ended U-tube of uniform cross-sectional area contains water (density 10^3 kgm^{-3}). Initially the water level stands at 0.29 m from the bottom in each arm. Kerosene oil (a water-immiscible liquid) of density 800 kgm^{-3} is added to the left arm until its length is 0.1 m, as shown in the schematic figure below. The ratio $\left(\frac{h_1}{h_2}\right)$ of the heights of the liquid in the two arms is

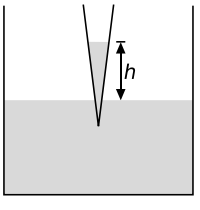


(A) $\frac{15}{14}$ (B) $\frac{35}{33}$

(C) $\frac{7}{6}$ (D) $\frac{5}{4}$

2. [JEE (Advanced) 2014]

A glass capillary tube is of the shape of truncated cone with an apex angle α so that its two ends have cross-sections of different radii. When dipped in water vertically, water rises in it to a height h, where the radius of its cross-section is b. If the surface tension of water is S, its density is ρ and its contact angle with glass is θ, the value of h will be (g is the acceleration due to gravity)



(A) $\frac{2S}{b\rho g}\cos(\theta-\alpha)$ (B) $\frac{2S}{b\rho g}\cos(\theta+\alpha)$

(C) $\frac{2S}{b\rho g}\cos\left(\theta-\frac{\alpha}{2}\right)$ (D) $\frac{2S}{b\rho g}\cos\left(\theta+\frac{\alpha}{2}\right)$

3. [JEE (Advanced) 2013]

One end of a horizontal thick copper wire of length $2L$ and radius $2R$ is welded to an end of another horizontal thin copper wire of length L and radius R. When the arrangement is stretched by applying forces at two ends, the ratio of the elongation in the thin wire to that in the thick wire is

(A) 0.25 (B) 0.50

(C) 2.00 (D) 4.00

4. [IIT-JEE 2012]

A thin uniform cylindrical shell, closed at both ends, is partially filled with water. It is floating vertically in water in half-submerged state. If ρ_c is the relative density of the material of the shell with respect to water, then the correct statement is that the shell is

(A) more than half-filled if ρ_c is less than 0.5

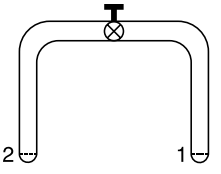
(B) more than half-filled if ρ_c is more than 1.0

(C) half-filled if ρ_c is more than 0.5

(D) less than half-filled if ρ_c is less than 0.5

5. [IIT-JEE 2008]

A glass tube of uniform internal radius (r) has a valve separating the two identical ends. Initially, the valve is in a tightly closed position. End 1 has a hemispherical soap bubble of radius r. End 2 has sub-hemispherical soap bubble as shown in figure. Just after opening the valve.



(A) air from end 1 flows towards end 2. No change in the volume of the soap bubbles

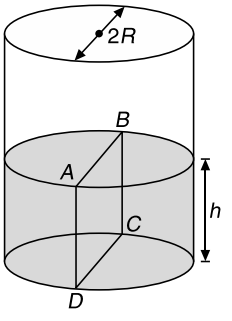
(B) air from end 1 flows towards end 2. Volume of the soap bubble at end 1 decreases

(C) no change occurs

(D) air from end 2 flows towards end 1. Volume of the soap bubble at end 1 increases

6. [IIT-JEE 2007]

Water is filled up to a height h in a beaker of radius R as shown in the figure. The density of water is ρ, the surface tension of water is T and the atmospheric pressure is p_0. Consider a vertical section $ABCD$ of the water column through a diameter of the beaker. The force on water on one side of this section by water on the other side of this section has magnitude



(A) $\left|2p_0Rh+\pi R^2\rho gh-2RT\right|$

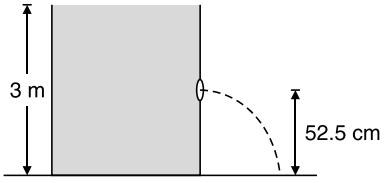
(B) $\left|2p_0Rh+R\rho gh^2-2RT\right|$

(C) $\left|p_0\pi R^2+R\rho gh^2-2RT\right|$

(D) $\left|p_0\pi R^2+R\rho gh^2+2RT\right|$

7. [IIT-JEE 2005]

Water is filled in a cylindrical container to a height of 3 m. The ratio of the cross-sectional area of the orifice and the beaker is 0.1. The square of the speed of the liquid coming out from the orifice is $\left(g = 10\ \text{ms}^{-2}\right)$



(A) $50\ \text{m}^2\text{s}^{-2}$ (B) $50.5\ \text{m}^2\text{s}^{-2}$

(C) $51\ \text{m}^2\text{s}^{-2}$ (D) $52\ \text{m}^2\text{s}^{-2}$

8. [IIT-JEE 2005]

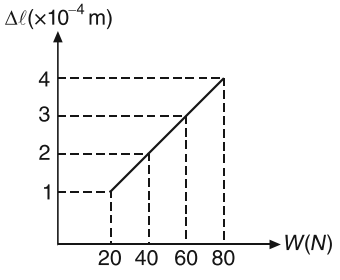
The pressure of a medium is changed from 1.01×10^5 Pa to 1.165×10^5 Pa and change in volume is 10% keeping temperature constant. The bulk modulus of the medium is

(A) 204.8×10^5 Pa (B) 102.4×10^5 Pa

(C) 51.2×10^5 Pa (D) 1.55×10^5 Pa

9. [IIT-JEE 2003]

The graph shows the extension (Δl) of a wire of length l m suspended from the top of a roof at one end and with a load W connected to the other end. If the cross-sectional area of the wire is $10^{-6}\ \text{m}^2$, calculate the Young's modulus of the material of the wire

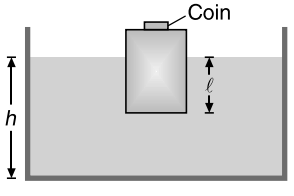


(A) $2\times10^{11}\ \text{Nm}^{-2}$ (B) $2\times10^{-11}\ \text{Nm}^{-2}$

(C) $3\times10^{12}\ \text{Nm}^{-2}$ (D) $2\times10^{13}\ \text{Nm}^{-2}$

10. [IIT-JEE 2002]

A wooden block, with a coin placed on its top, floats in water as shown in Figure.



The distance l and h are shown there. After sometime the coin falls into the water. Then

(A) l decreases and h increases

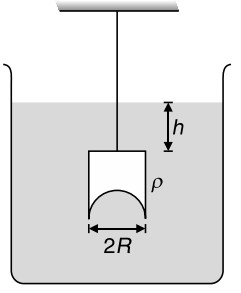
(B) l increases and h decreases

(C) both l and h increase

(D) both l and h decrease

11. [IIT-JEE 2001]

A hemispherical portion of radius R is removed from the bottom of a cylinder of radius R. The volume of the remaining cylinder is V and mass M. It is suspended by a string in a liquid of density ρ, where it stays vertical. The upper surface of the cylinder is at a depth h below the liquid surface. The force on the bottom of the cylinder by the liquid is



(A) Mg (B) $Mg - V\rho g$

(C) $Mg + \pi R^2 h\rho g$ (D) $\rho g\left(V + \pi R^2 h\right)$

12. [IIT-JEE 2000]

A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth $4y$ from the top. When the tank is completely filled with water, the quantities of water flowing out per second from both holes are the same. Then, R is equal to

(A) $\dfrac{L}{\sqrt{2\pi}}$ (B) $2\pi L$

(C) L (D) $\dfrac{L}{2\pi}$

13. [IIT-JEE 1999]

A closed compartment containing gas is moving with some acceleration in horizontal direction. Neglect effect of gravity. Then the pressure in the compartment is

(A) same everywhere

(B) lower in front side

(C) lower in rear side

(D) lower in upper side

14. [IIT-JEE 1998]

Water from a tap emerges vertically downwards with an initial speed of $1\ \text{ms}^{-1}$. The cross-sectional area of tap is $10^{-4}\ \text{m}^2$. Assume that the pressure is constant throughout and that the flow is steady, the cross-sectional area of stream 0.15 m below the tap is

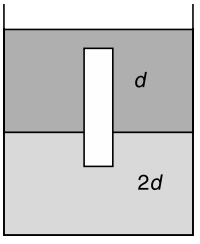
(A) $5\times10^{-4}\ \text{m}^2$ (B) $1\times10^{-4}\ \text{m}^2$

(C) $5\times10^{-5}\ \text{m}^2$ (D) $2\times10^{-5}\ \text{m}^2$

15. [IIT-JEE 1995]

A homogeneous solid cylinder of length $L\left(L < \dfrac{H}{2}\right)$, cross-sectional area $\dfrac{A}{5}$ is immersed such that it floats with its axis vertical at the liquid-liquid interface with length $\dfrac{L}{4}$ in the

denser liquid as shown in the figure. The lower density liquid is open to atmosphere having pressure P_0. Then, density D of solid is given by



(A) $\frac{5}{4}d$ (B) $\frac{4}{5}d$

(C) $4d$ (D) $\frac{d}{5}$

16. [IIT-JEE 1988]

A vessel contains oil (density $= 0.8\ \text{gcm}^{-3}$) over mercury (density $= 13.6\ \text{gcm}^{-3}$). A homogeneous sphere floats with half its volume immersed in mercury and the other half in oil. The density of the material of the sphere in gcm^{-3} is

(A) 3.3 (B) 6.4

(C) 7.2 (D) 12.8

17. [IIT-JEE 1983]

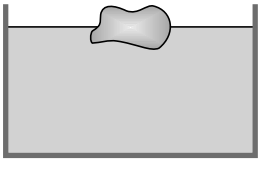
A U-tube of uniform cross-section is partially filled with a liquid I. Another liquid II which does not mix with liquid I is poured into one side. It is found that the liquid levels of the two sides of the tube are the same, while the level of liquid I has risen by 2 cm. If the specific gravity of liquid I is 1.1, the specific gravity of liquid II must be

(A) 1.12 (B) 1.1

(C) 1.05 (D) 1.0

18. [IIT-JEE 1982]

A body floats in a liquid contained in a beaker. The whole system as shown in figure falls freely under gravity. The upthrust on the body is



(A) ZERO

(B) equal to the weight of liquid displaced

(C) equal to the weight of the body in air

(D) equal to the weight of the immersed portion of the body

19. [IIT-JEE 1981]

The following four wires are made of the same material. Which of these will have the largest extension when the same tension is applied?

(A) Length = 50 cm, diameter = 0.5 mm

(B) Length = 100 cm, diameter = 1 mm

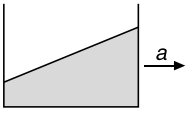
(C) Length = 200 cm, diameter = 2 mm

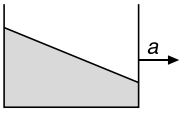
(D) Length = 300 cm, diameter = 3 mm

20. [IIT-JEE 1981]

A vessel containing water is given a constant acceleration a towards the right along a straight horizontal path. Which of the following diagrams represents the surface of the liquid?

(A)

(B) 

(C) 

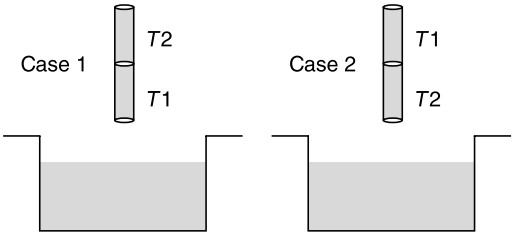
(D) None of these

Multiple Correct Choice Type Problems

(In this section each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct)

1. [JEE (Advanced) 2019]

A cylindrical capillary tube of 0.2 mm radius is made by joining two capillaries $T1$ and $T2$ of different materials having water contact angles of $0°$ and $60°$, respectively. The capillary tube is dipped vertically in water in two different configurations, Case I and II as shown in figure. Which of the following option(s) is (are) correct? [Surface tension of water is $0.075\ \text{Nm}^{-1}$, density of water is $1000\ \text{kgm}^{-3}$, take $g = 10\ \text{ms}^{-2}$]



(A) For case II, if the capillary joint is 5 cm above the water surface, the height of water column raised in the tube will be 3.75 cm. (Neglect the weight of the water in the meniscus)

(B) For case I, if the joint is kept at 8 cm above the water surface, the height of water column in the tube will be 7.5 cm. (Neglect the weight of the water in the meniscus)

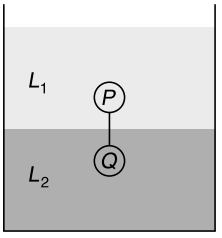
(C) For case I, if the capillary joint is 5 cm above the water surface, the height of water column raised in the tube will be more than 8.75 cm. (Neglect the weight of the water in the meniscus)

(D) The correction in the height of water column raised in the tube, due to weight of water contained in the meniscus, will be different for both cases.

2. [JEE (Advanced) 2015]

Two spheres P and Q for equal radii have densities ρ_1 and ρ_2, respectively. The spheres are connected by a massless string and placed in liquids L_1 and L_2 of densities σ_1 and σ_2 and viscosities η_1 and η_2, respectively. They float in equilibrium with the sphere P in L_1 and sphere Q in L_2 and the string being taut (see figure). If sphere P alone in L_2 has

terminal velocity $\vec{v}_P$ and Q alone in L_1 has terminal velocity $\vec{v}_Q$, then



(A) $\frac{|\vec{v}_P|}{|\vec{v}_Q|} = \frac{\eta_1}{\eta_2}$

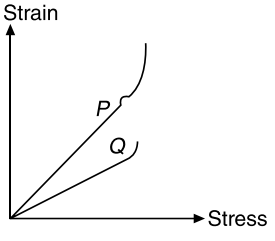
(B) $\frac{|\vec{v}_P|}{|\vec{v}_Q|} = \frac{\eta_2}{\eta_1}$

(C) $\vec{v}_P \cdot \vec{v}_Q > 0$

(D) $\vec{v}_P \cdot \vec{v}_Q < 0$

3. [JEE (Advanced) 2015]

In plotting stress versus strain curves for two materials P and Q, a student by mistake puts strain on the Y-axis and stress on the X-axis as shown in the figure. Then the correct statements is/are



(A) P has more tensile strength than Q

(B) P is more ductile than Q

(C) P is more brittle than Q

(D) The Young's modulus of P is more than that of Q

4. [JEE (Advanced) 2013]

A solid sphere of radius R and density ρ is attached to one end of a massless spring of force constant k. The other end of the spring is connected to another solid sphere of radius R and density 3ρ. The complete arrangement is placed in a liquid of density 2ρ and is allowed to reach equilibrium. The correct statement(s) is (are)

(A) the net elongation of the spring is $\frac{4\pi R^3 \rho g}{3k}$

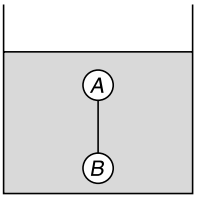
(B) the net elongation of the spring is $\frac{8\pi R^3 \rho g}{3k}$

(C) the light sphere is partially submerged

(D) the light sphere is completely submerged

5. [JEE (Advanced) 2011]

Two solid spheres A and B of equal volumes but of different densities d_A and d_B are connected by a string. They are fully immersed in a fluid of density d_F. They get arranged into an equilibrium state as shown in the figure with a tension in the string. The arrangement is possible only if



(A) $d_A < d_F$

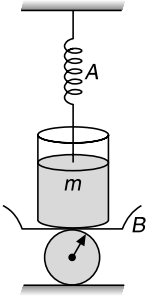
(B) $d_B > d_F$

(C) $d_A > d_F$

(D) $d_A + d_B = 2d_F$

6. [IIT-JEE 1985]

The spring balance A leads 2 kg with a mass m suspended from it. A balance B reads 5 kg when a beaker with liquid is put on the pan of the balance. The two balances are now so arranged that the hanging mass is inside the liquid in the beaker as shown in the figure. In this situation.



(A) the balance A will read more than 2 kg

(B) the balance B will read more than 5 kg

(C) the balance A will read less than 2 kg

(D) the balances A and B will read 2 kg and 5 kg respectively

Reasoning Based Questions

This section contains Reasoning type questions, each having four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Each question contains STATEMENT 1 and STATEMENT 2. You have to mark your answer as

Bubble (A) If both statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1.

Bubble (B) If both statements are TRUE but STATEMENT 2 is not the correct explanation of STATEMENT 1.

Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE.

Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 is TRUE.

1. [IIT-JEE 2008]

Statement-1: The stream of water flowing at high speed from a garden hose pipe tends to spread like a fountain when held vertically up, but tends to narrow down when held vertically down.

Statement-2: In any steady flow of an incompressible fluid, the volume flow rate of the fluid remains constant.

Matrix Match/Column Match Type Questions

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

If the correct matches are A → p, s and t; B → q and r; C → p and q; and D → s and t; then the correct darkening of bubbles will look like the following:

	p	q	r	s	t
A	●	●	○	●	●
B	●	●	●	○	○
C	●	●	○	○	○
D	●	○	○	●	●

1. [JEE (Advanced) 2014]

A person in a lift is holding a water jar, which has a small hole at the lower end of its side. When the lift is at rest, the water jet coming out of the hole hits the floor of the lift at a distance d of 1.2 m from the person. In the following, state of the lift's motion is given in List I and the distance where the water jet hits the floor of the lift is given in List II. Match the statements from List I with those in List II and select the correct answer using the code given below the lists.

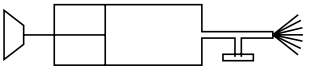
COLUMN-I	COLUMN-II
A. Lift is accelerating vertically up	p. $d = 1.2$ m
B. Lift is accelerating vertically down with an acceleration less than the gravitational acceleration.	q. $d > 1.2$ m
C. Lift is moving vertically up with constant speed.	r. $d < 1.2$ m
D. Lift is falling freely.	s. No water leaks out of the jar

Linked Comprehension Type Questions

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct. (For the sake of competitiveness there may be a few questions that may have more than one correct options)

Comprehension 1

A spray gun is shown in the figure where a piston pushes air out of nozzle. A thin tube of uniform cross-section is connected to the nozzle. The other end of the tube is in a small liquid container. As the piston pushes air through the nozzle, the liquid from the container rises into the nozzle and is sprayed out. For the spray gun shown, the radii of the piston and the nozzle are 20 mm and 1 mm respectively. The upper end of the container is open to the atmosphere.



1. [JEE (Advanced) 2014]

If the piston is pushed at a speed of 5 mms^{-1}, the air comes out of the nozzle with a speed of

(A) 0.1 ms^{-1} (B) 1 ms^{-1}

(C) 2 ms^{-1} (D) 8 ms^{-1}

2. [JEE (Advanced) 2014]

If the density of air is ρ_a and that of the liquid ρ_l, then for a given piston speed the rate (volume per unit time) at which the liquid is sprayed will be proportional to

(A) $\sqrt{\frac{\rho_a}{\rho_l}}$ (B) $\sqrt{\rho_a \rho_l}$

(C) $\sqrt{\frac{\rho_l}{\rho_a}}$ (D) ρ_l

Comprehension 2

When liquid medicine of density ρ is to be put in the eye, it is done with the help of a dropper. As the bulb on the top of the dropper is pressed, a drop forms at the opening of the dropper. We wish to estimate the size of the drop. We first assume that the drop formed at the opening is spherical because that requires a minimum increase in its surface energy. To determine the size, we calculate the net vertical force due to the surface tension T when the radius of the drop is R. When this force becomes smaller than the weight of the drop, the drop gets detached from the dropper.

3. [IIT-JEE 2010]

If the radius of the opening of the dropper is r, the vertical force due to the surface tension on the drop of radius R (assuming $r \ll R$) is

(A) $2\pi rT$ (B) $2\pi RT$

(C) $\frac{2\pi r^2 T}{R}$ (D) $\frac{2\pi R^2 T}{r}$

4. [IIT-JEE 2010]

If $r = 5 \times 10^{-4}$ m, $\rho = 10^3$ kgm^{-3}, $g = 10$ ms^{-2}, $T = 0.11$ Nm^{-1}, the radius of the drop when it detaches from the dropper is approximately

(A) 1.4×10^{-3} m

(B) 3.3×10^{-3} m

(C) 2.0×10^{-3} m

(D) 4.1×10^{-3} m

5. [IIT-JEE 2010]

After the drop detaches, its surface energy is

(A) 1.4×10^{-6} J (B) 2.7×10^{-6} J

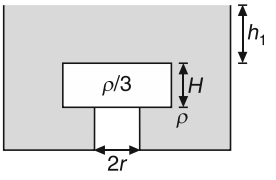
(C) 5.4×10^{-6} J (D) 8.1×10^{-9} J

Comprehension 3

A wooden cylinder of diameter $4r$, height H and density $\frac{\rho}{3}$ is kept on a hole of diameter $2r$ of a tank, filled with liquid of density ρ as shown in the figure.

6. [IIT-JEE 2006]

Now level of the liquid starts decreasing slowly. When the level of liquid is at a height h_1 above the cylinder the block starts moving up. At what value of h_1, will the block rise

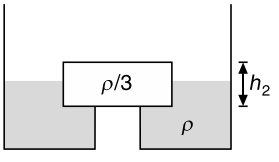


(A) $\frac{4h}{9}$ (B) $\frac{5h}{9}$

(C) $\frac{5h}{3}$ (D) remains same

7. [IIT-JEE 2006]

The block in the above question is maintained at the position by external means and the level of liquid is lowered. The height h_2 when this external force reduces to zero is



(A) $\frac{4h}{9}$ (B) $\frac{5h}{9}$

(C) remains same (D) $\frac{2h}{3}$

8. [IIT-JEE 2006]

If height h_2 of water level is further decreased, then

(A) cylinder will not move up and remains at its original position

(B) for $h_2 = \frac{h}{3}$, cylinder again starts moving up

(C) for $h_2 = \frac{h}{4}$, cylinder again starts moving up

(D) for $h_2 = \frac{h}{5}$ cylinder again starts moving up

Integer/Numerical Answer Type Questions

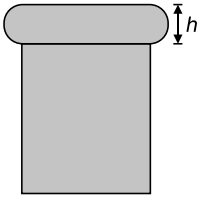
In this section, the answer to each question is a numerical value obtained after series of calculations based on the data provided in the question(s).

1. [JEE (Advanced) 2020]

A train with cross-sectional area S_t is moving with speed v_t inside a long tunnel of cross-sectional area $S_0 (S_0 = 4S_t)$. Assume that almost all the air (density ρ) in front of the train flows back between its sides and the walls of the tunnel. Also, the air flow with respect to the train is steady and laminar. Take the ambient pressure and that inside the train to be p_0. If the pressure in the region between the sides of the train and the tunnel walls is p, then $p_0 - p = \frac{7}{2N}\rho v_t^2$. The value of N is…….

2. JEE (Advanced) 2020

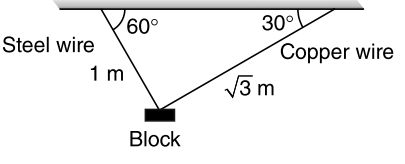
When water is filled carefully in a glass, one can fill it to a height h above the rim of the glass due to the surface tension of water. To calculate h just before water starts flowing, model the shape of the water above the rim as a disc of thickness h having semicircular edges, as shown schematically in Figure. When the pressure of water at the bottom of this disc exceeds what can be withstood due to the surface tension, the water surface breaks near the rim and water starts flowing from there. If the density of water, its surface tension and the acceleration due to gravity are 10^3 kgm^{-3}, 0.07 Nm^{-1} and 10 ms^{-2}, respectively, the value of h (in mm) is……



3. JEE (Advanced) 2019

A block of weight 100 N is suspended by copper and steel wires of same cross-sectional area 0.5 cm^2 and, length $\sqrt{3}$ m and 1 m, respectively. Their other ends are fixed on a ceiling as shown in figure. The angles subtended by copper and steel wires with ceiling are 30° and 60°, respectively. If elongation in copper wire is (Δl_c) and elongation in steel wire is (Δl_s), then the ratio $\frac{\Delta l_c}{\Delta l_s}$ is ……

[Young's modulus for copper and steel are 1×10^{11} Nm^{-2} and 2×10^{11} Nm^{-2}, respectively].



4. [JEE (Advanced) 2017]

A drop of liquid of radius $R = 10^{-2}$ m having surface tension $S = \frac{0.1}{4\pi}$ Nm^{-1} divides itself into K identical drops. In this process the total change in the surface energy $\Delta U = 10^{-3}$ J. If $K = 10^{\alpha}$, then the value of α is

5. [JEE (Advanced) 2016]

Consider two solid spheres P and Q each of density 8 gcm^{-3} and diameters 1 cm and 0.5 cm, respectively. Sphere P is

dropped into a liquid of density $0.8\ \text{gcm}^{-3}$ and viscosity $\eta = 3$ poiseulles. Sphere Q is dropped into a liquid of density $1.6\ \text{gcm}^{-3}$ and viscosity $\eta = 2$ poiseulles. The ratio of the terminal velocities of P and Q is

6. [JEE (Advanced) 2009]

Two soap bubbles A and B are kept in a closed chamber where the air is maintained at pressure $8\ \text{Nm}^{-2}$. The radii of bubbles A and B are 2 cm, respectively. Surface tension of the soap-water used to make bubbles is $0.04\ \text{Nm}^{-1}$. Find the ratio $\frac{n_B}{n_A}$, where n_A and n_B are the number of moles of air in bubbles A and B, respectively. [Neglect the effect of gravity]

7. [IIT-JEE 2009]

A cylindrical vessel of height $500\ \text{mm}$ has an orifice (small hole) at its bottom. The orifice is initially closed and water is filled in it upto height H. Now, the top is completely sealed with a cap and the orifice at the bottom is opened. Some water comes out from the orifice and the water level in the vessel becomes steady with height of water column being $200\ \text{mm}$. Find the fall in height (in mm) of water level due to opening of the orifice. [Take atmospheric pressure $= 1.0 \times 10^5\ \text{Nm}^{-2}$, density of water $= 1000\ \text{kgm}^{-3}$ and $g = 10\ \text{ms}^{-2}$. Neglect any effect of surface tension.]

ANSWER KEYS–TEST YOUR CONCEPTS AND PRACTICE EXERCISES

Test Your Concepts-I (Based on Young's Modulus, Longitudinal Stress and Strain)

1. $\frac{W_1+\frac{3W}{4}}{S}$
2. $T_C=\frac{mg}{4}$ and $T_S=\frac{mg}{2}$
3. $\Delta L_B=4$ mm , $\Delta L_C=9$ mm , $\Delta L_D=24$ mm
4. Wire B
5. $\frac{2mg}{3aY}$
6. Material of rod A is stronger
7. 1 mm
8. $\frac{\sigma_0 A_0}{\sin^2\theta}$
9. 6:3:4
10. 41.6 N, 0.136%
11. 2.6 mm
12. 2.0×10^{11} Nm^{-2}
13. 1, $\frac{13}{20}$
14. $\frac{\rho a_0 L^2}{2Y}$
15. (a) $F\left(1-\frac{x}{l}\right)$, (b) $\frac{F}{A}\left(1-\frac{x}{l}\right)$, (c) $\frac{Fl}{2AY}$, (d) $\frac{F}{2AY}$
16. (a) 4.4×10^{6} Nm^{-2} (b) 2.2×10^{-5} (c) 6.6×10^{-5} m
17. $Y\left(\frac{R-r}{r}\right)$, $Ybd\left(\frac{R-r}{r}\right)$

Test Your Concepts-II (Based on Elastic Energy, Energy Density and Poisson's Ratio)

1. 39.74 J
2. $\frac{F^2 l}{6AY}$
3. 0.015 J
4. 3×10^{8} Nm^{-2}, $\frac{5}{3}\times10^{-3}$, 3.34 mm, 60 mJ
5. 5×10^{-4} J
6. 2.25×10^{9} Nm^{-2}
7. 1×10^{8} Nm^{-2}
8. 2.5×10^{4} Jm^{-3} , 0.20 J

Test Your Concepts-III (Based on Shear Modulus, Tangential Stress, Shear Strain)

1. 333 Pa , 0.133 , 2.5 kPa
2. 500 Nm^{-2}
3. $\frac{\sigma^2 h^3}{2\eta}$
4. 5×10^{6} Nm^{-2} , 6.25×10^{-6} m

Test Your Concepts-IV (Based on Bulk's Modulus, Normal Stress and Volumetric Strain)

1. $\frac{mg}{3AB}$
2. 4.8×10^{9} Nm^{-2} , 2.1×10^{-10} m^2N^{-1}
3. 1.76×10^{9} Nm^{-2}
4. 2.525×10^{-4}%
5. 4.33 kgm^{-3}
6. 11.69 gcm^{-3}
7. (a) 4.4×10^{-5} atm^{-1} , (b) 22.7 atm
8. 23.5 kJm^{-3}
9. 1.55×10^{5} Nm^{-2}

Test Your Concepts-V (Based on Fluid Properties, Pressure and Pascal's Law)

1. 800 kgm^{-3}
2. 1039 kgm^{-3}
3. 20 N
4. No answer to be given (exclude 4 from answers)
5. 570 N
6. 52 atm
7. 300 cm^3
8. R
9. 2.5 km
10. 1.3 kN
11. 138 kPa
12. 1 cm , 25.2 cm
13. 2×10^{4} Nm^{-1} , Clockwise
14. 37.5 N
15. 0.83 gcm^{-3}
16. $\sin^{-1}\left[\frac{\rho_2 h}{(\rho_3-\rho_1)\ell}\right]$
17. $P_0+\frac{mg}{A}$
18. $4\rho g\ell R^2$

Test Your Concepts-VI (Based on Pressure in Accelerating Fluids)

1. $\frac{hg}{l}$
2. 10 cm , $\frac{5}{\pi}$ s^{-1}
3. $\frac{\omega^2 L^2}{2g}$
4. 4 ms^{-2}
5. $(hg + la)\rho$
6. $\frac{\omega^2 x^2}{2g}$

Test Your Concepts-VII (Based on Archimedes' Principle and Buoyancy)

1. 0.1
2. $\left(\frac{W}{W - W'}\right)\rho_w$
3. 33.3 g
4. 24 N, 12 ms^{-2}
5. 5 ms^{-2}, 2 s
6. $T_0\left(1 + \frac{a}{g}\right)$
7. 1.00625×10^5 Pa
8. 34.3°, 1.6 N (downwards)
9. 13 cm^3
10. ZERO
11. 3.92 N, 6 N

Test Your Concepts-VIII (Based on Viscosity and Terminal Speed)

1. 10^{-3} Nm^{-2}
2. 9 ms^{-1}
3. $\frac{4\pi R^2 gd}{3(6\pi\eta + K)}$
4. 4×10^{-3} N
5. 2×10^{-6} m
6. $\frac{2}{81} r^4 \left(\frac{\rho - 1}{\eta}\right)^2 g$
7. 0.033 cm
8. 1590, laminar
9. $\frac{64}{81}$
10. 6 : 1

Test Your Concepts-IX (Based on Equation of Continuity, Bernoulli's Theorem and Applications)

1. $\frac{v^2}{2g}$
2. 8.6 cm
3. 80 ms^{-1}
4. $h_2 = 5H$, $R_{max} = 10H$ and $h_2 = 6H$, $R_{max} = 8\sqrt{3}H$
5. 0.5 N
6. 10 ms^{-2}, 15.5 ms^{-1}
7. 4.51 ms^{-1}
8. $y = 0.4x^4$
9. 6.43×10^{-4} $m^3 s^{-1}$
10. 3 minute, 1.5 minute
11. 10 ms^{-1}, 4×10^4 Nm^{-2}
12. $P_0 + \rho g h\left(1 - \frac{R_1^2}{r^2}\right)$
13. $\sqrt{2} - 1$
14. $\frac{A(h - h_0)}{a\sqrt{2gh_0}}$
15. 12.3 ms^{-1}
16. 4 ms^{-1} , 7.2 N , $F_{min} = 0$ N , $F_{max} = 52.2$ N
17. $\frac{A}{a}\sqrt{\frac{2H}{g}}$
18. 4.9 N
19. 1.6×10^{-4} $m^3 s^{-1}$, 20 ms^{-1}
20. $\sqrt{6}H$
21. $v = \sqrt{2gh_1}$ (along the horizontal), $t = \sqrt{\frac{2h_2}{g}}$ and $R = 2\sqrt{h_1 h_2}$
22. 2.87 ms^{-1}
23. $\frac{A}{a}\sqrt{\frac{2H}{g}} - t_0$

Test Your Concepts-X (Based on Surface Tension, Surface Energy, Excess Pressure and Capillarity)

1. 440 $dynecm^{-2}$
2. $\frac{kx - mg}{2\pi(R + r)}$
3. 2.86 cm
4. 1.4 mm
5. $\frac{Rr}{R - r}$
6. $\frac{2Tl}{g}$
7. 6 cm
8. 9×10^{-7} J
9. 4.76 mm
10. 19 mJ
11. 144×10^{-7} J
12. 10 cm
13. 3.75×10^{-6} m
14. 1.35×10^{-3} J

Single Correct Choice Type Questions

1. C	**2.** D	**3.** A	**4.** B	**5.** B	**6.** A	**7.** B	**8.** D	**9.** B	**10.** D
11. D	**12.** A	**13.** C	**14.** D	**15.** C	**16.** B	**17.** B	**18.** A	**19.** D	**20.** D
21. A	**22.** A	**23.** A	**24.** A	**25.** B	**26.** D	**27.** C	**28.** D	**29.** C	**30.** B
31. B	**32.** A	**33.** C	**34.** A	**35.** C	**36.** D	**37.** B	**38.** C	**39.** C	**40.** C
41. D	**42.** A	**43.** A	**44.** A	**45.** C	**46.** C	**47.** D	**48.** A	**49.** A	**50.** C
51. A	**52.** A	**53.** B	**54.** C	**55.** B	**56.** D	**57.** C	**58.** D	**59.** C	**60.** A
61. B	**62.** A	**63.** C	**64.** D	**65.** C	**66.** A	**67.** D	**68.** B	**69.** D	**70.** A
71. B	**72.** B	**73.** B	**74.** A	**75.** C	**76.** B	**77.** D	**78.** D	**79.** B	**80.** A
81. C	**82.** D	**83.** B	**84.** A	**85.** B	**86.** B	**87.** B	**88.** A	**89.** D	**90.** D
91. A	**92.** C	**93.** C	**94.** A	**95.** D	**96.** C	**97.** D	**98.** B	**99.** D	**100.** D
101. B	**102.** B	**103.** B	**104.** C	**105.** B	**106.** B	**107.** A	**108.** C	**109.** D	**110.** D
111. C	**112.** A	**113.** D	**114.** B	**115.** A	**116.** B	**117.** C	**118.** A	**119.** A	**120.** C
121. C	**122.** B	**123.** B	**124.** A	**125.** D	**126.** B	**127.** D	**128.** C	**129.** A	**130.** C
131. D	**132.** B	**133.** B	**134.** A	**135.** A	**136.** B	**137.** C	**138.** B	**139.** D	**140.** A
141. D	**142.** B	**143.** D	**144.** B	**145.** D	**146.** C	**147.** A	**148.** D	**149.** B	**150.** D
151. B	**152.** C	**153.** B	**154.** B	**155.** C	**156.** B	**157.** B	**158.** C	**159.** D	**160.** C
161. D	**162.** D	**163.** C	**164.** A	**165.** A	**166.** C	**167.** C	**168.** D	**169.** D	**170.** C
171. D	**172.** B	**173.** C	**174.** D	**175.** A	**176.** D	**177.** C	**178.** C	**179.** A	**180.** C
181. D	**182.** A	**183.** A	**184.** A	**185.** C	**186.** D	**187.** C	**188.** B	**189.** D	**190.** D
191. C	**192.** C	**193.** C	**194.** D	**195.** D	**196.** A	**197.** D	**198.** D	**199.** C	**200.** A
201. C	**202.** C	**203.** A	**204.** B	**205.** A	**206.** A	**207.** B	**208.** B	**209.** A	**210.** C
211. A	**212.** C	**213.** A	**214.** C	**215.** B	**216.** D	**217.** C	**218.** B	**219.** D	**220.** D
221. D	**222.** B	**223.** B	**224.** A	**225.** A	**226.** B	**227.** C	**228.** D	**229.** A	**230.** C
231. B	**232.** C	**233.** C	**234.** A	**235.** C	**236.** B	**237.** C			

Multiple Correct Choice Type Questions

1. A, C	**2.** B, C	**3.** B, D	**4.** A, C	**5.** B, C, D
6. A, C, D	**7.** B, C	**8.** B, D	**9.** A, D	**10.** A, B, D
11. B, C, D	**12.** B, C	**13.** B, D	**14.** B, C, D	**15.** B, C, D
16. A, B, C	**17.** B, C	**18.** B, C	**19.** B, D	**20.** C, D
21. A, B, C	**22.** A, B, C, D	**23.** C	**24.** B, C, D	**25.** B, C
26. A, C	**27.** B, C	**28.** A, C, D	**29.** B, D	**30.** A, B, C
31. C, D	**32.** A, C, D	**33.** A, C	**34.** A, C	**35.** B, D
36. B, C	**37.** A, C, D	**38.** B, D	**39.** B, C	**40.** B, C
41. B, C	**42.** B, C	**43.** A, C	**44.** A, B, C	**45.** A, B, D
46. A, B, C				

Reasoning Based Questions

1. A	**2.** C	**3.** B	**4.** A	**5.** A	**6.** C	**7.** D	**8.** B	**9.** A	**10.** B
11. D	**12.** A	**13.** D	**14.** B	**15.** C	**16.** A	**17.** B	**18.** B	**19.** A	**20.** C
21. C	**22.** D	**23.** C	**24.** D	**25.** A	**26.** A	**27.** A	**28.** D	**29.** A	

Linked Comprehension Type Questions

1. D	**2.** B	**3.** B	**4.** A	**5.** B	**6.** C	**7.** A	**8.** A	**9.** C	**10.** B
11. C	**12.** B	**13.** C	**14.** C	**15.** D	**16.** A	**17.** C	**18.** B	**19.** C	**20.** C
21. C	**22.** C	**23.** D	**24.** B	**25.** B	**26.** C	**27.** B	**28.** D	**29.** B	**30.** A
31. C	**32.** B	**33.** A	**34.** C	**35.** A	**36.** C	**37.** A	**38.** B	**39.** B	**40.** C
41. B	**42.** C	**43.** D							

Matrix Match/Column Match Type Questions

1. A → (p, t)	B → (q)	C → (r)	D → (s)	
2. A → (t, r)	B → (p, r)	C → (q, r)	D → (s)	
3. A → (q)	B → (r)	C → (s)	D → (p)	
4. A → (p, q, r)	B → (q, s)	C → (q)	D → (q, s)	
5. A → (p, r, s, t)	B → (p, t)	C → (p, q, r, s)	D → (p, r, s, t)	
6. A → (s)	B → (p)	C → (p)	D → (p)	
7. A → (r)	B → (q)	C → (p)	D → (s)	
8. A → (p, q, s)	B → (p, r, s)	C → (p, q, s)	D → (p)	
9. A → (r)	B → (t)	C → (p)	D → (p)	E → (q)
10. A → (q)	B → (p, q, r, s)	C → (q)	D → (p, q, r, s)	
11. A → (p)	B → (q, r)	C → (s)	D → (s)	
12. A → (r)	B → (s)	C → (q)	D → (p)	
13. A → (p)	B → (q, r, s)	C → (p, s)	D → (q, r)	
14. A → (r)	B → (p)	C → (r)	D → (p)	
15. A → (r)	B → (s)	C → (p)	D → (q)	
16. A → (p)	B → (p, s)	C → (q, s)	D → (p, r)	
17. A → (q)	B → (p)	C → (r)	D → (t)	
18. A → (q)	B → (p)	C → (r)	D → (q)	

Integer/Numerical Answer Type Questions

1. 37	**2.** 260	**3.** 4.9, 54.3	**4.** 16	**5.** 8.5, 13600, 46
6. 14	**7.** 0	**8.** 0.15, 0.13	**9.** 4.1, 82	**10.** 2.3
11. 1.91	**12.** 1025	**13.** 11.3	**14.** 6	**15.** 0.125
16. 3.54	**17.** 14, lower string	**18.** 1.3	**19.** 0.91	**20.** 50
21. 12, 100	**22.** 1.24, 779, 1.5	**23.** 2	**24.** 27	**25.** 280
26. 4	**27.** 570	**28.** 20	**29.** 250.2	**30.** 23
31. 31	**32.** 145	**33.** 4.51	**34.** 10	**35.** 0.38
36. 10.8	**37.** 34.64	**38.** 847	**39.** 2.34	**40.** 26
41. 40				

ARCHIVE: JEE MAIN

1. A	**2.** C	**3.** A	**4.** C	**5.** B	**6.** B	**7.** B	**8.** B	**9.** 101	**10.** D
11. D	**12.** A	**13.** A	**14.** C	**15.** B	**16.** D	**17.** 4.00	**18.** B	**19.** A	**20.** D
21. D	**22.** C	**23.** B	**24.** C	**25.** B	**26.** C	**27.** A	**28.** A	**29.** D	**30.** *
31. D	**32.** C	**33.** C	**34.** A	**35.** C	**36.** A	**37.** B	**38.** C	**39.** A	**40.** C
41. D	**42.** A	**43.** *	**44.** A	**45.** C	**46.** A	**47.** C	**48.** C	**49.** D	**50.** D
51. D	**52.** C	**53.** D							

* No given option is correct.

ARCHIVE: JEE ADVANCED

Single Correct Choice Type Problems

1. B	**2.** D	**3.** C	**4.** A	**5.** B	**6.** B	**7.** A	**8.** D	**9.** A	**10.** D
11. D	**12.** A	**13.** B	**14.** C	**15.** A	**16.** C	**17.** B	**18.** A	**19.** A	**20.** C

Multiple Correct Choice Type Problems

1. A, B, D	**2.** A, D	**3.** A, B	**4.** A, D	**5.** A, B, D
6. B, C				

Reasoning Based Questions

1. A

Matrix Match/Column Match Type Questions

1. (A) → (p) (B) → (p) (C) → (p) (D) → (s)

Linked Comprehension Type Questions

1. C	**2.** A	**3.** C	**4.** A	**5.** B	**6.** C	**7.** A	**8.** A

Integer/Numerical Answer Type Questions

1. 9	**2.** 3.74	**3.** 2	**4.** 6	**5.** 3	**6.** 6	**7.** 6

CHAPTER

2 Heat and Thermodynamics

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Thermometry and Thermal Expansion
(b) Calorimetry
(c) Kinetic Theory of Gases (KTG) and Ideal Gas Equation
(d) Concept of Internal Energy, Degrees of Freedom and Molar Specific Heats for Ideal Gases
(e) Work Done and First Law of Thermodynamics (FLTD)
(f) Isochoric Processes
(g) Isobaric Processes
(h) Isothermal Processes
(i) Adiabatic Processes
(j) Polytropic Processes
(k) Cyclic Processes
(l) Heat Engine and Refrigerator
(m) Conduction and Convection
(n) Radiation and its Properties
(o) Stefan's Law
(p) Newton's Law of Cooling
(q) Solar Constant and Temperature of Sun
(r) Wien's Law

All this is followed by a variety of Exercise Sets (fully solved) which contain questions as per the latest JEE pattern. At the end of Exercise Sets, a collection of problems asked previously in JEE (Main and Advanced) are also given.

THERMOMETRY, THERMAL EXPANSION AND CALORIMETRY

HEAT

A form of energy producing in us the sensation of warmth and which is responsible for the change in thermal conditions of the body. It can also be thought as a form of energy, which flows due to the maintenance of an appropriate temperature difference between the two bodies.

THERMAL EQUILIBRIUM

A system is said to be in the state of thermal equilibrium is the macroscopic variables that characterize the system remains constant with time.

Consider an ideal gas enclosed with in a rigid closed container with fixed values of pressure, volume, temperature, mass and composition which remains constant with time. If the container is insulates from its surroundings, the container is in a state of thermodynamic equilibrium or thermal equilibrium.

Conceptual Note(s)

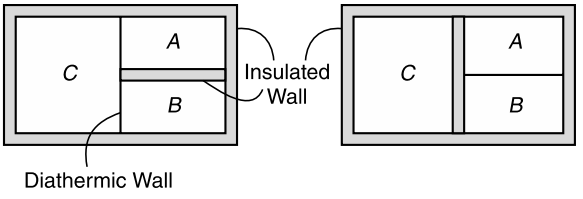
Equilibrium state of energy thermodynamic system is completely described by specific values of some macroscopic variables or state variables. The relation between the state variables is called the equation of state. The thermodynamic state variables are of two types (a) extensive and (b) intensive.

(a) Extensive variables depend on the quantities of system e.g., volume, mass etc.
(b) Intensive variables are independent of quantity of system e.g., pressure, density, etc.

ZEROTH LAW AND CONCEPT OF TEMPERATURE

Consider two system A and B separated by an adiabatic wall (an insulating wall that does not allow flow of energy (heat) through it), while each is in contact with a third sys-

tem C by a conducting wall or diathermic wall. The status of the systems changes until both A and B come to thermal equilibrium with respect to C. Now if the internal walls are interchanged, the state of the system will not change further. This means that A and B are also in thermal equilibrium with each other. This observation is the basis of **Zeroth Law of Thermodynamics**.



So, finally according to this Law, if two systems A and B are separately in thermal equilibrium with a third system C, then A and B must be in thermal equilibrium with each other. Hence to conclude there must exist certain scalar physical quantity which must be identical for all the systems in thermal equilibrium. This quantity (a scalar) is called **Temperature**.

Hence for the systems A, B and C in thermal equilibrium

$$T_A = T_B = T_C$$

It may be defined as that physical quantity which determines the degree of hotness of a body and the direction of heat flow.

THERMOMETRY AND THERMOMETERS

The branch dealing with measurement of temperature is called **Thermometry** and the devices used to measure temperature are called **Thermometers**. Any physical property, say X of the substance, which is a function of temperature, has to be taken in account for making these devices.

For a substance if X is a temperature dependent property varying linearly with temperature, then

$$\text{Temperature } T = \left(\frac{X_T - X_0}{X_{100} - X_0}\right) \times 100\ ^\circ\text{C}$$

TEMPERATURE SCALES

The centigrade (°C), Fahrenheit (°F), Kelvin (K), Reaumer (R), Rankine (R_n) are commonly used temperature scales. However, the following three scales are used very often while expressing the temperatures.

All these scales have Lower Fixed Point (L.F.P.) as melting point of ice and Upper Fixed Point (U.F.P.) as boiling point of water or the steam point. All of these scales are having (U.F.P. – L.F.P.) number of equal divisions.

The ice point is defined as the equilibrium temperature of a mixture of ice and water at a pressure of one atmosphere and the steam point is defined as the equilibrium temperature of water and steam at a pressure of one atmosphere.

The Celsius Scale

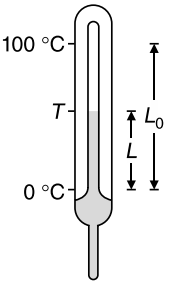
Devised by Anders Celsius in the year 1710, the interval between the lower fixed point and the upper fixed points is divided into 100 equal parts. Each division of the scale is called one degree centigrade or one degree Celsius i.e., 1 °C.

At normal pressure, the melting point of ice is 0 °C and is called the lower fixed point of the Celsius scale.

At normal pressure, the boiling point of water is 100 °C and is called the upper fixed point of the Celsius scale.

Assuming the cross-section of the thermometer capillary to be uniform and rate of expansion of the liquid with change in temperature to be constant, then we divide the distance between ice point and steam point into 100 equal parts and each part is then called as one degree. At any temperature the level of the liquid to the nearest mark can be easily compared. This scale was originally known as the centigrade scale because it has one hundred divisions between the principal reference marks.

A Celsius scale thermometer at temperature T, at which the liquid is extended to a distance L beyond the zero position is shown in Figure.



The temperature T is given by

$$T = \left(\frac{L}{L_0}\right) \times 100$$

Although the Celsius temperature scale is widely used, there is no logic behind choosing the ice point to be 0° and the steam point to be 100°.

The Fahrenheit Scale

This scale was devised by Gabriel Fahrenheit in the year 1717. The interval between the lower and the upper fixed points is divided into 180 equal parts. Each division of this scale is called one degree Fahrenheit (1 °F). On this scale, the melting point of ice at normal pressure is 32 °F. This is the lower fixed point. The boiling point of water at normal pressure is taken as 212 °F. This is the upper fixed point.

It is easy to transform temperature from one system into temperature on the other system. The general transformation formula used for this is

$$\frac{T_{\text{on X Scale}} - (\text{LFP})_{\text{on X Scale}}}{(\text{UFP})_{\text{on X Scale}} - (\text{LFP})_{\text{on X Scale}}} = \text{constant}$$

The above formula, when used to transform temperature T_C on Celsius scale to its value T_F on Fahrenheit scale gives

$$\frac{T_C - 0}{100 - 0} = \frac{T_F - 32}{212 - 32}$$

$$\Rightarrow \quad T_F = \frac{9}{5} T_C + 32$$

The Kelvin Scale

However, one temperature scale has more fundamental choice of zero point. This was given by Lord Kelvin from the study of gases. Kelvin scale uses intervals equal to those of the Celsius degree but with zero set at the lowest theoretical temperature that a gas can reach. The scale is based on the fact that a gas at 0 °C will lose 1/273.15 of its volume for a 1 °C drop in temperature. If this reduction in volume were to continue with decreasing temperature and if the gas did not liquefy, the volume would become zero at –273.15 °C. *This is a temperature called absolute zero.* The temperature scale based on this zero is the kelvin temperature scale. Since,

$$\frac{T_{\text{on X Scale}} - (\text{LFP})_{\text{on X Scale}}}{(\text{UFP})_{\text{on X Scale}} - (\text{LFP})_{\text{on X Scale}}} = \text{constant}$$

So, the relation between temperatures measured on Celsius scale T_C and Kelvin Scale T_K is given by

$$\frac{T_C - 0}{100 - 0} = \frac{T_K - 273.15}{373.15 - 273.15}$$

$$\Rightarrow \quad T_K = T_C + 273.15$$

This temperature in Kelvin scale is used with a unit of Kelvin (K) and it is not written with a degree sign. The temperature on Kelvin is also called absolute temperature.

The Reaumer Scale

This scale was devised by R. A. Reaumer in the year 1730. The interval between the lower and the upper fixed point is divided into 80 equal parts. Each division is called one degree Reaumer (1 °R). On this scale, the melting point of ice at normal pressure 0 °R. This is lower fixed point. The boiling point of water at normal pressure is 80 °R. This is the upper fixed point.

TEMPERATURE RELATION BETWEEN SCALES

All these temperatures (given in the table) are related to each other by the following relationship

Temperature Scale	°C	°F	K	°R	R_N	Any Scale
L.F.P.	0	32	273	0	492	T_L
U.F.P.	100	212	373	80	672	T_U
Number of Divisions (N)	100	180	100	80	180	$T_U - T_L$

$$\frac{T_C - 0}{100 - 0} = \frac{T_F - 32}{212 - 32} = \frac{T - 273}{373 - 273} = \frac{T_R - 0}{80 - 0} = \frac{T_{R_n} - 492}{672 - 492} = \frac{T - T_L}{T_U - T_L}$$

$$\Rightarrow \quad \frac{T_C - 0}{100} = \frac{T_F - 32}{180} = \frac{T - 273}{100} = \frac{T_R - 0}{80} = \frac{T_{R_n} - 492}{180} = \frac{T - T_L}{T_U - T_L}$$

The temperature increments on these scales are related to each other as

$$\frac{\Delta T_C}{100} = \frac{\Delta T_F}{180} = \frac{\Delta T}{100}$$

Problem Solving Technique(s)

(a) Rise in temperature is same for Celsius and Kelvin scale.

(b) Whenever, ratio of the temperatures has to be taken do not forget to convert the temperatures to the absolute Kelvin scale. Like for example if we are asked to take the ratio of the temperatures say $T_1 = 10$ °C and $T_2 = 5$ °C, then the ratio $\frac{T_1}{T_2} \neq 2$.

Instead of that the ratio is

$$\frac{T_1}{T_2} = \frac{10 + 273}{5 + 273} = \frac{283}{278} = 1.02$$

and this ratio happens to be very close to 1 (not 2). So please take care while expressing the temperature ratios.

(c) In both the Fahrenheit and Celsius temperature scale, the assignment of the zero point is arbitrary and temperatures below these zero points are achievable.

ILLUSTRATION 1

An iron piece is heated from 30 °C to 90 °C. Find the change in its temperature on the Fahrenheit and the kelvin scale.

SOLUTION

Since, we have seen that $\frac{\Delta T_C}{100} = \frac{\Delta T_F}{180} = \frac{\Delta T}{100}$

$$\Rightarrow \quad \Delta T_F = \frac{9}{5}\Delta T_C = \frac{9}{5}(90 - 30) = 108 \text{ °F}$$

Further, rise in temperature for both the celcius and the kelvin scale is the same, so $\Delta T = \Delta T_C = 60$ K

ILLUSTRATION 2

Express a temperature of 60 °F in degree celcius and in kelvin scale.

SOLUTION

Since, we know that $\frac{T_F - 32}{180} = \frac{T_C - 0}{100} = \frac{T - 273}{100}$

$$\Rightarrow \quad T_C = \frac{100}{180}(F - 32) = \frac{5}{9}(60 - 32) = 15.55 \text{ °C}$$

Since, $T = T_C + 273 = 15.55 + 273 = 288.55$ K

TYPES OF THERMOMETERS (OPTIONAL READING)

Liquid Thermometer

The functioning of these thermometers is based on the principle of expansion of liquids on heating e.g., mercury thermometer, alcohol thermometer etc. Mercury is preferably used in thermometers since it possesses low specific heat and high conductivity. The range of Hg thermometer is –30 °C to 357 °C.

Gas Thermometer

These thermometers are based on Gay Lussac's Law according to which temperature of the gas (T) is directly proportional to its pressure (P) at constant volume (V) i.e., $P \propto T$.

If P_0 be the pressure of the gas at 0 °C, P_{100} be the pressure of the gas at 100 °C and P_t be the pressure of the gas at t °C, all at constant volume, then

$$t = \left(\frac{P_t - P_0}{P_{100} - P_0}\right) \times 100\ ^\circ\text{C}$$

Gas thermometer measure temperature ranging from –268 °C to 1500 °C

Resistance Thermometers

These thermometers employ the variation of resistance of metals with the change in temperature. If R_t and R_0 are resistances of a metal at t °C and 0 °C then experimentally it has been checked that $R_t = R_o(1+\alpha t)$ where α is called **Temperature coefficient of resistance** and has unit $(^\circ\text{C})^{-1}$ or K^{-1}.

Platinum is used in resistance thermometers as it has high melting point and variation in resistance for pure platinum is quite large and is uniform throughout the range –200 °C to 1200 °C. The unknown temperature t is calculated from the equation

$$t = \left(\frac{R_t - R_0}{R_{100} - R_0}\right) \times 100\ ^\circ\text{C}$$

But if we do not know R_0 we calculate the resistance of metal R_1 and R_2 at temperatures t_1 and t_2 respectively.

$$\Rightarrow \quad R_1 = R_0(1+\alpha t_1) \qquad \ldots(1)$$

$$\Rightarrow \quad R_2 = R_0(1+\alpha t_2) \qquad \ldots(2)$$

Dividing (1) and (2) and rearranging, we get

$$\alpha = \frac{R_2 - R_1}{R_1 t_2 - R_2 t_1}$$

Thermocouple Thermometers

These types of thermometers are based on phenomenon of **Seebeck effect**.

According to this effect "when the distinct metals are joined (together called **thermocouple**) and a temperature difference is maintained across its two junctions then an e.m.f. is developed in the thermocouple". This developed e.m.f. is called **thermo-emf** and is a function of temperature.

$$\xi = At + Bt^2 \text{ (in volt)}$$

where A and B are constants and depend on the nature of metals selected to form a thermocouple. Thermocouple thermometers have range varying from –200 °C to 1600 °C with different thermocouples.

Total Radiation Pyrometers

Pyrometers are capable of measuring the temperatures (however high) irrespective of the distance of source from the pyrometer. These are based on **Stefan's Law** according to which "the radiant energy emitted per second per unit area (E) of a black body is directly proportional to the fourth power of absolute temperature (T) i.e. $E \propto T^4$

$$\Rightarrow \quad E = \sigma T^4$$

where σ is the constant of proportionality called **Stefan's Constant** whose value is $5.67 \times 10^{-8}\ \text{Wm}^{-2}\,\text{K}^{-4}$. The minimum temperature it measures is 800 °C.

Disappearing Filament Pyrometers

These are based on the principle that the filament of the bulb when seen through a red filter disappears when its temperature is equal to temperature of distant object emitting radiation. Its temperature measuring range varies from 600 °C to 2700 °C.

Vapour Pressure Thermometer

This thermometer is based on the fact that the saturated vapour pressure of a liquid depends upon its temperature and is related to temperature as

$$\log P = a + bT - \frac{c}{T}$$

where a, b and c are constants and T is temperature on absolute scale. On using liquid Helium, the minimum attainable temperature using this device is 0.71 K. Making use of other liquid vapours it can measure temperatures upto 122 K.

Magnetic Thermometer

It is used in measuring temperatures close to 0 K.

Problem Solving Technique(s)

The International Practical Temperature scale was developed in 1927 to provide a temperature scale which had an easy utilisation for practical purposes i.e., calibration of industrial and scientific instruments and was the best possible approximation to Kelvin scale.

This fixed point in thermometry is the triple point of water i.e., 273.16 K

THERMAL EXPANSION

It has been a general observation that heating a body is accompanied by an increase in the size of the body, be its length, breadth, thickness (obviously area and volume too). This phenomenon is called the **Phenomenon of Thermal Expansion**.

On heating a body if its length increases, we call it linear expansion, if its surface area increases, we call it **Superficial Expansion** or **Areal Expansion**, if its volume increases, we call it **Volume Expansion** or **Cubical Expansion**.

The coefficients of linear expansion, superficial expansion and volume expansion are denoted respectively by α, β and γ.

If L_0, A_0 and V_0 be the length, surface area and volume respectively at temperature T_0 and L, A and V be the respective quantities at temperature $(T_0 + \Delta T)$, where ΔT is the rise in temperature, then the average values of α, β and γ are

$$\langle \alpha \rangle = \frac{1}{L_0}\left(\frac{\Delta L}{\Delta T}\right), \langle \beta \rangle = \frac{1}{A_0}\left(\frac{\Delta A}{\Delta T}\right), \langle \gamma \rangle = \frac{1}{V_0}\left(\frac{\Delta V}{\Delta T}\right)$$

Their instantaneous values can be found by taking the limit $\Delta T \to 0$.

$$\alpha = \frac{1}{L_0}\lim_{\Delta T \to 0}\left(\frac{\Delta L}{\Delta T}\right) = \frac{1}{L_0}\left(\frac{dL}{dT}\right)$$

$$\beta = \frac{1}{A_0}\lim_{\Delta T \to 0}\left(\frac{\Delta A}{\Delta T}\right) = \frac{1}{A_0}\left(\frac{dA}{dT}\right)$$

$$\gamma = \frac{1}{V_0}\lim_{\Delta T \to 0}\left(\frac{\Delta V}{\Delta T}\right) = \frac{1}{V_0}\left(\frac{dV}{dT}\right)$$

For most substances, values of α, β and γ are independent of temperature as long as temperature variations are not large. For large temperature variations they start varying as a function of temperature.

If the length of a rod is L_0 at 0 °C and L_T at T °C, then for small α assumed to be a constant over the given temperature interval, we have

$$\alpha = \frac{L_T - L_0}{L_0 T}$$

$$\Rightarrow \quad L_T = L_0(1 + \alpha T)$$

This equation the most used one for solving problems related to linear expansion.

From above we also conclude that the coefficient of thermal expansion is defined as the change in dimension per unit original dimension for a unit rise of temperature. Unit of α, β, γ is $(°C)^{-1}$ or K^{-1}.

For the normal variations in the temperature, we have

$$\frac{\alpha}{1} = \frac{\beta}{2} = \frac{\gamma}{3}$$

So, we can conclude that $\beta = 2\alpha$ and $\gamma = 3\alpha$.

Problem Solving Technique(s)

(a) The values of α, β and γ are identical for both Celcius scale and Kelvin scale. This is due to the fact that rise in temperature is same on both the scales.

(b) If a solid object has a hole in it, what happens to the size of the hole, when the temperature of the object increases?

A common misconception is that if the object expands, the hole will shrink because material expands into the hole. But the truth is that if the object expands, the hole will expand too, because every linear dimension of an object changes in the same way when the temperature changes.

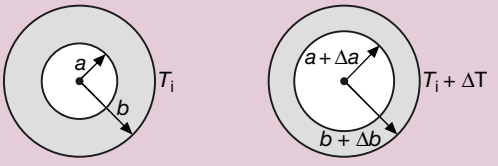


ILLUSTRATION 3

In an aluminium sheet, there is a hole of diameter 2 m and the sheet is mounted horizontally on a stand. On this hole, an iron sphere of diameter 2.004 m is placed. Initial temperature of this system is 25 °C. When the arrangement is heated, calculate the temperature at which the iron sphere will fall down through the hole in sheet. The coefficients of linear expansion for aluminium and iron are 2.4×10^{-5} and 1.2×10^{-5} respectively.

SOLUTION

Since coefficient of linear expansion for aluminium is more than that of iron, so it expands faster than iron. On heating the arrangement, at some higher temperature, say T, when the diameter of hole becomes exactly equal to that of the iron sphere, then the sphere will pass through the hole. So,

$$(\text{Diameter of Hole})_{Al} = (\text{Diameter of Sphere})_{\text{iron}}$$

$$\Rightarrow \quad 2[1 + \alpha_{Al}(T - 25)] = 2.004[1 + \alpha_{\text{iron}}(T - 25)]$$

$$\Rightarrow \quad 2\alpha_{Al}(T - 25) = 0.004 + 2.004\alpha_{\text{iron}}(T - 25)$$

$$\Rightarrow \quad T = \left(\frac{0.004}{2\alpha_{Al} - 2.004\alpha_{\text{iron}}} + 25\right) °C$$

$$\Rightarrow \quad T = \frac{0.004}{2 \times 2.4 \times 10^{-5} - 2.004 \times 1.2 \times 10^{-5}} + 25$$

$$\Rightarrow \quad T = 191.7 \text{ °C}$$

ILLUSTRATION 4

A rod AB of length l is pivoted at an end A and freely rotated in a horizontal plane at an angular speed ω about a vertical axis passing through A. If coefficient of linear

expansion of material of rod is α, find the percentage change in its angular velocity if temperature of system is increased by ΔT without mechanically disturbing the system.

SOLUTION

Since there is no mechanical disturbance, so no torque acts on the system and hence angular momentum is constant.

$$I\omega = \text{constant}$$

$$\Rightarrow \quad \Delta(I\omega) = 0$$

$$\Rightarrow \quad I\Delta\omega + \omega\Delta I = 0$$

$$\Rightarrow \quad \frac{\Delta\omega}{\omega} = -\frac{\Delta I}{I} \qquad \ldots(1)$$

where, $I = \frac{Ml^2}{3}$ is the moment of inertia of the rod about the specified axis of rotation.

On heating the rod, its length will change, so its moment of inertia also changes.

The fractional change in the length of the rod is

$$\frac{\Delta l}{l} = \alpha\Delta T \qquad \{\text{by definition}\}$$

Also, we have $I \propto l^2$

$$\Rightarrow \quad \frac{\Delta I}{I} = 2\frac{\Delta l}{l} \qquad \{\text{for small increments}\}$$

$$\Rightarrow \quad \frac{\Delta I}{I} = 2\alpha\Delta T \qquad \ldots(2)$$

From equations (1) and (2), we get

$$\frac{\Delta\omega}{\omega} = -\frac{\Delta I}{I} = -2\frac{\Delta l}{l} = -2\alpha\Delta T$$

So, percentage change in angular velocity of rod due to heating is

$$\left|\frac{\Delta\omega}{\omega}\right| \times 100\% = \frac{\omega - \omega'}{\omega} \times 100\%$$

$$\Rightarrow \quad \left|\frac{\Delta\omega}{\omega}\right| \times 100\% = (2\alpha\Delta T) \times 100\%$$

ILLUSTRATION 5

Two rods of lengths l_1 and l_2 are made of materials having coefficients of linear expansion α_1 and α_2 respectively. For what relation between above values, the difference in the lengths of the two rods is independent of the variation in temperature?

SOLUTION

The difference in length is

$$l_1 - l_2 = \text{constant} \qquad \{\text{assuming } l_1 > l_2\}$$

As $l_1 - l_2$ is independent of temperature, so we have

$$\Delta(l_1 - l_2) = 0$$

$$\Rightarrow \quad \Delta l_1 - \Delta l_2 = 0$$

Since, we know that $\Delta l = l\alpha\Delta T$

$$\Rightarrow \quad l_1\alpha_1\Delta T - l_2\alpha_2\Delta T = 0$$

$$\Rightarrow \quad l_1\alpha_1 = l_2\alpha_2$$

ILLUSTRATION 6

An iron wire and a copper wire are marked 100 m at 20 °C. Calculate the difference in lengths of the wires at 60 °C. Given that the coefficients of linear expansion for iron and copper are 1.2×10^{-5} K^{-1} and 1.7×10^{-5} K^{-1}.

SOLUTION

For iron wire, we have

$$l_{1i} = l_{0i}(1 + \alpha_i t_1) \qquad \ldots(1)$$

and $$l_{2i} = l_{0i}(1 + \alpha_i t_2) \qquad \ldots(2)$$

The elongation in the iron wire is

$$l_{2i} - l_{1i} = l_{0i}\alpha_i(t_2 - t_1) \qquad \ldots(3)$$

From equation (1), substituting $l_{0i} = \frac{l_{1i}}{(1 + \alpha_i t_1)}$ in equation (3), we get elongation in iron wire to be

$$l_{2i} - l_{1i} = \frac{l_{1i}\alpha_i(t_2 - t_1)}{(1 + \alpha_i t_1)} \qquad \ldots(4)$$

Similarly, elongation in copper wire is given by

$$l_{2c} - l_{1c} = \frac{l_{1c}\alpha_c(t_2 - t_1)}{(1 + \alpha_c t_1)} \qquad \ldots(5)$$

where, $l_{1i} = l_{1c} = l_1 = 100$ m

Subtracting (4) from (5), we get

$$l_{2c} - l_{2i} = l_1\frac{(\alpha_c - \alpha_i)(t_2 - t_1)}{(1 + \alpha_c t_1)(1 + \alpha_i t_1)} = 19.9 \text{ mm}$$

For low values of temperature t i.e., when $\alpha t < 1$, it is not necessary to reduce l_1 and l_2 to l_{01} and l_{02} at $t = 0$ °C. To a sufficiently high degree of accuracy, we can assume that $\Delta l = l\alpha\Delta t$. Under this assumption, the problem can be solved in a simpler and quicker way. Since

$$\Delta l_i = l_{1i}\alpha_i\Delta T \text{ and } \Delta l_c = l_{1c}\alpha_c\Delta T$$

where, $l_{1i} = l_{1c} = l_1$. So, we have

$$\Delta l = \Delta l_c - \Delta l_i = l_c(t_2 - t_1)(\alpha_c - \alpha_i) = 20 \text{ mm}$$

It can be seen that the deviation from a more exact value of 19.9 mm amounts to 0.1 mm, which corresponds to a relative error of

$$\text{Relative Error} = \frac{0.1}{19.9} \times 100\% = 0.5\%$$

ILLUSTRATION 7

A steel ball initially at a pressure of 10^5 Pa is heated from 20 °C to 120 °C keeping its volume constant. Find the final pressure inside the ball. Given that coefficient of linear expansion of steel is 1.1×10^{-5} °C^{-1} and Bulk modulus of steel is 1.6×10^{11} Nm^{-2}.

SOLUTION

On increasing temperature of ball by 100 °C (from 20 °C to 120 °C), the thermal expansion in its volume can be given as

$$\Delta V = V\gamma_s \Delta T = V(3\alpha_s)\Delta T \quad \ldots(1)$$

Since it is given that no change is volume is allowed, so the increment in volume due to thermal expansion is cancelled due to elastic compression by external pressure.
The Bulk's modulus of a material is defined as

$$B = \frac{\Delta P}{\Delta V/V}$$

The excess pressure inside the ball at 120 °C to keep its volume constant during the heating process is

$$\Delta P = B\left(\frac{\Delta V}{V}\right) = B(3\alpha_s \Delta T)$$

$$\Rightarrow \quad \Delta P = (1.6 \times 10^{11})\left[3(1.1 \times 10^{-5})(100)\right]$$

$$\Rightarrow \quad \Delta P = 5.28 \times 10^8 \text{ Nm}^{-2} = 5.28 \times 10^8 \text{ Pa}$$

ILLUSTRATION 8

A metal rod A of length 25 cm expands by 0.05 cm, when its temperature is raised from 0 °C to 100 °C. Another rod B of a different metal of length 40 cm expands by 0.04 cm for the same rise in temperature. A third rod C of length 50 cm made up of pieces of rods A and B placed end to end expands by 0.03 cm on heating from 0 °C to 50 °C. Find the lengths of each portion of the composite rod.

SOLUTION

For Rod A, we have $0.050 = 25\alpha_A(100)$

$$\Rightarrow \quad \alpha_A = 2 \times 10^{-5}(^\circ\text{C})^{-1} \quad \ldots(1)$$

Also, for Rod B, we have $0.040 = 40\alpha_B(100)$

$$\Rightarrow \quad \alpha_B = 10^{-5}(^\circ\text{C})^{-1} \quad \ldots(2)$$

Now, a third rod of length 50 cm is made from A and B, so

$$l_A + l_B = 50 \text{ cm} \quad \ldots(3)$$

$$\Rightarrow \quad \Delta l_A + \Delta l_B = 0.03 \text{ cm}$$

$$\Rightarrow \quad l_A\alpha_A(50) + l_B\alpha_B(50) = 0.03$$

$$\Rightarrow \quad l_A(2 \times 10^{-5})(50) + l_B(10^{-5})(50) = 0.03$$

$$\Rightarrow \quad 2l_A + l_B = \frac{0.03}{50 \times 10^{-5}}$$

$$\Rightarrow \quad 2l_A + l_B = \frac{3 \times 10^{-2} \times 10^5}{50} = 60 \text{ cm} \quad \ldots(4)$$

Subtracting (3) from (4), we get

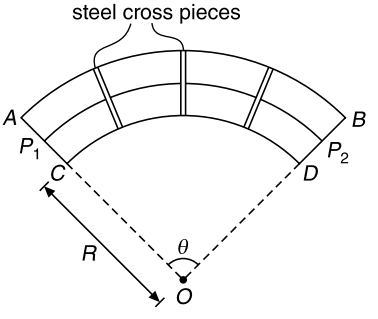
$$l_A = 10 \text{ cm and } l_B = 40 \text{ cm}$$

ILLUSTRATION 9

Two straight thin bars, one of brass and the other of steel are joined together side by side by short steel cross-pieces one cm long at 0 °C. When heated to 100 °C, the composite bar becomes bent into the arc of a circle. If α for brass is 19×10^{-6} (°C)$^{-1}$ and that for steel is 11×10^{-6} (°C)$^{-1}$, then calculate the radius of this circle.

SOLUTION

Since the expansion of brass is more than steel, hence the combination will bend with brass bar AB on the outside and steel wire CD on the inside as shown in Figure.



Let $OC = R$ and $\angle COD = \theta$

Length of steel cross pieces at 100 °C is

$$l_{\text{cross piece}} = 1(1 + 11 \times 10^{-6} \times 100) = 1.0011 \text{ cm}$$

So, $OA = (R + 1.0011)$ cm

Also, we see that $CD = R\theta$ $\ldots(1)$

and $AB = (R + 1.0011)\theta$ $\ldots(2)$

Since, $AB = L_0(1 + 19 \times 10^{-6} \times 100)$ $\ldots(3)$

and $CD = L_0(1 + 11 \times 10^{-6} \times 100)$ $\ldots(4)$

From equation (1), (2), (3) and (4), we get

$$\frac{AB}{CD}=\frac{R+1.0011}{R}=\frac{1+(19\times10^{-6})(100)}{1+(11\times10^{-6})(100)}$$

$$\Rightarrow \quad 1+\frac{1.001}{R}=\left(\frac{1+19\times10^{-4}}{1+11\times10^{-4}}\right)\left(\frac{1.0011}{R}\right)=\frac{8\times10^{-4}}{1+11\times10^{-4}}$$

$$\Rightarrow \quad R=(1.0011)\left(\frac{1+11\times10^{-4}}{8\times10^{-4}}\right)=1252.8 \text{ cm}$$

ILLUSTRATION 10

If a solid has coefficients of linear expansion α_x, α_y and α_z for three mutually perpendicular directions in a solid, what is the coefficient of volume expansion γ for the solid?

SOLUTION

Consider a cube with edges parallel to x, y and z of dimension l_0 at $T=0$. After a change in temperature $\Delta T=(T-0)$, the dimensions change to $l_x=l_0(1+\alpha_x T)$, $l_y=l_0(1+\alpha_y T)$ and $l_z=l_0(1+\alpha_z T)$.

The volume of the cuboid is given by

$$V=l_0^3(1+\alpha_x T)(1+\alpha_y T)(1+\alpha_z T)$$

$$\Rightarrow \quad V\approx V_0\left[1+(\alpha_x+\alpha_y+\alpha_z)T\right] \qquad \{\because V_0=l^3\}$$

So, coefficient of volume expansion is given by

$$\gamma=\alpha_x+\alpha_y+\alpha_z$$

ILLUSTRATION 11

A glass window is to be fitted in an aluminium frame. The temperature on the working day is 40 °C and the glass piece is a rectangle of sides 30 cm by 20 cm. Find the size of the aluminium frame so that in winters glass does not experience any stress when temperature drops to 0 °C. Given that coefficient of linear expansion for glass is 9×10^{-6} °C^{-1} and that for aluminium is 2.4×10^{-5} °C^{-1}.

SOLUTION

At 40 °C, the dimensions of glass in winters are

$$l=30(1-9\times10^{-6}\times40)=29.989 \text{ cm}$$

$$b=20(1-9\times10^{-6}\times40)=19.993 \text{ cm}$$

So, dimensions of aluminium frame at 40 °C are

$$l_1=29.989\times(1+2.4\times10^{-5}\times40) \text{ cm}=30.018 \text{ cm}$$

$$b_1=19.993(1+2.4\times10^{-5}\times40) \text{ cm}=20.012 \text{ cm}$$

ILLUSTRATION 12

An isosceles triangle is formed with a rod of length l_1 and coefficient of linear expansion α_1 for the base and two thin rods each of length l_2 and coefficient of linear expansion α_2 for the two pieces, if the distance between the apex and the midpoint of the base remain unchanged as the temperature is varied show that $\frac{l_1}{l_2}=2\sqrt{\frac{\alpha_2}{\alpha_1}}$.

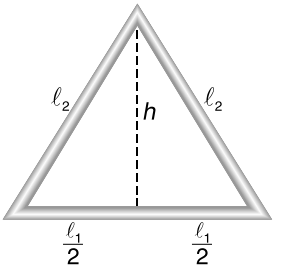
SOLUTION

According to the problem, we have h = constant

$$\Rightarrow \quad h^2=l_2^2-\frac{l_1^2}{4}=\text{constant}$$

$$\Rightarrow \quad \Delta\left(l_2^2\right)-\Delta\left(\frac{l_1^2}{4}\right)=0$$

$$\Rightarrow \quad 2l_2\Delta l_2-\frac{1}{4}2l_1\Delta l_1=0$$



Since, by definition, $\Delta l=l\alpha\Delta T$

$$\Rightarrow \quad \Delta l_2=l_2\alpha_2\Delta T \text{ and } \Delta l_1=l_1\alpha_1\Delta T$$

$$\Rightarrow \quad 2l_2(l_2\alpha_2\Delta T)=\frac{1}{4}(2l_1)(l_1\alpha_1\Delta T)$$

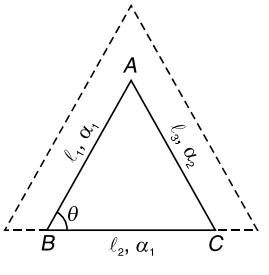
$$\Rightarrow \quad l_2^2\alpha_2=\frac{1}{4}l_1^2\alpha_1$$

$$\Rightarrow \quad \frac{l_1^2}{l_2^2}=4\left(\frac{\alpha_2}{\alpha_1}\right)$$

$$\Rightarrow \quad \frac{l_1}{l_2}=2\sqrt{\frac{\alpha_2}{\alpha_1}}$$

ILLUSTRATION 13

Three rods A, B and C for an equilateral triangle at 0 °C. Rods AB and BC have same coefficient of expansion α_1 and rod AC has α_2. If temperature of the system is increased to T °C, what is the change in angle θ formed by rods AB and BC?



SOLUTION

After increase of temperature, the triangle will not be equilateral. Let the lengths of the three rods be l_1, l_2 and l_3. From Cosine Law, we have

$$\cos\theta=\frac{l_1^2+l_2^2-l_3^2}{2l_1l_2}$$

$$\Rightarrow \quad 2l_1l_2\cos\theta=l_1^2+l_2^2-l_3^2 \qquad \ldots(1)$$

Differentiating equation (1), we get

$$2l_1\cos\theta dl_2 + 2l_2\cos\theta dl_1 - 2l_1l_2\sin\theta d\theta =$$

$$2l_1dl_1 + 2l_2dl_2 - 2l_3dl_3 \quad \ldots(2)$$

The changes in length of respective rods due to temperature rise are

$$dl_1 = l_1\alpha_1\Delta T$$

$$dl_2 = l_2\alpha_1\Delta T$$

$$dl_3 = l_3\alpha_2\Delta T \quad \ldots(3)$$

For an equilateral triangle, we have $\ell_1 = \ell_2 = \ell_3 = \ell$ and $\theta = 60°$. So, from equations (2) and (3), we get

$$2l^2\alpha_1\Delta T\cos 60° + 2l^2\alpha_1\Delta T\cos 60° - 2l^2\sin 60°d\theta =$$

$$2\ell^2\alpha_1\Delta T + 2\ell^2\alpha_1\Delta T - 2\ell^2\alpha_2\Delta T$$

$$\Rightarrow \quad \frac{\alpha_1}{2}\Delta T + \frac{\alpha_1}{2}\Delta T - \frac{\sqrt{3}}{2}d\theta = \alpha_1\Delta T + \alpha_1\Delta T - \alpha_2\Delta T$$

$$\Rightarrow \quad \alpha_1\Delta T - \frac{\sqrt{3}}{2}d\theta = 2\alpha_1\Delta T - \alpha_2\Delta T$$

$$\Rightarrow \quad (\alpha_2 - \alpha_1)\Delta T = \frac{\sqrt{3}}{2}d\theta$$

$$\Rightarrow \quad d\theta = \frac{2}{\sqrt{3}}(\alpha_2 - \alpha_1)T \qquad \{\because \Delta T = T\}$$

EFFECT OF TEMPERATURE ON PENDULUM CLOCKS

A pendulum clock consists of a metal bar attached with a bob at one end and fixed at the other end. The length of pendulum depends on temperature, and hence time period of clock depends on temperature. Let pendulum clock read correct time when its length is l_0. Its time period is $t_0 = 2\pi\sqrt{\frac{l_0}{g}}$. Now suppose that the temperature is raised by ΔT, then new time period becomes $t' = 2\pi\sqrt{\frac{l}{g}}$.

$$\Rightarrow \quad \frac{t'}{t} = \sqrt{\frac{l}{l_0}} \quad \ldots(1)$$

Since $l = l_0(1+\alpha\Delta T)$, so $\frac{l}{l_0} = 1+\alpha\Delta T$

$$\Rightarrow \quad \frac{t}{t_0} = (1+\alpha\Delta T)^{1/2} \qquad \{\text{from (1)}\}$$

For small $\alpha\Delta T$, we have $(1+\alpha\Delta T)^{1/2} \approx 1+\frac{1}{2}\alpha\Delta T$

$$\Rightarrow \quad \frac{t}{t_0} = 1+\frac{1}{2}\alpha\Delta T$$

$$\Rightarrow \quad \frac{t}{t_0} - 1 = \frac{t-t_0}{t_0} = \frac{1}{2}\alpha\Delta T$$

So, fractional loss in time $\frac{\Delta t}{t_0} = \frac{1}{2}\alpha\Delta T$

Number of seconds in one day is 86400 s

So, time lost per day is $\Delta t = \frac{1}{2}\alpha\Delta T \times 86400$ s

ILLUSTRATION 14

A pendulum gives correct time at 20 °C at a place where $g = 9.8\ \text{ms}^{-2}$. The pendulum consists of a light steel rod connected to a heavy ball. If it is taken to a different place where $g = 9.788\ \text{ms}^{-2}$. At what temperature will it give correct time? Coefficient of linear expansion of steel is $12\times10^{-6}\ °\text{C}^{-1}$.

SOLUTION

Since $t = 2\pi\sqrt{\frac{l}{g}}$ and for a pendulum to keep correct time, its period must be 2 s.

$$\Rightarrow \quad 2 = 2\pi\sqrt{\frac{l_0}{9.8}} \quad \ldots(1)$$

$$\Rightarrow \quad 2 = 2\pi\sqrt{\frac{l_0\left[1+12\times10^{-6}\times(T-20)\right]}{9.788}} \quad \ldots(2)$$

Equating (1) and (2), we get

$$\frac{l_0}{9.8} = \frac{l_0}{9.788}\left[1+12\times10^{-6}\times(T-20)\right]$$

$$\Rightarrow \quad \frac{9.788}{9.8} = 1+12\times10^{-6}\times(T-20)$$

$$\Rightarrow \quad \frac{9.788}{9.8} - 1 = 12\times10^{-6}\times(T-20)$$

$$\Rightarrow \quad -\frac{0.012}{9.8} = 12\times10^{-6}\times(T-20)$$

$$\Rightarrow \quad -\frac{12\times10^{-3}}{9.8} = 12\times10^{-6}(T-20)$$

$$\Rightarrow \quad T - 20 = -102$$

$$\Rightarrow \quad T = -82\ °\text{C}$$

ILLUSTRATION 15

A clock with a metallic pendulum is 5 seconds fast each day at a temperature of 15 °C and 10 seconds slow each day at a temperature of 30 °C. Find coefficient of linear expansion for the metal.

SOLUTION

The time lost per day is $\Delta t = \frac{1}{2}\alpha\Delta T \times 86400$ s

Let T_0 be the graduation temperature of clock.

At 15 °C, the clock is gaining time, so Δt = +5 s.

$$\Rightarrow \quad 5=\frac{1}{2}\alpha(T_0-15)\times 86400 \qquad \ldots(1)$$

At 30 °C, the clock is losing time, so $\Delta t = -10$ s

$$\Rightarrow \quad -10=\frac{1}{2}\alpha(T_0-30)\times 86400 \qquad \ldots(2)$$

Dividing equation (2) by (1), we get

$$2(T_0-15)=(30-T_0)$$

$$\Rightarrow \quad T_0=20\ ^\circ\text{C}$$

Substituting in equation (1), we get

$$5=\frac{1}{2}\alpha(20-15)\times 86400$$

$$\Rightarrow \quad \alpha=2.31\times 10^{-5}\ ^\circ\text{C}^{-1}$$

ILLUSTRATION 16

A pendulum clock and a digital clock both are synchronized to beep correct time at temperature 20 °C in the morning on 1st March, 2003. At 12.00 noon temperature increases to 40 °C and remains constant for three months. Now on 1st June, 2003, at 12:00 noon temperature drops to 10 °C and remains constant for a very long duration. Find the date and time on which both the clocks will again be synchronized for a moment.

SOLUTION

Since the digital clock (which is ideal) always keeps correct time. On increasing temperature on 1st March 12:00 noon, the pendulum clock slows down and start losing time. If α be the coefficient of linear expansion of the material of pendulum, then the fractional loss in time is

$$\frac{\Delta t}{t}=\frac{1}{2}\alpha\Delta T$$

In three months of March, April and May, we have 92 days, so the loss in time during these 92 days is given by

$$\Delta t_{92\text{ days}}=\frac{1}{2}\alpha(40-20)(92\times 86400)\text{ s} \qquad \ldots(1)$$

Now, on 1st June 12:00 noon, the temperature drops to 10 °C which is 10° less than the temperature at which clock keeps correct time. So, now the clock starts gaining time.

If after N days it gains exactly the time lost during previous three months i.e., 92 days then it shows right time at that moment.

Time gained by the clock in N days is

$$\Delta t_{N\text{ days}}=\frac{1}{2}\alpha(20-10)(N\times 86400)\text{ s} \qquad \ldots(2)$$

Equating equations (1) and (2), we get

$$|\Delta t_{92\text{ days}}|=|\Delta t_{N\text{ days}}|$$

$$\Rightarrow \quad \frac{1}{2}\alpha\times 20\times 92\times 86400=\frac{1}{2}\alpha\times 10\times N\times 86400$$

$$\Rightarrow \quad N=184\text{ days}$$

So, after 184 days from 1st June 2003, the pendulum clock will show correct time and both the clocks will be in synchronization for a moment and after 184 days means the date is 2nd December 2003 and time is 12:00 noon.

ILLUSTRATION 17

A clock with a metallic pendulum gains 6 seconds each day when the temperature is 20 °C and loses 6 second when the temperature is 40 °C. Find the coefficient of linear expansion of the metal.

SOLUTION

For simple pendulum, $t=2\pi\sqrt{\frac{l}{g}}$ i.e., $T\propto l^{\frac{1}{2}}$

$$\Rightarrow \quad \frac{\Delta t}{t}=\frac{1}{2}\frac{\Delta l}{l}=\frac{1}{2}\alpha\Delta T$$

Assuming that the clock gives correct time at temperature T_0, then we have

$$\frac{6}{24\times 3600}=\frac{1}{2}\alpha(T_0-20) \qquad \ldots(1)$$

and $$\frac{(-6)}{24\times 3600}=\frac{1}{2}\alpha(T_0-40) \qquad \ldots(2)$$

From equations (1) and (2), we get $T_0 = 30$ °C

$$\Rightarrow \quad \alpha=1.4\times 10^{-5}\ ^\circ\text{C}^{-1}$$

ERROR IN METAL SCALE DUE TO EXPANSION OR CONTRACTION

A metal scale expands with rise in temperature. Let a metal scale gives correct length L_0 at a certain temperature say T_1 °C. When the temperature of the scale is greater than T_1 °C, then the distance between any two scale divisions increases in the ratio $1:(1+\alpha T)$, where α is the coefficient of linear expansion of the material. If the reading of the scale is L cm, then the actual reading will be $L(1+\alpha T)$ cm and hence a correction of $+L\alpha T$ cm should be applied. Here L is the reading of scale at higher temperature.

When the scale is below T_1 °C, the distance between any two divisions contracts. If the scale reading be L cm, then the actual distance will be $L(1-\alpha T)$ and a correction $-L\alpha T$ should be applied. Here L is the length at lower temperature.

Problem Solving Technique(s)

It is clear from the above analysis that at higher temperature, the reading of the scale is lower than the true value in the ratio $1:(1+\alpha T)$ while at lower temperature, the scale reading is higher than the true distance in the ratio $(1+\alpha T):1$.

ILLUSTRATION 18

A meter scale is made of steel and measures correct length at 16 °C. What will be the percentage error if the scale is used

(a) on a summer day when temperature is 46 °C and

(b) on a winter day when the temperature is 6 °C?

Coefficient of linear expansion of steel is 11×10^{-6} °C^{-1}.

SOLUTION

(a) On a summer day the scale will measure less than the actual. Hence, the percentage error is

$$\% \text{ error} = -\left(\frac{\Delta l}{l}\right)\times 100$$

$$\Rightarrow \% \text{ error} = -(\alpha\Delta\theta)\times 100$$

$$\Rightarrow \% \text{ error} = -(11\times10^{-6})(46-16)\times 100$$

$$\Rightarrow \% \text{ error} = -0.033\%$$

(b) On a winter day the scale will measure more than the actual, hence the percentage error is

$$\% \text{ error} = +(\alpha\Delta\theta)\times 100$$

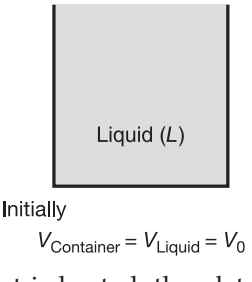
$$\Rightarrow \% \text{ error} = +(11\times10^{-6})(16-6)\times 100$$

$$\Rightarrow \% \text{ error} = +0.011\%$$

THERMAL EXPANSION IN LIQUIDS

Thermal expansion in liquids is also similar to that of volume expansion in solids and is governed by the similar relationship which is $V = V_0(1+\gamma\Delta T)$. γ for liquids is generally higher than that for solids.

Since liquids are always to be heated along with a container that contains them, so initially on heating the system (liquid + container), the level of liquid in container falls (since container expands more because it initially absorbs more heat compared to liquid which initially expands less). However, finally the liquid starts rising due to its faster expansion.



Consider a container (C) of volume V_0, coefficient of volume expansion γ_C to be filled up to the brim with a liquid (obviously of volume V_0), coefficient of volume expansion γ_L as shown in Figure.

When the complete arrangement is heated, then let the rise in temperature be ΔT. If V_L and V_C be the new respective volumes of the liquid and the container, then we have

For liquid, $V_L = V_0(1+\gamma_L\Delta T)$ …(1)

For container, $V_C = V_0(1+\gamma_C\Delta T)$ …(2)

So, volume of liquid that may overflow due to heating is

$$\Delta V = V_L - V_C = V_0(\gamma_L - \gamma_C)\Delta T \quad \ldots(3)$$

This expression can be written as

$$\Delta V = V_0\gamma_{\text{apparent}}\Delta T, \text{ where } \gamma_{\text{apparent}} = \gamma_{app} = \gamma_L - \gamma_C$$

(a) If $\gamma_L > \gamma_C$, then $\gamma_{app} > 0$ i.e., the level of liquid in the beaker will rise.

(b) If $\gamma_L < \gamma_C$, then $\gamma_{app} < 0$ i.e., the level of liquid in the beaker will fall.

(a) If $\gamma_L = \gamma_C$, then $\gamma_{app} = 0$ i.e., the level of liquid in the beaker will remain the same.

Conceptual Note(s)

Liquids generally increase in volume with increasing temperature and have volume expansion coefficients about ten times greater than these of solid. Water is an exception to this rule. From 0 °C to 4 °C water contracts and beyond 4 °C it expands. Hence density of water reaches a maximum value of 1000 kgm^{-3} at 4 °C.

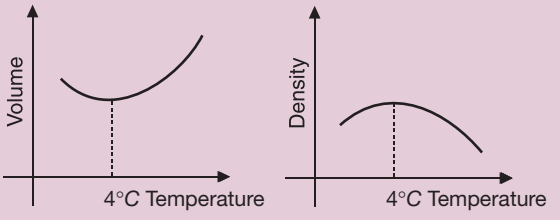


ILLUSTRATION 19

A long horizontal glass capillary tube open at both ends contains a mercury thread 1 m long at 0 °C. A scale is etched on the glass tube. This scale is correct at 0 °C. Find the length of mercury thread, as shown by this scale at 100 °C. Also find original length of thread at 100 °C. Given, $\gamma_{\text{mercury}} = 18.2\times10^{-5}$ K^{-1} and $\alpha_{\text{glass}} = 9\times10^{-6}$ K^{-1}.

SOLUTION

Let V_0 and V_t are volumes of mercury at 0 °C and t °C, A_0 and A_t are area of cross section of at 0 °C and at t °C. Then

$$V_0 = A_0 l_0 \quad \text{and} \quad V_t = A_t l_t$$

Since, $V_t = V_0(1+\gamma_{Hg}t)$

$$\Rightarrow A_t l_t = A_0 l_0(1+\gamma_{Hg}t)$$

$$\Rightarrow A_0 l_t(1+2\alpha_g t) = A_0 l_0(1+\gamma_{Hg}t)$$

$$\Rightarrow l_t = l_0\left(\frac{1+\gamma_{Hg}t}{1+2\alpha_g t}\right)$$

Expanding and neglecting negligible terms, we get

$$l_t = l_0\left[1+(\gamma_{Hg}-2\alpha_g)t\right]$$

$$\Rightarrow \quad l_{100} = (1)\left[1+(18.2-1.8)\times 10^{-5}\times 100\right]$$

$$\Rightarrow \quad l_{100} = 1.0164 \text{ m}$$

Let L be the required reading of thread on glass scale at temperature t. Then this section of the glass (scale) has length L at 0 °C and l_t at t °C. So,

$$l_t = L\left(1+\alpha_g t\right)$$

$$\Rightarrow \quad L = \frac{l_0\left[1+\left(\gamma_{Hg}-2\alpha_g\right)t\right]}{1+\alpha_g t} = l_0\left[1+\left(\gamma_{Hg}-3\alpha_g\right)t\right]$$

Substituting the values, we get

$$L = 1.0155 \text{ m}$$

ILLUSTRATION 20

A glass flask with volume 200 cm^3 is filled to the brim with mercury at 20 °C. How much mercury overflows when the temperature of the system is raised to 100 °C? The coefficient of linear expansion of the glass is 0.40×10^{-5} K^{-1}. Cubical expansion of mercury $= 18\times 10^{-5}$ K^{-1}.

SOLUTION

The coefficient of volume expansion for the glass is

$$\gamma_{\text{glass}} = 3\alpha_{\text{glass}} = 1.2\times 10^{-5} \text{ K}^{-1}$$

The glass flask of volume 200 cm^3 is filled to the brim with mercury, so we have

$$\Delta V = V_0\gamma_{\text{app}}\Delta T = V_0\left(\gamma_{\text{Hg}}-\gamma_{\text{glass}}\right)\Delta T$$

$$\Rightarrow \quad \Delta V = (200)\left[(18-1.2)\times 10^{-5}\right](100-20) \approx 2.7 \text{ cc}$$

ILLUSTRATION 21

The coefficient of apparent expansion of a liquid in a copper vessel is C and in a silver vessel S. The coefficient of volume expansion of copper is γ_C. Find the coefficient of linear expansion of silver.

SOLUTION

Apparent coefficient of volume expansion for liquid is

$$\gamma_{\text{app}} = \gamma_L - \gamma_S$$

$$\Rightarrow \quad \gamma_L = \gamma_{\text{app}} + \gamma_S$$

where γ_S is coefficient of volume expansion for solid vessel.

Since, γ_{app} for liquid in copper vessel is C, so when liquid is placed in copper vessel then

$$\gamma_L = C + \gamma_{\text{copper}} \qquad \ldots(1)$$

Also, γ_{app} for liquid in silver vessel is S, so when liquid is placed in silver vessel then

$$\gamma_L = S + \gamma_{\text{silver}} \qquad \ldots(2)$$

From equation (1) and (2), we get

$$C + \gamma_{\text{copper}} = S + \gamma_{\text{silver}}$$

$$\Rightarrow \quad \gamma_{\text{silver}} = C + \gamma_{\text{copper}} - S$$

$$\Rightarrow \quad \alpha_{\text{silver}} = \frac{\gamma_{\text{silver}}}{3} = \frac{C+\gamma_{\text{copper}}-S}{3}$$

ILLUSTRATION 22

A glass beaker holds exactly 1 litre at 0 °C. What is its volume at 50 °C? If the beaker is filled with mercury at 0 °C, what volume of mercury overflows when the temperature is 50 °C?

Given, $\alpha_g = 8.3\times 10^{-6}\,(°C)^{-1}$ and $\gamma_{Hg} = 1.82\times 10^{-4}\,(°C)^{-1}$

SOLUTION

The volume of beaker after the temperature change is,

$$V_{\text{beaker}} = V_0\left(1+3\alpha_g\Delta T\right)$$

$$\Rightarrow \quad V_{\text{beaker}} = (1)\left[1+3\times 8.3\times 10^{-6}\times 50\right]$$

$$\Rightarrow \quad V_{\text{beaker}} = 1.001 \text{ litre}$$

Volume of mercury at 50 °C is

$$V_{\text{mercury}} = V_0\left(1+\gamma_{Hg}\Delta T\right)$$

$$\Rightarrow \quad V_{\text{mercury}} = (1)\left[1+1.82\times 10^{-4}\times 50\right]$$

$$\Rightarrow \quad V_{\text{mercury}} = 1.009 \text{ litre}$$

The overflow is thus given by

$$\Delta V = 1.009 - 1.001 = 0.008 \text{ litre} = 8 \text{ ml}$$

ILLUSTRATION 23

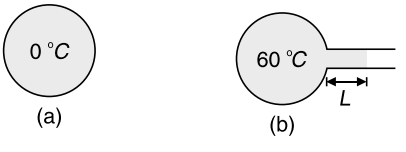
A glass bulb whose volume at 0 °C equal 10 cc is filled with mercury. It is then joined to stem of diameter 0.2 cm at 0 °C. What length of the stem will the liquid occupy at 60 °C?

Given, coefficient of volume expansion of mercury is 18×10^{-5} $°C^{-1}$ and coefficient of linear expansion of glass is 9×10^{-6} $°C^{-1}$.

SOLUTION

Volume of Hg at 60 °C is $V_{60} = V_0\left(1+60\gamma_{Hg}\right)$

Volume of glass bulb at 60 °C is $V_0\left(1+60\gamma_g\right)$



So, volume of Hg that will pass from the bulb into stem is $\Delta V = V_0\left(\gamma_{Hg}-\gamma_g\right)60$. The cross section of stem at 60 °C

is $A_{60} = A_0(1+60\beta_g)$ and let l be the length of mercury column in the stem at 60 °C. Then

$$l = \frac{V_0(\gamma_{Hg} - \gamma_g)60}{A_0(1+60\beta_g)}$$

where, $A_0 = \pi d^2/4 = \pi(0.2)^2/4 = 0.0314 \text{ cm}^2$

$$\Rightarrow \quad l = \frac{(10)(18-2.7)\times10^{-5}\times60}{(0.0314)(1+60\times1.8\times10^{-5})} = 2.92 \text{ cm}$$

ILLUSTRATION 24

A one litre closed flask contains some mercury. It is found that at different temperatures the volume of air inside the flask remains the same. What is the volume of mercury in flask? Given coefficient of linear expansion of glass $= 9\times10^{-6}$ °C^{-1}. Coefficient of volume expansion of Hg $= 1.8\times10^{-4}$ °C^{-1}.

SOLUTION

Since the volume of air in flask remains same. This is possible only when the expansion of flask is exactly the same as the expansion of mercury in the flask. So, coefficient of cubical expansion of glass is

$$\gamma_g = 3\alpha = 3\times9\times10^{-6} = 27\times10^{-6} \text{ °C}^{-1}$$

When the volume of air in flask is constant, then expansion of flask equals the expansion of mercury. Assuming x to be the volume of mercury in the flask, then

$$(1000)\gamma_g\Delta T = x\gamma_m\Delta T$$

$$\Rightarrow \quad x = V\left(\frac{\gamma_g}{\gamma_m}\right) = 1000\left(\frac{27\times10^{-6}}{1.8\times10^{-4}}\right) = 150 \text{ cm}^3$$

EXPANSION OF GASES

On heating, gases expand much greater than solids or liquids and equal volume of different gases expand equally when heated by the same amount. The change in pressure (P) and temperature (T) may change the volume of given mass of gas. It is important to note that

(a) while dealing with the coefficient of thermal expansion of a gas the pressure is assumed constant.

(b) all gases have same coefficient of volume expansion i.e. γ_V with volume variation given by $V = V_0(1+\gamma_V T)$ and the pressure coefficient of the gas is given by $\gamma_P = \frac{1}{273}$ with pressure variation given by $P = P_0(1+\gamma_P T)$.

(c) the coefficient of volume expansion for an ideal gas at constant pressure is equal to the reciprocal of the absolute temperature T. This can be shown by starting with the ideal gas equation

$$PV = nRT \qquad \ldots(1)$$

$$\Rightarrow \quad d(PV) = d(nRT)$$

$$\Rightarrow \quad PdV = nRdT$$

$$\Rightarrow \quad \gamma = \frac{1}{V}\left(\frac{dV}{dT}\right) = \frac{1}{V}\left(\frac{nR}{P}\right) = \frac{nR}{PV} = \frac{1}{T} \qquad \{\because \text{ of (1)}\}$$

CHANGE IN DENSITY OF SOLIDS AND LIQUIDS WITH TEMPERATURE

Since on heating, the mass of the bodies remains same and hence we conclude that heating of bodies is accompanied by an increase in volume and decrease in density (note this for water which has an anomalous behaviour from 0 °C to 4 °C and beyond 4 °C)

$$M = V_0\rho_0 = V\rho$$

Since, $V = V_0(1+\gamma\Delta T)$

$$\Rightarrow \quad \frac{\rho}{\rho_0} = \frac{V_0}{V} = \frac{1}{1+\gamma\Delta T} = (1+\gamma\Delta T)^{-1}$$

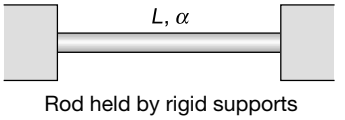
For $\gamma\Delta T << 1$, we have

$$\rho \approx \rho_0(1-\gamma\Delta T)$$

where, ρ is density at temperature T °C and ρ_0 is density at T_0 °C.

THERMAL STRESS IN A ROD FIXED BETWEEN TWO RIGID SUPPORTS

Consider a rod of length L_0 with Young's modulus Y which is supplied heat such that temperature increases from T_0 to $(T_0 + \Delta T)$ thus producing a stress in the rod, then thermal stress is F/A and thermal strain is $\Delta L/L_0 = \alpha\Delta T$.



Rod held by rigid supports

$$\Rightarrow \quad Y = \frac{\text{Thermal stress}}{\text{Thermal strain}} = \frac{F/A}{\Delta L/L_0}$$

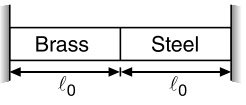
$$\Rightarrow \quad \text{Thermal Stress} = \frac{F}{A} = Y\left(\frac{\Delta L}{L_0}\right) = Y\alpha\Delta T$$

Problem Solving Technique(s)

Force which comes into play on heating a body is $F = YA\alpha\Delta T$.

ILLUSTRATION 25

Two rods, one of brass and other of steel of each of length l_0 having equal cross-sectional area are joined end to end are supported between two rigid vertical walls as shown in Figure.



Initially the rods are unstrained. If the temperature of system is raised by ΔT, calculate the displacement of the junction of two rods, if the coefficients of linear expansion and Young's modulus of brass and steel are α_b, $\alpha_s\,(<\alpha_b)$, Y_b and Y_s respectively.

SOLUTION

Since $\alpha_b > \alpha_s$, so the junction of the two rods is displaced towards right. Due to their elastic nature, each rod exerts an elastic restoring force on the other due to which the brass rod is expanded but less as compared to free expansion and steel is compressed due to the stress developed between the two rods.

Let x be the displacement of the interface.

For Brass Rod

Final length due to free thermal expansion only is $(l_0 + l_0\alpha_b\Delta T)$. However, the actual final length of the brass rod is $(l_0 + x)$ i.e., slightly less than the length of brass rod in free expansion.

Hence strain developed in the brass rod is

$$\left(\frac{\Delta l}{l_0}\right)_{\text{brass}} = \frac{(l_0 + l_0\alpha_b\Delta T) - (l_0 + x)}{l_0} = \frac{l_0\alpha_b\Delta T - x}{l_0}$$

For Steel Rod

Final length due to free thermal expansion only is $(l_0 + l_0\alpha_s\Delta T)$. However, the actual final length of the steel rod is $(l_0 - x)$ i.e., slightly less than the length of steel rod in free expansion.

Hence strain developed in the brass rod is

$$\left(\frac{\Delta l}{l_0}\right)_{\text{steel}} = \frac{(l_0 + l_0\alpha_s\Delta T) - (l_0 - x)}{l_0} = \frac{l_0\alpha_b\Delta T + x}{l_0}$$

However, the two rods are in contact, so stress developed in the two rods is same, so

$$(\text{Stress})_{\text{brass}} = (\text{Stress})_{\text{steel}}$$

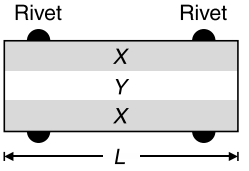
$$\Rightarrow \quad Y_b\left(\frac{\Delta l}{l_0}\right)_{\text{brass}} = Y_s\left(\frac{\Delta l}{l_0}\right)_{\text{steel}}$$

$$\Rightarrow \quad Y_b\left(\frac{\alpha_b l_b\Delta T - x}{l_0}\right) = Y_s\left(\frac{\alpha_s l_0\Delta T + x}{l_0}\right)$$

$$\Rightarrow \quad x = \frac{(Y_b\alpha_b - Y_s\alpha_s)l_0\Delta T}{Y_b + Y_s}$$

ILLUSTRATION 26

Two rods of material X sandwich another rod of material Y as shown in Figure.



At temperature T, the three rods are in a state of zero strain have length L and are riveted to each other. If the temperature of the system increases to $T + \Delta T$, find the final length of the system of the three rods. Given that the coefficients of linear expansions of the rods are α_x and $\alpha_y\,(>\alpha_x)$, Young's modulus for material X is Y_x and that for material Y is Y_y.

SOLUTION

Natural elongation in rod x and y respectively are

$$l_x = L(1+\alpha_x\Delta T) \text{ and } l_y = L(1+\alpha_y\Delta T)$$

Due to pivots, if final length is L_f, then strain developed in rods x and y is

$$(\text{Strain})_x = \frac{L_f - l_x}{l_x} \approx \frac{L_f - l_x}{L}$$

$$(\text{Strain})_y = \frac{l_y - L_f}{l_y} \approx \frac{l_y - L_f}{L}$$

There we can use $l_x \approx l_y \approx L$ as numerator is very small difference. Also, we note that stress on y is twice that of x, so

$$Y_y(\text{Strain})_y = 2Y_x(\text{Strain})_x$$

$$\Rightarrow \quad Y_y\left(L(1+\alpha_y\Delta T) - L_f\right) = 2Y_x\left(L_f - L(1+\alpha_x\Delta T)\right)$$

$$\Rightarrow \quad Y_yL + \alpha_yY_yL\Delta T - L_fY_y = 2Y_xL_f - 2Y_xL - 2Y_x\alpha_xL\Delta T$$

$$\Rightarrow \quad L_f = L\left[1 + \left(\frac{\alpha_yY_y + 2\alpha_xY_x}{Y_y + 2Y_x}\right)\Delta T\right]$$

CORRECTION FOR BAROMETRIC READING

The brass scale of a barometer is usually calibrated at 0 °C. A barometer is usually read at room temperature (say t °C). Therefore, corrections due to expansions of brass-scale and mercury must be taken into account.

CORRECTION FOR EXPANSION OF BRASS SCALE

If H is true height at 0 °C, then reading at t °C

$$H_t = H(1+\alpha t) \qquad \ldots(1)$$

α being linear coefficient of expansion for brass.

CORRECTION FOR EXPANSION OF MERCURY

If ρ_t is density of mercury at t °C, then pressure is

$$H_t\rho_t g = H(1+\alpha t)\rho_t g$$

If ρ_0 is density of mercury at 0 °C, then corresponding height of mercury at 0 °C, H_0 is given by

$$H_0\rho_0 g = H_t\rho_t g$$

$$H_0\rho_0 g = H(1+\alpha t)\rho_t g$$

But $\rho_t = \dfrac{\rho_0}{1+\gamma t}$ $\{\gamma$ being cubical expansion of mercury$\}$

$$\Rightarrow \quad H_0\rho_0 g = H(1+\alpha t)\left(\frac{\rho_0}{1+\gamma t}\right)g$$

$$\Rightarrow \quad H_0 = H\left(\frac{1+\alpha t}{1+\gamma t}\right) \text{ or } H_0 = H\{1-(\gamma-\alpha)t\}$$

EFFECT OF TEMPERATURE ON UPTHRUST

When a solid body is completely immersed in a liquid its apparent weight gets decreased due to an upthrust F acting on it by the liquid. The apparent weight is given by

$$W_{\text{app}} = W - F \qquad \ldots(1)$$

where F =upthrust = $V_s\rho_L g$

where V_S =volume of solid body and ρ_L = density of liquid

Now, as the temperature is increased V_S increases while ρ_L decreases. So, F may increase or decrease (or may remain constant also) depending upon the situation that which factor dominates on the other. We can write

$$F \propto V_S\rho_L$$

$$\Rightarrow \quad \frac{F'}{F} = \frac{V_S'}{V_S}\cdot\frac{\rho_L'}{\rho_L} = \left(\frac{V_S+\Delta V_S}{V_S}\right)\left(\frac{1}{1+\gamma_L\Delta T}\right)$$

$$\Rightarrow \quad \frac{F'}{F} = \left(\frac{V_S+\gamma_S V_S\Delta T}{V_S}\right)\left(\frac{1}{1+\gamma_L\Delta T}\right)$$

$$\Rightarrow \quad F' = F\left(\frac{1+\gamma_S\Delta T}{1+\gamma_L\Delta T}\right)$$

Now, if $\gamma_S > \gamma_L$, $F' > F$

$$\Rightarrow \quad W_{\text{app}}' < W_{\text{app}} \qquad \{\because \text{ of Equation (1)}\}$$

and vice versa and if $\gamma_S = \gamma_L$, $F' = F$ or $W_{\text{app}}' = W_{\text{app}}$

ILLUSTRATION 27

A solid body floats in liquid at temperature 0 °C and is completely submerged in it at 50 °C. What fraction f of volume of the body is submerged in the liquid at 0 °C, if $\gamma_s = 0.3\times10^{-5}\text{K}^{-2}$ and of the liquid, $\gamma_1 = 8.0\times10^{-5}\text{K}^{-1}$?

SOLUTION

In both cases, the weight of the body will be balanced by the force of buoyancy on it. At $t_0 = 0$ °C, the buoyancy is

$$F_b = fV_0\rho_0 g \qquad \ldots(1)$$

where V_0 is the volume of the body and ρ_0 is the density of the liquid at $t_0 = 0$ °C.

At $t = 50$ °C, the volume of the body becomes $V = V_0(1+\gamma_s t)$ and the density of the liquid is

$$\rho_1 = \frac{\rho_0}{1+\gamma_1 t}$$

The buoyancy in this case is

$$F_b = \frac{V_0\rho_0 g(1+\gamma_s t)}{1+\gamma_1 t} \qquad \ldots(2)$$

Equating equations (1) and (2), we get

$$f = \frac{1+\gamma_s t}{1+\gamma_1 t} = 96\%$$

ILLUSTRATION 28

A solid whose volume does not change with temperature floats in a liquid. For two different temperatures t_1 and t_2 of the liquid, fractions f_1 and f_2 of the volume of the solid remain submerged in the liquid. Find the coefficient of volume expansion of the liquid.

SOLUTION

With the rise in temperature, the liquid undergoes volume expansion, therefore the fraction of solid submerged in liquid increases. At temperature T_1 if density of liquid is ρ_1 and that at temperature T_2 is ρ_2, so we have

$$F_{\text{buoyant}} = V_{\text{imm}}\rho_{\text{liq}} g = mg$$

$$\Rightarrow \quad f_1V_0\rho_1 = f_2V_0\rho_2 = mg$$

$$\Rightarrow \quad \frac{f_1\rho_0}{1+\gamma T_1} = \frac{f_2\rho_0}{1+\gamma T_2}$$

$$\Rightarrow \quad f_1 + f_1T_2\gamma = f_2 + f_2T_1\gamma$$

$$\Rightarrow \quad \gamma = \frac{f_2-f_1}{f_1T_2-f_2T_1}$$

BIMETALLIC STRIP

Since each substance has its own characteristic average coefficient of expansion. For example, when the temperatures of a brass rod and a steel rod of equal length are raised by the same amount from some common initial value, the brass rod expands more than the steel rod because brass has a greater average coefficient of expansion than steel. Such type of bimetallic strip is found in practical devices such as **thermostats** to break or make electrical contact.

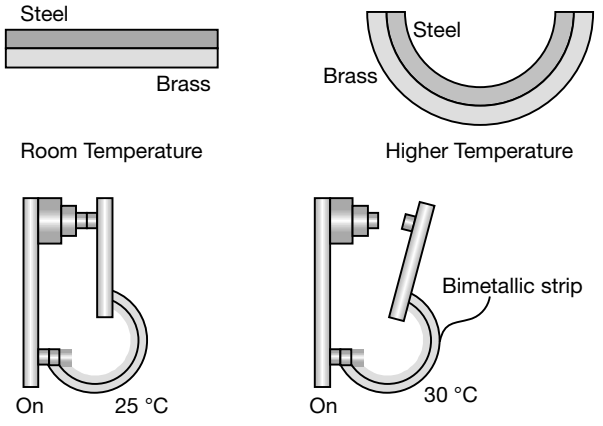


ILLUSTRATION 29

Two metal strips, each of length L and thickness $\left(\frac{d}{2}\right)$ at temperature T_0 are riveted together so that their ends coincide. One strip is made of metal A having coefficient of linear expansion α_A and the other with a coefficient α_B, where $\alpha_A > \alpha_B$. When this bimetallic strip is heated to a temperature $(T_0 + \Delta T)$, the bimetallic strip bends into the arc of the circle. Assuming that the thickness d of the bimetallic strip remains constant at new temperature, calculate the radius of the circle.

SOLUTION

Since $\alpha_A > \alpha_B$, so metal A will be on outer side and metal B on the inner side. Now,

$$L_A = r_A\theta = L(1+\alpha_A\Delta T) \quad \ldots(1)$$

$$L_B = r_B\theta = L(1+\alpha_B\Delta T) \quad \ldots(2)$$

Adding (1) and (2), we get

$$(r_A + r_B)\theta = L\left[2+(\alpha_A+\alpha_B)\Delta T\right]$$

$$\Rightarrow \quad r = \frac{r_A+r_B}{2} = \frac{L\left[2+(\alpha_A+\alpha_B)\Delta T\right]}{2\theta} \quad \ldots(3)$$

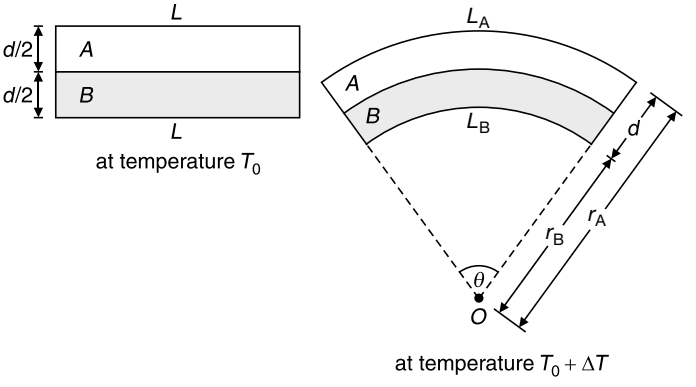
Also, $r_A - r_B = d$ $\ldots(4)$

Subtracting (2) from (1), we get

$$\theta(r_A - r_B) = L(\alpha_A - \alpha_B)\Delta T$$

$$\Rightarrow \quad \theta d = L(\alpha_A - \alpha_B)\Delta T \qquad \{\because\ r_A - r_B = d\}$$

$$\Rightarrow \quad \theta = \frac{L(\alpha_A-\alpha_B)\Delta T}{d} \quad \ldots(5)$$



Substituting (5) in (3), we get

$$r = \frac{L\left[2+(\alpha_A+\alpha_B)\Delta T\right]}{\left[\frac{2L(\alpha_A-\alpha_B)\Delta T}{d}\right]} = \frac{d}{2}\left[\frac{2+(\alpha_A+\alpha_B)\Delta T}{(\alpha_A-\alpha_B)\Delta T}\right]$$

Test Your Concepts-I

Based on Thermometry and Thermal Expansion

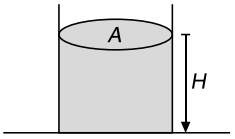
(Solutions on page H.75)

1. A 20 cm steel ruler is graduated to give correct measurements at 20 °C.
- **(a)** Will it give readings that are too long or too short at lower temperatures?
- **(b)** What will be the actual length of the ruler be when it is used in the desert at a temperature of 40 °C? Given, $\alpha_{steel} = 1.2\times10^{-5}\,(°C)^{-1}$.

2. **(a)** An aluminium measuring rod which is correct at 5 °C, measures a certain distance as 88.42 cm at 35 °C. Calculate the error in measuring the distance due to expansion of the rod.
- **(b)** If the aluminium rod measures the length of a steel rod as 88.42 cm at 35 °C, what is the correct length of the steel rod at 35 °C.
Given $\alpha_{Al} = 2.55\times10^{-5}\,(°C)^{-1}$

3. A steel tape measures the length of a copper rod at 90 cm when both are at 10 °C, the calibration temperature for the tape. What would the tape read for the length of the rod when both are at 30 °C? Given $\alpha_c = 1.7\times10^{-5}\,(°C)^{-1}$ and $\alpha_s = 1.2\times10^{-5}\,(°C)^{-1}$.

4. A liquid having coefficient of volume expansion γ is filled in a glass vessel. The coefficient of linear expansion of glass is α. When the arrangement is heated to raise the temperature of the liquid and the glass container by ΔT, expansion takes place in both. The expansion may be different or equal. Depending on the values of γ and α we may find that level of the liquid rises with respect to ground or it may fall with respect to ground. Find the relation between γ and α for which



(i) the fraction of volume of container occupied by liquid does not change with rise in temperature.

(ii) the level of the liquid in container does not change with respect to the container.

(iii) the level of the liquid in container does not change with respect to the ground.

5. The scale on a steel meter stick is calibrated at 15 °C. Find the error in the reading of 60 cm at 27 °C? Given, $\alpha_{steel} = 1.2\times10^{-5}\,(°C)^{-1}$.

6. In a mercury in glass thermometer the cross-section of the capillary is A_0 and volume of the bulb is V_0 at 0 °C. If mercury just fills the bulb at 0 °C, find that the length of mercury in the capillary at temperature t °C. Given that coefficient of cubical expansion of mercury is γ and coefficient of linear expansion of glass is α.

7. A second's pendulum clock has a steel wire. The clock is calibrated at 20 °C. How much time does the clock lose or gain in one week when the temperature is increased to 30 °C? Given, $\alpha_{steel} = 1.2\times10^{-5}\,(°C)^{-1}$.

8. A steel wire of cross-sectional area 0.5 mm^2 is held between two rigid clamps so that it is just taut at 20 °C. Calculate the tension in the wire at 0 °C. Given that Young's Modulus of steel is $Y_s = 2.1\times10^{12}$ $dynecm^{-2}$ and coefficient of linear expansion of steel is $\alpha_s = 1.1\times10^{-5}\ (°C)^{-1}$.

9. A steel rod 50 cm long has cross-sectional area of 0.8 cm^2. What force would be required to stretch this rod by the same amount as the expansion produced by heating it through 10 °C. Given, for steel, $\alpha = 10^{-5}\ K^{-1}$, $Y = 2\times10^{11}\ Nm^{-2}$.

10. Typical temperatures in the interior of the earth and sun are about 4×10^3 °C and 1.5×10^7 °C respectively. Calculate these temperatures on absolute scale. Also calculate the percentage error made in each case if the observer forgets to convert °C to K.

11. At 0 °C, three metal rods form an equilateral triangle. Two rods are of the same material, but the third is made of invar (its expansion is negligible). When the triangle is heated up to 100 °C, the angle formed between the two metal rods of the same material is $\frac{\pi}{3}-\theta$. Find the coefficient of linear expansion of the two metal rods.

12. Calculate the coefficient of volume expansion for an ideal gas at constant pressure.

13. Two thermometers, one marked in Fahrenheit the other in Celsius, are placed in a bath. At what temperature will both thermometers read the same?

14. A uniform solid brass cylinder of mass 0.50 kg and radius 0.030 m is placed on a friction less bearings and set to rotate about its geometrical axis with an angular velocity of 60 radian s^{-1}.

(a) Calculate the angular momentum of the cylinder and the work required to reach this state of rotation starting from rest.

(b) After the cylinder has reached the specified state of rotation, it is heated without any mechanical contact from room temperature of 20 °C to 100 °C, find the fractional change in the angular velocity of the cylinder. Given $\alpha_{Brass} = 2\times10^{-5}\ (°C)^{-1}$

15. In an alcohol-in-glass thermometer, the alcohol column has length 12.45 cm at 0.0 °C and length 21.30 cm at 100.0 °C. Calculate the temperature, if the column has length of

(a) 15.10 cm **(b)** 22.95 cm

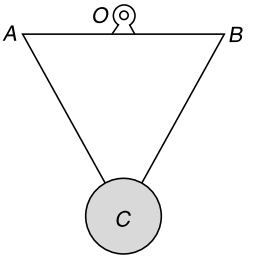
16. If the coefficient of linear expansion α is taken as a variable dependent on temperature T, show that the length L at temperature T is given by $L = L_0\left[1+\int_{T_0}^{T}\alpha(T)dT\right]$, where L_0 is the length at reference temperature T_0.

17. What should be the lengths of the steel rod and copper rod so that the length of the steel rod is 5 cm longer than the copper rod at all the temperatures?

Given $\alpha_{Cu} = 1.7\times10^{-5}\ (°C)^{-1}$ and $\alpha_{Steel} = 1.1\times10^{-5}\ (°C)^{-1}$

18. If the volume of a block of metal changes by 0.12% when it is heated by 20 °C. Calculate the coefficient of linear expansion of the metal.

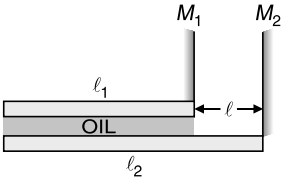
19. A compensated pendulum is a pendulum made up of two or more metals arranged such that the effective length of centre of mass of the whole body remains unchanged. Such a situation is shown in Figure in which the compensated pendulum is in the form of an isosceles triangle of base length $l_1 = 5$ cm and coefficient of linear expansion $\alpha_1 = 18\times10^{-6}$ and side length l_2 and coefficient of linear expansion $\alpha_2 = 12\times10^{-6}$. Find l_2 so that the distance of centre of mass of the bob from suspension centre O may remain the same at all the temperature.



20. A circular hole in an aluminium plate is 2.54 cm in diameter at 0 °C. What is the diameter when the temperature of the plate is raised to 100 °C. Given $\alpha_{Al} = 2.3\times10^{-5}\,(°C)^{-1}$.

21. If the triple point of neon and carbon dioxide are observed at 24.57 K and 216.55 K respectively, then express these temperatures on the Celsius and Fahrenheit scales.

22. An optical engineering firm needs to ensure that the separation between two mirrors M_1 and M_2 is unaffected by temperature changes. The mirrors are attached to the ends of two bars of different materials that are welded together at one end as shown in figure. The surfaces of the bars in contact are lubricated. Show that the distance ℓ does not change with temperature if $\alpha_1\ell_1 = \alpha_2\ell_2$, where α_1 and α_2 are the respective thermal coefficients of temperature. Also determine the required lengths ℓ_1 and ℓ_2, in terms of α_1, α_2 and ℓ.



CALORIMETRY: HEAT, WORK AND MECHANICAL EQUIVALENT OF HEAT

Experimentally it has been observed that the work done (W) produces a proportional amount of heat (Q) i.e.

$$W \propto Q$$
$$W = JQ$$

where J is called **Joule's Mechanical Equivalent of Heat** whose value is $4.186\ \text{Jcal}^{-1}$ $\left(\text{or } 4.2\ \text{Jcal}^{-1}\right)$

Problem Solving Technique(s)

While writing above relation W must be expressed in joule and Q must be expressed in calorie i.e., if

$Q = 1$ calorie, $W = 4.18$ joule

i.e., a work of 4.18 joule has to be done to produce a heat of 1 calorie. Hence 1 calorie $\equiv$ 4.18 joule

CONCEPT OF GRAM SPECIFIC HEAT

As a student of Physics, you can always think and understand that different masses of bodies made of same material require different amounts of heat to be supplied to them to raise their temperature by the same amount. Similarly, equal masses of bodies of same material require different amounts of heat to be supplied to them to raise their temperature by different amounts and also equal masses of bodies of different material require different amount of heat to be supplied to them to raise their temperature by same amount. (Think of some practical examples!)

From above, we conclude that amount of heat supplied Q to a body is directly proportional to mass m of body and rise in temperature ΔT of the body i.e., $Q \propto m\Delta T$

$$\Rightarrow \quad Q = mc\Delta T$$

where, c is called the **"gram specific heat"** or **"specific heat"** of the material of the body which gains heat or loses heat. c is different for different materials (NOTE THIS!) and is a constant for small temperature variations whereas, for large temperature variations it starts varying as a function of temperature (and hence cannot be taken as a constant). Further if an infinitesimal amount of heat dQ is supplied to a body of mass m and rise in temperature is dT then $dQ = mcdT$

$$\Rightarrow \quad c = \frac{1}{m}\frac{dQ}{dT} = \frac{1}{m}\lim_{\Delta T \to 0}\frac{\Delta Q}{\Delta T}$$

Conceptual Note(s)

(a) Specific heat of water $c_{\text{water}} = 1\,\text{calg}^{-1}(^\circ\text{C})^{-1}$

(b) Specific heat of ice $c_{\text{ice}} = 0{\cdot}5\,\text{calg}^{-1}(^\circ\text{C})^{-1}$

(c) $1\,\text{calg}^{-1}(^\circ\text{C})^{-1} = 4200\ \text{Jkg}^{-1}(^\circ\text{C})^{-1} = 4200\ \text{Jkg}^{-1}\text{K}^{-1}$

THERMAL CAPACITY OR HEAT CAPACITY

The amount of heat required to raise the temperature of a body through 1 unit. $(\Delta T = 1$ unit change$)$ i.e., if $\Delta T = 1$ unit then Heat capacity $= mc$

So, heat capacity of a body of mass m, gram specific heat c is the product of m and c.

WATER EQUIVALENT (w)

Consider a body having mass m and gram specific c. Let dQ be the amount of heat supplied to the body such that the rise in temperature is dT.

The amount (or mass) of water (w) to which this same amount of heat dQ should be supplied such that the rise in temperature of water is also dT is called **the water equivalent of the body**.

For Body $\quad dQ = mcdT$ $\quad \ldots(1)$

For Water $\quad dQ = wc_{\text{water}}dT$

Since $c_{\text{water}} = 1\ \text{calg}^{-1}(^\circ\text{C})^{-1}$

$$\Rightarrow \quad dQ = wdT \qquad \ldots(2)$$

Using (1) and (2), the water equivalent w is given by

$$w = mc$$

"water equivalent (w) of a body of mass m, gram specific heat c is the product of m and c."
SI unit of water equivalent is kg and cgs unit is gram (g).

A MISCONCEPTION

Two bodies A and B of masses m_1 and m_2 at temperatures T_1 and T_2 with gram specific heats c_1, and c_2 are placed in contact with each other. As a result of contact heat must flow from the body at higher temperature (say A) to the body at lower temperature and ultimately the equilibrium temperature should be $\frac{T_1+T_2}{2}$.

Now let's give this thing a thought from an example of everyday life.

Consider a person of mass 60 kg holding a piece of ice of mass 6 g in his palm. As a result of this heat must flow from the body of person to ice and ultimately the equilibrium temperature should become (in approximation) $\frac{37+0}{2} = 18.5\,°C$

At this body temperature can you think a person to survive? This means that holding a mere 6 g ice piece on the palm may prove to be fatal for the person. CAN THAT BE SO?

MISSING CONCEPT

Maximum students have this "misconception" that when two bodies at temperatures T_1 and T_2 are placed in contact the equilibrium temperature must be $\frac{T_1+T_2}{2}$, **which is FALSE (in most of the situations)**, but under some specific predefined conditions it can be true (see CASE-3 in next article).

MISCONCEPTION REMOVAL

It is that we must say heat flows from the palm of the person to ice till equilibrium is attained which is always governed by the equation.

$$\begin{pmatrix}\text{Heat Lost by}\\ \text{Body at Higher}\\ \text{Temperature}\end{pmatrix} = \begin{pmatrix}\text{Heat Gained by}\\ \text{Body at Lower}\\ \text{Temperature}\end{pmatrix}$$

This is also called as the Law of Calorimetry in which

$$\Sigma Q_{\text{isolated}} = 0 \text{ OR } \Sigma Q_{\text{lost}} = \Sigma Q_{\text{gained}}$$

BODIES PLACED IN CONTACT

Consider two bodies A and B of masses m_1 and m_2, at temperatures T_1 and having gram specific heats c_1 and c_2. When placed in contact, if equilibrium temperature is T_{eq} then Heat lost by A = Heat gained by B

$$\Rightarrow \quad m_1c_1(T_1 - T_{eq}) = m_2c_2(T_{eq} - T_2)$$

$$\Rightarrow \quad T_{eq} = \frac{m_1c_1T_1 + m_2c_2T_2}{m_1c_1 + m_2c_2}$$

CASE-1: SAME BODIES WITH UNEQUAL MASSES

If bodies are of identical materials, then $c_1 = c_2$

$$\Rightarrow \quad T_{eq} = \frac{m_1T_1 + m_2T_2}{m_1 + m_2}$$

CASE-2: DIFFERENT BODIES WITH EQUAL MASSES

If bodies are not of identical materials $(c_1 \neq c_2)$
But equal masses, then

$$T_{eq} = \frac{T_1c_1 + T_2c_2}{c_1 + c_2}$$

CASE-3: SAME BODIES WITH EQUAL MASSES

If bodies are of identical materials, $c_1 = c_2$ and equal masses i.e., $m_1 = m_2$. Then,

$$T_{eq} = \frac{T_1 + T_2}{2}$$

CASE-4: BODIES HAVING EQUAL WATER EQUIVALENT OR HEAT CAPACITIES

If bodies have equal water equivalents or the heat capacities, then we have $m_1c_1 = m_2c_2$. Hence, we get

$$T_{eq} = \frac{m_1c_1T_1 + m_2c_2T_2}{m_1c_1 + m_2c_2} = \frac{T_1 + T_2}{2}$$

Problem Solving Technique(s)

If a number of hot and cold liquids having no chemical affinity for each other are mixed together then equilibrium condition or temperature can be found by using Law of Calorimetry according to which

$$\Sigma Q_{\text{isolated}} = 0 \text{ OR } \Sigma Q_{\text{lost}} = \Sigma Q_{\text{gained}}$$

ILLUSTRATION 30

The molar heat capacity of a certain substance varies with temperature T as $C = a + bT$ where $a = 27.2\ \text{Jmol}^{-1}\text{K}^{-1}$ and $b = 4 \times 10^{-3}\ \text{Jmol}^{-1}\text{K}^{-2}$. How much heat is necessary to change the temperature of 2 mol of this substance from 27 °C to 427 °C?

SOLUTION

Initial temperature $T_i = 27 + 273 = 300\ \text{K}$

Final temperature $T_f = 427 + 273 = 700\ \text{K}$

Since, $Q = \int_{T_i}^{T_f} nCdT = \int_{T_i}^{T_f} (a + bT)\,dT$

$$\Rightarrow \quad Q = \int_{300}^{700} (2)\left(27.2 + 4\times10^{-3}\ \mathrm{T}\right)\cdot dT$$

$$\Rightarrow \quad Q = \left(54\cdot 4T + 4\times10^{-3}T^2\right)\Big|_{300}^{700}$$

$$\Rightarrow \quad Q = 23360\ \mathrm{J}$$

ILLUSTRATION 31

Three liquids P, Q and R are given. It is observed that 4 kg of P at 60 °C and 1 kg of R at 50 °C, when mixed produce a resultant temperature of 55 °C. A mixture of 1 kg of P at 60 °C and 1 kg of Q at 50 °C shows a temperature of 55 °C. Find the resulting temperature when 1 kg of Q at 60 °C is mixed with 1 kg of R at 50 °C.

SOLUTION

Let c_P, c_Q and c_R be the gram specific heats of liquids P, Q and R respectively.
When P and R are mixed, then

$$\Rightarrow \quad 4c_P(60-55) = (1)c_R(55-50)$$

$$\Rightarrow \quad 4c_P = c_R \qquad \ldots(1)$$

When P and Q are mixed, then

$$(1)c_P(60-55) = (1)c_Q(55-50)$$

$$\Rightarrow \quad c_P = c_Q \qquad \ldots(2)$$

When Q and R are mixed, then

$$(1)c_Q(60-T) = (1)c_R(T-50)$$

$$\Rightarrow \quad c_Q(60-T) = 4c_Q(T-50) \qquad \{\because \text{of (1) and (2)}\}$$

$$\Rightarrow \quad 60-T = 4(T-50)$$

$$\Rightarrow \quad T = 52\ ^\circ\mathrm{C}$$

ILLUSTRATION 32

A metal container of mass 500 g contains 200 g of water at 20 °C. A block of iron also of mass 200 g at 100 °C is dropped into water. Find the equilibrium temperature of the water. Given that specific heats of metal of container and iron and that of water are 910 $\mathrm{Jkg^{-1}K^{-1}}$, 470 $\mathrm{Jkg^{-1}K^{-1}}$ and 4200 $\mathrm{Jkg^{-1}K^{-1}}$ respectively.

SOLUTION

Here container and water are at 20 °C, thus when iron block is dropped into water it will loose energy to it and its temperature will fall. If equilibrium temperature is T_0, then we have

heat lost by iron block = heat gained by water plus container

$$m_i s_i(100-T_0) = m_\omega s_\omega(T_0-20) + m_C s_C(T_0-20)$$

$$\Rightarrow \quad T_0 = \frac{m_i s_i(100) + m_\omega s_\omega\left(20 + m_c s_c(20)\right)}{m_i s_i + m_\omega s_\omega + m_c s_c}$$

$$\Rightarrow \quad T_0 = \frac{0.2\times470\times100 + 0.2\times4200\times20 + 0.5\times910\times20}{0.2\times470 + 0.2\times4200 + 0.5\times910}$$

$$\Rightarrow \quad T_0 = \frac{9400 + 16800 + 9100}{94 + 840 + 455}$$

$$\Rightarrow \quad T_0 = \frac{35300}{1329} = 26.56\ ^\circ\mathrm{C}$$

CONCEPT OF LATENT HEAT OR THE HEAT OF TRANSFORMATION

The word Latent is from the Latin **"latere"**, meaning hidden or concealed.

A substance usually undergoes a change in temperature when heat is transferred between the substance and its surroundings. However, situations arise when the flow of heat does not change the temperature of body. This situation is arising when the physical characteristics of the substance change from one form to another, referred to as **Phase Change**. All such phase changes involve a change in internal energy. The energy required is called the **Heat of Transformation**.
The heat required to change the phase of a given mass m of a pure substance is given as

$$Q = mL$$

where, L is the latent heat of substance and depends upon nature of phase change as well as properties of a substance.

The **Latent Heat of Fusion**, L_f, is used when the phase change is from solid to liquid. The **Latent Heat of Vapourisation** is used when the phase change is from liquid to gas (vapours).
Latent heat of fusion of ice is $L_{\text{ice}} = 80\ \mathrm{calg^{-1}}$.
Latent heat of vaporisation of water is also called as latent heat of steam $L_{\text{steam}} = 540\ \mathrm{calg^{-1}}$.

When gas (or vapours) cools, it eventually returns to the liquid phase or condenses. The heat per unit mass given up is called ***Latent Heat of Condensation*** *which equals the latent heat of vaporisation. Likewise, when a liquid cools it eventually solidifies and so the* ***Latent Heat of Solidification*** *equals the Latent Heat of Fusion.*

Conceptual Note(s)

(a) ICE-WATER MIXING

Suppose water at temperature T_w °C be mixed with ice at 0 °C and let the equilibrium temperature of the mixture be $T_{\text{mix}} = T$, then we observe that firstly the ice will melt at 0 °C and then it's temperature rises

from 0 °C to T so as to attain thermal equilibrium. By Law of Calorimetry, we have

Heat lost by water = Heat gained by ice

$$\Rightarrow \quad m_w c_w (T_w - T_{mix}) = m_i L_i + m_i c_w (T_{mix} - 0^\circ)$$

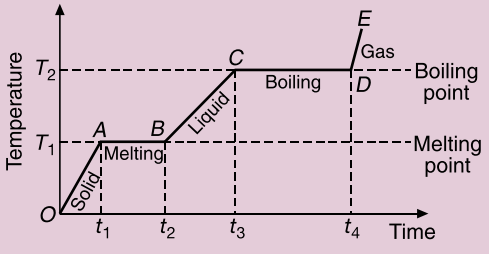
$$\Rightarrow \quad T_{mix} = T = \frac{m_w T_w - \frac{m_i L_i}{c_w}}{m_w + m_i}$$

(i) If $m_w = m_i$ then $T_{mix} = \dfrac{\left(T_w - \frac{L_i}{c_w}\right)}{2}$

(ii) By using this formulae, if $T_{mix} < T_i$, then take $T_{mix} = 0$ °C

(b) HEATING CURVE

Let heat be supplied at constant rate $P\left(=\frac{dQ}{dt}\right)$ to a given mass m of a solid and a graph be plotted between temperature and time. The graph is called Heating Curve. From the curve, following conclusions can be made.



(i) In the region OA, temperature of solid is changing with time so

$$Q_1 = mc_s\Delta T$$

$$\Rightarrow \quad P\Delta t = mc_s\Delta T$$

$$\Rightarrow \quad c_s = \frac{P\Delta t}{m\Delta T} = \frac{P}{m\left(\frac{\Delta T}{\Delta t}\right)}$$

Since, $\dfrac{\Delta T}{\Delta t}$ is the slope of temperature-time curve, so we have

$$c_s \propto \frac{1}{\text{Slope of line } OA}$$

So, the specific heat (or thermal capacity) is inversely proportional to the slope of temperature-time curve.

(ii) In the region AB temperature is constant, so it represents change of state i.e., melting of solid with melting point T_1. At A, melting starts and at B all the solid is converted into liquid. So between A and B the substance is partly solid and partly liquid. If L_F be the latent heat of fusion, then $Q_2 = mL_F$

$$\Rightarrow \quad L_F = \frac{P(t_2 - t_1)}{m} \qquad \{\because Q_2 = P(t_2 - t_1)\}$$

$$\Rightarrow \quad L_F \propto \text{length of line } AB$$

So, latent heat of fusion is proportional to the length of line of zero slope.

(iii) In the region BC temperature of liquid increases, so specific heat (or thermal capacity) of liquid will be inversely proportional to the slope of line BC, so

$$c_L \propto \frac{1}{\text{Slope of line } BC}$$

(iv) In the region CD, we observe that temperature is constant, so it represents the change of state, i.e., boiling with boiling point T_2. At C, all substance is in liquid state while at D it is in vapour state and between C and D it is partly liquid and partly gas. The length of line CD is proportional to latent heat of vapourisation L_V. So, we have

$$L_V \propto \text{length of line } CD$$

(v) The line DE represents the gaseous state of substance with its temperature increasing linearly with time. The reciprocal of slope of line will be proportional to specific heat or thermal capacity of substance in vapour state.

ILLUSTRATION 33

How much heat is supplied to convert 40 g of ice at –10 °C to steam at 100 °C?

SOLUTION

To convert 40 g of ice at –10 °C to 40 g steam at 100 °C, we have the following steps.

STEP-1

Heat supplied, Q_1 to convert 40 g of ice at –10 °C to 40 g of ice at 0 °C is

$$Q_1 = (40)(0.5)(0-(-10)) \text{ cal} = 200 \text{ cal}$$

STEP-2

Heat supplied, Q_2 to convert 40 g of ice at 0 °C to 40 g of water at 0 °C is

$$Q_2 = (40)(80) \text{ cal} = 3200 \text{ cal}$$

STEP-3

Heat supplied, Q_3 to convert 40 g of water at 0 °C to 40 g of water at 100 °C is

$$Q_3 = (40)(1)(100) \text{ cal} = 4000 \text{ cal}$$

STEP-4

Heat supplied, Q_4 to convert 40 g of water at 100 °C to 40 g of steam at 100 °C is

$$Q_4 = (40)(540)\ \text{cal} = 21600\ \text{cal}$$

So, total heat supplied is

$$Q = Q_1 + Q_2 + Q_3 + Q_4$$

$$\Rightarrow \quad Q = 200 + 3200 + 4000 + 21600 = 29000\ \text{cal} = 29\ \text{kcal}$$

ILLUSTRATION 34

In an industrial process 10 kg of water per hour is to be heated from 20 °C to 80 °C. To do this, steam at 150 °C is passed from a boiler into a copper coil immersed in water. The steam condenses in the coil and is returned to the boiler as water at 90 °C. How many kg of steam is required per hour?

Given that specific heat of steam is $1\ \text{calg}^{-1}(^\circ\text{C})^{-1}$ and latent heat of steam is $540\ \text{calg}^{-1}$

SOLUTION

Heat required by 10 kg water to increase its temperature from 20 to 80 °C in one hour is given by

$$Q_1 = (mc\Delta T)_{\text{water}} = (10\times10^3)(1)(80-20)$$

$$Q_1 = 600\ \text{kcal}$$

If m gram of steam is condensed per hour, the heat released by steam in converting into water at 90 °C

$$Q_2 = mc_s(150-100) + mL_v + mc_w(100-90)$$

i.e., $$Q_2 = m(1\times50 + 540 + 1\times10) = 600m\ \text{cal}$$

$$\{\because\ c_s = c_w = 1\ \text{calg}^{-1}\ ^\circ\text{C}\}$$

According to given problem $Q_2 = Q_1$

$$600m = 600\times10^3$$

$$\Rightarrow \quad m = 1\times10^3\ \text{g} = 1\ \text{kg}$$

ILLUSTRATION 35

In a pitcher when water is filled some water comes to outer surface slowly through its porous walls and gets evaporated. Most of the latent heat needed for evaporation is taken from water inside and hence this water is cooled down. Assume that 10 kg water is taken in the pitcher and 12 g water comes out and gets evaporated per minute. Neglecting heat transfer by convection and radiation to surroundings, calculate the time in which temperature of water in pitcher decreases by 5 °C.

SOLUTION

It is given that 12 g water is evaporated per minute, thus heat required per minute for it is

$$Q = mL_V = 12\times540 = 6480\ \text{calmin}^{-1}$$

After time t, mass of water inside the pitcher is

$$m = [10(1000) - 12t]\ \text{gram}$$

In a further time dt, mass of vapourised water is $dm = 12dt$ and let the temperature of water inside pitcher falls by dT. So, we get

$$(10000 - 12t)(1)(-dT) = (12dt)540$$

$$\Rightarrow \quad -dT = 6480\left(\frac{dt}{10000-12t}\right) \qquad \ldots(1)$$

Integrating equation (1), we get

$$-\int_{T_0}^{T_0-5} dT = 12\times540\int_0^t \frac{dt}{10000-12t}$$

$$\Rightarrow \quad 5 = 6480\left[\left(-\frac{1}{12}\right)\ln(10000-12t)\right]_0^t$$

$$\Rightarrow \quad -\frac{5}{540} = \ln\left(\frac{10000-12t}{10000}\right)$$

$$\Rightarrow \quad e^{-5/540} = \frac{10000-12t}{10000}$$

$$\Rightarrow \quad 10000e^{-5/540} = 10000 - 12t$$

$$\Rightarrow \quad 12t = 10000\left(1-e^{-5/540}\right)$$

$$\Rightarrow \quad t = \frac{10000}{12}(1-0.991) \approx 7.5\ \text{minute}$$

Students should note that this method will give the correct answer if the mass of water inside the pitcher nearly remains constant i.e., when the rate of evaporation is very as in this case.

ILLUSTRATION 36

Ice at 0 °C is added to 200 g of water initially at 70 °C in a vacuum flask. When 50 g of ice has been added and has all melted, the temperature of the flask and contents is 40 °C. When a further 80 g of ice has been added and has all melted, the temperature of the whole becomes 10 °C. Neglecting heat lost to the surroundings, calculate the latent heat of fusion of ice? (Specific heat of water is $1\ \text{calg}^{-1}\ ^\circ\text{C}$) and water equivalent of flask.

SOLUTION

If L is the latent heat of ice and W is the water equivalent of flask, According to Principle of Calorimetry

Heat gained = Heat lost

$$\Rightarrow \quad m'L + m'C\Delta T = (m+W)C\ \Delta T_{\text{water}}$$

$$\Rightarrow \quad 50(L + 1\times(40-0)) = (200+W)\times1\times(70-40)$$

$$\Rightarrow \quad 5L = 3W + 400 \qquad \ldots(1)$$

Now the system contains $(200+50)$ g of water at 40 °C, so when further 80 g of ice is added, then

$$80(L+1\times(10-0))=(250+W)\times1\times(40-10)$$

$$\Rightarrow \quad 8L=3W+670 \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$L=90 \text{ calg}^{-1} \text{ and } W=\left(\frac{50}{3}\right) \text{g}$$

ILLUSTRATION 37

A thermally insulated piece of metal is heated under atmospheric pressure by an electric current so that it receives electric energy at a constant power P. This leads to an increase of the absolute temperature T of the metal with time t as $T=T_0\left[1+a\left(t-t_0\right)\right]^{1/4}$, where a, t_0 and T_0 are constants. Determine the heat capacity at constant pressure $C_P(T)$ of the metal.

SOLUTION

Let the temperature increase in a small time interval dt be dT. During this time interval the metal receives an energy Pdt. The heat capacity is the ratio between the energy supplied and the temperature increase, so

$$C_P=\frac{Pdt}{dT}=\frac{P}{dT/dt}$$

From the given equation,

$$\frac{dT}{dt}=\left(\frac{T_0}{4}\right)a\left(1+a\left(t-t_0\right)\right)^{-3/4}=T_0\frac{a}{4}\left(\frac{T_0}{T}\right)^3$$

$$\Rightarrow \quad C_P=\frac{P}{dT/dt}=\left(\frac{4P}{aT_0^4}\right)T^3$$

We must note that, at low but not extremely low temperatures heat capacities of metals $C_P \propto T^3$.

Test Your Concepts-II

Based on Calorimetry

(Solutions on page H.77)

1. In a container of negligible mass 30 g of steam at 100 °C is added to 200 g of water that has a temperature of 40 °C. If no heat is lost to the surroundings, what is the final temperature of the system? Also find masses of water and steam in equilibrium. Take $L_v=539 \text{ calg}^{-1}$ and $c_{\text{water}}=1 \text{ calg}^{-1}(^\circ\text{C})^{-1}$
2. A copper cube of mass 200 g slides down a rough inclined plane of inclination 37° at a constant speed. Assuming that the loss in mechanical energy goes into the copper block as thermal energy, find the increase in temperature of the block as it slides down through 60 cms. Specific heat capacity of copper is equal to 420 $\text{Jkg}^{-1}\text{K}^{-1}$. Take $g=10 \text{ ms}^{-2}$.
3. A pitcher contains 1 kg water at 40 °C. It is given that the rate of evaporation of water from the surface of pitcher is 0.1 gs^{-1}. Calculate the time in which water inside the pitcher cools down to 30 °C. Given that latent heat of vaporization of water is 540 calg^{-1} and specific heat of water is 1 $\text{calg}^{-1}\ ^\circ\text{C}^{-1}$.
4. A vertical close cardboard tube 150 cm long is filled with lead shots to a depth of 30 cm. What is the least number of times the tube has to be inverted end to end to warm the lead shots by 2 °C. Given c_{lead} is $0.031 \text{ calg}^{-1}{}^\circ\text{C}^{-1}$.
5. A bullet of mass 10 g travelling at a speed of 100 ms^{-1} strikes a fixed wooden target. One half of the kinetic energy is transformed into heat to the bullet and the other half is transformed into heat to the wooden block. Find the rise in temperature of the bullet if c_{bullet} is $0.030 \text{ calg}^{-1}{}^\circ\text{C}^{-1}$.
6. The temperature of a body rises by 44 °C when a certain amount of heat is given to it. The same heat when supplied to 22 g of ice at −8 °C, raises its temperature to 16 °C. Find the water equivalent of the body.
   {Given: $S_{\text{water}}=1 \text{ calg}^{-1}\ ^\circ\text{C}^{-1}$ and $L_f=80 \text{ calg}^{-1}$, $S_{\text{ice}}=0.5 \text{ calg}^{-1}\ ^\circ\text{C}^{-1}$}
7. When 400 J of heat is added to a 0.1 kg sample of metal, its temperature increases by 20 °C. Calculate the specific heat of the metal.
8. 1 kg of ice at 0 °C is mixed with 1 kg of steam at 100 °C. Find the equilibrium temperature and the final composition of the mixture. Given that latent heat of fusion of ice is $3.36\times10^5 \text{ Jkg}^{-1}$ and latent heat of vaporization of water is $2.27\times10^6 \text{ Jkg}^{-1}$ and specific heat of water is $4200 \text{ Jkg}^{-1}{}^\circ\text{C}^{-1}$.
9. 10 g of water at 70 °C is mixed with 5 g of water at 30 °C. Find the temperature of the mixture in equilibrium.
10. When a block of metal of specific heat $0.1 \text{ calg}^{-1}{}^\circ\text{C}^{-1}$ and weighing 110 g is heated to 100 °C and then quickly transferred to a calorimeter containing 200 g of liquid at 10 °C, the resulting temperature is 18 °C. On repeating the experiment with 400 g of same liquid in the same calorimeter at the same initial temperature, the resulting temperature is 14.5 °C. Calculate specific heat of the liquid and water equivalent of the calorimeter.
11. The temperatures of equal masses of three different liquids A, B and C are 12 °C, 19 °C and 28 °C respectively. The temperature when liquids A and B are

mixed is $16\ °C$ and when liquids *B* and *C* are mixed is $23\ °C$. What should be the temperature when liquids *A* and *C* are mixed?

12. A lead bullet just melts when stopped by an obstacle. Assuming 25% heat to be absorbed by the obstacle, find the velocity of the bullet if its initial temperature is $27\ °C$. Given melting point of lead is $327\ °C$, c_{lead} is $0.03\ \text{calg}^{-1}°\text{C}^{-1}$, $L_{bullet} = 6\ \text{calg}^{-1}$ and $J = 4.2\ \text{joulecal}^{-1}$.

13. How should $1\ \text{kg}$ of water at $50\ °C$ be divided in two parts so that if one part is turned into ice at $0\ °C$, then it would release sufficient amount of heat to vaporize the other part. Given that latent heat of fusion of ice is $3.36\times10^5\ \text{Jkg}^{-1}$, latent heat of vapourisation of water is $22.5\times10^5\ \text{Jkg}^{-1}$ and specific heat of water is $4200\ \text{Jkg}^{-1}\text{K}^{-1}$.

14. A steel drill making $180\ \text{rpm}$ is used to drill a hole in a block of steel. The mass of the steel block and the drill is $180\ g$ each. If the entire mechanical work is used up in producing heat and the rate of rise of temperature of the block is $0.5\ °\text{Cs}^{-1}$. Find

(a) the rate of working of drill in watt.

(b) the couple required to drive the drill

(Given: Specific heat of steel is $0.10\ \text{calg}^{-1}°\text{C}^{-1}$)

15. A lead bullet strikes against a steel armour plate with a velocity of $300\ \text{ms}^{-1}$. If the bullet loses all its velocity after the impact, find the rise in temperature of the bullet assuming that the heat produced is equally shared by bullet and the target. Given $c_{bullet} = 0.03\ \text{calg}^{-1}°\text{C}^{-1}$.

16. An aluminium container of mass $100\ \text{g}$ contains $200\ \text{g}$ of ice at $-20\ °C$. Heat is added to the system at the rate 100 calorie per second. What is temperature of the system after four minutes? Specific heat of ice $= 0.5\ \text{calg}^{-1}\ °\text{C}^{-1}$ and latent heat of fusion of ice $= 80\ \text{calg}^{-1}$ and specific heat of aluminium is $0.215\ \text{calg}^{-1}\ °\text{C}^{-1}$.

17. In an experiment of measuring specific heat of water, a stream of water flows at a steady rate of $5\ \text{gs}^{-1}$ over an electrical heater dissipating $135\ \text{W}$ and a temperature rise of $5\ \text{K}$ is observed. On increasing the rate of flow to $10\ \text{gs}^{-1}$, the same temperature rise is produced with a dissipation of $240\ \text{W}$. Find the specific heat of water. Assume heat loss to the surrounding in both cases is the same.

18. How much heat is required to convert $8\ \text{g}$ of ice at $-15\ °C$ to steam at $100\ °C$? Given that $c_{ice} = 0.5\ \text{calg}^{-1}(°\text{C})^{-1}$, $L_{ice} = 80\ \text{cal g}^{-1}$, $L_{steam} = 540\ \text{cal g}^{-1}$, $c_{water} = 1\ \text{cal g}^{-1}\ (°\text{C})^{-1}$.

19. $5\ \text{g}$ ice at $0\ °C$ is mixed with $5\ \text{g}$ of steam at $100\ °C$. What is the final temperature?

20. A copper calorimeter of mass $100\ \text{g}$ contains $200\ \text{g}$ of a mixture of ice and water. Steam at $100\ °C$ under normal pressure is passed into the calorimeter and the temperature of the mixture is allowed to rise to $50\ °C$. If the mass of the calorimeter and its contents is now $330\ \text{g}$, what was the ratio of ice and water in the beginning? Neglect heat losses. Given that

Specific heat of copper $= 420\ \text{Jkg}^{-1}\ \text{K}^{-1}$

Specific heat of water $= 4200\ \text{Jkg}^{-1}\ \text{K}^{-1}$

Latent heat of fusion of ice $= 3.36\times10^5\ \text{Jkg}^{-1}$ and

Latent heat of condensation of steam $= 22.5\times10^5\ \text{Jkg}^{-1}$

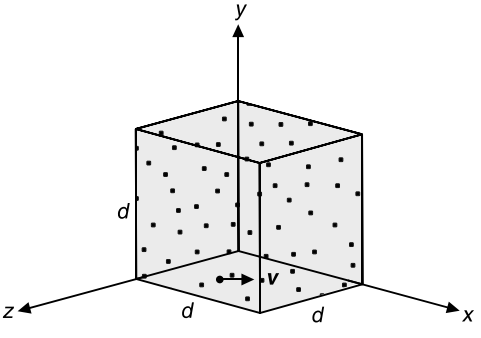
KINETIC THEORY OF GASES (KTG)

KINETIC THEORY OF GASES

Since a gas is made up of large number of molecules executing random motion, so the characteristics of a gas depends on the properties of motion of these molecules and this analysis of a gas at the microscopic level is known as Kinetic Theory of Gases (KTG) in which we relate the microscopic properties (like velocity of the molecule) to the macroscopic properties (like pressure, temperature, volume etc).

MOLECULAR MODEL FOR THE PRESSURE OF AN IDEAL GAS

Let us begin by developing a microscopic model of an ideal gas which shows that the pressure which a gas exerts on the walls of its container is a consequence of the collisions of the gas molecules with the walls of container. The following assumptions will be made



(a) The number of molecules is large, and the average separation between them is large compared with their dimensions. *So, molecules occupy negligible volume in comparison to volume of container.*

(b) The molecules obey Newton's Laws of Motion, but considered collectively move in a random fashion. *By random fashion we mean that molecules move in all the directions with equal probability and with various speeds. This distribution in velocities doesn't change with time, despite the collisions between molecules.*

(c) The forces between molecules are negligible except during a collision. *The forces between molecules are short range, so that the only time the molecules interact with each other is during a collision.*

(d) The molecules undergo elastic collisions with each other and with the walls of the container. *So, the molecules are considered to be structureless (i.e., point masses), and in the collisions both momentum and kinetic energy are conserved.*

(e) The gas under consideration is a pure gas i.e., *all molecules of the gas are identical.*

(f) Ideally, all gases obey the ideal gas equation i.e.,

$$PV = nRT$$

where, P is the pressure of gas, V is the volume of gas, T is the temperature of gas and R is Universal Gas Constant having value

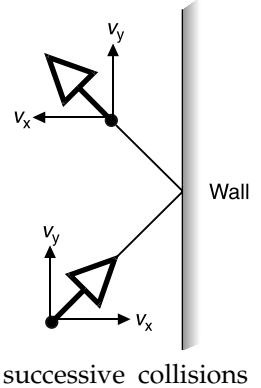
$$R = 8.314 \text{ Jmol}^{-1}\text{K}^{-1}$$

(g) Typically, a real gas obeys ideal gas equation only at very high temperature and very low pressure. This is because at very high temperatures, the kinetic energy of the molecules is so large that any interaction between the molecules of the gas can be neglected. Also, at very low pressure, the separation between the gas molecules is very large and hence again the interaction between the molecules of the gas can be neglected.

Let's derive an expression for the pressure of an ideal gas consisting of N molecules in a container of volume $V (= d^3)$. The container is assumed to be in the cubic shape with each edge of length d. Consider the collision of one molecule moving with velocity v towards the right hand face of the system (box). The molecule has velocity components v_x, v_y and v_z along x, y and z axes respectively. When the molecule collides elastically with the wall its x component of velocity gets reversed whereas y and z components remaining unaltered.

The x component of momentum before collision is mv_x and after collision is $-mv_x$. So, change in momentum of molecule along x axis is

$$\Delta p_x = -mv_x - mv_x = -2mv_x$$



The momentum delivered to the wall for each collision is $2mv_x$, since the momentum of the system (molecule+container) is conserved.

For a molecule to make two successive collisions with the same wall it must travel a distance $2d$ in the x direction in time Δt. So

$$\Delta t = \frac{2d}{v_x}$$

If F is the magnitude of the average force exerted by a molecule on the wall in time Δt, then

$$F\Delta t = \Delta p_x = 2mv_x$$

$$\Rightarrow \quad F = \frac{2mv_x}{\Delta t} = \frac{2mv_x}{2d/v_x} = \frac{mv_x^2}{d} \qquad \ldots(1)$$

Total force on the wall is the sum of all such terms for all particles. To get the total pressure P on the wall we have

$$P = \frac{\Sigma F}{A} = \frac{m}{d^3}\left(v_{x_1}^2 + v_{x_2}^2 + ...\right) \quad ...(2)$$

where v_{x_1}, v_{x_2} ..... refer to the x components of velocities for particles 1, 2, ...

The average value of v_x^2 is given by

$$\langle v_x^2 \rangle = \frac{v_{x_1}^2 + v_{x_2}^2 + ... + v_{x_N}^2}{N}$$

and the volume of container is given by $V = d^3$ using (2), we have

$$P = \frac{Nm}{V}\langle v_x^2 \rangle \quad ...(3)$$

Further for any particle moving with velocity v

$$v^2 = v_x^2 + v_y^2 + v_z^2 \quad ...(4)$$

$$\Rightarrow \quad \langle v^2 \rangle = \langle v_y^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle \quad ...(5)$$

As there is no preferred direction for the molecules, so average values of v_x^2, v_y^2 and v_z^2 are equal and hence

$$\langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle = \frac{\langle v^2 \rangle}{3}$$

Substituting (5) in (3), we have

$$P = \frac{1}{3}\frac{Nm}{V}\langle v^2 \rangle = \frac{2}{3V}\left[N\left\{\frac{1}{2}m\langle v^2 \rangle\right\}\right] \quad ...(6)$$

Average Kinetic Energy of each molecule is $\langle K \rangle = \frac{1}{2}m\langle v^2 \rangle$, so total Kinetic Energy associated with the system of N molecules is

$$E = N\langle K \rangle = N\left(\frac{1}{2}m\langle v^2 \rangle\right) \quad ...(7)$$

Using equation (6) and (7), we get

$$P = \frac{2}{3}\frac{E}{V}$$

$$\Rightarrow \quad PV = \frac{2}{3}E$$

The empirical equation of state for an ideal gas is

$$PV = Nk_BT \quad ...(8)$$

where the Boltzmann constant is

$$k_B = \frac{R}{N_A} = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Substituting (8) in (6), we have

$$\frac{Nk_BT}{V} = \frac{2}{3}\frac{N}{V}\frac{1}{2}m\langle v^2 \rangle$$

$$\Rightarrow \quad \frac{1}{2}m\langle v^2 \rangle = \frac{3}{2}k_BT$$

$$\Rightarrow \quad \langle v^2 \rangle = \frac{3k_BT}{m}$$

Root Mean Square (RMS) velocity of a molecule of mass m at temperature

$$v_{\text{rms}} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{3RT}{M}}$$

where, m is mass of one molecule of gas and M is the molar mass of gas or mass of 1 mole of gas.

Conceptual Note(s)

(a) To conclude, with this simplified model of ideal gas, we have arrived at an important result that relates the macroscopic quantities of pressure, volume and temperature to a microscopic quantity like rms speed. So, we get a key link between the microscopic world of gas molecules and the macroscopic world.

(b) In the derivation of this result, note that we have not accounted for collisions between gas molecules. When these collisions are considered, the results **does not alter as collisions do not affect the momenta of the particles, with no net effect on the walls**. Bravo!!! This is consistent with one of our initial assumptions, namely, that the distribution of velocities doesn't change with time.

(c) In addition, although our result was derived for a cubical container whereas the result is valid for a container of any shape.

(d) The number density of molecules in gas is $n_0 = \frac{N}{V}$ and each molecule is assumed to be moving in random direction with rms speed v_{rms}. Due to the random character of motion of the gas molecules, it can be assumed that towards "each face" of the cubical container, $\frac{n_0}{6}$ molecules are moving with this speed. Due to this the number of collisions N_C of the molecules with "a wall" of the cubical container per square metre of its surface can be written as

$$N_C = \left(\frac{n_0}{6}\right)v_{\text{rms}}$$

KINETIC INTERPRETATION OF TEMPERATURE

From the point of view of kinetic theory, temperature is a quantity characterizing the average kinetic energy of translatory motion of the molecules of an ideal gas. Since we know that

$$PV = \frac{2}{3}N_A\left(\frac{1}{2}m\langle v^2 \rangle\right)$$

Comparing above equation with $PV = N_Ak_BT$, we can find the average kinetic energy of a molecule as

$$K_{av} = \frac{1}{2}mv_{rms}^2 = \frac{3}{2}k_BT$$

For an ideal gas, the absolute temperature is a measure of the average translational kinetic energy of the molecules.

Note that the kinetic energy calculated above is associated with the random translational motion of a single molecule. It does not include any orderly motion imposed. The rms speed of molecules varies with temperature as

$$v_{rms} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{3RT}{M}}$$

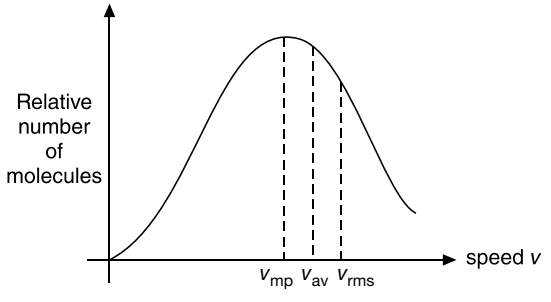
MAXWELL BOLTZMANN LAW FOR DISTRIBUTION OF MOLECULAR SPEEDS

Since the molecules in a gas are in random motion, so many molecules of gas may have speeds less than the average speed (or rms speed) and others have speeds greater than the average speed (or rms speed).

As an example, the rms speed of an oxygen molecule in a gas sample at 300 K is

$$v_{rms} = \sqrt{\frac{3(8.314)(300)}{32/1000}} \approx 483 \text{ ms}^{-1}$$

This does not mean that the speed of every molecule in the sample is 483 ms^{-1}. Many molecules in the sample may have speeds less than 483 ms^{-1} and many may molecules in the sample may have speed more than 483 ms^{-1}. *This conclusion was made by Maxwell on the basis of kinetic theory and the experiments done in which Maxwell plotted a graph of variation of speeds of molecules with the relative number of molecules as shown in Figure.*



Maxwell derived an equation giving the distribution of molecules in different speeds. If dN represents the number of molecules with speeds between v and $v+dv$, then based on statistical approach, the number of molecules dN having speed between v and $v+dv$ is given by

$$dN = 4\pi N\left(\frac{m}{2\pi k_BT}\right)^{3/2} v^2 e^{-mv^2/2k_BT}dv$$

The probability that a molecule possesses speed v is

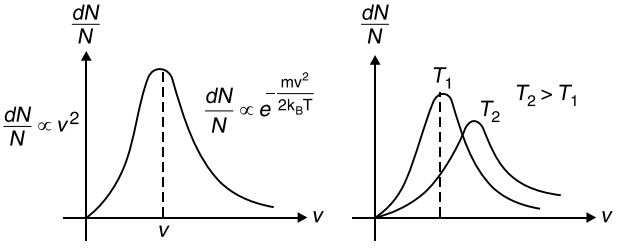
$$\frac{dN}{N} = f(v)dv = 4\pi\left(\frac{m}{2\pi k_BT}\right)^{3/2} v^2 e^{-mv^2/2k_BT}dv$$

where, $f(v)$ is the number of gas molecules per unit range of speeds.

The expected number of molecules $N(v_1, v_2)$ in the gas sample lying within speeds between v_1 to v_2 is obtained by integrating $Nf(v)dv$ from v_1 to v_2 i.e.,

$$N(v_1, v_2) = \int_{v_1}^{v_2} f(v)dv$$

The speed v_p at which $\frac{dN}{dv}$ is maximum is called the most probable speed (v_{mp}). Maxwell distribution curve is shown in Figure (a). The curves show the dN increases with increase of speed, becomes maximum and then it decreases.



At low speeds, $dN \propto v^2$ and hence this part of curve is parabolic. At high speeds $dN \propto e^{-mv^2/2k_BT}$, so this part of the curve is decreasing exponentially.

The speed at which dN is maximum is called the most probable speed and denoted by v_{mp}. The value of v_{mp} decreases with increase of temperature. Figure (b) represents dN verses v curves at two different temperatures T_1 and $T_2\,(>T_1)$.

AVERAGE VELOCITY OF GAS MOLECULES

Since all molecules of the gas in container have random motion due to repeated collisions, so if $\vec{v}_1$, $\vec{v}_2$ ….. $\vec{v}_N$ are the instantaneous velocity vectors of all N molecules of a gas, then average velocity vector of these N molecules is zero i.e.,

$$\langle \vec{v} \rangle = \frac{\vec{v}_1 + \vec{v}_2 + ... + \vec{v}_N}{N} = 0$$

Hence, the average velocity of molecules in a gas is always taken as zero. However, average speed of the gas molecules will not be zero.

AVERAGE SPEED OF GAS MOLECULES

Due to the random motion of the gas molecules, average velocity of all the molecules of a gas is zero. However, it is not same in case of mean or average speed, which is obtained by averaging the speed of the molecules. If can be simply defined as

$$v_{av} = \langle v \rangle = \frac{|\vec{v}_1| + |\vec{v}_2| + ... |\vec{v}_N|}{N}$$

Using Maxwell's distribution function, we can calculate the mean speed as

$$v_{av} = \langle v \rangle = \frac{1}{N}\int_0^\infty v f(v)\, dv = \frac{1}{N}\int_0^\infty v dN = \sqrt{\frac{8RT}{\pi M}}$$

ROOT-MEAN-SQUARE VELOCITY OF GAS MOLECULES

As the name implies this is the square root of mean of squares of velocities of all the molecules of a gas. Mean of squares can be simply written as

$$v_{\text{rms}} = \sqrt{\langle v^2 \rangle} = \sqrt{\left(\frac{|\vec{v}_1|^2 + |\vec{v}_2|^2 + ... |\vec{v}_N|^2}{N}\right)}$$

and from Maxwells Law, we can write

$$v_{\text{rms}} = \sqrt{\frac{1}{N}\int_0^\infty v^2 f(v)\, dv} = \sqrt{\frac{1}{N}\int_0^\infty v^2 dN} = \sqrt{\frac{3RT}{M}}$$

MOST PROBABLE SPEED OF GAS MOLECULES

The speed v_{mp} at which $\frac{dN}{dv}$ is maximum is called the most probable speed (v_{mp}). This is the speed which maximum number of molecules have and this speed corresponds to maxima in the Maxwell's distribution curve. So at most probable speed, we have

$$\frac{dN}{dv} = 0$$

$$\Rightarrow \quad v_{\text{mp}} = \sqrt{\frac{2RT}{M}}$$

So we conclude that, $v_{\text{rms}} > v_{\text{mean}} > v_{\text{mp}}$

Conceptual Note(s)

(a) $v_{\text{rms}} > v_{av} > v_{mp}$ i.e., $R > A > M$

One can easily remember above relation by the word *RAM*, where *R* stands for rms, *A* for Average and *M* for most probable

(b) $v_{\text{rms}} : v_{av} : v_{mp} = \sqrt{3} : \sqrt{\frac{8}{\pi}} : \sqrt{2} = \sqrt{3} : \sqrt{2.5} : \sqrt{2}$

(c) Speed of sound in a gas is given by

$$v_{\text{sound}} = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{\gamma k_B T}{m}} = \sqrt{\frac{\gamma P}{\rho}}$$

i.e., the speed of sound in a gas is of the same order as rms speed of its molecules. In nutshell, we can write

$$v = \sqrt{\frac{ART}{M}} = \sqrt{\frac{Ak_B T}{m}} = \sqrt{\frac{AP}{\rho}}$$

where A is a number whose value is given by

$A = 3$ for v_{rms}, $A = \frac{8}{\pi}$ for v_{av}

$A = 2$ for v_{mp}, $A = \gamma$ for v_{sound}

(d) Note that in the above expression the molecular mass M should be taken in kilogram (not in gram). For example, at NTP the rms speed of hydrogen gas is given by

$$(v_{\text{rms}})_{H_2} = \sqrt{\frac{3\times 8.31\times 273}{2\times 10^{-3}}} = 1840 \text{ ms}^{-1} \qquad \text{(Correct)}$$

$$\neq \sqrt{\frac{3\times 8.31\times 273}{2}} \qquad \text{(Incorrect)}$$

MEAN FREE PATH

If $\lambda_1, \lambda_2, \lambda_3, ..., \lambda_N$ are N free paths, then mean free path

$$\lambda = \frac{\lambda_1 + \lambda_2 + ... + \lambda_N}{N} = \frac{\bar{v}t}{N} = \frac{\langle v \rangle t}{N}$$

where, $\bar{v} = \langle v \rangle$ is average speed.

Also $\lambda = \frac{1}{\sqrt{2}\pi D^2 n}$ where, n is the number density of molecules i.e.,

$$n = \frac{\text{Number of molecules}}{\text{Volume}} = \frac{N}{V}$$

and D is the molecular diameter. Since for one mole of gas sample, we have

$$n = \frac{N_A}{V} = \frac{N_A}{RT/P} = \frac{P}{k_B T}$$

$$\Rightarrow \quad \lambda = \frac{k_B T}{\sqrt{2}\pi D^2 P}$$

ILLUSTRATION 38

The speeds (in ms^{-1}) for 8 molecules 2, 4, 5, 5, 8, 9, 12 and 15. Calculate their

(a) average speed and **(b)** rms speed

SOLUTION

(a) The average speed is given by

$$v_{av} = \frac{1}{N}(v_1 + v_2 + ... + v_N)$$

$$\Rightarrow \quad v_{av} = \frac{(2+4+5+5+8+9+12+15)\ \text{ms}^{-1}}{8}$$

$$\Rightarrow \quad v_{av} = 7.5\ \text{ms}^{-1}$$

(b) To find the rms speed we first find the average of v^2:

$$\langle v^2 \rangle = \frac{1}{N}\left(v_1^2 + v_2^2 + v_3^2 + \ldots\ldots\ldots + v_N^2\right)$$

$$\Rightarrow \quad \langle v^2 \rangle = \frac{\left(2^2+4^2+5^2+5^2+8^2+9^2+12^2+15^2\right)}{8}$$

$$\Rightarrow \quad \langle v^2 \rangle = 73\ \text{ms}^{-2}$$

Then, $v_{\text{rms}} = \langle \sqrt{v^2} \rangle = 8.54\ \text{ms}^{-1}$

In general the average speed is not equal to the rms speed.

ILLUSTRATION 39

In a certain region of space, on an average, there are only 5 molecules per cubic centimetre. The gas has a temperature of 3 K. Calculate average pressure of this very dilute gas if $k_B = 1.38\times10^{-23}\ \text{Jmol}^{-1}\text{K}^{-1}$.

SOLUTION

According to ideal gas equation $PV = nRT$

$$\Rightarrow \quad P = \left(\frac{nN_Ak_BT}{V}\right) \qquad \{\because R = N_Ak\}$$

Also, the number of molecules $N = nN_A$

$$\Rightarrow \quad PV = Nk_BT$$

$$\Rightarrow P = \frac{N}{V}k_BT$$

where, N/V is the number of molecules per unit volume. Since,

$$\frac{N}{V} = 5\ \text{moleculescm}^{-3} = \frac{5\ \text{molecules}}{(10^{-2}\ \text{m})^3}$$

$$\Rightarrow \quad \frac{N}{V} = 5\times10^6\ \text{moleculesm}^{-3}$$

$$\Rightarrow \quad P = \left(5\times10^6\right)\left(1.38\times10^{-23}\right)(3) = 2.07\times10^{-16}\ \text{Pa}$$

ILLUSTRATION 40

Find the average translational kinetic energy of molecules in air at 300 K.

SOLUTION

Although air consists of a mixture of gases, they all have the same average translational kinetic energy.

$$K_{av} = \frac{3}{2}k_BT = \frac{3}{2}\left(1.38\times10^{-23}\right)(300) = 6.21\times10^{-21}\ \text{J}$$

ILLUSTRATION 41

Calculate the root mean square speed of smoke particles of mass 5×10^{-17} kg in their Brownian motion in air at NTP. Take $k_B = 1.38\times10^{-23}\ \text{JK}^{-1}$.

SOLUTION

According to Kinetic Theory of Gases

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3k_BT}{m}}$$

Here $T = 273$ K, $m = 5\times10^{-17}$ kg, $k_B = 1.38\times10^{-23}\ \text{JK}^{-1}$

$$\Rightarrow \quad v_{\text{rms}} = \sqrt{\frac{3\times1.38\times10^{-23}\times273}{5\times10^{-17}}}$$

$$\Rightarrow \quad v_{\text{rms}} = 1.5\times10^{-2}\ \text{ms}^{-1} = 1.5\ \text{cms}^{-1}$$

ILLUSTRATION 42

Two vessels having equal volume contain molecular hydrogen at one atmosphere and helium at two atmospheres respectively. What is the ratio of rms speed of hydrogen molecule to that of helium molecule, assume that both the samples are at the same temperature?

SOLUTION

According to Kinetic Theory of Gases, $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

$$\Rightarrow \quad \frac{v_H}{v_{He}} = \sqrt{\frac{M_{He}}{M_H}} = \sqrt{\frac{4}{2}} = \sqrt{2}$$

Conceptual Note(s)

If one uses the relation

$$\frac{v_H}{v_{He}} = \sqrt{\frac{P_H}{M_H}\times\frac{M_{He}}{P_{He}}} = \sqrt{\frac{1}{2}\times\frac{4}{2}} = 1 \text{ i.e., } v_H = v_{He}$$

This solution is **wrong** as in it M is **not** molecular weight but total mass of gas. Now as pressure of helium is double that of hydrogen at same temperature and volume, in accordance with Avogadro's Law the number of moles of helium will be double that of hydrogen.

$$\Rightarrow \quad \frac{v_H}{v_{He}} = \sqrt{\frac{P_H}{nM_H}\times\frac{(2nM_{He})}{P_{He}}} = \sqrt{\frac{1}{2}\times\frac{2\times4}{2}} = \sqrt{2}$$

ILLUSTRATION 43

At what temperature will the rms speed of oxygen molecule will be sufficient for escaping from the earth? Given that escape velocity from surface of earth is $v_e = 11.2\ \text{kms}^{-1}$, mass of oxygen molecule is $m = 2.76\times10^{-26}$ kg and Boltzmann's constant is $k_B = 1.38\times10^{-23}\ \text{JK}^{-1}$.

SOLUTION

If the temperature is T, according to kinetic theory of gases translational $KE = \left(\frac{3}{2}\right)k_B T$

The oxygen molecule will escape from earth if

$$\frac{3}{2}k_B T > \frac{1}{2}mv_e^2$$

$$\Rightarrow \quad T > \frac{mv_e^2}{3k_B}$$

$$\Rightarrow \quad T > \frac{2.76\times10^{-26}\times\left(11.2\times10^3\right)^2}{3\times1.38\times10^{-23}}$$

$$\Rightarrow \quad T > 8.36\times10^4 \text{ K}$$

ILLUSTRATION 44

Calculate the number of molecules per unit volume of air at 0 °C, pressure of 1.013×10^5 Pa and the average distance between molecules if Boltzmann constant is $k_B = 1.38\times10^{-23}$ JK^{-1}.

SOLUTION

Pressure of gas is given as $P = \frac{2}{3}E$, where E is the translational kinetic energy per unit volume.

$$\Rightarrow \quad P = \frac{2}{3}\left(\frac{3}{2}k_B T\times n_0\right) = n_0 k_B T$$

$$\Rightarrow \quad n_0 = \frac{P}{k_B T} = \frac{1.013\times10^5}{1.38\times10^{-23}\times273}$$

$$\Rightarrow \quad n_0 = 2.688\times10^{25} \text{ m}^{-3}$$

Average separation between molecules

$$d = \left(\frac{1}{n_0}\right)^{\frac{1}{3}} = 3.31\times10^{-9} \text{ m}$$

ILLUSTRATION 45

One gram mole NO_2 at 57 °C and 2 atm pressure is kept in a vessel. Assuming the molecules to be moving with rms velocity, find the number of collisions per second, which the molecules make with one square meter area of the vessel wall.

SOLUTION

Since, $P = \frac{nRT}{V}$ …(1)

Also, we know that $P = \frac{F}{A}$

For $A = 1$ m^2, we have $P = F$ (numerically)

So, $P = F = \frac{dp}{dt} = \left(2mv_{\text{rms}}\right)f$ …(2)

So, from (1) and (2), we get

$$P = \frac{nRT}{V} = 2mv_{rms}f$$

where, f is the number of collisions per unit area per unit time and m is the mass of one gas molecules $= \frac{M}{N_A}$

$$\Rightarrow \quad P = 2\frac{M}{N_A}\sqrt{\frac{3RT}{M}}f$$

$$\Rightarrow \quad f = \frac{PN_A}{2}\sqrt{\frac{1}{3MRT}}$$

Substituting the values, we get

$$f = \frac{2\times10^5\times6.02\times10^{23}}{2}\sqrt{\frac{1}{3\times46\times10^{-3}\times8.31\times330}}$$

$$\Rightarrow \quad f = 3.1\times10^{27} \text{ s}^{-1}$$

ILLUSTRATION 46

A cubical box of side 1 meter contains helium gas (atomic weight 4) at a pressure of 100 Nm^{-2}. During an observation time of 1 second, an atom travelling with the root mean square speed parallel to one of the edges of the cube, was found to make 500 hits with a particular wall, without any collision with other atoms. If $R = \frac{25}{3}$ Jmol^{-1}K^{-1} and $k = 1.38\times10^{-23}$ JK^{-1}, then calculate the

(a) temperature of the gas.
(b) average kinetic energy per atom.
(c) total mass of helium gas in the box.

SOLUTION

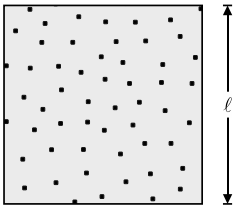
Given that, Volume of the box is 1 m^3, pressure of the gas is 100 Nm^{-2}.

If T be the temperature of the gas

(a) Time between two consecutive collisions with one wall is

$$\Delta t = \frac{1}{500} \text{ sec}$$

This time is equal to $\frac{2l}{v_{\text{rms}}}$, where l is the side of the cube.



$$\Rightarrow \quad \Delta t = \frac{2l}{v_{\text{rms}}} = \frac{1}{500}$$

$$\Rightarrow \quad v_{\text{rms}} = 1000 \text{ ms}^{-1} \qquad \{\because\ l = 1 \text{ m}\}$$

$$\Rightarrow \quad \sqrt{\frac{3RT}{M}} = 1000$$

$$\Rightarrow \quad T = \frac{(1000)^2 M}{3R} = \frac{(10)^6\left(4\times10^{-3}\right)}{3(25/3)} = 160 \text{ K}$$

(b) Average kinetic energy per atom is $\frac{3}{2}k_BT$

$$\Rightarrow \langle E\rangle = \frac{3}{2}(1.38\times10^{-23})(160)\ \text{J}$$

$$\Rightarrow \langle E\rangle = 3.312\times10^{-21}\ \text{J}$$

(c) Since, $PV = nRT = \left(\frac{m}{M}\right)RT$

So, mass of helium gas in the box is given by

$$m = \frac{PVM}{RT}$$

Substituting the values, we get

$$m = \frac{(100)(1)(4\times10^{-3})}{(25/3)(160)} = 3\times10^{-4}\ \text{kg}$$

ILLUSTRATION 47

Calculate the root mean square speed and mean kinetic energy of one mole of hydrogen at STP (Given that density of hydrogen is $0.09\ \text{kgm}^{-3}$). If mass of a molecule of hydrogen is 3.33×10^{-27} kg, then calculate the value of Avogadro's number and Boltzmann's constant. Take $R = 8.3\ \text{Jmol}^{-1}\text{K}^{-1}$.

SOLUTION

Since, the pressure of a gas is given by $P = \frac{1}{3}\rho v_{\text{rms}}^2$

$$\Rightarrow \quad v_{\text{rms}} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3\times0.76\times13.6\times10^3\times9.8}{0.09}}$$

$$\Rightarrow \quad v_{\text{rms}} = 1837\ \text{ms}^{-1} = 1.837\ \text{kms}^{-1}$$

Kinetic energy is $K = N_A\left(\frac{1}{2}mv_{\text{rms}}^2\right) = \frac{1}{2}Mv_{\text{rms}}^2$

where, $M = 2\ \text{g} = 2\times10^{-3}$ kg

$$\Rightarrow \quad K = \frac{1}{2}\times2\times10^{-3}\times(1837)^2 = 3374.56\ \text{J}$$

Mass of one molecule of hydrogen is $m = 3.33\times10^{-27}$ kg and molar mass of hydrogen is $M = 2\times10^{-3}$ kg. So, the Avogadro's number N_A which is the number of molecules in one-gram molecule of hydrogen is given by

$$N_A = \frac{M}{m} = \frac{2\times10^{-3}}{3.34\times10^{-27}} = 3\times10^{23}\ \text{molecules}$$

$$\Rightarrow \quad k_B = \frac{R}{N_A} = \frac{8.3}{6\times10^{23}} = 1.38\times10^{-23}\ \text{Jmol}^{-1}\text{K}^{-1}$$

ILLUSTRATION 48

In a container, the molecular density of an enclosed gas is 10^{26} molecules per cubic centimetre, each of mass 3×10^{-27} kg. If rms velocity of the gas molecules is $2000\ \text{ms}^{-1}$, calculate the number of molecules hitting per square meter of the container wall every second and the pressure exerted on the walls of container by the molecules.

SOLUTION

Number of molecules hitting per square meter of container wall is given as

$$N_C = \left(\frac{n}{6}\right)v_{\text{rms}} = \frac{1}{6}\times10^{26}\times2000$$

$$\Rightarrow \quad N_C = 3.33\times10^{28}\ \text{per second}$$

and pressure is $P = \frac{1}{3}\rho v_{\text{rms}}^2$

$$\Rightarrow \quad P = \frac{1}{3}(3\times10^{-27})(10^{26})(2000)^2 = 4\times10^5\ \text{Nm}^{-2}$$

ILLUSTRATION 49

Calculate the temperature at which rms velocity of a gas molecule is same as that of a molecule of another gas at 47 °C. Molecular weight of first and second gases are 64 and 32 respectively.

SOLUTION

Since $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

For first and second gas if temperature is T_1 and T_2 respectively, then

$$(v_{\text{rms}})_1 = (v_{\text{rms}})_2$$

$$\Rightarrow \quad \frac{T_1}{M_1} = \frac{T_2}{M_2}$$

$$\Rightarrow \quad \frac{T_1}{64\times10^{-3}} = \frac{320}{32} \qquad \{\because T_2 = 47+273 = 320\ \text{K}\}$$

$$\Rightarrow \quad T_1 = 640\ \text{K}$$

ILLUSTRATION 50

A vessel of volume $V = 16.62$ litre contains a mixture of hydrogen and helium at a temperature 17 °C and pressure 6 atmosphere. The mass of the mixture is 10 g. Find the

(a) ratio of mass of hydrogen to that of helium.

(b) ratio of rate of collisions of hydrogen molecule per unit area of the container walls to that of helium molecules.

SOLUTION

(a) $PV = (n_1+n_2)RT = \left(\frac{m_1}{M_1}+\frac{m_2}{M_2}\right)RT$

$$\Rightarrow \quad \frac{m_1}{2}+\frac{m_2}{4} = \frac{6\times10^5\times16.62\times10^{-3}}{8.31\times300}$$

$$\Rightarrow \quad 2m_1+m_2 = 16 \qquad \ldots(1)$$

Also, $m_1 + m_2 = 10$ …(2)

Solving equations (1) and (2), we get

$m_1 = 6$ g and $m_2 = 4$ g

$\Rightarrow \frac{m_1}{m_2} = \frac{3}{2}$

(b) Number of moles of H_2, $n_1 = 3$ and number of moles of He, $n_2 = 1$

Since, $P = \frac{nRT}{V} = 2mv_{rms}f$ …(3)

where f is the number of collisions of the molecules per unit area per unit time.

Since, $m = \frac{M}{N_A}$ and $v_{rms} = \sqrt{\frac{3RT}{M}}$

Substituting these values in equation (3), we get

$f \propto \frac{n}{\sqrt{M}}$

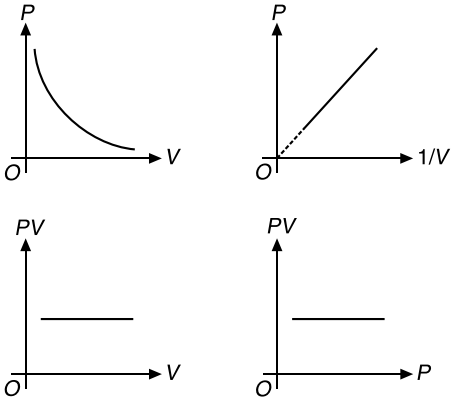
$\Rightarrow \frac{f_1}{f_2} = \frac{n_1}{n_2}\sqrt{\frac{M_2}{M_1}} = \frac{3}{1}\sqrt{\frac{4}{2}} = 3\sqrt{2}$

GAS LAWS

The gases such as hydrogen, oxygen and helium etc. which cannot be liquified easily are called **permanent gases**. The gases whose molecules are point masses (mass without having volume) and do not attract each other are called **ideal or perfect gases**. Assuming permanent gases to be ideal, through experiment it has been observed that, these gases irrespective of their nature obey the following laws.

BOYLE'S LAW

For a given mass of an ideal gas at constant temperature, the volume of a gas is inversely proportional to its pressure. i.e. $V \propto \frac{1}{P}$



So, PV = constant, for given temperature and given mass of gas.

CHARLE'S LAW

At constant pressure, the volume of a given quantity of a gas is directly proportional to the absolute temperature i.e., $V \propto T$ or $\frac{V}{T}$ = constant, for a given mass of gas at constant pressure

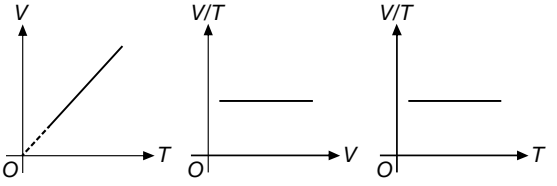


ILLUSTRATION 51

A glass container encloses a gas at a pressure of 8×10^5 Pa and 300 K temperature. The container walls can bear a maximum pressure of 10^6 Pa. If the temperature of container is gradually increased, find the temperature at which container will break.

SOLUTION

At constant volume, we have $\frac{P_1}{T_1} = \frac{P_2}{T_2}$

$\Rightarrow \quad T_2 = \left(\frac{P_2}{P_1}\right)T_1$

Since, $P_1 = 8 \times 10^5$ Pa, $T_1 = 300$ K, $P_2 = 10^6$ Pa

$\Rightarrow \quad T_2 = \frac{10^6}{8 \times 10^5} \times 300 = 375$ K

GAY-LUSSAC'S LAW

For a given mass of an ideal gas at constant volume, pressure of a gas is directly proportional to its absolute temperature i.e., $P \propto T$

$\Rightarrow \quad \frac{P}{T}$ = constant, where m and V = constant.

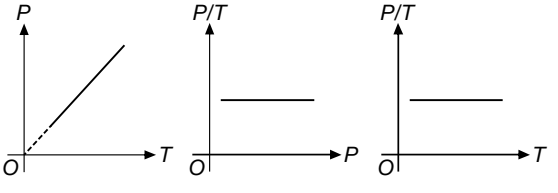
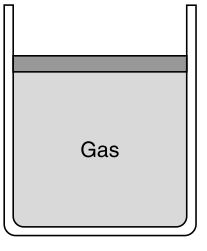


ILLUSTRATION 52

A gas is enclosed inside a cylindrical container shown in Figure.



Its initial volume is V and temperature is T. No external pressure is applied on the light piston, hence gas pressure must be equal to the atmospheric pressure. If gas temperature is doubled, find its final volume. If in its final state, the piston is clamped and temperature is again doubled, find the final pressure of the gas.

SOLUTION

Initially $P = \text{constant}$, so we have $V \propto T$
So, when the gas temperature is doubled, then gas volume is also doubled i.e., $V' = 2V$
Now, when piston is clamped after attaining the final state, then $V' = \text{constant} = 2V$, so we have $P \propto T$.
Now, when its temperature is doubled from $2T$ to $4T$, then the pressure also becomes double the value and hence $P' = 2P$

AVOGADRO'S LAW

At same pressure and same temperature, equal volumes of all gases contain equal number of molecules. At STP or NTP (0 °C and 1 atm i.e., 273 K and 1.01×10^5 Nm^{-2}), we have 1 mole of an ideal gas $\equiv N_A\,(=6.02\times10^{23})$ molecules of gas$\equiv$ 22.4 litre of the gas $\equiv M$ (molecular weight) gram of the gas.

DALTON'S LAW OF PARTIAL PRESSURES

If an enclosure contains a number of non-reacting gases, each gas behaves independent of the other and the total pressure exerted by the gases on the walls of enclosure is the sum of pressures exerted by individual gases i.e., $P = P_1 + P_2 + P_3 + ...$

ILLUSTRATION 53

A vessel of volume 2×10^{-3} m^3 contains 0.1 mol of hydrogen gas and 0.2 mol of helium. If the temperature of the mixture is 300 K, calculate the pressure due to component gases and the mixture. $\left(R = 8.31\ \text{Jmol}^{-1}\text{K}^{-1}\right)$

SOLUTION

As according to ideal gas equation $PV = nRT$,

$$P_H = n_H\frac{RT}{V} = (0.1)\left(\frac{8.31\times300}{2\times10^{-3}}\right) = 1.25\times10^5\ \text{Pa}$$

$$P_{He} = n_{He}\frac{RT}{V} = (0.2)\left(\frac{8.31\times300}{2\times10^{-3}}\right) = 2.50\times10^5\ \text{Pa}$$

and by Dalton's Law of partial pressures,

$$P = P_H + P_{He} = (1.25+2.5)\times10^5 = 3.75\times10^5\ \text{Pa}$$

GRAHAM'S LAW OF DIFFUSION

At constant temperature and pressure, the rate of diffusion of a gas is inversely proportional to the square root of its density

i.e., Rate of diffusion $\propto \frac{1}{\sqrt{\rho}}$, where ρ is the density of gas.

IDEAL GAS EQUATION

Combining first four laws (i.e., Boyle's Law, Charle's Law, Gay-Lussac's Law and Avogadro's Law) we get one single equation for an ideal gas, i.e.

$$PV = RT$$

$$PV = nRT \text{ for } n \text{ moles of gas}$$

$$\Rightarrow \quad \frac{PV}{nT} = R = \text{constant} \qquad \ldots(1)$$

Here, R is the Universal Gas Constant having value

$$R = 8.31\ \text{Jmol}^{-1}\text{K}^{-1} = 2\ \text{calmol}^{-1}\text{K}^{-1}$$

For 1 mole of a gas $n = 1$ i.e., $PV = RT$

$$\Rightarrow \quad \frac{PV}{T} = R = \text{constant}$$

ILLUSTRATION 54

A vessel of volume 20 litre contains a mixture of hydrogen and helium gas at 20 °C and a pressure of 2 atm. The mass of the mixture is 5 g. Calculate the ratio of mass of hydrogen gas to helium gas.

SOLUTION

If m_1 be the mass of hydrogen gas and m_2 be the mass of helium gas, then applying the ideal gas equation $PV = nRT$, we get

$$(2)(20) = \left(\frac{m_1}{2} + \frac{m_2}{4}\right)(0.082)(293)$$

$$\Rightarrow \quad 2m_1 + m_2 = 6.66 \qquad \ldots(1)$$

Also, $m_1 + m_2 = 5$ $\qquad \ldots(2)$

Solving equations (1) and (2), we get

$$m_1 = 1.66\ \text{g and } m_2 = 3.34g$$

$$\Rightarrow \quad \frac{m_1}{m_2} = \frac{1}{2}$$

ILLUSTRATION 55

In a closed container of volume 10^{-3} m^3, O_2 gas is filled at temperature 400 K and pressure 1.5 atm. A small hole is made in the container from which gas leaks out to open atmosphere. After some time, the temperature and pressure of container become equals to that of surrounding. Find the mass of gas that leaks out from the container. (Atmospheric temperature = 300 K)

SOLUTION

If initially, there is mass m_1 of O_2 gas in the container, then from ideal gas equation, we get

$$PV = \left(\frac{m_1}{32}\right)RT$$

$$\Rightarrow \quad m_1 = \frac{32PV}{RT} = \frac{32(1.5\times10^5)(10^{-3})}{(8.314)(400)} = 1.443 \text{ g}$$

Finally, if m_2 mass of O_2 is left in the container, then from ideal gas equation, we get

$$PV = \left(\frac{m_2}{32}\right)RT$$

$$\Rightarrow \quad m_2 = \frac{32PV}{RT} = \frac{32(10^5)(10^{-3})}{(8.314)(300)} = 1.283 \text{ g}$$

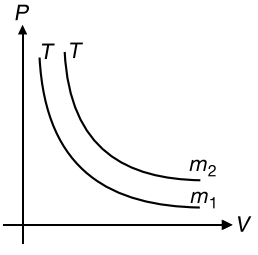
So, mass of gas leaked out is

$$\Delta m = m_1 - m_2$$

$$\Rightarrow \quad \Delta m = 1.443 - 1.283 = 0.16 \text{ g}$$

ILLUSTRATION 56

The P-V diagram for two gases having different masses m_1 and m_2 are drawn (as shown) at constant temperature T. Which of m_1 and m_2 is smaller?



SOLUTION

Since, $PV = nRT = \left(\frac{m}{M}\right)RT$

$$\Rightarrow \quad m = (PV)\left(\frac{M}{RT}\right)$$

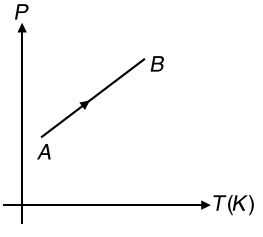
$\Rightarrow \quad m \propto PV$, when T =constant

From the graph we observe that $P_2V_2 > P_1V_1$ (for same P or V).

So, $m_2 > m_1$.

ILLUSTRATION 57

For the given mass of an ideal gas the P-T diagram is shown in figure. What can you conclude regarding the change in volume (whether it is constant, increasing or decreasing)?



SOLUTION

When volume is constant, P-T graph is a straight line passing through origin. The given line does not pass through origin, hence volume is not constant. From Ideal Gas Equation, we get

$$V = (nR)\left(\frac{T}{P}\right)$$

Now, to see the volume of the gas we will have to see whether $\frac{T}{P}$ is increasing or decreasing.

From the given graph we can write the P-T equation as,

$$P = aT + b \qquad (y = mx + c)$$

where a and b are positive constants.

$$\Rightarrow \quad \frac{P}{T} = a + \frac{b}{T}$$

Now since, $T_B > T_A$

$$\Rightarrow \quad \frac{b}{T_B} < \frac{b}{T_A}$$

$$\Rightarrow \quad \left(\frac{P}{T}\right)_B < \left(\frac{P}{T}\right)_A$$

$$\Rightarrow \quad \left(\frac{T}{P}\right)_B > \left(\frac{P}{T}\right)_A$$

$$\Rightarrow \quad V_B > V_A$$

Thus, as we move from A to B, volume of the gas is increasing.

ILLUSTRATION 58

Equal masses of a gas are sealed in two vessels, one of volume V_0 and other of volume $2V_0$. If the first vessel is at temperature 300 K and the other is at 600 K. Find the ratio of pressures in the two vessels.

SOLUTION

As number of moles in the two vessels are equal, we have

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

It is given that $V_1 = V_0$, $V_2 = 2V_0$ and $T_1 = 300$ K, $T_2 = 600$ K

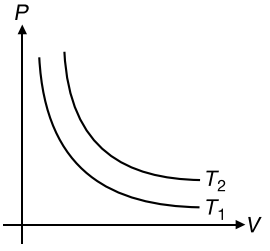
$$\Rightarrow \quad \frac{P_1V_0}{300} = \frac{P_2(2V_0)}{600}$$

$$\Rightarrow \quad P_1 = P_2$$

$$\Rightarrow \quad \frac{P_1}{P_2} = 1$$

ILLUSTRATION 59

P-V diagrams for two gases of same mass are drawn at two different temperatures T_1 and T_2. Explain, which of T_1 or T_2 is greater.



SOLUTION

According to the ideal gas equation, we have $PV = nRT$

$$\Rightarrow \quad T = \frac{PV}{nR}$$

Here mass of the gas is constant, which implies that number of moles are constant, i.e., $T \propto PV$.
In the given diagram product of P and V for T_2 is more than T_1 at all points (keeping either P or V same for both graphs). Hence, we conclude that

$$T_2 > T_1$$

ILLUSTRATION 60

Two glass bulbs of equal volume are connected by a narrow tube and are filled with a gas at 0 °C and a pressure of 76 cm of mercury. One of the bulbs is then placed in melting ice and the other is placed in water bath maintained at 62 °C. What is the new value of the pressure inside the bulbs? The volume of the connecting tube is negligible.

SOLUTION

Assuming the volume of each bulb to be V, then the number of moles present in each bulb initially is

$$n = \frac{PV}{RT} = \frac{(76\text{ cm})V}{R(273)}$$

Total number of moles in the two connected bulbs is

$$2n = \frac{2\times(76\text{ cm})V}{R(273)}$$

When one bulb is placed in melting ice (273 K) and other is placed in water at 62 °C = 335 K, still the total number of moles in the two bulbs will be equal to the initial moles i.e., $2n$. If P_f be the final pressure in the two bulbs, then we have

$$\frac{P_i(2V)}{T_i} = \frac{P_f V}{T_{f_1}} + \frac{P_f V}{T_{f_2}}$$

$$\Rightarrow \quad \frac{(76\text{ cm})(2V)}{273} = \frac{P_f V}{273} + \frac{P_f V}{335}$$

$$\Rightarrow \quad P_f = \frac{2\times 76}{273} \times \frac{273\times 335}{608}$$

$$\Rightarrow \quad P_f = 83.75\text{ cm of Hg}$$

ILLUSTRATION 61

Find the minimum attainable pressure of one mole of an ideal gas if during the expansion, its temperature and volume are related as $T = T_0 + \alpha V^2$ where T_0 and α are positive constants.

SOLUTION

Given that one mole of gas is used, so we have

$$PV = RT$$

$$\Rightarrow \quad P = \frac{RT}{V} = \frac{R}{V}\left(T_0 + \alpha V^2\right) \qquad \left\{\because T = T_0 + \alpha V^2\right\}$$

The pressure P will be minimum, when

$$\frac{dP}{dV} = 0$$

$$\Rightarrow \quad \frac{dP}{dV} = -\frac{T_0}{V^2} + \alpha = 0$$

$$\Rightarrow \quad V = \sqrt{\frac{T_0}{\alpha}}$$

Thus, pressure of gas is minimum when its volume is $V = \sqrt{\frac{T_0}{\alpha}}$. At this volume its temperature is given as

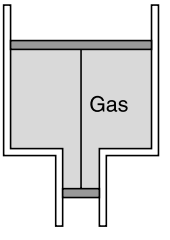
$$T = T_0 + \alpha V^2$$

$$\Rightarrow \quad T = T_0 + \alpha\left(\sqrt{\frac{T_0}{\alpha}}\right)^2 = 2T_0$$

Thus, minimum value of pressure is

$$P_{\min} = \frac{RT}{V} = \frac{R(2T_0)}{\sqrt{T_0/\alpha}} = 2R\sqrt{T_0\alpha}$$

ILLUSTRATION 62



A smooth vertical tube having two different cross sections is open from both the ends but closed by two sliding frictionless pistons tied with an inextensible string as shown in Figure. One mole of an ideal gas is enclosed between the pistons. The difference in cross-sectional areas of the two pistons is given ΔS. The masses of pistons are m_1 and m_2 for larger and smaller one respectively. Calculate the temperature through which the tube has to be raised so that the pistons get displaced through a distance l. Take atmospheric pressure to be P_0.

SOLUTION

Let initial pressure of gas be P, S_1 and S_2 be the cross-sectional areas of the larger and smaller piston respectively, then for equilibrium of the two pistons we have

$$P_0S_1 + m_1g + T = PS_1 \quad \{\text{For larger piston}\}$$

where, T is the tension in string

$$PS_2 + m_2g = T + P_0S_2 \quad \{\text{For smaller piston}\}$$

Adding these equations and rearranging, we get

$$P_0(S_1 - S_2) + m_1g + m_2g = P(S_1 - S_2)$$

$$\Rightarrow \quad P_0 + \left(\frac{m_1 + m_2}{\Delta S}\right)g = P \quad \ldots(1)$$

If gas temperature is increased from T_1 to T_2 the volume of gas increases from V to $V + l\Delta S$, where l is the displacement of pistons. Then from ideal gas equation, we have

for initial state, $PV = RT_1$ …(2)

for final state, $P(V + l\Delta S) = RT_2$ …(3)

From equation (1), we see that pressure of gas does not change as it does not depend on temperature
Subtracting equation (2) from (3), we get

$$Pl\Delta S = R(T_2 - T_1)$$

$$\Rightarrow \quad T_2 - T_1 = \frac{Pl\Delta S}{R} = \left[P_0 + \left(\frac{m_1 + m_2}{\Delta S}\right)g\right]\frac{l\Delta S}{R}$$

$$\Rightarrow \quad T_2 - T_1 = \left[P_0\Delta S + (m_1 + m_2)g\right]\frac{l}{R}$$

ILLUSTRATION 63

A vessel of volume 30 litres contains ideal gas at a temperature 63 °C. After a portion of the gas has been let out, the pressure in the vessel decreased by 0.415 bar. If density of gas at STP is 1.3 gltr^{-1}, find the mass of gas released from the vessel.

SOLUTION

At STP, from ideal gas equation, we have $P = \frac{\rho RT}{M}$

$$\Rightarrow \quad 10^5 = \frac{1.3 \times 8.314 \times 273}{M}$$

$$\Rightarrow \quad M = 2.95 \times 10^{-3} \text{ kg}$$

Since $P = \frac{nRT}{V}$, so $\Delta P = \frac{(\Delta n)RT}{V}$

$$\Rightarrow \quad 0.415 \times 10^5 = \frac{(\Delta n)(8.314)(336)}{30 \times 10^{-3}}$$

$$\Rightarrow \quad \Delta n = 0.4456 \text{ mole}$$

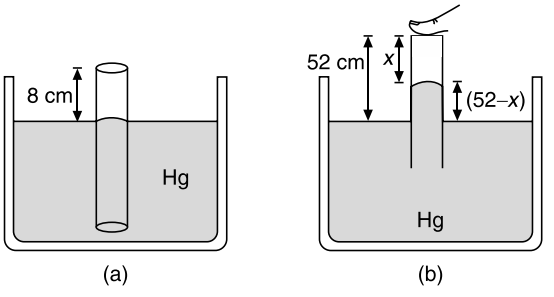
$$\Rightarrow \quad m = M(\Delta n) = 13.14 \text{ g}$$

ILLUSTRATION 64

An open glass tube is immersed in mercury in such a way that the length of 8 cm extends above the mercury level. Now the open end of the tube is closed by a finger and raised further by 44 cm. Calculate the length of air column above mercury in the tube. Take atmospheric pressure to be 76 cm of mercury. Neglect capillary effect.

SOLUTION

The initial state of the tube is shown in Figure (a). Now the tube is closed at the top and raised up further by 44 cm so that its length above mercury now becomes $8 + 44 = 52$ cm as shown in Figure (b).



Initially tube is open to atmosphere, so pressure of air inside the tube is 76 cm of Hg. If A be the cross-sectional area of the tube then initial volume of air in tube is $8A$.
When upper end of tube is closed and raised up, then let the air column be of length x, so final volume of enclosed air is xA. Pressure at the mercury level in the container is equal to atmospheric pressure i.e., 76 cm of Hg, hence pressure at the mercury air interface inside the tube is

$$76 - (52 - x) = (24 + x) \text{ cm of Hg}$$

Since during the process, temperature of system remains constant, so we have $P_1V_1 = P_2V_2$

$$\Rightarrow \quad (76)(8A) = (24 + x)(xA)$$

$$\Rightarrow \quad 608 = 24x + x^2$$

$$\Rightarrow \quad x^2 + 24x - 608 = 0$$

$$\Rightarrow \quad x = 15.4 \text{ cm or } x = -39.4 \text{ cm}$$

Hence the acceptable final value of air column is 15.4 cm.

VANDER WAAL'S EQUATION

Perfect gas equation $PV = RT$ is most closely obeyed at high temperature and low pressure where the intermolecular forces and the volume of gas molecules become negligible. It fails at high pressure and low temperature, because actual gas molecules possess finite size and intermolecular attraction, Vander Waal introduced these two corrections in perfect gas equation and obtained the following equation for real gases

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

where a and b are Vander Waal's constants, depending on the nature of the gas.

Vander Waal's equation for n moles may be expressed as

$$\left(P+\frac{an^2}{V^2}\right)(V-nb)=nRT$$

CRITICAL TEMPERATURE AND PRESSURE (T_C)

Critical Temperature is the temperature below which a gas can be liquefied by increasing the pressure alone. The critical temperature is different for different gases, being 31 °C for CO_2, –118 °Cfor oxygen, –240 °C for hydrogen and –268 °C for Helium.

The minimum pressure required to liquify a gas at critical temperature is called critical pressure (P_C). Lesser is the critical temperature, lesser is the critical pressure being 72.8 atm for CO_2 and 2.26 atm for helium. The values of critical temperature, critical pressure and critical volume are

$$T_C=\frac{8a}{27Rb},\ P_C=\frac{a}{27b^2},\ V_C=3b$$

BOYLE TEMPERATURE (T_B)

The Boyle temperature is that temperature

(a) below which the product PV first decreases, becomes minimum and then increases with increase of pressure P.

(b) above which the product PV increases with increase of pressure P.

So, Boyle temperature is $T_B=\frac{a}{Rb}$

Since $T_C=\frac{8a}{27Rb}$, so $T_B=\frac{27}{8}T_C$

VARIATION OF PRESSURE WITH HEIGHT OR ELEVATION ABOVE THE SEA LEVEL

Consider the atmospheric pressure at sea level $(y=0)$ to be P_0. Let us find the air pressure with elevation in the earth's atmosphere, assuming the temperature to be constant throughout.

We begin with the pressure relation that we have already discussed in the chapter of fluid mechanics i.e.,

$$\frac{dP}{dy}=-\rho g$$

Since $PV=RT$, where $V=\frac{M}{\rho}$

So, the density ρ is given by,

$$\rho=\frac{PM}{RT} \qquad \ldots(1)$$

$$\Rightarrow \quad \frac{dP}{dy}=-\left(\frac{PM}{RT}\right)g$$

$$\Rightarrow \quad \int_{P_0}^{P}\frac{dP}{P}=-\int_0^y\left(\frac{Mg}{RT}\right)dy \qquad \ldots(2)$$

$$\Rightarrow \quad \int_{P_0}^{P}\frac{dP}{P}=-\frac{Mg}{RT}\int_0^y dy \qquad \{\because\ T=\text{constant}\}$$

$$\Rightarrow \quad P=P_0e^{-\left(\frac{Mg}{RT}\right)y}$$

So, the pressure of air decreases exponentially with height (y). Also from equation (1), we get

$$\rho=\frac{PM}{RT}=\frac{MP_0}{RT}e^{-\left(\frac{Mg}{RT}\right)y}=\rho_0e^{-\left(\frac{Mg}{RT}\right)y}$$

i.e., density also varies exponentially with height.

Problem Solving Technique(s)

If in some problems $T \neq$ constant, but it is given as function of elevation y, then equation (2), becomes

$$\int_{P_0}^{P}\frac{dP}{P}=-\frac{Mg}{R}\int_0^y\frac{dy}{T}$$

where T is the given function of y, so it is inside the integration as done in the following Illustrations.

VARIATION OF PRESSURE WITH DEPTH BELOW THE SEA LEVEL

Below the sea level, the pressure at a depth h of water is

$$P=P_0+\rho_w gh$$

The P versus y or P versus h graph is shown in figure, so

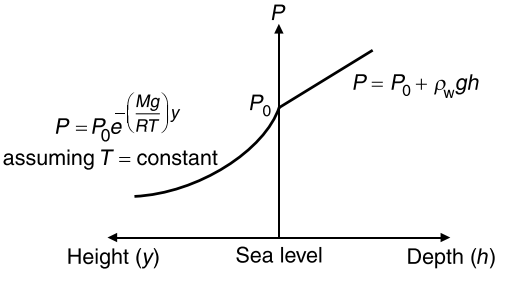


ILLUSTRATION 65

An ideal gas of molar mass M is contained in a tall vertical cylindrical vessel whose base area is A and height h. The temperature of the gas is T, its pressure on the bottom base is P_0. Assuming the temperature and the free-fall acceleration g to be independent of the height, find the mass of gas in the vessel.

SOLUTION

At a height y, let us consider an elemental layer of width dy. If mass in this layer is dm, then

$$dm = (Ady)\rho = Ady\left(\frac{PM}{RT}\right) = \frac{MA}{RT}\left(P_0 e^{-\frac{Mgy}{RT}}\right)dy$$

Total mass is given by

$$m = \int dm = \frac{P_0 MA}{RT}\int_0^h e^{-\frac{Mgy}{RT}}\,dy$$

$$\Rightarrow \quad m = \frac{P_0 MA}{RT}\left(-\frac{RT}{Mg}\right)e^{\frac{-Mgy}{RT}}\Big|_0^h = \frac{P_0 A}{g}\left(1 - e^{-\frac{Mgh}{RT}}\right)$$

ILLUSTRATION 66

The atmospheric pressure at sea level $(h = 0)$ is P_0. If the temperature of atmosphere varies with height h as $T = T_0 - ah$ where a and T_0 are positive constants of proper dimensions. Find the pressure function with height h. Take molecular mass M of the air and acceleration due to gravity g to be constant.

SOLUTION

Since, $\frac{dP}{dh} = -\rho g = -\left(\frac{PM}{RT}\right)g$

$$\Rightarrow \quad \frac{dP}{dh} = -\left\{\frac{PM}{R(T_0 - ah)}\right\}g$$

$$\Rightarrow \quad \frac{dP}{P} = -\left(\frac{Mg}{R}\right)\frac{dh}{T_0 - ah}$$

$$\Rightarrow \quad \int_{P_0}^{P}\frac{dP}{p} = -\left(\frac{Mg}{R}\right)\int_0^h \frac{dh}{T_0 - ah}$$

$$\Rightarrow \quad \log_e\left(\frac{P}{P_0}\right) = \frac{Mg}{Ra}\log_e\left(\frac{T_0 - ah}{T_0}\right)$$

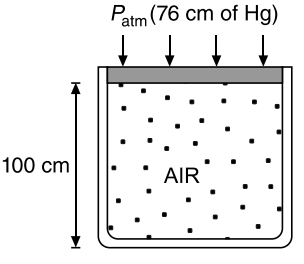
$$\Rightarrow \quad P = P_0\left(\frac{T_0 - ah}{T_0}\right)^{\frac{Mg}{Ra}}$$

Test Your Concepts-III

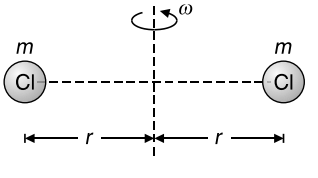
Based on Kinetic Theory of Gases and Ideal Gas Equation

(Solutions on page H.80)

1. Two equal bulbs are joined by a very narrow tube and the whole system is initially filled with a gas at NTP and sealed. What will be the pressure of the gas when one of the bulbs is immersed in boiling water and the other in ice?
2. The rms velocity of hydrogen molecules at a certain temperature is $300\ \text{ms}^{-1}$. If the temperature is doubled and hydrogen gas dissociates into atomic hydrogen. Find the rms speed of the gas molecules now.
3. A beam of particles each of mass m_0 and speed v is directed along the x-axis. The beam strikes an area $1\,\text{mm}$ square with 10^{15} particles striking per second. Find the pressure on the area due to the beam if the particles stick to the area when they hit. Evaluate for an electron beam in a television tube where $m_0 = 9.1\times10^{-31}$ kg and $v = 8\times10^{7}\ \text{ms}^{-1}$.
4. There are two containers, each of volume V containing ideal gases. The pressure and temperature of the gases in the two vessels are p_1, T_1 and p_2, T_2 respectively. If the vessels are now connected by a thin long tube of negligible volume, the final temperature of the two after mixing is T. Find the final pressure of the gas.
5. A uniform tube closed at one end, contains a pallet of mercury $10\ \text{cm}$ long. When the tube is kept vertically with the closed end of tube upward, the length of air column trapped by mercury and the closed end of the tube is $20\ \text{cm}$. If the tube is inverted so that its open end becomes upward, then calculate the final length of the air column trapped by mercury and closed end of the tube. Take atmospheric pressure to be $76\ \text{cm}$ of Hg.
6. A vertical cylinder of height $100\ \text{cm}$ contains air at a constant temperature and its top is closed by a frictionless piston at atmospheric pressure ($76\ \text{cm}$ of Hg) as shown in Figure. If mercury is slowly poured on the piston, then due to weight of mercury, the air inside gets compressed. Calculate the maximum height of the mercury column which can be collected on the piston.



7. In a crude model of a rotating diatomic molecule of chlorine (Cl_2), the two Cl atoms are 2×10^{-10} m apart and rotate about their centre of mass with angular speed $\omega = 2\times10^{12}\ \text{rads}^{-1}$. What is the rotational kinetic energy of one molecules of Cl_2, which has a molar mass of $70\ \text{gmol}^{-1}$?



8. Prove that the pressure of an ideal gas is numerically equal to two third of the mean translational kinetic energy per unit volume of the gas.

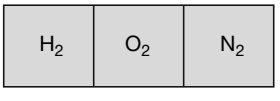
9. A rubber balloon is filled with air at 2×10^5 Nm^{-2} pressure at a temperature 20 °C. When its temperature is increased to 40 °C, the volume of balloon is increased by 2%. Find the final air pressure inside the balloon at 40 °C.

10. A barometer tube 90 cm contains some air above the mercury. The reading is 74.8 cm when the true atmospheric pressure is 76 cm and the temperature is 30 °C. If the reading is observed to be 75.4 cm on a day when the temperature is 10 °C, what is the true pressure?

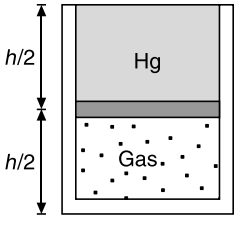
11. An ideal gas is enclosed between the closed end of a uniform cross-sectional tube and a pellet of mercury of length $h = 10$ cm. The length of the tube occupied by the gas when held vertical with the closed end upward is $l_1 = 40$ cm. When it is turned through $\theta = 60°$ the length occupied by the gas is only $l_2 = 38$ cm. Calculate the frictional force between the wall of the tube and mercury pellet. The radius of the tube is $r = 2$ mm. Take $g = 10$ ms^{-2} and density of mercury 13600 kgm^{-3}.

12. A perfectly conducting vessel of volume $V = 0.4$ m^3 contains an ideal gas at a temperature $T = 273$ K. The density of the gas at STP is $\rho = 1.2$ kgm^{-3}. A portion of the gas is let out and the pressure of the gas falls by $\Delta P = 0.24$ atm. Find the mass of the gas which escapes from the vessel.

13. A vessel of volume $V = 30$ litre is separated into 3 equal parts by stationary semipermeable partitions. The left, middle and right parts are filled with $m_1 = 30$ g of hydrogen (H_2), $m_2 = 160$ g of oxygen (O_2), and $m_3 = 70$ g of nitrogen (N_2) respectively. The left partition lets only hydrogen, while the right partition lets through hydrogen and nitrogen. What will be the pressure in each part of the vessel after the equilibrium has been set in if the vessel is kept at a constant temperature $T = 300$ K?



14. A vertical hollow cylinder of height 1.52 m is fitted with a movable piston of negligible mass and thickness. The lower part of cylinder contains an ideal gas and the upper part is filled with mercury as shown in Figure.



Initially the temperature of system is 300 K and the lengths of gas and mercury columns are equal. Calculate the temperature to which system is raised so that half of mercury overflows. Take atmospheric pressure to be 76 cm of Hg and ignore thermal expansion of mercury.

15. Assuming the temperature and the molar mass of air, as well as the free fall acceleration, to be independent of height, find the difference in heights at which the air densities at the temperature 0 °C differ $\left(M = 28\ gmol^{-1}\right)$

(a) e times; **(b)** by $\eta = 2\%$

16. An ideal gas of molar mass M is contained in a very tall vertical cylindrical vessel in the uniform gravitational field in which the free-fall acceleration equals g. Assuming the gas temperature to be the same and equal to T, calculate the height at which the centre of gravity of the gas is located.

17. The mass of an oxygen molecule is 5.28×10^{-26} kg. Its mean or average velocity at NTP is 4.25×10^2 ms^{-1}. Calculate the average kinetic energy of an oxygen molecule at 0 °C.

18. A vessel contains a mixture of nitrogen $(m_1 = 7\ g)$ and carbon dioxide $(m_2 = 11\ g)$ at a temperature $T = 290$ K and pressure $p_0 = 1$ atm. Find the density of this mixture, assuming the gases to be ideal.

19. A vessel of volume 5 litre contains 1.4 g of N_2 at 1500 K and 0.4 g of He. If 30% of the nitrogen molecules are dissociated into atoms then find the gas pressure.

20. A glass tube, which is closed at one end, is completely submerged with open end downwards in a vessel of mercury, so that air column of length 10 cm is trapped inside the tube. To what height must the upper end of the tube be raised above the level of mercury in the vessel so that the level of mercury in the tube coincides with that in the vessel? Also calculate the mass of the air, if the temperature remains constant at 27 °C. Given that area of cross-section of tube is 1.0 cm^2, atmospheric pressure is 1.013×10^5 Pa i.e., 76 cm of Hg and molecular weight of air is 29.

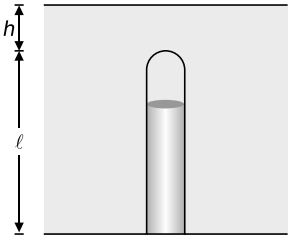
21. Show that for any distribution of speeds for an ideal gas $v_{rms} \geq v_{av}$.

22. A horizontal cylinder closed from one end is rotated with a constant angular velocity ω about a vertical axis passing through the open end of the cylinder. The outside air pressure is p_0, the temperature is T and the molar mass of air is M. Find the air pressure as a function of the distance r from the rotation axis. The molar mass is assumed to be independent of r.

23. A very tall cylindrical vessel with an ideal gas of molar mass M filled in it is placed in a uniform gravitational field. If the temperature of gas varies with height such

that the gas density remains same throughout the container, then calculate the temperature gradient in the container dT/dh.

24. A glass tube with a length $\ell = 50$ cm and a cross section $a = 0.5$ cm^2 is soldered at one end and is submerged into water $(\rho = 1000\ \text{kgm}^{-3})$ as shown in figure. What force F should be applied to hold the tube under the water if the distance from the surface of the water to the soldered end is $h = 10$ cm and the atmospheric pressure $P_0 = 1.01 \times 10^5$ Nm^{-2}. The weight of the tube is $W = 1.47 \times 10^{-1}$ N. Take $g = 9.8$ ms^{-2}. Assume that temperature of the gas inside the tube remains constant.



25. An ideal gas of molar mass M is located in the uniform gravitational field in which the free-fall acceleration is equal to g. Find the gas pressure as a function of height h, if $p = p_0$ at $h = 0$ and the temperature varies with height as

(a) $T = T_0(1 - ah)$

(b) $T = T_0(1 + ah)$

CONCEPT OF INTERNAL ENERGY (U)

In thermodynamics, the internal energy of a system is the energy contained within the system, excluding the kinetic energy of motion of the system as a whole and the potential energy of the system as a whole due to external force fields.

It keeps account of the gains and losses of energy of the system that are due to changes in its internal state. The internal energy of a system can be changed by transfers of matter or heat or by doing work. So, internal energy is possessed by the system due to molecular configuration and molecular motion. In other words, the internal energy possessed by a system may also be said as the mean energy of '**disordered**' motion of molecules. The energy due to molecular configuration is called **internal potential energy** (U_P) and the energy due molecular motion is called **internal kinetic energy** (U_K). The total internal energy possessed by the system is then given by

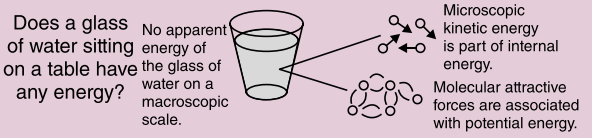
$$U = U_P + U_K$$

Conceptual Note(s)

Internal energy is defined as the energy associated with the random, disordered motion of molecules. It is separated in scale from the macroscopic ordered energy associated with moving objects. It refers to the invisible microscopic energy on the atomic and molecular scale.

FOR EXAMPLE

At room temperature, a glass of water sitting on a table has no apparent energy, either potential or kinetic . However, on the microscopic scale the liquid is a mass of high speed molecules traveling at hundreds of meters per second. When we superimpose an ordered large scale motion on the water as a whole (may be by tossing/splashing the water across the room) then this microscopic energy would not necessarily be changed.


Does a glass of water sitting on a table have any energy?
No apparent energy of the glass of water on a macroscopic scale.
Microscopic kinetic energy is part of internal energy.
Molecular attractive forces are associated with potential energy.

DEGREES OF FREEDOM f

Degrees of freedom (f) is defined as the total number of possible independent significant ways in which a system can have energy.

The independent motions can be translational, rotational or vibrational or any combination of them. A particle moving in a straight line can have only one degree of freedom. i.e., translational degree of freedom. Similarly, a particle moving in a plane have two translational degrees of freedom and a particle free to move in space have three translational degrees of freedom.

Monatomic gas molecules can be considered as point masses moving in space and they will have only three translational degrees of freedom. However, this does not hold good with polyatomic gases.

In a polyatomic molecule we have to take rotational motion of atoms about their centre of mass and the vibrational motion of the atoms relative to each other. Thus, we must write the molecular kinetic energy as

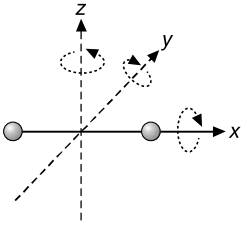
$$E = \begin{pmatrix}\text{translational}\\ \text{energy}\end{pmatrix} + \begin{pmatrix}\text{rotational}\\ \text{energy}\end{pmatrix} + \begin{pmatrix}\text{vibrational}\\ \text{energy}\end{pmatrix}$$

As the temperature of a gas increases, the different components of the molecular energy also increase. But each component increases in a different manner. Vibrational energy depends on the strength of the bonding of the atoms in the

molecule. The stronger the ionic bonds, the more difficult it is to increase the vibrational energy. On the other hand, the rotational energy depends on the geometry (moment of inertia) of the molecule. But since vibrational energy comes into picture only at high temperatures, vibrational degrees of freedom are not taken into account unless stated in the question.

(a) A **monatomic** gas has 3 degrees of freedom (all translational). Although a monatomic molecules can also rotate but due to its small moment of inertia rotational kinetic energy is insignificant.

(b) A **diatomic** gas such as H_2, O_2 etc. are made up of two atoms joined rigidly to one another through a bond. This cannot only move bodily, but also rotate about any one of the three co-ordinate axes. However, its moment of inertia about the axis joining two atoms is negligible. Hence it can have 3 rotational motions out of which only two rotational motions are significant. Thus, a diatomic molecule has five degrees of freedom, three translational and two rotational.



(c) A nonlinear polyatomic gas molecule such as H_2O, NH_3 etc. can rotate about any of three co-ordinate axes. Hence it has six degrees of freedom, three translational and three rotational. However, a linear polyatomic gas molecule like CO_2 has five degrees of freedom three translational and two rotational (like a diatomic molecule).

For a molecule with n atoms there will be $3n$ degrees of freedom. These can be attributed as follows:

Type	Translation	Rotation	Vibration
Linear	3	2	$3n-5$
Non-linear	3	3	$3n-6$

There is no general way in which one can find the degrees of freedom. However, if a system has A number of independent particles and R is the number of independent restrictions between them then the number of degrees of freedom (f) is

$$f = 3A - R$$

(a) For a monatomic gas molecule

$A = 1$ and $R = 0$

Hence for monatomic gas molecule $f = 3$.

(b) For di-atomic gas molecule

At normal temperature $A = 2$ and $R = 1$

(as the bond existing between its atoms must be rigid).

So, for diatomic gas molecule $f = 5$.

POLYATOMIC GASES

A polyatomic gas molecule should also have three translational degrees of freedom and the number of rotational degrees of freedom depending on the geometry of the molecule. If the molecule is a linear one like CO_2 then it has only two rotational degrees of freedom and if it is non-linear molecule like H_2O, CH_4 etc. then it has three rotational degrees of freedom because about all three co-ordinate axes, some significant moment of inertia exists.

Number of vibrational degrees of freedom in complex polyatomic molecules varies in different ways. There is no simple theoretical way to calculate the exact number of active vibrational degrees of freedom in a polyatomic molecule.

Hence, the total internal energy of n moles of a polyatomic gas at a temperature T for a linear molecule is given by

$$U = \frac{f}{2}nRT = \left(\frac{5+x}{2}\right)nRT \qquad \ldots(1)$$

where x are the number of vibrational degrees of freedom.

For a non-linear molecule which has x number of vibrational degrees of freedom, the total internal energy of n moles of a polyatomic gas at a temperature T is given by

$$U = \frac{f}{2}nRT = \left(\frac{6+x}{2}\right)nRT \qquad \ldots(2)$$

THEOREM OF EQUIPARTITION OF ENERGY

According to this theorem, "the energy of a system in thermal equilibrium is equally divided amongst all degrees of freedom."

Since, energy associated with a monatomic gas molecule is

$$\frac{1}{2}m\langle v^2\rangle = \frac{3}{2}k_BT$$

and $$\frac{1}{3}\langle v^2\rangle = \langle v_x^2\rangle = \langle v_y^2\rangle = \langle v_z^2\rangle$$

$$\Rightarrow \quad \frac{1}{2}m\langle v_x^2\rangle = \frac{1}{2}m\langle v_y^2\rangle = \frac{1}{2}m\langle v_z^2\rangle = \frac{1}{2}k_BT.$$

So, we observe, energy associated with each degree of freedom/molecule is $\frac{1}{2}k_BT$

Energy associated with each degree of freedom per mole is $\frac{1}{2}RT$, where T is the absolute temperature.

So, for a gas with f degrees of freedom, energy associated with 1 mole of gas that is its Internal Energy is

$$U = f\left(\frac{1}{2}RT\right)$$

So, according to Theorem of Equipartition of Energy,

"The energy of a system in thermal equilibrium is equally divided amongst all the degrees of freedom and the energy associated with each degree of freedom per mole is $\frac{1}{2}RT$ or energy associated with each degree of freedom per molecule is $\frac{1}{2}k_BT$".

So, the internal energy of 1 mole of monatomic gas is

$$U = \frac{3}{2}RT$$

So, the internal energy of 1 mole of diatomic gas is

$$U = \frac{5}{2}RT$$

In general, we can say that the internal energy of n mole of a polyatomic gas with f degrees of freedom is

$$U = n\left(\frac{f}{2}RT\right)$$

MOLAR SPECIFIC HEAT OF THE GAS (C)

Consider a container containing m gram of gas of molecular mass M. If n is the number of moles of gas in container, dQ is the heat supplied and rise in temperature is dT then

$$c = \frac{1}{m}\left(\frac{dQ}{dT}\right)$$

Further $n = \frac{m}{M}$, so $m = nM$

$$\Rightarrow \quad c = \frac{1}{Mn}\frac{dQ}{dT}$$

$$\Rightarrow \quad C = Mc = \frac{1}{n}\left(\frac{dQ}{dT}\right)$$

$$\begin{pmatrix}\text{Molar}\\ \text{Specific}\\ \text{Heat } (C) \text{ of}\\ \text{the Gas}\end{pmatrix} = \begin{pmatrix}\text{Molar}\\ \text{Mass } (M)\\ \text{of}\\ \text{the Gas}\end{pmatrix} \times \begin{pmatrix}\text{Gram}\\ \text{Specific}\\ \text{Heat } (c) \text{ of}\\ \text{the Gas}\end{pmatrix}$$

Conceptual Note(s)

(a) Please understand that every thermodynamic process has a molar specific heat C defined as

$$C = Mc = \frac{1}{n}\left(\frac{dQ}{dT}\right)$$

where M is the molar mass of the gas and c is its gram specific heat.

(b) A gas can be supplied heat at constant volume by keeping it inside a rigid closed container OR at constant pressure by keeping it inside a container which has a frictionless piston attached to it.

(c) When heat is supplied to the gas at constant pressure, then the molar specific heat is called as molar specific heat at constant pressure, and is denoted by C_P.

(d) When heat is supplied to the gas at constant volume, then the molar specific heat is called as molar specific heat at constant volume and is denoted by C_V.

SPECIFIC HEAT AT CONSTANT VOLUME (CV)

When heat is supplied to gas at constant volume the entire heat supplied just increases the internal energy of gas and does nothing else (i.e., no external work is done by the gas), so we have

$$C_V = \frac{1}{n}\left(\frac{dQ}{dT}\right)_V$$

If dU is the change in internal energy of gas at constant volume then, heat supplied (dQ) equals the increase in internal energy (dU), so

$$C_V = \frac{1}{n}\left(\frac{dQ}{dT}\right)_V = \frac{1}{n}\left(\frac{dU}{dT}\right)$$

C_V FOR AN IDEAL GAS

Since the internal energy of n mole of a polyatomic gas with f degrees of freedom is $U = n\left(\frac{f}{2}RT\right)$

By definition we have

$$C_V = \frac{1}{n}\frac{dU}{dT} = \frac{1}{n}\frac{d}{dT}\left(\frac{nfRT}{2}\right)$$

$$\Rightarrow \quad C_V = \frac{fR}{2}$$

As we know that the internal energy of 1 mole of monatomic gas is $U = \frac{3}{2}RT$

$$\Rightarrow \quad C_V = \frac{dU}{dT} = \frac{3R}{2}$$

Problem Solving Technique(s)

Please note that for any kind of thermodynamic process, if heat dQ is supplied, then the rise in internal energy is $dU = nC_V dT$. However, $dQ = dU$, only when the heat is supplied at constant volume.

SPECIFIC HEAT AT CONSTANT PRESSURE (C_P)

$$C_P = \frac{1}{n}\left(\frac{dQ}{dT}\right)_P$$

When heat is supplied to the gas at constant pressure a part of heat increases the internal energy of gas and remaining does an external work (pushing the piston is at the expense of this heat supplied).

So, to increase the internal energy of the gas by the same amount (as in case of heat supplied at constant volume) more amount of heat has to be supplied thus making us to conclude that

$$C_P > C_V$$

INTERNAL ENERGY (U) FOR AN IDEAL GAS: REVISITED

Since internal energy is possessed by the system due to molecular configuration and molecular motion. As discussed in the earlier text, the energy due to molecular configuration is called **internal potential energy** (U_P) and the energy due molecular motion is called **internal kinetic energy** (U_K). So, the total internal energy possessed by the system is given by

$$U = U_P + U_K$$

In case of an ideal gas, the intermolecular forces are zero, so the potential energy for an ideal gas is zero and hence the total kinetic energy of the ideal gas is its internal energy U. Please note that a gas may possess kinetic energy due to

(a) Translational Motion (due to translatory motion of molecules)
(b) Rotational Motion and
(c) Vibrational Motion

For an ideal gas, the internal energy U depends upon temperature T only and is directly proportional to it. So, we have $U \propto T$
Further the change in the internal is given by

$$\Delta U = \Delta U_P + \Delta U_K$$

Since for an ideal gas, no interaction exists between the molecules (meaning U_P= constant) and hence $\Delta U_P = 0$.

$$\Delta U = \Delta U_K = nC_V \Delta T = nC_V \left(T_f - T_i\right)$$

For infinitesimal temperature increments, we have

$$dU = nC_V dT$$

The Table below shows Molar Heat Capacities of Various Gases.

MOLAR HEAT CAPACITY (cal mole^{-1} K^{-1})				
GAS	C_V	C_P	$C_P - C_V$	$\gamma = \frac{C_P}{C_V}$
MONATOMIC GASES				
He	2.98	4.97	1.99	1.67
Ar	2.98	4.97	1.99	1.67

(*Continued*)

MOLAR HEAT CAPACITY (cal mole^{-1} K^{-1})				
DIATOMIC GASES				
H_2	4.88	6.87	1.99	1.41
N_2	4.96	6.95	1.99	1.40
O_2	5.04	7.03	1.99	1.40
CO	5.02	7.01	1.99	1.40
Cl_2	6.15	8.29	2.14	1.35
POLYATOMIC GASES				
CO_2	6.80	8.83	2.03	1.30
SO_2	7.50	9.65	2.15	1.29
H_2O	6.46	8.46	2.00	1.30
CH_4	6.48	8.49	2.01	1.31

RELATION BETWEEN C_P AND C_V

For 1 mole of gas which is supplied heat at constant volume

$$dQ = dU = (1)C_V dT \quad \ldots(1)$$

For 1 mole of gas supplied heat at constant pressure

$$dQ = \begin{pmatrix}\text{Increase in}\\ \text{Internal Energy}\end{pmatrix} + \begin{pmatrix}\text{External Work}\\ \text{Done by Piston}\end{pmatrix}$$

$$\Rightarrow \quad dQ = dU + PdV \qquad \{\because dW = Fdx = PAdx = PdV\}$$

$$\Rightarrow \quad C_P dT = C_V dT + PdV \quad \ldots(2)$$

Since, we know that, for a 1 mole of ideal gas $PV = RT$

$$\Rightarrow \quad d(PV) = d(RT)$$

$$\Rightarrow \quad PdV + VdP = RdT \quad \ldots(3)$$

But as external work is done by piston at constant pressure only i.e., $dP = 0$. Using (3), we have

$$PdV = RdT \quad \ldots(4)$$

Put (4) in (2), we have

$$C_P dT = C_V dT + RdT$$

After simplification, we have

$$C_P - C_V = R \qquad \text{(Called **Mayer's Relation**)}$$

where, $R = 8.314$ Jmole^{-1} K^{-1}

$$R = 1.99 \text{ calmole}^{-1} \text{ K}^{-1} \approx 2 \text{ calmole}^{-1} \text{ K}^{-1}$$

C_P and C_V are approximately equal for solids and liquids.

C_P FOR AN IDEAL GAS

For an ideal gas, according to Mayer's relation, we have

$$C_P - C_V = R \text{ and } C_V = \frac{fR}{2}$$

where f is the degrees of freedom of the ideal gas.

$$\Rightarrow \quad C_P = C_V + R = \left(\frac{f}{2}+1\right)R$$

Since $C_P - C_V = R$, so we must note that $C_P > C_V$.

ADIABATIC RATIO (γ) FOR AN IDEAL GAS

The ratio of the molar specific heat at constant pressure to the molar specific heat at constant volume i.e., $\frac{C_P}{C_V}$ is denoted by γ and is called as the Adiabatic Ratio. Hence,

$$\gamma = \frac{C_P}{C_V} > 1$$

For an ideal gas having f degrees of freedom, we have

$$C_V = \frac{fR}{2} \text{ and } C_P = \left(\frac{f}{2}+1\right)R$$

$$\Rightarrow \quad \gamma = \frac{C_P}{C_V} = 1 + \frac{2}{f}$$

This relation helps us to find the value γ for monatomic, diatomic, polyatomic gases.

RELATION OF C_P AND C_V WITH γ

Since $C_P - C_V = R$

$$\Rightarrow \quad \frac{C_P}{C_V} - \frac{C_V}{C_V} = \frac{R}{C_V}$$

$$\Rightarrow \quad \gamma - 1 = \frac{R}{C_V}$$

$$\Rightarrow \quad C_V = \frac{R}{\gamma - 1} \text{ and } C_P = \frac{\gamma R}{\gamma - 1}$$

Problem Solving Technique(s)

(a) Mixture of Gases (at same temperatures)

Following results are helpful for a mixture of gases.

(i) The internal energy is

$$U_{mix} = U_1 + U_2 + \ldots$$

(ii) The molar mass is

$$M_{mix} = \frac{n_1 M_1 + n_2 M_2 + \ldots}{n_1 + n_2 + \ldots}$$

(iii) The pressure is

$$P_{mix} = P_1 + P_2 + \ldots$$

(iv) The molar specific heat at constant volume is

$$(C_V)_{mix} = \frac{n_1 C_{V1} + n_2 C_{V2} + \ldots}{n_1 + n_2 + \ldots}$$

(v) The molar specific heat at constant pressure is

$$(C_P)_{mix} = (C_V)_{mix} + R = \frac{n_1 C_{P_1} + n_2 C_{P_2} + \ldots.}{n_1 + n_2 + \ldots.}$$

(vi) The ratio of specific heat at constant pressure to the specific heat at constant volume is

$$\gamma_{mix} = \left(\frac{C_P}{C_V}\right)_{mix}$$

(b) Expressions of U, C_P, C_V and γ for a monatomic, diatomic and polyatomic gas

For one mole of an ideal gas $U = \frac{f}{2}RT$

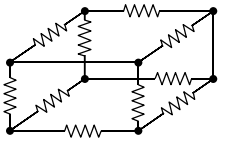
$$\Rightarrow \quad C_V = \frac{dU}{dT} = \frac{fR}{2}$$

$$\Rightarrow \quad C_P = C_V + R = \left(1 + \frac{f}{2}\right)R$$

$$\Rightarrow \quad \gamma = \frac{C_P}{C_V} = 1 + \frac{2}{f}$$

Gas	f	U	$C_V = \frac{fR}{2}$	$C_P = \left(1+\frac{f}{2}\right)R$	$\gamma = 1 + \frac{2}{f}$
Monatomic	3	$\frac{3}{2}RT$	$\frac{3R}{2}$	$\frac{5R}{2}$	1.67
Diatomic	5	$\frac{5}{2}RT$	$\frac{5R}{2}$	$\frac{7R}{2}$	1.4
Polyatomic	6	$3RT$	$3R$	$4R$	1.33

DULONG AND PETIT'S LAW



In crystalline solids (assumed to be like monatomic gas), the atoms are arranged in a three dimensional array, called a lattice. Each atom in a lattice can vibrate along three mutually perpendicular directions, each of which has two degrees of freedom. One corresponding to vibrational K.E. and the other vibrational P.E.

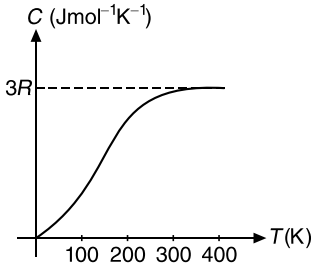
Thus, each atom has a total of six degrees of freedom. The volume of a solid does not change significantly with temperature and so there is little difference between C_V and C_P for a solid. The molar heat capacity is expected to be,

$$C = \frac{fR}{2} = \frac{6R}{2} = 3R \qquad \{\text{ideal crystalline solid}\}$$

Its numerical value is

$$C \approx 25 \text{ Jmol}^{-1}\text{K}^{-1} \approx 6 \text{ calmol}^{-1}\text{K}^{-1}$$

This result was first found experimentally by **Dulong and Petit**. Figure shows that the **Dulong and Petit Law** is obeyed quite well at high $(> 250 \text{ K})$ temperatures. At low temperatures, the heat capacities decrease.



Conceptual Note(s)

VARIATION OF SPECIFIC HEAT OF SOLIDS "DULONG AND PETIT'S LAW"

Dulong and Petit's Law states that average molar specific heat, of a chemically pure crystalline solid, is constant at room temperature and is, nearly, equal to $3R$ $\left(\approx 6 \text{ cal mole}^{-1} \text{ K}^{-1}\right)$ where R is gas constant for one mole of the substance.

ILLUSTRATION 67

A vessel contains a mixture of 7 g of nitrogen and 11 g of carbon dioxide at temperature $T = 290$ K. If pressure of the mixture $P = 1$ atm, calculate its density $\left(R = 8.31 \text{ Jmol}^{-1}\text{K}^{-1}\right)$.

SOLUTION

As molecular weights of N_2 and CO_2 are 28 and 44, and $n = \left(\frac{m}{M}\right)$, $n_N = \frac{7}{28} = \frac{1}{4}$ and $n_C = \frac{11}{44} = \frac{1}{4}$

$$\Rightarrow \quad n = n_N + n_C = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

Now as according to gas equation $PV = nRT$,

$$V = \frac{nRT}{P} = \left(\frac{1}{2}\right)\frac{8.31 \times 290}{1.01 \times 10^5} = 1.19 \times 10^{-2} \text{ m}^3$$

Mass of the gas is given by

$$m = 7 + 11 = 18 \text{ g} = 18 \times 10^{-3} \text{ kg}$$

$$\Rightarrow \quad \rho = \left(\frac{m}{V}\right) = \frac{(18 \times 10^{-3})}{(1.19 \times 10^{-2})} = 1.5 \text{ kgm}^{-3}$$

ILLUSTRATION 68

Two perfect gases at absolute temperatures T_1 and T_2 are mixed. There is no loss of energy. Find the temperature of the mixture if the masses of the molecules are m_1 and m_2 and the number of molecules in the gases are n_1 and n_2.

SOLUTION

According to the kinetic theory of gases, the kinetic energy of an ideal gas molecule at temperature T is given by $KE = \left(\frac{3}{2}\right)kT$. And as there is no force of attraction among the molecules of a perfect gas, PE of the molecule is zero. So, the energy of a molecule of perfect gas,

$$E = KE + PE = \frac{3}{2}kT + 0 = \frac{3}{2}kT$$

Now if T is the temperature of the mixture, by conservation of energy, i.e.,

$$n_1E_1 + n_2E_2 = (n_1 + n_2)E$$

we have $n_1\left(\frac{3}{2}kT_1\right) + n_2\left(\frac{3}{2}kT_2\right) = (n_1 + n_2)\frac{3}{2}kT$

i.e., $T = \frac{n_1T_1 + n_2T_2}{n_1 + n_2}$

ILLUSTRATION 69

Nitrogen gas of mass 15 g is enclosed in a vessel at 300 K. Calculate the amount of heat required to double the root mean square velocity of these molecules.

SOLUTION

The rms speed of gas molecules is given by

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

Since, v_{rms} is directly proportional to $\sqrt{T}$, so to double rms speed, the temperature has to be raised four times i.e., $T_f = 1200$ K.

Since the nitrogen gas $(f = 5)$ is enclosed in a vessel, so it cannot expand or it cannot loose any energy to external system. Hence, amount of heat required to double the rms speed of nitrogen is

$$Q = \Delta U = nC_V\Delta T = \left(\frac{15}{28}\right)\left(\frac{5}{2}(8.314)\right)(1200 - 300)$$

$$\Rightarrow \quad Q \approx 10021 \text{ J}$$

ILLUSTRATION 70

One mole of a monatomic gas is mixed with 3 moles of a diatomic gas. What is the molar specific heat of the mixture at constant volume? $(R = 8.31 \text{ Jmol}^{-1}\text{K}^{-1})$

SOLUTION

For monatomic gas $(C_V)_1 = \frac{3R}{2}$, while for diatomic gas $(C_V)_2 = \frac{5R}{2}$, so by conservation of energy, we get

$$(C_V)_{\text{mix}} = \frac{n_1(C_V)_1 + n_2(C_V)_2}{n_1 + n_2} = \frac{1\left(\frac{3R}{2}\right) + 3\left(\frac{5R}{2}\right)}{1+3}$$

$$\Rightarrow \quad (C_V)_{\text{mix}} = \frac{9R}{4} = \frac{9}{4} \times 8.31 = 18.7 \text{ Jmol}^{-1}\text{K}^{-1}$$

ILLUSTRATION 71

In a thermally isolated container, 1 g of helium having rms velocity 1000 ms^{-1} and 4 g of oxygen having rms velocity 1000 ms^{-1} are mixed. Calculate rms velocities of helium and oxygen molecules after equilibrium is attained. Take $R = 25/3$ $\text{Jmol}^{-1}\text{K}^{-1}$.

SOLUTION

The rms velocity of helium molecules is 1000 ms^{-1}. If temperature of this gas is T_1, then

$$v_{\text{rms}} = \sqrt{\frac{3RT_1}{M_1}}$$

$$\Rightarrow \quad 1000 = \sqrt{\frac{3 \times 8.314 \times T_1}{2 \times 10^{-3}}}$$

$$\Rightarrow \quad T_1 = \frac{(2 \times 10^{-3})(10^6)}{3(25/3)} = 80 \text{ K}$$

Also, the rms velocity of oxygen molecules is 1000 ms^{-1}. If temperature of this gas is T_2, then

$$v_{\text{rms}} = \sqrt{\frac{3RT_2}{M_2}}$$

$$\Rightarrow \quad 1000 = \sqrt{\frac{3 \times 8.314 \times T_2}{32 \times 10^{-3}}}$$

$$\Rightarrow \quad T_2 = \frac{(32 \times 10^{-3})(10^6)}{3(25/3)} = 1280 \text{ K}$$

It is given that 1 g of *He* and 4 g of O_2 is mixed. If their number of moles are n_1 and n_2, then

$$n_1 = \frac{1}{2} \text{ and } n_2 = \frac{4}{32} = \frac{1}{8}$$

When gases at different temperature are mixed at constant volume (or in a container), the total internal energy of system remains constant before and after mixing, so in this case, if final temperature of mixture is T_f, then

$$T_f = \frac{f_1 n_1 T_1 + f_2 n_2 T_2}{f_1 n_1 + f_2 n_2}$$

where, $f_1 = 3$, $f_2 = 5$, $n_1 = 1/2$, $n_2 = 1/8$, $T_1 = 80$ K and $T_2 = 1280$ K

Thus, final temperature of mixture is given as

$$\Rightarrow \quad T_f = \frac{(3)(1/2)(80) + (5)(1/8)(1280)}{3(1/2) + 5(1/8)}$$

$$\Rightarrow \quad T_f = \frac{120 + 800}{2.125} \approx 433 \text{ K}$$

Thus, final rms velocity of *He* gas molecules is

$$v_{\text{rms}} = \sqrt{\frac{3RT_f}{M_1}} = \sqrt{\frac{3(25/3)(433)}{2 \times 10^{-3}}} = 2326.5 \text{ ms}^{-1}$$

Final rms velocity of O_2 gas molecules is

$$v_{\text{rms}} = \sqrt{\frac{3RT_f}{M_2}} = \sqrt{\frac{3(25/3)(433)}{32 \times 10^{-3}}} = 581.6 \text{ ms}^{-1}$$

Problem Solving Technique(s)

C_P, C_V AND γ OF AN IDEAL GAS

For an ideal gas, we have to keep the following points in our mind.

(a) Molar specific heat is the heat required to raise the temperature of 1 mole of a gas by 1 °C.

(b) The **adiabatic exponent** (γ) is defined as the ratio of the specific heat of the gas at constant pressure to the specific heat of the gas at constant volume i.e., $\gamma = \frac{C_P}{C_V}$. γ is dimensionless and $\gamma > 1$.

(c) Molar specific heat at constant volume, in terms of the adiabatic exponent is

$$C_V = \frac{dU}{dT} = \frac{R}{\gamma - 1}$$

(d) Molar specific heat at constant pressure, in terms of the adiabatic exponent is

$$C_P = C_V + R = \frac{R\gamma}{\gamma - 1}$$

(e) In general, for n moles of an ideal gas, we have $C_V = \frac{1}{n}\left(\frac{dU}{dT}\right)$.

(f) Please note that the specific heat(s) is/are always expressed per mole of the sample of the gas. So, the answer for the specific heat must remain the same whether the calculations are done by taking one mole or n moles of the gas sample.

(g) U is the internal energy of 1 mole of the gas.

For 1 mole of the gas, we have $dU = (1)C_V dT = C_V dT$ and for n moles of the gas sample we have $dU = nC_V dT$.

ILLUSTRATION 72

1 g mole of oxygen at 27 °C and 1 atmospheric pressure is enclosed in a vessel.

(a) Assuming the molecules to be moving with v_{rms}, find the number of collisions per second which the molecules make with one square meter area of the vessel wall.

(b) The vessel is next thermally insulated and moved with a constant speed v_0. It is then suddenly stopped. The process results in a rise of temperature of the gas by 1 °C. Calculate the speed v_0. Given, $k_B = 1.38\times10^{-23}$ JK^{-1} and $N_A = 6.02\times10^{23}$ mol^{-1}.

SOLUTION

(a) Mass of one oxygen molecule is

$$m = \frac{M}{N_A} = \frac{32}{6.02\times10^{23}} \text{ g}$$

$$\Rightarrow \quad m = 5.316\times10^{-23} \text{ g} = 5.316\times10^{-26} \text{ kg}$$

Since, $v_{rms} = \sqrt{\frac{3k_BT}{m}}$

$$\Rightarrow \quad v_{rms} = \sqrt{\frac{3\times1.38\times10^{-23}\times300}{5.316\times10^{-26}}} = 483.35 \text{ ms}^{-1}$$

Change in momentum per collision is

$$\Delta p = mv_{rms} - (-mv_{rms}) = 2mv_{rms}$$

$$\Rightarrow \quad \Delta p = (2)(5.316\times10^{-26})(483.35)$$

$$\Rightarrow \quad \Delta p = 5.14\times10^{-23} \text{ kgms}^{-1}$$

Now, suppose n particles strike per second per unit area of vessel wall, then force applied on the vessel wall is

$$F = n\Delta p = (n)(5.14\times10^{-23}) \text{ N} \qquad \left\{\because F_{ext} = \frac{dp}{dt}\right\}$$

Since, $P = \frac{F}{A}$, for unit area $F = P$

$$\Rightarrow \quad (n)(5.14\times10^{-23}) = 1.01\times10^5$$

$$\Rightarrow \quad n = 1.965\times10^{27} \text{ per second}$$

(b) When the vessel is stopped the ordered motion of the vessel converts into disordered motion and temperature of the gas is increased. So, we have

$$N_A\left(\frac{1}{2}mv_0^2\right) = \Delta U \qquad \ldots(1)$$

Since, $U = \frac{5}{2}RT$ $\{\text{for } O_2\}$

$$\Rightarrow \quad \Delta U = \frac{5}{2}R\Delta T$$

where m is the mass of one gas molecule given by

$$m = 5.316\times10^{-26} \text{ kg}$$

Substituting these values in equation (1), we get

$$v_0 = \sqrt{\frac{5R\Delta T}{N_Am}} = \sqrt{\frac{5\times8.31\times1}{32\times10^{-3}}} \quad \{\because N_Am = 32\times10^{-3} \text{ kg}\}$$

$$\Rightarrow \quad v_0 = 36 \text{ ms}^{-1}$$

ILLUSTRATION 73

A cubical vessel of side 1 m contains one mole of nitrogen at a temperature of 300 K. If the molecules are assumed to move with the rms speed, then find the number of collisions per second which the molecules may make with the wall of the vessel.

Further, if the vessel is now thermally insulated, moved with a constant speed v and then suddenly stopped, then this results in rise of temperature by 2 °C, calculate v.

SOLUTION

Given that volume of container is $V = 1$ m^3, temperature of gas is $T = 300$ K and amount of gas is $n = 1$ mole

Let n_0 be the number of molecules per unit volume, then we have $n = 1$ mole

$$\Rightarrow \quad n_0 = 6.023\times10^{23} \text{ m}^{-3}$$

The rms velocity of molecules at 300 K temperature is

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$\Rightarrow \quad v_{rms} = \sqrt{\frac{3\times8.314\times300}{28\times10^{-3}}} \approx 517 \text{ ms}^{-1}$$

Also, we know from our knowledge of kinetic theory of gases, that the number of collisions taking place per second per square meter of wall is given by

$$N = \frac{n_0v}{6} = \frac{(6.023\times10^{23})(517)}{6} \approx 5.2\times10^{25} \text{ s}^{-1}\text{m}^{-2}$$

Now, when the container is moving at speed v, the kinetic energy associated with one mole of nitrogen gas molecules is

$$K = \frac{1}{2}(N_Am_{N_2})v^2 = \frac{1}{2}Mv^2 = \frac{1}{2}(28\times10^{-3})v^2$$

When the container is suddenly stopped, this kinetic energy is transformed into thermal energy and increases the internal energy of gas (because container is insulated). If temperature increment of gas is ΔT, then rise in its internal energy is

$$\Delta U = \left(\frac{nf}{2}\right)R\Delta T = \frac{5}{2}(nR\Delta T)$$

$$\Rightarrow \quad \Delta U = \left(\frac{5}{2}\right)(1)(8.314)(2) = 41.57 \text{ J}$$

Applying law of conservation of energy, we get

$$\frac{1}{2}(28\times10^{-3})v^2 = 41.57$$

$$\Rightarrow \quad v = \sqrt{\frac{41.57\times2}{28\times10^{-3}}} \approx 54.5 \text{ ms}^{-1}$$

ILLUSTRATION 74

An adiabatic vessel contains 3 mole of a diatomic gas. Moment of inertia of each molecule is 2.76×10^{-46} kgm^2 and root mean square angular velocity is 5×10^{12} rads^{-1}. Another adiabatic vessel contains 5 mole of a monatomic gas at a temperature 470 K. Assume the gases to be ideal, calculate root mean square angular velocity of diatomic molecules when the two vessels are connected by a thin tube of negligible volume. Boltzmann constant $k_B = 1.38\times10^{-23}$ JK^{-1}.

SOLUTION

According to law of equipartition of energy, energy associated with each degree of freedom per molecule is $\frac{1}{2}k_BT$. Since a diatomic gas molecule has two rotational degrees of freedom, so its rotational energy must be $2\left(\frac{1}{2}k_BT\right) = k_BT$.

If initial temperature of diatomic gas is T_1, then

$$\frac{1}{2}I\omega_{\text{rms}}^2 = k_BT_1$$

$$\Rightarrow \quad T_1 = \frac{I\omega_{\text{rms}}^2}{2k_B} = \frac{(2.76\times10^{-46})(5\times10^{12})^2}{2(1.38\times10^{-23})}$$

$$\Rightarrow \quad T_1 = 250 \text{ K}$$

When the two vessels containing diatomic and monatomic gases are connected, these gases exchange this thermal energy but no energy is lost to surrounding because the vessels are adiabatic. This mixing of gases takes place at constant volume, so total internal energy of the system remains constant. If T_f be the final temperature of the system, then

$$T_f = \frac{n_1f_1T_1 + n_2f_2T_2}{n_1f_1 + n_2f_2} \qquad \ldots(1)$$

For a diatomic gas, $f_1 = 5$, $n = 3$ and $T_1 = 250$ K

For monatomic gas, $f_2 = 3$, $n_2 = 5$ and $T_2 = 470$ K

Thus, from equation (1), we get

$$\Rightarrow \quad T_f = \frac{(5)(3)(250)+(3)(5)(470)}{(5)(3)+(3)(5)} = 360 \text{ K}$$

Thus, final mixture of the two gases is at temperature 360 K. If final rms angular velocity of diatomic gas molecules is $(\omega_{\text{rms}})_f = \omega_f$, then from law of equipartition of energy, we get

$$\frac{1}{2}I\omega_f^2 = k_BT$$

$$\Rightarrow \quad \omega_f = \sqrt{\frac{2k_BT}{I}} = \sqrt{\frac{2\times1.38\times10^{-23}\times360}{2.76\times10^{-46}}}$$

$$\Rightarrow \quad \omega_f = 6\times10^{12} \text{ rads}^{-1}$$

Test Your Concepts-IV

Based on Internal Energy, Degrees of Freedom and Molar Specific Heats for Ideal Gases

(Solutions on page H.84)

1. Two perfect monatomic gases at absolute temperatures T_1 and T_2 are mixed. There is no loss of energy. Find the temperature of the mixture if the number of moles in the gases are n_1 and n_2.
2. The ratio of translational and rotational kinetic energy of a gas at 100 K is 3 : 2. Calculate the internal energy of one mole gas at this temperature. Take $R = 8.3$ Jmol^{-1}K^{-1}.
3. A tank used for filling helium balloons has a volume of 0.3 m^3 and contains 2 mol of helium gas at 20 °C. Assuming that the helium behaves like an ideal gas, calculate the
   (a) total translational kinetic energy of the molecules of the gas.
   (b) average kinetic energy per molecule.
4. If Avogadro's number is $N_A = 6.02\times10^{23}$ and Boltzmann's constant is $k_B = 1.38\times10^{-23}$ JK^{-1}, then calculate the
   (a) average kinetic energy of translation of an oxygen molecule at 27 °C
   (b) total kinetic energy of an oxygen molecule at 27 °C and
   (c) total kinetic energy in joule of one mole of oxygen at 27 °C.
5. Calculate the change in internal energy of 3 mole of helium gas when its temperature is increased by 2 K.
6. A cylindrical container of volume V is divided in two equal parts by a fixed diathermic partition. Identical gases are filled in the two parts at initial pressure and temperature p_1, T_1 and p_2, $T_2 (> T_1)$ respectively. After a long time when the two gases are in thermal

equilibrium, calculate the final temperature and pressure of the gases in the two parts. Also calculate the amount of heat supplied by gas in one part to that in other part in if gas taken is monatomic.

7. Calculate γ for a gaseous mixture consisting of 2 mole of O_2 and 3 mole of CO_2. The gases are assumed to be ideal.

8. The temperature of a gas consisting of rigid diatomic molecules is 300 K. Calculate the approximate angular root mean square velocity of a rotating molecule if its moment of inertia is equal to 2.1×10^{-39} kgm^2.

9. An insulated gas jar of mass M which is equal to the molecular mass of the enclosed gas moves with certain kinetic energy. When the jar is stopped, the increment of temperature of the gas is ΔT. Find the kinetic energy of the system before stopping. Take the adiabatic exponent of the gas to be γ.

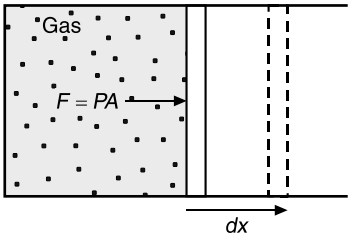
FIRST LAW OF THERMODYNAMICS AND THERMODYNAMIC PROCESSES

THERMODYNAMICS

Thermodynamics is the study of the relations between heat, work, temperature, and energy. Thermodynamics is concerned with the work done by a system and the heat it exchanges with its surroundings. Alternatively, it is the study of changes that occur in some part of the universe (we designate as the system) and then everything else (outside the system) is the surrounding. A real or imagined boundary may separate the system from its surroundings. *A collection of properties such as pressure, volume, temperature and some other properties to be discussed later characterize the thermodynamical state of a system. The laws of thermodynamics describe how the energy in a system changes and whether the system can perform any useful work on its surroundings.*

CONCEPT OF WORK

Consider a frictionless piston of area A attached to a container. On heating, the piston is displaced through dx. If dW is the work done then



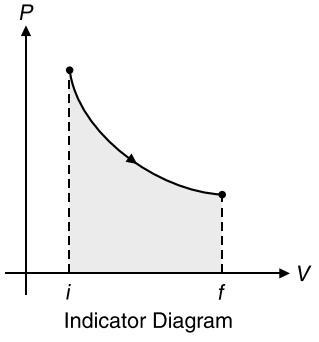
$$dW = Fdx = (PA)dx = P(Adx) \qquad \{\because F = PA\}$$

$$\Rightarrow \quad dW = PdV \qquad \{\because Adx = dV\}$$

To get total work done from initial state i to final state f (along a specified path)

$$W = \int_i^f PdV$$

Also work done is equal to the area under a curve in an **INDICATOR DIAGRAM** called P-V **diagram.**



Indicator Diagram

Now, there are two methods, using which we can calculate the work done by the gas.

Method 1: To be used when P-V relation (dependence of pressure on volume) is known.

Suppose $P = f(V)$ (i.e., pressure is a function of volume) then

$$dW = PdV = f(V)dV$$

$$\Rightarrow \quad W = \int dW = \int_{V_i}^{V_f} f(V)dV$$

Problem Solving Technique(s)

In the above discussion we have seen that

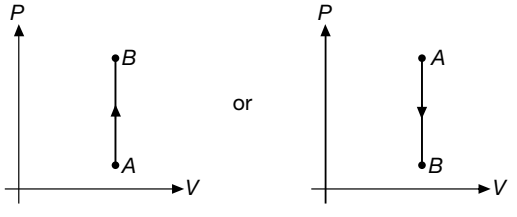
$$W = \int_{V_i}^{V_f} PdV$$

From this equation it seems as if work done can be calculated only when P-V equation is known and the limits V_i and V_f are known to us. **But it is not so.** We can calculate work done even if we know the limits of temperature.

Method 2: To be used when P-V graph is given.

In this method, the work done by a gas is equal to the area under P-V graph (also called as Indicator Diagram). Following different cases are possible.

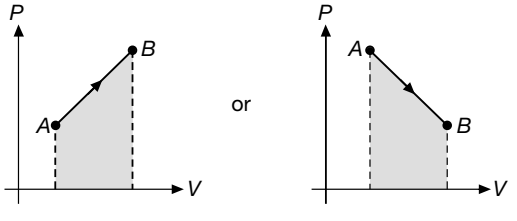
CASE-1: When volume is constant



V =constant

$$\Rightarrow \quad W_{AB} = 0$$

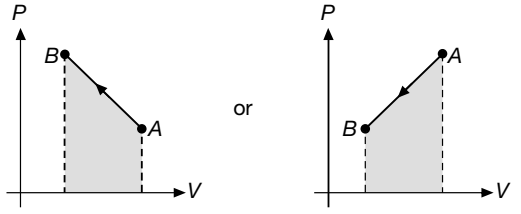
CASE-2: When volume is increasing (i.e., expansion)



V is increasing

$$\Rightarrow \quad W_{AB} > 0 \text{ and } W_{AB} = \text{Shaded area}$$

CASE-3: When volume is decreasing (i.e., compression)

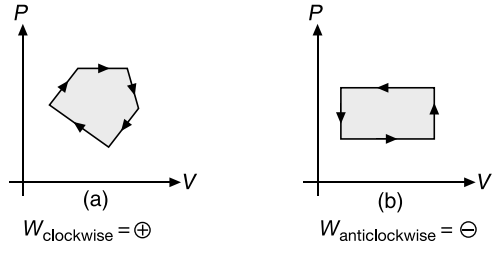


V is decreasing

$\Rightarrow$ $W_{AB} < 0$ and W_{AB} = –Shaded area

CASE-4: Cyclic process

It consists of a series of thermodynamic processes transferring heat and work, while varying pressure, volume, temperature and other state variables, eventually returning the system to its initial state.

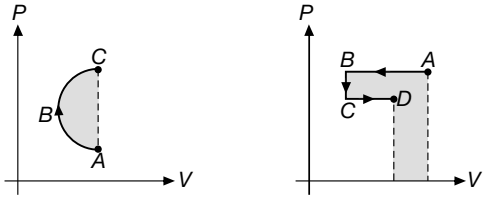


Cyclic process

$W_{\text{clockwise cycle}}$ = +Shaded area {in figure (a)}

$W_{\text{anticlockwise cycle}}$ = –Shaded area {in figure (b)}

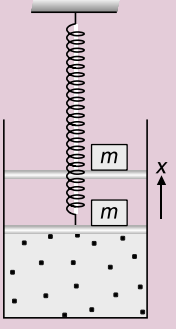
CASE-5: Incomplete cycle



W_{ABC} = +Shaded area W_{ABCD} = –Shaded area

Problem Solving Technique(s)

(a) Sometimes the piston (which is assumed to be light) is attached to a spring of force constant k and a mass m is placed over the piston. The area of the piston is A. The gas expands. To make the calculation easy we assume that initially the spring was in its natural length. We are required to find the work done by the gas. As the piston is assumed to be light, net force on it at every instant is zero.



$$PA = kx + mg + P_0A$$

$$\Rightarrow P = P_0 + \frac{kx}{A} + \frac{mg}{A}$$

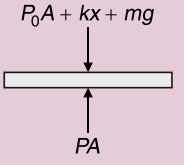
Since, $dW = PdV = P(Adx) = (AP_0 + kx + mg)dx$

$$\Rightarrow W = \int_0^x PdV$$

$$\Rightarrow W = \int_0^x (AP_0 + kx + mg)dx$$

$$\Rightarrow W = P_0Ax + \frac{1}{2}kx^2 + mgx \qquad \{\text{as } Ax = \Delta V\}$$

$$\Rightarrow W = P_0\Delta V + \frac{1}{2}kx^2 + mgx$$



The result can be stated in a different manner as under. The gas does work

(i) W_1 against the atmospheric pressure P_0 (which is constant)

(ii) W_2 against the spring force kx (which varies linearly with x) and

(iii) W_3 against the gravity force mg (which is again constant).

So, W_1 =Work done against P_0 is $P_0\Delta V$

W_2 =Work done against kx is $\frac{1}{2}kx^2$v and W_3 = Work done against mg is mgx

Hence,

$$W_{\text{Total}} = W_1 + W_2 + W_3 = P_0\Delta V + \frac{1}{2}kx^2 + mgx$$

(b) From point number (1). We may conclude that work done by a gas is zero if the other side of the piston is vacuum.

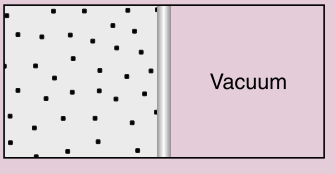


ILLUSTRATION 75

The temperature of n moles of an ideal gas is increased from T_0 to $2T_0$ through a process $P = \frac{\alpha}{T}$. Calculate the work done by the gas.

SOLUTION

From ideal gas equation we have, $PV = nRT$ …(1)

Given that $P = \frac{\alpha}{T}$ …(2)

Dividing (1) by (2), we get

$$V = \frac{nRT^2}{\alpha}$$

$$\Rightarrow \quad dV = \frac{2nRT}{\alpha}dT$$

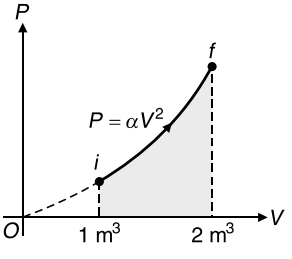
Since, $W = \int_{V_i}^{V_f} P\,dV$

$$\Rightarrow \quad W = \int_{T_0}^{2T_0}\left(\frac{\alpha}{T}\right)\left(\frac{2nRT}{\alpha}\right)dT = 2nRT_0$$

So, work done is calculated without substituting the limits of volume.

ILLUSTRATION 76

A sample of ideal gas is expanded to twice its original volume of 1 m^3 in a quasi-static process for which $P = \alpha V^2$, with $\alpha = 6$ atmm^{-6} as shown in figure. How much work is done by the expanding gas. Given 1 atm = 10^5 Nm^{-2}.



SOLUTION

Since $dW = PdV$

$$\Rightarrow \quad W = \int_1^2 \alpha V^2 dV = 6\times10^5\left(\frac{V^3}{3}\Big|_1^2\right)$$

$$\Rightarrow \quad W = \left(\frac{6\times10^5}{3}\right)(8-1) = 14\times10^5 \text{ J}$$

HEAT VERSUS WORK

Heat is the energy transferred between two bodies as a consequence of a temperature difference between them. In contrast, **work** is a mode of energy transfer in which the point of application of a force moves through a displacement and is not associated with a temperature difference. Both heat and work are "energy in transit" from one body to another during the operation of some process, once the process stops, heat and work have no meaning.

MECHANICAL EQUIVALENT OF HEAT

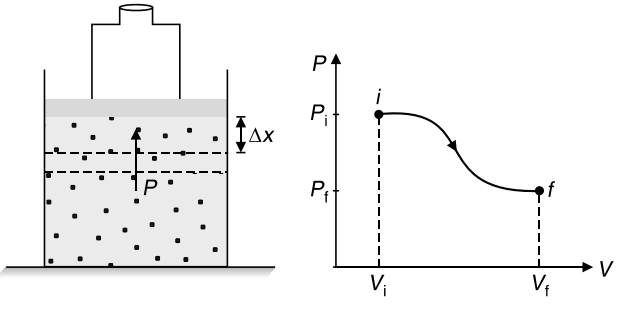
It has been concluded from Joule's experiment that the mechanical work required to produce a given change in temperature is in fixed proportion to the heat required for the same change in temperature. This constant factor is called the **mechanical equivalent** of heat.

1 calorie = 4.186 J

Thus, a change in the state of a system produced by the addition of 1 calorie of heat may also be produced by the performance of 4.186 J of work on the system.

THERMODYNAMIC WORK

Figure shows a gas confined to a cylinder by a weight on a movable piston. Our system is the gas, whereas the cylinder and the piston form the environment. If the piston is allowed to move upward, the gas expands and does work on it. To calculate the work done by the gas, we assume that the process is **quasistatic**. In a **quasistatic process** the thermodynamics variables (P, V, T, n, etc.) of the system and its surroundings change infinitely slowly. Thus, the system is always arbitrarily close to an equilibrium state, in which it has a well-defined volume, and the whole system is characterized by single value of the macroscopic variables. To ensure that the piston moves very slowly, there must be some force, for example, provided by a weight, directed opposite to that due to the pressure. If the piston were to move suddenly, the rapid expansion would involve turbulence and the pressure would not be uniquely defined.



When the piston rises by dx, the work dW done by the gas is $dW = Fdx = (PA)dx$ where A is the cross-sectional area of the piston. Since the change in volume of the gas is $dV = Adx$, the work may be expressed as

(Quasistatic) $\qquad dW = PdV$

As a quasistatic process evolves, P and V are always uniquely defined. This allows us to depict the process on a PV diagram such as figure. When the system is taken quasistatically from the equilibrium state i to another equilibrium state f, the total work done by the system is

$$W = \int_{V_i}^{V_f} PdV$$

In figure the work is represented by the area under the curve. If $V_f > V_i$, the work done by the gas is **positive**. If the

volume decreases, the work done by the gas is **negative**. This may be interpreted as positive work done on the gas by the environment. The work done depends not only on the initial and final states but also on the details of the process, that is, the thermodynamic path between the states. Therefore, we need to know how the pressure varies with the volume.

FIRST LAW OF THERMODYNAMICS

Consider a system that consists of a gas enclosed by a piston in a cylinder. Suppose the system is taken quasistatically from an initial state P_i, V_i, T_i to a final state P_f, V_f, T_f. At each step the work done and heat exchanged are measured. We know that both the total work done W and the total heat transfer Q to or from the system depend on the thermodynamic path. However, the difference $Q - W$, is the same for all paths between the given initial and final equilibrium states, and it is equal to the change in internal energy ΔU of the system.

$$\Delta U = Q - W$$

$$đQ = dU + đW$$

In the above statement, Q is **positive** when heat enters the system and W is **positive** when **work** is done by the system on its surroundings.

The equation, is the mathematical statement of the **First Law of Thermodynamics**. It states that the internal energy of a system changes when work is done on the system (or by it), and when it exchanges heat with the environment.

Note that the First Law is valid for all processes, quasistatic or not. However, if friction is present, or the process is not quasistatic, the internal energy U is uniquely defined only at the initial and final equilibrium states.

The First Law establishes the existence of internal energy U as a state function – one that depends only on the thermodynamic state of the system.

In the macroscopic approach of thermodynamics, there is no need to specify the physical nature of the internal energy. The experimental results are sufficient to prove that such a function exists. The internal energy is the sum of all possible kinds of energies stored in the system – mechanical, electrical, magnetic, chemical, nuclear, and so on. It does not include the kinetic and potential energies associated with the centre of mass of the system.

MISCONCEPTION BETWEEN HEAT AND INTERNAL ENERGY

Confusion between heat and internal energy arises from erroneous statements that refer to the "heat content" of a body. Even correct terms like "the heat capacity of a body" can mislead one to believe that heat is somehow stored in a system. This is not correct.

The physical quantity possessed by a system is **internal energy**, which is the sum of all the kind of energy in the system. As the First Law indicates, U may be changed either by heat exchange or by work. The internal energy is a state function that depends on the equilibrium state of a system, whereas Q and W depend on the thermodynamic path between two equilibrium states. That is, Q and W are associated with processes. The heat absorbed by a system will increase its internal energy, only some of which is average translatory kinetic energy. It is therefore incorrect to say that heat is the energy of the random motion.

FIRST LAW OF THERMODYNAMICS: REVISITED

When the Law of Conservation of Energy was first introduced it was stated that the mechanical energy (kinetic + potential) of a system is conserved in the absence of nonconservative forces such as friction. That is, the changes in the internal energy of the system were not included in this mechanical model.

The First Law of Thermodynamics is a **generalisation of the Law of Conservation of Energy** that includes possible changes in internal energy. It is a universally valid law that can be applied to all kinds of processes. Furthermore it provides us with a connection between the microscopic and macroscopic worlds.

When a system undergoes an infinitesimal change in state, where a small amount of heat, $đQ$, is transferred and a small amount of work, $đW$, is done, the internal energy also changes by a small amount dU. Thus, for infinitesimal processes we can write

$$đQ = dU + đW$$

$đQ$ and $đW$ are not true differential quantities i.e., $đQ$ and $đW$ are path functions and can never be expressed as the difference of final value and initial value whereas, dU is a true differential i.e. a state function and can always be expressed as the difference of final value and initial value.

Problem Solving Technique(s)

RIGHT SIGN CONVENTIONS SHOULD BE FOLLOWED WHILE USING THE FIRST LAW

(a) Heat gained by the system is positive
(b) Heat lost by the system is negative
(c) Gain in internal energy is positive
(d) Loss in internal energy is negative
(e) Work done by the system is positive (i.e., during expansion)
(f) Work done on the system is negative (i.e., during compression)

ILLUSTRATION 77

When a system goes from state A to state B, it is supplied with 400 J of heat and it does 100 J of work.

(a) For this transition, what is the system's change in internal energy?

(b) If the system moves from B to A, what is the change in internal energy.

(c) If in moving from A to B along a different path in which $W'_{AB} = 400$ J of work is done on the system, how much heat does it absorb?

SOLUTION

(a) According to $FLTD$, we have

$$\Delta U_{AB} = Q_{AB} - W_{AB} = (400 - 100)\text{ J}$$

$$\Rightarrow \quad \Delta U_{AB} = 300\text{ J}$$

(b) Consider a closed path that passes through the state A and B, then internal energy is a state function and so ΔU is zero for a closed path.

$$\Rightarrow \quad \Delta U = \Delta U_{AB} + \Delta U_{BA} = 0$$

$$\Rightarrow \quad \Delta U_{BA} = -\Delta U_{AB}$$

$$\Rightarrow \quad \Delta U_{BA} = -300\text{ J}$$

(c) The change in internal energy is the same for any path, so

$$\Delta U_{AB} = \Delta U'_{AB} = Q'_{AB} - W'_{AB}$$

$$\Rightarrow \quad 300\text{ J} = Q'_{AB} - (-400\text{ J})$$

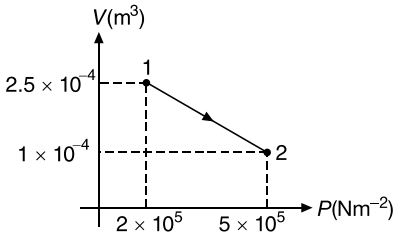
So, the heat exchanged is

$$Q'_{AB} = -100\text{ J}$$

The negative sign indicates that heat is lost by the system in this process.

ILLUSTRATION 78

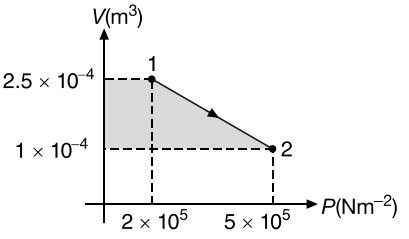
A gas is taken from state-1 to state-2 along the path shown in Figure.



If 70 cal of heat is extracted from the gas in the process, calculate the change in internal energy of the system.

SOLUTION

Since work done is equal to the area under the P-V graph, so in this case work done will be the negative shaded area as shown in Figure. Negative because volume is decreasing.



$$\Rightarrow \quad W = -\frac{1}{2} \times 1.5 \times 10^{-4} \times (2+5) \times 10^5 = -52.5\text{ J}$$

Since, 70 cal of heat is extracted in the process, so

$$Q = -70\text{ cal} = 70 \times 4.2\text{ J} = -294\text{ J}$$

From First Law of Thermodynamics (FLTD), we get

$$Q = W + \Delta U$$

$$\Rightarrow \quad \Delta U = Q - W = (-294) - (-52.5) = -241.5\text{ J}$$

Hence, the internal energy of gas decreases by 241.5 J in the given process.

ILLUSTRATION 79

For an ideal gas if the molar heat capacity varies as $C = C_V + 3aT^2$. Find the equation of the process in the variables $(T,\ V)$ where a is a constant.

SOLUTION

From First Law of Thermodynamics, $dQ = dU + dW$

$$\Rightarrow \quad CdT = C_V dT + PdV$$

$$\Rightarrow \quad (C_V + 3aT^2)dT = C_V dT + PdV$$

$$\Rightarrow \quad 3aT^2 dT = PdV = \left(\frac{RT}{V}\right)dV \qquad \{\because\ PV = RT\}$$

$$\Rightarrow \quad \left(\frac{3a}{R}\right)TdT = \frac{dV}{V}$$

Integrating, we get

$$\left(\frac{3aT^2}{2R}\right) = \log_e V - \log_e C$$

where C is a positive constant

$$\Rightarrow \quad \log_e\left(\frac{V}{C}\right) = \frac{3aT^2}{2R}$$

$$\Rightarrow \quad \frac{V}{C} = e^{\frac{3aT^2}{2R}}$$

$$\Rightarrow \quad Ve^{-\frac{3aT^2}{2R}} = \text{constant}$$

ILLUSTRATION 80

A vertical cylinder of cross-sectional area A contains one mole of an ideal monatomic gas under a piston of mass M. At a certain instant, a heater that transmits to the gas an

amount of heat q per unit time is switched on under the piston. Determine the velocity v of the piston under the condition that the gas pressure under piston is constant and equal to P_0 and the gas under the piston is thermally insulated.

SOLUTION

Let v be the speed of piston (upwards). According to First Law of Thermodynamics, we have

$$Q = \Delta U + W$$

where, $\Delta U = (1)C_V \Delta T = \frac{3}{2}R\Delta T$

$$W = P\Delta V = PA\Delta x$$

where, $P = P_0 + \frac{Mg}{A}$

Also, $PV = (1)RT$

Since P =constant, so $P\Delta V = R\Delta T$

$$\Rightarrow \quad Q = P\Delta V + \frac{3}{2}P\Delta V = \frac{5}{2}PA\Delta x$$

Further, $Q = q\Delta t$

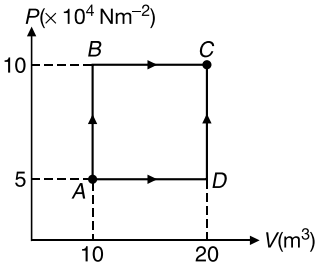
$$\Rightarrow \quad q\Delta t = \frac{5}{2}PA\Delta x$$

$$\Rightarrow \quad q = \frac{5}{2}\left(P_0 + \frac{Mg}{A}\right)A\frac{\Delta x}{\Delta t} = \frac{5}{2}(P_0A + Mg)v$$

$$\Rightarrow \quad v = \frac{2}{5}\frac{q}{(P_0A + Mg)}$$

ILLUSTRATION 81

A sample of 2 kg monatomic helium (assumed ideal) is taken through the process ABC and another sample of 2 kg of the same gas is taken through the process ADC (shown in figure). Given molecular mass of helium is 4.



(a) What is the temperature of helium in each of the states A, B, C and D?

(b) Is there any way of telling afterwards which sample of helium went through the process ABC and which went through the process ADC? Write Yes or No.

(c) How much is the heat involved the process ABC and ADC?

SOLUTION

Number of gram moles of He is given by

$$n = \frac{m}{M} = \frac{2\times10^3}{4} = 500$$

(a) $V_A = 10\ \text{m}^3$, $P_A = 5\times10^4\ \text{Nm}^{-2}$

$$\Rightarrow \quad T_A = \frac{P_AV_A}{nR} = \frac{(10)(5\times10^4)}{(500)(8.31)} = 120.34\ \text{K}$$

Similarly, $V_B = 10\ \text{m}^3$, $P_B = 10\times10^4\ \text{Nm}^{-2}$

$$\Rightarrow \quad T_B = \frac{(10)(10\times10^4)}{(500)(8.31)}\ \text{K} = 240.68\ \text{K}$$

Similarly, $V_C = 20\ \text{m}^3$, $P_C = 10\times10^4\ \text{Nm}^{-2}$

$$\Rightarrow \quad T_C = \frac{(20)(10\times10^4)}{(500)(8.31)}\ \text{K} = 481.36\ \text{K}$$

and $V_D = 20\ \text{m}^3$, $P_D = 5\times10^4\ \text{Nm}^{-2}$

$$\Rightarrow \quad V_D = \frac{(20)(5\times10^4)}{(500)(8.31)}\ \text{K} = 240.68\ \text{K}$$

(b) No, it is not possible to tell afterwards which sample went through the process ABC or ADC. But during the process if we note down the work done in both the processes, then the process which requires more work goes through process ABC.

(c) In the process ABC

$$\Delta U = nC_V\Delta T = n\left(\frac{3}{2}R\right)(T_C - T_A)$$

$$\Rightarrow \quad \Delta U = (500)\left(\frac{3}{2}\right)8.31(481.36 - 120.34)\ \text{J}$$

$$\Rightarrow \quad \Delta U = 2.25\times10^6\ \text{J}$$

and W =area under

$$BC = (20-10)(10)\times10^4\ \text{J} = 10^6\ \text{J}$$

So, by $FLTD$, we get

$$Q_{ABC} = \Delta U + W = (2.25\times10^6 + 10^6)\ \text{J}$$

$$\Rightarrow \quad Q_{ABC} = 3.25\times10^6\ \text{J}$$

In the process ADC, ΔU will be same (because it depends on initial and final temperatures only.

Since, W = Area under $AD = (20-10)(5\times10^4)$ J

$$\Rightarrow \quad W = 0.5\times10^6\ \text{J}$$

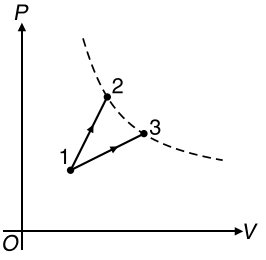
So, by $FLTD$, we get

$$Q_{ADC} = \Delta U + W = (2.25\times10^6 + 0.5\times10^6)\ \text{J}$$

$$\Rightarrow \quad Q_{ADC} = 2.75\times10^6\ \text{J}$$

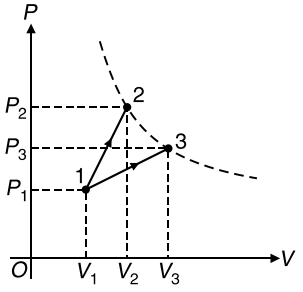
ILLUSTRATION 82

A gas takes part in two thermal processes in which it is heated from the same initial state to the same final temperature. The P-V diagram for these two processes are indicated by straight lines 1-3 and 1-2 in figure. Find out in which process the amount of heat supplied to the gas is larger.



SOLUTION

Since the temperature remains constant, $\Delta T = 0$



Now, $W_{12} = \frac{1}{2}(P_1 + P_2)(V_2 - V_1)$

and $W_{13} = \frac{1}{2}(P_1 + P_3)(V_3 - V_1)$

$\Rightarrow \quad W_{12} - W_{13} = \frac{1}{2}\left[P_1(V_2 - V_3) + (P_3 - P_1)V_1\right]$

Since, $V_2 - V_3 < 0$ and $P_3 - P_1 < 0$, so

$W_{12} - W_{13} < 0$

$\Rightarrow \quad W_{13} > W_{12}$

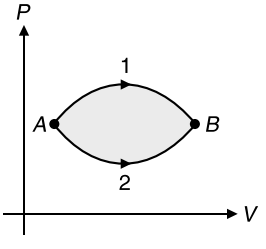
$\Rightarrow \quad Q_{13} > Q_{12}$

Test Your Concepts-V

Based on Work Done and First Law of Thermodynamics

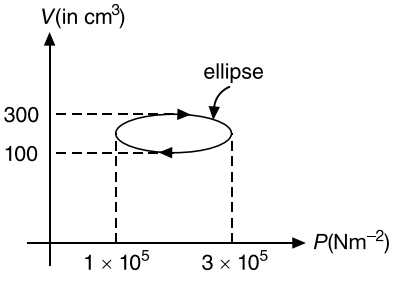
(Solutions on page H.85)

1. A certain amount of an ideal gas passes from state *A* to *B* first by means of process 1, then by means of process 2. In which of the process is the amount of heat absorbed by the gas greater.

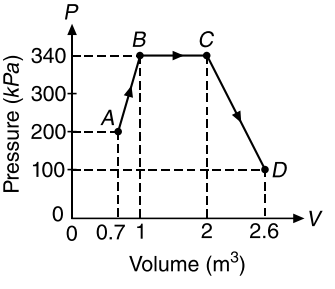


2. For a thermodynamical system, the pressure, the volume and the temperature are related to each other as new gas law given by $P = \frac{\alpha T^2}{V}$, where α is a constant. Calculate work done by the system in this process when pressure remains constant and its temperature changes from T_0 to $2T_0$.

3. Calculate the heat absorbed by a system in going through the cyclic process shown in figure.



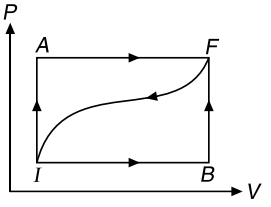
4. The *PV* diagram for a gas confined to a cylinder with the help of a piston is shown in Figure. Calculate the work done by the gas to expand from *A* to *C* and from *A* to *D* along the curve.



5. The table given below gives some values for different processes. All the data is in joule. Calculate the unknown values using First Law of Thermodynamics.

Process	Q	W	U_i	U_f	ΔU
1.	35	–15	–	–10	–
2.	–15	–	–	60	–20
3.	–	–20	80	–	40

6. When a thermodynamic system is taken from an initial state *I* to a final state *F* along the path *IAF*, as shown in figure, the heat energy absorbed by the system is 55 J and the work done by the system is 25 J. If the same system is taken along the path *IBF*, the heat energy absorbed by the system is 35 J.



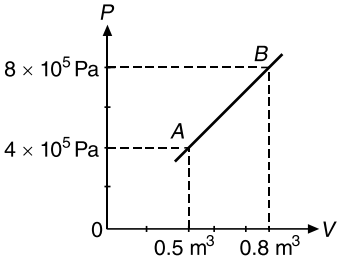
(a) Calculate the work done along the path *IBF*.

(b) If for the curved path *FI*, work done on the system is 15 J, how much heat energy gained or lost by the system along this path.

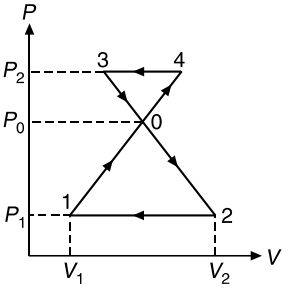
(c) If internal energy at *I* is 10 J, calculate the internal energy at *F*.

(d) If internal energy is 20 J at the point *B*, then calculate the heat gained (or heat lost) for the processes *BF* and *IB*.

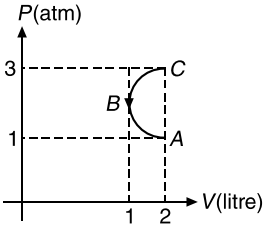
7. The volume of a monatomic ideal gas increases linearly with pressure, as shown in the figure. Calculate the increase in internal energy, work done by the gas, and the heat supplied to the gas.



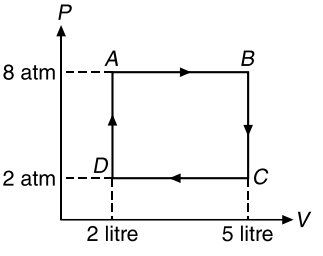
8. Calculate the work done by an ideal gas during a cyclic process 14321 shown in Figure. Take $P_1 = 10^5$ Pa, $P_0 = 3\times10^5$ Pa, $P_2 = 4\times10^5$ Pa, $V_2 - V_1 = 100$ litre and segments 43 and 21 of the cycle to be parallel to the *V*-axis.



9. The *PV* diagram shown in figure for a thermodynamic process is a semicircle. Calculate the work done on the gas in the process *ABC*.



10. The *P-V* diagram of the thermodynamic process of an ideal gas is shown in Figure. Calculate the work done in the processes $A \to B$, $B \to C$, $C \to D$, and $D \to A$. Also calculate the work done in the complete cycle. Take $1\,\text{atm} = 1.0\times10^5\ \text{Nm}^{-2}$.



ISOLATED SYSTEM

Consider first an isolated system for which there is no heat exchange and no work is done on the external environment. In this case $Q = 0$ and $W = 0$, so from the First Law we conclude $\Delta U = 0$

$\Rightarrow \quad U = \text{constant}$

The internal energy of an isolated system is constant.

ISOCHORIC PROCESS

A process in which volume remains constant. e.g., a gas heated in a rigid container i.e., $V = \text{constant}$
In this case

$$đW = PdV = 0$$

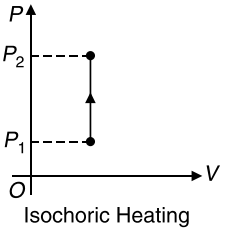
So, no external work is done in an Isochoric process.

{for numerical problems}

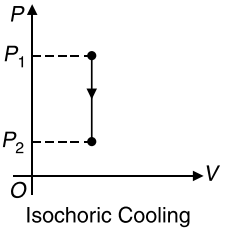
According to First Law of Thermodynamics $đQ = dU + đW$

$\Rightarrow \quad đQ = dU = nC_V dT$

i.e., in an isochoric process the entire heat supplied just increases the internal energy of the gas responsible for the increase in temperature of gas.



Isochoric Heating



Isochoric Cooling

Also, we must note that, in case of an isochoric process volume of the system remains constant

i.e., $V = \text{constant}$

$$\Rightarrow \quad \frac{P}{T} = \text{constant}$$

Since the boundary of the system does not displace because volume is constant, therefore, $W = 0$
The change in internal energy is given by

$$\Delta U = nC_V \Delta T = \frac{nR}{\gamma - 1}\Delta T$$

From First Law, we have $Q = W + \Delta U$

$$\Rightarrow \quad Q = \Delta U = \frac{nR}{\gamma - 1}\Delta T = \frac{P_f V_f - P_i V_i}{\gamma - 1}$$

ILLUSTRATION 83

Gaseous hydrogen initially at STP in a container of volume 5×10^{-5} m^3 is cooled by 55 K. Calculate the change in internal energy and amount of heat lost by the gas.

SOLUTION

Initially at STP, the gas pressure is $P = P_{atm} = 10^5$ Pa, gas temperature is $T = 273$ K and gas volume is $V = 5 \times 10^{-3}$ m^3. If n mole of gas is there, then

$$PV = nRT$$

$$\Rightarrow \quad n = \frac{PV}{RT} = \frac{10^5 \times 5 \times 10^{-3}}{8.314 \times 273} = 0.22 \text{ mole.}$$

Change in internal energy in the process is

$$\Delta U = \left(\frac{nf}{2}\right) R\Delta T$$

$$\Rightarrow \quad \Delta U = \frac{5}{2} \times 0.22 \times 8.314 \times (-55) = -251.5 \text{ J}$$

The negative sign indicates that there is a loss in internal energy.
Since gas is enclosed in a container, so its volume remains constant during the process and hence work done in this process is zero. Applying FLTD, we get

$$Q = \Delta U + W = -251.5 + 0 = -251.5 \text{ J}$$

The negative sign indicates that heat is lost by the system.

ILLUSTRATION 84

An ideal gas has specific heat at constant pressure to be $C_p = \frac{5}{2}R$. The gas is kept in a closed vessel of volume 0.0083 m^3 at a temperature of 300 K and a pressure of 1.6×10^6 Nm^{-2}. An amount of 2.49×10^4 J of heat energy is supplied to the gas. Calculate the final temperature and pressure of gas. Given $R = 8.3$ Jmol^{-1}K^{-1}.

SOLUTION

Here nothing is said about the number of moles of ideal gas. So, let the number of moles of gas be n. Then from ideal gas equation, we have

$$n = \frac{PV}{RT} = \frac{(1.6 \times 10^6)(0.0083)}{(8.3)(300)} = \frac{16}{3}$$

Since $C_P = \frac{5R}{2}$, so $C_V = \frac{3R}{2}$ $\quad \{\because C_P - C_V = R\}$

Since at constant volume, we have

$$Q = \Delta U = nC_V \Delta T$$

$$\Rightarrow \quad 2.49 \times 10^4 = \left(\frac{16}{3}\right)\left(\frac{3R}{2}\right)\Delta T$$

$$\Rightarrow \quad \Delta T = T_f - T_i = 375 \text{ K}$$

$$\Rightarrow \quad T_f = 300 + 375 = 675 \text{ K}$$

Further at constant volume $\frac{P_i}{T_i} = \frac{P_f}{T_f}$

$$\Rightarrow \quad P_f = \left(\frac{T_f}{T_i}\right)P_i = \left(\frac{675}{300}\right)(1.6 \times 10^6)$$

$$\Rightarrow \quad P_f = 3.6 \times 10^6 \text{ Nm}^{-2}$$

ILLUSTRATION 85

Consider two thermally insulated vessels, one with 0.025 mole of helium and other with n mole of hydrogen. Initially both the gases are at room temperature. Now equal amount of heat is supplied to both the vessels. It is found that in both the gases, temperature rises by same amount. Find the number of moles of hydrogen in second vessel

SOLUTION

Since the gases are enclosed in closed vessels, so the heating process can be taken as isochoric i.e. $W = 0$.
Also, heat supplied to both vessels is same, so applying FLTD, we get

$$Q = n_1 C_{V_1} \Delta T = n_2 C_{V_2} \Delta T$$

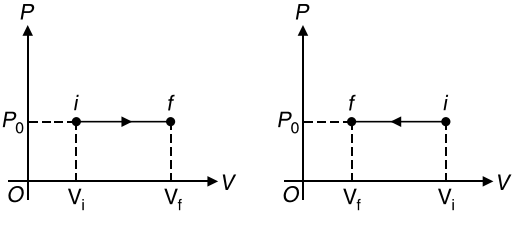
For He, $C_{V_1} = \frac{3}{2}R$ and for H_2, $C_{V_2} = \frac{5}{2}R$]

$$\Rightarrow \quad 0.025 \times \frac{3}{2} R\Delta T = n\frac{5}{2} R\Delta T$$

$$\Rightarrow \quad n = \frac{0.025 \times 3}{5} = 0.015 \text{ mole}$$

ISOBARIC PROCESS

In an isobaric process the pressure of the system remains constant i.e., P = constant.



The work done is given by

$$W = \int PdV = P_0 \int_{V_i}^{V_f} dV = P_0 \left(V_f - V_i\right)$$

Using gas equation $PV = nRT$

We get, $W = P_0 \left(V_f - V_i\right) = nR\left(T_f - T_i\right)$

$$\Rightarrow \quad W_{\text{isob}} = P\Delta V = nR\Delta T$$

Since the change in internal energy is independent of the path followed, therefore

$$\Delta U = nC_V \Delta T = \frac{nR}{\gamma - 1}\Delta T = \frac{nR}{\gamma - 1}\left(T_f - T_i\right)$$

Using First Law of Thermodynamics,

$$Q = W + \Delta U$$

$$Q = nR\left(T_f - T_i\right) + \frac{nR}{\gamma - 1}\left(T_f - T_i\right)$$

$$\Rightarrow \quad Q = \frac{n\gamma R}{\gamma - 1}\left(T_f - T_i\right) = nC_P \Delta T$$

So, for isobaric process, we have

$$Q = nC_p \Delta T = nC_p \left(T_f - T_i\right), \text{ where } C_p = \frac{\gamma R}{\gamma - 1}$$

Problem Solving Technique(s)

(a) $C_p - C_v = R$

(b) $\dfrac{C_p}{C_v} = \gamma$

(c) A process in which pressure is constant. e.g., boiling of water at atmospheric pressure. So,

$\Rightarrow W_{\text{isobarric}} = P\left(V_v - V_\ell\right) = P\Delta V$

where, V_v is volume of vapours, V_ℓ is volume of liquid. Also, the heat that must be transferred to the liquid to vapourise all of it is equal to mass (m) of liquid times the latent heat of vapourisation L_v of liquid, that is, $dQ = mL_v$, so, According to First Law of Thermodynamics $dU = mL_v - P\left(V_v - V_\ell\right)$

ILLUSTRATION 86

Find the ratio of $\dfrac{Q}{\Delta U}$ and $\dfrac{Q}{W}$ in an isobaric process. Given that the ratio of molar heat capacities $\dfrac{C_P}{C_V} = \gamma$.

SOLUTION

In an isobaric process P =constant. Therefore, $C = C_P$

$$\Rightarrow \quad \frac{Q}{\Delta U} = \frac{nC_P \Delta T}{nC_V \Delta T} = \frac{C_P}{C_V} = \gamma$$

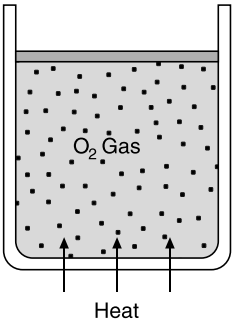
Now, $\dfrac{Q}{W} = \dfrac{Q}{\Delta Q - \Delta U}$ $\quad \{\because\ Q = \Delta U + W\}$

$$\Rightarrow \quad \frac{Q}{W} = \frac{nC_P \Delta T}{nC_P \Delta T - nC_V \Delta T} = \frac{C_P}{C_P - C_V}$$

$$\Rightarrow \quad \frac{Q}{W} = \frac{\dfrac{C_P}{C_V}}{\dfrac{C_P}{C_V} - 1} = \frac{\gamma}{\gamma - 1}$$

ILLUSTRATION 87

A cylindrical container contains oxygen gas and is closed by a movable frictionless piston of mass 50 kg having cross-sectional area 100 cm^2 as shown in Figure. Some heat is supplied to the cylinder at atmospheric pressure of 10^5 Pa so that the piston is slowly displaced up by 20 cm. Calculate the amount of heat supplied to the gas.



SOLUTION

Since the piston is open to atmosphere, so the gas is under a constant pressure given by

$$P_{\text{gas}} = P_{\text{atm}} + \frac{Mg}{A} = 10^5 + \frac{50 \times 10}{100 \times 10^{-4}} \text{ Pa}$$

It is given that the piston moves out by 20 cm, so the work done in the process is

$$W = P_{\text{gas}} \Delta V, \text{ where}$$

$$\Delta V = A\Delta x = \left(100 \times 10^{-4}\right)\left(20 \times 10^{-2}\right) = 2 \times 10^{-3} \text{ m}^3$$

$$\Rightarrow \quad W = 1.5 \times 10^5 \times 2 \times 10^{-3} = 300 \text{ J}$$

For a process is which gas pressure is constant, work done can also be given by

$$W = P_{\text{gas}} \Delta V = nR\Delta T$$

$$\Rightarrow \quad nR\Delta T = 300 \text{ J}$$

So $Q = nC_P \Delta T$, where $C_P = \dfrac{7R}{2}$

$$\Rightarrow \quad Q = n\left(\frac{7}{2}R\right)\Delta T = \frac{7}{2}\left(nR\Delta T\right)$$

$$\Rightarrow \quad Q = \frac{7}{2} \times 300 = 1050 \text{ J}$$

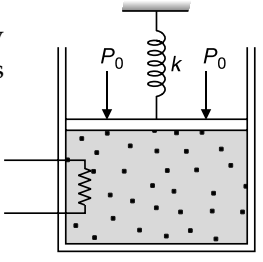
ILLUSTRATION 88

Two moles of an ideal monatomic gas are confined within a cylinder by a massless and frictionless spring loaded piston of cross-sectional area $4 \times 10^{-3}\ \text{m}^2$. Initially the spring is in its relaxed state. Now the gas is heated by an electric heater, placed inside the cylinder, for some time. During

this time, the gas expands and does 50 J of work in moving the piston through a distance 0.10 m. The temperature of the gas increases by 50 K. Calculate the spring constant and the heat supplied by the heater. $\left(P_0 = 1.0\times10^5\ \text{Nm}^{-2}\right)$

SOLUTION

When the piston has displaced by x, the pressure inside the cylinder is given by



$$P = P_0 + \frac{kx}{A}$$

$$\Rightarrow \quad W = \int PdV = \int\left(P_0 + \frac{kx}{A}\right)A\,dx$$

$$\Rightarrow \quad W = \left(P_0Ax + \frac{1}{2}kx^2\right)$$

$$\Rightarrow \quad 50 = 10^5\times4\times10^{-3}\times0.1 + \frac{1}{2}k\times(0.1)^2$$

$$\Rightarrow \quad k = 2000\ \text{Nm}^{-1}$$

Also, change in internal energy is given by

$$\Delta U = nC_v\Delta T = 2\times\frac{3R}{2}\times50 = 150\times8.3 = 1245\ \text{J}$$

From First Law of Thermodynamics, we have

$$Q = \Delta U + W$$

$$Q = 1245 + 50 = 1295\ \text{J}$$

ILLUSTRATION 89

A cylinder with a piston contains 0.2 kg of water at 100 °C . What is the change in internal energy of the water when it is converted to steam at 100 °C at a constant pressure of 1 atm? The density of water is $\rho_0 = 10^3\ \text{kgm}^{-3}$ and that of steam is $\rho_s = 0.6\ \text{kgm}^{-3}$. The latent heat of vaporization of water is $L_y = 2.26\times10^6\ \text{Jkg}^{-1}$.

SOLUTION

The heat transfer to the water is

$$Q = mL_V = (0.2\ \text{kg})\left(2.26\times10^6\ \text{Jkg}^{-1}\right)$$

$$\Rightarrow \quad Q = 4.52\times10^5\ \text{J}$$

The work done by the water when it expands against the piston at constant pressure is

$$W = P(V_s - V_w) = P\left(\frac{m}{\rho_s} - \frac{m}{\rho_w}\right)$$

$$\Rightarrow \quad W = \left(1.01\times10^5\ \text{Nm}^2\right)\left(\frac{0.2\ \text{kg}}{0.6\ \text{kgm}^{-3}} - \frac{0.2\ \text{kg}}{1000\ \text{kgm}^{-3}}\right)$$

$$\Rightarrow \quad W = 3.36\times10^4\ \text{J}$$

The change in internal energy is

$$\Delta U = Q - W = 452\ \text{kJ} - 33.6\ \text{kJ} = 418.4\ \text{kJ}$$

ILLUSTRATION 90

(a) One mole of oxygen is heated from 0 °C at constant pressure, till its volume becomes 10% more than the initial volume. Find the heat required. The specific heat of O_2 under these conditions is $0.22\ \text{calg}^{-1}\text{K}^{-1}$.

(b) If the same amount of heat is supplied to the gas at constant volume, calculate the final temperature.

SOLUTION

(a) Since, the specific heat of oxygen is given to be $0.22\ \text{calg}^{-1}\text{K}^{-1}$, so molar specific heat of oxygen is

$$C_P = (32)(0.22)\ \text{calmol}^{-1}\text{K}^{-1}$$

At constant pressure, we have $\frac{V}{T}$ =constant

So, 10% increase in volume increases the temperature by 10% i.e., $T_i = 273\ \text{K}$ and $T_f = 300.3\ \text{K}$

$$\Rightarrow \quad \Delta T = 27.3\ \text{K}$$

Since, $Q = nC_P\Delta T$

$$\Rightarrow \quad Q = (1)(32)(0.22)(27.3) = 192\ \text{cal}$$

(b) Since, again $Q = 192$ cal

$$\Rightarrow \quad Q = nC_V\Delta T, \text{ where } C_V = C_P - R$$

Since, $C_P = (32)(0.22)\ \text{calmol}^{-1}\text{K}^{-1}$

$$\Rightarrow \quad C_P = 7\ \text{calmol}^{-1}\text{K}^{-1}$$

$$\Rightarrow \quad C_V = C_P - R = 5\ \text{calmol}^{-1}\text{K}^{-1}$$

$$\left\{\because\ R = 2\ \text{calmol}^{-1}\text{K}^{-1}\right\}$$

$$\Rightarrow \quad 192 = (1)(5)\Delta T$$

$$\Rightarrow \quad \Delta T = T_f - T_i = 38.4\ \text{K}$$

$$\Rightarrow \quad T_f = 311.4\ \text{K}$$

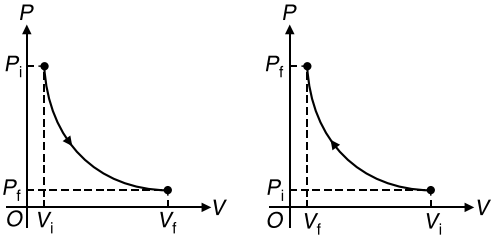
ISOTHERMAL PROCESS

To a novice we can explain that any process taking place gradually can be taken to be an example of isothermal process e.g., melting process.

Mathematically, a process in which T is constant or $dT = 0$ is an isothermal process i.e., for such a process $dU = 0$ and hence an isothermal process is not accompanied by any change in internal energy. So, the First Law of Thermodynamics takes the form $\bar{d}Q = \bar{d}W$ i.e., the entire heat supplied just does an external work.

The equation of state governing isothermal process is

$$PV = \text{constant}.$$



For an isothermal process, PV = constant, so

$$d(PV) = 0$$

$$\Rightarrow \quad PdV + VdP = 0$$

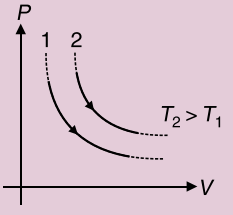
$$\Rightarrow \quad \frac{dP}{dV} = -\frac{P}{V}$$

$$\Rightarrow \quad (\text{Slope})_{\text{isot}} = \frac{dP}{dV} = -\frac{P}{V}$$

Problem Solving Technique(s)

(a) Two Isotherms never intersect each other.

(b) Isotherm far from the PV axis has more temperature compared to the isotherm close to the PV axis.



WORK DONE IN AN ISOTHERMAL PROCESS

Since $W = \int dW = \int_i^f PdV$

$$\Rightarrow \quad W_{\text{isot}} = nRT\int_{V_1}^{V_2}\frac{dV}{V} \qquad \{\because PV = nRT\}$$

$$\Rightarrow \quad W_{\text{isot}} = nRT\log_e\left(\frac{V_2}{V_1}\right) = 2.303\ nRT\log_{10}\left(\frac{V_2}{V_1}\right)$$

where, $\frac{V_2}{V_1}$ is called the **Expansion Ratio**.

Since $P_1V_1 = P_2V_2$

$$\Rightarrow \quad W_{\text{isot}} = nRT\log_e\left(\frac{P_1}{P_2}\right) = 2.303\ nRT\log_{10}\left(\frac{P_1}{P_2}\right)$$

Since temperature of the system remains constant, therefore, there is no change in internal energy.

$$\Delta U = nC_v\Delta T = 0$$

Using First Law of Thermodynamics,

$$Q = W + \Delta U$$

$$\Rightarrow \quad Q = W = nRT\ln\left|\frac{V_f}{V_i}\right|$$

ILLUSTRATION 91

Three moles of an ideal gas at 300 K are isothermally expanded to five times its volume and heated at this constant volume so that the pressure is raised to its initial value before expansion. In the whole process 83.14 kJ heat is required. Calculate the ratio $\left(\frac{C_P}{C_V}\right)$ of the gas. ($\log_e 5 = 1.61$ and R = 8.31 $\text{Jmol}^{-1}\ \text{K}^{-1}$)

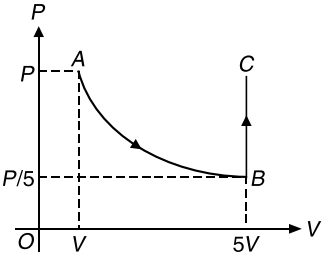
SOLUTION

According to First Law of Thermodynamics, $Q = \Delta U + W$

For an isothermal change,

$$T = \text{constant},\ U = \text{constant},\ \Delta U = 0$$

and $W = nRT\log_e\left|\frac{V_f}{V_i}\right|$



$$\Rightarrow \quad W = 3\times 8.31\times 300\times\log_e(5) = 12.03\ \text{kJ}$$

$$\Rightarrow \quad Q_{\text{isothermal}} = 0 + 12.03 = 12.03\ \text{kJ} \qquad \ldots(1)$$

For isochoric change, V = constant

$$\Rightarrow \quad W = \int PdV = 0$$

So, $\Delta U = nC_V\Delta T = 3C_V\Delta T \qquad \{\because\ n = 3\}$

Applying gas equation between points A and C

$$\frac{PV}{300} = \frac{P(5V)}{T_C}$$

$$\Rightarrow \quad T_C = 1500\ \text{K}$$

So that $\Delta T = T_C - T_B = 1500 - 300 = 1200$

$$\Rightarrow \quad \Delta U = 3C_V\times 1200 = 3.6\ C_V\ \text{kJ}$$

Note: To find T_C you can apply gas equation between points B and C also and hence,

$$Q_{\text{isochoric}} = 3.6\ C_V + 0 = 3.6\ C_V\ \text{kJ} \qquad \ldots(2)$$

According to given problem,

$$Q_{\text{isothermal}} + Q_{\text{isochoric}} = 83.14\ \text{kJ}$$

Using equation (1) and (2), we get

$$12.03 + 3.6\ C_V = 83.14$$

$$\Rightarrow \quad C_V = \left(\frac{71.11}{3.6}\right) = 19.75\ \text{J}$$

Thus $C_P = C_V + R = 19.75 + 8.3 = 28.05\ \text{Jmol}^{-1}\ \text{K}^{-1}$

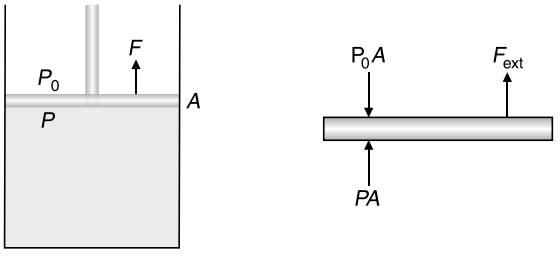
$$\Rightarrow \quad \gamma = \frac{C_P}{C_V} = \frac{28.05}{19.75} = 1.42$$

ILLUSTRATION 92

One mole of a gas is put under a weightless piston of a vertical cylinder at temperature T. The space over the piston opens into atmosphere. How much work should be performed by some external force to increase isothermally the volume under the piston to twice the volume (neglect friction of piston)?

SOLUTION

Drawing the Free Body Diagram for the piston, we get



Now according to Newton's Second Law, we have for piston $F_{net} = m_{piston}a$

where a is acceleration of the piston. Since, $m_{piston} \to 0$

$$\Rightarrow \quad F_{net} = F_{ext} + PA - P_0A = 0$$

So, work done by external force is

$$W_{ext} = \int dW = \int F_{ext}dx = \int (P_0 - P)Adx$$

$$\Rightarrow \quad W_{ext} = \int (P_0 - P)dV = P_0\int_V^{2V} dV - \int_V^{2V} PdV$$

For an ideal gas $PV = RT$, so

$$W_{ext} = P_0V - RT\int_V^{2V}\frac{dV}{V} = P_0V - RT\left(\log_e V\Big|_V^{2V}\right)$$

$$\Rightarrow \quad W_{ext} = P_0V - RT\log_e(2) \qquad \ldots(1)$$

Since temperature is constant, T, so we have

$PV = RT$ (finally)

$P_0V = RT$ (initially)

So, from (1) we get $W_{ext} = RT - RT\log_e(2)$

$$\Rightarrow \quad W_{ext} = RT\left(1 - \log_e(2)\right)$$

ILLUSTRATION 93

A piston can freely move inside a horizontal cylinder closed from both ends. Initially, the piston separates the inside space of the cylinder into two equal parts each of volume V_0, in which an ideal gas is contained under the same pressure P_0 and at the same temperature. What work has to be performed in order to increase isothermally the volume of one part of gas is η times compared to that of the other by slowly moving the piston?

SOLUTION

As the piston is displaced externally some external work is done in the process. If piston is displaced towards right, the gas on left side expands and does some work. Similarly, gas on right is compressed and work is done on it. Work done by external agent is

$$W_{ext} = W_{\text{by gas in left part}} + W_{\text{on gas in right part}} \qquad \ldots(1)$$

It is given that initial volume of both the parts is V_0 and in the process final volume of one part is η times that of the other part. If the final volume of right part is V then that of left part will become ηV. Since total volume of container is $2V_0$, so we have $V + \eta V = 2V_0$

$$\Rightarrow \quad V = \frac{2V_0}{\eta + 1}$$

For gas in left part, work done by gas during isothermal expansion is

$$W_{\text{by gas}} = nRT\ln\frac{V_2}{V_1}$$

$$\Rightarrow \quad W_{\text{by gas}} = P_0V_0\ln\left(\frac{\eta V}{V_0}\right) = P_0V_0\ln\left(\frac{2\eta}{\eta+1}\right) \qquad \ldots(2)$$

Similarly, for gas in right part, work done on the gas is isothermal compression is

$$W_{\text{on gas}} = nRT\ln\frac{V_2}{V_1} = P_0V_0\ln\frac{V}{V_0}$$

$$\Rightarrow \quad W_{\text{on gas}} = P_0V_0\ln\left(\frac{2}{\eta+1}\right) \qquad \ldots(3)$$

Now from equations (1), (2) and (3), we get

$$W_{ext} = W_{\text{by gas in left part}} + W_{\text{on gas in right part}}$$

$$\Rightarrow \quad W_{ext} = P_0V_0\ln\left(\frac{2\eta}{\eta+1}\right) + P_0V_0\ln\left(\frac{2}{\eta+1}\right)$$

$$\Rightarrow \quad W_{ext} = P_0V_0\ln\left[\frac{4\eta}{(\eta+1)^2}\right]$$

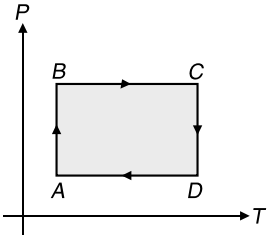
Test Your Concepts-VI

Based on Isochoric, Isobaric and Isothermal Processes

(Solutions on page H.86)

1. What is the heat input needed to raise the temperature of two moles of helium gas from 0 °C to 100 °C.
(a) at constant volume
(b) at constant pressure
(c) what is the work done by the gas in part
Give your answer in terms of R.

2. Plot P-V, V-T and ρ-T graph corresponding to the P-T graph for an ideal gas shown in figure.



3. One mole of an ideal gas is heated from 0 °C to 100 °C at a constant pressure of 1 atm. Calculate the work done in the process. Take $1\,\text{atm} = 10^5\ \text{Nm}^{-2}$.

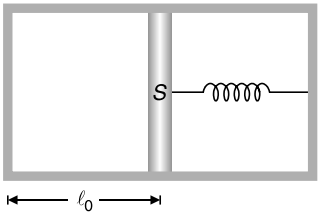
4. Two moles of a certain gas at a temperature $T_0 = 300$ K were cooled isochorically so that the pressure of the gas becomes half the initial pressure. Then as a result of isobaric process, the gas is allowed to expand till its temperature goes back to the initial value. Find the total amount of heat absorbed by gas in this process. Take $R = 8.3\ \text{Jmol}^{-1}\text{K}^{-1}$.

5. One mole of a certain ideal gas is given 500 cal heat. As a result the temperature rises by 72 K at constant pressure. Find the work performed by the gas, the change of internal energy and the value of γ.

6. An ideal di-atomic gas is heated at constant pressure such that it performs a work $W = 2.0$ J. Find the amount of heat supplied.

7. Three moles of an ideal gas $\left(C_P = \frac{7}{2}R\right)$ at pressure P_0 and temperature T_0 is isothermally expanded to twice its initial volume, it is then compressed at a constant pressure to its original volume.
(a) Sketch P-V and P-T diagram for complete process.
(b) Calculate net work done by the gas.
(c) Calculate net heat supplied to the gas during complete process.
Give all answers in terms of gas constant R.

8. A piston S of mass M, area of cross section A can slide inside a cylinder without friction. The walls of the cylinder are adiabatic and the piston is diathermic. Length of the cylinder is $2\ell_0$. Initial pressure of each chamber is P_0 and volume of both the chambers are equal. Initially spring is unstretched and has spring constant k. The spring is fixed with piston and wall of the cylinder as shown in the figure.



Find out its time period when piston is given small displacement.

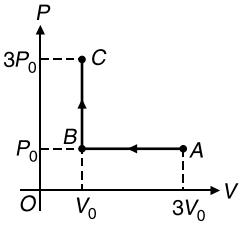
9. A vertical hollow cylinder contains an ideal gas. The gas is enclosed in the cylinder by a 5 kg movable piston having a cross-sectional area of $5\times10^{-3}\ \text{m}^2$. The gas is now heated from 300 K to 350 K and the piston rises by 0.1 m. Now, the piston clamped in this position and gas is cooled back to 300 K. Calculate the difference between the heat energy added during heating and the heat energy lost during cooling. Take $1\,\text{atm} = 10^5\ \text{Nm}^{-2}$ and $g = 10\ \text{ms}^{-2}$.

10. A vessel of volume V is evacuated by means of a piston air pump. One piston stroke removes the volume ΔV of air. How many strokes are needed to reduce the pressure in the vessel by x times the original pressure. The process is assumed to be isothermal and the gas is ideal.

11. Four moles of a monatomic ideal gas are at pressure $3\times10^5\ \text{Nm}^{-2}$ and temperature 100 K (state A). It is heated isobarically to temperature 400 K (state B). Next it undergoes isothermal expansion to pressure $1\times10^5\ \text{Nm}^{-2}$ (state C). It is then cooled isobarically to 100 K (state D). Finally, it is compressed isothermally to return to initial state A. Draw P-T, P-V and V-T diagrams for the whole process.

12. In a gaseous system, a gas expands from $10^{-4}\ \text{m}^3$ to $2\times10^{-4}\ \text{m}^3$ while its pressure remains constant at $10^5\ \text{Ntm}^{-2}$. Calculate the amount of heat absorbed by the gas in the expansion. $[\gamma = 1.67]$

13. P-V diagram of an ideal gas for a process ABC is as shown in the figure.



(a) Find total heat absorbed or released by the gas during the process ABC.

(b) Change in internal energy of the gas during the process *ABC*.
(c) Plot pressure versus density graph of the gas for the process *ABC*.

14. One mole of an ideal gas is contained under a weightless piston of a vertical cylinder at a temperature *T*. The space over the piston opens into the atmosphere. What work has to be performed to increase isothermally the gas volume under the piston *n* times by slowly raising the piston? Friction is negligible.

15. If 70 calorie of heat is required to raise the temperature of 2 mole of an ideal gas at constant pressure from 30 to 35 °C. If $R = 2$ calmol^{-1}K^{-1}, calculate
(a) the work done by the gas
(b) increase in internal energy of the gas

ADIABATIC PROCESS

Any process carried out suddenly is an example of an Adiabatic process e.g., bursting of a cycle tube.
Mathematically a process in which the system and surroundings do not exchange any heat with each other i.e. $dQ = 0$ and in such a process all P, V and T must change simultaneously such that $C = 0$.

EQUATION OF STATE FOR AN ADIABATIC PROCESS

According to First Law of Thermodynamics

$$dQ = dU + dW$$

$$\Rightarrow \quad 0 = dU + dW = nC_V dT + PdV \qquad \ldots(1)$$

For an Ideal Gas, we have $PV = nRT$

$$\Rightarrow \quad d(PV) = nRdT$$

$$\Rightarrow \quad PdV + VdP = nRdT$$

$$\Rightarrow \quad dT = \frac{PdV + VdP}{nR} \qquad \ldots(2)$$

Using (2) in (1), we get

$$\Rightarrow \quad \left(\frac{C_V}{R} + 1\right)PdV + \frac{C_V VdP}{R} = 0$$

$$\Rightarrow \quad (C_V + R)PdV + C_V VdP = 0$$

$$\Rightarrow \quad C_P PdV + C_V VdP = 0$$

$$\Rightarrow \quad \gamma PdV + VdP = 0 \qquad \left\{\because \frac{C_P}{C_V} = \gamma\right\}$$

$$\Rightarrow \quad \gamma \int_{V_1}^{V_2} \frac{dV}{V} = -\int_{P_1}^{P_2} \frac{dP}{P}$$

$$\Rightarrow \quad \gamma \log_e\left(\frac{V_2}{V_1}\right) = -\log_e\left(\frac{P_2}{P_1}\right)$$

$$\Rightarrow \quad \log_e\left\{\left(\frac{V_2}{V_1}\right)^{\gamma}\right\} = \log_e\left(\frac{P_1}{P_2}\right)$$

$$\Rightarrow \quad P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

$$\Rightarrow \quad PV^{\gamma} = \text{constant}$$

For an Adiabatic process the equation of state is/are

$$\left.\begin{aligned} PV^{\gamma} &= \text{constant} \\ TV^{\gamma-1} &= \text{constant} \\ T^{\gamma}P^{1-\gamma} &= \text{constant} \end{aligned}\right\} \text{ where } \gamma = \frac{C_P}{C_V}$$

ILLUSTRATION 94

An ideal monatomic gas at 300 K expands adiabatically to twice its volume. What is the final temperature?

SOLUTION

For an ideal monatomic gas, we have $\gamma = \frac{5}{3}$

For an adiabatic process, $TV^{\gamma-1} = \text{constant}$

$$\Rightarrow \quad T_f V_f^{\gamma-1} = T_i V_i^{\gamma-1}$$

$$\Rightarrow \quad T_f = T_i\left(\frac{V_i}{V_f}\right)^{\gamma-1} = (300)\left(\frac{1}{2}\right)^{\frac{5}{3}-1}$$

$$\Rightarrow \quad T_f = 189 \text{ K}$$

WORK DONE IN AN ADIABATIC PROCESS

Since, $dQ = 0$

$$\Rightarrow \quad dW = -dU = -nC_V dT$$

$$\Rightarrow \quad W_{\text{adiabatic}} = -nC_V \int_{T_1}^{T_2} dT = -nC_V(T_2 - T_1)$$

Further, $C_V = \frac{R}{\gamma - 1}$ $\qquad \left\{\because C_P - C_V = R \text{ and } \gamma = \frac{C_P}{C_V}\right\}$

$$\Rightarrow \quad W_{\text{adiabatic}} = \frac{nR}{1-\gamma}(T_2 - T_1)$$

$$\Rightarrow \quad W_{\text{adiabatic}} = \frac{1}{1-\gamma}(nRT_2 - nRT_1)$$

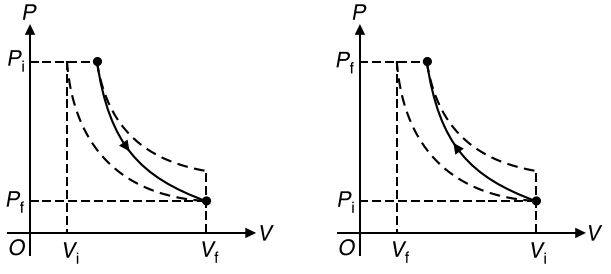
Since $P_2V_2 = nRT_2$ and $P_1V_1 = nRT_1$

$$\Rightarrow \quad W_{\text{adiabatic}} = \frac{1}{1-\gamma}(P_2V_2 - P_1V_1)$$

$$\Rightarrow \quad W = -\Delta U$$

Work done by the system is equal to the decrease in internal energy.
Work done on the system is equal to the increase in internal energy.

INDICATOR DIAGRAM FOR AN ADIABATIC PROCESS



For an adiabatic process, we have $PV^{\gamma} = \text{constant}$

$$\Rightarrow \quad d\left(PV^{\gamma}\right) = 0$$

$$\Rightarrow \quad P\left(\gamma V^{\gamma-1} dV\right) + V^{\gamma} dP = 0$$

$$\Rightarrow \quad \gamma P\left(\frac{V^{\gamma}}{V}\right) dV + V^{\gamma} dP = 0$$

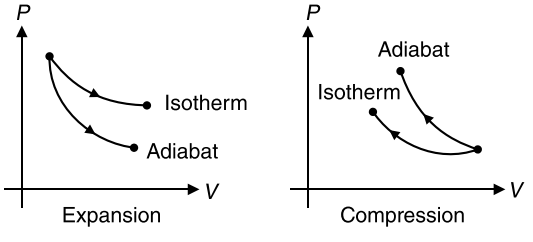
$$\Rightarrow \quad \frac{dP}{dV} = -\gamma\left(\frac{P}{V}\right)$$

Since, $\left(\text{Slope}\right)_{\text{isot}} = -\dfrac{P}{V}$

$$\Rightarrow \quad \left(\text{Slope}\right)_{\text{adia}} = \gamma\left(\text{Slope}\right)_{\text{isot}}$$

Since $\gamma > 1$, so $\left(\text{Slope}\right)_{\text{adia}} > \left(\text{Slope}\right)_{\text{isot}}$

Due to this, during expansion an isotherm lies above the adiabat and during compression an adiabat lies above the isotherm.



Also, we note that the more the value of γ, the more steep will be the curve for adiabatic process as shown in Figure.

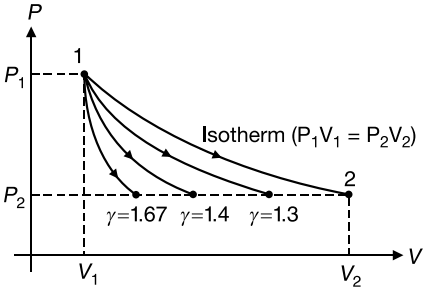


ILLUSTRATION 95

A sample of diatomic gas with $\gamma = 1.5$ is compressed from a volume of 1600 cc to 400 cc adiabatically. The initial pressure of gas was 1.5×10^5 Pa. Find the final pressure and work done by the gas in the process.

SOLUTION

We know in an adiabatic process, pressure and volume of gas of its different sates are related as

$$P_1V_1^{\gamma} = P_2V_2^{\gamma}$$

$$\Rightarrow \quad P_2 = \left(\frac{V_1}{V_2}\right)^{\gamma} P_1$$

$$\Rightarrow \quad P_2 = \left(\frac{1600}{400}\right)^{1.5} \times 1.5\times10^5$$

$$\Rightarrow \quad P_2 = (4)^{1.5} \times 1.5\times10^5 = 1.2\times10^6 \text{ Pa}$$

For an adiabatic process work done by a gas is

$$W = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$$

$$\Rightarrow \quad W = \frac{1.5\times10^5 \times 1600\times10^{-6} - 1.2\times10^6 \times 400\times10^{-6}}{1.5-1}$$

$$\Rightarrow \quad W = \frac{240-480}{0.5} = -480 \text{ J}$$

Here work done by gas comes out a negative value thus we can state that as gas is being compressed, work is done on the gas and so work done by gas is –480 J.

ILLUSTRATION 96

Two identical gases whose adiabatic exponent is γ are filled in two identical containers at equal pressure. In both the containers the volume of gas is doubled. In first container it is done by an isothermal process and in second container it is done by an adiabatic process. Find the condition for which the work done by the gas in the two expansion process is same.

SOLUTION

In first container, work done is

$$W_1 = W_{\text{isot}} = nRT_1 \ln(2) \qquad \ldots(1)$$

In second container, work done is

$$W_2 = W_{\text{ad}} = \frac{nR\left(T_2 - T_1\right)}{1-\gamma} \qquad \ldots(2)$$

where, $T_2 = T_1(2)^{1-\gamma}$ $\qquad \left\{\because T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}\right\}$

According to the problem ,we have $W_1 = W_2$

$$\Rightarrow \quad nRT_1 \ln(2) = \frac{nRT_1\left(2^{1-\gamma} - 1\right)}{\gamma - 1}$$

$$\Rightarrow \quad (\gamma - 1)\ln(2) = 1 - 2^{1-\gamma}$$

ILLUSTRATION 97

A piston divides a closed gas cylinder into two parts. Initially the piston is kept pressed such that one part has a pressure P_0 and volume $5V_0$ and the other part has pressure $8P_0$ and volume V_0; the piston is now left free. Find the new pressure and volume for the isothermal and adiabatic process $(\gamma = 1.5)$.

SOLUTION

Final pressure will be same on both sides. Let it be P, with volume V, on the left side and $(6V_0 - V)$ on the right side.

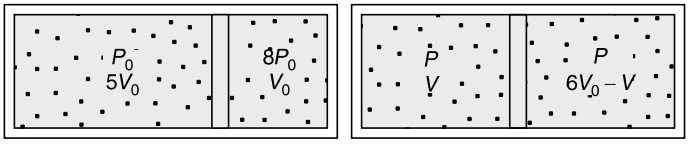
CASE-1: Isothermal Process

For the gas enclosed in the left chamber,

$$P_0 \times 5V_0 = PV \quad \ldots(1)$$

while for the gas in the right chamber,

$$8P_0 \times V_0 = P(6V_0 - V) \quad \ldots(2)$$



After solving, we get

$$V = \frac{30}{13}V_0$$

$$\Rightarrow \quad P = \frac{13}{6}P_0$$

$$\Rightarrow \quad (6V_0 - V) = \frac{48}{13}V_0$$

CASE-2: Adiabatic Process

$$P_0(5V_0)^\gamma = P(V)^\gamma \quad \ldots(3)$$

and for the gas in the right chamber,

$$8P_0(V_0)^\gamma = P(6V_0 - V)^\gamma \quad \ldots(4)$$

Dividing (4) by (3), we get

$$\left(\frac{6V_0 - V}{V}\right)^\gamma = \frac{8}{5^\gamma}$$

$$\Rightarrow \quad \frac{6V_0}{V} = 1 + \frac{4}{5} \text{ i.e., } V = \frac{10}{3}V_0$$

Substituting it in equation (3),

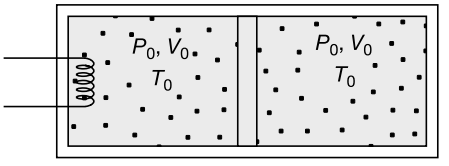
$$P = P_0\left(\frac{5V_0 \times 3}{10V_0}\right)^{\frac{3}{2}} = \frac{3\sqrt{3}}{2\sqrt{2}}P_0 = 1.84P_0$$

$$\Rightarrow \quad P = 1.84P_0;\ V = \left(\frac{10}{3}\right)V_0$$

$$\Rightarrow \quad (6V_0 - V) = \left(\frac{8}{3}\right)V_0$$

ILLUSTRATION 98

A rectangular box as shown in the figure has a partition which can slide without friction along the length of the box. Initially each of the two chambers of the box has one mole of monatomic ideal gas $\left(\gamma = \frac{5}{3}\right)$ at a pressure P_0, volume V_0 and temperature T_0. The chamber on the left is slowly heated by an electric heater.



The walls of the box and the partition are thermally insulated. The gas in the left chamber expands, pushing the partition until the final pressure in both chambers becomes $\left(\frac{243}{32}\right)P_0$. Calculate

(a) the final temperature of the gas in each chamber
(b) the work done by the gas in the right chamber

SOLUTION

(a) As no heat is given to the right chamber and it is thermally insulated, so the change in the right chamber is adiabatic. And if V_R is the final volume of the gas in the right chamber,

$$P_0V_0^{5/3} = \left(\frac{243}{32}\right)P_0V_R^{5/3}$$

$$\Rightarrow \quad V_R = \frac{8}{27}V_0$$

Now applying the gas equation to the gas enclosed in the right chamber
(before and after compression),

$$\frac{P_0V_0}{T_0} = \left(\frac{243}{32}\right)P_0 \times \left(\frac{8}{27}\right)V_0 \times \frac{1}{T_R}$$

$$\Rightarrow \quad T_R = \frac{9}{4}T_0$$

Now for the gas enclosed in the left side, the final volume

$$V_L = 2V_0 - V_R = 2V_0 - \frac{8}{27}V = \frac{46}{27}V_0$$

And from the gas equation

$$\left(\frac{P_0V_0}{T_0}\right) = \left(\frac{243}{32}\right)P_0 \times \left(\frac{46}{27}\right)V_0 \times \frac{1}{T_L}$$

$$\Rightarrow T_L = \frac{207}{16}T_0 = 13T_0$$

(b) Work done by the gas in the right chamber is under adiabatic condition.

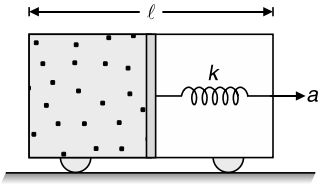
$$W_A = \frac{nR(T_F - T_I)}{1-\gamma} = \frac{1(8.3)(T_0 - 9T_0/4)}{(5/3)-1}$$

$$\Rightarrow \quad W_A = -\frac{3}{2}\times 8.3 \times \frac{5}{4}T_0 = -15.5T_0 \text{ J}$$

Negative sign means that work is done on the gas.

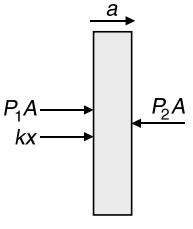
ILLUSTRATION 99

An adiabatic piston of mass m equally divides an insulator container of volume V_0 and length l. A light spring connects the piston to the right wall. Mass of the piston is m. In equilibrium pressure on both sides of the piston is P_0. The container starts moving with acceleration a towards the right. Find the stretch x of the spring when acceleration of the piston equals acceleration of container. (Assume that $x \ll l$). The gas in the container has the adiabatic exponent (ratio of C_P and C_V) γ.



SOLUTION

Free body diagram of piston is shown in figure below.



From Newton's Second Law, we get

$$P_1A + kx - P_2A = ma \qquad \ldots(1)$$

where A is the area of cross-section of the piston and x is the stretch of the spring.
Under adiabatic conditions, we have

$$P_0\left(\frac{V_0}{2}\right)^\gamma = P_1\left(\frac{V_0}{2} - Ax\right)^\gamma \qquad \ldots(2)$$

and $$P_0\left(\frac{V_0}{2}\right)^\gamma = P_2\left(\frac{V_0}{2} + Ax\right)^\gamma \qquad \ldots(3)$$

From Equation (2), we get

$$P_1 = P_0\left(\frac{V_0}{V_0 - 2Ax}\right)^\gamma = P_0\left(1 - \frac{2x}{l}\right)^{-\gamma} \qquad \{\because V_0 = Al\}$$

Since, $x \ll l$, so we get

$$\left(1 - \frac{2x}{l}\right)^{-\gamma} \cong 1 + \frac{2\gamma x}{l}$$

$$\Rightarrow \quad P_1 = P_0\left(1 + \frac{2\gamma x}{l}\right)$$

Similarly, $P_2 = P_0\left(1 - \frac{2\gamma x}{l}\right)$

Substituting the values of P_1 and P_2 in Equation (1), we get

$$P_0A\left(1 + \frac{2\gamma x}{l}\right) + kx - P_0A\left(1 - \frac{2\gamma x}{l}\right) = ma \qquad \ldots(4)$$

Since, $V_0 = Al$, so $A = \frac{V_0}{l}$

Substituting $A = \frac{V_0}{l}$ in Equation (4), we get

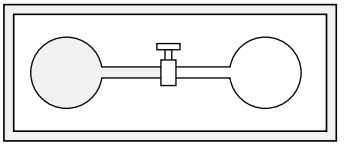
$$\frac{P_0V_0}{l}\left(1 + \frac{2\gamma x}{l}\right) + kx - \frac{P_0V_0}{l}\left(1 - \frac{2\gamma x}{l}\right) = ma$$

$$\left(\frac{4\gamma P_0V_0}{l^2}\right)x + kx = ma$$

$$\Rightarrow \quad x = \frac{ma}{k + \frac{4\gamma P_0V_0}{l^2}}$$

ADIABATIC FREE EXPANSION

We now consider what happens when a gas is allowed to expand adiabatically without doing any work. Figure shows two vessels connected by a tube with a stopcock. Initially, one vessel is filled with gas while the other is evacuated. The system is thermally insulated, that is, $Q = 0$.



When the stopcock is opened the gas quickly expands to fill the second chamber. The uncontrolled expansion is not quasistatic and cannot be depicted on a PV diagram. Since the gas does no work, $W = 0$. From the First Law we conclude that

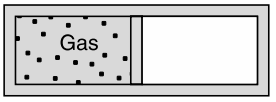
$$\Delta U = 0$$

In an adiabatic free expansion the internal energy of any gas (ideal or real) does not change.

ILLUSTRATION 100

A cylindrical container of volume V whose walls are adiabatic is taken. Initially a light adiabatic piston divides the container in two equal parts as shown in Figure. In the left part, n moles of an ideal gas with adiabatic exponent γ is filled at temperature T_0 and in the right part there is vacuum. If the piston is released, the gas fills the whole container. Calculate the final pressured and temperature of gas. Now if the piston is slowly displaced externally

back to its initial position, then calculate the final pressure and temperature of gas.



SOLUTION

When the piston is released the gas expands to fill the complete volume of container. Since there is nothing in the other part of cylinder, so this is the case of free expansion of gas. Hence no work is done by the gas.

Also, the container is thermally insulated from surroundings, so the gas temperature remains constant and hence according to Boyle's Law, as the volume of gas is doubled, then its final pressure is reduced to half. So, we have

$$P_f = \frac{P_i}{2}$$

Initial pressure P_i of gas is given by applying gas law, so we get $P_i = \frac{nRT_0}{V/2}$ and $P_f = \frac{P_i}{2} = \frac{nRT_0}{V}$.

Now if piston is displaced back to its initial position, then the process is an adiabatic compression of gas in which gas volume decreases to half i.e., $\frac{V}{2}$.

For an adiabatic process, we have

$$P_1V_1^{\gamma} = P_2V_2^{\gamma}$$

where $P_1 = \frac{nRT_0}{V}$, $V_1 = V$ and $V_2 = \frac{V}{2}$

$$\Rightarrow \quad P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma} = P_1(2)^{\gamma} = (2)^{\gamma}\left(\frac{nRT_0}{V}\right)$$

Similarly, for an adiabatic process, gas volume and temperature in different states are related as

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$\Rightarrow \quad T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

where $T_1 = T$, $V_1 = V$ and $V_2 = \frac{V}{2}$

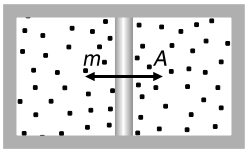
$$\Rightarrow \quad T_2 = T_0(2)^{\gamma-1}$$

Test Your Concepts-VII

Based on Adiabatic Process

(Solutions on page H.89)

1. In a cylinder filled with an ideal gas and closed from both ends there is a piston of mass m and cross-sectional area A. In equilibrium position the piston divides the cylinder into two equal parts, each with volume V_0. The gas pressure is P_0. The piston is slightly displaced from the equilibrium position and released. Find the time period of the oscillation, assuming the processes in the gas to be adiabatic and friction negligible. $\left(\text{Take } \frac{C_P}{C_V} = \gamma\right)$



2. One mole of a gas is isothermally expanded at 27 °C till the volume of doubled. Then it is adiabatically compressed to its original volume. Calculate the total work done if $\gamma = 1.4$, $R = 25/3$ Jmol^{-1}K^{-1}, $\ln(2) \approx 0.7$ and $(2)^{0.4} \approx 1.3$.

3. Two moles of a gas $\left(\gamma = \frac{5}{3}\right)$ are initially at temperature 27 °C and occupy a volume of 20 litres. The gas is first expanded at constant pressure until the volume is doubled. Then it is subjected to an adiabatic change until the temperature returns to its initial value.
(a) Sketch the process on a P-V diagram.
(b) What are final volume and pressure of the gas.
(c) What is the work done by the gas.

4. One litre of an ideal gas $(\gamma = 1.5)$ at 300 K temperature and 10^5 Pa pressure, is suddenly compressed to half of its original volume. Calculate the final temperature of the gas. The gas is then cooled isobarically to 300 K and then it is expanded isothermally to achieve its original volume of 1 litre. Calculate the work done by the gas in each process and also calculate the total work done in the cycle. Take $2 \approx (1.41)^2$ and $\ln 2 \approx 0.7$

5. There are two vessels, each of them containing one mole of an ideal monoatomic gas. Initial volume of each gas in each vessel is 8.3×10^{-3} m^3 at 27 °C. Equal amount of heat is supplied to each vessel. In one of the vessels, the volume of gas is doubled without change in its internal energy, whereas the volume of gas is held constant in the other vessel. The vessels are now connected to allow free mixing of the gas. Find the final temperature and pressure of the combined gas system.

6. One mole of oxygen, initially at temperature $T = 300$ K is compressed adiabatically so that its pressure increases $\eta = 10$ times. Find the final temperature and work done on it. Take $(10)^{1/7} \approx 1.4$ and $R = 8.3$ Jmol^{-1}K^{-1}.

7. On a P-V diagram starting from an initial state (P_0, V_0) plot an adiabatic expansion to $2V_0$, an isothermal expansion to $2V_0$ and an isobaric expansion to $2V_0$.

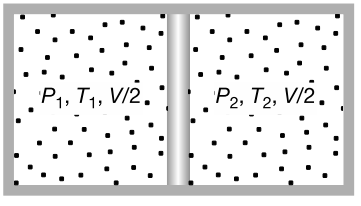
(a) Use this graph to determine in which process the least work is done by the system.

(b) Plot the same processes on a P-T diagram starting from (P_0, T_0).

Assume the gas to be a monatomic gas.

8. During the adiabatic expansion of 2 moles of a gas, the change in internal energy was found to be -100 J. Calculate the work done by the gas in the process.

9. The figure shows an insulated cylinder of volume V containing monatomic gas in both the compartments. The piston is diathermic.

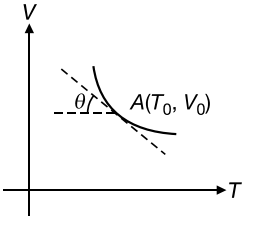


(a) Initially the piston is kept fixed and the system is allowed to acquire a state of thermal equilibrium. If the initial pressures and temperatures are as shown, calculate the final temperature and the final pressure.

(b) Now the pin which was keeping the piston fixed is removed and the piston set free to move. The piston is allowed to slide slowly, such that a state of mechanical equilibrium is also achieved $(T_1P_2 > P_1T_2)$. Calculate the final volume of gas in each compartment.

10. The volume of an ideal diatomic gas with $\gamma = 1.5$ is changed adiabatically from 16 litre to 12 litre. Find the ratio of the final and initial pressure and temperatures.

11. A gas is undergoing an adiabatic process. At a certain stage A, the values of volume and temperature are (V_0, T_0). From the details given in the graph, find the value of C_P and C_V.



POLYTROPIC PROCESS

A process in which all P, V and T change simultaneously such that C is a non-zero constant is called a polytropic process. The equation of for a polytropic process is given by

$$PV^x = \text{constant} \qquad \ldots(1)$$

where $(x \neq 1 \text{ or } \gamma)$ is a polytropic process.

For an ideal gas, we have $P = \frac{RT}{V}$, so equation (1) can be re-written as

$$\left(\frac{nRT}{V}\right)V^x = \text{constant}$$

$$\Rightarrow \quad TV^{x-1} = \text{constant}$$

Similarly, we have $v = \frac{nRT}{P}$ and hence equation (1) can again be written as

$$T^x P^{1-x} = \text{constant}$$

MOLAR SPECIFIC HEAT OF A POLYTROPIC PROCESS

Since, we know that $C = \frac{dQ}{dT}$ (for 1 mole of gasd)

$$\Rightarrow \quad C = \frac{dU + dW}{dT} \qquad \{\because dQ = dU + dW\}$$

$$\Rightarrow \quad C = \frac{dU}{dT} + P\frac{dV}{dT} = C_V + P\frac{dV}{dT} \qquad \ldots(1)$$

Since, $TV^{x-1} = \text{constant}$

Taking derivative w.r.t. T, we get

$$T(x-1)V^{x-2}\frac{dV}{dT} + V^{x-1} = 0$$

$$\Rightarrow \quad (x-1)\frac{T}{V}\frac{dV}{dT} + 1 = 0$$

$$\Rightarrow \quad \frac{dV}{dT} = -\left[\frac{V}{T(x-1)}\right]$$

Substituting in equation (1), we get

$$C = C_V + P\left[-\frac{V}{T(x-1)}\right]$$

$$\Rightarrow \quad C = C_V - \frac{PV}{T(x-1)}$$

$$\Rightarrow \quad C = C_V - \frac{RT}{(x-1)T} \qquad \{\text{where } T \neq 0\}$$

$$\Rightarrow \quad C = C_V + \frac{R}{1-x}$$

Problem Solving Technique(s)

A polytropic process is a kind of general process having constant specific heat. It can also be used to denote many special process like

(i) For Isothermal Process, $PV = \text{constant}$
So, $x = 1$ and hence $C \to \infty$

(ii) For Adiabatic Process, $PV^{\gamma} = \text{constant}$
So, $x = \gamma$ and hence $C = 0$

(iii) For Isochoric Process, $V = \text{constant}$
So, $x \to \infty$ and hence $C = C_V$

(iv) For Isochoric Process, $P = \text{constant}$
So, $x = 0$ and hence $C = C_V + R = C_P$

WORK DONE IN A POLYTROPIC PROCESS

The equation for a polytropic process is

$$PV^x = \text{constant}$$

So, work done in a polytropic process is

$$W_{\text{poly}} = \int_{V_1}^{V_2} PdV = \int_{V_1}^{V_2} kV^{-x}dV = k\left(\frac{V^{-x+1}}{-x+1}\right)\Bigg|_{V_1}^{V_2}$$

$$\Rightarrow \quad W_{\text{poly}} = k\left(\frac{V^{-x+1}}{-x+1}\right)\Bigg|_{V_1}^{V_2} = \frac{k}{1-x}\left(V_2^{-x+1} - V_1^{-x+1}\right)$$

$$\Rightarrow \quad W_{\text{poly}} = \frac{1}{1-x}\left[\left(kV_2^{-x}\right)V_2 - \left(kV_1^{-x}\right)V_1\right] \qquad \ldots(1)$$

Since $kV_2^{-x} = P_2$ and $kV_1^{-x} = P_1$

$$W_{\text{poly}} = \left(\frac{nR}{1-x}\right)\Delta T = \frac{P_2V_2 - P_1V_1}{1-x}, \text{ where } x \neq 1$$

INDICATOR DIAGRAM FOR A POLYTROPIC PROCESS

For a polytropic process, we have $PV^x = \text{constant}$

$$\Rightarrow \quad d\left(PV^x\right) = 0$$

$$\Rightarrow \quad P\left(xV^{x-1}dV\right) + V^x dP = 0$$

$$\Rightarrow \quad Px\left(\frac{V^x}{V}\right)dV + V^x dP = 0$$

$$\Rightarrow \quad \frac{dP}{dV} = -x\left(\frac{P}{V}\right)$$

Since, $\left(\text{Slope}\right)_{\text{isot}} = -\frac{P}{V}$

$$\Rightarrow \quad \left(\text{Slope}\right)_{\text{poly}} = x\left(\text{Slope}\right)_{\text{isot}}$$

So, for polytropic processes having x more than 1, the curve will be steeper than the isotherm (i.e., PV curve for isothermal process).
So, for polytropic processes having x less than 1, the curve will have less slope than the isotherm (i.e., PV curve for isothermal process) as shown in Figure.

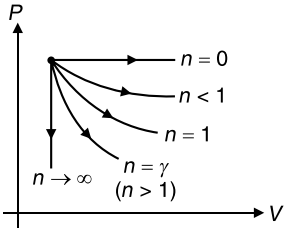


ILLUSTRATION 101

1 mole of an ideal monatomic gas is expanded till the temperature of the gas is doubled under the process $V^2T = \text{constant}$. The initial temperature of the gas is 400 K. Calculate the total work done in the process, in terms of R.

SOLUTION

Since, $T_i = 400$ K and $T_f = 2T_i = 800$ K

$$\Rightarrow \quad \Delta T = T_f - T_i = 400 \text{ K}$$

$$\Rightarrow \quad \Delta U = nC_V\Delta T$$

$$\Rightarrow \quad \Delta U = (1)\left(\frac{3}{2}R\right)(400) = 600R$$

The given process is $V^2T = \text{constant}$

Substituting $T = \frac{PV}{R}$, we get

$$PV^3 = \text{constant}$$

Comparing this equation with equation of a polytropic process i.e., $PV^x = \text{constant}$ we observe that $x = 3$ and so molar heat capacity of polytropic process is

$$C = C_V + \frac{R}{1-x} = \frac{3R}{2} + \frac{R}{1-3} \qquad \left\{\because C_V = \frac{3R}{2}\right\}$$

$$\Rightarrow \quad C = \frac{3}{2}R - \frac{R}{2} = R$$

Since, $Q = nC\Delta T = (1)(R)(400) = 400R$

Now from First Law, we have

$$W = Q - \Delta U = -200R$$

Problem Solving Technique(s)

Work done in a polytropic process $PV^x = \text{constant}$ is

$$W_{\text{poly}} = \left(\frac{nR}{1-x}\right)\Delta T = \frac{P_2V_2 - P_1V_1}{1-x}, \text{ where } x \neq 1$$

Molar specific of the process is

$$C = C_V + \frac{R}{1-x} = \frac{R}{\gamma-1} + \frac{R}{1-x}$$

If f be the degrees of freedom of the gas, then

$$C_V = \frac{fR}{2}$$

For the above problem, we have $x = 3$ and the gas is monatomic, so we get

$$W = \left(\frac{1\times R}{1-3}\right)(400) = -200R \text{ and } C = \frac{3R}{2} + \frac{R}{1-3} = R$$

ILLUSTRATION 102

An ideal diatomic gas with $C_V = \frac{5R}{2}$ occupies a volume V_i at a pressure P_i. The gas undergoes a process in which the pressure is proportional to the volume. At the end of the process, it is found that the rms speed of the gas molecules has doubled from its initial value. Determine the amount of energy transferred to the gas by heat.

SOLUTION

Given that $P \propto V$, so $PV^{-1} = \text{constant}$

As we know, molar heat capacity for the polytropic process $PV^x = \text{constant}$ is,

$$C = \frac{R}{\gamma-1} + \frac{R}{1-x} = C_V + \frac{R}{1-x}$$

In the given problem, we have

$$C_V = \frac{5R}{2} \text{ and } x = -1, \text{ so we get}$$

$$\Rightarrow \quad C = \frac{5R}{2} + \frac{R}{2} = 3R \qquad \ldots(1)$$

At the end of the process rms speed is doubled. Since $v_{rms} \propto \sqrt{T}$, so the temperature must have become four times. Since

$$Q = nC\Delta T = nC\left(T_f - T_i\right) = nC\left(4T_i - T_i\right)$$

$$\Rightarrow \quad \Delta Q = 3nCT_i = 3n(3R)T_i = 9\left(nRT_i\right)$$

$$\Rightarrow \quad \Delta Q = 9P_iV_i$$

ILLUSTRATION 103

A gas undergoes a process such that $P \propto \frac{1}{T}$. If the molar heat capacity for this process is $C = 33.24\ \text{Jmol}^{-1}\text{K}^{-1}$, find the degrees of freedom of the molecules of the gas.

SOLUTION

Since, $P \propto \frac{1}{T}$

$$\Rightarrow \quad PT = \text{constant} \qquad \ldots(1)$$

For one mole of an ideal gas, we have

$$PV = RT \qquad \ldots(2)$$

From Equations (1) and (2), we get

$$P^2V = \text{constant}$$

$$\Rightarrow \quad PV^{\frac{1}{2}} = \text{constant} \qquad \ldots(3)$$

In the polytropic process PV^x =constant, the molar heat capacity is given by

$$C = \frac{R}{\gamma-1} + \frac{R}{1-x}$$

The given process is given by

$$PV^{\frac{1}{2}} = \text{constant}$$

$$\Rightarrow \quad x = \frac{1}{2}$$

$$\Rightarrow \quad C = \frac{R}{\gamma-1} + \frac{R}{1-\frac{1}{2}} = \frac{R}{\gamma-1} + 2R$$

Since $C = 33.24\ \text{Jmol}^{-1}\text{K}^{-1}$

$$\Rightarrow \quad 33.24 = R\left(\frac{1}{\gamma-1} + 2\right) = 8.31\left(\frac{1}{\gamma-1} + 2\right)$$

Solving this we get

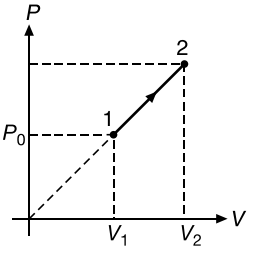
$$\gamma = 1.5$$

Since, $\gamma = 1 + \frac{2}{f}$

So, degree of freedom is $f = \frac{2}{\gamma-1} = \frac{2}{1.5-1} = 4$

ILLUSTRATION 104

n moles of a monatomic ideal gas undergoes a thermodynamic process along a path from 1 to 2 as shown in Figure. The gas pressure at 1 is P_0. Calculate the amount of heat supplied to gas in this process and work done by the gas in the process if the gas expands from volume V_1 to V_2.



SOLUTION

From the indicator diagram shown in figure, we observe that the PV curve is a straight line passing through origin, so we have

$$P = kV$$

$$\Rightarrow \quad PV^{-1} = k = \text{constant} \qquad \ldots(1)$$

So, we can say that the process is a polytropic with the value of polytropic constant $x = -1$. The molar specific heat of the gas is

$$C = C_V + \frac{R}{1-x} = C_V + \frac{R}{1-(-1)}$$

$$\Rightarrow \quad C = \frac{3R}{2} + \frac{R}{2} = 2R \qquad \left\{\because (C_V)_{\text{monatomic}} = \frac{3R}{2}\right\}$$

So, heat supplied is

$$Q = nC\Delta T = nC(T_2 - T_1) = n(2R)(T_2 - T_1) \qquad \ldots(2)$$

Since $P = kV$ (from graph) and from ideal gas equation $P = \frac{nRT}{V}$, so we get

$$nRT = kV^2$$

So, $T_2 = \frac{kV_2^2}{nR}$ and $T_1 = \frac{kV_1^2}{nR}$

Substituting in equation (2), we get

$$Q = 2nR\left(\frac{k}{nR}\right)(V_2^2 - V_1^2) = 2k(V_2^2 - V_1^2) \qquad \ldots(3)$$

For the given process, from equation (1), we get

$$P_1 = P_0 = kV_1$$

$$\Rightarrow \quad k = \frac{P_0}{V_1}$$

Substituting in equation (3), we get

$$Q = 2P_0\left(\frac{V_2^2 - V_1^2}{V_1}\right)$$

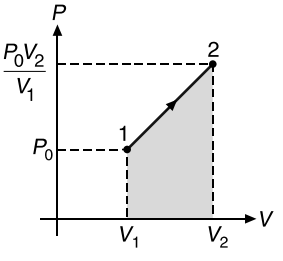
Work done in a polytropic process is given by

$$W = n(C - C_V)(T_2 - T_1)$$

$$\Rightarrow \quad W = n\left(2R - \frac{3R}{2}\right)\left(\frac{V_2^2 P_0}{nRV_1} - \frac{P_0 V_1}{nR}\right)$$

$$\Rightarrow \quad W = \frac{P_0}{2}\left(\frac{V_2^2 - V_1^2}{V_1}\right)$$

Work done in the process can also be obtained by the calculating the shaded area under the PV graph. So



$$W = \frac{1}{2}\left(P_0 + P_0\frac{V_2}{V_1}\right)(V_2 - V_1)$$

$$\Rightarrow \quad W = \frac{P_0}{2V_1}(V_2^2 - V_1^2)$$

ILLUSTRATION 105

One mole of an ideal gas, whose adiabatic exponent equal to γ, is expanded so that the amount of heat transferred to the gas is equal to the decrease in internal energy. Find

(a) the molar heat capacity of the gas in this process,
(b) the equation of the process in the variables T, V
(c) the work performed by one mole of the gas when its volume increases η times if the initial temperature of gas is T_0.

SOLUTION

(a) Since the amount of heat supplied is equal to the decrease in internal energy of gas, so for this process, we have

$$dU = -dQ$$

$$\Rightarrow \quad nC_V dT = -nC dT$$

$$\Rightarrow \quad C = -C_V = -\frac{R}{\gamma - 1} \qquad \ldots(1)$$

(b) According to FLTD, we have

$$dQ = dU + dW$$

$$\Rightarrow \quad dQ = -dQ + dW \qquad \{\because dU = -dQ\}$$

$$\Rightarrow \quad 2dQ = dW$$

Since, $dQ = -nC_V dT$

$$\Rightarrow \quad -\frac{2nR}{\gamma - 1}dT = PdV \qquad \ldots(2)$$

From ideal gas equation, we have $PV = nRT$

$$\Rightarrow \quad PdV + VdP = nRdT \qquad \ldots(3)$$

Substituting value of dT from equation (3) in (2), we get

$$PdV + VdP = \left(\frac{1-\gamma}{2}\right)PdV$$

$$\Rightarrow \quad \left(\frac{1+\gamma}{2}\right)PdV = -VdP$$

$$\Rightarrow \quad \left(\frac{1+\gamma}{2}\right)\frac{dV}{V} = -\frac{dP}{P}$$

Integrating this equation, we get

$$\left(\frac{1+\gamma}{2}\right)\int\frac{dV}{VP} = -\int\frac{dP}{P}$$

$$\Rightarrow \quad \ln V^{\left(\frac{1+\gamma}{2}\right)} = -\ln P + C$$

$$\Rightarrow \quad \ln V^{\left(\frac{1+\gamma}{2}\right)} + \ln P = C$$

$$\Rightarrow \quad PV^{\left(\frac{1+\gamma}{2}\right)} = \text{constant} \qquad \ldots(4)$$

Since we require process equation in T and V, so substituting $P = \frac{nRT}{V}$ (from ideal gas equation) in equation (4), we get

$$\left(\frac{nRT}{V}\right)V^{\left(\frac{1+\gamma}{2}\right)} = \text{constant}$$

$$\Rightarrow TV^{\left(\frac{\gamma-1}{2}\right)} = \text{constant} \quad \ldots(5)$$

In terms of P and V, the process equation (4) can be directly obtained by the molar specific heat of the process by calculating the polytropic constant x. Since C is a constant not depending on pressure, volume or temperature of gas thus we can say that this process is a polytropic process whose molar specific heat can be given as

$$C = \frac{R}{\gamma-1} + \frac{R}{1-x} = -\frac{R}{\gamma-1}$$

where x is the polytropic constant for the process.

$$\Rightarrow \frac{R}{1-x} = -\frac{2R}{\gamma-1}$$

$$\Rightarrow 1-x = -\frac{1}{2}(\gamma-1)$$

$$\Rightarrow 2-2x = 1-\gamma$$

$$\Rightarrow x = \frac{1+\gamma}{2}$$

So, the process equation of this thermodynamic process in P and V can simply be written as

$$PV^x = \text{constant}$$

$$\Rightarrow PV^{\left(\frac{1+\gamma}{2}\right)} = \text{constant}$$

which is same as equation (5)

(c) Work done by a gas in a polytropic process is

$$W_{\text{poly}} = \frac{nR}{1-x}(T_2 - T_1), \text{ where } x = \frac{1+\gamma}{2}$$

$$\Rightarrow W_{\text{poly}} = \frac{2nR}{1-\gamma}(T_2 - T_0)$$

Now T_2 is the temperature when volume becomes η times i.e., ηV_0. Since,

$$TV^{\left(\frac{\gamma-1}{2}\right)} = \text{constant}$$

$$\Rightarrow T_2(\eta V_0)^{\left(\frac{\gamma-1}{2}\right)} = T_0 V_0^{\left(\frac{\gamma-1}{2}\right)}$$

$$\Rightarrow T_2 = T_0\left(\frac{1}{\eta}\right)^{\left(\frac{\gamma-1}{2}\right)} = T_0\eta^{\left(\frac{1-\gamma}{2}\right)}$$

$$\Rightarrow W = \frac{2R}{1-\gamma}\left[T_0(\eta)^{\frac{1-\gamma}{2}} - T_0\right] \quad \{\because n = 1 \text{ mole}\}$$

$$\Rightarrow W = \frac{2RT_0}{1-\gamma}\left[(\eta)^{\frac{1-\gamma}{2}} - 1\right]$$

ILLUSTRATION 106

A gas consisting of monatomic molecules (having three degrees of freedom) was expanded in a polytropic process so that the rate of collisions of the molecules against the vessel's wall did not change. Calculate the molar heat capacity of the gas in the process.

SOLUTION

Since the rate of collisions with vessel wall is

$$N_C = \frac{1}{6}n_0 v_{rms} = \text{constant}$$

where, n_0 is the number density of molecules.

$$\Rightarrow \frac{1}{6}\left(\frac{N}{V}\right)\sqrt{\frac{3RT}{m}} = \text{constant}$$

$$\Rightarrow \frac{\sqrt{T}}{V} = \text{constant}$$

$$\Rightarrow T = kV^2$$

$$\Rightarrow \frac{dT}{dV} = 2kV$$

Molar specific of the gas is given by

$$C = C_V + \frac{PdV}{ndT}, \text{ where } C_V = \frac{3R}{2}$$

$$\Rightarrow C = \frac{3R}{2} + \left(\frac{RT}{V}\right)\frac{dV}{dT} = \frac{3R}{2} + \frac{R(kV^2)}{V}\frac{1}{2kV}$$

$$\Rightarrow C = \frac{3R}{2} + \frac{R}{2} = 2R$$

Test Your Concepts-VIII

Based on Polytropic Process

(Solutions on page H.91)

1. Three moles of a diatomic gas is taken at temperature T. Its volume is varied according to the law $V = \alpha T^{-2}$ where α is a positive constant during the process. If the final temperature of the gas is found to be $2T$, find the heat supplied to the gas.

2. An ideal gas has an adiabatic exponent γ. In some process its molar heat capacity varies as $C = \frac{\alpha}{T}$, where α is a constant. Calculate the

(a) work performed by one mole of the gas during its heating from the temperature T_0 to the temperature η times higher

(b) equation of the process in terms of pressure and volume.

3. Find the maximum attainable temperature of an ideal gas in a process, $P = P_0 - \alpha V^2$, where P_0, α are constants and V is the volume of one mole of gas.

4. An ideal gas has a molar heat capacity C_V at constant volume. Find the molar heat capacity of this gas as a function of its volume V, if the gas separately undergoes the processes $T = T_0 e^{\alpha V}$ and $P = P_0 e^{\alpha V}$, where P_0, T_0 and α are positive constants.

5. Consider one mole of an ideal gas whose volume changes with temperature as $V = \frac{\alpha}{T}$, where α is constant. Find the amount of heat required to raise its temperature by ΔT, if its adiabatic constant is γ.

6. An ideal gas consisting of rigid diatomic molecules was expanded in a polytropic process so that the rate of collisions of the molecules against the vessel's wall did not change. Calculate molar heat capacity of the gas in this process.

7. Consider one mole of an ideal gas whose pressure changes with volume as $P = \alpha V$, where α is constant. If it is expanded such that its volume increases η times, find the change in its internal energy, work done by the gas and heat capacity of the gas in terms of C_V and R.

8. An ideal gas is taken through a process in which the process equation is given as $P = kV^{\alpha}$, where k and α are positive constants. Calculate the value of α for which molar specific heat in this process becomes zero.

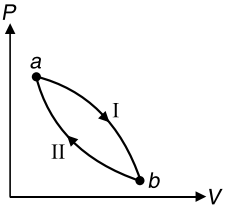
9. Two moles of an ideal monatomic gas undergoes the process $P = \alpha T^{\frac{1}{2}}$, where α is a constant. If $R = 25/3$ Jmol^{-1}K^{-1}, then calculate

(a) the work done by the gas if its temperature increases by 60 K.

(b) the molar specific heat of the gas.

CYCLIC PROCESS

Engines operate in cycles, in which the system – for example, a gas – periodically returns to its initial state. In figure, the system goes from state a to state b via path I, for which $W_I > 0$, and returns to its initial state via path II, for which $W_{II} < 0$. The net work done by the system is the area enclosed by the curve. In a clockwise traversal the network is positive.



Since the system returns to its initial state, the change in internal energy in one complete cycle is zero, that is, $\Delta U = 0$.

From the First Law we see that

$$Q = W$$

The net work done by the system in each cycle, $W = W_I + W_{II}$, is equal to the net heat input per cycle.

This result is of importance in the discussion of steam engines and diesel engines, for instance, in which the influx of heat is used to perform mechanical work.

The efficiency of a cyclic process is

$$\eta = \frac{W_{\text{total}}}{Q_{\text{input}}} = \frac{W_{\text{total}}}{\Sigma Q_{\text{positive}}}$$

EFFICIENCY OF A CYCLIC PROCESS

In a cyclic process, we have

$$\Delta U = 0$$

and hence from First Law, we get $Q_{\text{net}} = W_{\text{net}}$

Now, let us understand the meaning of efficiency of a cycle. Suppose 100 J of heat is supplied to a system (in our case it is an ideal gas) and the system does 60 J of work. Then efficiency of the cycle is 60%. Thus, efficiency (η) of a cycle can be defined as

$$\Rightarrow \quad \eta = \left(\frac{\text{Useful work done}}{\text{Heat supplied}}\right)_{\text{during the cycle}} \times 100\%$$

$$\Rightarrow \quad \eta = \frac{W_{\text{Total}}}{|Q_{+\text{ve}}|} \times 100\% = \frac{|Q_{+\text{ve}}| - |Q_{-\text{ve}}|}{|Q_{+\text{ve}}|} \times 100\%$$

$$\Rightarrow \quad \eta = \left(1 - \left|\frac{Q_{-\text{ve}}}{Q_{+\text{ve}}}\right|\right) \times 100\%$$

Thus, $\eta = \frac{W_{\text{Total}}}{|Q_{+\text{ve}}|} \times 100 = \left(1 - \left|\frac{Q_{-\text{ve}}}{Q_{+\text{ve}}}\right|\right) \times 100\%$

HEAT ENGINE

A heat engine is a device which converts Thermal energy to other useful forms of energy such as mechanical energy, electrical energy. In other words, any device that transforms heat partly into work or mechanical energy is called a heat engine. Usually, a quantity of matter inside the engine undergoes inflow and outflow of heat, expansion and compression, and sometimes change of phase. We call this matter the working substance of the engine. In internal-combustion engines the working substance is a mixture of air and fuel; in a steam turbine it is water.

The simplest kind of engine to analyse is one in which the working substance undergoes a cyclic process, a sequence of processes that eventually leaves the substance in the same state in which it started. In a steam turbine the water is recycled and used over and over. Internal-combustion engines do not use the same air over and over,

but we can still analyse them in terms of cyclic processes that approximate their actual operation.

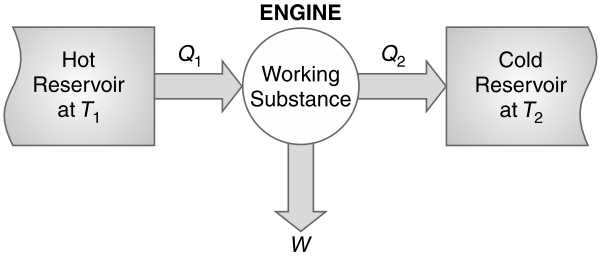
All heat engines absorb heat from a source at a relatively high temperature, perform some mechanical work, and discard or reject some heat at a lower temperature. As far as the engine is concerned, the discarded heat is wasted. In internal-combustion engine the waste heat is that discarded in the hot exhaust gases and the cooling system; in a steam turbine it is the heat that must flow out of the used steam to condense and recycle the water.

When a system is carried through a cyclic process, its initial and final internal energies are equal. For any cyclic process, the First Law of Thermodynamics requires that

$$U_2 - U_1 = 0 = Q - W$$

$$\Rightarrow \quad Q = W$$

That is, the net heat flowing into the engine in a cyclic process equals the net work done by the engine.
The block diagram of a heat engine is shown in Figure.



When we talk of a heat engine, we happen to think of two bodies with which the working substance of the engine can interact.

One of these, called the hot reservoir, represents the heat source, it can give the working substance large amounts of heat at a constant temperature T_1 without appreciably changing its own temperature. The other body, called the cold reservoir, can absorb large amounts of discarded heat from the engine at a constant lower temperature T_2.

e.g. In a steam-turbine system the flames and hot gases in the boiler are the hot reservoir, and the cold water and air used to condense and cool the used steam are the cold reservoir.

We denote the quantities of heat transferred from the hot and cold reservoirs to be Q_1 and Q_2, respectively. Following our sign convention, a quantity of heat Q is positive when heat is transferred into the working substance and is negative when heat leaves the working substance. Thus, in a heat engine, Q_1 is positive but Q_2 is negative.

When an engine repeats the same cycle over and over, Q_1 and Q_2 represent the quantities of heat absorbed and rejected by the engine during one cycle. The net heat Q absorbed per cycle is

$$Q = Q_1 - Q_2$$

The useful output of the engine is the net work W done by the working substance.

From the First Law, we have $W = Q = Q_1 - Q_2$

Ideally, we would like to convert all the heat Q_1 into work. In that case, we would have $Q_1 = W$ and $Q_2 = 0$. Experience shows that this is impossible; there is always some heat wasted, and Q_2 is never zero. This is also the statement of Second Law of Thermodynamics (discussed afterwards).

By Law of Conservation of Energy, we get

$$Q_1 = Q_2 + W$$

$$\Rightarrow \quad W = Q_1 - Q_2$$

$$\Rightarrow \quad \eta = \frac{W}{Q_1} = 1 - \frac{Q_1}{Q_2}$$

Since $\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$

$$\Rightarrow \quad \eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

According to this, efficiency is 100% if $Q_2 = 0$, that is, no heat is rejected to the cold reservoir or sink that is the entire heat absorbed must be converted to mechanical work, which according to Second Law of Thermodynamics is impossible.

In practice, heat engines convert only a fraction of absorbed heat into mechanical work. For example, a good automobile engine has an efficiency of about 20% and diesel engines have efficiencies ranging from 35% to 40%.

On the basis of this fact, the Kelvin-Plank form of Second Law of Thermodynamics states that "no heat engine, operating in a cycle, can absorb thermal energy from a reservoir and perform an equal amount of work. This is equivalent to say that is impossible to construct a perpetual motion machine."

ILLUSTRATION 107

A Carnot engine, whose efficiency is 40%, takes in heat from a source maintained at a temperature of 500 K. It is desired to have an engine of efficiency 60%, then calculate the intake temperature for the same exhaust temperature.

SOLUTION

Efficiency of Carnot engine, $\eta = 1 - \frac{T}{T_1}$

where T_1 is the temperature of the source and T_2 is the temperature of the sink.

For 1st case, $\eta = 40\%$, $T_1 = 500$ K

$$\Rightarrow \quad \frac{40}{100} = 1 - \frac{T_2}{500}$$

$$\Rightarrow \quad T_2 = \frac{3}{5} \times 500 = 300 \text{ K}$$

For 2nd case, $\eta = 60\%$, $T_2 = 300$ K

$$\Rightarrow \quad \frac{60}{100} = 1 - \frac{300}{T_1}$$

$$\Rightarrow \quad T_1 = \frac{5}{2} \times 300 = 750 \text{ K}$$

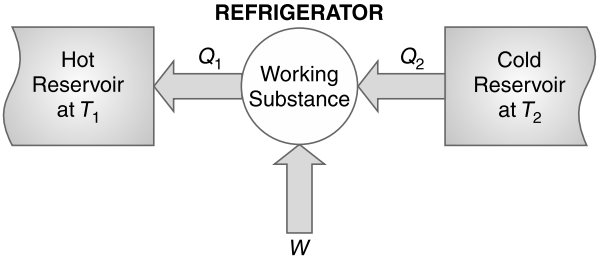
REFRIGERATOR OR HEAT PUMP

It is a heat Engine running in reverse.
According to Law of Conservation of Energy

$$Q_1 = Q_2 + W$$

Coefficient of performance

$$\beta = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} \qquad (\beta > 1)$$



In practice it is desirable to carry out this process with minimum amount of work. If it could be accompanied without doing any work, we would have a **"perfect refrigerator"** which is again in violation with Second Law of Thermodynamics

On the basis of this fact, the Rudolph-Clausius form of Second Law of Thermodynamics states that "it is impossible to construct a cyclical machine that produces no other effect than to transfer heat continuously from one body at lower temperature to another body at a higher temperature."

RELATION BETWEEN η AND β

Since $\eta = 1 - \frac{T_2}{T_1}$ …(1)

$$\Rightarrow \quad \eta = \frac{T_1 - T_2}{T_1} \qquad \text{…(2)}$$

Also, $\beta = \frac{T_2}{T_1 - T_2}$ …(3)

Multiplying (2) and (3), we get

$$\eta\beta = \frac{T_2}{T_1} = 1 - \eta \qquad \{\because \text{ of (1)}\}$$

$$\Rightarrow \quad \eta(\beta + 1) = 1$$

$$\Rightarrow \quad \eta = \frac{1}{1+\beta}$$

For η to be MAXIMUM, β must be MINIMUM i.e., 1

$$\Rightarrow \quad \eta_{max} = \frac{1}{1+1} = \frac{1}{2} = 50\%$$

ILLUSTRATION 108

If minimum possible work is done by a refrigerator in converting 100 g of water at 0 °C to ice, how much heat is released to the surrounding at temperature 27 °C. Take latent heat of ice to be 80 calg^{-1}.

SOLUTION

Since $\frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$, where, $Q_2 = mL_{ice} = 8000$ cal

Also, $T_2 = 273$ K and $T_1 = 300$ K

$$\Rightarrow \quad \frac{8000}{W} = \frac{273}{300 - 273} = \frac{273}{27}$$

$$\Rightarrow \quad W = 791.2 \text{ cal}$$

$$\Rightarrow Q_1 = Q_2 + W = 791.2 + 8000 = 8791.2 \text{ cal}$$

ILLUSTRATION 109

A Carnot engine having an efficiency of 10% is being used as a refrigerator. If the work done on the refrigerator is 10 J, then calculate the amount of heat absorbed from the reservoir at lower temperature.

SOLUTION

Since, $\eta = \frac{1}{10} = \frac{1}{1+\beta}$

Also by definition, we have

$$\beta = \frac{Q_2}{W} = 9$$

$$\Rightarrow \quad Q_2 = 9W = 90 \text{ J}$$

REVERSIBLE PROCESS

A process which can be retraced back in the opposite direction in such a way that the system passes through the same states as in direct process and finally the system and the surroundings acquire the initial conditions.

CONDITIONS FOR A PROCESS TO BE REVERSIBLE

(a) The change must take place at a very slow rate.
(b) There should be no loss of energy due to conduction, convection or dissipation of energy against any resistance, like friction, viscosity etc.
(c) No heat should be converted into magnetic or electric energy.
(d) The system must always be in thermal and chemical equilibrium with the surroundings.

EXAMPLE:

(a) Fusion of ice is a reversible process.
(b) Vaporisation of water is a reversible process.
(c) Temperature of two junctions of a thermocouple get reversed on reversing the direction of current in it. Thus Peltier effect a reversible process.
(d) Gradual compression and extension of a spring is a reversible process.

IRREVERSIBLE PROCESS

The process which cannot be retraced back in the opposite direction is defined as irreversible process. The system does not pass through the same intermediate states as in the direction process. Almost all processes of nature are irreversible.

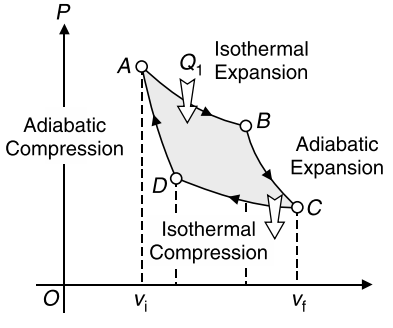
EXAMPLE:

(a) Work done against friction.
(b) Joule heating effect i.e. heat produced in condition by an electric current.
(c) Diffusion of gases into one another.
(d) Magnetic hysteresis.

CARNOT ENGINE/CYCLE

Carnot devised an ideal engine which is based on a reversible cycle of four operations in succession.

(a) Isothermal Expansion
(b) Adiabatic Expansion
(c) Isothermal Compression
(d) Adiabatic Compression



If T_1 and T_2 are absolute temperatures of source and sink, then
Work done in Carnot cycle

$$W = \text{Area enclosed by cycle } ABCDA$$

$$\text{Also } W = R(T_1 - T_2)\log_e\left(\frac{V_f}{V_i}\right) \qquad \{\text{for 1 mole of gas}\}$$

Heat absorbed from source

$$Q_1 = W_1 = RT_1 \log_e\left(\frac{V_f}{V_i}\right)$$

Since efficiency, $\eta = \dfrac{W}{Q_1}$

$$\Rightarrow \quad \eta = \frac{R(T_1 - T_2)\log_e \dfrac{V_f}{V_i}}{RT_1 \log_e \dfrac{V_f}{V_i}} = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1}$$

So, Efficiency of Carnot Engine,

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

Conceptual Note(s)

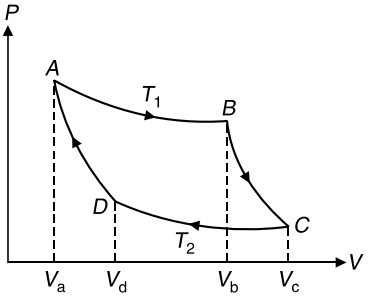
In a Carnot cycle $\dfrac{Q_1}{T_1} = \dfrac{Q_2}{T_2}$

If an engine consists of a number of sources and sinks, then

$$\left(\sum \frac{Q}{T}\right)_{\text{source}} = \left(\sum \frac{Q}{T}\right)_{\text{sink}}$$

ILLUSTRATION 110

Two different adiabatic parts for the same gas intersects two isotherms at T_1 and T_2 as shown in the P-V diagram in the figure. How does the ratio $\dfrac{V_a}{V_d}$ compare with the ratio of $\dfrac{V_b}{V_c}$?



SOLUTION

For adiabatic change $TV^{\gamma-1} = \text{constant}$
For adiabatic process BC

$$T_1 V_b^{\gamma-1} = T_2 V_c^{\gamma-1} \qquad \ldots(1)$$

and for adiabatic process DA,

$$T_1 V_a^{\gamma-1} = T_2 V_d^{\gamma-1} \qquad \ldots(2)$$

Dividing equation (2) by (1),

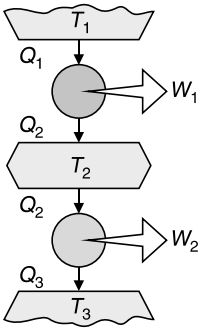
$$\left(\frac{V_a}{V_b}\right)^{\gamma-1} = \left(\frac{V_d}{V_c}\right)^{\gamma-1}$$

$$\Rightarrow \quad \frac{V_a}{V_d} = \frac{V_b}{V_c}$$

Hence, both ratios are same.

HEAT ENGINES IN SERIES

Let there be three reservoirs at temperatures T_1, T_2 and T_3 $(T_1 > T_2 > T_3)$ as shown in Figure.



There are two heat engines in this. The

(a) first engine working between temperatures T_1 and T_2, and

(b) second engine working between temperatures T_2 and T_3.

Let Q_1 be heat taken by first engine from a reservoir (T_1). If second engine takes heat Q_2 rejected by first heat engine; then the two engines are said to be in series. Let W_1 and W_2 be work done per cycle by engines. Then the efficiency of combination

$$\eta = \frac{W_1 + W_2}{Q_1} = 1 - \frac{T_3}{T_1}$$

ILLUSTRATION 111

Two Carnot engines A and B are operated in series. The first one, A, receives heat at $T_1 = 600$ K and rejects to a reservoir at temperature T_2. The second engine B receives heat rejected by the first engine and, in turn, rejects to a heat reservoir at $T_3 = 400$ K. Calculate the temperature T_2 if the work outputs of the two engines are equal.

SOLUTION

Since, $\frac{Q_1}{T_1} = \frac{Q_2}{T_2} = \frac{Q_3}{T_3}$

Also, $W_1 = Q_1 - Q_2$ and $W_2 = Q_2 - Q_3$

Given that $W_1 = W_2$, so we have

$\Rightarrow \quad Q_1 - Q_2 = Q_2 - Q_3$

$\Rightarrow \quad 2Q_2 = Q_1 + Q_3$

$\Rightarrow \quad 2T_2 = T_1 + T_3$

$\Rightarrow \quad T_2 = \frac{T_1 + T_3}{2} = 500$ K

ILLUSTRATION 112

Three Carnot engines operate in series between a heat source at a temperature T_1 and a heat sink at temperature T_4. There are two other reservoirs at temperature T_2 and T_3 such that $T_1 > T_2 > T_3 > T_4$. If the three engines are equally efficient, then calculate T_2 and T_3 in terms of T_1 and T_4.

SOLUTION

Since all engines have equal efficiency, so we have

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{T_3}{T_2} = 1 - \frac{T_4}{T_3}$$

$$\Rightarrow \quad \frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3} \qquad \ldots(1)$$

$\Rightarrow \quad T_2 = \sqrt{T_1 T_3}$ and $T_3 = \sqrt{T_2 T_4}$

Substituting value of T_2 in equation (1), we get

$$\Rightarrow \quad \frac{\sqrt{T_1 T_3}}{T_1} = \frac{T_4}{T_3}$$

$$\Rightarrow \quad \frac{T_1 T_3}{T_1^2} = \frac{T_4^2}{T_3^2}$$

$$\Rightarrow \quad T_3^3 = T_1 T_4^2$$

$$\Rightarrow \quad T_3 = \left(T_1 T_4^2\right)^{1/3}$$

Similarly, we get $T_2 = \left(T_4 T_1^2\right)^{1/3}$

ILLUSTRATION 113

An ideal gas is taken through a cyclic thermodynamic process through four steps. The amounts of heat involved in these steps are $Q_1 = 5960$ J, $Q_2 = -5585$ J, $Q_3 = -2980$ J and $Q_4 = 3645$ J respectively. The corresponding quantities of work involved are $W_1 = 2200$ J, $W_2 = -825$ J, $W_3 = -1100$ and W_4 respectively. Find the value of W_4. What is the efficiency of the cycle?

SOLUTION

In a cyclic process $dU = 0$

Therefore, $dQ = dW$

$$\Rightarrow \quad Q_1 + Q_2 + Q_3 + Q_4 = W_1 + W_2 + W_3 + W_4$$

Hence, $W_4 = (Q_1 + Q_2 + Q_3 + Q_4) - (W_1 + W_2 + W_3)$

$$\Rightarrow \quad W_4 = (5960 - 5585 - 2980 + 3645) - (2200 - 825 - 1100)$$

$$\Rightarrow \quad W_4 = 765 \text{ J}$$

Efficiency, $\eta = \frac{W_{\text{total}}}{\Sigma Q_{\oplus}} \times 100\%$

$$\Rightarrow \quad \eta = \left\{\frac{W_1 + W_2 + W_3 + W_4}{Q_1 + Q_4}\right\} \times 100$$

$$\Rightarrow \quad \eta = \left\{\frac{(2200 - 825 - 1100 + 765)}{5960 + 3645}\right\} \times 100$$

$$\Rightarrow \quad \eta = \frac{1040}{9605} \times 100 = 10.82\%$$

Since, from Conservation of Energy, we have

$$W_{net} = Q_{+ve} - Q_{-ve}\ (in\ a\ cycle)$$

$$\Rightarrow \quad \eta = \frac{W_{net}}{Q_{+ve}} \times 100 = \frac{(Q_{+ve} - Q_{-ve})}{Q_{+ve}} \times 100$$

$$\Rightarrow \quad \eta = \left(1 - \frac{Q_{-ve}}{Q_{+ve}}\right) \times 100$$

In the above question

$$Q_{-ve} = |Q_2| + |Q_3| = (5585 + 2980)\ \text{J} = 8565\ \text{J}$$

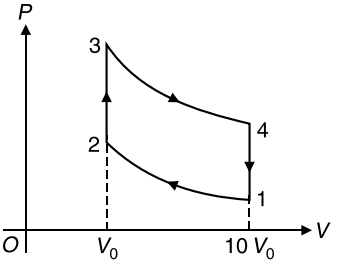
and $Q_{+ve} = Q_1 + Q_4 = (5960 + 3645)\ \text{J} = 9605\ \text{J}$

$$\Rightarrow \quad \eta = \left(1 - \frac{8565}{9605}\right) \times 100$$

$$\Rightarrow \quad \eta = 10.82\%$$

ILLUSTRATION 114

One mole of oxygen undergoes a cyclic process in which volume of the gas changes 10 times within the cycle, as shown in the figure. Processes: $1-2$ and $3-4$ are adiabatic, $2-3$ and $4-1$ are isochoric. Find the efficiency of the process.



SOLUTION

$$W_{12} = \frac{R}{1-\gamma}(T_2 - T_1)$$

Since, $T_2 = V_2^{\gamma-1} = T_1 V_1^{\gamma-1}$

$$\Rightarrow \quad T_2 = T_1 \alpha^{\gamma-1} \text{ where } \alpha = \frac{V_1}{V_2}$$

Thus, $W_{12} = \frac{RT_2}{1-\gamma}\left(1 - \frac{1}{\alpha^{\gamma-1}}\right)$

Similarly, $T_3 = T_4 \alpha^{\gamma-1}$

and $W_{34} = \frac{RT_3}{1-\gamma}\left(\frac{1}{\alpha^{\gamma-1}} - 1\right)$

Also $W_{23} = 0, W_{41} = 0$

$$\Rightarrow \quad W_{\text{net}} = W_{12} + W_{23} + W_{34} + W_{41}$$

$$\Rightarrow \quad W_{\text{net}} = \frac{R(T_2 - T_3)}{1-\gamma}\left(1 - \frac{1}{\alpha^{\gamma-1}}\right)$$

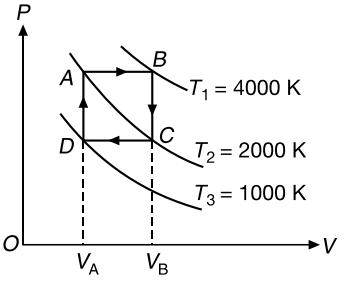
$$Q_{\text{in}} = \frac{R}{\gamma-1}(T_3 - T_2) = \frac{R}{1-\gamma}(T_2 - T_3)$$

Since $\eta = \frac{W_{\text{net}}}{Q_{\text{in}}} = 1 - \frac{1}{\alpha^{\gamma-1}}$, where $\alpha = 10, \gamma = 1.4$

$$\Rightarrow \quad \eta = 60\%$$

ILLUSTRATION 115

Three isotherms at temperature $T_1 = 4000$ K, $T_2 = 2000$ K, $T_3 = 1000$ K are shown in Figure. When one mole of an ideal monatomic gas is taken through the paths AB, BC, CD and DA, calculate the change in internal energy ΔU, work done by the gas W and heat Q absorbed by the gas along each path. Also efficiency for the complete cycle $ABCDA$ if it is given that $V_A = 1\ \text{m}^3$ and $V_B = 2\ \text{m}^3$. Take $R = 8.3\ \text{Jmol}^{-1}\text{K}^{-1}$



SOLUTION

Change in its internal energy of one mole of monatomic gas is $\Delta U = C_V \Delta T$, where C_V is the molar specific heat of the gas at constant volume. For monatomic gas, $C_V = \left(\frac{3}{2}\right)R$. So, along path BA, we have

$$\Delta U_{AB} = C_V (T_B - T_A)$$

$$\Rightarrow \quad \Delta U_{AB} = \left(\frac{3R}{2}\right)(4000 - 2000) = 3000R$$

$$\Rightarrow \quad \Delta U_{AB} = (3000)(8.3) = 24900\ \text{J}$$

Similarly, $\Delta U_{BC} = C_V (T_C - T_B)$

$$\Delta U_{BC} = \left(\frac{3R}{2}\right)(2000 - 4000) = -3000R$$

$$\Rightarrow \quad \Delta U_{BC} = -(3000)(8.3) = -24900\ \text{J}$$

$$\Delta U_{CD} = C_V (T_D - T_C)$$

$$\Rightarrow \quad \Delta U_{CD} = \left(\frac{3R}{2}\right)(1000 - 2000) = -1500R$$

$$\Rightarrow \quad \Delta U_{CD} = -(1500)(8.3) = -12450\ \text{J}$$

$$\Delta U_{DA} = C_V (T_A - T_D)$$

$$\Rightarrow \quad \Delta U_{DA} = \left(\frac{3R}{2}\right)(2000 - 1000) = 1500R$$

$$\Rightarrow \quad \Delta U_{DA} = (1500)(8.3) = 12450\ \text{J}$$

So, we observe that for the complete cyclic process

$$\Delta U = 0$$

Work done by gas during path AB at constant pressure is

$$W_{AB} = P(V_2 - V_1) = R(T_1 - T_2)$$

$$\Rightarrow \quad W_{AB} = (8.3)(2000) = 16600 \text{ J}$$

For both paths BC and DA, V = constant, so work done is zero.
Work done by gas during path CD at constant pressure is

$$W_{CD} = R(T_3 - T_2) = -1000R$$

$$\Rightarrow \quad W_{CD} = -(1000)(8.3) = -8300 \text{ J}$$

Net work done by the gas during the cycle is

$$W = 16600 - 8300 = 8300 \text{ J}$$

For process AB, we have

$$Q_{AB} = C_P(T_1 - T_2) = \left(\frac{5R}{2}\right)(2000)$$

$$\Rightarrow \quad Q_{AB} = 5000R = 5000(8.3) = 41500 \text{ J}$$

Since, Q_{AB} is positive, so heat is being absorbed during this process.
For process BC, we have

$$Q_{BC} = C_V(T_2 - T_1) = \left(\frac{3R}{2}\right)(-2000)$$

$$\Rightarrow \quad Q_{BC} = -3000R = -3000(8.3) = -24900 \text{ J}$$

Since, Q_{BC} is negative, so heat is being rejected during this process.
For process CD, we have

$$Q_{CD} = C_P(T_3 - T_2) = \left(\frac{5R}{2}\right)(-1000)$$

$$\Rightarrow \quad Q_{CD} = -2500R = -2500(8.3) = -20750 \text{ J}$$

Since, Q_{CD} is negative, so heat is being rejected during this process.
For process DA, we have

$$Q_{DA} = C_V(T_1 - T_3) = \left(\frac{3R}{2}\right)(3000)$$

$$\Rightarrow \quad Q_{DA} = 4500R = 4500(8.3) = 37350 \text{ J}$$

Since, Q_{DA} is positive, so heat is being absorbed during this process.
Efficiency of the complete cycle is

$$\eta = \frac{W_{\text{total}}}{\Sigma Q_{\oplus}} = \frac{W_{AB} + W_{CD}}{Q_{AB} + Q_{DA}}$$

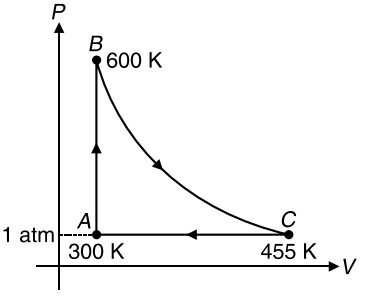
$$\Rightarrow \quad \eta = \left(\frac{8300}{41500 + 37350}\right) \times 100\% = 10.53\%$$

ILLUSTRATION 116

The P-V diagram of 0.2 mol of a diatomic ideal gas is shown in figure. Process BC is adiabatic. The value of γ for this gas is 1.4.

(a) Find the pressure and volume at points A, B and C.
(b) Calculate ΔQ, ΔW and ΔU for each of the three processes.
(c) Find the thermal efficiency of the cycle.

Take 1 atm $= 1 \times 10^5$ Nm^{-2}



SOLUTION

(a) $P_A = P_C = 1 \text{ atm} = 1.01 \times 10^5$ Nm^{-2}

Process AB is an isochoric process, so we have

$$P \propto T$$

$$\Rightarrow \quad \frac{P_B}{P_A} = \frac{T_B}{T_A}$$

$$\Rightarrow \quad P_B = \left(\frac{T_B}{T_A}\right)P_A = \left(\frac{600}{300}\right)(1 \text{ atm}) = 2 \text{ atm}$$

$$\Rightarrow \quad P_B = 2.02 \times 10^5 \text{ Nm}^{-2}$$

From ideal gas equation, we get

$$V = \frac{nRT}{P}$$

$$\Rightarrow \quad V_A = V_B = \frac{nRT_A}{P_A}$$

$$\Rightarrow \quad V_A = \frac{(0.2)(8.31)(300)}{(1.01 \times 10^5)} = 5 \times 10^{-3} \text{ m}^3$$

$$\Rightarrow \quad V_A = 5 \text{ litre}$$

and $V_C = \dfrac{nRT_C}{P_C} = \dfrac{(0.2)(8.31)(455)}{(1.01 \times 10^5)}$

$$\Rightarrow \quad V_C = 7.6 \times 10^{-3} \text{ m}^3$$

$$\Rightarrow \quad V_C \approx 7.6 \text{ litre}$$

State	P	V
A	1 atm	5 lt.
B	2 atm	5 lt.
C	1 atm	7.6 lt.

(b) **Process** AB is an isochoric process. Hence,

$$W_{AB} = 0$$

$$\Rightarrow \quad Q_{AB} = \Delta U_{AB} = nC_V \Delta T = n\left(\frac{5}{2}R\right)(T_B - T_A)$$

$$\Rightarrow \quad Q_{AB} = (0.2)\left(\frac{5}{2}\right)(8.31)(600-300) \approx 1246 \text{ J}$$

Process BC is an adiabatic process. Hence,

$$Q_{BC} = 0$$

$$\Rightarrow \quad W_{BC} = -\Delta U_{BC}$$

Since, $\Delta U_{BC} = nC_V \Delta T = nC_V (T_C - T_B)$

$$\Rightarrow \quad \Delta U_{BC} = (0.2)\left(\frac{5}{2}R\right)(455-600)$$

$$\Rightarrow \quad \Delta U_{BC} = (0.2)\left(\frac{5}{2}\right)(8.31)(-145) \text{ J}$$

$$\Rightarrow \quad \Delta U_{BC} \approx -602 \text{ J}$$

$$\Rightarrow \quad W_{BC} = -\Delta U_{BC} = 602 \text{ J}$$

Process CA is an isobaric process. Hence,

$$Q_{CA} = nC_P \Delta T = n\left(\frac{7}{2}R\right)(T_A - T_C)$$

$$\Rightarrow \quad Q_{CA} = (0.2)\left(\frac{7}{2}\right)(8.31)(300-455)$$

$$\Rightarrow \quad Q_{CA} \approx -902 \text{ J}$$

Also, $\Delta U_{CA} = nC_V \Delta T$

$$\Rightarrow \quad \Delta U_{CA} = \frac{Q_{CA}}{\gamma} \qquad \left\{\because \ \gamma = \frac{C_P}{C_V}\right\}$$

$$\Rightarrow \quad \Delta U_{CA} = -\frac{902}{1.4} \approx -644 \text{ J}$$

From FLTD, we get

$$W_{CA} = Q_{CA} - \Delta U_{CA} = -258 \text{ J}$$

(c) Efficiency of the cycle

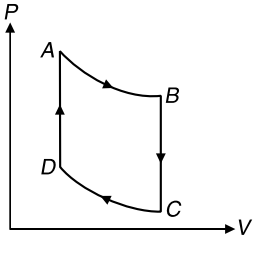
$$\eta = \frac{W_{\text{Total}}}{|Q_{+\text{ve}}|} \times 100\%$$

$$\Rightarrow \quad \eta = \frac{344}{1246} \times 100\%$$

$$\Rightarrow \quad \eta = 27.6\%$$

ILLUSTRATION 117

One mole of a monatomic ideal gas is taken through the cycle shown in figure:



$A \to B$: adiabatic expansion
$B \to C$: cooling at constant volume
$C \to D$: adiabatic compression
$D \to A$: heating at constant volume

The pressure and temperature at A, B etc., are denoted by P_A, T_A, P_B, T_B etc., respectively. Given that $T_A = 1000$ K, $P_B = \left(\frac{2}{3}\right)P_A$ and $P_C = \left(\frac{1}{3}\right)P_A$, calculate the following quantities:

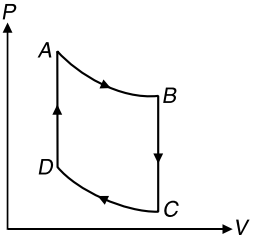
(a) The work done by the gas in the process $A \to B$
(b) The heat lost by the gas in the process $B \to C$
(c) The temperature T_D

Given that $\left(\frac{2}{3}\right)^{2/5} = 0.85$

SOLUTION

Given $T_A = 1000\,\text{K}$, $P_B = \frac{2}{3}P_A$ and $P_C = \frac{1}{3}P_A$

Number of moles, $n = 1$ and $\gamma = \frac{C_P}{C_V} = \frac{5}{3}R$ (monatomic)



(a) $A \to B$ is an Adiabatic Process, therefore

$$P_A^{1-\gamma} T_A^{\gamma} = P_B^{1-\gamma} T_B^{\gamma}$$

$$\Rightarrow \quad T_B = T_A \left(\frac{P_A}{P_B}\right)^{\frac{1-\gamma}{\gamma}} = (1000)\left(\frac{3}{2}\right)^{\frac{1-5/3}{5/3}}$$

$$\Rightarrow \quad T_B = (1000)\left(\frac{3}{2}\right)^{-\frac{2}{5}} = (1000)\left(\frac{2}{3}\right)^{\frac{2}{5}}$$

$$\Rightarrow \quad T_B = (1000)(0.85)$$

$$\Rightarrow \quad T_B = 850 \text{ K}$$

Now, work done in the process $A \to B$ is

$$W_{AB} = \frac{R}{1-\gamma}(T_B - T_A) = \frac{831}{1-\left(\frac{5}{3}\right)}(850-1000)$$

$$\Rightarrow \quad W_{AB} = 1869.75 \text{ J}$$

(b) $B \to C$ is an Isochoric Process, so V =constant, hence

$$\Rightarrow \quad \frac{T_B}{T_C} = \frac{P_B}{P_C}$$

$$\Rightarrow \quad T_C = \left(\frac{P_C}{P_B}\right)T_B = \left(\frac{\left(\frac{1}{3}\right)P_A}{\left(\frac{2}{3}\right)P_A}\right)850 \text{ K}$$

$\Rightarrow T_C = 425\ \text{K}$

In an Isochoric Process, we have

$$Q_{BC} = \Delta U = nC_V\Delta T = (1)\left(\frac{3}{2}R\right)(T_C - T_B)$$

$$\Rightarrow Q_{BC} = \left(\frac{3}{2}\right)(8.31)(425-850)$$

$$\Rightarrow Q_{BC} = -5297.6\,\text{J}$$

Therefore, heat lost in the process BC is 5297.6 J

(c) CD and AB are Adiabatic Process. Therefore, we have

$$P_C^{1-\gamma}T_C^{\gamma} = P_D^{1-\gamma}T_D^{\gamma}$$

$$\Rightarrow \frac{P_C}{P_D} = \left(\frac{T_D}{T_C}\right)^{\frac{\gamma}{1-\gamma}} \quad \ldots(1)$$

and $P_A^{1-\gamma}T_A^{\gamma} = P_B^{1-\gamma}T_B^{\gamma}$

$$\Rightarrow \frac{P_A}{P_B} = \left(\frac{T_B}{T_A}\right)^{\frac{\gamma}{1-\gamma}} \quad \ldots(2)$$

Multiplying equations (1) and (2), we get

$$\frac{P_CP_A}{P_DP_B} = \left(\frac{T_DT_B}{T_CT_A}\right)^{\frac{\gamma}{1-\gamma}} \quad \ldots(3)$$

Processes BC and DA are Isochoric i.e., V =constant, so

$$\frac{P_C}{P_B} = \frac{T_C}{T_B} \text{ and } \frac{P_A}{P_D} = \frac{T_A}{T_D}$$

Multiplying these two equations, we get

$$\frac{P_CP_A}{P_DP_B} = \frac{T_CT_A}{T_BT_D} \quad \ldots(4)$$

From equations (3) and (4), we get

$$\left(\frac{T_DT_B}{T_CT_A}\right)^{\frac{\gamma}{1-\gamma}} = \left(\frac{T_CT_A}{T_BT_D}\right)$$

$$\Rightarrow \left(\frac{T_CT_A}{T_DT_B}\right)^{\frac{1-\gamma}{\gamma}} = \left(\frac{T_CT_A}{T_BT_D}\right)$$

$$\Rightarrow \frac{T_CT_A}{T_BT_D} = 1$$

$$\Rightarrow T_D = \frac{T_CT_A}{T_B} = \frac{(425)(1000)}{850}\ \text{K}$$

$$\Rightarrow T_D = 500\ \text{K}$$

ILLUSTRATION 118

Three moles of an ideal gas $\left(C_p = \frac{7}{2}R\right)$ at pressure, P_A and temperature T_A is isothermally expanded to twice its initial volume. It is then compressed at constant pressure to its original volume. Finally, gas is compressed at constant volume to its original pressure P_A.

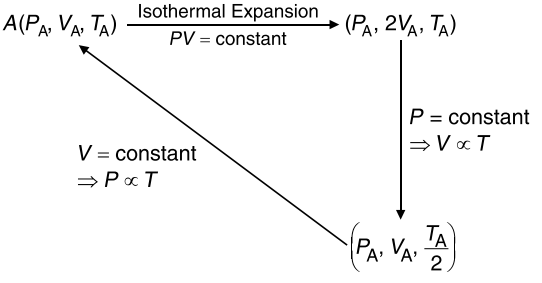
(a) Sketch P-V and P-T diagrams for the complete process.

(b) Calculate the net work done by the gas, and net heat supplied to the gas during the complete process.

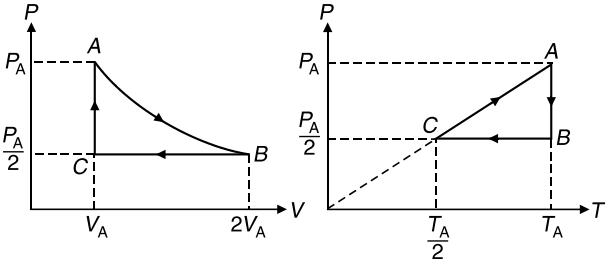
Take $\log_e(2) = 0.7$

SOLUTION

(a)



The P-V and P-T diagram are shown.



(b) Since, the process is cyclic, so

$$\Delta U = 0$$

From $FLTD$, we get

$$Q = W = W_{AB} + W_{BC} + W_{CA}$$

where, $W_{AB} = (3)RT_A\log_e(2)$

$$W_{BC} = P_A\Delta V = nR\Delta T$$

$$\Rightarrow W_{BC} = (3)R\left(\frac{T_A}{2} - T_A\right)$$

$$\Rightarrow W_{BC} = -\frac{3RT_A}{2}$$

and $W_{CA} = 0$ {Isochoric Process}

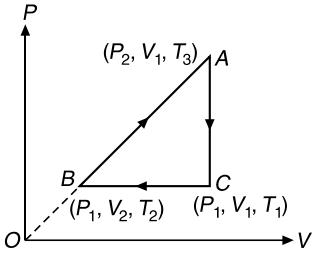
$$\Rightarrow W_{\text{net}} = W = 3RT_A\log_e(2) - \frac{3RT_A}{2}$$

$$\Rightarrow W_{\text{net}} = 0.6RT_A$$

$$\Rightarrow Q_{\text{net}} = W_{\text{net}} = 0.6RT_A$$

ILLUSTRATION 119

An ideal gas with the adiabatic exponent γ goes through a cycle shown in Figure within which the absolute temperature varies n fold in each process. Calculate the efficiency of this cycle.



SOLUTION

According to the problem, if we take $T_2 = T$, then

$$T_1 = nT \text{ and } T_3 = n^2T$$

Equation for process BA is $P = kV$ i.e., $PV^{-1} = k$ i.e., $x = -1$, so specific heat for this process is

$$C = C_V + \frac{R}{1-x} = C_V + \frac{R}{2}$$

$$\Rightarrow \quad C = \frac{R}{\gamma-1} + \frac{R}{2} = \frac{R}{2}\left(\frac{\gamma+1}{\gamma-1}\right)$$

Heat supplied to one mole of gas in process BA is

$$Q_{BA} = C\Delta T = \frac{R}{2}\left(\frac{\gamma+1}{\gamma-1}\right)(n^2-1)T$$

Work done by gas equals the area enclosed by the cyclic process, so

$$W_{\text{total}} = \frac{1}{2}(P_2 - P_1)(V_1 - V_2)$$

$$\Rightarrow \quad W_{\text{total}} = \frac{R}{2}(n^2T - nT - nT + T)$$

$$\Rightarrow \quad W_{\text{total}} = \frac{RT}{2}(n-1)^2$$

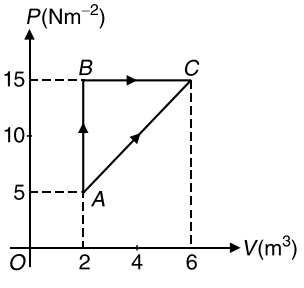
Efficiency of cycle is

$$\eta = \frac{\text{Work done}}{\text{Heat input}} = \frac{W}{Q_{BA}}$$

$$\Rightarrow \quad \eta = \frac{\frac{RT}{2}(n-1)^2}{\frac{RT}{2}\left(\frac{\gamma+1}{\gamma-1}\right)(n^2-1)} = \left(\frac{\gamma-1}{\gamma+1}\right)\left(\frac{n-1}{n+1}\right)$$

ILLUSTRATION 120

In the given figure, an ideal gas changes its state from A to state C by two paths ABC and AC.



(a) Find the path along which work done is the least

(b) The internal energy of gas at A is 10 J and amount of heat supplied to change its state to C through the path AC is 200 J. Calculate the internal energy at C.

(c) The internal energy of gas at state B is 20 J. Find the amount of heat supplied to the gas to go from A to B.

SOLUTION

Since the work done $W = \int P\,dV$ = area under $P-V$ curve, so

(a) $W_{ABC} = W_{AB} + W_{BC}$

i.e. $W_{ABC} = 0 + 15\times 4 = 60$ J

and $W_{AC} = \frac{1}{2}(5+15)\times(6-2) = 40$ J

thus, the work done along AC is least.

(b) According to First Law of Thermodynamics,

$$dQ = dU + dW$$

So, for path AC,

$$(U_C - U_A) = dQ - dW = 200 - 40 = 160 \text{ J}$$

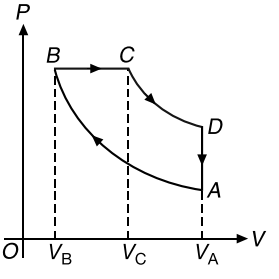
$$\Rightarrow \quad U_C = 160 + U_A = 160 + 10 = 170 \text{ J}$$

(c) For path AB, First Law of Thermodynamics yields

$$dQ = (U_B - U_A) + 0 = 20 - 10 = 10 \text{ J}$$

ILLUSTRATION 121

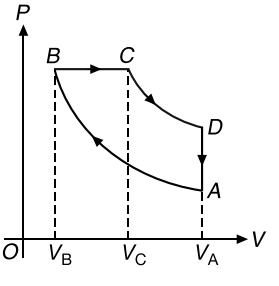
One mole of a diatomic ideal gas $(\gamma = 1.4)$ is taken through a cyclic process as shown in figure starting from point A.



In this cycle, the process AB is an adiabatic compression, BC is isobaric, CD an adiabatic expansion and DA is isochoric. The volume ratios are $\frac{V_A}{V_B} = 16$ and $\frac{V_C}{V_B} = 2$. The temperature at A is $T_A = 300$ °K. Calculate the temperature of the gas at the point B and D. Also calculate efficiency of the cycle.

SOLUTION

The respective cyclic process is shown in Figure.



The expansion and compression ratio are given to be $\frac{V_A}{V_B} = 16$ and $\frac{V_C}{V_B} = 2$

For an adiabatic process AB, we have

$$T_A V_A^{\gamma-1} = T_B V_B^{\gamma-1}$$

$$\Rightarrow \quad T_B = T_A\left(\frac{V_A}{V_B}\right)^{\gamma-1}$$

$$\Rightarrow \quad T_B = 300\times(16)^{0.4} \approx 909 \text{ K}$$

Similarly, for isobaric process BC, we have

$$\frac{T_C}{V_C} = \frac{T_B}{V_B}$$

$$\Rightarrow \quad T_C = T_B\left(\frac{V_C}{V_B}\right) \qquad \ldots(1)$$

$$\Rightarrow \quad T_C = 909\times 2 = 1818 \text{ K}$$

Similarly, for adiabatic process CD we have

$$T_C V_C^{\gamma-1} = T_D V_D^{\gamma-1}$$

$$\Rightarrow \quad T_D = T_C\left(\frac{V_C}{V_D}\right)^{\gamma-1}$$

Since we see that $V_D = V_A$, so

$$\frac{V_C}{V_D} = \frac{V_C}{V_A} = \frac{V_C}{V_B}\times\frac{V_B}{V_A} = 2\times\frac{1}{16} = \frac{1}{8}$$

So, from equation (1), we get

$$T_D = 1818\times\left(\frac{1}{8}\right)^{0.4} \approx 791 \text{ K}$$

The efficiency of cycle can be given as

$$\eta = 1-\left|\frac{Q_{\text{out}}}{Q_{\text{in}}}\right|$$

Processes AB and CD, both being adiabatic processes, no heat exchange takes place.

In isobaric process BC, the temperature of gas increases, so heat Q_1 absorbed by the system is given by

$$Q_1 = nC_P(T_C - T_B)$$

Similarly, in isochoric process DA, the temperature of gas decreases, so heat Q_2 rejected by the system is given by

$$Q_2 = nC_P(T_D - T_A)$$

The efficiency of any cyclic process is

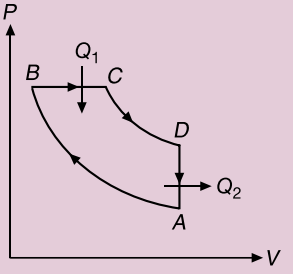
$$\eta = 1-\left|\frac{Q_{\text{out}}}{Q_{\text{in}}}\right| = 1-\frac{Q_2}{Q_1}$$

$$\Rightarrow \quad \eta = 1-\frac{C_V(T_D - T_A)}{C_P(T_C - T_B)} = 1-\frac{T_D - T_A}{\gamma(T_C - T_B)}$$

$$\Rightarrow \quad \eta = 1-\frac{791-300}{(1.4)(1818-909)}$$

$$\Rightarrow \quad \eta = 1-\frac{491}{1272.6} = 0.614 = 61.4\%$$

Conceptual Note(s)



While solving problems for calculating efficiency of a cyclic process, students must first identify the paths of heat input i.e. paths in which heat is being absorbed and the paths of heat output i.e. paths in which heat is being or heat is being rejected. For the discussed problem, heat Q_1 is being absorbed in the path BC and heat Q_2 is being rejected in the path DA. So we have shown an incoming arrow on the path BC and an outgoing arrow on the path DA as shown in figure.

LIMITATION OF FIRST LAW AND INTRODUCTION TO SECOND LAW OF THERMODYNAMICS

The First Law of Thermodynamics is just the Law of Conservation of energy generalised to include heat as a form of energy, that is, according to this law an increase in one form of energy must be accompanied by a decrease in some other form of energy and it puts no restrictions on the type of energy conversions that can occur. Furthermore it makes no distinction between heat and work. According to First Law, the internal energy of a body may be increased by either doing work on it or adding heat to it. But a major difference between heat and work exists which is not evident from the first law. For example, it is possible to convert work into heat completely whereas in practice it is impossible to convert the heat completely to work without changing the surroundings.

To overcome this situation the Second Law of Thermodynamics was introduced. This law establishes which processes in nature do or do not occur. Of all the processes permitted by the first law only certain types of energy conversions can take place. The following are some examples of processes that are consistent with the First Law of Thermodynamics but proceed in an order governed by the Second Law of Thermodynamics.

(a) When two bodies at different temperature are placed in contact with each other heat always flows from hotter body to colder but not vice-versa.

(b) Salt dissolves spontaneously in water, but extracting it back requires the use of some external agent.

(c) A rubber ball dropped onto the ground bounces several time and eventually comes to rest. The opposite process does not occur.

(d) The oscillations of a pendulum will slowly decrease in amplitude due to collisions with air molecules and friction at the point of suspension. Eventually the

pendulum will come to rest. Thus, the initial mechanical energy of the pendulum gets converted to thermal energy. The reverse transformation of energy doesn't occur.

These are all examples of Irreversible Processes, that is a category of processes occurring only in one direction. None of these processes occur in the opposite temporal order; if they did, they would violate the Second Law of Thermodynamics. This one way nature of Thermodynamic processes in fact establishes a direction of time. (which moves forward but never backward).

The Second Law of Thermodynamics, which can be stated in many equivalent way, has a lot of practical applications. For an engineering point of view, the most important application is the limited efficiency of heat engines. That is, the Second Law says that a machine capable of converting the thermal energy continuously into other forms of energy completely, can never be constructed.

THE SECOND LAW OF THERMODYNAMICS: REVISITED

Experimental evidence suggests strongly that it is impossible to build a heat engine that converts heat completely to work, that is, an engine with 100% thermal efficiency. This impossibility is the basis of one statement of the **Second Law of Thermodynamics**, as follows.

It is impossible for any system to undergo a process in which it absorbs heat from a reservoir at a single temperature and converts the heat completely into mechanical work, with the system ending in the same state in which it began.

The basis of the Second Law of Thermodynamics lies in the difference between the nature of internal energy and that of macroscopic mechanical energy. In a moving body the molecules have random motion, but superimposed on this is a coordinated motion of every molecule in the direction of the body's velocity. The kinetic energy associated with this coordinated macroscopic motion is what we call the kinetic energy of the moving body. The kinetic and potential energies associated with the random motion constitute the internal energy.

If the Second Law were not true, we could power an automobile or run a power plant by cooling the surrounding air. Neither of these impossibilities violates the First Law Thermodynamics. The Second Law, therefore, is not a deduction from the first but stands by itself as a separate law of nature. The First Law denies the possibility of creating or destroying energy; the Second Law limits the availability of energy and the ways in which it can be used and converted.

It is impossible for any process to have as its sole result the transfer of heat from a cooler to a hotter body.

ENTROPY

Entropy is a measure of disorder of molecular motion of a system. Greater is the disorder, greater is the entropy. The change in entropy i.e.

$$dS = \frac{\text{Heat absorbed by system}}{\text{Absolute temperature}}$$

or $$dS = \frac{dQ}{T}$$

This relation is called the mathematical form of Second Law of Thermodynamics.

Conceptual Note(s)

dQ is a path function but $\frac{dQ}{T}$ called Entropy is a state function.

$$\Rightarrow \int dS = S_{\text{final}} - S_{\text{initial}} = \Delta S = \frac{dQ}{T}$$

CALCULATION OF ENTROPY

For Solids and Liquids

(a) When heat given to a substance changes its state at constant temperature, then change in entropy

$$dS = \frac{dQ}{T} = \pm\frac{mL}{T}$$

where positive sign refers to heat absorption and negative sign to heat evolution.

(b) When heat given to a substance raises its temperature from T_1 to T_2, then change in entropy

$$dS = \int \frac{dQ}{T} = \int_{T_1}^{T_2} mc\frac{dT}{T} = mc\log_e\left(\frac{T_2}{T_1}\right)$$

$$\Rightarrow \Delta S = 2.3mc\log_{10}\left(\frac{T_2}{T_1}\right)$$

For a Perfect Gas

Perfect gas equation for n moles is $PV = nRT$

$$\Delta S = \int \frac{dQ}{T} = \int \frac{nC_V dT + PdV}{T} \qquad \{\because dQ = dU + dW\}$$

$$\Rightarrow \Delta S = \int \frac{nC_V dT + \frac{nRT}{V}dV}{T} \qquad \{\because PV = nRT\}$$

$$\Rightarrow \Delta S = nC_V\int_{T_1}^{T_2}\frac{dT}{T} + nR\int_{V_1}^{V_2}\frac{dV}{V}$$

$$\Rightarrow \quad \Delta S = nC_V \log_e\left(\frac{T_2}{T_1}\right) + nR\log_e\left(\frac{V_2}{V_1}\right)$$

Similarly in terms of T and P,

$$\Delta S = nC_P \log_e\left(\frac{T_2}{T_1}\right) - nR\log_e\left(\frac{P_2}{P_1}\right)$$

and in terms of P and V

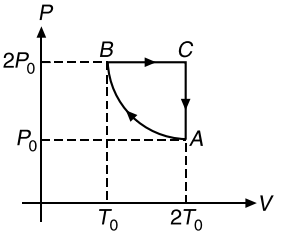
$$\Delta S = nC_V \log_e\left(\frac{P_2}{P_1}\right) + nC_P \log_e\left(\frac{V_2}{V_1}\right)$$

Test Your Concepts-IX

Based on Cyclic Process, Heat Engine and Refrigerator

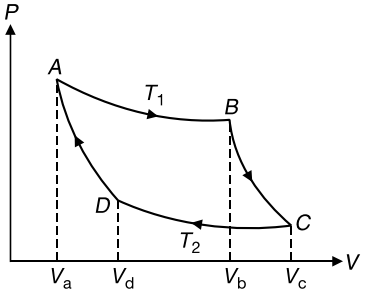
(Solutions on page H.93)

1. Two moles of an ideal monoatomic gas is taken through a cycle *ABCA* whose *PT* diagram is shown in Figure.



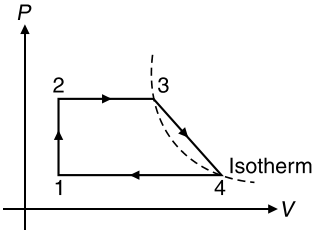
During the process *AB*, pressure and temperature of the gas vary such that PT = constant. If $T_0 = 300$ K, calculate work done by the gas in process *AB* and the heat absorbed or released by the gas in each of the process.

2. Two different adiabatic paths for the same gas intersect two isotherms at T_1 and T_2 as shown in the given *P-V* diagram. Calculate the efficiency of the process.

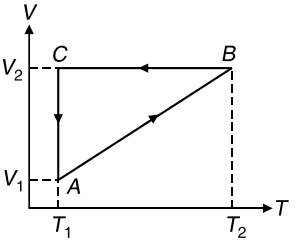


CYCLIC

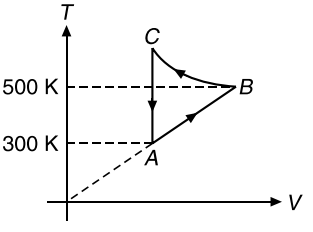
3. n moles of an ideal gas is made to undergo the cycle $1 \to 2 \to 3 \to 4 \to 1$ as shown in figure. Process $3 \to 4$ is a straight line. The gas temperatures in states 1, 2 and 3 are T_1, T_2 and T_3 respectively. Also points 3 and 4 lie on the same isotherm. Determine the work done by the gas during the cycle.



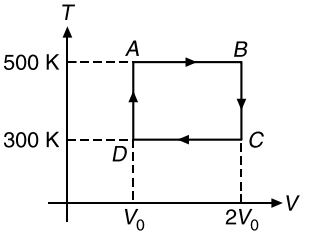
4. A cyclic process *ABCA* shown in the *V-T* diagram is performed with a constant mass of an ideal gas. Show the same process on a *P-V* diagram. (In the figure, *CA* is parallel to the *V*-axis and *BC* is parallel to the *T*-axis).



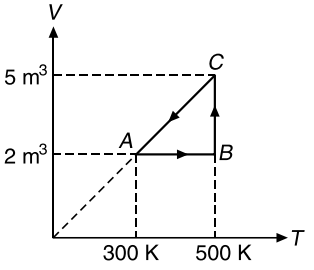
5. Consider the cyclic process *ABCA* shown in Figure. An ideal gas of 2 moles undergoes this process and a total of 1200 J heat is rejected by the gas in the complete cycle. Calculate work done by the gas during the process *BC*.



6. The *VT* graph for a thermodynamic process in which gas temperature changes from 300 K to 500 K during isochoric part of the cycle and volume of gas is doubled during isothermal part of the cycle is shown in Figure. This graph is plotted for 2 moles of a gas. Calculate total heat supplied by gas in the complete cycle.

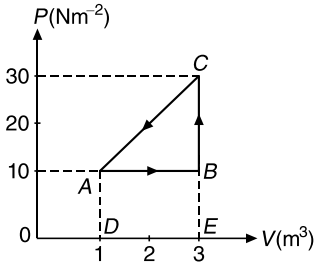


7. A process *ABCA* is performed on one mole of an ideal gas whose *VT* diagram is shown in Figure. Calculate the net heat supplied to the gaseous system during the process. Take $R = 8.3\ \text{Jmol}^{-1}\text{K}^{-1}$ and $\ln(2.5) \approx 0.9$



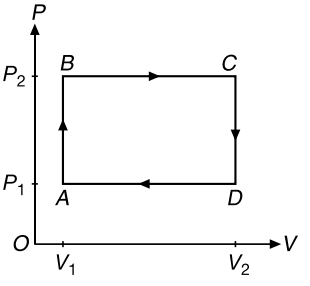
8. An ideal gas whose ratio of specific heats is γ goes through a direct (clockwise) cycle consisting of adiabatic, isobaric and isochoric lines. Find the efficiency of the cycle if in the adiabatic process the volume of ideal gas increase *n* fold.

9. An ideal gas is taken round a cyclic thermodynamic process *ABCA* as shown in Figure.



If the internal energy of the gas at point *A* is assumed zero while at *B* it is 50 J and heat absorbed by the gas in process *BC* is 90 J, then calculate the internal energy of gas at point *C*, heat energy absorbed by gas in process *AB*, heat energy rejected or absorbed by gas in the process *CA* and net work done by the gas in the complete cycle *ABCA*.

10. An ideal gas whose adiabatic exponent is equal to γ goes through a cycle consisting of two isochoric and two isobaric lines as shown in figure. find the efficiency of such a cycle, if the absolute temperature of the gas rises *n* times both in isochoric heating and in isobaric expansion.



TRANSFER OF HEAT

MODES OF HEAT TRANSFER

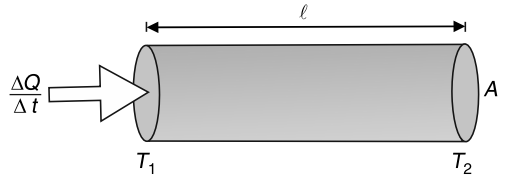
The transfer of heat between a system and its surroundings can occur through these three transfer mechanisms.

(a) Conduction
(b) Convection
(c) Radiation

HEAT CONDUCTION

It is the process in which heat is transferred from the place of higher temperature to the place of lower temperature due to molecular vibration of particles without their actual motion.

To start with the conduction process every cross section of body absorbs heat handing over excess heat to next section but at a later stage when steady state is reached then temperature of all the parts of the body becomes constant.



Consider a cylindrical metal bar of area A length l. Across this area and length is maintained a temperature difference as shown and remaining portion (i.e. curved portion) is kept insulated from surroundings to avoid leakage of heat. At steady state the rate of flow of heat $\frac{\Delta Q}{\Delta t}$ (or heat current) becomes constant.

Experimentally it has been observed that heat current is directly proportional to

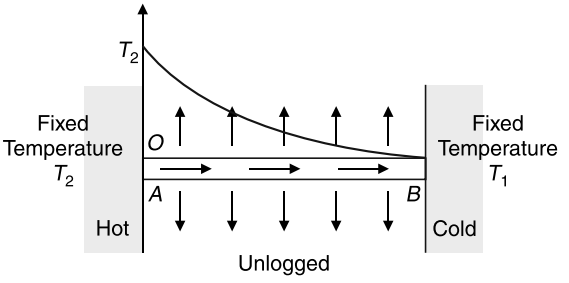
(a) the temperature gradient $\left(\frac{T_1 - T_2}{l}\right)$ maintained across the faces
(b) area A perpendicular to temperature gradient through which heat enters.

$$\frac{\Delta Q}{\Delta t} = KA\left(\frac{T_1 - T_2}{l}\right)$$

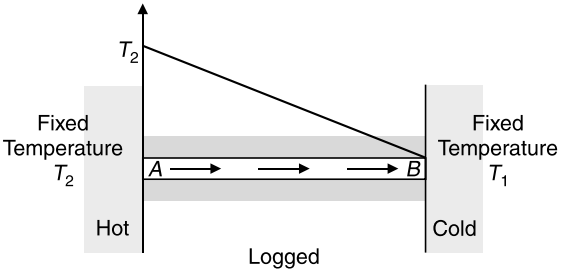
where K is **coefficient of thermal conductivity** of the bar.

Also, we can write $\frac{dQ}{dt} = -KA\left(\frac{dT}{dx}\right)$

Please note that the negative sign indicates the decrease in temperature along the direction of heat flow.



Here $\frac{dQ}{dt}$ is the time rate of heat transfer across the area A, $\frac{dT}{dx}$ is called the **temperature gradient**, and k is a constant of proportionality called the **thermal conductivity**. We choose the direction of heat flow to be the direction in which x increases; since heat flows in the direction of decreasing T, we introduce a minus sign in equation, (i.e., we wish $\frac{dQ}{dt}$ to be positive when $\frac{dT}{dx}$ is negative).



The phenomenon of heat conduction also shows that the concepts of heat and temperature are distinctly different. Different rods, having the same temperature difference between their ends, may transfer entirely different quantities of heat in the same time.

ILLUSTRATION 122

A copper rod 2 m long has a circular cross-section of radius 1 cm. One end is kept at 100 °C and the other at 0 °C and the surface is insulated so that negligible heat is lost through the surface. Calculate the

(a) thermal resistance of the bar.
(b) thermal current H.
(c) temperature gradient $\frac{dT}{dx}$ and
(d) temperature 25 cm from the hot end.

Thermal conductivity of copper is 401 $Wm^{-1}K^{-1}$

SOLUTION

(a) Thermal resistance $R_{Th} = \frac{l}{kA} = \frac{l}{k(\pi r^2)}$

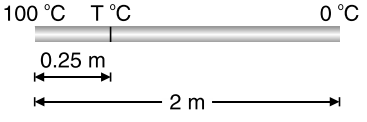
$$\Rightarrow \quad R = \frac{(2)}{(401)(\pi)(10^{-2})^2} = 15.9 \text{ KW}^{-1}$$

(b) Thermal current, $H = \frac{\Delta T}{R} = \frac{100}{15.9} = 6.3$ W

(c) Temperature gradient

$$\frac{\Delta T}{\Delta x} = \frac{0-100}{2} = -50 \text{ Km}^{-1} = -50 \text{ °Cm}^{-1}$$

(d) Let T (in °C) be the temperature at 25 cm from the hot end, then we have

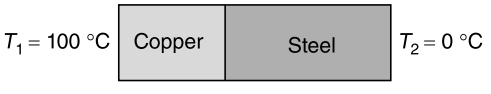


$$\frac{100-T}{0.25} = \frac{100-0}{2}$$

$$\Rightarrow \quad T = 87.5 \text{ °C}$$

ILLUSTRATION 123

A copper bar of length 75 cm and a steel bar of length 125 cm, both having circular cross-section with diameter 2 cm are joined together end to end as shown in Figure.



The free ends of the copper and steel bars are maintained at 100 °C and 0 °C respectively. The surfaces of the bars are thermally insulated. Calculate the temperature of the copper steel junction and the heat transmitted per unit time across the junction if coefficient of thermal conductivity of copper is 9.2×10^{-2} kcalm^{-1}°C^{-1}s^{-1} and that of steel is 1.1×10^{-2} kcalm^{-1}s^{-1}°C^{-1}.

SOLUTION

If T be the temperature of junction, then in steady state, the rate of flow of heat in the copper bar must be same as that in the steel bar i.e.,

$$\frac{dQ}{dt} = \frac{k_c A(T_1 - T)}{l_c} = \frac{k_s A(T - T_2)}{l_s}$$

$$\Rightarrow \quad \left(\frac{9.2\times10^{-2}}{1.1\times10^{-2}}\right)\left(\frac{125}{75}\right) = \frac{T-0}{100-T}$$

$$\Rightarrow \quad T = 93.3 \text{ °C}$$

The heat flowing through the junction per second is

$$\frac{dQ}{dt} = \frac{k_c A(T_1 - T)}{l_c} = \frac{(9.2\times10^{-2})(3.14\times10^{-4})(100-93.3)}{0.75}$$

$$\Rightarrow \quad \frac{dQ}{dt} = 2.58\times10^{-4} \text{ Js}^{-1}$$

ILLUSTRATION 124

A rigid box contains one mole of a monatomic ideal gas. The walls of the box have total surface area A, thickness D and thermal conductivity k. Initially the gas is at a temperature T_0 and pressure P_0. The temperature of the surroundings is $\frac{T_0}{2}$. Find the temperature and pressure of the gas as a function of time.

SOLUTION

Since, the box is rigid, so V =constant

$$\Rightarrow \quad \frac{dQ}{dt} = \frac{dU}{dt}$$

Since, $dU = (1)C_V dT$ and $\frac{dQ}{dt} = -kA\frac{\Delta T}{\Delta x}$

$$\Rightarrow \quad \frac{dQ}{dt} = -C_V\left(\frac{dT}{dt}\right) = \frac{kA\left(T - \frac{T_0}{2}\right)}{D}$$

$$\Rightarrow \quad \int_{T_0}^{T} -\left(\frac{dT}{T - \frac{T_0}{2}}\right) = \frac{kA}{DC_V}\int_0^t dt$$

$$\Rightarrow \quad T = \frac{T_0}{2}\left(1 + e^{-\frac{kAt}{DC_V}}\right)$$

For a monatomic gas, $C_V = \frac{3R}{2}$

$$\Rightarrow \quad T = \frac{T_0}{2}\left(1 + e^{-\left(\frac{2kA}{3DR}\right)t}\right)$$

Since, V =constant, so $\frac{P_0}{T_0} = \frac{P}{T}$

$$\Rightarrow \quad P = \frac{P_0}{2}\left(1 + e^{-\left(\frac{2kA}{3DR}\right)t}\right)$$

ILLUSTRATION 125

A cylindrical brass boiler of radius 15 cm and thickness 1 cm is filled with water and placed on an electric heater. If the water boils at the rate of 200 gs^{-1}, estimate the temperature of the heater filament. Given that the thermal conductivity of brass is 109 Js^{-1}m^{-1}°C^{-1} and latent heat of vaporisation of water is 2.256×10^3 Jg^{-1}.

SOLUTION

Since water is boiling at the rate of 200 gs^{-1}, the rate at which heat energy is supplied by the heater to water is

$$\frac{Q}{t} = \frac{mL}{t} = \left(\frac{m}{t}\right)L = 4.512\times10^5 \text{ Js}^{-1} \qquad \ldots(1)$$

Radius of the boiler is $r = 15$ cm $= 0.15$ m
So, base area of the boiler is

$$A = \pi r^2 = 3.142 \times (0.15)^2 = 0.0707 \text{ m}^2$$

Thickness of brass is $d = 1$ cm $= 1 \times 10^{-2}$ m

Thermal conductivity of brass is 109 $\text{Js}^{-1}\text{m}^{-1}{}^\circ\text{C}^{-1}$

Temperature of boiling water is $T_w = 100\ ^\circ\text{C}$

If T_f is the temperature of the filament, then the rate at which heat energy is transmitted through the base is given by

$$\frac{Q}{t} = \frac{kA(T_f - T_w)}{d} \quad \ldots(2)$$

Using (1) and substituting the values of k, A, T_w and d in equation (2), we get

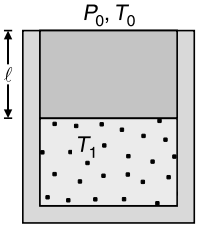
$$\frac{(109)(0.0707)(T_f - 100)}{1 \times 10^{-2}} = 4.512 \times 10^5$$

$$\Rightarrow \quad T_f - 100 = 585.5$$

$$\Rightarrow \quad T_f = 685.5\ ^\circ\text{C}$$

ILLUSTRATION 126

n moles of a monatomic gas at temperature T_1 are trapped under a piston having area A, length l, density ρ and thermal conductivity K. P_0 and T_0 are pressure and temperature of the atmosphere. Find length of the gas column as a function of time. Neglect friction and heat loss through the walls of container and sides of piston.



SOLUTION

If P be the pressure of the gas, then for equilibrium of piston, we have

$$P_0 A + \rho l g A = \rho A$$

$$\Rightarrow \quad P = P_0 + \rho l g = \text{constant}$$

If T is the temperature of gas at any time t, then the rate of heat flow through the piston is

$$\frac{dQ}{dt} = \frac{KA(T - T_\theta)}{l} = -nC_P\left(\frac{dT}{dt}\right) = -\frac{5}{2}nR\frac{dT}{dt}$$

$$\Rightarrow \quad \frac{dT}{T - T_0} = -\frac{2KA}{5nlR}dt$$

$$\Rightarrow \quad \int_{T_1}^{T} \frac{dT}{T - T_0} = -\frac{2KA}{5nlR}\int_0^t dt$$

$$\Rightarrow \quad T = T_0 + (T_1 - T_0)e^{-\frac{2KAt}{5nlR}}$$

Now if x is length of gas column at this instant, then

$$P(Ax) = nRT$$

$$\Rightarrow \quad x = \frac{nRT}{PA} = \frac{nR}{A(P_0 + \rho l g)}\left[T_0 + (T_1 - T_0)e^{-\frac{2KAt}{5nlR}}\right]$$

WIEDEMANN FRANZ LAW

This law deals with the physical ideas about flow of heat through conductors and insulators at microscopic level. According to the modern electronic theory of thermal conduction, the flow of heat in a body from hotter part to colder part is due to the motion of free electrons.

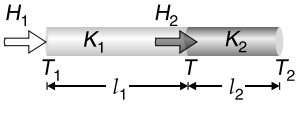
The electrons in the outermost orbit of an atom in a metal are loosely bound to the nucleus. When a metallic rod is heated at one end, the atoms acquire greater kinetic energy and their amplitude of vibration increases. A part of this energy is gained by the electrons in the outermost orbit. These electrons drift away from the atom and move towards the cooler portion of rod. These energetic electrons collide against atoms in cooler portion of the rod and impart their energy to these atoms. These atoms also get agitated and the electrons in their outermost orbits are made free. Thus, the heat energy is transported from the hotter to the colder parts by the motion of free electrons. It may be mentioned here that a good conductor of heat is also a good conductor of electricity.

In the year 1853, Wiedemann and Franz established a relationship between thermal conductivity K and electrical conductivity σ. The ratio of thermal and electrical conductivities is the same for the metals at a particular temperature and is proportional to the absolute temperature of the metal. If T is the absolute temperature, then $\frac{K}{\sigma} \propto T$

$$\Rightarrow \quad \frac{K}{\sigma T} = \text{constant}$$

TEMPERATURE OF JUNCTION

Let two rods of thermal conductivities K_1, K_2, lengths l_1, l_2 and equal cross-sectional area A connected in series. In steady state the temperatures of ends of rod are T_1 and T_2 and the temperature of junction is T.



In steady state the rate of flow of heat in both rods is the same. Therefore

$$\left(\frac{Q}{t}\right)_1 = \left(\frac{Q}{t}\right)_2$$

$$\Rightarrow \quad \frac{K_1A(T_1-T)}{l_1}=\frac{K_2A(T-T_2)}{l_2}$$

$$\Rightarrow \quad T=\frac{(K_1/l_1)T_1+(K_2/l_2)T_2}{(K_1/l_1)+(K_2/l_2)}=\frac{K_1T_1l_1+K_2T_2l_1}{K_1l_2+K_2l_1}$$

THERMOMETRIC CONDUCTIVITY (OR DIFFUSIVITY)

The thermometric conductivity or diffusivity is defined as the ratio of the coefficient of thermal conductivity to the thermal capacity per unit volume of the material. We have thermal capacity is mc. So, thermal capacity per unit volume is $\frac{mc}{V}=\rho c$, where ρ is density of substance. So, diffusivity D is

$$D=\frac{K}{\rho c}$$

THERMAL RESISTANCE

The hindrance offered by a body to the flow of heat is called its thermal resistance and is given by

$$R=\frac{\text{Temperature Difference }(\Delta T)}{\text{Heat Current }(H)}=\frac{\Delta T}{H}=\frac{l}{KA}$$

where l is length of rod area A having temperature difference ΔT across its ends. The unit of thermal resistance is $^\circ\text{Cscal}^{-1}$.

If different rods are connected in series, then heat flowing per second is same i.e., $H_1=H_2=H_3$

and net thermal resistance is

$$R_S=R_1+R_2+R_3$$

If different rods are connected in parallel, then net thermal resistance R is given by

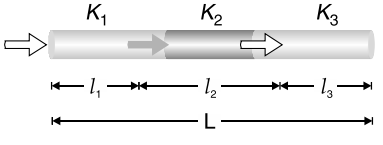
$$\frac{1}{R_P}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$$

and temperature difference ΔT is the same.

SLABS OF DIFFERENT MATERIALS IN SERIES

Let three slabs each of cross-sectional area A, lengths l_1, l_2, l_3 and conductivities K_1, K_2, K_3 respectively be connected in series. Then in series

$$R_S=R_1+R_2+R_3$$



If K_s is equivalent conductivity, then from relation $R=\frac{l}{KA}$, we get

$$\frac{L}{K_SA}=\frac{l_1}{K_1A}+\frac{l_2}{K_2A}+\frac{l_3}{K_3A}$$

$$\Rightarrow \quad \frac{l_1+l_2+l_3}{K_S}=\frac{l_1}{K_1}+\frac{l_2}{K_2}+\frac{l_3}{K_3}$$

For 3 slabs of equal length

$$\frac{3l}{K_S}=\frac{l}{K}+\frac{l}{K_2}+\frac{l}{K_3}$$

$$\Rightarrow \quad \frac{1}{K_S}=\frac{\frac{1}{K_1}+\frac{1}{K_2}+\frac{1}{K_3}}{3}$$

Special case: For two slabs of equal length

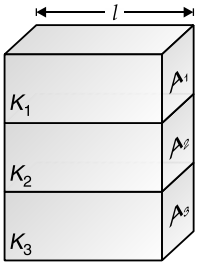
$$\frac{2l}{K_S}=\frac{l}{K_1}+\frac{l}{K_2}$$

$$\Rightarrow \quad K_S=\frac{2K_1K_2}{K_1+K_2}$$

SLABS OF DIFFERENT MATERIALS IN PARALLEL

Let three slabs each of length l, areas A_1, A_2, A_3 and thermal conductivities K_1, K_2, K_3 respectively be connected in parallel. Then in parallel arrangement

$$\frac{1}{R_P}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$$



If K_P is equivalent conductivity, then from relation $R=\frac{l}{KA}$, we get

$$\frac{K_PA}{l}=\frac{K_1A_1}{l}+\frac{K_2A_2}{l}+\frac{K_3A_3}{l}$$

$$\Rightarrow \quad K_P=\frac{K_1A_1+K_2A_2+K_3A_3}{A}$$

$$\Rightarrow \quad K_P=\frac{K_1A_1+K_2A_2+K_3A_3}{A_1+A_2+A_3}$$

For three slabs of equal areas

$$K_P=\frac{K_1+K_2+K_3}{3}$$

Special Case: For two slabs of equal area

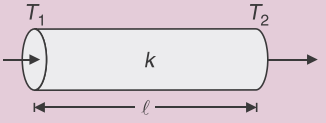
$$K_P = \frac{K_1 + K_2}{2}$$

Problem Solving Technique(s)

Thermal and Electrical Conductivity. There exists an useful analogue between thermal conductivity and electrical conductivity

Thermal Conduction

$$\frac{dQ}{dt} = kA\left(\frac{T_1 - T_2}{\ell}\right)$$



(a) Heat flows from higher temperature to lower temperature. The rate of heat flow is called the heat current.

$$I = \frac{dQ}{dt}$$

(b) Thermal resistance is defined as

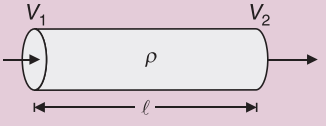
$$R_T = \frac{\ell}{kA}$$

(c) Ohm's Law for heat conduction may be stated as

$$I = \frac{T_1 - T_2}{R_T}$$

Electrical Conduction

$$\frac{dq}{dt} = \frac{A}{\rho}\left(\frac{V_1 - V_2}{\ell}\right)$$



(a) Charges flow from higher potential to lower potential

(b) The rate of charge flow is called the electric current

$$I = \frac{dq}{dt}$$

(c) Electrical resistance is defined as

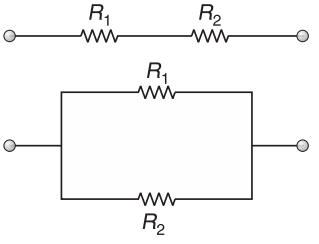
$$R_E = \frac{\rho\ell}{A}$$

(d) Ohm's Law for electric conduction may be stated as

$$I = \frac{V_1 - V_2}{R_E}$$

Using the above analogy, a problem of heat conduction may be transformed into a problem of electrical conduction and can be easily solved using the formulae of electric circuits.

For series and parallel combination, the equivalent electrical or the equivalent thermal resistance is given by

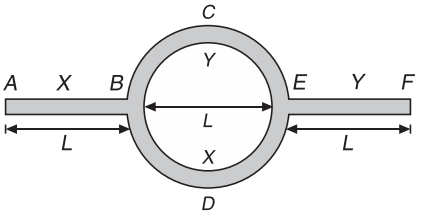


$$R = R_1 + R_2 \qquad \text{(series)}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \qquad \text{(parallel)}$$

ILLUSTRATION 127

Rods made of materials X and Y are connected as shown in Figure.



The cross-sectional area of all the rods is same. If the end A is maintained at 80 °C and the end F is maintained at 10 °C. Calculate the temperatures of junctions B and E in steady state. Given that thermal conductivity of material X is double that of Y.

SOLUTION

The thermal resistances of different rods are

$$R_{AB} = \frac{L}{k_X A},\ R_{EF} = \frac{L}{k_Y A},\ R_{BDE} = \frac{\pi L/2}{k_X A},\ R_{BCE} = \frac{\pi L/2}{k_Y A}$$

If temperature of junction B and E are taken as T_B and T_E, then in steady state, at the junction B, we have

$$\frac{T_A - T_B}{R_{AB}} = \frac{T_B - T_E}{R_{BCE}} + \frac{T_B - T_E}{R_{BDE}}$$

$$\Rightarrow \quad \frac{80 - T_B}{L/k_X A} = \frac{T_B - T_E}{\pi L/2k_Y A} + \frac{T_B - T_E}{\pi L/2k_X A}$$

Since, $k_X = 2k_Y$

$$\Rightarrow \quad 80 - T_B = \left(\frac{T_B - T_E}{\pi}\right) + \left(\frac{T_B - T_E}{2\pi}\right)$$

$$\Rightarrow \quad 80 - T_B = \frac{3}{2\pi}(T_B - T_E) \qquad \ldots(1)$$

Similarly, at junction E, we have

$$\frac{T_B - T_E}{R_{BCE}} + \frac{T_B - T_E}{R_{BDE}} = \frac{T_E - 10}{R_{EF}}$$

$$\Rightarrow \quad \frac{T_B - T_E}{\pi L/2k_Y A} + \frac{T_B - T_E}{\pi L/2k_X A} + \frac{T_E - 10}{L/k_Y A}$$

$$\Rightarrow \quad \frac{2(T_B - T_E)}{\pi} + \frac{4(T_B - T_E)}{\pi} = T_E - 10$$

$$\Rightarrow \quad \frac{3}{4\pi}(T_B - T_E) = T_E - 10 \qquad \ldots(2)$$

Solving equations (1) and (2), we get

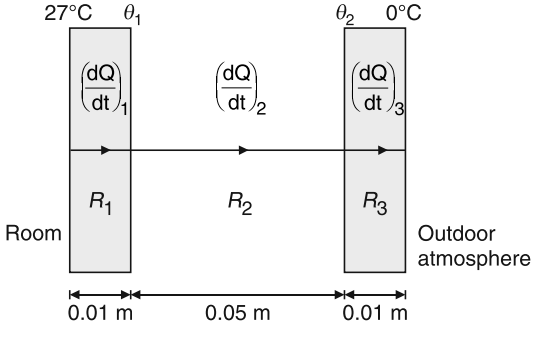
$T_E = 19.74\ ^\circ\text{C}$ and $T_B = 60.52\ ^\circ\text{C}$

ILLUSTRATION 128

A double-pane window used for insulating a room thermally from outside consists of two glass sheets each of area $1\ \text{m}^2$ and thickness 0.01 m separated by a 0.05 m thick stagnant air space. In the steady state, the room glass inter-face and the glass-outdoor interface are at constant temperatures of 27 °C and 0 °C respectively. Calculate the rate of heat flow through the window pane. Also find the temperatures of other interfaces. Given thermal conductivities of glass and air as 0.8 and 0.08 $\text{W m}^{-1}\ \text{K}^{-1}$ respectively.

SOLUTION

Let θ_1 and θ_2 be the temperatures of the two interfaces as shown in figure.



Thermal resistance, $R = \dfrac{l}{KA}$

$$\Rightarrow \quad R_1 = R_3 = \frac{(0.01)}{(0.8)(1)} = 0.0125\ \text{KW}^{-1} \text{ or } ^\circ\text{CW}^{-1}$$

and $\quad R_2 = \dfrac{(0.05)}{(0.08)(1)} = 0.625\ \text{KW}^{-1}$ or $^\circ\text{CW}^{-1}$

Now the rate of heat flow $\left(\dfrac{dQ}{dt}\right)$ will be equal from all the three sections and since rate of heat flow is given by

$$\frac{dQ}{dt} = \frac{\text{Temperature difference}}{\text{Thermal resistance}}$$

and $\left(\dfrac{dQ}{dt}\right)_1 = \left(\dfrac{dQ}{dt}\right)_2 = \left(\dfrac{dQ}{dt}\right)_3$

Therefore, $\dfrac{27 - \theta_1}{0.0125} = \dfrac{\theta_1 - \theta_2}{0.625} = \dfrac{\theta_2 - 0}{0.0125}$

Solving this equation, we get

$\theta_1 = 26.48\ ^\circ\text{C}$ and $\theta_2 = 0.52\ ^\circ\text{C}$

and $\quad \dfrac{dQ}{dt} = \dfrac{27 - \theta_1}{0.0125}$

$$\Rightarrow \quad \frac{dQ}{dt} = \frac{27 - 26.48}{0.0125} = 41.6\ \text{W}$$

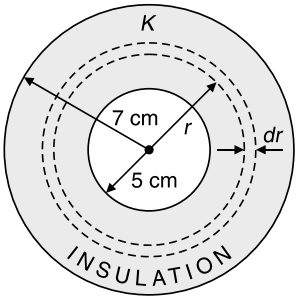
ILLUSTRATION 129

A steam pipe of radius 5 cm carries steam at 100 °C. The pipe is covered by a jacket of insulating material 2 cm thick having a thermal conductivity 0.07 $\text{Wm}^{-1}\text{K}^{-1}$. If the temperature at the outer wall of the pipe jacket is 20 °C, how much heat is lost through the jacket per metre length in an hour?

SOLUTION

Thermal resistance per meter length of an element at distance r of thickness dr is

$$dR = \frac{dr}{k(2\pi r)} \qquad \left\{\because\ R = \frac{l}{kA}\right\}$$



Total resistance $R = \displaystyle\int_{1=5\ \text{cm}}^{2=7\ \text{cm}} dR$

$$\Rightarrow \quad R = \frac{1}{2\pi k}\left[\int_{5\times10^{-2}}^{7\times10^{-2}} \frac{dr}{r}\right] = \frac{1}{2\pi k}\log_e\left(\frac{7}{5}\right)$$

$$\Rightarrow \quad R = \frac{1}{(2\pi)(0.07)}\log_e(1.4) = 0.765\ \text{KW}^{-1}$$

Since, Heat current $H = \dfrac{\text{Temperature difference}}{\text{Thermal resistance}}$

$$\Rightarrow \quad H = \frac{(100 - 20)}{0.765} = 104.6\ \text{watt}$$

So, heat lost Q in one hour is

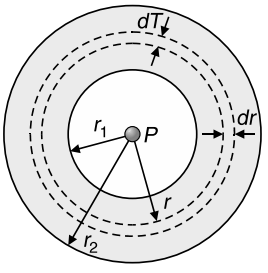
$$Q = Ht = (104.6\ \text{Js}^{-1})(3600\ \text{s}) = 3.76\times10^5\ \text{J}$$

ILLUSTRATION 130

A point source of heat of power P is placed at the centre of a spherical shell of mean radius R. The material of the shell has thermal conductivity K. Calculate the thickness of the shell if temperature difference between the outer and inner surfaces of the shell in steady state is T.

SOLUTION

Consider a concentric spherical shell of radius r and thickness dr as shown in Figure.



In steady state, the rate of heat flow (heat current) through this shell will be

$$H = \frac{\Delta T}{R} = \frac{(-dT)}{\dfrac{dr}{(K)(4\pi r^2)}} \qquad \left\{\because\ R = \frac{l}{KA}\right\}$$

$$\Rightarrow \quad H = -(4\pi K r^2)\frac{dT}{dr}$$

Here negative sign is used because with increase in r, T decreases.

$$\Rightarrow \quad \int_1^2 \frac{dr}{r^2} = -\frac{4\pi K}{H}\int_{T_1}^{T_2} dT$$

This equation gives,

$$H = \frac{4\pi K r_1 r_2 (T_1 - T_2)}{(r_2 - r_1)}$$

In steady state, $H = P$, $r_1 r_2 \approx R^2$ and $T_1 - T_2 = T$

So, thickness of shell is

$$r_2 - r_1 = \frac{4\pi K R^2 T}{P}$$

INGEN-HAUZ EXPERIMENT

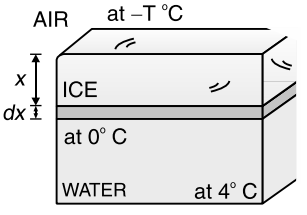
It is used to compare thermal conductivities of different materials. If l_1 and l_2 are the lengths of wax melted on rods, then the ratio of thermal conductivities is

$$\frac{K_1}{K_2} = \frac{l_1^2}{l_2^2}$$

So, in this experiment, we observe that the thermal conductivity K is proportional to the square of the length i.e., $K \propto (\text{length})^2$

GROWTH OF ICE ON PONDS

When the atmospheric temperature falls below $0\ ^\circ C$ (say $-T\ ^\circ C$), the cold air above water extracts heat from the water. As a result the water begins to freeze into the ice layers. Consider at any time the thickness of ice is x and further layer of ice of thickness dx is formed in time dt. If ρ is density of ice and L be the latent heat of fusion, then



$$\frac{dQ}{dt} = KA\frac{\Delta T}{\Delta x} = KA\left\{\frac{0-(-T)}{x}\right\}$$

$$\Rightarrow \quad dQ = KA\left(\frac{T}{x}\right)dt$$

Further, $dQ = (dm)L = (Adx)\rho L$

$$\Rightarrow \quad Adx\rho L = KA\left(\frac{T}{x}\right)dt$$

$$\Rightarrow \quad dt = \frac{\rho L}{KT}x\,dx$$

$$\Rightarrow \quad t = \int_0^t dt = \frac{\rho L}{KT}\int_{x_1}^{x_2} x\,dx = \frac{\rho L}{KT}\left(\frac{x_2^2}{2} - \frac{x_1^2}{2}\right)$$

$$\Rightarrow \quad t = \frac{\rho L}{2KT}(x_2^2 - x_1^2)$$

where, L is latent heat of fusion of ice, K is coefficient of thermal conductivity of ice and T is temperature of air below $0\ ^\circ C$.

Take care and do not apply a negative sign for putting value of temperature in formula and also do not convert it to absolute scale.

Since ice is a poor conductor of heat, so the rate of increase of thickness of ice on ponds decreases with time.

ILLUSTRATION 131

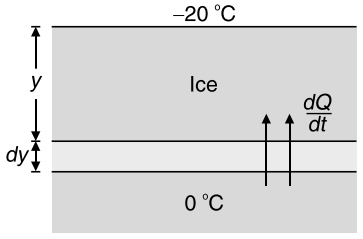
The thickness of ice on a lake is 5 cm and the temperature of air is $-20\ ^\circ C$. Calculate how long will it take for the thickness of ice to be doubled. Thermal conductivity of ice is $0.005\ \text{calcm}^{-1}\text{s}^{-1}{}^\circ\text{C}^{-1}$, density of ice is $0.92\ \text{gcc}^{-1}$ and latent heat of ice is $80\ \text{calg}^{-1}$.

SOLUTION

Since, $\dfrac{dQ}{dt} = L\left(\dfrac{dm}{dt}\right)$

$$\Rightarrow \quad \frac{KA[0-(-20)]}{y} = LA\rho\left(\frac{dy}{dt}\right)$$

where, A is the area of lake.



$$\Rightarrow \quad \int_5^{10} y\,dy = \frac{20K}{\rho L}\int_0^t dt$$

$$\Rightarrow \quad t = \frac{\rho L}{20K}\left(\frac{10^2}{2} - \frac{5^2}{2}\right)$$

$$\Rightarrow \quad t = \frac{(0.92)(80)}{(20)(0.005)}(50 - 12.5) = 27600 \text{ s}$$

$$\Rightarrow \quad t = 7 \text{ hr } 40 \text{ minute}$$

ILLUSTRATION 132

A rod of length L with thermally insulated lateral surface consists of a material whose heat conductivity coefficient varies with temperature as $K = \frac{\alpha}{T}$, where α is a constant. The ends of the rod are kept at temperatures T_1, and $T_2\,(T_1 > T_2)$. Find the function $T(x)$ where x is the distance from the end whose temperature is T, and the heat flow density.

SOLUTION

METHOD-I:

Since rate of flow of heat is $q = -KA\frac{dT}{dx}$

So, heat flow density is $H = \frac{q}{A} = -K\frac{dT}{dx}$

$$\Rightarrow \quad H\int_0^L dx = -\int_{T_1}^{T_2}\frac{\alpha}{T}dT$$

$$\Rightarrow \quad HL = \alpha \log_e\left|\frac{T_1}{T_2}\right|$$

Hence $H = \frac{\alpha}{L}\log_e\left|\frac{T_1}{T_2}\right|$ …(1)

Once again, $H\int_0^x dx = -\alpha\int_{T_1}^{T}\frac{dT}{T}$

$$\Rightarrow \quad Hx = \alpha \log_e\left|\frac{T_1}{T}\right|$$

$$\Rightarrow \quad \frac{\alpha x}{L}\log_e\left|\frac{T_1}{T_2}\right| = \alpha \log_e\left|\frac{T_1}{T}\right| \qquad \{\text{from equation (1)}\}$$

$$\Rightarrow \quad T = T_1\left(\frac{T_2}{T_1}\right)^{\frac{x}{L}}$$

METHOD-I :

The rate of heat flow through a conductor is given by $q = \frac{dQ}{dt} = \frac{KA\Delta T}{L}$ where K is the average conductivity.

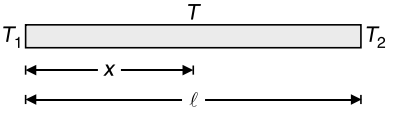
$$K_{av} = \frac{1}{T_2 - T_1}\int_{T_1}^{T_2} K\,dT = \frac{\alpha \log_e|T_2/T_1|}{T_2 - T_1}$$

$$\Rightarrow \quad q = \frac{\alpha \log_e|T_2/T_1|}{(T_2 - T_1)}\left[\frac{(T_1 - T_2)A}{l}\right]$$

So, heat flow density is given by

$$\frac{q}{A} = \frac{\alpha}{l}\log_e\left|\frac{T_1}{T_2}\right| \qquad \ldots(1)$$

Let T be the temperature at a distance x from the left end as shown in the Figure.



Then, $K'_{av} = \frac{\alpha \log_e|T/T_1|}{T - T_1}$ and heat flow density is

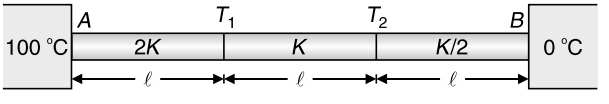
$$\frac{q'}{A} = \frac{\alpha}{x}\ln\left|\frac{T_1}{T}\right| \qquad \ldots(2)$$

Equating (1) and (2) we get

$$T = T_1\left(\frac{T_2}{T_1}\right)^{\frac{x}{L}}$$

ILLUSTRATION 133

Three rods of same length l and cross-sectional area A are joined in series between two heat reservoirs as shown in the figure. Their conductivities are $2K$, K and $\frac{K}{2}$ respectively. Assuming that the conductors are logged from the surroundings find the temperatures T_1 and T_2 at the junction in the steady state condition.



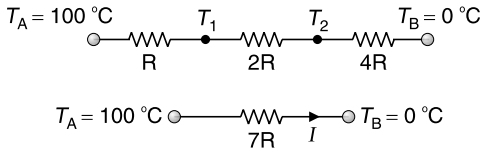
SOLUTION

The thermal resistance of the three conductors are

$$R_1 = \frac{l}{2KA},\ R_2 = \frac{l}{KA},\ R_3 = \frac{2l}{KA}$$

If $R_1 = R$, then $R_2 = 2R$ and $R_3 = 4R$

Thus, the electric analogy of the heat conduction system is shown in the Figure.



The equivalent resistance of the system is

$$R_{eq} = R + 2R + 4R = 7R$$

The heat current or thermal current is

$$I = \frac{T_A - T_B}{R_{eq}} = \frac{100 - 0}{7R} = \frac{100}{7R}$$

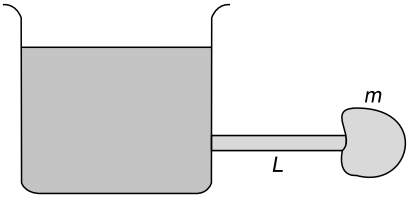
The temperature T_1 and T_2 of the junction are

$$T_1 = T_A - IR = 100 - \left(\frac{100}{7R}\right)R = \frac{600}{7}\ ^\circ C$$

$$T_2 = T_A - I(R + 2R) = 100 - \left(\frac{100}{7R}\right)(3R) = \frac{400}{7}\ ^\circ C$$

ILLUSTRATION 134

Figure shows a water tank at a constant temperature. T_0 and a small body of mass m and specific heat c at a temperature T_1. Given that $T_1 < T_0$. A metal rod of length L, cross-sectional area A, having thermal conductivity k is placed between the tank and the body to connect them. Calculate temperature of body as a function of time. Assume the heat capacity of rod to be negligible.



SOLUTION

It is given that the temperature T_0 of water tank is constant and is very large, so heat is conducted through the rod to the small body.

At some instant t, when the temperature of small body is T, then rate of heat flow through the rod is

$$\frac{dQ}{dt} = \frac{kA(T_0 - T)}{L} \qquad \ldots(1)$$

Equation (1) is used for heat conduction in steady state.

Since the heat capacity of the rod through which heat is conducting is given to be negligible, so it does not absorb any heat and hence we can assume that it is always in steady state.

Let the temperature of small body rise through dT on absorbing dQ amount of heat, so we have

$$dQ = mcdT$$

Now from equation (1), we get

$$mc\frac{dT}{dt} = \frac{kA}{L}(T_0 - T)$$

$$\Rightarrow \quad \frac{dT}{T_0 - T} = \frac{kA}{mcL}dt$$

$$\Rightarrow \quad \int_{T_1}^{T}\frac{dT}{T_0 - T} = \frac{kA}{mcL}\int_0^t dt$$

$$\Rightarrow \quad -\ln\left(\frac{T_0 - T}{T_0 - T_1}\right) = \frac{kA}{mcL}t$$

$$\Rightarrow \quad \frac{T_0 - T}{T_0 - T_1} = e^{-\frac{kAt}{mcL}}$$

$$\Rightarrow \quad T = T_0 - (T_0 - T_1)e^{-\frac{kAt}{mcL}}$$

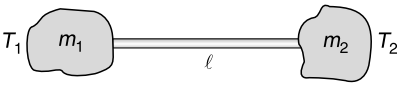
ILLUSTRATION 135

When two bodies of masses m_1 and m_2 with specific heats s_1 and s_2 at absolute temperature $(T_1)_0$ and $(T_2)_0$ less than $(T_1)_0$ are connected by a rod of length l and cross-sectional area A with thermal conductivity k. Calculate the temperature difference of the bodies after time t. Neglect any heat loss due to radiation at any surface.

SOLUTION

At an instant t, after connecting the two bodies, let their temperatures be T_1 and T_2 respectively $(T_1 > T_2)$. At this instant, heat must be flowing from m_1 to m_2 because m_1 is at higher temperature.

If dQ is the amount of heat flowing through the rod from m_1 to m_2, then because of flow of heat, if dT_1 is fall in temperature of m_1 and dT_2 is rise in temperature of m_2, then we have



$$dQ = -m_1 s_1 dT_1 \qquad \ldots(1)$$

and $$dQ = m_2 s_2 dT_2 \qquad \ldots(2)$$

Since this heat dQ is conducted through the rod, so

$$\frac{dQ}{dt} = \frac{kA(T_1 - T_2)}{l}$$

$$\Rightarrow \quad dQ = \frac{kA}{l}(T_1 - T_2)dt \qquad \ldots(3)$$

Since this expression for dQ is given as a function of $(T_1 - T_2)$, so from equations (1) and (2), we get

$$\frac{dQ}{m_1 s_1} = -dT_1 \qquad \ldots(4)$$

and $\frac{dQ}{m_2 s_2} = dT_2$ …(5)

So, on adding equations (4) and (5), we get

$$dQ\left(\frac{1}{m_1 s_1}+\frac{1}{m_2 s_2}\right) = -d(T_1 - T_2) \quad \ldots(6)$$

Now from equation (3) and (6), we get

$$\frac{kA(T_1 - T_2)}{l}\left(\frac{1}{m_1 s_1}+\frac{1}{m_2 s_2}\right)dt = -d(T_1 - T_2)$$

$$\Rightarrow \quad \frac{d(T_1 - T_2)}{T_1 - T_2} = -\frac{kA}{l}\left(\frac{1}{m_1 s_1}+\frac{1}{m_2 s_2}\right)dt$$

Integrating the expression within proper limits, we get

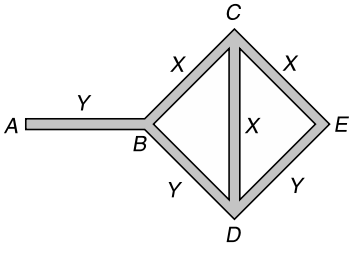
$$\int_{(T_1)_0-(T_2)_0}^{T_1-T_2} \frac{d(T_1 - T_2)}{(T_1 - T_2)} = -\int_0^t \frac{kA}{l}\left(\frac{1}{m_1 s_1}+\frac{1}{m_2 s_2}\right)dt$$

$$\Rightarrow \quad \ln\left[\frac{T_1 - T_2}{(T_1)_0 - (T_2)_0}\right] = -\frac{kA}{l}\left(\frac{1}{m_1 s_1}+\frac{1}{m_2 s_2}\right)dt$$

$$\Rightarrow \quad (T_1 - T_2) = \left[(T_1)_0 - (T_2)_0\right] e^{-\frac{kA}{l}\left(\frac{1}{m_1 s_2}+\frac{1}{m_2 s_2}\right)t}$$

ILLUSTRATION 136

Three rods of material X and three rods of material Y are connected as shown in Figure.



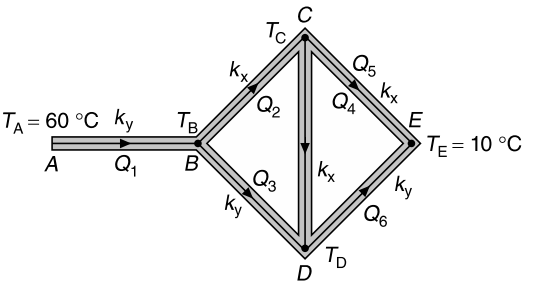
All rods have identical lengths and cross-sectional areas. If the end A is maintained at 60 °C and the junction E at 10 °C, calculate the temperature of junctions B, C and D. The thermal conductivity of X is 9.2×10^{-2} kcalm^{-1}s^{-1}°C^{-1} and that of Y is 4.6×10^{-2} kcalm^{-1}s^{-1}°C^{-1}.

SOLUTION

Let k_x and k_y be the thermal conductivities of X and Y respectively and let T_B, T_C and T_D be the temperatures of junctions B, C and D respectively. According to the problem, we have

$$T_A = 60\ °C \text{ and } T_E = 10\ °C$$

In steady state, the rate at which heat enters a junction is equal to the rate at which it leaves that junction. The direction of thermal current also called as rate of flow of heat is denoted by arrows shown in Figure.



At junction B, we have

$$\frac{k_y A(60 - T_B)}{l} = \frac{k_x A(T_B - T_C)}{l} + \frac{k_y A(T_B - T_D)}{l}$$

$$\Rightarrow \quad k_y(T_A - T_B) = k_x(T_B - T_C) + k_y(T_B - T_D)$$

$$\Rightarrow \quad 60 - T_B = \frac{k_x}{k_y}(T_B - T_C) + k_y(T_B - T_D)$$

Given that $\frac{k_x}{k_y} = \frac{9.2\times10^{-2}}{4.6\times10^{-2}} = 2$, so we get

$$(60 - T_B) = 2(T_B - T_C) + (T_B - T_D)$$

$$\Rightarrow \quad 4T_B - 2T_C - T_D = 60 \quad \ldots(1)$$

At junction C, we have

$$\frac{k_x A(T_B - T_C)}{l} = \frac{k_x A(T_C - T_D)}{l} + \frac{k_x A(T_C - 10)}{l}$$

$$\Rightarrow \quad -T_B + 3T_C - T_D = 10 \quad \ldots(2)$$

At junction D, we have

$$\frac{k_y A(T_B - T_D)}{d} + \frac{k_x A(T_C - T_D)}{d} = \frac{k_y A(T_D - 10)}{d}$$

$$\Rightarrow \quad (T_B - T_D) + \frac{k_x}{k_y}(T_C - T_D) + (T_D - 10)$$

Since $\frac{k_x}{k_y} = 2$, so we get

$$(T_B - T_D) + 2(T_C - T_D) = (T_D - 10)$$

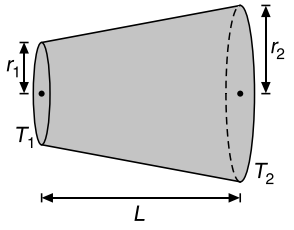
$$\Rightarrow \quad T_B + 2T_C - 4T_D = -10 \quad \ldots(3)$$

Solving equation (1), (2) and (3), we get

$$T_B = 30\ °C \text{ and } T_C = T_D = 20\ °C$$

ILLUSTRATION 137

Calculate the heat current through a frustum of a cone made of material having coefficient of thermal conductivity k, whose two ends are maintained at temperatures T_1 and $T_2\,(>T_1)$ respectively as shown in Figure.

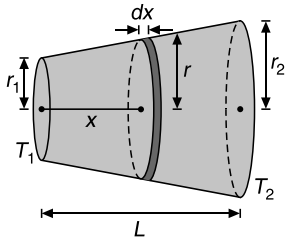


SOLUTION

Let us consider an elemental disc at a distance x from left face. If r be the radius of this disc, then

$$r = r_1 + \left(\frac{r_2 - r_1}{L}\right)x$$

Thermal resistance of this elemental disc is given by



$$dR_{Th} = \frac{dx}{kA_{disc}} = \frac{dx}{k\pi\left[r_1 + \left(\frac{r_2 - r_1}{L}\right)x\right]^2}$$

Total thermal resistance of frustum is

$$R_{Th} = \int dR_{Th} = \frac{1}{k\pi}\int_0^l \frac{dx}{\left[r_1 + \left(\frac{r_2 - r_1}{L}\right)x\right]^2}$$

$$\Rightarrow \quad R_{Th} = \frac{L}{k\pi(r_2 - r_1)}\left[\frac{1}{r_1 + \left(\frac{r_2 - r_1}{L}\right)x}\right]\Bigg|_0^l$$

$$\Rightarrow \quad R_{Th} = \frac{L}{k\pi(r_2 - r_1)}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{L}{k\pi r_1 r_2}$$

Hence heat current through the frustum is

$$\frac{dQ}{dt} = \frac{T_2 - T_1}{R_{Th}} = \frac{k\pi r_1 r_2 (T_2 - T_1)}{L}$$

CONVECTION

Mode of transfer of heat by means of migration of material particles of the medium. It is of two types.

Natural Convection

This arises due to difference of densities at two places and is a consequence of gravity because on account of gravity the hot light particles rise up and cold heavy particles try settling down. It mostly occurs on heating a liquid/fluid.

Forced Convection

If a fluid is forced to move to take up heat from a hot body then the convection process is called forced convection. In this case Newton's Law of cooling holds good according to which rate of loss of heat from a hot body due to a moving fluid is directly proportional to the surface area of body and excess temperature of body over its surroundings i.e.

$$H \propto A(T - T_0)$$

$$H = -hA(T - T_0)$$

where, h =constant of proportionality, T is temperature of body and T_0 is temperature of surrounding. Further

$$H = \frac{dQ}{dt} = -hA(T - T_0)$$

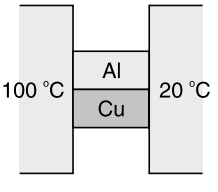
$$\Rightarrow \quad \frac{dT}{dt} = -\frac{hA}{mc}(T - T_0) \qquad \{\because dQ = mcdT\}$$

Test Your Concepts-X

Based on Conduction

(Solutions on page H.96)

1. Two metal cubes with 3 cm edges of copper and aluminium are arranged as shown in figure. Calculate the



(a) total thermal current from one reservoir to the other

(b) ratio of the thermal current carried by the copper cube to that carried by the aluminium cube. Thermal conductivity of copper is 401 $Wm^{-1}K^{-1}$ and that of aluminium is 237 $Wm^{-1}K^{-1}$.

2. Three rods of copper, brass and steel are welded together to form a Y-shaped structure. The cross-sectional area of each rod is 4 cm^2. The end of copper rod is maintained at 100 °C and the ends of the brass and steel rods at 80 °C and 60 °C, respectively. Assume that there is no loss of heat

from the surfaces of the rods. The lengths of rods are: copper 46 cm brass 13 cm and steel 12 cm.

(a) What is the temperature of the junction point?

(b) What is the heat current in the copper rod? $k(Cu) = 0.92$, $k(\text{steel}) = 0.12$ and $k(\text{brass}) = 0.26$ and all values of k are in $\text{calcm}^{-1}\text{s}^{-1}(^\circ\text{C})^{-1}$.

3. A metallic cylindrical vessel whose inner and outer radii are r_1 and r_2 is filled with ice at 0 °C. The mass of the ice in the cylinder is m. Circular portions of the cylinder are sealed with completely adiabatic walls. The vessel is kept in air. Temperature of the air is 50 °C. How long will it take for the ice to melt completely? Thermal conductivity of the cylinder is k, its length is ℓ and latent heat of fusion is L.

4. One end of a copper rod of uniform cross section and of length 1.5 m is kept in contact with ice and the other end with water at 100 °C. At what point along its length should a temperature of 200 °C be maintained so that in steady state, the mass of ice melting be equal to that of the steam produced in the same interval of time? Assume that the whole system is insulated from the surroundings. Latent heat of fusion of ice and vaporisation of water are 80 calg^{-1} and 540 calg^{-1} respectively.

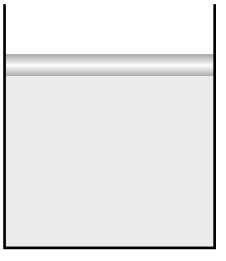
5. Three rods each of same length and cross-section are joined in series. The thermal conductivity of the materials are k, $2k$ and $3k$ respectively. If the one end is kept at 200 °C and the other end is kept at 100 °C, what would be the temperature of the junctions in the steady state? Assume that no heat is lost due to radiation from the sides of the rods.

6. Two chunks of metal with heat capacities C_1 and C_2 are interconnected by a rod of length ℓ and cross-sectional area A and fairly low conductivity k. The whole system is thermally insulated from the environment. At a moment $t = 0$, the temperature difference between two chunks of metal equals $(\Delta T)_0$. Assuming the heat capacity of the rod to be negligible, find the temperature difference between the chunks as a function of time.

7. A uniform metal bar 100 cm long insulated on the sides, has its ends exposed to ice and steam respectively. If there is a layer of water 0.1 mm thick at each end, calculate the temperature gradient in the bar.
Thermal conductivity of the metal is 1.04 $\text{calcm}^{-1}\text{s}^{-1}(^\circ\text{C})^{-1}$ thermal conductivity of the water is 0.0014 $\text{calcm}^{-1}\text{s}^{-1}(^\circ\text{C})^{-1}$.

8. A cylindrical block of length 0.4 m and area of cross-section 0.04 m^2 is placed coaxially on a thin metal disc of mass 0.4 kg and of the same cross-section. The upper face of the cylinder is maintained at a constant temperature of 400 K and the initial temperature of the disc is 300 K. If the thermal conductivity of the material of the cylinder is 10 $\text{Wm}^{-1}\text{K}^{-1}$ and the specific heat of the material of the disk is 600 $\text{Jkg}^{-1}\text{K}^{-1}$, how long will it take for the temperature of the disc to increase to 350 K? Assume for purpose of calculation the thermal conductivity of the disc to be very high and the system to be thermally insulated except for the upper face of the cylinder.

9. The shown container has all insulating surface except bottom through which heat can flow. The bottom has area of cross-section A, thickness a and thermal conductivity k. The movable piston is insulating and massless and no leakage is possible through it. Initially (at time $t = 0$) the gas (monatomic) inside the container is at temperature T_0 and volume V_0. The surrounding temperature is $T_S\,(> T_0)$ and pressure P_0. Find



(a) the temperature of the container as a function of time.

(b) the height of piston from the bottom as a function of time if initial height is h_0.

10. Two identical adiabatic vessels A and B each containing n mole of a monatomic and diatomic gas respectively. Both the vessels are connected by a rod of length ℓ and cross-sectional area A. Thermal conductivity of rod material is k and lateral surface of the rod is insulated. At any time $t = 0$, the temperature of the gases in the vessel are T_1 and T_2 respectively $(T_1 > T_2)$. Neglect heat capacity of the rod and the vessels, find the time when the temperature difference of the vessels becomes half the initial temperature difference.

RADIATION

Mode of transfer of heat energy in which the heat travels from one point to the other in the form of electromagnetic radiations. *This heat energy is also called radiant energy*.

The process of radiation does not need any material medium for heat transfer. The radiant energy emitted by a body travels in the space just like light.

Conceptual Note(s)

Thermal radiation, once emitted, is just like an electromagnetic light wave. It, therefore, obeys all the laws of wave theory. The wavelengths are still small compared to the dimensions of usual obstacles encountered, so the rules of geometrical optics are valid, i.e., it travels in a straight line, casts shadow, is reflected and refracted at the change of medium, etc.

PREVOST'S THEORY OF HEAT EXCHANGE

Prevost's theory of heat exchange states that *"each body radiates energy to the surroundings and receives radiant energy from the surroundings"*. The radiation from each body is emitted regardless of the presence or absence of other bodies. So, all bodies above zero kelvin, absorb and emit energy in the form of radiations. The rise and fall of temperatures, which is observed in a body, is due to the exchange of radiant energy with the surrounding bodies.

The amount of radiant energy emitted per unit time depends on the nature of the emitting surface, its area and its temperature.

The rate of emission of radiant energy is faster at higher temperatures than that at lower temperatures.

If a body radiates more than what it absorbs, its temperature falls. If a body radiates less than what it absorbs, its temperature rises. In thermal equilibrium with the surroundings, the body radiates and absorbs energy at the same rate.

So a hotter body placed in a room, will radiate at a faster rate than the rate at which it absorbs, due to which the body suffers a net loss of thermal energy in any given time and hence the temperature of the body falls.

Similarly, when a cold body is kept in warm surroundings, then radiant energy lost to the surroundings is less than the radiant energy absorbed from the surroundings, due to which there is a net increase in the thermal energy of the body and hence the temperature of the body rises.

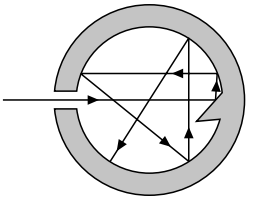
PERFECTLY BLACK BODY

A body which absorbs all the radiation falling on it is called a blackbody. A black body emits radiation at the fastest rate. The radiation emitted by a blackbody is also called blackbody radiation.

The radiation inside an enclosure with its inner walls maintained at a constant temperature has the same properties as the blackbody radiation and is also called blackbody radiation. *A blackbody is also called an ideal radiator.*

A perfect blackbody, absorbing 100% of the radiation falling on it, is only an ideal concept. Among the materials, lampblack is close to a blackbody. It reflects only about 1% of the radiation falling on it.

The most simple and commonly used black body was designed by Ferry. It consists of an enclosure painted black from inside and a small hole is made in its wall, then the hole acts as a very good blackbody because any radiation that falls on the hole and goes inside the enclosure will have a little or no chance to come out of the hole again. This is because it gets absorbed after multiple reflections inside the enclosure and the wedge shaped obstruction inside the enclosure (directly opposite to the hole) ensures that the incoming radiation does not get reflected directly back to the hole as shown in Figure.



ABSORPTANCE, REFLECTANCE AND TRANSMITTANCE

Consider an amount of heat radiation Q incident on a substance. This incident heat gets divided into three parts.

Heat Absorbed

A part of the incident heat is kept by the substance and said to be absorbed. Let this amount be Q_1.

Absorptive power (or absorptance) of the substance is defined as the ratio between amount of heat absorbed by it to the total amount of heat incident upon it.

$$a = \frac{Q_1}{Q} \qquad \ldots(1)$$

Heat Reflected

A part of the incident heat is sent back into the same medium from which it is coming. This portion of heat, say Q_2 is said to be reflected.

Reflective power (or reflectance) of a substance is defined as the ratio between the amount of the heat reflected by the substance to the total amount of heat incident upon it.

$$r = \frac{Q_2}{Q} \qquad \ldots(2)$$

Heat Transmitted

Another part Q_3 of the incident heat radiations pass through body and gets transmitted.

Transmittive power (or transmittance) of a substance is defined as the ratio between amount of heat transmitted by the body to the total amount of heat incident upon it.

$$t = \frac{Q_3}{Q} \qquad \ldots(3)$$

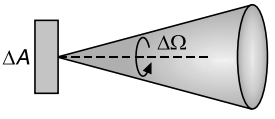
Adding equation (1), (2) and (3), we have

$$a + r + t = \frac{Q_1}{Q} + \frac{Q_2}{Q} + \frac{Q_3}{Q} = \frac{Q}{Q} = 1$$

$$\Rightarrow \quad a + r + t = 1$$

EMISSIVE POWER

All bodies emit heat radiations at all temperatures. The energy emitted by them comprises of all wavelengths ranging from zero to infinity. Amount of energy emitted by them depends upon the nature and area of the body and also upon the time for which the emission is considered.

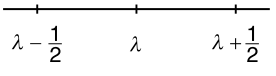


Consider a small area ΔA of a body emitting thermal radiation. Consider a small solid angle $\Delta\Omega$ about the normal to the radiating surface. Let the energy radiated by the area ΔA of the surface in the solid angle $\Delta\Omega$ in time Δt be ΔU, then emissive power of the body is defined as the energy radiated per second per unit area per unit solid angle along the normal to the radiating surface. Mathematically, we have

$$E = \frac{\Delta U}{(\Delta t)(\Delta A)(\Delta\Omega)}$$

Monochromatic Emittance

Monochromatic emittance or monochromatic emissive power or spectral emissive power e_λ of a body at a temperature T for a wavelength λ, is defined as the energy radiated, in vacuum, per unit time, per unit area per unit range of wave length i.e. lying between $\left(\lambda - \frac{1}{2}\right)$ to $\left(\lambda + \frac{1}{2}\right)$.



e_λ for a body will be different for different values of λ and for different values of T.

Radiant Emittance

Radiant emittance of body at a temperature T is defined as the total amount of energy (for all wavelengths) radiating per unit time, per unit area by the body. If dE is the amount of energy radiated per second per unit area for wavelength range $d\lambda$ then $dE = e_\lambda d\lambda$. So, radiant emittance E is given by

$$E = \int_0^E dE = \int_0^\infty e_\lambda d\lambda$$

Units of E in CGS system are $\text{erg cm}^{-2}\text{s}^{-1}$ and in SI system are $\text{Jm}^{-2}\text{s}^{-1}$.

EMISSIVITY

The emissivity e of a body, at a particular temperature, is the ratio of the emissive power of a body to the emissive power of a black body at the same temperature. It is a unitless quantity. Mathematically, we have

$$e = \frac{E_{\text{body}}}{E_{\text{black body}}} = \frac{E}{E_0}$$

Its value lies between 0 and 1.

ENERGY DENSITY (*U*)

It deals with the amount of heat energy present, at any point, in the medium surrounding the source.

Spectral Energy Density

Spectral energy density, at any point, is defined as the radiant energy per unit volume, around that point, for wavelengths lying in a unit range around λ i.e., between $\left(\lambda - \frac{1}{2}\right)$ and $\left(\lambda + \frac{1}{2}\right)$. It depends upon the value of λ as well as temperature of the source of heat.

Total Energy Density

Total energy density at any point is defined as the radiant energy per unit volume, around the point, for all wavelengths taken together.

If du is the amount of energy per unit volume for wavelengths lying in between $\left(\lambda - \frac{d\lambda}{2}\right)$ to $\left(\lambda + \frac{d\lambda}{2}\right)$, i.e. in wavelength range $d\lambda$, then

$$du = u_\lambda d\lambda$$

$$\Rightarrow \quad u = \int_0^u du = \int_0^\infty u_\lambda d\lambda$$

Conceptual Note(s)

Emissive power and radiant emittance are constants of the source while the energy density varies from point to point.

ABSORPTIVE POWER

Absorptive power of a body is defined as the fraction of the incident radiation that is absorbed by the body. If we denote the absorptive power by a,

$$a = \frac{\text{Energy Absorbed}}{\text{Energy Incident}}$$

As all the radiation incident on a blackbody is absorbed, the absorptive power of a blackbody is unity. *Please note that the absorptive power is a dimensionless quantity but the emissive power is not.*

KIRCHHOFF'S LAW OF HEAT RADIATIONS

Consider two bodies, one polished and the other painted black, having equal surface areas, suspended in a room. After a sufficiently long time, the temperature of both the bodies will be same as the room temperature. Since the bodies have equal surface areas, so equal amount of radiation falls on the two surfaces. The polished body reflects a large part of radiation but absorbs a little, whereas the black painted body absorbs a large part of radiation but reflects a little part. Since the temperature of each body remains constant, so we conclude that the polished surface radiates at a slower rate and the black-painted surface radiates at a faster rate, hence making us conclude that *good absorbers of radiation are also good emitters.*

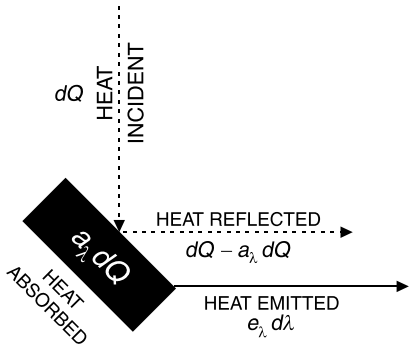
This can also be concluded by studying Kirchhoff's Law of Heat radiation which states that "at any temperature, the ratio of emissive power e_λ of a body to its absorptive power a_λ, for a particular wavelength, is always constant and is equal to the emissive power of a perfect black body (E_λ) for that wavelength."

Hence, $\frac{e_\lambda}{a_\lambda} = \text{constant}$

If E_λ and A_λ are the emissive power and absorptive power for a perfect black body, then $\frac{e_\lambda}{a_\lambda} = \frac{E_\lambda}{A_\lambda}$. For a perfect black body, since $A_\lambda = 1$ So,

$$\frac{e_\lambda}{a_\lambda} = E_\lambda$$

Let dQ be a small amount of heat incident per second per unit area on a body. Incident heat radiations have their wavelengths spread in a very narrow range $d\lambda$, i.e., between $\left(\lambda - \frac{d\lambda}{2}\right)$ and $\left(\lambda + \frac{d\lambda}{2}\right)$.



Therefore, heat energy absorbed per second per unit area of the body (retained by the body) $= a_\lambda dQ$ where, a_λ is the absorptive power of the body for that wavelength.

The body emits certain amount of energy by virtue of its own temperature. If e_λ is the emissive power of the body for that wavelength. Then

$$\begin{pmatrix}\text{Energy radiated per second}\\ \text{per unit area of the body}\end{pmatrix} = e_\lambda d\lambda$$

$$\begin{pmatrix}\text{Total amount of heat}\\ \text{going away from body}\end{pmatrix} = (1 - a_\lambda)dQ + e_\lambda d\lambda$$

In equilibrium, this heat must be same as that incident on the body. This is in accordance with the Law of Conservation of Energy. So, we have

$$dQ = (1 - a_\lambda)dQ + e_\lambda d\lambda$$

$$\Rightarrow \quad dQ = dQ - a_\lambda dQ + e_\lambda d\lambda \qquad \ldots(1)$$

$$\Rightarrow \quad e_\lambda d\lambda = a_\lambda dQ$$

In the case of a perfect black body,

$$e_\lambda = E_\lambda \text{ and } a_\lambda = 1$$

$$\Rightarrow \quad E_\lambda d\lambda = 1(dQ)$$

$$\Rightarrow \quad dQ = E_\lambda d\lambda$$

Substituting for dQ, we get $e_\lambda d\lambda = a_\lambda (E_\lambda d\lambda)$

$$\Rightarrow \quad \frac{e_\lambda}{a_\lambda} = E_\lambda$$

ILLUSTRATION 138

Radiant energy of 100 units is incident on a surface. This incident radiant energy has 20 units of wavelength λ_1, 30 units are of wavelength λ_2 and the remaining of other wavelengths. The total 60 units of energy is absorbed by the surface. In this 60 units, 5 units are of λ_1 and 25 units are of λ_2. Find a, a_{λ_1} and a_{λ_2}.

SOLUTION

Total absorptive power $a = \frac{60}{100} = 0.6$

Spectral absorptive power for λ_1 is $a_{\lambda_1} = \frac{5}{20} = 0.25$

Spectral absorptive power for λ_2 is $a_{\lambda_2} = \frac{25}{30} = 0.83$

From this example it is clear that total absorption power of the surface is only 0.6 whereas a_{λ_2} is $0.83 (> 0.6)$ i.e. the surface is good absorber of wavelength λ_2.

APPLICATIONS OF KIRCHHOFF'S LAW

According to Kirchhoff's Law, the ratio of emissive power to the absorptive power of a body is a constant quantity. It means a body having greater emissive power must have

greater absorptive power also, i.e., good absorbers must be good radiators. A number of application based upon this fact are described below.

(a) Let a polished metal with a dark spot on it be heated to a high temperature and taken into a dark room. The spot begins to shine brightly indicating that good absorbers are good radiators.

(b) Let a piece of decorated china be heated to a high temperature and taken into room. The coloured decorations which appeared dark (due to their greater absorptive powers) begin to shine brightly inside a dark room (due to their greater emissive powers).

(c) Kirchhoff's Law provides an explanation to the Fraunhofer lines (a set of dark lines present in solar spectrum).

(d) Outer walls of thermos flask are made shining. These shining walls neither absorb more heat from the surrounding nor radiate more heat to the surroundings. Thus, the temperature of bodies inside the flask is preserved.

Conceptual Note(s)

Kirchhoff's Law of thermal radiations implies that ratio between e_λ and a_λ for any-body is constant

Quantity $(= E_\lambda)$. Alternative it can be expressed in the form that the substances

which are good absorbers must be good radiators (or emitters).

STEFAN'S LAW OR STEFAN'S-BOLTZMANN LAW OR STEFAN'S FOURTH POWER LAW

According to this law, the radiant energy emitted per second i.e., $u = -\frac{dQ}{dt}$ by a perfect black body placed in surroundings at 0 K is directly proportional to the fourth power of the absolute temperature (T) i.e.,

$$u = -\frac{dQ}{dt} = \sigma A T^4 \qquad \ldots(1)$$

where, σ a universal constant called the Stefan-Boltzmann constant (or Stefan's constant) whose value is given by

$$\sigma = 5.67 \times 10^{-8}\ \text{Wm}^{-2}\text{K}^{-4}$$

The negative sign in equation (1) denotes the loss in energy of the body in the form of radiation.

A body which is not a blackbody will emit radiant energy less than that given by equation (1). It will however be proportional to T^4. The energy emitted per second per unit area by such a body can be written as

$$u' = -\frac{dQ}{dt} = e\sigma A T^4 \qquad \ldots(2)$$

where e is a constant for the given surface having a value lying between 0 and 1. This constant is called the emissivity of the surface. It is zero for completely reflecting surface and is unity for a blackbody.

According to Kirchhoff's law, we have

$$\frac{E_{\text{body}}}{E_{\text{black body}}} = a \qquad \ldots(3)$$

where a is the absorptive power of the body. The emissive power E is proportional to the energy radiated per unit time i.e. u. Substituting equations (1) and (2) in (3), we get

$$\frac{e\sigma A T^4}{\sigma A T^4} = a$$

$$\Rightarrow \quad e = a$$

Hence, emissivity and absorption power have the same value.

If a body of emissivity e is kept in thermal equilibrium with surroundings at temperature T_0 *(also called as Ambient Temperature)*, then the rate at which radiant energy is absorbed by the body equals the rate at which the radiant energy is emitted by the body. Hence the radiant energy absorbed or emitted per second by the body is

$$u = \left|\frac{dQ}{dt}\right| = e\sigma A T_0^4$$

However, if the temperature of body is changed to $T\,(> T_0)$ but the ambient temperature remains T_0, then the radiant energy emitted per second by the body is

$$u_{\text{emitted}} = e\sigma A T^4$$

The radiant energy absorbed by the body per second is

$$u_{\text{absorbed}} = e\sigma A T_0^4$$

Hence the net loss in thermal energy of the body per unit time is

$$\Delta u = u_{\text{emitted}} - u_{\text{absorbed}} = e\sigma A\left(T^4 - T_0^4\right)$$

Similarly, if a black body at temperature T is placed in surroundings at temperature $T_0\,(< T)$, then energy radiated per second per unit area is

$$-\frac{dQ}{dt} = \sigma A\left(T^4 - T_0^4\right)$$

ILLUSTRATION 139

A sphere, a cube and a thin circular plate all made of the same material and having the same mass are initially heated to 200 °C. Which of these objects will cool fastest and which one slowest when left in air at room temperature?

SOLUTION

When body cools by radiation, according to Stefan's Law,

$$\frac{dT}{dt} = -\frac{eA\sigma}{mc}\left(T^4 - T_0^4\right)$$

Here m, c,e, T and T_0 are same for all bodies.

$$\Rightarrow \quad \frac{dT}{dt} \propto A \qquad \text{where } A \text{ is the surface area}$$

Since for a given mass area of plate is maximum while of sphere is minimum the disc will cool fastest while sphere slowest.

ILLUSTRATION 140

A body which has a surface area $5\ \text{cm}^2$ and a temperature of $727\ ^\circ\text{C}$ radiates $300\ \text{J}$ of energy each minute. Calculate the emissivity of the body if Stefan's constant is $\sigma = 5.67\times10^{-8}\ \text{Wm}^{-2}\text{K}^{-4}$.

SOLUTION

According to Stefan's Law, we have energy radiated per second by a body is $P = e\sigma AT^4$, where

$$P = \left(\frac{300\ \text{J}}{60\ \text{s}}\right) = 5\ \text{Js}^{-1},\ T = 273 + 727 = 1000\ \text{K}$$

$$\Rightarrow \quad e = \frac{5}{\left(5.67\times10^{-8}\right)\left(5\times10^{-4}\right)\left(10^3\right)^4} = \frac{1}{5.67} = 0.18$$

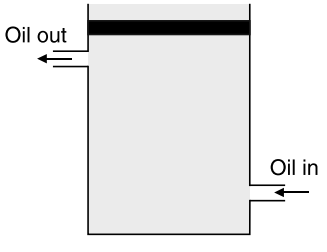
ILLUSTRATION 141

The top of an insulated cylindrical container is covered by a disc having emissivity 0.6 and conductivity $0.167\ \text{WKm}^{-1}$ and thickness 1 cm. The temperature is maintained by circulating oil as shown.

(a) Find the radiation loss to the surroundings in Jm^{-2}s if temperature of the upper surface of disc is $127\ ^\circ\text{C}$ and temperature of surroundings is $27\ ^\circ\text{C}$.

(b) Also find the temperature of the circulating oil. Neglect the heat loss due to convection.

$$\left(\text{Given: } \sigma = \frac{17}{3}\times10^{-8}\ \text{Wm}^{-2}\text{K}^{-4}\right)$$



SOLUTION

(a) Rate of heat loss per unit area due to radiation is given by

$$I = e\sigma\left(T^4 - T_0^4\right)$$

where, $T = 127 + 273 = 400\ \text{K}$

and $T_0 = 27 + 273 = 300\ \text{K}$

$$\Rightarrow \quad I = 0.6\times\frac{17}{3}\times10^{-8}\left((400)^4 - (300)^4\right)$$

$$\Rightarrow \quad I = 59\ \text{Wm}^{-2}$$

(b) Let θ be the temperature of the oil, then

$$\begin{pmatrix}\text{Rate of Heat Flow}\\ \text{Through Conduction}\end{pmatrix} = \begin{pmatrix}\text{Rate of Heat Loss}\\ \text{Due to Radiation}\end{pmatrix}$$

$$\Rightarrow \quad \frac{\text{Temperature Difference}}{\text{Thermal Resistance}} = 595A$$

$$\Rightarrow \quad \frac{(\theta - 127)}{\left(\frac{l}{KA}\right)} = 595A$$

where, A is the area of the disc, K is the coefficient of thermal conductivity and l is the thickness (or length) of disc.

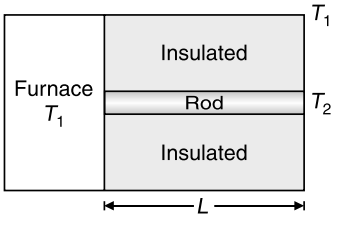
$$\Rightarrow \quad (\theta - 127)\frac{K}{l} = 595$$

$$\Rightarrow \quad \theta = 595\left(\frac{l}{K}\right) + 127 = \frac{595\times10^{-2}}{0.167} + 127$$

$$\Rightarrow \quad \theta = 162.6\ ^\circ\text{C}$$

ILLUSTRATION 142

One end of a rod of length L and cross-sectional area A is kept in a furnace of temperature T_1. The other end of the rod is kept at a temperature T_2. The thermal conductivity of the material of the rod is K and emissivity of the rod is e. It is given that $T_2 = T_S + \Delta T$, where $\Delta T \ll T_S$, T_S being the temperature of the surroundings. If $\Delta T = \alpha\left(T_1 - T_S\right)$, find the proportionality constant α. Consider that heat is lost only by radiation at the end where the temperature of the rod is T_2.



SOLUTION

$$\text{Since, } \begin{pmatrix}\text{Rate of Heat}\\ \text{Conduction}\\ \text{Through Rod}\end{pmatrix} = \begin{pmatrix}\text{Rate of Heat}\\ \text{Lost from Right}\\ \text{End of the Rod}\end{pmatrix}$$

$$\Rightarrow \quad \frac{KA\left(T_1 - T_2\right)}{L} = eA\sigma\left(T_2^4 - T_S^4\right) \qquad \ldots(1)$$

Given that $T_2 = T_S + \Delta T$

$$\Rightarrow \quad T_2^4 = (T_S + \Delta T)^4 = T_S^4\left(1+\frac{\Delta T}{T_S}\right)^4$$

Using Binomial Expansion, we get

$$T_2^4 = T_S^4\left(1+4\frac{\Delta T}{T_S}\right) \qquad \{\because\ \Delta T \ll T_S\}$$

$$\Rightarrow \quad T_2^4 - T_S^4 = 4(\Delta T)\left(T_S^3\right)$$

Substituting in Equation (1), we have

$$\frac{K(T_1 - T_S - \Delta T)}{L} = 4e\sigma T_S^3 \Delta T$$

$$\Rightarrow \quad \frac{K(T_1 - T_S)}{L} = \left(4e\sigma T_S^3 + \frac{K}{L}\right)\Delta T$$

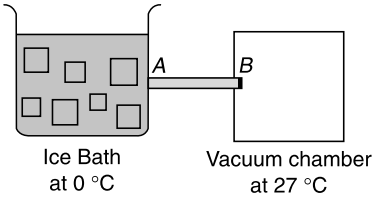
$$\Rightarrow \quad \Delta T = \frac{K(T_1 - T_S)}{4e\sigma L T_S^3 + K} = \alpha(T_1 - T_S)$$

Comparing with the given relation, we get proportionality constant α to be

$$\alpha = \frac{K}{4e\sigma L T_S^3 + K}$$

ILLUSTRATION 143

A cylindrical rod of 50 cm length having cross-sectional area 1 cm^2 is used as a conducting material between an ice bath at 0 °C and a vacuum chamber at 27 °C as shown in Figure.



The end of rod which is inside the vacuum chamber behaves like a black body and is at temperature 27 °C in steady state. Find the thermal conductivity of the material of rod and rate at which ice is melting in the ice bath. Given that latent heat of fusion of ice is 3.35×10^5 Jkg^{-1}.

SOLUTION

In steady state, heat absorbed per second by the end of the rod which is in vacuum chamber by radiation is fully conducted per second to the ice bath through the rod.

$$\Rightarrow \quad \frac{kA(T_B - T_A)}{L} = \sigma A\left(T_{vc}^4 - T_B^4\right)$$

$$\Rightarrow \quad \frac{k(27\ ^\circ\text{C} - 0\ ^\circ\text{C})}{0.5} = 5.67\times10^{-8}\left[(300)^4 - (290)^4\right]$$

$$\Rightarrow \quad k = \frac{(5.67\times10^{-8})\left[(300)^4 - (290)^4\right](0.5)}{27}$$

$$\Rightarrow \quad k = 1.08\ \text{Wm}^{-1}{}^\circ\text{C}$$

So, the rate of heat obtained by the ice bath is

$$\frac{dQ}{dt} = \frac{kA(T_A - T_B)}{l} = \frac{1.08\times1\times10^{-4}\times27}{0.5}$$

$$\Rightarrow \quad \frac{dQ}{dt} = 5.83\times10^{-3}\ \text{Js}^{-1}$$

This heat is used to melt the ice in ice bath. If m is the mass of ice being melted per second, then

$$\frac{dQ}{dt} = mL$$

$$\Rightarrow \quad 5.83\times10^{-3} = m\times3.35\times10^5$$

$$\Rightarrow \quad m = 1.74\times10^{-8}\ \text{kgs}^{-1}$$

ILLUSTRATION 144

A solid copper sphere, at an initial temperature of 200 K, having density ρ, emissivity e, gram specific heat c and radius r is suspended inside a chamber whose walls are almost at 0 K. Calculate the time required for the temperature of the sphere to drop to 100 K.

SOLUTION

According to Stefan's Law the rate of loss of energy due to radiation

$$P = eA\sigma T^4 \qquad \ldots(1)$$

Now if the rate of change of temperature of a body of mass m and specific heat c is $\left(\frac{dT}{dt}\right)$ the rate of loss of heat will be

$$\frac{dQ}{dt} = mc\frac{dT}{dt} \qquad \ldots(2)$$

As Equation (1) and (2) represents the same quantity of energy

$$-mc\frac{dT}{dt} = e\sigma AT^4$$

The negative sign shows that temperature decreases with time. Since $m = \left(\frac{4}{3}\right)\pi r^3\rho$ and $A = 4\pi r^2$,

$$-\frac{dT}{dt} = \left(\frac{3e\sigma}{\rho cr}\right)T^4$$

$$\Rightarrow \quad -\int_0^t dt = \frac{r\rho c}{3e\sigma}\int\frac{dT}{T^4}$$

$$\Rightarrow \quad t = \frac{r\rho c}{9e\sigma}\left(\frac{1}{T^3}\right)_{200}^{100} = \frac{7r\rho c}{72e\sigma}\times10^{-6}\ \text{s}$$

NEWTON'S LAW OF COOLING (SPECIAL CASE OF STEFAN'S LAW)

This law is the special case of Stefan's law applicable for comparable temperatures of body T and surroundings T_0 i.e., $T \approx T_0$ but $T - T_0 \neq 0$.

According to Newton's Law of Cooling (NLC), "for comparable temperatures of the body and surroundings, the rate of loss of heat radiated by a body (or the rate of cooling) is proportional to average excess temperature".

This relation can be established using Stefan's Law, according to which

$$-\frac{dQ}{dt} = \sigma A\left(T^4 - T_0^4\right)$$

Negative sign indicates loss of heat and A is the surface area of the body.

$$\Rightarrow \quad -\frac{dQ}{dt} = \sigma A\left(T^2 + T_0^2\right)\left(T + T_0\right)\left(T - T_0\right) \qquad \ldots(1)$$

For comparable temperatures of body and surroundings, we have $T \approx T_0$ but $T - T_0 \neq 0$, so

$$T + T_0 = 2T_0 \text{ and } T^2 + T_0^2 = 2T_0^2$$

$$\Rightarrow \quad -\frac{dQ}{dt} = \left(4\sigma A T_0^3\right)\left(T - T_0\right) \qquad \ldots(2)$$

$$\Rightarrow \quad -\frac{dQ}{dt} \propto \left(T - T_0\right)$$

where T is mean temperature of body and T_0 that of surroundings.

If a body cools from temperature T_i to a temperature T_f when placed in surroundings at temperature T_0such that $T_i > T_f > T_0$, then $\left(T - T_0\right)$ is called the average excess temperature given by

$$\left(T - T_0\right) = \left(T_{av} - T_0\right) = \left(\frac{T_i + T_f}{2}\right) - T_0$$

If m is the mass of body, c its specific heat and dT the change in temperature, then $dQ = mc\,dT$

So from equation (2), we get

$$-mc\left(\frac{dT}{dt}\right) \propto A\left(T - T_0\right)$$

For a spherical body

$$A = 4\pi r^2,\ m = V\rho = \frac{4}{3}\pi r^3 \rho$$

So, rate of loss of heat by the body

$$\frac{dQ}{dt} = k\left(T - T_0\right)4\pi r^2$$

where k is constant of proportionality and depends on emissive power of body.

Rate of cooling $-\frac{dT}{dt}$ is given by

$$-mc\frac{dT}{dt} = k\left(T - T_0\right)A$$

$$\Rightarrow \quad -\left(\frac{4}{3}\pi r^3 \rho\right)c\frac{dT}{dt} = k\left(T - T_0\right)4\pi r^2$$

$$\Rightarrow \quad -\frac{dT}{dt} = \frac{3k\left(T - T_0\right)}{r\rho c} \propto \frac{1}{r\rho c}$$

SOLUTION TO DIFFERENTIAL EQUATION INVOLVED IN NEWTON'S LAW OF COOLING

According to Newton's Law of Cooling

$$-\frac{1}{A}\frac{dQ}{dt} = 4\sigma T_0^3\left(T - T_0\right)$$

$$\Rightarrow \quad -\frac{1}{A}mc\left(\frac{dT}{dt}\right) = 4\sigma T_0^3\left(T - T_0\right)$$

$$\Rightarrow \quad -\frac{dT}{T - T_0} = \frac{4\sigma A T_0^3}{mc}dt$$

$$\Rightarrow \quad -\int_{T_1}^{T_2}\frac{dT}{T - T_0} = \frac{4\sigma A T_0^3}{mc}\int_0^t dt$$

$$\Rightarrow \quad \log_e\left(T - T_0\right)\Big|_{T_1}^{T_2} = -\left(\frac{4\sigma A T_0^3}{mc}\right)t$$

$$\Rightarrow \quad \log_e\left(T_2 - T_0\right) - \log_e\left(T_1 - T_0\right) = -\left(\frac{4\sigma A T_0^3}{mc}\right)t$$

$$\Rightarrow \quad \log_e\left(\frac{T_2 - T_0}{T_1 - T_0}\right) = -\left(\frac{4\sigma A T_0^3}{mc}\right)t \qquad \ldots(1)$$

$$\Rightarrow \quad t = \frac{mc}{4\sigma A T_0^3}\log_e\left(\frac{T_1 - T_0}{T_2 - T_1}\right)$$

Alternately from (1), we get

$$\frac{T_2 - T_0}{T_1 - T_0} = e^{-kt}, \text{ where } k = \frac{4\sigma A T_0^3}{mc}$$

$$\Rightarrow \quad T_2 = T_0 + \left(T_1 - T_0\right)e^{-kt} \qquad \ldots(2)$$

Equation (2), shows that rate of fall of temperature also falls exponentially with time.

ILLUSTRATION 145

A body cools in 7 minutes from 60 °C to 40 °C. What will be its temperature after the next 7 minutes? The temperature of surroundings is 10 °C.

SOLUTION

METHOD-I:

According to Newton's Law of Cooling,

$$\left(\frac{T_1 - T_2}{t}\right) = K\left(\frac{T_1 + T_2}{2}\right) - T_0$$

So that $\left(\frac{60-40}{7}\right)=K\left(\frac{60+40}{2}\right)-10$

$\Rightarrow \quad K=\frac{1}{14}$ …(1)

Now if after cooling from 40 °C for 7 minutesthe temperature of the body becomes T, according to Newton's Law of Cooling

$$\left(\frac{40-T}{7}\right)=K\left(\frac{T+40}{2}\right)-10$$

which in the light of equation (1), i.e., gives

$$\left(\frac{40-T}{7}\right)=\frac{1}{14}\left(\frac{20+T}{2}\right)$$

i.e. $160-4T=20+T$

$\Rightarrow \quad T=28\ ^\circ\text{C}$

METHOD-II (NOT ADVISABLE):

According to Newton's Law of Cooling, we have

$$-\frac{dT}{dt}=K\left(T-T_0\right)$$

$$\Rightarrow \quad \int_0^t dt=\frac{1}{K}\int_{T_1}^{T_2}-\frac{dT}{\left(T-T_0\right)}$$

$$\Rightarrow \quad t=\frac{1}{K}\log_e\left(\frac{T_1-T_0}{T_2-T_0}\right)$$

In first case, $7=\frac{1}{K}\log_e\left(\frac{60-10}{40-10}\right)$ …(1)

In second case, $7=\frac{1}{K}\log_e\left(\frac{40-10}{T-10}\right)$ …(2)

$$\Rightarrow \quad \log\left(\frac{50}{30}\right)=\log\left(\frac{30}{T-10}\right)$$

$$\Rightarrow \quad \frac{5}{3}=\frac{30}{T-10}$$

$\Rightarrow \quad 5T-50=90$

$\Rightarrow \quad T=28\ ^\circ\text{C}$

ILLUSTRATION 146

A calorimeter of water equivalent 100 g cools in air in 18 min from 60 °C to 40 °C. When a block of metal of mass 60 g is heated to 60 °C and placed inside the calorimeter. Assume heat loss only by radiation and Newton's Law of cooling to be valid. Find the specific heat of metal if now the system cools from 60 °C to 40 °C in 20 min.

SOLUTION

According to Newton's Law of Cooling, we have

$$\frac{T_i-T_f}{t}=\frac{4\sigma AT_0^3}{mc}\left(\frac{T_i+T_f}{2}-T_0\right)$$

$$\Rightarrow \quad \frac{T_i-T_f}{t}=\frac{k}{w}\left(\frac{T_i+T_f}{2}-T_0\right)$$

where, $w=mc$ is the water equivalent (or the heat capacity of the body) and $k=4\sigma AT_0^3$ is the constant which depends on surrounding temperature and surface area of body exposed to surrounding.

Since the calorimeter of water equivalent 100 g takes 18 min to cool down from 60° to 40 °C, so we have

$$\frac{60-40}{18}=\frac{k}{100}\left(\frac{60+40}{2}-T_0\right)$$

$$\Rightarrow \quad \frac{10}{9}=\frac{k}{100}\left(50-T_0\right) \qquad \ldots(1)$$

Now when metal block of specific heat c be placed in the calorimeter, then it takes 20 min to cool down from 60 °C to 40 °C, so we have

$$\frac{60-40}{20}=\frac{k}{100+60c}\left(\frac{60+40}{2}-T_0\right)$$

$$\Rightarrow \quad 1=\frac{k}{100+60c}\left(50-T_0\right) \qquad \ldots(2)$$

Dividing equation (1) by (2), we get

$$\frac{10}{9}=\frac{100+60c}{100}$$

$\Rightarrow \quad 100=900+540c$

$$\Rightarrow \quad c=\frac{100}{540}=0.185\ \text{calg}^{-1}\text{K}^{-1}$$

ILLUSTRATION 147

A metal ball of 1 kg is heated by a 20 W heater in a room at 20 °C. After sometime temperature of ball becomes steady at 50 °C. Calculate the rate of loss of heat to the surrounding when its temperature was 30 °C. Also find the rate at which it loses heat to the surrounding when its temperature was 30 °C.

SOLUTION

According to the problem, when ball is at 50 °C, its temperature becomes steady. So, the rate at which heat is being lost by the ball equals the rate at which heat is being supplied to the ball by the heater i.e. 20 W. So, from Newton's Law of cooling, we get

$$20=k\left(50-20\right)$$

$$\Rightarrow \quad k=\frac{2}{3} \qquad \ldots(1)$$

When ball was at 30 °C, then according to Newton's Law of cooling, we have

$$\left|\frac{dQ}{dt}\right| = k(30-20) = 10k \qquad \ldots(2)$$

$$\Rightarrow \quad \frac{dQ}{dt} = 10\times\frac{2}{3} = \frac{20}{3}\ \text{W}$$

SOLAR CONSTANT AND TEMPERATURE OF SUN

Solar constant is defined as the amount of energy received from the sun by the earth per minute per of surface placed normally to the sun's rays at mean distance of the earth from the sun in the absence of atmosphere. The value of solar constant is

$$S = 2\ \text{calmin}^{-1}\text{cm}^{-2} = 1.4\ \text{kWm}^{-2}$$

The temperature T of sun is given by

$$T^4 = \frac{S}{\sigma}\left(\frac{r}{R}\right)^2$$

where, S is the solar constant, σ is Stefan's constant, r is the mean distance of earth from sun and R is the radius of sun. This formula can be derived as follows.

Let R and T be the radius and absolute temperature of the sun. Considering the sun as a black body, the heat radiated per unit time is given by

$$u = -\frac{dQ}{dt} = \sigma T^4\left(4\pi R^2\right)$$

The above radiated energy would spread over a surface area of $4\pi r^2$. So, energy received per unit time per unit surface area is

$$S = \frac{\sigma T^4\left(4\pi R^2\right)}{4\pi r^2} = \sigma T^4\left(\frac{r}{R}\right)^2 = \frac{\sigma T^4 R^2}{r^2}$$

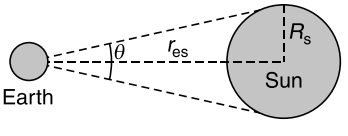
$$\Rightarrow \quad T^4 = \frac{Sr^2}{\sigma R^2} = \frac{S}{\sigma}\left(\frac{r}{R}\right)^2$$

ILLUSTRATION 148

The earth receives solar energy at the rate of 2 $\text{calmin}^{-1}\text{cm}^{-2}$. Assuming the radiation to be black body in nature, estimate the surface temperature of the sun. Given that $\sigma = 5.67\times10^{-8}\ \text{Wm}^{-2}\text{K}^{-4}$ and angular diameter of the sun 32 minute of arc.

SOLUTION

Let the surface temperature of sun be T_s, then total energy radiated by sun per second is given by



$$u = -\frac{dQ}{dt} = \sigma T_s^4\left(4\pi R_s^2\right)$$

Energy received per second by the earth per unit area is given by

$$u_{\text{earth}} = \frac{|u|}{4\pi r_{es}^2} = \sigma T_s^4\left(\frac{R_s}{r_{es}}\right)^2 \qquad \ldots(1)$$

Since the angular diameter of sun as observed from earth is 32 minute of arc, so we gave

$$R_s = r_{es}\left(\frac{\theta}{2}\right)$$

$$\frac{R_s}{r_{es}} = \frac{\theta}{2} = \frac{1}{2}\left(\frac{32}{60}\right) = 4.655\times10^{-3}\ \text{radian}$$

Since, we are given that

$$u_{\text{earth}} = \frac{2(4.2)(10^4)}{60}\ \text{Js}^{-1}\text{m}^{-2} \qquad \ldots(2)$$

From equations (1) and (2), we get

$$\frac{2\times4.2\times10^4}{60} = \left(5.67\times10^{-8}\right)T_s^4\left(4.655\times10^{-3}\right)^2$$

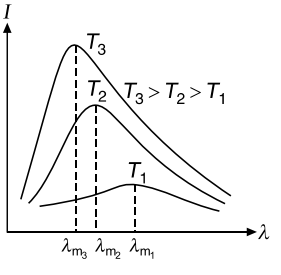
$$\Rightarrow \quad T_s^4 = \frac{2\times4.2\times10^4}{(60)\left(5.67\times10^{-8}\right)\left(4.655\times10^{-3}\right)^2} = 1.14\times10^{15}$$

$$\Rightarrow \quad T_s = 5810.67\ \text{K}$$

BLACK BODY RADIATION SPECTRUM AND WIEN'S LAW (OR WIEN'S DISPLACEMENT LAW)

It has been observed that the radiation emitted by a body is a mixture of waves of different wavelengths. However, only a small range of wavelength contributes significantly to the total radiation, e.g. the radiation emitted by a body at 300 K i.e., room temperature, has significant contribution from wavelengths around 9550 nm which lies in the long infrared region (visible light has a range of about 380 – 780 nm). *When temperature of the emitter increases, this dominant wavelength is observed to decrease.* At around 1100 K, the radiation has a good contribution from red region of wavelengths and the object appears red. At temperatures around 3000 K, the radiation contains enough shorter wavelengths and so the object appears white. Even at such a high temperature most of the significant contribution comes from wavelengths around 950 nm.

The relative importance of different wavelengths in a thermal radiation can be studied qualitatively from the intensity I vs wavelength λ graph shown in Figure.



In this graph, the intensity of radiation in the neighborhood of a given wavelength is plotted against the wavelength at different temperatures. It is observed that, as the temperature is increased, the wavelength λ_m corresponding to the maximum intensity of radiation decreases.

Actually, this wavelength λ_m is observed to be inversely proportional to the absolute temperature of the body emitting the radiation. So,

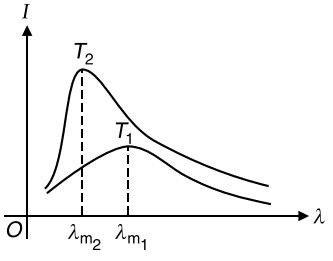
$$\lambda_m T = b$$

where b is a constant.

This equation is known as the Wien's Displacement Law. For a blackbody, the constant b appearing in equation is measured to be 0.288 cmK and is known as the Wein's constant. So, we have

$$b = 2.888\times10^{-3} \text{ kelvinmetre} = 2.888\times10^{-3} \text{ Km}$$

Graphically it can be shown that $\lambda_{m_1} T_1 = \lambda_{m_2} T_2$



Since $T_2 > T_1$, so $\lambda_{m_2} < \lambda_{m_1}$

Also, it is observed that as the temperature of the body increases, the wavelength at which the radiant intensity is maximum shifts toward left. It is called **Wein's displacement** Law.

In solar radiations $\lambda_m = 4753$ Å

Hence temperature of sun is approximately

$$T = \frac{b}{\lambda_m} = \frac{2.89\times10^{-3}}{4.753\times10^{-7}} = 6080 \text{ K}$$

Also, it has been observed that, area under the curve represents the radiant intensity of radiation at a particular temperature. Since total radiant energy is proportional to fourth power of T, area should also be proportional to fourth power of absolute temperature T. Hence

$$\text{Area} \propto T^4$$

ILLUSTRATION 149

A black body at 1500 K emits maximum energy of wavelength 2200 Å. If sun emits maximum energy of wavelength 5500 Å, calculate temperature of sun.

SOLUTION

According to Wein's displacement law, we have

$$\lambda_{m_1} T_1 = \lambda_{m_2} T_2$$

$$\Rightarrow \quad (2200\times10^{-10})1500 = (5500\times10^{-10})T_2$$

$$\Rightarrow \quad T_2 = \left(\frac{220}{55}\right)(1500) = 6000 \text{ K}$$

ILLUSTRATION 150

Two bodies A and B have thermal emissivity of 0.01 and 0.81 respectively. The outer surface areas of the two bodies are same. The two bodies emit total radiant power at the same rate. The wavelength λ_B corresponding to maximum spectral radiancy from B is shifted from the wavelength corresponding to maximum spectral radiancy in the radiation from A by 1 μm. If the temperature of A is 5802 K, calculate

(a) the temperature of $B$and
(b) the wavelength λ_B.

SOLUTION

(a) $P_A = P_B$

$$\Rightarrow \quad e_A \sigma A_A T_A^4 = e_B \sigma A_B T_B^4$$

$$\Rightarrow \quad T_B = \left(\frac{e_A}{e_B}\right)^{\frac{1}{4}} T_A \qquad \{\because A_A = A_B\}$$

Substituting the values, we get

$$T_B = \left(\frac{0.01}{0.81}\right)^{\frac{1}{4}} (5802) = 1934 \text{ K}$$

(b) According to Wein's Displacement Law, we have

$$\lambda_A T_A = \lambda_B T_B$$

$$\Rightarrow \quad \lambda_B = \left(\frac{5802}{1934}\right)\lambda_A = 3\lambda_A \qquad \ldots(1)$$

Also, we are given that

$$\lambda_B - \lambda_A = 1 \ \mu\text{m}$$

$$\Rightarrow \quad \lambda_B - \left(\frac{1}{3}\right)\lambda_B = 1 \ \mu\text{m}$$

$$\Rightarrow \quad \lambda_B = 1.5 \ \mu\text{m}$$

ILLUSTRATION 151

The radiant emittance of a black body is $R = 250 \text{ kWm}^{-2}$. At what wavelength will the emissivity of this black body be maximum? Take $b = 2.9\times10^{-3}$ Km and $\sigma = 5.67\times10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$.

SOLUTION

According to Stefan's law, the energy radiated per unit time per unit area is given by $R = \sigma T^4$, so

$$T = \left(\frac{R}{\sigma}\right)^{\frac{1}{4}}$$

$$\Rightarrow \quad T = \left(\frac{250\times10^{3}}{5.67\times10^{-8}}\right)^{1/4} = 1.449\times10^{3}\ \text{K} = 1449\ \text{K}$$

According to Wein's displacement law, the maximum spectral radiance will be at wavelength λ_m given by

$$\lambda_m = \frac{b}{T} = \frac{2.89\times10^{-3}}{1449} = 19.9447\times10^{-7}\ \text{m} = 19944.7\ \text{Å}$$

Test Your Concepts-XI

Based on Radiation

(Solutions on page H.98)

1. A liquid takes 5 minutes to cool from 80 °C to 50 °C. How much time will it take to cool from 60 °C to 30 °C ? The temperature of surrounding is 20 °C.

2. An electric heater of power 1 kW emits thermal radiations the surface area of heating element of heat is 200 cm^2. If this heating element is treated like a black body, calculate the temperature at its surface. Assume its temperature is very much higher than its surroundings.

3. One end *A* of a metallic rod of length 10 cm is inserted in a furnace whose temperature is 827 °C. Curved surface of rod is insulated. The room temperature is 27 °C. When the steady state is attained, temperature of other end *B* of rod is 702 °C. Calculate thermal conductivity of rod if Stefan's constant is 5.67×10^{-8} $Wm^{-2}K^{-4}$.

4. A metal sphere of radius $r = 4$ cm is coated black. Density of the metal is 9×10^{3} kgm^{-3} and its specific heat capacity is $4.1\,kJkg^{-1}(°C)^{-1}$. How much time is required for the sphere to cool down from 800 K to 300 K? Stefan's constant $\sigma = 5.67\times10^{-8}$ $Wm^{-2}K^{-4}$.

5. A copper ball of diameter *d* was placed in an evacuated vessel whose walls are kept at the absolute zero temperature. The initial temperature of the ball is T_0. Assuming the surface of the ball to be absolutely black, find how soon its temperature decreases η times. Take specific heat of copper *c*, density of copper ρ and emissivity *e*.

6. A solid metallic sphere of diameter 20 cm and mass 10 kg is heated to a temperature of 327 °C and suspended in a box inside which a constant temperature of 27 °C is maintained. Find the rate at which the temperature of the sphere will fall with time. Stefan's constant 5.67×10^{-8} $Wm^{-2}K^{-4}$ and specific heat of metal 420 $Jkg^{-1}°C^{-1}$.

7. End *A* of a rod *AB* of length $\ell_0 = 5$ m and cross-sectional area $A = 1\ m^2$ is maintained at some constant temperature. The heat conductivity of the rod is varying with the distance *x* from, the end *A* as $k = k_0(1+ax)$, where $k_0 = 11.4$ $Wm^{-1}K^{-1}$ and $a = 0.2$ ms^{-1}. The other end *B* of this rod is radiating energy at the rate of $P = 4560$ W and the wavelength with maximum energy density emitted from this end is 14500 Å. Express the variation of temperature as a function of *x* and determine the temperature of the end *A*. Assume that except the ends, the rod is thermally insulated and Wein's constant $b = 2.9\times10^{-3}$ Km.

8. A cube of mass 1 kg and volume 125 cm^3 is placed in an evacuated chamber at 27 °C. Initially temperature of block is 227 °C. Assume block behaves like a black body, find the rate of colling of block if specific heat of the material of block is 400 $Jkg^{-1}k$.

9. A block having some emissivity is maintained at 500 K temperature in a surrounding of 300 K temperature. It is observed that, to maintain the temperature of the block, 210 W external power is required to be supplied to it. If instead of this block a black body of same geometry and size is used, 700 W external power is required for the same. Calculate the emissivity of the material of the block.

10. A black walled metal container of negligible heat capacity is filled with water. The container has sides of length 10 cm. It is placed in an evacuated chamber at 27 °C. How long will it take for the temperature of water to change from 30 °C to 29 °C.

11. A calorimeter of negligible heat capacity contains 100 g water at 40 °C. The water cools to 35 °C in 5 minutes. If the water is now replaced by a liquid of same volume as that of water at same initial temperature, it cools to 35 °C in 2 minutes. Given specific heats of water and that liquid are 4200 $Jkg^{-1}°C$ and 2100 $Jkg^{-1}°C$ respectively. Find the density of the liquid.

12. If the filament of a 114.75 W bulb has an area 0.25 cm^2 and behaves as a perfect black body. Find the wavelength corresponding to the maximum in its energy distribution. Given $\sigma = 5.7\times10^{-8}$ $Wm^{-2}K^{-4}$, $b = 2.89\times10^{-3}$ Km

SOLVED PROBLEMS

PROBLEM 1

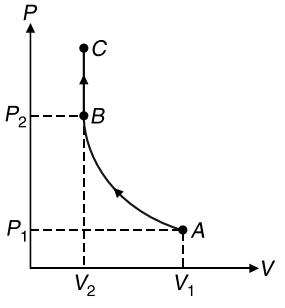
Two moles of an ideal monatomic gas initially at pressure P_1 and volume V_1 undergo an adiabatic compression until its volume is V_2. Then the gas is given heat Q at constant volume V_2.

(a) Sketch the complete process on a P-V diagram.
(b) Find the total work done by the gas, the total change in internal energy and the final temperature of the gas.

[Give your answer in terms of P_1, V_1, V_2, Q and R]

SOLUTION

(a) The P-V diagram for the complete process is shown below.



Process $A \to B$ is adiabatic compression and process $B \to C$ is isochoric.

(b) Total work done by the gas

Process AB

Since $W_{ad} = \dfrac{P_f V_f - P_i V_i}{1-\gamma}$

$$\Rightarrow \quad W_{AB} = \frac{P_A V_A - P_B V_B}{\gamma - 1}$$

$$\Rightarrow \quad W_{AB} = \frac{P_1 V_1 - P_2 V_2}{\frac{5}{3} - 1}, \ \gamma = \frac{5}{3} \text{ for monatomic gas}$$

Since, $P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$

$$\Rightarrow \quad P_2 = P_1 \left(\frac{V_1}{V_2}\right)^{\gamma}$$

$$\Rightarrow \quad W_{AB} = \frac{P_1 V_1 - P_1 \left(\frac{V_1}{V_2}\right)^{\gamma} V_2}{2/3}$$

$$\Rightarrow \quad W_{AB} = \frac{3}{2} P_1 V_1 \left[1 - \left(\frac{V_1}{V_2}\right)^{\gamma - 1}\right]$$

$$\Rightarrow \quad W_{AB} = -\frac{3}{2} P_1 V_1 \left[\left(\frac{V_1}{V_2}\right)^{\frac{5}{3}-1} - 1\right]$$

$$\Rightarrow \quad W_{AB} = -\frac{3}{2} P_1 V_1 \left[\left(\frac{V_1}{V_2}\right)^{2/3} - 1\right]$$

Since, process BC is Isochoric, so

$$W_{BC} = 0 \qquad \{\because V = \text{constant}\}$$

$$\Rightarrow \quad W_{\text{Total}} = W_{AB} + W_{BC} = -\frac{3}{2} P_1 V_1 \left[\left(\frac{V_1}{V_2}\right)^{2/3} - 1\right]$$

Total change in internal energy

Since, process AB is Adiabatic, so

$$Q_{AB} = 0$$

From $FLTD$, we have

$$Q = \Delta U + W$$

$$\Rightarrow \quad Q_{AB} = 0 = \Delta U_{AB} + W_{AB}$$

$$\Rightarrow \quad \Delta U_{AB} = -W_{AB} = \frac{3}{2} P_1 V_1 \left[\left(\frac{V_1}{V_2}\right)^{2/3} - 1\right]$$

Since, process BC is Isochoric, so

$$W_{BC} = 0$$

So, from $FLTD$, we get

$$\Delta U_{BC} = Q_{BC} = Q \qquad \{\text{given}\}$$

$$\Rightarrow \quad \Delta U_{\text{Total}} = \Delta U_{AB} + \Delta U_{BC} = \frac{3}{2} P_1 V_1 \left[\left(\frac{V_1}{V_2}\right)^{2/3} - 1\right] + Q$$

Final temperature of the gas

Since, $\Delta U_{\text{Total}} = nC_V \Delta T = 2\left(\dfrac{R}{\gamma - 1}\right)(T_C - T_A)$

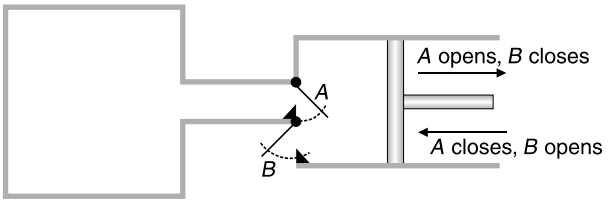
$$\Rightarrow \quad \frac{3}{2} P_1 V_1 \left[\left(\frac{V_1}{V_2}\right)^{2/3} - 1\right] + Q = \frac{2R}{\left(\frac{5}{3} - 1\right)}\left(T_C - \frac{P_A V_A}{2R}\right)$$

$$\Rightarrow \quad \frac{3}{2} P_1 V_1 \left[\left(\frac{V_1}{V_2}\right)^{2/3} - 1\right] + Q = 3R\left(T_C - \frac{P_1 V_1}{2R}\right)$$

$$\Rightarrow \quad T_C = \frac{Q}{3R} + \frac{P_1 V_1}{2R}\left(\frac{V_1}{V_2}\right)^{2/3} = T_{\text{final}}$$

PROBLEM 2

A vessel of volume V having pressure P_0 is evacuated by means of a piston air pump as shown in figure. Valve A opens (B closed) when the piston moves outward and A closes (B opens) when the piston moves inward. Each stroke of the piston captures the volume ΔV. How many strokes are needed to reduce the pressure in the vessel η times? The process is assumed to be isothermal and ideal.



SOLUTION

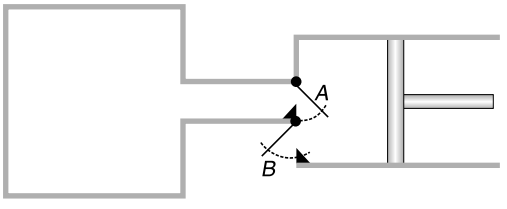
When the piston moves outward, the volume of the gas is $(V+\Delta V)$ and pressure P_1, such that

$$P_1(V+\Delta V)=P_0V \quad \ldots(1)$$

During return stroke of piston volume ΔV is ejected. The moment the return stroke begins, valve A is closed, the remaining gas has pressure P_1 and volume V. For second stroke, we have

$$P_2(V+\Delta V)=P_1V \quad \ldots(2)$$

$$\Rightarrow \quad P_2=\frac{P_1V}{(V+\Delta V)}=P_0\left(\frac{V}{V+\Delta V}\right)^2$$



Similarly, $P_3=P_0\left(\frac{V}{V+\Delta V}\right)^2$

After N^{th} stroke, $P_N=P_0\left(\frac{V}{V+\Delta V}\right)^N \quad \ldots(3)$

$$\Rightarrow \quad N=\frac{\log_e\left(\frac{P_N}{P_0}\right)}{\log_e\left(\frac{V}{V+\Delta V}\right)} \quad \ldots(4)$$

Since, $\frac{P_N}{P_0}=\frac{1}{\eta}$

$$\Rightarrow \quad N=\frac{\log_e\left(\frac{1}{\eta}\right)}{\log_e\left(\frac{V}{V+\Delta V}\right)}$$

Conceptual Note(s)

Note that the mass of the gas ejected in each stroke is not the same. From Ideal Gas equation, we have

$P_{N-1}V=\frac{m_{N-1}}{M}RT$ {mass after $N-1$ strokes}

$P_NV=\frac{m_N}{M}RT$ {mass after N strokes}

Mass of gas ejected in the N^{th} stroke is

$$m_{N-1}-m_N=\frac{MV}{RT}(P_{N-1}-P_N) \quad \ldots(5)$$

After N^{th} stroke, we have from Boyle's Law,

$$P_{N-1}V=P_N(V+\Delta V)$$

$$\Rightarrow \quad (P_{N-1}-P_N)V=P_N(\Delta V) \quad \ldots(6)$$

From equations (5) and (6), we have

$$m_{N-1}-m_N=\frac{M(\Delta V)}{RT}P_N=\frac{\Delta V}{V}m_N$$

$$\Rightarrow \quad \frac{V}{m_N}-\frac{V}{m_{N-1}}=\frac{\Delta V}{V}\frac{V}{m_N-1} \quad \ldots(7)$$

The specific volume of gas $\frac{V}{m}$ is volume of a unit mass of gas.

$$\Rightarrow \quad \frac{\frac{V}{m_N}-\frac{V}{m_{N-1}}}{\left(\frac{V}{m_{N-1}}\right)}=\frac{\Delta V}{V}=\text{constant}$$

The fractional change in specific volume is constant in this process i.e.,

$$\left(\frac{\Delta V}{V}\right)=\left(\frac{m_{N-1}-m_N}{m_N}\right)$$

PROBLEM 3

Find the equations of the process for an ideal gas in terms of the variables T and V if the molar heat capacity varies as

(a) $C=C_V+\alpha T$

(b) $C=C_V+\beta V$ and

(c) $C=C_V+aP$

where α, β and a are constants.

SOLUTION

Since, $C=\frac{dQ}{dT}$ (for 1 mole of an ideal gas)

$$\Rightarrow \quad C=\frac{dU+PdV}{dT}$$

$$\Rightarrow \quad C=C_V+P\left(\frac{dV}{dT}\right)$$

$$\Rightarrow \quad C=C_V+\frac{RT}{V}\left(\frac{dV}{dT}\right) \quad \ldots(1)$$

(a) Since, we are given that

$$C=C_V+\alpha T \quad \ldots(2)$$

So, comparing (1) and (2), we get

$$\alpha T = \frac{RT}{V}\frac{dV}{dT}$$

$$\Rightarrow \frac{\alpha}{R}dT = \frac{dV}{V}$$

$$\Rightarrow \int\frac{dV}{V} = \frac{\alpha}{R}\int dT$$

$$\Rightarrow \log_e V = \frac{\alpha T}{R} + \text{constant}$$

$$\Rightarrow Ve^{-\frac{\alpha T}{R}} = \text{constant}$$

(b) Similarly comparing

$$C = C_V + \frac{RT}{V}\frac{dV}{dT}$$

and $C = C_V + \beta V$, we get

$$\beta V = \frac{RT}{V}\frac{dV}{dT}$$

$$\frac{\beta}{R}\frac{dT}{T} = \frac{dV}{V^2}$$

Integrating, we get

$$\int\frac{dV}{V^2} = \frac{\beta}{R}\int\frac{dT}{T}$$

$$\Rightarrow -\frac{1}{V} = \frac{\beta}{R}\log_e T + \text{constant}$$

$$\Rightarrow Te^{\frac{R}{\beta V}} = \text{constant}$$

(c) Similarly comparing

$$C = C_V + P\frac{dV}{dT}$$

and $C = C_V + aP$, we get

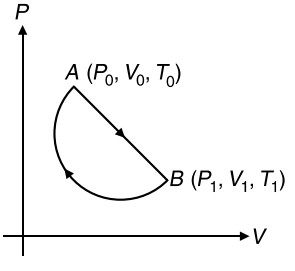
$$\Rightarrow aP = P\frac{dV}{dT}$$

$$\Rightarrow \frac{dV}{dT} = a$$

$$\Rightarrow V = aT + \text{constant}$$

PROBLEM 4

An ideal monatomic gas is taken through a cyclic process in which it expands linearly from a state $(P_0,\ V_0,\ T_0)$ to a state $(P_1,\ V_1,\ T_1)$ followed by an adiabatic compression back to the original state. Given that $P_0 = 32,\ V_0 = 8,\ P_1 = 1$ and $V_1 = 64$ in SI units. Calculate the thermal efficiency of the cycle.



SOLUTION

The P-V equation for the process AB is found by using $y - y_1 = m(x - x_1)$, where $m = \frac{y_2 - y_1}{x_2 - x_1}$

Since, AB process passes through $(8,\ 32)$ so we get

$$P - 32 = \left(\frac{1-32}{64-8}\right)(V-8)$$

$$\Rightarrow P = -\frac{31}{56}V + \frac{255}{7}$$

So, the P-V equation for the process AB can be written as,

$$P = aV + b, \text{ where } a = -\frac{31}{56} \text{ and } b = \frac{255}{7}$$

The work done by the gas is the area under the curve and is given by

$$W_{AB} = \frac{1}{2}(P_0 + P_1)(V_1 - V_0) = 924 \text{ J}$$

The work done by the gas in the adiabatic compression is given by

$$W_{BA} = \frac{P_BV_B - P_AV_A}{\gamma - 1} = \frac{P_1V_1 - P_0V_0}{\gamma - 1}$$

$$\Rightarrow W_{BA} = \frac{64 - 32\times 8}{\frac{5}{3} - 1}$$

$$\Rightarrow W_{BA} = -288 \text{ J}$$

Thus, the net work done by the gas during this cycle is

$$W_{\text{net}} = W_{AB} + W_{BA} = 924 - 288 = 636 \text{ J}$$

From the First Law net Q for the whole cyclic process is also 636 J $\{\because \Delta U = 0\}$

Along the linear path AB, heat both enters and leaves the system. There is a transition point $(P_m,\ V_m,\ T_m)$ which represents the point on this path at which the heat flow reverses direction changing from heat input to heat output. So, first of all we must find Q as a function of V i.e., the heat input as a function of V and then find the particular volume V_m at which Q is a maximum. Using the First Law of Thermodynamics, we have

$$Q = Q(V) = W + \Delta U \qquad \ldots(1)$$

The P-V equation along the path AB is,

$$P = aV + b$$

Multiplying both sides by V, we get

$$PV = aV^2 + bV$$

Since, $Q = Q(V) = W + \Delta U$

$$\Rightarrow \quad Q(V) = \frac{1}{2}(P_0 + P)(V - V_0) + nC_V\Delta T$$

$$\Rightarrow \quad Q(V) = \frac{1}{2}(P_0 + P)(V - V_0) + \frac{3nR}{2}(T - T_0)$$

$$\Rightarrow \quad Q(V) = \frac{1}{2}(P_0 + aV + b)(V - V_0) + \frac{3}{2}(PV - P_0V_0)$$

$$\Rightarrow \quad Q(V) = \frac{1}{2}(P_0 + aV + b)(V - V_0) + \frac{3}{2}(aV^2 + bV - P_0V_0) \qquad \ldots(2)$$

Now, $\frac{dQ(V)}{dV} = 0$

$$\Rightarrow \quad aV - \frac{aV_0}{2} + \frac{1}{2}(P_0 + b) + \frac{3}{2}(2aV + b) = 0$$

$$\Rightarrow \quad 4aV = \frac{aV_0}{2} - \frac{1}{2}(P_0 + b) - \frac{3b}{2}$$

$$\Rightarrow \quad V = \frac{V_0}{8} - \frac{1}{8a}(P_0 + b) - \frac{3b}{8a}$$

Substituting the values, we get

$$V_m = \frac{8}{8} + \frac{56}{8\times 31}\left(32 + \frac{255}{7}\right) + \frac{3\times 255}{7}\times\frac{56}{8\times 31}$$

$$\Rightarrow \quad V_m = 41.13 \text{ SI units}$$

From equation (2), we get

$$Q_{+ve} = \frac{1}{2}\left(32 - \frac{31}{56}\times 41.13 + \frac{255}{7}\right)(41.13 - 8) + \frac{3}{2}\left[\left(-\frac{31}{56}\right)(41.13)^2 + \left(\frac{255}{7}\right)(41.13) - 32\times 8\right]$$

$$\Rightarrow \quad Q_{+ve} \approx 1215 \text{ J}$$

Efficiency of the cycle is

$$\eta = \left(\frac{W_{\text{Total}}}{Q_{+ve}}\right)\times 100$$

$$\Rightarrow \quad \eta = \frac{636}{1215}\times 100$$

$$\Rightarrow \quad \eta = 52\%$$

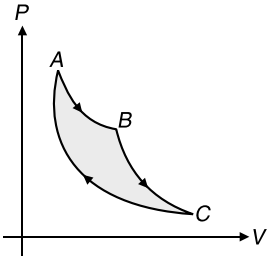
PROBLEM 5

An ideal gas expands isothermally along AB and does 700 J of work.

(a) How much heat does the gas exchange along AB.

(b) The gas then expands adiabatically along BC and does 400 J of work. When the gas returns to A along CA, it exhausts 100 J of heat to its surroundings. How much work is done on the gas along this path.

(c) Find the efficiency of the given cycle.



SOLUTION

(a) AB is an isothermal process. Hence,

$$\Delta U_{AB} = 0$$

$$\Rightarrow \quad Q_{AB} = W_{AB} = 700 \text{ J} \qquad \{\because\ Q_{AB} = \Delta U_{AB} + W_{AB}\}$$

(b) BC is an adiabatic process. Hence,

$$Q_{BC} = 0$$

Since, $W_{BC} = 400$ J

$$\Rightarrow \quad \Delta U_{BC} = -W_{BC} = -400 \text{ J}$$

ABC is a cyclic process and internal energy is a state function. Therefore,

$$(\Delta U)_{\text{whole cycle}} = 0 = \Delta U_{AB} + \Delta U_{BC} + \Delta U_{CA}$$

So, from First Law of Thermodynamics, we have

$$Q_{AB} + Q_{BC} + Q_{CA} = W_{AB} + W_{BC} + W_{CA}$$

Substituting the values, we get

$$700 + 0 - 100 = 700 + 400 + W_{CA}$$

$$\Rightarrow \quad W_{CA} = -500 \text{ J}$$

Negative sign implies that work is done on the gas. We can summarise the values in different processes in the table below.

Process	Q(J)	W(J)	ΔU(J)
AB	700	700	0
BC	0	400	–400
CA	–100	–500	400
For complete cycle	600	600	0

Since, total work done is 600 J, so we can also conclude that area of the closed curve is 600 J.

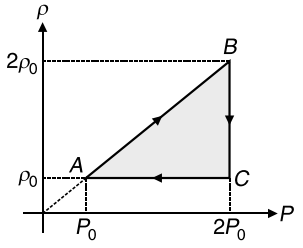
(c) From table we observe that Q_{+ve} during the cycle is 700 J, while the total work done in the cycle is 600 J.

So, efficiency $\eta = \frac{W_{\text{total}}}{|Q_{+ve}|}\times 100 = \left(\frac{600}{700}\right)\times 100$

$$\Rightarrow \quad \eta = 85.71\%$$

PROBLEM 6

The density versus pressure graph of one mole of an ideal monatomic gas undergoing a cyclic process is shown in figure. The molecular mass of the gas is M.



(a) Find the work done in each process.
(b) Find heat rejected by gas in one complete cycle.
(c) Find the efficiency of the cycle.

SOLUTION

(a) As $n=1$, $m=M$, also we know that $\rho=\dfrac{PM}{RT}$

For process AB, we observe that $\rho \propto P$ so it is an isothermal process T is constant, so $\rho=\dfrac{PM}{RT}$

$$\Rightarrow\ W_{AB}=RT_A\log_e\left(\frac{P_A}{P_B}\right)=RT_A\log_e\left(\frac{1}{2}\right)$$

$$\Rightarrow\ W_{AB}=-\frac{P_0M}{\rho_0}\log_e(2)$$

Also, $\Delta U_{AB}=0$

$$\Rightarrow\ Q_{AB}=W_{AB}=-\frac{P_0M}{\rho_0}\log_e(2)$$

Process BC is an isobaric process (P =constant), so

$$W_{BC}=P_B(V_C-V_B)=2P_0\left(\frac{M}{\rho_C}-\frac{M}{\rho_B}\right)=\frac{2P_0M}{2\rho_0}=\frac{P_0M}{\rho_0}$$

So, $\Delta U_{BC}=C_V\Delta T$

$$\Rightarrow\ \Delta U_{BC}=\left(\frac{3}{2}R\right)\left(\frac{2P_0M}{\rho_0R}-\frac{2P_0M}{2\rho_0R}\right)=\frac{3P_0M}{2\rho_0}$$

So, by FLTD, we get

$$Q_{BC}=W_{BC}+\Delta U_{BC}=\frac{5P_0M}{2\rho_0}$$

Process CA has ρ =constant, so V =constant

So, the process CA is an isochoric process, hence

$$W_{CA}=0$$

$$\Rightarrow\ \Delta U_{CA}=C_V\Delta T$$

$$\Rightarrow\ \Delta U_{CA}=\left(\frac{3}{2}R\right)(T_A-T_C)$$

$$\Rightarrow\ \Delta U_{CA}=\left(\frac{3}{2}R\right)\left(\frac{P_0M}{\rho_0R}-\frac{2P_0M}{\rho_0R}\right)$$

$$\Rightarrow\ \Delta U_{CA}=-\frac{3P_0M}{2\rho_0}$$

So, by FLTD, we get

$$Q_{CA}=\Delta U_{CA}=-\frac{3P_0M}{2\rho_0}$$

(b) Heat rejected by the gas in one complete cycle is

$$Q=Q_{-\text{ve}}=-\frac{P_0M}{\rho_0}\log_e(2)-\frac{3P_0M}{2\rho_0}$$

$$\Rightarrow\ Q=-\frac{P_0M}{\rho_0}\left(\log_e(2)+\frac{3}{2}\right)$$

(c) Efficiency of the cycle (in fraction)

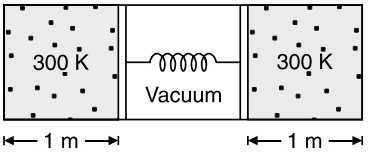
$$\eta=\frac{\text{Total work done}}{\text{Heat supplied}}=\frac{\text{Total work done}}{\text{Heat absorbed}}$$

$$\Rightarrow\ \eta=\frac{W_{\text{total}}}{Q_{+\text{ve}}}=\frac{\frac{P_0M}{\rho_0}\left(1-\log_e(2)\right)}{\frac{5}{2}\left(\frac{P_0M}{\rho_0}\right)}$$

$$\Rightarrow\ \eta=\frac{2}{5}\left(1-\log_e(2)\right)$$

PROBLEM 7

Consider the shown diagram where the two chambers separated by piston-spring arrangement contain equal amounts of certain ideal gas. Initially when the temperatures of the gas in both the chambers are kept at 300 K. The compression in the spring is 1 m. The temperature of the left and the right chambers are now raised to 400 K and 500 K respectively. If the pistons are free to slide, find the final compression in the spring.



SOLUTION

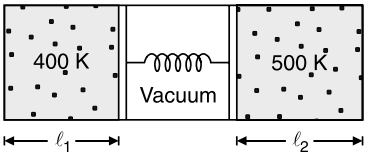
Let l_1 and l_2 be the final lengths of the two parts, then from gas equation, we have

$$\frac{P_0(Al_0)}{T_0}=\frac{P(Al_1)}{T_1}=\frac{P(Al_2)}{T_2} \qquad \ldots(1)$$

Considering the equilibrium of piston in initial and final states, we have

$$P_0A=kx_0 \text{ and } PA=kx$$

$$\Rightarrow\ \frac{P}{P_0}=\frac{x}{x_0} \qquad \ldots(2)$$



Now, $\begin{pmatrix}\text{Decrease}\\ \text{in length}\\ \text{of spring}\end{pmatrix} = \begin{pmatrix}\text{Total increase}\\ \text{in the lengths of}\\ \text{the two chambers}\end{pmatrix}$

$$\Rightarrow \quad x - x_0 = (l_1 + l_2) - 2l_0 \qquad \ldots(3)$$

From equation (1), we get

$$l_1 = \frac{P_0 l_0 T_1}{PT_0} \text{ and } l_2 = \frac{P_0 l_0 T_2}{PT_0}$$

Using equation (2), we get

$$l_1 = \frac{x_0 l_0 T_1}{xT_0} \text{ and } l_2 = \frac{x_0 l_0 T_2}{xT_0}$$

Substituting these in equation (3), we have

$$x - x_0 = \frac{x_0 l_0}{xT_0}(T_1 + T_2) - 2l_0$$

Substituting the values and solving for x, we get

$$x \approx 1.3 \text{ m}$$

PROBLEM 8

A 5 m long cylindrical steel wire with radius 2×10^{-3} m is suspended vertically from a rigid support and carries a bob of mass 100 kg at the other end. If the bob gets snapped, calculate the change in temperature of the wire ignoring losses. (For the steel wire, Young's modulus is 2.1×10^{11} Pa, Density is 7860 kgm^{-3}, Specific heat is 420 $\text{Jkg}^{-1}\text{K}^{-1}$).

SOLUTION

According to the question

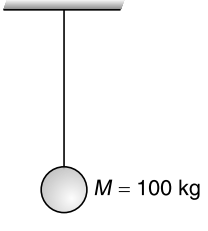
Length of the wire, $l = 5$ m

Radius of the wire, $r = 2\times10^{-3}$ m

Density of wire, $\rho = 7860 \text{ kgm}^{-3}$

Young's modulus, $Y = 2.1\times10^{11} \text{ Nm}^{-2}$

and Specific heat, $c = 420 \text{ Jkg}^{-1}\text{K}^{-1}$



Mass of wire, $m = (\text{density})(\text{volume}) = (\rho)(\pi r^2 l)$

$$\Rightarrow \quad m = (7860)(\pi)(2\times10^{-3})^2(5) \text{ kg} = 0.494 \text{ kg}$$

Since, $\frac{\text{Energy}}{\text{Volume}} = \frac{1}{2}(\text{Stress})(\text{Strain})$

So, elastic potential energy stored in the wire is

$$U = \frac{1}{2}(\text{Stress})(\text{Strain})(\text{Volume})$$

$$\Rightarrow \quad U = \frac{1}{2}\left(\frac{Mg}{\pi r^2}\right)\left(\frac{\Delta l}{l}\right)(\pi r^2 l) = \frac{1}{2}(Mg)\Delta l \quad \left\{\because \Delta l = \frac{Fl}{AY}\right\}$$

$$\Rightarrow \quad U = \frac{1}{2}(Mg)\frac{(Mgl)}{(\pi r^2)Y} = \frac{1}{2}\frac{M^2 g^2 l}{\pi r^2 Y}$$

Substituting the values, we get

$$\Rightarrow \quad U = \frac{1}{2}\frac{(100)^2(10)^2(5)}{(3.14)(2\times10^{-3})^2(2.1\times10^{11})} = 0.9478 \text{ J}$$

When the bob gets snapped, this energy is utilised in raising the temperature of the wire.

So, $U = mc\Delta T$

$$\Rightarrow \quad \Delta T = \frac{U}{mc} = \frac{0.9478}{0.494(420)} \text{ °C or K}$$

$$\Rightarrow \quad \Delta T = 4.568\times10^{-3} \text{ °C}$$

PROBLEM 9

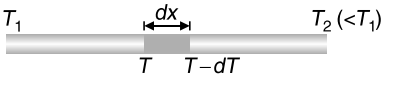
A rod of length l with thermally insulated lateral surface of area A consists of a material whose heat conduction coefficient varies with temperature as $k = \frac{k_0}{a+bT}$. The two ends of the rod are at temperatures T_1 and $T_2\,(<T_1)$.

(a) Find the heat transferred across the rod.

(b) Express the temperature of the rod at a point as a function of, x, distance from the hot end.

SOLUTION

(a) $H = -kA\frac{dT}{dx}$



$$\Rightarrow \quad Hdx = -\frac{k_0}{a+bT}AdT$$

$$\Rightarrow \quad H\int_0^l dx = -k_0 A\int_{T_1}^{T_2}\frac{dT}{a+bT}$$

$$\Rightarrow \quad H = \frac{Ak_0}{bl}\log_e\left(\frac{a+bT_1}{a+bT_2}\right)$$

(b) Further $H\int_0^x dx = -k_0 A\int_{T_1}^{T}\frac{dT}{a+bT}$

Substituting value of H from part (a) and integrating the above equation, we get

$$T=\frac{1}{b}\left[(a+bT_1)\left(\frac{a+bT_2}{a+bT_1}\right)^{x/l}-a\right]$$

PROBLEM 10

In the glass tube, half portion is filled by a liquid A and the other half by liquid B. The temperature of the whole system is increased by ΔT. It is given that initial volume of liquid A and $B=\frac{V}{2}$. Find out ΔV_A and ΔV_B if the coefficient of volume expansion of liquids A and B are γ_A and γ_B respectively and bulk modulii of the two liquids are k_A and k_B respectively. Neglect the expansion of glass tube.

SOLUTION

Total change in volume due to heating is given by

$$\Delta V_A+\Delta V_B=\frac{V}{2}\gamma_A\Delta T+\frac{V}{2}\gamma_B\Delta T$$

$$\Rightarrow\quad \Delta V_A+\Delta V_B=\frac{V}{2}(\gamma_A+\gamma_B)\Delta T \qquad \ldots(1)$$

This increase in volume causes stress (pressure) on the liquids (since the glass tube does not expand).
Let P be this excess pressure then total decrease in volume due to this excess pressure is

$$\Delta V_A+\Delta V_B=\frac{k_AV}{2P}+\frac{k_BV}{2P} \qquad \left\{\because\ k=\frac{\Delta P}{\left(\frac{\Delta V}{V}\right)}\right\}$$

Since, the tube does not expand, so we have

$$\frac{V}{2}\Delta T(\gamma_A+\gamma_B)=(k_A+k_B)\frac{V}{2P}$$

$$\Rightarrow\quad P=\frac{k_A+k_B}{(\gamma_A+\gamma_B)\Delta T}$$

$$\Rightarrow\quad (\Delta V_A)_{\text{total}}=\frac{V}{2}\gamma_A\Delta T-\frac{k_AV}{2(k_A+k_B)}(\gamma_A+\gamma_B)\Delta T$$

$$\Rightarrow\quad (\Delta V_A)_{\text{total}}=\frac{V}{2}\left(\frac{\gamma_Ak_B-\gamma_Bk_A}{k_A+k_B}\right)\Delta T$$

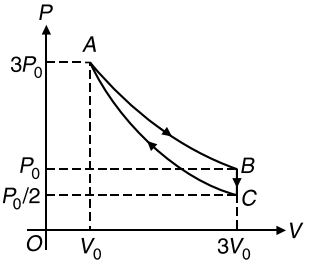
Similarly, $(\Delta V_B)_{\text{total}}=\frac{V}{2}\left(\frac{\gamma_Bk_A-\gamma_Ak_B}{k_A+k_B}\right)\Delta T$

PROBLEM 11

One mole of a monatomic gas is taken through the cycle $ABCA$ as shown in the figure. In the process $C\to A$ the gas obeys the relation

$$Q_{C\to A}+W_{C\to A}=0$$

where $Q_{C\to A}$ is the heat supplied in the process $C\to A$ and $W_{C\to A}$ is the work done by the gas in that process, AB is an isothermal process.



(a) Find the work done and change in internal energy in each process and also find the heat exchanged in each process.

(b) Find the average molar specific heat for processes AB and CA.

SOLUTION

(a) For process AB, $\Delta U_{AB}=0$

For isothermal process,

$$W_{AB}=nRT_A\log_e\left(\frac{V_B}{V_A}\right)$$

$$\Rightarrow\quad W_{AB}=P_AV_A\log_e(3)=3P_0V_0\log_e(3)$$

So from FLTD, we get

$$Q_{AB}=W_{AB}=3P_0V_0\log_e(3)$$

For process BC, $W_{BC}=0$ {isochoric}

$$\Rightarrow\quad \Delta U_{BC}=nC_V\Delta T$$

$$\Rightarrow\quad \Delta U_{BC}=(1)\left(\frac{R}{\gamma-1}\right)(T_C-T_B)$$

$$\Rightarrow\quad \Delta U_{BC}=\left(\frac{1}{5/3-1}\right)\left(\frac{3}{2}P_0V_0-3P_0V_0\right)$$

$$\Rightarrow\quad \Delta U_{BC}=-\frac{9P_0V_0}{4}$$

So from FLTD, we get

$$Q_{BC}=\Delta U_{BC}=-\frac{9P_0V_0}{4}$$

Since for a cyclic process, $\Delta U_{\text{Total}}=0$

$$\Rightarrow\quad \Delta U_{CA}=+\frac{9P_0V_0}{4}$$

Further, $Q_{AB}+Q_{BC}+Q_{CA}=W_{AB}+W_{BC}+W_{CA}$ $\{\because\ \Delta U=0\}$

Since we have, $Q_{CA}=-W_{CA}$ {given}

$$\Rightarrow\quad 2W_{CA}=Q_{AB}+Q_{BC}-W_{AB}-W_{BC}=-\frac{9P_0V_0}{4}$$

$$\Rightarrow\quad W_{CA}=-\frac{9P_0V_0}{8}$$

So from FLTD, we get

$$Q_{CA}=W_{CA}+\Delta U_{CA}=\frac{9P_0V_0}{8}$$

The Q, W and ΔU values for different processes are summarised in the table below.

Process	Q	W	ΔU
AB	$3P_0V_0\log_e(3)$	$P_0V_0\log_e(3)$	0
BC	$-\frac{9P_0V_0}{4}$	0	$-\frac{9P_0V_0}{4}$
CA	$\frac{9P_0V_0}{8}$	$-\frac{9P_0V_0}{8}$	$\frac{9P_0V_0}{4}$

(b) For isothermal process, $C_{AB} \to \infty$

For process CA, $Q_{CA} = \Delta U_{CA} + W_{CA}$

Also, $W_{CA} = -Q_{CA}$ {given}

$\Rightarrow \quad 2Q_{CA} = \Delta U_{CA}$

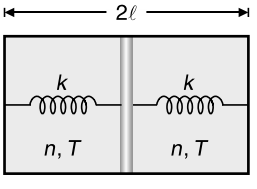
$\Rightarrow \quad 2C\Delta T = C_V\Delta T$

$\Rightarrow \quad C = \frac{C_V}{2} = \left(\frac{1}{2}\right)\left(\frac{3}{2}R\right)$

$\Rightarrow \quad C = \frac{3}{4}R$

PROBLEM 12

A thin heat insulating piston divides a horizontal cylindrical vessel of length $2l$ into two equal parts. Each part contains n moles of ideal monatomic gas at a temperature T. Two springs of spring constant k each are connected to the piston on either side as shown in figure. When heat Q is supplied to the gas in the right part, the piston is displaced to the left by a distance $x = \frac{l}{2}$. The left part is in contact with a thermostat at temperature T all the time. Determine the heat Q' given away to the thermostat.



SOLUTION

Net heat added to the system is $Q - Q'$. From First Law of Thermodynamics, we have

$$Q = \Delta U + W \Rightarrow Q - Q' = \Delta U + U_S \qquad \ldots(1)$$

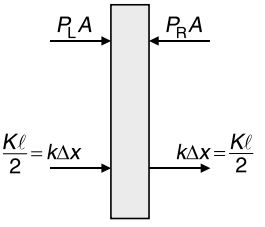
where ΔU is the change in internal energy of the system and U_S is the elastic potential energy stored in the spring. The work done by the gas changes the potential energy of the spring.

The temperature in the left part has not changed as it is in contact with a thermostat and piston wall is non-conducting. Hence the change in the internal energy of the gas is due to the heating of the gas in the right part by ΔT. So, we have

$$\Delta U = nC_V\Delta T = \frac{3}{2}nR\Delta T \qquad \ldots(2)$$

Since the gas is monatomic, so $C_V = \frac{3}{2}R$

Forces acting on the piston are shown in Figure.



In equilibrium, we have

$$P_LA + 2k\Delta X = P_RA \qquad \ldots(3)$$

When the piston has moved by a distance $x = \frac{l}{2}$, the volume of the left part becomes $V_L = A\left(l - \frac{l}{2}\right)$ and the volume of the right part becomes $V_R = A\left(l + \frac{l}{2}\right)$. From Ideal Gas Equation, we have

$$P_L = \frac{nRT}{A\left(l - \frac{l}{2}\right)} \qquad \ldots(4)$$

$$P_R = \frac{nR(T + \Delta T)}{A\left(l + \frac{l}{2}\right)} \qquad \ldots(5)$$

Substituting expression for P_L and P_R in equation (3), we get

$$\frac{2nR(T + \Delta T)}{3Al} = \frac{2nRT}{Al} + \frac{kl}{A} \qquad \ldots(6)$$

Solving for ΔT, we now get

$$\Delta T = 2T + \frac{3kl^2}{2nR}$$

From equation (1), we have

$$Q' = Q - \Delta U - U_S$$

$$\Rightarrow \quad Q' = Q - \frac{3}{2}nR\left[2T + \frac{3kl^2}{2nR}\right] - \frac{kl^2}{4}$$

$$\Rightarrow \quad Q' = Q - 3nRT - \frac{5}{2}kl^2$$

PROBLEM 13

A solid body X of heat capacity C is kept in an atmosphere whose temperature is $T_A = 300$ K. At time $t = 0$, the temperature of X is $T_0 = 400$ K. It cools according to Newton's Law of Cooling. At time t_1 its temperature is found to be

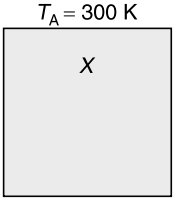
350 K. At this time (t_1) the body X is connected to a large body Y at atmospheric temperature T_A through a conducting rod of length L, cross-sectional area A and thermal conductivity K. The heat capacity of Y is so large that any variation in its temperature may be neglected. The cross-sectional area A of the connecting rod is small compared to the surface area of X. Find the temperature of X at time $t = 3t_1$.

SOLUTION

In the first part of the question $(t \le t_1)$

At $t = 0$, $T_x = T_0 = 400$ K and at $t = t_1$, $T_x = T_1 = 350$ K

Temperature of atmosphere, $T_A = 300$ K (constant)



Since, this cools down according to Newton's Law of Cooling, so we have

Rate of Cooling $\propto$ Temperature Difference

$$\Rightarrow \quad \left(-\frac{dT}{dt}\right) = k(T - T_A)$$

$$\Rightarrow \quad \frac{dT}{T - T_A} = -kdt$$

$$\Rightarrow \quad \int_{T_o}^{T_1} \frac{dT}{T - T_A} = -k\int_0^{t_1} dt$$

$$\Rightarrow \quad \log_e\left(\frac{T_1 - T_A}{T_o - T_A}\right) = -kt_1$$

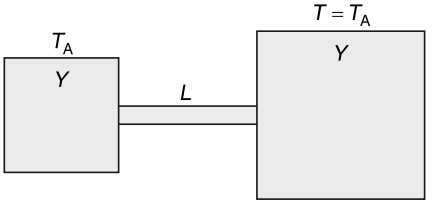
$$\Rightarrow \quad kt_1 = -\log_e\left(\frac{350 - 300}{400 - 300}\right)$$

$$\Rightarrow \quad kt_1 = \log_e(2) \qquad \ldots(1)$$

In the second part, body X cools by radiations (according to Newton's Law) as well as by conduction $(t > t_1)$, so

$$\left(\begin{array}{c}\text{Rate of}\\ \text{Cooling}\end{array}\right) = \left(\begin{array}{c}\text{Cooling by}\\ \text{Radiation}\end{array}\right) + \left(\begin{array}{c}\text{Cooling by}\\ \text{Conduction}\end{array}\right)$$

$$\Rightarrow \quad \left(-\frac{dT}{dt}\right) = k(T - T_A) + \frac{KA}{CL}(T - T_A) \qquad \ldots(2)$$



Since, in conduction, $\frac{dQ}{dt} = \frac{KA(T - T_A)}{L} = C\left(-\frac{dT}{dt}\right)$

$$\Rightarrow \quad \left(-\frac{dT}{dt}\right) = \frac{KA}{LC}(T - T_A)$$

Where C is the heat capacity of body X,

$$\Rightarrow \quad \left(-\frac{dT}{dt}\right) = \left(k + \frac{KA}{CL}\right)(T - T_A) \qquad \ldots(3)$$

Let at $t = 3t_1$, temperature of X becomes T_2.

Therefore, from equation (3), we get

$$\int_{T_1}^{T_2} \frac{dT}{T - T_A} = -\left(k + \frac{KA}{LC}\right)\int_{t_1}^{3t_1} dt$$

$$\Rightarrow \quad \log_e\left(\frac{T_2 - T_A}{T_1 - T_A}\right) = -\left(k + \frac{KA}{LC}\right)(2t_1)$$

$$\Rightarrow \quad \log_e\left(\frac{T_2 - T_A}{T_1 - T_A}\right) = -\left(2kt_1 + \frac{2KA}{LC}t_1\right)$$

Since, from equation (1), we have

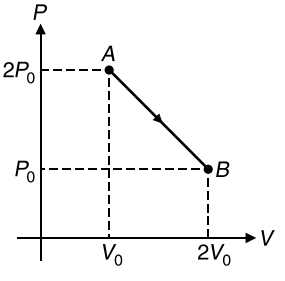
$$kt_1 = \log_e(2)$$

$$\Rightarrow \quad \log_e\left(\frac{T_2 - 300}{350 - 300}\right) = -2\log_e(2) - \frac{2KAt_1}{LC}$$

$$\Rightarrow \quad T_2 = \left(300 + 12.5e^{\frac{-2KAt_1}{CL}}\right) K$$

PROBLEM 14

P-V diagram of n moles of an ideal gas is as shown in figure. Find the maximum temperature between A and B.



SOLUTION

For given number of moles of a gas, we have

$$T \propto PV \qquad \{\because PV = nRT\}$$

Although we observe that $(PV)_A = (PV)_B$

$$\Rightarrow \quad T_A = T_B$$

However still it is not an isothermal process. Because in an isothermal process P-V graph is a rectangular hyperbola while here it is a straight line. So, to see the behaviour of temperature first we will find either T-V equation or T-P

equation and from that equation we can judge how the temperature varies. From the graph we will firstly write P-V equation and then we will convert it either to T-V equation or to T-P equation.
From the graph the P-V equation can be written as,

$$P = -\left(\frac{P_0}{V_0}\right)V + 3P_0 \qquad \{\because\ y = -mx + c\}$$

$$\Rightarrow \quad PV = -\left(\frac{P_0}{V_0}\right)V^2 + 3P_0V$$

Since $PV = nRT$, so we get

$$nRT = 3P_0V - \left(\frac{P_0}{V_0}\right)V^2$$

$$\Rightarrow \quad T = \frac{1}{nR}\left[3P_0V - \left(\frac{P_0}{V_0}\right)V^2\right]$$

This is the required T-V equation. This is quadratic in V. Hence, T-V graph is a parabola. Now, to find maximum value of T we have

$$\frac{dT}{dV} = 0$$

$$\Rightarrow \quad 3P_0 - \left(\frac{2P_0}{V_0}\right)V = 0$$

$$\Rightarrow \quad V = \frac{3}{2}V_0$$

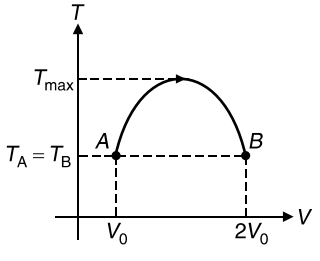
Further $\frac{d^2T}{dV^2} < 0$, at $V = \frac{3}{2}V_0$

Hence, T is maximum at $V = \frac{3}{2}V_0$ and this maximum value is

$$T_{max} = \frac{1}{nR}\left[(3P_0)\left(\frac{3V_0}{2}\right) - \left(\frac{P_0}{V_0}\right)\left(\frac{3V_0}{2}\right)^2\right]$$

$$\Rightarrow \quad T_{max} = \frac{9P_0V_0}{4nR}$$

Thus, T-V graph is shown in figure, where



$$T_A = T_B = \frac{2P_0V_0}{nR}$$

and $T_{max} = \frac{9P_0V_0}{4nR} = 2.25\left(\frac{P_0V_0}{nR}\right)$

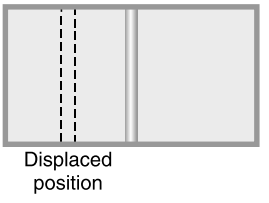
PROBLEM 15

A heat conducting piston can freely move inside a closed thermally insulated cylinder with an ideal gas. In equilibrium the piston divides the cylinder into two equal parts, the gas temperature being equal to T_0. The piston is slowly displaced. Find the gas temperature as a function of the ratio η of the volumes of the greater and smaller sections. The adiabatic exponent of the gas is equal to γ.

SOLUTION

As the piston is heat conducting, the temperature on both the parts is equal at each instant. The system is isolated, so there is no heat transfer to or from the system. From First Law of Thermodynamics $(FLTD)$, we have

$$Q = \Delta U + W = 0$$



From the condition of the problem, we get

$$V_0 + v = \eta(V_0 - v)$$

where v is the displaced volume

$$\Rightarrow \quad v = \left(\frac{\eta - 1}{\eta + 1}\right)V_0 \qquad \ldots(1)$$

The work done by an external agent in displacing the piston slowly increases the internal energy of both the compartments is

$$F_{ext}dx = 2nC_VdT = (P_1 - P_2)dV \qquad \ldots(2)$$

where P_1 and P_2 are the respective pressures of the two compartments.
From Ideal Gas Equation, we have

$$P_1 = \frac{nRT}{V_0 - v},\ P_2 = \frac{nRT}{V_0 + v} \qquad \ldots(3)$$

On substituting equation (3) in (2), we get

$$\left(\frac{nRT}{V_0 - v} - \frac{nRT}{V_0 + v}\right)dv = 2nC_VdT$$

$$\Rightarrow \quad \frac{R}{C_V}\int_0^V \frac{n}{(V_0^2 - v^2)}dv = \int_{T_0}^T \frac{dT}{T}$$

Since $C_V = \frac{R}{\gamma - 1}$, so

$$(\gamma - 1)\int_0^V \frac{v}{V_0^2 - v^2}dv = \log_e\left(\frac{T}{T_0}\right)$$

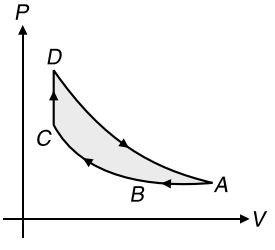
$$\Rightarrow \quad \log_e\left(\frac{T}{T_0}\right)=\left[\frac{\gamma-1}{2}\right]\left[-\log_e\left(V_0^2-v^2\right)\right]_0^v$$

$$\Rightarrow \quad \log_e\left(\frac{T}{T_0}\right)=\log_e\left[\frac{V_0^2}{V_0^2-v^2}\right]^{\frac{\gamma-1}{2}}$$

$$\Rightarrow \quad T=T_0\left[\frac{(\eta+1)^2}{4\eta}\right]^{\frac{\gamma-1}{2}}$$

PROBLEM 16

Helium is used as working substance in an engine working on the cycle as shown in figure. Processes A-B, B-C, C-D and D-A are isobaric, adiabatic, isochoric and isothermal respectively. The ratio of maximum to minimum volume of helium during the cycle is $8\sqrt{2}$ and that of maximum to minimum absolute temperature is 4. Calculate efficiency of the cycle.



SOLUTION

Let $V_C = V_D = V_0$ (minimum), so $V_A = 8\sqrt{2}V_0$ (maximum)

Since, process AB is isobaric, so we have

$$\frac{V_A}{T_A}=\frac{V_B}{T_B} \qquad \ldots(1)$$

Since $V_A > V_B$, so $T_A > T_B$

Further, process BC is adiabatic compression. Hence, $T_C > T_B$

Since, process CD is isochoric, so we have

$$\frac{P_C}{T_C}=\frac{P_D}{T_D}$$

Since $P_D > P_C$, so $T_D > T_C$

The process DA is isothermal. Hence, $T_A = T_D$

Hence, during complete cycle the temperature at B is minimum while the temperature at A and D has the maximum value. So let, $T_B = T_0$ (minimum),

Then, $T_A = T_D = 4T_0$ (maximum)

From equation (1), we have

$$V_B=\left(\frac{T_B}{T_A}\right)V_A=\left(\frac{1}{4}\right)\left(8\sqrt{2}V_0\right)=2\sqrt{2}V_0$$

Since, BC is an adiabatic process, so

$$T_C V_C^{\gamma-1}=T_B V_B^{\gamma-1}$$

$$\Rightarrow \quad T_C=T_B\left(\frac{V_B}{V_C}\right)^{\gamma-1}=T_0\left(\frac{2\sqrt{2}V_0}{V_0}\right)^{\frac{5}{3}-1}$$

$$\Rightarrow \quad T_C=2T_0$$

Now let us make a table for W and Q for different processes.

Process	W	Q
AB (isobaric)	$P(V_B-V_A)$ $=nR(T_B-T_A)$ $=-3nRT_0$	Q is negative
BC (adiabatic)	$\frac{nR(T_B-T_c)}{\gamma-1}$ $=\frac{nR(T_0-2T_0)}{\left(\frac{5}{3}\right)-1}$ $=-\frac{3}{2}nRT_0$	0
CD (isochoric)	0	$nC_V(T_D-T_C)$ $=(n)\left(\frac{3}{2}R\right)(4T_0-2T_0)$ $=3nRT_0$
DA (isothermal)	$nR(4T_0)\log_e\left(\frac{8\sqrt{2}V_0}{V_0}\right)$ $=14nRT_0\log_e(2)$	$14nRT_0\log_e(2)$

So, $W_{\text{total}} = 14nRT_0\log_e(2)-3nRT_0-\frac{3}{2}nRT_0$

$$\Rightarrow \quad W_{\text{total}}=5.2nRT_0$$

and $Q_{+\text{ve}} = 14nRT_0\log_e(2)+3nRT_0$

$$\Rightarrow \quad Q_{+\text{ve}}=12.7nRT_0$$

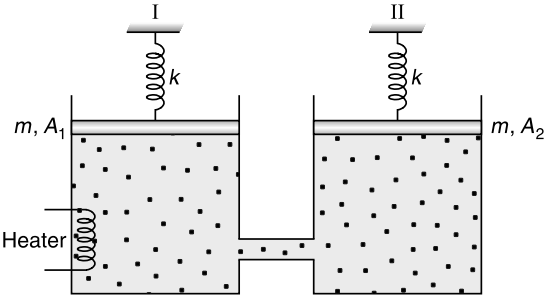
So, efficiency of cycle is

$$\eta=\frac{W_{\text{total}}}{Q_{+\text{ve}}}\times 100=\left(\frac{5.2}{12.7}\right)\times 100$$

$$\Rightarrow \quad \eta=43\%$$

PROBLEM 17

Two vertical cylinders with thermally insulated walls and pistons of same mass fitted on the top as shown in the figure. The cylinders enclose n moles of a monatomic gas. The initial pressure is P and it is given that Spring (I) is relaxed. An electric heater slowly supplies heat to the system. Finally, Spring (II) becomes relaxed.



(a) What is the initial state of Spring (II)? Given that $A_2 < A_1$.
(b) What is the final pressure?
(c) If the initial volume is V, what is the final volume?

SOLUTION

(a) Let P_0 be the atmospheric pressure, then considering the equilibrium of Piston I, we get

$$PA_1 = mg + P_0A_1$$

$$\Rightarrow \quad P = P_0 + \frac{mg}{A_1} \qquad \ldots(1)$$

Now consider the forces acting on Piston II. According to given condition, Spring II is relaxed, finally. Before that we have to determine the state of spring by comparing the downward and upward forces.
Net upward force acting on Piston II is

$$F_{up} = PA_2 = P_0A_2 + mg\frac{A_2}{A_1} \qquad \{\because \text{ of equation (1)}\}$$

Net downward force acting on Piston II is

$$F_{down} = mg + P_0A_2$$

Since, $A_2 < A_1$, so net upward force is less than net downward force. Hence to keep the piston in equilibrium, the spring must apply an upward force on it, therefore the spring is stretched initially and the spring force is

$$kx = mg\left(1 - \frac{A_2}{A_1}\right)$$

where x is the extension in the Spring II.

(b) When Spring II is relaxed, let P' be the pressure. Then from the equilibrium condition of piston of area A_2 from equation (1), we have

$$P' = \frac{mg}{A_2} + P_0 = P - \frac{mg}{A_1} + \frac{mg}{A_2}$$

$$\Rightarrow \quad P' = P + mg\left(\frac{A_1 - A_2}{A_1A_2}\right)$$

(c) When the pressure increases, Spring I is compressed. Let x' be the compression, then from the condition of equilibrium of piston of area A_1, we have

$$P'A_1 = kx' + mg + P_0A_1$$

$$\Rightarrow \quad kx' = P'A_1 - mg - P_0A_1$$

$$\Rightarrow kx' = \left[P + mg\left(\frac{A_1 - A_2}{A_1A_2}\right)\right]A_1 - mg - \left(P - \frac{mg}{A_1}\right)A_1$$

$$\Rightarrow \quad kx' = mg\left(\frac{A_1}{A_2} - 1\right)$$

Change in volume of the gas is $\Delta V = x'A_1 + xA_2$

$$\Rightarrow \quad \Delta V = \frac{mg}{k}\left(\frac{A_1}{A_2} - 1\right)A_1 + \frac{mg}{k}\left(1 - \frac{A_1}{A_2}\right)A_2$$

$$\Rightarrow \quad \Delta V = \frac{mg}{k}\left(\frac{A_1^2}{A_2} - A_1 + A_2 - \frac{A_2^2}{A_1}\right)$$

Final volume is

$$V_f = V + \Delta V = V + \frac{mg}{k}\left(\frac{A_1^2}{A_2} - A_1 + A_2 - \frac{A_2^2}{A_1}\right)$$

$$\Rightarrow \quad V_f = V + \frac{mg}{k}\frac{(A_1 - A_2)(A_1^2 + A_2^2)}{A_1A_2}$$

PROBLEM 18

A cubical rigid container of edge $l = 10$ cm, thickness of wall $d = 4$ mm and thermal conductivity $k = 8.31 \times 10^{-3}$ $Wm^{-1}K^{-1}$ has one mole of a monatomic gas at temperature $T_0 = 400$ K. If the temperature of the surrounding is $T_s = 300$ K, find the

(a) time when pressure becomes $\frac{5}{6}$ of initial pressure.
(b) temperature at that instant.

SOLUTION

(a) From the First Law of Thermodynamics, we have

$$dQ = dU + dW$$

Since, volume is constant, therefore, $dW = 0$. So, for 1 mole of gas, we have

$$\left(\frac{dQ}{dt}\right) = C_V\left(-\frac{dT}{dt}\right) \qquad \ldots(1)$$

Further from the equation of heat conduction, we have

$$\left(\frac{dQ}{dt}\right) = \frac{kA(T - T_s)}{d} \qquad \ldots(2)$$

From equations (1) and (2), we get

$$\frac{dT}{dt} = \frac{kA(T_s - T)}{C_Vd} = \frac{2kA(T_s - T)}{3Rd}$$

$$\Rightarrow \quad \int_{T_0}^{T}\frac{dT}{T_s - T} = \frac{2kA}{3Rd}\int_0^t dt$$

$$\Rightarrow \quad T = T_s - (T_s - T_0)e^{-\frac{2kAt}{3Rd}} \qquad \ldots(3)$$

As the volume remains constant, so

$$\frac{P}{T}=\frac{P_0}{T_0}$$

$$\Rightarrow\ P=\left(\frac{P_0}{T_0}\right)T$$

$$\Rightarrow\ P=\left(\frac{P_0}{T_0}\right)\left(T_s-(T_s-T_0)e^{-\frac{2kAt}{3Rd}}\right) \qquad \ldots(4)$$

Given, $\frac{P}{P_0}=\frac{5}{6}$, $A=6l^2$

$$\Rightarrow\ \frac{2kA}{3Rd}=\frac{4kl^2}{Rd}=\frac{4\times8.31\times10^{-3}\times(0.1)^2}{(8.31)(4\times10^{-3})}=10^{-2}\ \text{s}^{-1}$$

Substituting the values in equation (4)

$$\frac{5}{6}=\left(\frac{1}{400}\right)\left(300+(100)e^{-10^{-2}t}\right)$$

Solving this equation, we get

$$t\approx 110\ \text{sec.}$$

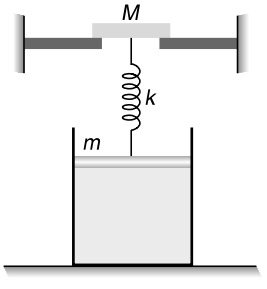
(b) As $P\propto T$ $\{V=\text{constant}\}$

So, temperature will also remain $\frac{5}{6}$th of its initial value, hence

$$T=\frac{5}{6}T_0=\left(\frac{5}{6}\right)(400)=333.33\ \text{K}$$

PROBLEM 19

0.01 moles of an ideal diatomic gas is enclosed in an adiabatic cylinder of cross-sectional area $A=10^{-4}\ \text{m}^2$. In the arrangement shown, a block of mass $M=0.8\ \text{kg}$ is placed on a horizontal support and another block of mass $m=1\ \text{kg}$ is suspended from a spring of stiffness constant $k=16\ \text{Nm}^{-1}$. Initially, the spring is relaxed and the volume of the gas is $V=1.4\times10^{-4}\ \text{m}^3$.



(a) Find the initial pressure of the gas.

(b) If block m is gently pushed down and released it oscillates harmonically, find its angular frequency of oscillation.

(c) When the gas in the cylinder is heated up the piston starts moving up and the spring gets compressed so that the block M is just lifted up. Determine the heat supplied. Take atmospheric pressure $p_0=10^5\ \text{Nm}^{-2}$, $g=10\ \text{ms}^{-2}$.

SOLUTION

(a) $P=P_0+\frac{mg}{A}=10^5+\frac{(1)(10)}{10^{-4}}=2\times10^5\ \text{Nm}^{-2}$

(b) For adiabatic process, $\left(\frac{dP}{dV}\right)=-\gamma\left(\frac{P}{V}\right)$

$$\Rightarrow\ |(dP)|=\left(\frac{7}{5}\right)\left(\frac{2\times10^5}{1.4\times10^{-4}}\right)(Ax)$$

Net restoring force is $F=-[|dP|A+kx]$

$$\Rightarrow\ m\frac{d^2x}{dt^2}=-\left[\left(\frac{7}{5}\right)\left(\frac{2\times10^5}{1.4\times10^{-4}}\right)(10^{-4})^2+16\right]x$$

$$\Rightarrow\ \frac{d^2x}{dt^2}=-36x=-\omega^2x$$

$$\Rightarrow\ \omega=6\ \text{rads}^{-1}$$

(c) Block M will start moving up when

$$kx=Mg$$

$$\Rightarrow\ x=\frac{Mg}{k}=\frac{0.8\times10}{16}=0.5\ \text{m}$$

Now, $\Delta V=Ax=5\times10^{-5}\ \text{m}^3$

and $W_{\text{gas}}=P_0\Delta V+\frac{1}{2}kx^2+mgx$

$$\Rightarrow\ W_{\text{gas}}=(10^5)(5\times10^{-5})+\frac{1}{2}\times16\times(0.5)^2+(1)(10)(0.5)$$

$$\Rightarrow\ W_{\text{gas}}=12\ \text{J}$$

Since, $\Delta U=nC_V\Delta T$

where, $T_i=\frac{P_iV_i}{nR}=\frac{(2\times10^5)(1.4\times10^{-4})}{(0.01)(8.31)}=337\ \text{K}$

and $T_f=\frac{P_fV_f}{nR}=\frac{\left(P_0+\frac{kx}{A}+\frac{mg}{A}\right)(1.9\times10^{-4})}{(0.01)(8.31)}$

$$\Rightarrow\ T_f=\frac{\left(2\times10^5+\frac{16\times0.5}{10^{-4}}\right)(1.9\times10^{-4})}{(0.01)(8.31)}$$

$$\Rightarrow\ T_f=640.3\ \text{K}$$

$$\Rightarrow\ \Delta U=(0.01)\left(\frac{5}{2}\right)(8.31)(640.3-337)$$

$$\Rightarrow\ \Delta U=63\ \text{J}$$

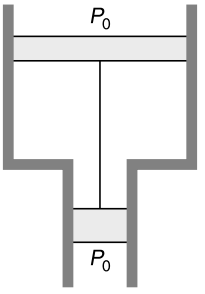
So, from $FLTD$, we get

$$Q=\Delta U+W=63+12=75\ \text{J}$$

PROBLEM 20

A smooth vertical cylinder has two different cross-sections open from both ends and equipped with two pistons of different cross-section areas. Each piston slides within a respective section as shown in figure.

One mole of an ideal gas is enclosed between the pistons tied with a non-stretchable thread. The cross-section area of the upper piston is ΔA greater than that of the lower one. The combined mass of the two pistons is equal to m. The atmospheric pressure outside is P_0. By how much kelvin must the gas between the pistons be heated to shift the piston through l unit.



SOLUTION

Let A_1, A_2 denote the cross-section area of the lower and upper piston and l_1, l_2 the lengths of string in respective sections. Initial volume of the gas enclosed is

$$V_i = l_1A_1 + l_2A_2$$

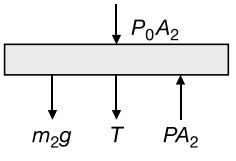
If the piston shifts upward by l, then final volume of the gas enclosed is

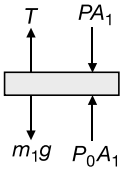
$$V_f = (l_1 - l)A_1 + (l_2 + l)A_2$$

The change in volume is

$$\Delta V = lA_1 - lA_2 = l\Delta A \qquad \ldots(1)$$

Let us draw the free body diagrams for the two pistons.





Now we consider the equilibrium of the upper and lower pistons, then

For Lower Piston, $PA_1 + m_1g = T + P_0A_1$ …(2)

For Upper Piston, $PA_2 = P_0A_2 + T + m_2g$ …(3)

From equations (2) and (3), we get

$$P(A_1 - A_2) = P_0(A_1 - A_2) + (m_1 + m_2)g$$

$$\Rightarrow \quad P\Delta V = (P_0\Delta A + mg)l$$

when $m = m_1 + m_2$ is the combined mass of the piston, as given in the problem.

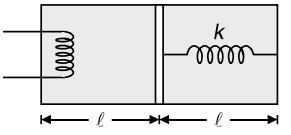
After temperature rises by ΔT, from Ideal Gas Equation applied for one mole of gas, we have

$$\Delta T = \frac{P\Delta V}{R}$$

$$\Rightarrow \quad \Delta T = \frac{(P_0\Delta A + mg)l}{R}$$

PROBLEM 21

An adiabatic cylinder of length 1 m and area of cross section 10^{-2} m^2 is closed at both ends. A freely moving nonconducting thin piston divides the cylinder into two equal parts. Each part contains 28 g of N_2. The natural length of the spring connected to the piston and right wall of the cylinder is $l = 50$ cm and $k = \sqrt{2}\times10^3 \ \frac{\text{N}}{\text{m}}$. Initially one-third molecules of the nitrogen in the right part are dissociated into atoms. If initial pressure in each part is $P_0 = \sqrt{2}\times10^5$ Nm^{-2}, then to compress the spring by $\frac{3l}{4}$, calculate the



(a) work done by the gas in right part.
(b) heat supplied by the heater.

SOLUTION

(a) Initial volume of gas in the right part is $V_0 = Al$. To compress the spring by $\frac{3l}{4}$, the final volume becomes $V = \frac{V_0}{4} = \frac{Al}{4}$, so for the gas in the right part, we have

$$P_0(Al)^\gamma = P'\left(\frac{Al}{4}\right)^\gamma \qquad \ldots(1)$$

The right part contains 28 g of N_2, i.e., 1 mol of N_2 and $\frac{1}{3}$rd of molecules are dissociated into atoms. So, in the right part $n_1 = \frac{2}{3}$ moles are of diatomic gas $\left(C_{V_1} = \frac{5}{2}R\right)$ and $n_2 = 2\times\frac{1}{3} = \frac{2}{3}$ moles are of monatomic gas $\left(C_{V_2} = \frac{3}{2}R\right)$. The C_V of the mixture is given by

$$C_V = \frac{n_1C_{V_1} + n_2C_{V_2}}{n_1 + n_2} = \frac{\left(\frac{2}{3}\right)\left(\frac{5}{2}R\right) + \left(\frac{2}{3}\right)\left(\frac{3}{2}R\right)}{\left(\frac{2}{3} + \frac{2}{3}\right)}$$

$$\Rightarrow \quad C_V = 2R$$

Since, $C_P = C_V + R = 3R$

$$\Rightarrow \quad \gamma = \frac{C_P}{C_V} = 1.5$$

Hence, from equation (1), we get

$$P' = 8P_0$$

Work done during compression by the gas in right part is given by

$$W_R = \frac{P_0V_0 - P'V'}{\gamma - 1}$$

$$\Rightarrow \quad W_R = \frac{P_0(Al) - (8P_0)\left(\frac{Al}{4}\right)}{1.5 - 1} = -2P_0Al$$

$$\Rightarrow \quad W_R = -(2)\left(\sqrt{2}\times 10^5\right)\left(10^{-2}\right)(0.5) = -1414 \text{ J}$$

Here negative sign implies that work is done on the gas

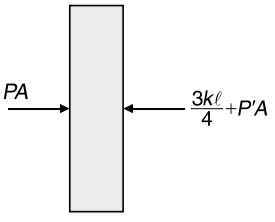
(b) Considering equilibrium of piston, we get

$$PA = \frac{3kl}{4} + P'A$$

So, pressure on the left part is

$$P = 8P_0 + \frac{3kl}{4A}$$

Initial temperature $T_0 = \frac{P_0Al}{R}$



and final temperature

$$T = \frac{(P)\left(\frac{5}{4}Al\right)}{R} = \frac{5Al}{4R}\left(8P_0 + \frac{3kl}{4A}\right)$$

Increase in internal energy of the gas in left part is

$$\Delta U_L = C_v\Delta T = \left(\frac{3}{2}R\right)(T - T_0)$$

$$\Rightarrow \quad \Delta U_L = \left(\frac{3}{2}R\right)\left[\frac{5Al}{4R}\left(8P_0 + \frac{3kl}{4A}\right) - \frac{P_0Al}{R}\right]$$

$$\Rightarrow \quad \Delta U_L = \frac{27}{2}P_0Al + \frac{45}{32}kl^2$$

Work done by gas in the left part to compress the spring and the gas in right part is given by

$$W_L = \frac{1}{2}k\left(\frac{3}{4}l\right)^2 + |W_R|$$

$$\Rightarrow \quad W_L = \frac{9}{32}kl^2 + 2P_0Al$$

Hence, heat supplied by heating coil is

$$Q = W_L + \Delta U_L$$

$$\Rightarrow \quad Q = \frac{9}{32}kl^2 + 2P_0Al + \frac{27}{2}P_0Al + \frac{45}{32}kl^2$$

Substituting the values, we get

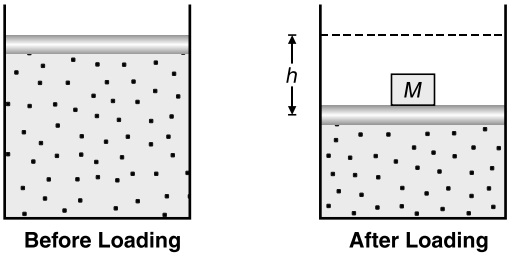
$$Q = 11555 \text{ J}$$

PROBLEM 22

A vertical cylinder of volume V has n moles of an ideal monatomic gas. The walls of the cylinder are thermally insulated, the piston is weightless. When a mass M is placed on the piston, the piston is displaced by a distance h. What is the final temperature of the gas after the piston has been displaced, the area of the piston is A and the atmospheric pressure is P_0?

SOLUTION

Since the gas is thermally insulated, the entire work done on the gas is equal to change in internal energy. When the piston is displaced by h, the volume becomes $(V - Ah)$. The net pressure on the piston is $P_0 + \left(\frac{Mg}{A}\right)$. From Ideal Gas Equation, we have



Before loading,

$$P_0V = nRT_1 \qquad \ldots(1)$$

After loading,

$$\left[P_0 + \left(\frac{Mg}{A}\right)\right](V - Ah) = nRT_2 \qquad \ldots(2)$$

The change in internal energy is given by

$$\Delta U = nC_V\Delta T = \frac{3}{2}nR(T_2 - T_1)$$

Work done on the system is Mgh

$$\Rightarrow \quad W = -Mgh$$

Therefore, from First Law of Thermodynamics, we have

$$Q = W + \Delta U$$

Since, the walls of the cylinder are thermally insulated, so

$$Q = 0$$

$$\Rightarrow \quad 0 = -Mgh + \Delta U$$

$$\Rightarrow \quad Mgh = \frac{3}{2}nR(T_2 - T_1) \qquad \ldots(3)$$

Subtracting equation (1) from equation (2), we get

$$\left[P_0V + \left(\frac{Mg}{A}\right)V - P_0Ah - Mgh - P_0V\right] = nR(T_2 - T_1)$$

$$\Rightarrow \quad nR(T_2 - T_1) = \left[\left(\frac{Mg}{A}\right)V - P_0Ah - Mgh\right] \qquad \ldots(4)$$

Substituting $nR(T_2 - T_1)$ from equation (4) in equation (3), we get

$$\frac{MgV}{A} - Mgh - P_0Ah = \frac{2}{3}Mgh$$

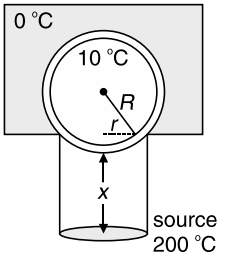
$$\Rightarrow \quad h = \frac{MgV}{A\left(P_0A + \frac{5}{3}Mg\right)}$$

Substituting h in equation (2) and solving for T_2, we get

$$T_2 = \frac{(P_0A + mg)(3P_0A - 2Mg)V}{(3P_0A + 5Mg)nAR}$$

PROBLEM 23

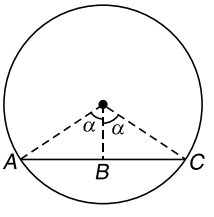
A spherical shell made from iron is placed at a distance x above a circular heat source of radius r. Radius of shell is R. The shell is surrounded by cold bath (0 °C) except for the part which is just above the source. The temperature of the source is 200 °C. In equilibrium position the temperature of the inner part of the shell is 10 °C. If the temperature at a distance x from the source is $\frac{1}{x}$ times the temperature of the source, find x at equilibrium. The thickness of the shell is t.



SOLUTION

The heat is flowing into the shell through the part just above the source and the remaining part is conducting heat into the bath.

At equilibrium these two rates are equal. The surface area through which the heat is going into, i.e., the area ABC is,



$$A_1 = 2\pi R^2(1 - \cos\alpha)$$

where, $\cos\alpha = \frac{\sqrt{R^2 - r^2}}{R}$

$$\Rightarrow \quad A_1 = 2\pi R^2\left[1 - \sqrt{1 - \left(\frac{r}{R}\right)^2}\right]$$

So, $A_2 = 4\pi R^2 - A_1 = 2\pi R^2\left[1 + \sqrt{1 - \left(\frac{r}{R}\right)^2}\right]$

Temperature at the outer surface of part ABC is

$$T_1 = \frac{200}{x}\ ^\circ\text{C}$$

Temperature of inner surface of shell is $T_2 = 10$ °C and temperature of the bath is $T_3 = 0$ °C

Since, $\left(\frac{dQ}{dt}\right)_{\text{in}} = \left(\frac{dQ}{dt}\right)_{\text{out}}$

$$\Rightarrow \quad \frac{kA_1(T_1 - T_2)}{t} = \frac{kA_2(T_2 - T_3)}{t}$$

Substituting the values and the solving, we get

$$x = 10\left[1 - \sqrt{1 - \left(\frac{r}{R}\right)^2}\right]$$

PROBLEM 24

A gaseous mixture enclosed in a vessel of volume V consists of one gram mole of gas A with $\gamma\left(= \frac{C_p}{C_v} = \frac{5}{3}\right)$ and another gas B with $\gamma = \frac{7}{5}$ at a certain temperature T. The gram molecular weights of the gases A and B are 4 and 32 respectively. The gases A and B do not react with each other and are assumed to be ideal. The gaseous mixture follows the equation $PV^{19/13}$ = constant, in adiabatic process

(a) Find the number of gram moles of the gas B in the gaseous mixture.

(b) Compute the speed of sound in the gaseous mixture at 300 K

(c) If T is raised by 1 K from 300 K, find the percentage change in the speed of sound in the gaseous mixture.

(d) The mixture is compressed adiabatically to $\frac{1}{5}$ of its initial volume V. Find the change in its adiabatic compressibility in terms of the given quantities.

SOLUTION

(a) Number of moles of gas A are $n_A = 1$ (given)

Let the number of moles of gas B be $n_B = n$

Then, $\begin{pmatrix}\text{Internal}\\ \text{Energy of}\\ \text{the Mixture}\end{pmatrix} = \begin{pmatrix}\text{Internal}\\ \text{Energy of}\\ \text{Gas } A\end{pmatrix} + \begin{pmatrix}\text{Internal}\\ \text{Energy of}\\ \text{Gas } B\end{pmatrix}$

Since, $U = \frac{nRT}{\gamma - 1}$

$$\Rightarrow \quad \frac{(n_A + n_B)RT}{\gamma_{\text{mixture}} - 1} = \frac{n_ART}{\gamma_A - 1} + \frac{n_BRT}{\gamma_B - 1} \qquad \ldots(1)$$

Since, the mixture obeys the law $PV^{19/13}$ = constant (in adiabatic process).

Therefore, $\gamma_{\text{mixture}} = \frac{19}{13}$ $\qquad \{\because\ PV^\gamma = \text{constant}\}$

Substituting the values in equation (1), we get

$$\frac{1+n}{\left(\frac{19}{13}\right) - 1} = \frac{1}{\left(\frac{5}{3}\right) - 1} + \frac{n}{\left(\frac{7}{5}\right) - 1}$$

Solving this, we get $n = 2$

Conceptual Note(s)

For $\gamma_{mixture}$ we can directly use the formula

$$\frac{n}{\gamma_{mixture}-1}=\frac{n_1}{\gamma_1-1}+\frac{n_2}{\gamma_2-1}$$

(b) Molecular weight of the mixture will be given by

$$M=\frac{n_AM_A+n_BM_B}{n_A+n_B}=\frac{(1)(4)+2(32)}{1+2}$$

$$\Rightarrow\quad M=22.67\text{ g}$$

Speed of sound in a gas is given by

$$v=\sqrt{\frac{\gamma RT}{M}}$$

Therefore, in the mixture of the gas, we have

$$v=\sqrt{\frac{(19/13)(8.31)(300)}{22.67\times10^{-3}}}\text{ ms}^{-1}=401\text{ ms}^{-1}$$

(c) $v\propto\sqrt{T}$

$$\Rightarrow\quad v=KT^{1/2}\qquad\ldots(2)$$

$$\Rightarrow\quad\frac{dv}{dT}=\frac{1}{2}KT^{-1/2}$$

$$\Rightarrow\quad dv=K\left(\frac{dT}{2\sqrt{T}}\right)$$

$$\Rightarrow\quad\frac{dv}{v}=\frac{K}{v}\left(\frac{dT}{2\sqrt{T}}\right)\frac{K}{v}=\frac{1}{\sqrt{T}}\qquad\{\text{from equation (2)}\}$$

$$\Rightarrow\quad\frac{dv}{v}=\frac{1}{\sqrt{T}}\left(\frac{dT}{2\sqrt{T}}\right)=\frac{1}{2}\left(\frac{dT}{T}\right)$$

$$\Rightarrow\quad\frac{dv}{v}\times100=\frac{1}{2}\left(\frac{1}{300}\right)\times100=0.167$$

Therefore, percentage change in speed is 0.167%

(d) $\text{Compressibility}=\dfrac{1}{\text{Bulk modulus}}=\beta(\text{say})$

Since, adiabatic Bulk modulus is given by

$$B_{ad}=\gamma P\qquad\left\{\because\ B=-\frac{dP}{dV/V}\right\}$$

So, adiabatic compressibility will be given by

$$\beta_{ad}=\frac{1}{\gamma P}\text{ and }\beta'_{ad}=\frac{1}{\gamma P'}=\frac{1}{\gamma P(5)^{\gamma}}$$

Since, $PV^{\gamma}=P'\left(\dfrac{V}{5}\right)^{\gamma}$

$$\Rightarrow\quad P'=P(5)^{\gamma},\text{ where }\gamma=\gamma_{mixture}=\frac{19}{13}$$

$$\Rightarrow\quad\Delta\beta=\beta'_{ad}-\beta_{ad}=-\frac{1}{\gamma P}\left[1-\left(\frac{1}{5}\right)^{\gamma}\right]$$

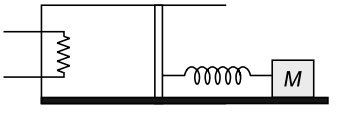
$$\Rightarrow\quad\Delta\beta=\frac{-V}{\gamma(n_A+n_B)RT}\left[1-\left(\frac{1}{5}\right)^{\gamma}\right]\qquad\left\{\because\ P=\frac{nRT}{V}\right\}$$

$$\Rightarrow\quad\Delta\beta=\frac{-V}{\left(\frac{19}{13}\right)(1+2)(8.31)(300)}\left[1-\left(\frac{1}{5}\right)^{\frac{19}{13}}\right]$$

$$\Rightarrow\quad\Delta\beta=-8.27\times10^{-5}\text{ V}$$

PROBLEM 25

An adiabatic cylinder has 8 g of helium. A light smooth adiabatic piston is connected to a light spring of force constant 300 Nm^{-1}. The other end of the spring is connected with a block of mass 10 kg kept on a rough horizontal surface of coefficient of friction $\mu=0.3$. Area of cross-section of cylinder is $A=25\text{ cm}^2$. Initially the spring is in a relax position and the temperature of the gas is 400 K, the gas is heated slowly for some time by means of an electric heater so as the block M just starts moving. Find

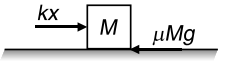


(a) the work done by the gas
(b) the final temperature
(c) heat supplied by the heater

Take 1 atm = 10^5 Nm^{-2} and $g=10\text{ ms}^{-2}$

SOLUTION

(a) The block M just starts moving when,



$$kx=\mu(Mg)$$

$$\Rightarrow\quad x=\frac{\mu Mg}{k}=\frac{(0.3)(10)(10)}{300}=0.1\text{ m}$$

The force exerted by the gas on the piston is

$$F=(P_0A+kx)$$

Work done by the gas is $W=\int_0^x F\,dx$

$$\Rightarrow\quad W=\int_0^x(P_0A+kx)\,dx=P_0Ax+\frac{1}{2}kx^2$$

$$\Rightarrow\quad W=(10^5)(25\times10^{-4})(0.1)+\left(\frac{1}{2}\right)(300)(0.1)^2$$

$\Rightarrow \quad W = 25 + 1.5 = 26.5 \text{ J}$

(b) The initial temperature is $T_i = 400$ K. Let the final temperature be T_f. Then,

$$P_0V_0 = nRT_i \text{ and } PV' = nRT_f$$

where $P = P_0 + \frac{kx}{A}$ and $V' = V_0 + Ax$

Hence, $nRT_f = \left(P_0 + \frac{kx}{A}\right)(V_0 + Ax)$

$$\Rightarrow \quad nRT_f = nRT_i + P_0Ax + kx^2 + \frac{kxnRT_i}{P_0A}$$

$$\Rightarrow \quad T_f = T_i + \left(\frac{P_0Ax + kx^2}{nR}\right) + \left(\frac{kxT_i}{P_0A}\right)$$

$$\Rightarrow \quad T_f = 400 + \frac{(25+3)}{2\times 8.31} + \frac{(300)(0.1)(400)}{(10^5)(25\times 10^{-4})}$$

$$\Rightarrow \quad T_f = 449.68 \text{ K}$$

(c) Change in internal energy is given by

$$\Delta U = nC_V\Delta T = \frac{3}{2}nR\left(T_f - T_i\right)$$

$$\Rightarrow \quad \Delta U = \left(\frac{3}{2}\right)(2)(8.31)(449.68 - 400) \text{ J}$$

$$\Rightarrow \quad \Delta U = 1238.52 \text{ J}$$

From First Law of Thermodynamics, we have

$$Q = \Delta U + W = 1238.52 + 26.5 \cong 1265 \text{ J}$$

PROBLEM 26

A wire of length 1 m and radius 10^{-3} m is carrying a current. At equilibrium its temperature is 900 K while that of surrounding is 300 K. The resistivity of the material of wire at 300 K is 10^{-7} Ωm and its temperature coefficient of resistance is 7.8×10^{-3} per °C. Find the current in the wire. Give $\sigma = 5.67\times 10^{-8}$ $\text{Wm}^{-2}\text{K}^{-4}$. Take emissivity for wire surface $e = 1$.

SOLUTION

In steady state,

$$\begin{pmatrix}\text{Power generated by}\\ \text{the resistance}\end{pmatrix} = \begin{pmatrix}\text{Power Radiated in}\\ \text{the Atmosphere}\end{pmatrix}$$

$$\Rightarrow \quad I^2R = \sigma A_1\left(T^4 - T_0^4\right)$$

If ρ be the electrical resistivity of the wire, then

$$R = \frac{\rho_{900\text{ K}}\, l}{A_2}$$

$$\Rightarrow \quad I^2\frac{\rho_{900\text{ K}}\, l}{A_2} = \sigma A_1\left(T^4 - T_0^4\right) \qquad \ldots(1)$$

where, $A_1 = 2\pi rl$, $A_2 = \pi r^2$

Since, $\rho_{900\text{ K}} = \rho_{300\text{ K}}(1 + \alpha\Delta T)$

$$\Rightarrow \quad \rho_{900\text{ K}} = 10^{-7}\left[1 + 7.8\times 10^{-3}\times(900 - 300)\right]$$

$$\Rightarrow \quad \rho_{900\text{ K}} = 5.68\times 10^{-7} \text{ Ωm}$$

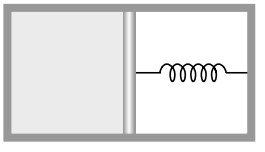
Substituting in equation (1), we have

$$I^2\left(\frac{5.68\times 10^{-7}}{\pi\times 10^{-6}}\right) = 5.67\times 10^{-8}\times 2\times\pi\times$$

$$10^{-3}\times 1\left[(900)^4 - (300)^4\right]$$

$$\Rightarrow \quad I = 35.73 \text{ A}$$

PROBLEM 27



A non-conducting piston separates a thermally insulated vessel into two parts and it moves in the vessel without friction. The left part of the vessel contains one mole of an ideal monatomic gas and the right part is empty. The piston is connected to the right wall of vessel, through a spring whose length in free state is equal to the length of the vessel. Calculate the heat capacity of the system neglecting the heat capacities of vessel, piston and spring.

SOLUTION

If Q be the heat supplied to the system, then this supplied heat will only increase the internal energy of the system because there is no friction and the vessel is thermally insulated, so $Q = \Delta U$

$$\Rightarrow \quad Q = \Delta U_{\text{spring}} + \Delta U_{\text{gas}} \qquad \ldots(1)$$

$$\text{where } \Delta U_{\text{spring}} = \frac{1}{2}k\left(x_2^2 - x_1^2\right) \qquad \ldots(2)$$

$$\text{and } \Delta U_{\text{gas}} = (1)C_V\Delta T = \frac{3R}{2}\left(T_2 - T_1\right) \qquad \ldots(3)$$

where x_1 and x_2 are compression in the spring at temperatures T_1 and T_2 respectively.

Now in equilibrium, we have $PA = kx$

$$\Rightarrow \quad x = \frac{PA}{k}$$

For one mole of an ideal gas, we have $PV = RT$

$$\Rightarrow \quad P(Ax) = RT$$

But $PA = kx$

$$\Rightarrow \quad kx^2 = RT$$

$$\Rightarrow \quad \frac{1}{2}k\left(x_2^2 - x_1^2\right) = \frac{R}{2}\left(T_2 - T_1\right)$$

$$\Rightarrow \quad \Delta U_{\text{spring}} = \frac{R}{2}\left(T_2 - T_1\right) \qquad \ldots(4)$$

$$\Rightarrow \quad Q = \Delta U = \frac{1}{2}k\left(x_2^2 - x_1^2\right) + \frac{3R}{2}\left(T_2 - T_1\right)$$

$$\Rightarrow \quad Q = \Delta U = \frac{R}{2}\left(T_2 - T_1\right) + \frac{3R}{2}\left(T_2 - T_1\right)$$

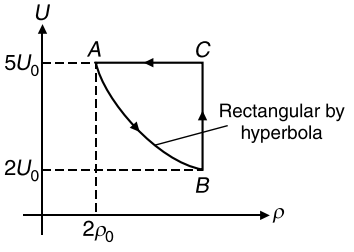
$$\Rightarrow \quad Q = \Delta U = 2R\left(T_2 - T_1\right)$$

But $C = \dfrac{Q}{\Delta T} = \dfrac{\Delta U}{\Delta T}$ {by definition}

$$\Rightarrow \quad C = 2R$$

PROBLEM 28

Figure shows the variation of the internal energy U with the density ρ of one mole of an ideal monatomic gas for a thermodynamic cycle $A \to B \to C \to A$. Assume the process $A \to B$ to be a part of a rectangular hyperbola.



(a) Draw the P-V diagram of the above process.
(b) Find the total amount of heat absorbed by the system for the cyclic process.
(c) Find the work done in the process AB.

SOLUTION

(a) Process $A \to B$ is part of a rectangular hyperbola, so

$$U\rho = \text{constant}$$

From Ideal Gas Equation, we have

$$PV = RT$$

$$\Rightarrow \quad P\left(\frac{M}{\rho}\right) = RT$$

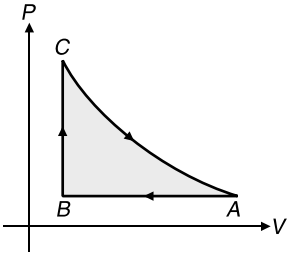
$$\Rightarrow \quad \frac{P}{\rho} = \frac{RT}{M}$$

Since, internal energy is function of temperature, so $U = aT$, where a is a constant

$$\Rightarrow \quad U\rho = aT\frac{PM}{RT} = \text{constant}$$

$$\Rightarrow \quad P = \text{constant}$$

Therefore, the process $A \to B$ is isobaric.
Process $B \to C$ is isochoric because density is constant.
Process $C \to A$ is isothermal because internal energy is constant. Now P-V diagram is shown below.



(b) Total heat absorbed by the system, is

$$Q = Q_{A\to B} + Q_{B\to C} + Q_{C\to A}$$

Since process $A \to B$ is isobaric, so

$$Q_{A\to B} = C_P\Delta T = C_P\frac{\Delta U}{C_v} = -5U_0$$

As process $B \to C$ is isochoric, so

$$Q_{B\to C} = \Delta U = 3U_0$$

As process $C \to A$ is isothermal, so

$$Q_{C\to A} = nRT\log_e\left(\frac{V_A}{V_C}\right)$$

$$\Rightarrow \quad Q_{C\to A} = R\left(\frac{10U_0}{3R}\right)\log_e\left(\frac{\rho_C}{\rho_A}\right)$$

Since, $\rho_A = 2\rho_0$, $\rho_C = \rho_B = 5\rho_0$

$$\Rightarrow \quad Q_{C\to A} = \frac{10U_0}{3}\log_e\left(\frac{5}{2}\right)$$

$$\Rightarrow \quad Q = -2U_0 + \frac{10U_0}{3}\log_e(2.5)$$

$$\Rightarrow \quad Q = \left(\frac{10}{3}\log_e(2.5) - 2\right)U_0$$

(c) From First Law of Thermodynamics, we get

$$W_{A\to B} = \Delta Q_{A\to B} - \Delta U_{A\to B}$$

$$\Rightarrow \quad W_{A\to B} = -5U_0 - \left(-3U_0\right)$$

$$\Rightarrow \quad W_{A\to B} = -2U_0$$

PRACTICE EXERCISES

SINGLE CORRECT CHOICE TYPE QUESTIONS

This section contains Single Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. The efficiency of a Carnot engine operating between the temperatures T_1 and T_2 $(T_1 > T_2)$ is η_0. The temperature of sink is decreased by α K and efficiency is η_1. The temperature of source is increased by α K and efficiency is η_2.

(A) $\eta_1 = \eta_2 = \eta_0$ (B) $\eta_1 > \eta_2 > \eta_0$

(C) $\eta_1 < \eta_2 < \eta_0$ (D) $\eta_1 > \eta_0 > \eta_2$

2. An ideal monatomic gas at 27 °C is compressed adiabatically to $\frac{8}{27}$ of its original volume. Then the rise in temperature is

(A) 450 °C (B) 375 °C

(C) 225 °C (D) 405 °C

3. One mole of an ideal gas undergoes a process $P = \frac{P_0}{1+\left(\frac{V_0}{V}\right)^2}$.

Here P_0 and V_0 are constants. Change in temperature of the gas when volume is changed from $V = V_0$ to $V = 2V_0$ is

(A) $-\frac{2P_0V_0}{5R}$ (B) $\frac{11P_0V_0}{10R}$

(C) $-\frac{5P_0V_0}{4R}$ (D) P_0V_0

4. The temperature of an isolated body of mass m, gram specific heat c falls from T_1 to T_2 in time t.

(A) $t = \frac{mc}{\sigma}\left(\frac{1}{T_2^3} - \frac{1}{T_1^3}\right)$ (B) $t = \frac{2mc}{\sigma}\left(\frac{1}{T_2^3} - \frac{1}{T_1^3}\right)$

(C) $t = \frac{mc}{3\sigma}\left(\frac{1}{T_2^3} - \frac{1}{T_1^3}\right)$ (D) $t = \frac{3\sigma}{mc}\left(\frac{1}{T_2^3} - \frac{1}{T_1^3}\right)$

5. Two moles of helium are mixed with n moles hydrogen. The root mean square (rms) speed of the gas molecules in the mixture is $\sqrt{2}$ times the speed of sound in the mixture. Then value of n is

(A) 1 (B) 3

(C) 2 (D) $\frac{3}{2}$

6. When a gas A is introduced into an evacuated flask kept at 25 °C, the pressure is found to be one atmosphere. If an equal mass of another gas B is then added to the same flask, the total pressure becomes 3 atm. Assuming ideal gas behaviour, calculate the ratio of molecular weights $M_A : M_B$

(A) 1:1 (B) 2:1

(C) 1:2 (D) 3:1

7. An ideal gas $(\gamma = 1.5)$ is expanded adiabatically. How many times has the gas to be expanded to reduce the root mean square velocity of molecules 2 times

(A) 4 times (B) 16 times

(C) 8 times (D) 2 times

8. A solid copper sphere (density ρ and specific heat capacity c) of radius r at an initial temperature 200 K is suspended inside a chamber whose walls are at almost 0 K. The time required (in μs) for the temperature of the sphere to drop to 100 K is

(A) $\frac{72}{7}\frac{r\rho c}{\sigma}$ (B) $\frac{7}{72}\frac{r\rho c}{\sigma}$

(C) $\frac{27}{7}\frac{r\rho c}{\sigma}$ (D) $\frac{7}{27}\frac{r\rho c}{\sigma}$

9. A vertical U-tube contains a liquid. When the two arms are maintained at different temperatures 50 °C and 60 °C, the levels of liquid in the two arms are 49 cm and 50 cm respectively. The coefficient of volume expansion of the liquid is

(A) 1.2×10^{-3} °C^{-1} (B) 1.4×10^{-3} °C^{-1}

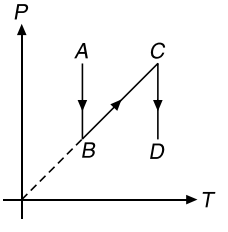
(C) 2.3×10^{-3} °C^{-1} (D) 1.8×10^{-3} °C^{-1}

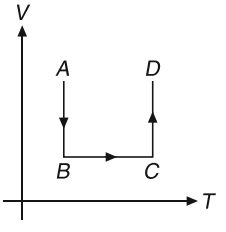
10. Two metallic spheres P and Q of the same surface finish are taken. Weight of P is twice that of Q. Both the spheres are heated to the same temperature and are left in a room to cool by radiation. The ratio of the rate of cooling of P to that of Q is

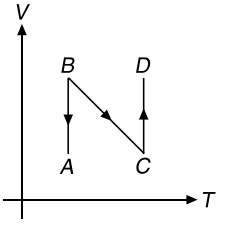
(A) $1:\sqrt{2}$ (B) $\sqrt{2}:1$

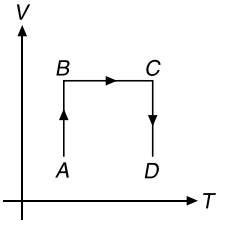
(C) $1:(2)^{\frac{1}{3}}$ (D) $2^{\frac{1}{3}}:1$

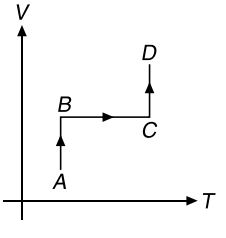
11. Select the corresponding V-T diagram for the P-T diagram shown in Figure.



(A)


(B)


(C)


(D)


12. Water is being boiled in a flat bottomed kettle placed on a stove. The area of the bottom is $300\ cm^2$ and the thickness is $2\ mm$. If the amount of steam produced is $1\ g\,min^{-1}$, then the difference of the temperature between the inner and the outer surfaces of the bottom is (thermal conductivity of the material of the kettle= $0.5\ cal\ cm^{-1}s^{-1}\ °C^{-1}$, latent heat of the steam is equal to $540\ calg^{-1}$)

(A) 12 °C (B) 1.2 °C
(C) 0.12 °C (D) 0.012 °C

13. In a given process of an ideal gas and $dQ < 0$. Then for the gas

(A) the temperature will decrease
(B) the volume will increase
(C) the pressure will remain constant
(D) the temperature will increase

14. $2\ kg$ of ice at –20 °C is mixed with $5\ kg$ of water at 20 °C in an insulating vessel having a negligible heat capacity. Assuming that the specific heat of water, ice and latent heat of ice respectively are $1\ kcalkg^{-1}(°C^{-1})$, $0.5\ kcalkg^{-1}(°C^{-1})$ and $80\ kcalkg^{-1}$, then the final mass of water (in kg) left in the container is

(A) 2 (B) 3
(C) 6 (D) 12

15. In case of Boyle's Law, if the pressure increases by 1%, the percentage of decrease in volume is

(A) 1% (B) $\left(\frac{100}{101}\right)\%$
(C) $\left(\frac{1}{10}\right)\%$ (D) $\left(\frac{1}{100}\right)\%$

16. A body of mass $25\ kg$ is dragged on a rough horizontal floor for one hour with a speed of $2\,kmh^{-1}$. The coefficient of friction for the surfaces in contact is 0.5 and half the heat produced is absorbed by the body. If specific heat of body is $0.1\ calg^{-1}(°C)^{-1}$ and $g = 9.8\ ms^{-2}$, then the rise in temperature of body is

(A) 39 K (B) 59.5 K
(C) 84.5 K (D) 11.6 K

17. A mixture of $4\ g$ helium and $28\ g$ nitrogen is enclosed in a vessel at a constant temperature $300\ K$. The quantity of heat absorbed by the mixture to double the root-mean velocity of its molecule is

(A) $4500R$ (B) $1800R$
(C) $7200R$ (D) $3600R$

18. The energy density $\frac{U}{V}$ of an ideal monatomic gas is related to its pressure P as

(A) $\frac{U}{V} = 3P$ (B) $\frac{U}{V} = \frac{3}{2}P$
(C) $\frac{U}{V} = \frac{P}{3}$ (D) $\frac{U}{V} = \frac{5}{2}P$

19. A uniform solid brass sphere is rotating with angular speed ω_0 about a diameter. If its temperature is now increased by 100 °C. What will be its new angular speed. $\left(\text{Given } \alpha_B = 2\times10^{-5} \text{ per } °C\right)$

(A) $1.1\ \omega_0$ (B) $1.01\ \omega_0$
(C) $0.996\ \omega_0$ (D) $0.824\ \omega_0$

20. In an ideal heat pump, heat from inside at $277\ K$ is transferred to a room at $300\ K$. The amount of heat (in joule) which will be delivered to the room for each joule of electrical energy consumed is

(A) 12 J (B) 1 J
(C) 25 J (D) 13 J

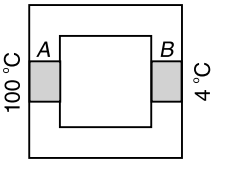
21. Which of the following statement is true?

(A) In a cyclic process, the internal energy of the gas increases.
(B) Free expansion is a reversible process.
(C) In a cyclic process, work done by the system is non-zero.
(D) Efficiency of Carnot engine is 100% if temperature of sink is 0 °C.

22. An insulator container contains 4 moles of an ideal diatomic gas at temperature T . Heat Q is supplied to this gas, due to which 2 moles of the gas are dissociated into atoms but temperature of the gas remains constant. Then

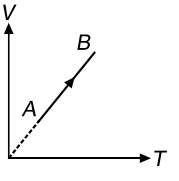
(A) $Q = 2RT$ (B) $Q = RT$
(C) $Q = 3RT$ (D) $Q = 4RT$

23. A closed cubical box made of perfectly insulating material has walls of thickness $8\ cm$ and the only way for heat to enter or leave the box is through two solid metal plugs A and B, each of cross-sectional area $12\ cm^2$ and length $8\ cm$ fixed in the opposite walls of the box as shown in the figure. Outer surface A is kept at 100 °C while the outer surface B is kept at 4 °C. The thermal conductivity of the material of the plugs is $0.5\ cals^{-1}cm^{-1}(°C)^{-1}$. A source of energy generating $36\ cals^{-1}$ is enclosed inside the box. The equilibrium temperature of the inner surface of the box (assuming that it is same at all points on the inner surface) is



(A) 38 °C (B) 57 °C
(C) 76 °C (D) 85 °C

24. An ideal monatomic gas undergoes the process AB as shown in Figure.



If the heat supplied and the work done in the process are Q and W respectively, the ratio $Q:W$ is

(A) 5 : 2 (B) 5 : 3
(C) 3 : 2 (D) 2 : 1

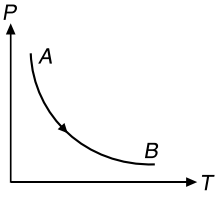
25. The thermal conductivities of copper, mercury and glass are, respectively, K_C, K_M and K_G such that $K_C > K_M > K_G$. If the same quantity of heat flows per second per unit area of each and the corresponding temperature gradients are X_C, X_M and X_G, then

(A) $X_C = X_M = X_G$ (B) $X_C > X_M > X_G$
(C) $X_C < X_M < X_G$ (D) $X_M > X_C > X_G$

26. Sixty percent of given sample of oxygen gas when raised to a high temperature dissociates into atoms. Ratio of its initial heat capacity (at constant volume) to the final heat capacity (at constant volume) will be

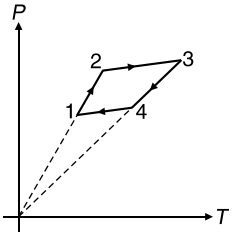
(A) $\frac{8}{7}$ (B) $\frac{25}{26}$
(C) $\frac{10}{7}$ (D) $\frac{25}{27}$

27. Two moles of an ideal monatomic gas is expanded according to relation $PT = \text{constant}$ from its initial state (P_0, V_0) to the final state, due to which its pressure becomes half of the initial pressure. The change in internal energy is



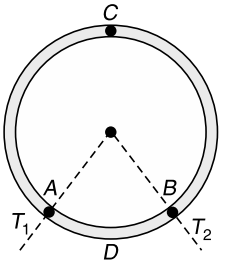
(A) $\frac{3}{4}P_0V_0$ (B) $\frac{3}{2}P_0V_0$
(C) $\frac{9}{2}P_0V_0$ (D) $\frac{5}{2}P_0V_0$

28. In a certain region of space there are n number of molecules per unit volume. The temperature of the gas is T. If k_B is the Boltzmann's Constant and R is the Universal Gas Constant, then the pressure of the gas will be



(A) nRT (B) nk_BT
(C) $\frac{nT}{k_B}$ (D) $\frac{nT}{R}$

29. A ring consisting of two parts ADB and ACB of same conductivity K carries an amount of heat H. The ADB part is now replaced with another metal keeping the temperatures T_1 and T_2 constant. The heat carried increases to $2H$. The conductivity of the new ADB part if it is given that $\frac{ACB}{ADB} = 3$, is

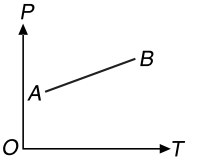


(A) $\frac{7}{3}K$ (B) $2K$
(C) $\frac{5}{2}K$ (D) $3K$

30. The specific heat of many solids at low temperatures varies with absolute temperature T according to the relation $S = AT^3$, where A is a constant. The heat energy required to raise the temperature of a mass m of such a solid from $T = 0$ to $T = 20$ K is

(A) 4×10^4 mA (B) 2×10^3 mA
(C) 8×10^6 mA (D) 2×10^6 mA

31. In the following graph for a given mass of gas, as temperature increases, volume



(A) remains constant (B) decreases
(C) increases (D) becomes zero

32. Three identical rods of same material are joined to form an equilateral triangle. The temperature of end A and B is maintained constant as $\sqrt{3}T$ and T. The ratio of $\frac{T_C}{T_B}$ will be (assuming no loss of heat from surfaces)

(A) $\frac{1+\sqrt{3}}{2}$ (B) $\frac{1-\sqrt{3}}{2}$
(C) $\frac{1+\sqrt{2}}{2}$ (D) $\frac{1-\sqrt{2}}{2}$

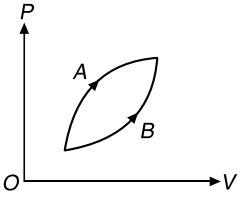
33. In the process PV =constant, pressure (P) versus density (ρ) graph of an ideal gas is

(A) a straight line parallel to P-axis
(B) a straight line parallel to ρ-axis
(C) a straight line passing through origin
(D) a parabola

34. A mass m of steam at 100 °C is to be passed into a vessel containing 10 g of ice and 100 g of water at 0 °C so that all the ice is melted and the temperature is raised to 5 °C. Neglecting heat absorbed by the vessel, we get

(A) $m = 2.1$ g (B) $m = 4.2$ g
(C) $m = 6.3$ g (D) $m = 8.4$ g

35. Two processes A and B for a given sample of a gas are shown in Figure.
Let, Q_1 be the amount of heat absorbed in process A, Q_2 be the amount of heat absorbed in process B, ΔU_1 be the change in internal energy in process A and ΔU_2 be the change in internal energy in process B, then



(A) $Q_1 = Q_2$, $\Delta U_1 = \Delta U_2$ (B) $Q_1 > Q_2$, $\Delta U_1 > \Delta U_2$
(C) $Q_1 < Q_2$, $\Delta U_1 < \Delta U_2$ (D) $Q_1 > Q_2$, $\Delta U_1 = \Delta U_2$

36. The end A of rod AB of length 1 m is maintained at 80 °C and the end B at 0 °C. The temperature at a distance of 60 cm from the end A is

(A) 16 °C (B) 32 °C
(C) 48 °C (D) 64 °C

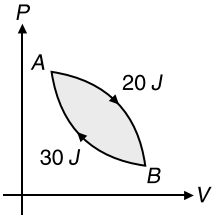
37. Three liquids A, B and C are at temperatures of 60 °C, 55 °C and 50 °C respectively. 4 g of A mixed with 3 g of C gives 55 °C and 2 g of A mixed with 3 g of B gives 57 °C. The temperature of the mixture when equal masses of B and C are mixed is

(A) 52.1 °C (B) 55 °C
(C) 52.5 °C (D) 53 °C

38. A closed system receives 200 kJ of heat at constant volume. It then rejects 100 kJ of heat while it has 50 kJ of work done on it at constant pressure. If an adiabatic process can be found which will restore the system to its initial state, the work done by the system during the process is

(A) 100 kJ (B) 50 kJ
(C) 150 kJ (D) 200 kJ

39. In a cyclic process shown in the figure an ideal gas is adiabatically taken from B to A, the work done on the gas during the process $B \to A$ is 30 J, when the gas is taken from $A \to B$ the heat absorbed by the gas is 20 J. The change in internal energy of the gas in the process $A \to B$ is

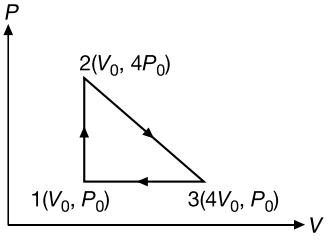


(A) 20 J (B) –30 J
(C) 50 J (D) –10 J

40. An air bubble doubles in radius on rising from the bottom of a lake to its surface. Assuming that the bubble rises slowly and the atmospheric pressure to be equal to a column of water of height H, the depth of the lake is

(A) $4H$ (B) $5H$
(C) $7H$ (D) $14H$

41. One mole of an ideal diatomic gas is taken through the cycle as shown in Figure. The average molecular speed of the gas in the states 1, 2 and 3 are in the ratio

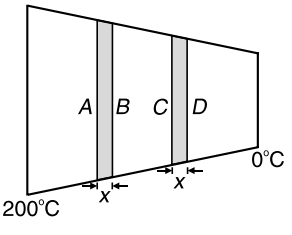


(A) 1 : 2 : 2 (B) $1 : \sqrt{2} : \sqrt{2}$
(C) 1 : 1 : 2 (D) 1 : 2 : 4

42. A graph is plotted with $\frac{PV}{T}$ on y-axis and mass of the gas along x-axis for different gases. The graph is

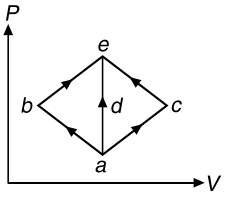
(A) a straight line parallel to x-axis for all the gases
(B) a straight line passing through origin with a slope having a constant value for all the gases
(C) a straight line passing through origin with a slope having different values for different gases
(D) a straight line parallel to y-axis for all the gases

43. Two ends of a conducting rod of varying cross-section are maintained at 200 °C and 0 °C respectively. In steady state



(A) temperature difference across AB and CD are equal
(B) temperature difference across AB is greater than that of across CD
(C) temperature difference across AB is less than that of across CD
(D) temperature difference may be equal or different depending on the thermal conductivity of the rod

44. The initial state of an ideal gas is represented by the point a on the P-V diagram and its final state by the point e. The gas goes from the state a to the state e by three quasi stationary processes represented by (i) abe, (ii) ace, (iii) ade. The heat absorbed by the gas is



(A) the same in all the processes.
(B) the same in processes abe and ace
(C) less in process abe than in ade
(D) less in process ace than in ade

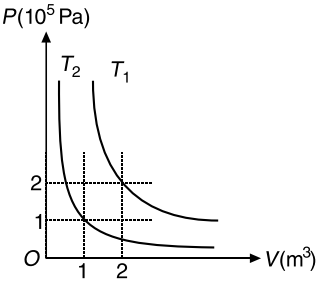
45. The efficiency of a heat engine is η and the coefficient of performance of a heat pump is β.

(A) $\eta = \frac{1}{\beta}$ (B) $\eta\beta = \frac{1}{2}$
(C) $\eta = \frac{1}{\beta + 1}$ (D) $\eta = \frac{1}{\beta - 1}$

46. According to the result obtained in PROBLEM 45, the maximum efficiency of a heat engine can be

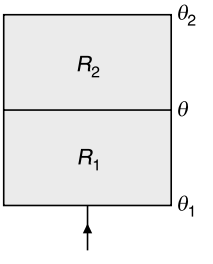
(A) 100% (B) 50%
(C) 25% (D) 12.5%

47. The graphs shows two isotherms for a fixed mass of an ideal gas. The ratio of rms speed of the molecules at temperatures T_1 to that at T_2 is



(A) $2\sqrt{2}$ (B) $\sqrt{2}$
(C) 2 (D) 4

48. Consider the two insulating sheets with thermal resistances R_1 and R_2 as shown in figure. The temperature θ is



(A) $\frac{\theta_1\theta_2 R_1 R_2}{(\theta_1 + \theta_2)(R_1 + R_2)}$ (B) $\frac{\theta_1 R_1 + \theta_2 R_2}{R_1 + R_2}$
(C) $\frac{(\theta_1 + \theta_2) R_1 R_2}{R_1^2 + R_2^2}$ (D) $\frac{\theta_1 R_2 + \theta_2 R_1}{R_1 + R_2}$

49. Two containers of equal volume contain identical gases at pressures P_1 and P_2 and absolute temperatures T_1 and T_2 respectively. The vessels are joined and the gas reaches a common pressure P and a common temperature T. Then

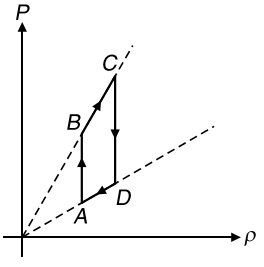
(A) $P=\left(\frac{P_1}{T_1}+\frac{P_2}{T_2}\right)T$ (B) $P=\frac{1}{2}\left(\frac{P_1}{T_1}+\frac{P_2}{T_2}\right)T$

(C) $P=\left(\frac{P_1T_2+P_2T_1}{T_1^2+T_2^2}\right)T$ (D) $P=\left(\frac{P_1T_2+P_2T_1}{T_1^2-T_2^2}\right)T$

50. Two cylinders A and B are taken. The cylinder A is fitted with a moveable piston and cylinder B is fitted with a fixed piston and both contain equal amounts of an ideal monatomic gas at same temperature. If same amount of heat is given to the gas in each cylinder and temperature of the gas in A rises by 15 K, then the rise in temperature of the gas in B is

(A) 30 K (B) 25 K
(C) 15 K (D) 20 K

51. Pressure versus density graph of an ideal gas is shown in figure



(A) during the process AB work done by the gas is positive
(B) during the processes AB work done by the gas is negative
(C) during the process BC internal energy of the gas is increasing
(D) None of the above

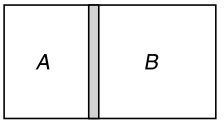
52. A spherical black body of radius r_0 radiates a power P at temperature T_0. Another spherical black body of radius $\frac{r_0}{2}$ and at temperature $2T_0$ emits a power

(A) P (B) $2P$
(C) $4P$ (D) $8P$

53. A gas undergoes a process in which its pressure P and volume V are related as $PV^n=$ constant, where n is a constant. If $\gamma=\frac{C_P}{C_V}$, then molar heat capacity for the gas in this process is zero for

(A) $n=\gamma$ (B) $n=\gamma-1$
(C) $n=\gamma+1$ (D) $n=1-\gamma$

54. Two different ideal gases are filled in two parts A and B of a container separated by a fixed diathermic (conducting) separator as shown in Figure.
The rms speed of the molecules in part A is equal to the mean speed of molecules in the part B. Then ratio of molar mass of gas in part A to that in part B is



(A) 1:1 (B) $\pi:2$
(C) 2:3 (D) $3\pi:8$

55. One mole of an ideal gas with heat capacity C_P at constant pressure undergoes the process $T=T_0+\alpha V$ where T_0 and α are constants. If its volume increases from V_1 to V_2, the amount of heat transferred to the gas is

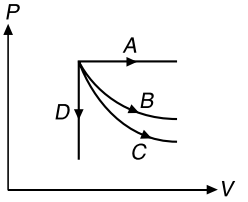
(A) $C_PRT_0\ln\left(\frac{V_2}{V_1}\right)$

(B) $\alpha C_P\frac{(V_2-V_1)}{RT_0}\ln\left(\frac{V_2}{V_1}\right)$

(C) $\alpha C_P(V_2-V_1)+RT_0\ln\left(\frac{V_2}{V_1}\right)$

(D) $RT_0\ln\left(\frac{V_2}{V_1}\right)-\alpha C_P(V_1-V_2)$

56. The P-V diagram shows four different possible reversible processes performed on a monatomic ideal gas. The process B is isothermal and process C is adiabatic. For which process(es) does the temperature of the gas decrease?



(A) Process A only
(B) Process C only
(C) Only processes C and D
(D) Only processes A and C

57. If gas molecules undergo inelastic collision with the walls of the container

(A) the temperature of the gas will decrease
(B) the pressure of the gas will increase
(C) neither the temperature nor the pressure will change
(D) the temperature of the gas will increase

58. n moles of an ideal gas undergo a process in which the temperature changes with volume as $T=KV^2$. The work done by the gas as the temperature changes from T_0 to $4T_0$ is

(A) $3nRT_0$ (B) $\left(\frac{5}{2}\right)nRT_0$
(C) $\left(\frac{3}{2}\right)nRT_0$ (D) ZERO

59. When a block of iron floats in mercury at 0 °C, fraction k_1 of its volume is submerged, while at the temperature 60 °C, at fraction k_2 is seen to be submerged. If the coefficient of volume expansion of iron is γ_{Fe} and that of mercury is γ_{Hg}, then the ratio $\frac{k_1}{k_2}$ can be expressed as

(A) $\frac{1+60\gamma_{Fe}}{1+60\gamma_{Hg}}$ (B) $\frac{1-60\gamma_{Fe}}{1+60\gamma_{Hg}}$
(C) $\frac{1+60\gamma_{Fe}}{1-60\gamma_{Hg}}$ (D) $\frac{1+60\gamma_{Hg}}{1+60\gamma_{Fe}}$

60. A and B are two points on a uniform metal ring whose centre is O. The angle $AOB=\theta$. A and B are maintained at two different constant temperatures. When $\theta=180°$, the rate of total heat flow from A to B is 1.2 W. When $\theta=90°$, this rate will be

(A) 0.6 W (B) 0.9 W
(C) 1.6 W (D) 1.8 W

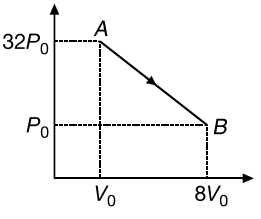
61. Water of volume 2 liter in a container is heated with a coil of 1 kW at 27 °C. The lid of the container is open and energy dissipates at rate of 160 Js^{-1}. The time in which temperature will rise from 27 °C to 77 °C if specific heat of water is given to be 4.2 $kJkg^{-1}$ is

(A) 8 min 20 s (B) 6 min 2 s
(C) 7 min (D) 14 min

62. A sample of ideal gas $(\gamma = 1.4)$ is heated at constant pressure. If an amount of 100 J heat is supplied to the gas, the work done by the gas is

(A) 56.28 J (B) 42.12 J
(C) 36.23 J (D) 28.57 J

63. The molar heat capacity of 1 mole of monatomic gas for the process AB shown in Figure is



(A) positive, $3.3R$
(B) negative, $3.3R$
(C) zero
(D) negative,

64. In Ingen Hausz experiment the wax melts up to 5 cmand 10 cm on bars A and B, respectively. The ratio of the thermal conductivities of A and B is

(A) 1 : 2 (B) 1 : 4
(C) 1 : 8 (D) 1 : 16

65. The work done by 1 mole of Vander Waal's gas, during its isothermal expansion from volume V_1 to V_2 at temperature T is

(A) $RT\ln\left(\frac{V_2}{V_1}\right)$ (B) $RT\ln\left(\frac{V_2-b}{V_1-b}\right)$

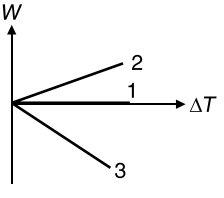
(C) $a\left(\frac{1}{V_2}-\frac{1}{V_1}\right)$ (D) $RT\ln\left(\frac{V_2-b}{V_1-b}\right)+a\left(\frac{1}{V_2}-\frac{1}{V_1}\right)$

(where a and b are Vander Waal's constants)

66. Two spheres of same size are made of the same material but one is solid and the other is hollow. They are heated to the same temperature.

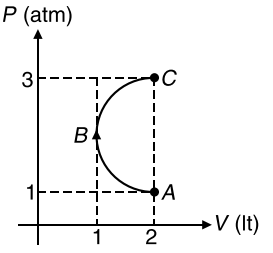
(A) Both spheres expand equally
(B) The solid sphere expands more
(C) The hollow sphere expands more
(D) Data is insufficient to arrive at a conclusion

67. For an ideal gas, a graph is shown between work done by the gas vs temperature change for three processes. Processes 1, 2 and 3 respectively are



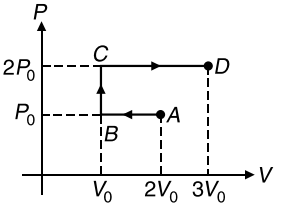
(A) Isobaric, adiabatic, isochoric
(B) Adiabatic, isobaric, isochoric
(C) Isochoric, adiabatic, isobaric
(D) Isochoric, isobaric, adiabatic

68. In the P-V diagram shown in figure ABC is a semicircle. The work done in the process ABC is



(A) zero
(B) $\frac{\pi}{2}$ atmlt
(C) $-\frac{\pi}{2}$ atmlt
(D) 4 atmlt

69. P-V diagram of an ideal gas is as shown in figure. Work done by the gas in the process $ABCD$ is



(A) $4\,P_0V_0$
(B) $2P_0V_0$
(C) $3P_0V_0$
(D) P_0V_0

70. Water of volume 2 liter in a container is heated with a coil of 1 kW at 27 °C. The lid of the container is open and energy dissipates at rate of 160 Js^{-1}. The time in which temperature will rise from 27 °C to 77 °C if specific heat of water is given to be 4.2 $kJkg^{-1}$ is

(A) 8 min 20 s (B) 6 min 2 s
(C) 7 min (D) 14 min

71. A Carnot engine working between 300 K and 600 K has a work output of 800 J per cycle. The amount of heat energy supplied to the engine from the source per cycle is

(A) 1200 J (B) 2400 J
(C) 1600 J (D) 3200 J

72. Four spheres A, B, C and D of different metals but of same radius are kept at same temperature. The ratio of their densities and specific heats are 2 : 3 : 5 : 1 and 3 : 6 : 2 : 4. Which sphere will show the fastest rate of cooling (initially)

(A) A (B) B
(C) C (D) D

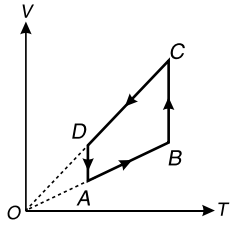
73. An engine uses a cyclic process consisting of four different processes. The heat exchange during these processes is $Q_1 = 500$ J, $Q_2 = -100$ J, $Q_3 = 150$ J, $Q_4 = -50$ J. The efficiency of the cycle is

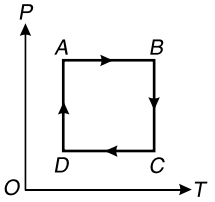
(A) 77% (B) 63%
(C) 84% (D) 51%

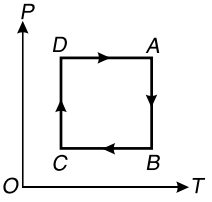
74. Internal energy of n_1 moles of hydrogen at temperature T is equal to the internal energy of n_2 moles of helium at temperature $2T$. Then the ratio $\frac{n_1}{n_2}$ is

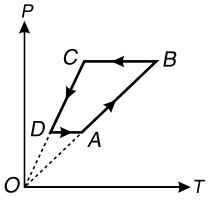
(A) $\frac{3}{5}$ (B) $\frac{2}{3}$
(C) $\frac{6}{5}$ (D) $\frac{3}{7}$

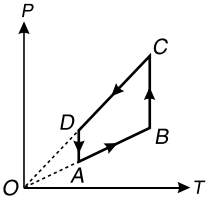
75. A cyclic process is shown on the V-T diagram. The same process on a P-T diagram is shown by



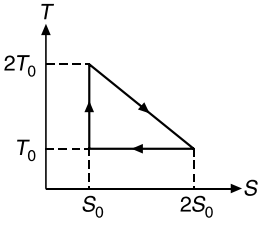
(A) 

(B) 

(C) 

(D) 

76. The temperature-entropy diagram of a reversible engine cycle is given in the figure. Its efficiency is



(A) $\frac{1}{3}$

(B) $\frac{2}{3}$

(C) $\frac{1}{2}$

(D) $\frac{1}{4}$

77. The average degrees of freedom per molecule for a gas is 6. The gas performs 25 J of work when it expands at constant pressure. The heat absorbed by gas is

(A) 75 J (B) 100 J

(C) 150 J (D) 125 J

78. The ends of two rods of different materials, having thermal conductivities, radii of cross-section and lengths in the ratio 1 : 2, are maintained at the same temperature difference. If the rate of flow of heat in the larger rod is 4 $cals^{-1}$, that in the shorter rod in $cals^{-1}$ will be

(A) 1 (B) 2

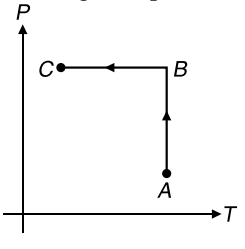
(C) 8 (D) 16

79. On a hypothetical scale X, the ice point is 40° and the steam point is 120°. For another scale Y, the ice point and steam point are –30° and 130° respectively. If X reads 50°, then Y would read

(A) –5° (B) –8°

(C) –10° (D) –12°

80. Ideal gas is taken through the process shown in the figure



(A) In process AB, work done by system is positive

(B) In process AB, heat is rejected

(C) In process AB, internal energy increases

(D) In process AB internal energy decrease and in process BC, internal energy increases

81. The pressure and density of a diatomic gas $\left(\gamma = \frac{7}{5}\right)$ change adiabatically from (P_1, ρ_1) to (P_2, ρ_2). If $\left(\frac{\rho_2}{\rho_1}\right) = 32$, then $\left(\frac{P_1}{P_2}\right)$ is

(A) 128 (B) $\frac{1}{128}$

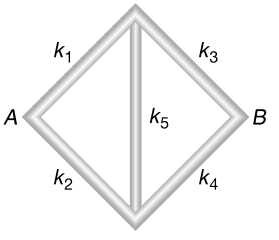
(C) 32 (D) None of these

82. If pressure and temperature of an ideal gas is doubled and volume is halved, the number of molecules of the gas

(A) remain constant (B) become half

(C) become two times (D) become four times

83. Five rods having thermal conductivities k_1, k_2, k_3, k_4 and k_5 are arranged as shown. The points A and B are maintained at different temperatures such that no thermal current flows through the central rod.



(A) $k_1 k_4 = k_2 k_3$

(B) $k_1 = k_3, k_2 = k_4$

(C) $k_1 k_3 = k_2 k_4$

(D) $\frac{k_1}{k_4} = \frac{k_3}{k_2}$

84. A freezer has coefficient of performance 6. If 4×10^6 J of work is done on the freezer, then the mass of water at 0 °C that gets converted into ice cubes at 0 °C is

(A) ≈ 5 kg (B) ≈ 3.6 kg

(C) ≈ 54 kg (D) ≈ 72 kg

85. 120 g of ice at 0 °C is mixed with 100 g of water at 80 °C Latent gas of fusion is 80 $calg^{-1}$ and specific heat of water is 1 $calg^{-1}$ °C. The final temperature of the mixture is

(A) 0 °C (B) 40 °C

(C) 20 °C (D) 10 °C

86. In the PROBLEM 85, mass of ice and water in the mixture when thermal equilibrium is attained is

(A) 40 g ice, 180 g water

(B) 60 g ice, 160 g water

(C) 20 g ice, 200 g water

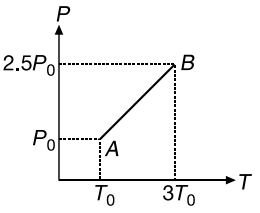
(D) 10 g ice, 100 g water

87. Temperature of 1 mole of an ideal gas is increased from 300 K to 310 K under isochoric process. Heat supplied to the gas in this process is $Q = 25R$ where R = universal gas constant. What amount of work has to be done by the gas if temperature of the gas decreases from 310 K to 300 K adiabatically

(A) $10R$ (B) $50R$

(C) $25R$ (D) $\frac{25}{2}R$

88. In the following graph for a given mass of an ideal gas, the ratio of density at B to that at A is

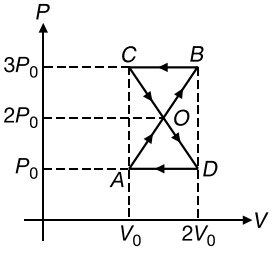


(A) 5 : 6
(B) 3 : 4
(C) 6 : 5
(D) 4 : 3

89. A black body radiates power P and maximum energy is radiated by it around a wavelength λ_0. The temperature of the black body is now changed such that it radiates maximum energy around the wavelength $\frac{3\lambda_0}{4}$. The power radiated by it now is

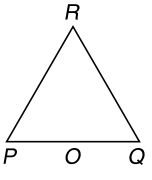
(A) $\frac{256}{81}P$ (B) $\frac{16}{9}P$
(C) $\frac{64}{27}P$ (D) $\frac{4}{3}P$

90. A thermodynamic system undergoes cyclic process $ABCDA$ as shown in figure. The work done by the system is



(A) P_0V_0
(B) $2\,P_0V_0$
(C) $\frac{P_0V_0}{2}$
(D) zero

91. Three rods of equal length l are joined to form an equilateral triangle PQR. O is the midpoint of PQ. Distance OR remains same for small change in temperature. Coefficient of linear expansion for PR and RQ is α_2 but that for PQ is α_1. Then

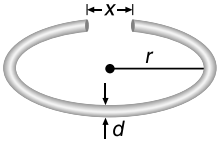


(A) $\alpha_2 = 3\alpha_1$
(B) $\alpha_2 = 4\alpha_1$
(C) $\alpha_1 = 3\alpha_2$
(D) $\alpha_1 = 4\alpha_2$

92. Ideal monatomic gas is taken through a process $dQ = 2dU$. The molar heat capacity for the process is (where dQ is heat supplied and dU is changed in internal energy)

(A) $5R$ (B) $3R$
(C) R (D) None of these

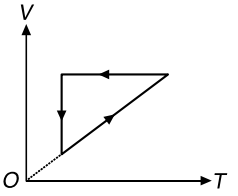
93. A cylindrical metal rod of length L_0 is shaped into a ring with a small gap as shown. On heating the system



(A) x decreases, r and d increase
(B) x and r increase, d decreases
(C) x, r and d all increase
(D) Data insufficient to arrive at a conclusion

94. A sample of nitrogen gas was taken through the cyclic process shown in Figure. What is the ratio of energy released by the gas in the isochoric process to work done by the gas in the isobaric process?

(A) 1.5
(B) 2.5
(C) 0.4
(D) 0.66



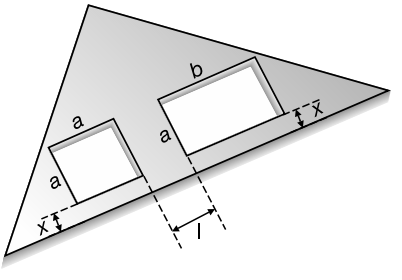
95. A wire of length L_0 is supplied heat to raise its temperature by$T$. If γ is the coefficient of volume expansion of the wire and Y is the Young's modulus of the wire then the energy density stored in the wire is

(A) $\frac{1}{2}\gamma^2T^2Y$ (B) $\frac{1}{3}\gamma^2T^2Y$
(C) $\frac{1}{18}\frac{\gamma^2T^2}{Y}$ (D) $\frac{1}{18}\gamma^2T^2Y$

96. Which one of the following would raise the temperature of 20 g of water at 30 °C most when mixed with (Specific heat of water is 1 cal $g^{-1}(°C)^{-1}$

(A) 20 g of water at 40 °C (B) 40 g of water at 35 °C
(C) 10 g of water at 50 °C (D) 4 g of water at 80 °C

97. A triangular plate has two cavities, one square and one rectangular as shown. The plate is heated.



(A) a increases, b decreases
(B) a and b both increase
(C) a and b increase, x and l decrease
(D) a, b, x and l all increase

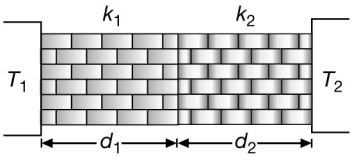
98. The three rods described in PROBLEM 276 are placed individually, with their ends kept at the same temperature difference. The rate of heat flow through C is equal to the rate of combined heat flow through A and B. k_C must be equal to

(A) $k_A + k_B$ (B) $\frac{k_Ak_B}{k_A + k_B}$
(C) $\frac{1}{2}(k_A + k_B)$ (D) $2\left(\frac{k_Ak_B}{k_A + k_B}\right)$

99. A closed hollow insulated cylinder, filled with gas at 0 °C, is divided into two portions with a movable light insulated piston initially at the middle. The gas on one side of the piston is heated to 100 °C and the piston is seen to move by 5 cm. The length of the cylinder is

(A) 132 cm (B) 87.4 cm
(C) 38.6 cm (D) 64.6 cm

100. Two walls of thickness d_1 and d_2, thermal conductivities k_1 and k_2 respectively are in contact. If the temperatures at the outer surfaces are T_1 and T_2 respectively, then the temperature at the interface in steady state is



(A) $\frac{k_1T_1 + k_2T_2}{k_1 + k_2}$ (B) $\frac{k_1T_1d_2 + k_2T_2d_1}{k_1d_2 + k_2d_1}$

(C) $\frac{k_1T_1d_1 + k_2T_2d_2}{k_1d_1 + k_2d_2}$ (D) $\frac{T_1 + T_2}{2}$

101. A gas is expanded to double its volume by two different processes. One is isobaric and the other is isothermal. Let W_1 and W_2 be the respective work done, then

(A) $W_2 = W_1 \log_e(2)$ (B) $W_2 = \frac{W_1}{\log_e(2)}$

(C) $W_2 = \frac{W_1}{2}$ (D) data is insufficient

102. A steel tape measures the length of a copper rod as 90 cm when both are at 10 °C, the calibration temperature, for the tape. What would the tape read for the length of the rod when both are at 30 °C. Given $\alpha_{steel} = 1.2 \times 10^{-5}$ per °C and $\alpha_{Cu} = 1.7 \times 10^{-5}$ per °C

(A) 89.00 cm (B) 90.21 cm

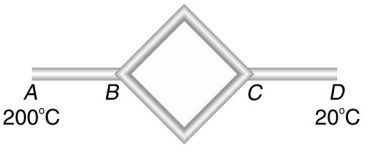
(C) 89.80 cm (D) 90.01 cm

103. Two monatomic ideal gases at absolute temperature T_1 and T_2 are mixed. The masses of the molecules are m_1 and m_2 and the number of molecules in the gases are N_1 and N_2 respectively. The temperature of mixture will be

(A) $\frac{T_1 + T_2}{2}$ (B) $\frac{N_1m_1T_1 + N_2m_2T_2}{N_1m_1 + N_2m_2}$

(C) $\frac{N_1T_1 + N_2T_2}{N_1 + N_2}$ (D) $\frac{m_1T_1 + m_2T_2}{m_1 + m_2}$

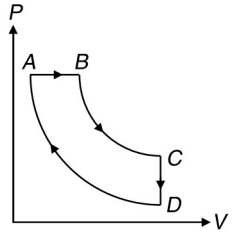
104. Six identical conducting rods are joined as shown in figure. Points A and D are maintained at temperatures 200 °C and 20 °C respectively. The temperature of junction B will be

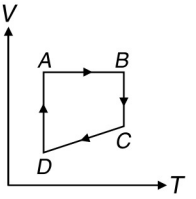


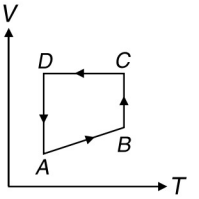
(A) 120 °C (B) 100 °C

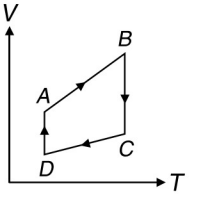
(C) 140 °C (D) 80 °C

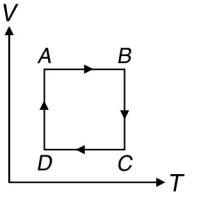
105. A cycle process $ABCD$ is shown in the P-V diagram. Which of the following curves represent the same process?



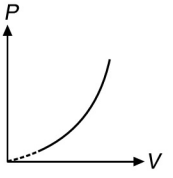
(A) 

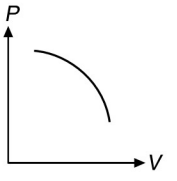
(B) 

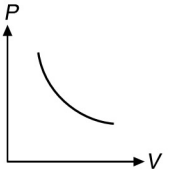
(C) 

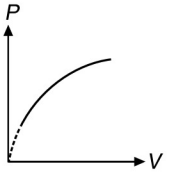
(D) 

106. An ideal gas follows a process PT = constant. The correct graph between pressure and volume is

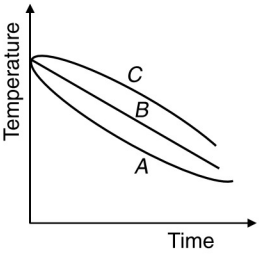
(A) 

(B) 

(C) 

(D) 

107. A block of steel heated to 100 °C is left in a room to cool. Which of the curves shown in the figure represents the decrease of temperature with time?



(A) A (B) B

(C) C (D) None of these

108. 1 g of water on evaporation at atmospheric pressure forms 1671 cm^3 of steam. Heat of vaporisation at this pressure is 540 $calg^{-1}$. The increase in internal energy is

(A) 250 cal (B) 500 cal

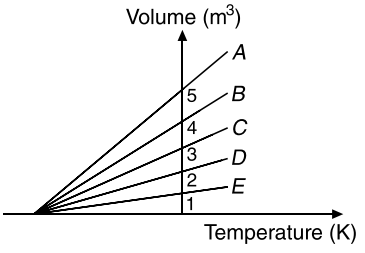
(C) 1000 cal (D) 1500 cal

109. A steel rod of length L_0 has a cross-sectional area A. The force required to stretch this rod by the same amount as the expansion produced by heating it through ΔT is (Coefficient of linear expansion of steel is α and Young's Modulus for steel is Y)

(A) $YA\alpha\Delta T$ (B) $\frac{1}{2}YA\alpha\Delta T$

(C) $2YA\alpha\Delta T$ (D) None of these

110. The expansion of an ideal gas of mass m at a constant pressure P is given by the straight line D. Then the expansion of the same ideal gas of same mass at a pressure $0.5P$ is given by the straight line

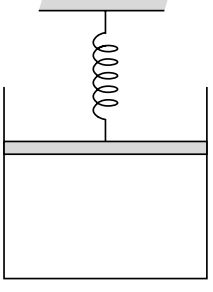


(A) E (B) C
(C) B (D) A

111. One end of a conducting rod is maintained at temperature $50\ ^{o}C$ and at the other end ice is melting at $0\ ^{o}C$. The rate of melting of ice is doubled if

(A) the temperature is made $200\ ^{o}C$ and the area of cross-section of the rod is doubled
(B) the temperature is made $100\ ^{o}C$ and length of the rod is made four times
(C) area of cross section of rod is halved and length is doubled
(D) the temperature is made $100\ ^{o}C$ and area of cross-section of rod and length both are doubled

112. One mole of an ideal monatomic gas is contained in a piston-cylinder arrangement as shown in Figure.



The gas is initially at a pressure of 1 atm and temperature $27\ ^{\circ}C$. The very light piston has cross-sectional area $0.005\ m^2$ and is connected to an undeformed spring of spring constant $10^4\ Nm^{-1}$. The work done by the gas when pressure is increased to 3 atm is

(A) 100 J (B) 50 J
(C) 150 J (D) 125 J

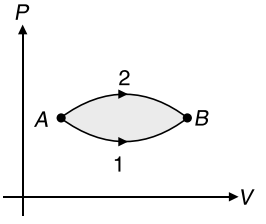
113. Hot water cools from $60\ ^{\circ}C$ to $50\ ^{\circ}C$ in the first 10 minutes and to $42\ ^{\circ}C$ in the next 10 minutes. The temperature of the surroundings is

(A) $5\ ^{\circ}C$ (B) $10\ ^{\circ}C$
(C) $15\ ^{\circ}C$ (D) $20\ ^{\circ}C$

114. If a body coated black at 600 K surrounded by atmosphere at 300 K has cooling rate r_0, the same body at 900 K, surrounded by the same atmosphere, will have cooling rate equal to

(A) $\frac{16}{3}r_0$ (B) $\frac{81}{16}r_0$
(C) $16r_0$ (D) $4r_0$

115. The figure shows two paths for the change of state of a gas from A to B. The ratio of molar heat capacities in Path 1 and Path 2 is

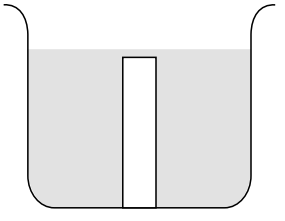


(A) >1 (B) <1
(C) 1 (D) Data Insufficient

116. A gas is enclosed in a vessel of volume V at a pressure P. It is being pumped out of the vessel by means of a piston pump with a stroke volume v. Pressure of the gas in the vessel after n strokes is

(A) $\frac{P}{n}$ (B) $P\left(\frac{v}{V}\right)^n$
(C) $P\left(\frac{v}{V+v}\right)^n$ (D) $P\left(\frac{V}{V+v}\right)^n$

117. In a cylindrical glass container, a solid silica is placed vertically at its bottom and remaining space is filled with mercury up to the top level of the silica as shown in Figure. Assume that the volume of the silica remains unchanged due to variation in temperature. The coefficient of cubical expansion of mercury is γ and coefficient of linear expansion of glass is α. If the top surface of silica and mercury remain at the same level with the variation in temperature, then the ratio of volume of silica to the volume of mercury is equal to



(A) $\frac{\gamma}{2\alpha}$ (B) $\frac{\gamma}{3\alpha}$
(C) $\frac{\gamma}{2\alpha}-1$ (D) $\frac{\gamma}{3\alpha}-1$

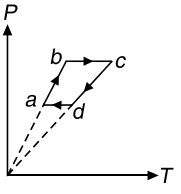
118. A body cools from $50\ ^{\circ}C$ to $49.9\ ^{\circ}C$ in 5 s. Assuming temperature of surroundings to be $30\ ^{\circ}C$ and Newton's law of cooling to be valid, the time it will take to cool from $40\ ^{\circ}C$ to $39.9\ ^{\circ}C$ is

(A) 2.5 s (B) 5 s
(C) 10 s (D) 20 s

119. At room temperature, the rms speed of the molecules of a certain diatomic gas is found to be $1930\ ms^{-1}$. The gas is

(A) H_2 (B) F_2
(C) O_2 (D) Cl_2

120. Two moles of an monatomic gas undergoes a cyclic process as shown in Figure. If the respective temperatures at points a, b, c and d are 400 K, 800 K, 1600 K and 800 K, then the work done per cycle is



(A) 3.2 kJ
(B) 4.8 kJ
(C) 6.6 kJ
(D) 8.3 kJ

121. A gas is supplied heat such that its temperature, pressure and volume, all three at any instant change simultaneously such that the work is being done at the expense of the internal energy of the gas. If C be the molar heat capacity of the gas then

(A) $C = 0$ (B) $C = \frac{dU}{dT}$
(C) $C = \frac{dW}{dT}$ (D) Data Insufficient

122. Two solid spheres of radii R_1 and R_2 are made of the same material and have similar surfaces. These are raised to the same temperature and then allowed to cool under identical conditions. The ratio of their initial rates of loss of heat are

(A) $\frac{R_1}{R_2}$ (B) $\frac{R_2}{R_1}$
(C) $\frac{R_1^2}{R_2^2}$ (D) $\frac{R_2^2}{R_1^2}$

123. In PROBLEM 122, the ratio of the initial rates of cooling (i.e. rates of fall of temperature) is equal to

(A) $\frac{R_1}{R_2}$ (B) $\frac{R_2}{R_1}$
(C) $\frac{R_1^2}{R_2^2}$ (D) $\frac{R_2^2}{R_1^2}$

124. One mole of an ideal gas at temperature T was cooled isochorically till the gas pressure fell from P to $\frac{P}{n}$. Then, by an isobaric process, the gas was restored to the initial temperature. The net amount of heat absorbed by the gas in the process is

(A) $RT\left(1-\frac{1}{n}\right)$ (B) $\frac{RT}{n}$
(C) $RT(n-1)$ (D) nRT

125. The specific heats of an ideal gas at constant pressure and constant volume are 525 $\text{Jkg}^{-1}(^\circ\text{C})^{-1}$ and 315 $\text{Jkg}^{-1}(^\circ\text{C})^{-1}$ respectively. Its density at NTP is

(A) 0.64 kgm^{-3} (B) 2.62 kgm^{-3}
(C) 1.20 kgm^{-3} (D) 1.75 kgm^{-3}

126. Two metal rods having same length and area of cross-section are fixed end to end between two rigid supports. The coefficients of linear expansion of the rods are α_1 and α_2 and their respective Young's moduli are Y_1 and Y_2. The system is now cooled and it is observed that the junction between the rods does not shift at all for the condition.

(A) $Y_1\alpha_2 = Y_2\alpha_1$ (B) $Y_1\alpha_1^2 = Y_2\alpha_2^2$
(C) $Y_1\alpha_1 = Y_2\alpha_2$ (D) $Y_1\alpha_2^2 = Y_2\alpha_1^2$

127. Five molecules have speed 2 kms^{-1}, 1.5 kms^{-1}, 1.6 kms^{-1}, 1.6 kms^{-1} and 1.2 kms^{-1} respectively. The most probable speed is

(A) 2 kms^{-1} (B) 1.58 kms^{-1}
(C) 1.6 kms^{-1} (D) 1.5 kms^{-1}

128. A steel scale measures the length of a copper rod as L cm when both are at 20 °C, the calibration temperature for the scale. If the coefficients of linear expansion for steel and copper are α_s and α_c respectively, what would be the scale reading (in cm) when both are at 21 °C?

(A) $L\left(\frac{1+\alpha_c}{1+\alpha_s}\right)$ (B) $L\frac{\alpha_c}{\alpha_s}$
(C) $L\frac{\alpha_s}{\alpha_c}$ (D) L

129. Unit mass of a liquid of volume V_1 completely turns into a gas of volume V_2 at constant atmospheric pressure P_0 and temperature T. The latent heat of vaporization is L. Then the change in internal energy of the gas is

(A) L (B) $L + P(V_2 - V_1)$
(C) $L - P(V_2 - V_1)$ (D) ZERO

130. How much heat energy should be added to a mixture of 10 g of hydrogen and 40 g of He to change the temperature by 50 °C kept in a closed vessel

(A) 2500 cal (B) 2750 cal
(C) 2000 cal (D) None of these

131. A monatomic gas undergoes a process given by $2dU + 3dW = 0$, then the process is

(A) isobaric (B) adiabatic
(C) isothermal (D) None of these

132. Molar heat capacity of a monatomic gas during a process obeying $P\sqrt{T} = \text{constant}$ is

(A) $-3R$ (B) $2.5R$
(C) $3R$ (D) $2R$

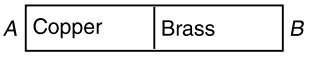
133. The volume thermal expansion coefficient of an ideal gas at constant pressure is (Here T =absolute temperature of gas)

(A) T (B) T^2
(C) $\frac{1}{T}$ (D) $\frac{1}{T^2}$

134. An ideal gas mixture filled inside a balloon expands according to the relation $PV^{2/3} = \text{constant}$. The temperature inside the balloon is

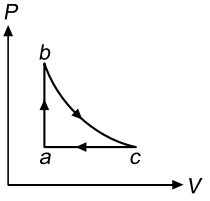
(A) increasing (B) decreasing
(C) constant (D) Cannot be defined

135. Two rods of copper and brass $(K_C > K_B)$ of same length and area of cross-section are joined as shown. End A is kept at 100 °C and end B at 0 °C. The temperature at the junction



(A) will be more than 50 °C
(B) will be less than 50 °C
(C) will be 50 °C
(D) may be more or less than 50 °C depending upon the size of rods

136. In the following cycle carried out on a diatomic gas, bc is an isothermal process. The gas absorbs 5000 J of heat as its temperature increases from 300 K to 800 K in going from a to b. The quantity of heat rejected by the gas during the process ca is



(A) 3000 J
(B) 5000 J
(C) 7000 J
(D) 10000 J

137. The volume of a block of metal changes by 0.12% when heated through 20 °C. Then α is

(A) $2\times10^{-5}(°C)^{-1}$ (B) $4\times10^{-5}(°C)^{-1}$
(C) $6\times10^{-5}(°C)^{-1}$ (D) $8\times10^{-5}(°C)^{-1}$

138. A black body at 227 °C radiates heat at the rate of 5 $cals^{-1}cm^{-2}$. The rate of heat radiated in $cals^{-1}cm^{-2}$ at 727 °C is

(A) 40 (B) 80
(C) 160 (D) 240

139. If the amount of heat given to the system be 35 J and the amount of work done on the system be 15 J, then the change in the internal energy of the system is

(A) –50 J (B) 20 J
(C) 30 J (D) 50 J

140. Which one of the following samples of gases has the largest internal energy?

(A) 1 mole of helium occupying 1 m^3 at 600 K
(B) 56 g of nitrogen at 10^7 Nm^{-2} and 300 K
(C) 32 g of oxygen at 1 atm and 300 K
(D) 12×10^{23} molecules of argon occupying 4 m^3 at 450 K

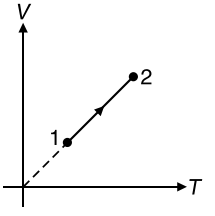
141. A monatomic ideal gas, initially at temperature T_1, is enclosed in a cylinder fitted with a frictionless piston. The gas is allowed to expand to a temperature T_2 by releasing the piston suddenly. If L_1 and L_2 be the lengths of the gas column before and after the expansion respectively, then $\frac{T_1}{T_2}$ is given by

(A) $\left(\frac{L_1}{L_2}\right)^{\frac{2}{3}}$ (B) $\frac{L_1}{L_2}$
(C) $\frac{L_2}{L_1}$ (D) $\left(\frac{L_2}{L_1}\right)^{\frac{2}{3}}$

142. The quantity $\frac{PV}{kT}$ (k =Boltzmann's constant) represents

(A) number of moles of the gas
(B) total mass of the gas
(C) number of molecules in the gas
(D) density of the gas

143. Volume versus temperature graph of two moles of helium gas is as shown in figure. The ratio of heat absorbed and the work done by the gas in process 1-2 is



(A) 3
(B) $\frac{5}{2}$
(C) $\frac{5}{3}$
(D) $\frac{7}{2}$

144. A wall has two layers A and B, each made of different material. Both the layers have the same thickness. The thermal conductivity of the material of A is twice that of B. Under thermal equilibrium the temperature difference across the wall is 36 °C. The temperature difference across the layer A is

(A) 6 °C (B) 12 °C
(C) 18 °C (D) 24 °C

145. A metal ball immersed in water weighs W_1 at 0 °C and W_2 at 50 °C. The coefficient of cubical expansion of metal is less than that of water. Then

(A) $W_1 > W_2$ (B) $W_1 < W_2$
(C) $W_1 = W_2$ (D) data is insufficient

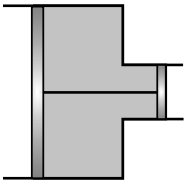
146. A linear accelerator consists of a hundred brass discs tightly fitted into a steel tube. At 40 °C the diameter of each disc is 10.02 cm. The system is assembled by cooling the discs in dry ice at –60 °C to enable them to slide into the close fitting tube. If the coefficient of expansion of brass is 2×10^{-5} $°C^{-1}$, the diameter of each disc in dry ice will be

(A) 9.94 cm (B) 9.96 cm
(C) 9.98 cm (D) 10.00 cm

147. A system is taken from state A to state B along two different paths 1 and 2. The work done on the system along these two paths is W_1 and W_2 respectively. The heat absorbed by the system along these two paths is Q_1 and Q_2 respectively. The internal energy at A and B is U_A and U_B respectively

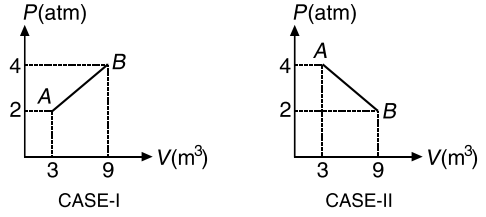
(A) $W_1 = W_2 = U_B - U_A$
(B) $Q_1 = Q_2 = U_A - U_B$
(C) $Q_1 + W_1 = Q_2 + W_2 = U_A + U_B$
(D) $Q_1 + W_2 = Q_2 + W_1 = U_B - U_A$

148. Two pistons can move freely inside a horizontal cylinder having two sections of unequal cross-sections. The pistons are joined by an inextensible, light string and same gas is enclosed between the pistons. On cooling the system, the pistons will



(A) move to the left
(B) move to the right
(C) remain stationary
(D) follow (A) or (C) depending upon the initial pressure of the gas

149. A given sample of gas can be taken from state A to state B by two different processes shown in Figure. Heat added to the system will be



(A) more in CASE-1
(B) more in CASE-2
(C) same in both cases and non-zero
(D) zero in both cases

150. A gas undergoes a change of state during which 100 J of heat is supplied to it and it does 20 J of work. The system is brought back to its original state through a process during which 20 J of heat is released by the gas. The work done by the gas in the second process is

(A) 60 J (B) 40 J
(C) 80 J (D) 20 J

151. Two rods of length L_2 and coefficient of linear expansion α_2 are connected freely to a third rod of length L_1 of coefficient of linear expansion α_1 to form an isosceles triangle. The arrangement is supported on the knife edge at the midpoint of L_1 which is horizontal. The apex of the isosceles triangle is to remain at a constant distance from the knife edge if

(A) $\frac{L_1}{L_2}=\frac{\alpha_2}{\alpha_1}$ (B) $\frac{L_1}{L_2}=\sqrt{\frac{\alpha_2}{\alpha_1}}$

(C) $\frac{L_1}{L_2}=2\frac{\alpha_2}{\alpha_1}$ (D) $\frac{L_1}{L_2}=2\sqrt{\frac{\alpha_2}{\alpha_1}}$

152. 1 gof ice at 0 °C is mixed with 1 g of steam at 100 °C. After thermal equilibrium is attained the temperature of the mixture is

(A) 1 °C (B) 50 °C
(C) 81 °C (D) 100 °C

153. In PROBLEM 152, the maximum mass of ice that can be taken to get the equilibrium temperature (as calculate above) is

(A) 1 g (B) 2 g
(C) 3 g (D) 4 g

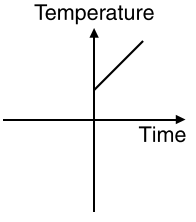
154. Which statements is true (T) or false (F)
(i) A refrigerator is reverse of a heat engine.
(ii) A heat pump is the same as a refrigerator.
(iii) The coefficient of performance of a refrigerator can be infinite.
(iv) In a heat engine, heat cannot be fully converted to work.

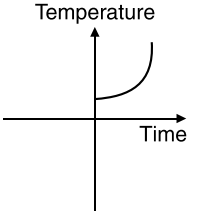
(A) i-F, ii-F, iii-F, iv-T (B) i-T, ii-T, iii-F, iv-T
(C) i-T, ii-F, iii-T, iv-T (D) i-F, ii-T, iii-F, iv-F

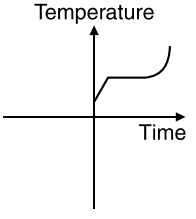
155. In a 10 metre deep lake, the bottom is at a constant temperature of 4 °C. The air temperature is constant at –4 °C. The thermal conductivity of ice is 3 times that of water. Neglecting the expansion of water on freezing, the maximum thickness of ice will be

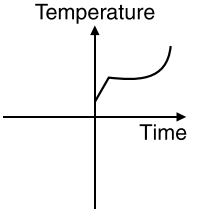
(A) 7.5 m (B) 6 m
(C) 5 m (D) 2.5 m

156. Liquid oxygen at 50 K is heated to 300 K at constant pressure of 1 atm. The rate of heating is constant. Which of the following graphs represents the variation of temperature with time?

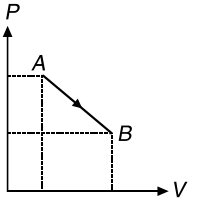
(A) 

(B) 

(C) 

(D) 

157. An ideal monatomic gas undergoes a process AB as shown in Figure.



If $P_A = 2P_B = 10^6$ Nm^{-2} and $V_B = 4V_A = 0.4$ m^3, then the heat absorbed by the gas in this process is

(A) 350 kJ (B) 375 kJ
(C) 425 kJ (D) 500 kJ

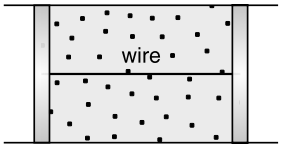
158. During adiabatic process pressure (P) versus density (ρ) equation is

(A) $P\rho^{\gamma}$ = constant (B) $P\rho^{-\gamma}$ = constant

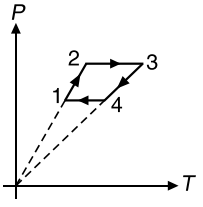
(C) $P^{\gamma}\rho^{1+\gamma}$ = constant (D) $P^{\frac{1}{\gamma}}\rho^{\gamma}$ = constant

159. A cylindrical tube of uniform cross-sectional area A is fitted with two air tight frictionless pistons. The pistons are connected to each other by a metallic wire. Initially the pressure of the gas is P_0 and temperature is T_0. Atmospheric pressure is also P_0. Now the temperature of the gas is increased to $2T_0$, the tension in the wire will be

(A) $2P_0A$
(B) P_0A
(C) $P_0A/2$
(D) $4P_0A$



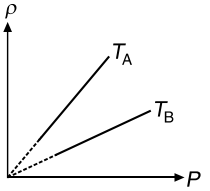
160. Three moles of an ideal monatomic gas performs a cycle $1 \to 2 \to 3 \to 4 \to 1$ as shown. The gas temperature in different states are $T_1 = 400$ K, $T_2 = 800$ K, $T_3 = 2400$ K and $T_4 = 1200$ K. The work done by the gas during the cycle is



(A) $1200R$ (B) $3600R$
(C) $2400R$ (D) $2000R$

161. In the given Figure isothermal graphs for an ideal gas

(A) $T_A > T_B$
(B) $T_B > T_A$
(C) $T_A = T_B$
(D) $T_A \geq T_B$



162. If 2 moles of an ideal monatomic gas at temperature T_0 is mixed with 4 moles of another ideal monatomic gas at temperature $2T_0$, then the temperature of the mixture is

(A) $\frac{5}{3}T_0$ (B) $\frac{3}{2}T_0$
(C) $\frac{4}{3}T_0$ (D) $\frac{5}{4}T_0$

163. An aluminium measuring rod, which is correct at 5 °C measures the length of a line as 80 cm at 45 °C. If thermal coefficient of linear expansion of aluminium is 2.50×10^{-5} per °C. The correct length of the line is

(A) 80.08 cm (B) 79.92 cm
(C) 81.12 cm (D) 79.62 cm

164. Two rods of the same material have diameters in the ratio 1 : 2 and length in the ratio 2 : 1. If the temperature difference between their ends is the same, the ratio of the heats conducted by them in a given time is

(A) 1 : 4 (B) 4 : 1
(C) 1 : 8 (D) 8 : 1

165. A polished metal plate with a rough black spot on it is heated to about 1400 K and quickly taken to dark room. Then the spot

(A) will appear brighter than the plate
(B) will appear darker than the plate
(C) and the plate will appear equally bright
(D) the plate will not be visible in dark room

166. A stationary vertical cylindrical container of very large height filled with a gas of molar mass M at constant Temperature. The pressure at the bottom is P_1 and at the top is P_2. If the acceleration due to gravity is assumed to be constant for the whole cylinder, which is equal to g. Then the height of the cylinder is

(A) $\frac{RT}{Mg}\ln\frac{P_1}{P_2}$ (B) $\frac{RT}{2Mg}\ln\frac{P_1}{P_2}$
(C) $\frac{2RT}{Mg}\ln\frac{P_1}{P_2}$ (D) $\frac{RT}{3Mg}\ln\frac{P_1}{P_2}$

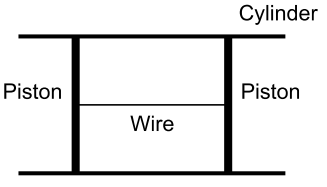
167. Two identical rods of the same material are joined in series. Under a temperature difference, a certain quantity of heat flows through the combination in four minutes. If the two rods are joined in parallel, the same quantity of heat will flow through the combination under the same temperature difference in

(A) 1 min (B) 2 min
(C) 8 min (D) 16 min

168. The molar heat capacity in a process of a diatomic gas if it does a work of $\frac{Q}{4}$ when a heat of Q is supplied to it is

(A) $\frac{2}{5}R$ (B) $\frac{5}{2}R$
(C) $\frac{10}{3}R$ (D) $\frac{6}{7}R$

169. Two air tight frictionless pistons connected to each other by a metallic wire are fitted in a uniform cylindrical vessel of cross-sectional area 1 m^2 as shown in Figure.



Initially the vessel contains an ideal gas at atmospheric pressure P_0 at temperature 27 °C. If the temperature of the gas is doubled on absolute scale, the tension in the wire will be about

(A) 1 N (B) 10^5 N
(C) 2×10^5 N (D) zero

170. The coefficient of linear expansion for a certain metal varies with temperature as $\alpha(T)$. If L_0 is the initial length of the metal and the temperature of metal is changed from T_0 to $T\,(< T_0)$. Then,

(A) $L = L_0 \int_{T_0}^{T} \alpha(T)\,dT$ (B) $L = L_0\left[1 - \int_{T}^{T_0} \alpha(T)\,dT\right]$

(C) $L = L_0\left[1 - \int_{T_0}^{T} \alpha(T)\,dT\right]$ (D) $L > L_0$

171. Water in a lake is changing into ice at 0 °C when the atmospheric temperature is 10 °C. If the time taken for 1 cm thick ice layer to be formed is 7 hour, the time required for the thickness of ice to increase from 1 cm to 2 cm is

(A) 7 hour (B) 14 hour
(C) < 7 hour (D) > 14 hour

172. Four rods of same material but different radii r and lengths l are used to connect two reservoirs of heat at different temperatures. The one which will conduct most heat is

(A) $r = 2$ cm, $l = 0.5$ m (B) $r = 2$ cm, $l = 2$ m

(C) $r = 1$ cm, $l = 1$ m (D) $r = 0.5$ cm, $l = 0.5$ cm

173. Two slabs A and B having lengths l_1 and l_2, respectively, and having same cross-section have thermal conductivities K_1 and K_2 respectively. They are placed in contact and constant temperature difference is maintained across the combination. The ratio of the quantities of heat flowing through A and B in a given time is

(A) $\frac{K_1}{l_1} : \frac{K_2}{l_2}$ (B) $\frac{K_1}{l_2} : \frac{K_2}{l_1}$

(C) $K_1 l_1 : K_2 l_2$ (D) $1:1$

174. If the ratio of number of moles of hydrogen gas to oxygen gas is two, the ratio of total translational kinetic energy of hydrogen gas molecules to that of oxygen molecules at 20 °C is

(A) $1:1$ (B) $2:1$

(C) $1:8$ (D) $1:16$

175. The temperature of a room heated by a heater is 20 °C when outside temperature is –20 °C and it is 10 °C when the outside temperature is –40 °C. The temperature of the heater is

(A) 80 °C (B) 100 °C

(C) 40 °C (D) 60 °C

176. The loss in weight of a solid when immersed in a liquid at 0 °C is W_0 and at t °C is W. If the coefficients of volume expansion of the solid and the liquid be γ_s and γ_l respectively, then

(A) $W = W_0\left[(\gamma_s - \gamma_l)\right]$ (B) $W = W_0\left[1 + (\gamma_s - \gamma_l)t\right]$

(C) $W = \frac{W_0 t}{\gamma_l - \gamma_s}$ (D) $W = W_0\left[1 - (\gamma_s - \gamma_l)t\right]$

177. The temperatures of the source and the sink in Carnot engine are 400 K and 300 K respectively. If the engine receives 600 cal of heat from the source per cycle, the heat rejected to the sink per cycle is

(A) 150 cal (B) 300 cal

(C) 400 cal (D) 450 cal

178. The absolute temperature T of a gas is plotted against its pressure P for two different constant volumes V_1 and V_2 where $V_1 > V_2$. T is plotted along x-axis and P along y-axis.

(A) Slope for curve corresponding to volume V_1 is greater than that corresponding to volume V_2

(B) Slope for curve corresponding to volume V_2 is greater than that corresponding to volume V_1

(C) Slope for both curves are equal

(D) Slope for both curves are unequal such that they intersect at $T = 0$

179. An ideal heat engine having 40% efficiency is operating with a heat sink at 47 °C. The temperature of heat source is

(A) 260 °C (B) 78 °C

(C) 527 °C (D) 117 °C

180. Two spherical black bodies of radii r_1 and r_2 and with surface temperatures T_1 and T_2 respectively radiate the same power. $\frac{r_1}{r_2}$ must be equal to

(A) $\left(\frac{T_1}{T_2}\right)^2$ (B) $\left(\frac{T_2}{T_1}\right)^2$

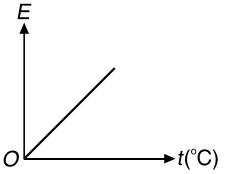
(C) $\left(\frac{T_1}{T_2}\right)^4$ (D) $\left(\frac{T_2}{T_1}\right)^4$

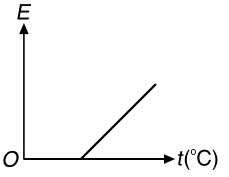
181. The ratio of specific heat of a gas at constant pressure to that at constant volume is γ. The change in internal energy of a mass of gas when the volume changes from V to $2V$ at constant pressure P is

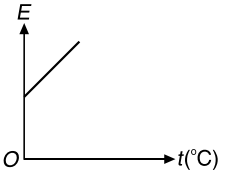
(A) $\frac{R}{\gamma - 1}$ (B) PV

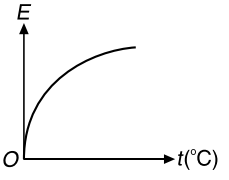
(C) $\frac{PV}{\gamma - 1}$ (D) $\frac{\gamma PV}{\gamma - 1}$

182. The graph which represent the variation of mean kinetic energy of molecules with temperature t (in celsius) is

(A) 

(B) 

(C) 

(D) 

183. When a copper sphere is heated percentage change

(A) is maximum in radius

(B) is maximum in volume

(C) is maximum in density

(D) is equal in radius, volume and density

184. 70 calories of heat is required to raise the temperature of 2 moles of an ideal gas at constant pressure from 40 °C to 45 °C $\left(R = 2\ \text{calmol}^{-1}(\text{°C})^{-1}\right)$. The gas may be

(A) H_2 (B) He

(C) CO_2 (D) NH_3

185. A rod of length 20 cm made of a metal A expands by 0.075 cm when its temperature is raised from 0 °C to 100 °C. Another rod of a different metal B having the same length expands by 0.045 cm for the same change in temperature. A third rod of the same length is composed of two parts, one of metal A and the other of metal B. This rod expands by 0.060 cm for the same change in temperature. The portion made of metal A has length

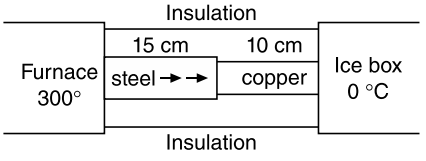
(A) 20 cm (B) 10 cm

(C) 15 cm (D) 18 cm

186. A body cools from 50 °C to 49.9 °C in 5 s. How long will it take to cool from 40 °C to 39.9 °C? Assume temperature of surroundings to be 30 °C and Newton's Law of Cooling is valid

(A) 2.5 s (B) 10 s
(C) 20 s (D) 5 s

187. The area of the cross section of the steel rod is twice that of the copper rod in the above Figure. What is approximate temperature of the steel-copper junction in the steady state of system? Thermal conductivity of steel = 50.2 $Js^{-1}mK$ Thermal conductivity of copper = 385 $Js^{-1}mK$



(A) 44 °C (B) 33 °C
(C) 120 °C (D) 84 °C

188. 3.2 kg of ice at −10 °C just melts with a mass m of steam
(A) $m = 400$ g (B) $m = 800$ g
(C) $m = 425$ g (D) $m = 900$ g

189. A planet having average surface temperature T_0 is at an average distance d from the sun. Assuming that the planet receives radiant energy from the sun only and it loses radiant energy only from its surface and neglecting all other atmospheric effects we conclude
(A) $T_0 \propto d^2$ (B) $T_0 \propto d^{-2}$
(C) $T_0 \propto d^{\frac{1}{2}}$ (D) $T_0 \propto d^{-\frac{1}{2}}$

190. One mole of a gas expands according to the relation $V = kT^{2/3}$. The work done to increase the temperature of gas by 30 °C is
(A) $10R$ (B) $20R$
(C) $40R$ (D) $60R$

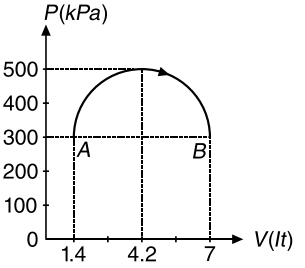
191. A gas has volume V and pressure p. The total translational kinetic energy of all the molecules of the gas is
(A) $\frac{3}{2}pV$ only if the gas is monatomic
(B) $\frac{3}{2}pV$ only if the gas is diatomic
(C) $> \frac{3}{2}pV$ if the gas is diatomic
(D) $\frac{3}{2}pV$ in all cases

192. An electrical steam generator has rated power 1000 W and heat converting efficiency is 0.9. Water enters the inlet at 20 °C and steam comes out from the outlet at 100 °C. Latent heat of steam is 2250 Jg^{-1}. The mass of water entering the device per second is
(A) 1.5 g (B) 0.5 g
(C) 0.35 g (D) 0.39 g

193. Change in internal energy of an ideal gas is given by $\Delta U = nC_V\Delta T$. This is applicable for (C_V =molar heat capacity at constant volume)
(A) isochoric process only
(B) all processes
(C) a process where ΔT is positive
(D) all the processes except isothermal process

194. A gas expands in a piston cylinder device from V_1 to V_2, the process being described by $P = \frac{a}{V} + b$ where P is in Nm^{-2} and V is in cubic metre. The work done in the process is
(A) $a\log_e\left(\frac{V_1}{V_2}\right) + b(V_2 - V_1)$
(B) $-a\log_e\left(\frac{V_2}{V_1}\right) - b(V_2 - V_1)$
(C) $-a\log_e\left(\frac{V_1}{V_2}\right) - b(V_2 - V_1)$
(D) $a\log_e\left(\frac{V_2}{V_1}\right) + b(V_2 - V_1)$

195. A sample of an ideal gas is in a cylinder fitted with a piston. As 5.76 kJ of heat is supplied to the gas to raise its temperature, the pressure the gas adjusted so that the state of the gas changes from point A to point B along the semicircle in P-V graph shown in Figure. The change in internal energy of the gas is



(A) 3.2 kJ (B) 1.6 kJ
(C) 6.4 kJ (D) 4.2 kJ

196. The root mean square (rms) speed of hydrogen molecules at a certain temperature is 300 ms^{-1}. If the temperature is doubled and hydrogen gas dissociates into atomic hydrogen the r.m.s. speed will become
(A) 424.26 ms^{-1} (B) 300 ms^{-1}
(C) 600 ms^{-1} (D) 50 ms^{-1}

197. A black body is at a temperature of 2880 K. The energy of radiation emitted by this object with wavelength between 499 nm and 500 nm is U_1, between 999 nm and 1000 nm is U_2 and between 1499 nm and 1500 nm is U_3. The Wein constant, $b = 2.88 \times 10^6$ nm-K. Then
(A) $U_1 = 0$ (B) $U_3 = 0$
(C) $U_1 > U_2$ (D) $U_2 > U_1$

198. Three rods A, B and C of same length and cross-sectional area are joined in series. Their thermal conductivities are in the ratio 1 : 2 : 1.5. If the open ends of A and C are at 200 °C and 18 °C, respectively. At equilibrium, the temperature at the junction of A and B is
(A) 74 °C (B) 116 °C
(C) 156 °C (D) 148 °C

199. A gas has a molar specific heat C_V at constant volume. n moles of this gas is heated such that rise in temperature is ΔT. The internal energy changes by an amount $nC_V\Delta T$

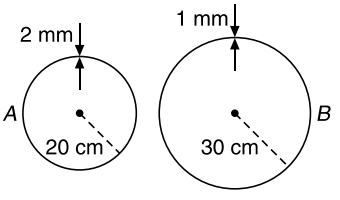
(A) only if ΔT occurs at constant pressure.
(B) only if ΔT occurs at constant volume.
(C) in any non-adiabatic process.
(D) in any known process.

200. Three samples of the same gas A, B, and C $\left(\gamma = \frac{3}{2}\right)$ have initially equal volume. Now the volume of each sample is doubled. The process is adiabatic for A isobaric for B and isothermal for C. If the final pressures are equal for all three samples, the ratio of their initial pressures are

(A) $2\sqrt{2}:2:1$ (B) $2\sqrt{2}:1:2$
(C) $\sqrt{2}:1:2$ (D) $2:1:\sqrt{2}$

201. Two hollow spheres A and B of different materials are filled with ice and kept in the same room. Ratio of thermal conductivity of A to that of B is $1:2$. If time taken for complete melting of ice in sphere A is 80 min, then time taken by ice in sphere B to melt completely is

(A) 30 minutes
(B) 60 minutes
(C) 80 minutes
(D) 160 minutes



202. The mass of 1 litre of He under a pressure of 2 atmosphere and a temperature of 27 °C is

(A) 0.16 g (B) 0.32 g
(C) 0.48 g (D) 0.64 g

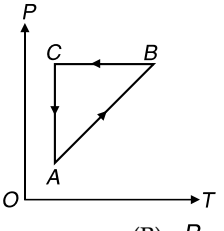
203. If two rods of length L and $2L$ having coefficients of linear expansion α and 2α respectively are connected so that total length becomes $3L$, the average coefficient of linear expansion of the composite rod equals

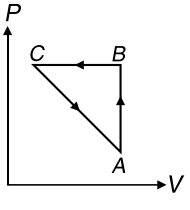
(A) $\frac{3}{2}\alpha$ (B) $\frac{5}{2}\alpha$
(C) $\frac{5}{3}\alpha$ (D) None of these

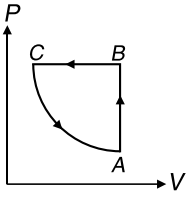
204. 8 g of steam condenses when passed through 80 g of water initially at 10 °C. If the temperature of water rises to 60 °C, latent heat of steam is

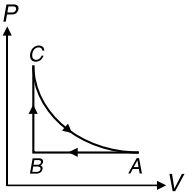
(A) 460 calg^{-1} (B) 540 calg^{-1}
(C) 240 calg^{-1} (D) 500 calg^{-1}

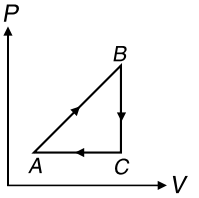
205. A cyclic process is shown on the p-T diagram. Which of the curves show the same process on a p-V diagram?



(A)


(B)


(C)


(D)


206. The intensity of radiation emitted by the Sun has its maximum value at a wavelength of 510 nm and that emitted by the North Star has the maximum value at 350 nm. If these stars behave like black bodies, then the ratio of the surface temperatures of the Sun and the North Star is

(A) 1.46 (B) 0.69
(C) 1.21 (D) 0.83

207. A monatomic gas expands at constant pressure on heating. The percentage of heat supplied that increases the internal energy of the gas and that is involved in the expansion is

(A) 75%, 25% (B) 25%, 75%
(C) 60%, 40% (D) 40%, 60%

208. A gas mixture contains N molecules each of mass m of gas A and $2N$ molecules each of mass $3m$ of gas B. If the rms speed of molecules of gas B is V and the mean square of x component of the velocity of molecules of gas A is v^2, then $\frac{v^2}{V^2}$ equals

(A) 4 (B) 3
(C) 2 (D) 1

209. Temperature of an ideal gas is 300 K. The change in temperature of the gas when its volume changes from V to $2V$ in the process $P = aV$ (Here a is a positive constant) is

(A) 900 K (B) 1200 K
(C) 600 K (D) 300 K

210. 20 g of ice at –10 °C is dropped into a calorimeter containing 20 g of water at 20 °C. Given that the specific heat of water is twice that of ice, at equilibrium, the calorimeter has

(A) 16.25 g of ice and 23.75 g of water
(B) 12.5 g of ice and 27.5 g of water
(C) 20 g of ice and 20 g of water
(D) 40 g of water

211. The temperature of source and sink of a Carnot Engine are 327 °C and 27 °C, respectively. The efficiency of the engine is

(A) $1-\left(\frac{27}{327}\right)$ (B) $\frac{27}{327}$
(C) 0.5 (D) 0.7

212. Three rods A, B and C have the same dimensions. Their thermal conductivities are k_A, k_B, and k_C respectively. A and B are placed end to end, with their free ends kept at certain temperature difference. C is placed separately with its ends kept at same temperature difference. The two arrangements conduct heat at the same rate. k_C must be equal to

(A) $k_A + k_B$ (B) $\frac{k_A k_B}{k_A + k_B}$

(C) $\frac{1}{2}(k_A + k_B)$ (D) $\frac{2k_A k_B}{k_A + k_B}$

213. An ideal gas at pressure P and volume V is expanded to volume $2V$. **COLUMN-I** represents the thermodynamic process used during expansion. **COLUMN-II** represents the work during these process in the random order

COLUMN-I	COLUMN-II
i. Isobaric	p. $\frac{PV(1-2^{1-\gamma})}{\gamma-1}$
ii. Isothermal	q. PV
iii. Adiabatic	r. $PV \ln 2$

(A) i-r, ii-q, iii-p (B) i-q, ii-p, iii-r
(C) i-q, ii-r, iii-p (D) i-p, ii-q, iii-r

214. The least amount of work that must be done to freeze one gram of water at 0 °C by means of a refrigerator placed in surrounding at 27 °C is
(A) 7.9 cal (B) 11.9 cal
(C) 13.9 cal (D) 15.9 cal

215. Certain amount of an ideal gas are contained in a closed vessel. The vessel is moving with a constant velocity v. The molecular mass of gas is M. The rise in temperature of the gas when the vessel is suddenly stopped is $\left(\gamma = \frac{C_P}{C_V}\right)$

(A) $\frac{Mv^2}{2R(\gamma+1)}$ (B) $\frac{Mv^2(\gamma-1)}{2R}$

(C) $\frac{Mv^2}{2R\gamma}$ (D) $\frac{Mv^2\gamma}{2R(\gamma-1)}$

216. According to Debye's T^3 Law, the specific heat of many solids at low temperature T varies according to the relation $c = \alpha T^3$, where α is a constant. The heat energy required to raise the temperature of 4 kg mass from $T = 1$ K to $T = 3$ K is
(A) 208α (B) 20α
(C) 80α (D) 8α

217. The relation between U, p and V for a gas in an adiabatic process is given by relation $U = a + bPV$. Find the value of adiabatic exponent (γ) of this gas

(A) $\frac{b+1}{b}$ (B) $\frac{b+1}{a}$

(C) $\frac{a+1}{b}$ (D) $\frac{a}{a+b}$

218. Earth receives 1400 Wm^{-2} of solar power. If all the solar energy falling on a lens of area 0.2 m^2 is focussed on to a block of ice of mass 280 g, the time taken to melt the ice will be (Take latent heat of fusion of ice is 3.3×10^5 Jkg^{-1})
(A) 303 s (B) 330 s
(C) 33 s (D) 5.5 s

219. The ratio $\frac{C_p}{C_V} = \gamma$ for a gas. Its molecular weight is M. Its specific heat capacity at constant pressure is

(A) $\frac{R}{\gamma-1}$ (B) $\frac{\gamma R}{\gamma-1}$

(C) $\frac{\gamma R}{M(\gamma-1)}$ (D) $\frac{\gamma RM}{(\gamma-1)}$

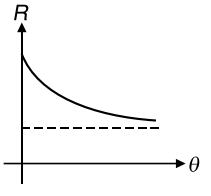
220. Let 1 litre of a gas at 127 °C be cooled to 27 °C at constant pressure. Change in its volume will be
(A) 750 ml (B) –750 ml
(C) 250 ml (D) –250 ml

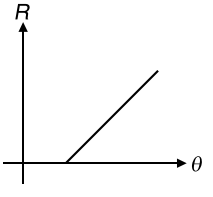
221. Two container having the same volume contain different gases at the same temperature but one exerting twice the pressure exerted by the other. The ratio of the number of molecules in two gases is
(A) 1 : 2 (B) 2 : 1
(C) $\sqrt{2}$: 1 (D) 1 : $\sqrt{2}$

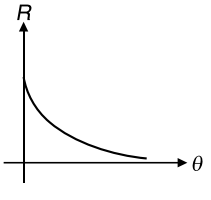
222. A Carnot engine takes 300 cal of heat at 500 K and rejects 150 cal of heat to the sink. The temperature of the sink is
(A) 1000 K (B) 750 K
(C) 250 K (D) 125 K

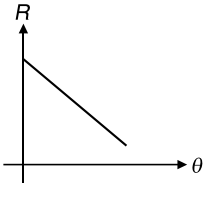
223. Heat can enter a thermos-flask only through a cork fitted in its opening. Cork has an area 100 cm^2 and thickness 4 cm. Its thermal conductivity is 0.008 $cals^{-1}cm^{-1}°C^{-1}$. If the outside temperature is 30 °C, time taken by 200 g of ice to melt is about
(A) 45 minutes (B) 55 minutes
(C) 30 minutes (D) 35 minutes

224. Temperature of a body θ is slightly more than the temperature of the surrounding θ_0. Its rate of cooling (R) versus temperature of body (θ) is plotted, its shape would be

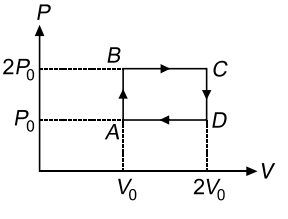
(A) 

(B) 

(C) 

(D) 

225. Helium gas goes through a cycle $ABCDA$ as shown in Figure. Efficiency of this cycle is nearly



(A) 15.4%
(B) 9.1%
(C) 10.5%
(D) 12.5%

226. The freezing point on a thermometer is marked as –20° and the boiling point as 130°. A temperature of human body (34 °C) on this thermometer will be read as

(A) 31° (B) 51°
(C) 20° (D) None of these

227. An object is at the temperature of 400 °C. At what temperature would it radiate twice as fast? The temperature of the surroundings may be assumed to be negligible

(A) 200 °C (B) 200 K
(C) 200 F (D) 800 K

228. When 10 moles of an ideal diatomic gas having initial volume V_0 and temperature T_0 respectively is allowed to expand at constant pressure by supplying $3.5RT_0$ amount of heat, then the final volume and temperature of the gas are

(A) $1.1V_0, 1.1T_0$ (B) $0.9V_0, 0.9T_0$
(C) $1.1V_0, 0.9T_0$ (D) $0.9V_0, 1.1T_0$

229. A gas thermometer is used as a standard thermometer for measurement of temperature. When the gas container of the thermometer is immersed in water at its triple point 273.16 K, the pressure in the gas thermometer reads 3×10^4 Nm^{-2}. When the gas container of the same thermometer is immersed in another system, the gas pressure reads 3.5×10^4 Nm^{-2}. The temperature of this system is

(A) 234.13 K (B) –38.86 °C
(C) 318.69 °C (D) 45.5 °C

230. The temperature at which the velocity of oxygen will be half of that of hydrogen at N.T.P. is

(A) 1092 °C (B) 1492 K
(C) 273 K (D) 819 °C

231. A cooking pot should have

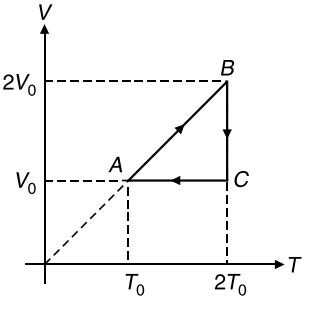
(A) high specific heat and low conductivity
(B) high specific heat and high conductivity
(C) low specific heat and low conductivity
(D) low specific heat and high conductivity

232. Two vessels A and B of different materials are similar in shape and size. The same quantity of ice filled in them melts in times t_1 and t_2 respectively. The ratio of the thermal conductivities of A and B is

(A) $t_1 : t_2$ (B) $t_2 : t_1$
(C) $t_1^2 : t_2^2$ (D) $t_2^2 : t_1^2$

233. An ideal monatomic gas undergoes a cyclic process $ABCA$ as shown in the figure. The ratio of heat absorbed during AB to the work done on the gas during BC is

(A) $\frac{5}{2\log_e 2}$
(B) $\frac{5}{3}$
(C) $\frac{5}{4\log_e 2}$
(D) $\frac{5}{6}$



234. Two rods of same length 20 cm, of metals A and B, expand by 0.02 cm and 0.04 cm respectively when their temperature is raised from 20 °C to 60 °C. A composite rod of length 20 cm is made by joining two rods of metal A and B. This rod expands by 0.025 cm for same change in temperature. Length of part made of metal A is

(A) 20 cm (B) 10 cm
(C) 15 cm (D) 18 cm

235. The molar specific heat of an ideal gas (C)

(A) cannot be negative
(B) must lie in the range starting from C_V and terminating at C_P i.e. $C_V \le C \le C_P$
(C) must equal either $C_P$or C_V
(D) may have any value lying between $-\infty$ and ∞

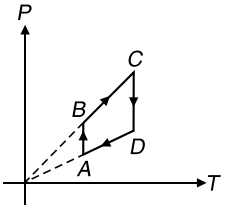
236. A gaseous mixture of 4 g of oxygen and 2 g of helium expands adiabatically and reversibly from 20 litre to 30 litre. The initial temperature is 117 °C. The final temperature (in kelvin) is

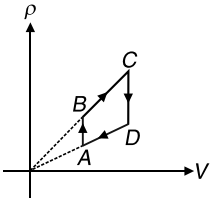
(A) $390\times\left(\frac{2}{3}\right)^{\frac{10}{17}}$ (B) $390\times\left(\frac{2}{3}\right)^{\frac{27}{17}}$
(C) $390\times\left(\frac{2}{3}\right)$ (D) $390\times\left(\frac{3}{2}\right)^{\frac{10}{17}}$

237. A column of liquid is contained in a horizontal tube which is open at both ends. The change in temperature does not alter the length of this liquid column in the tube. If α be the coefficient of linear expansion of the material of the tube and γ be the coefficient of volume expansion of the liquid then

(A) $\gamma = 3\alpha$ (B) $\gamma = 2\alpha$
(C) $\gamma = \alpha$ (D) $\gamma = \frac{1}{2}\alpha$

238. Pressure versus temperature graph of an ideal gas is as shown in figure corresponding density (ρ) versus volume (V) graph will be



(A)


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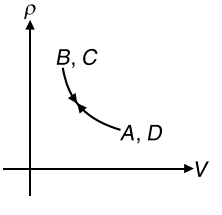

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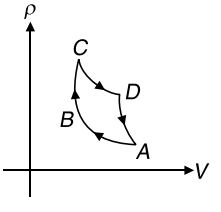

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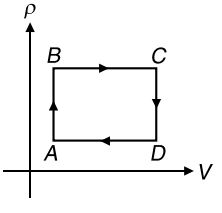


239. A faulty thermometer has its fixed points marked as 10° and 90°. Temperature of a bdy as measured by the faulty thermometer is the correct temperature of the body on celsius scale. The temperature of body is

(A) 60 °C (B) 55 °C
(C) 50 °C (D) 52 °C

240. An object is cooled from 75 °C to 65 °C in 2 minute in a room at 30 °C. The time taken to cool the same object from 55 °C to 45 °C in the same room is

(A) 5 minute (B) 3 minute
(C) 4 minute (D) 2 minute

241. The amount of heat required to raise the temperature of 1 mole of a monatomic gas from 20 °C to 30 °C at constant volume is H. Then the amount of heat required to raise the temperature of 2 mole of a diatomic gas from 20 °C to 25 °C at constant pressure is

(A) $2H$ (B) $\frac{4}{3}H$
(C) $\frac{5}{3}H$ (D) $\frac{7}{3}H$

242. A mass of gas is first expanded isothermally and then compressed adiabatically to its original volume. Further simplest operation performed on the gas to restore it to its original state is

(A) an isobaric cooling to bring its temperature to initial value.
(B) an isochoric cooling to bring its pressure to its initial value.
(C) an isobaric heating to bring its temperature to initial value.
(D) an isochoric heating to bring its temperature to initial value.

243. A cylinder of radius R made of a material of thermal conductivity k_1 is surrounded by a cylindrical shell of inner radius R and outer radius $2R$ made of a material of thermal conductivity k_2. The two ends of the combined system are maintained at two different temperatures. There is no loss of heat across the cylindrical surface and the system is in the steady state. The effective conductivity of the system is

(A) $k_1 + k_2$ (B) $\frac{k_1 k_2}{k_1 + k_2}$
(C) $\frac{k_1 + 3k_2}{4}$ (D) $\frac{3k_1 + k_2}{4}$

244. In a container of negligible heat capacity 100 g of a liquid at 20 °C is heated. Specific heat of liquid varies with temperature T as $c = (100T + 500)\ \text{Jkg}^{-1}\ ^\circ\text{C}^{-1}$, where T is in °C. The amount of heat required to raise the temperature of the liquid to 40 °C is

(A) 5000 J (B) 6000 J
(C) 7000 J (D) 8000 J

245. P-V diagram of a diatomic gas is a straight line passing through origin. The molar heat capacity of the gas in the process will be

(A) $4R$ (B) $2.5R$
(C) $3R$ (D) $\frac{4R}{3}$

246. If $\alpha_{\text{brass}} = 18 \times 10^{-6}\,(^\circ\text{C})^{-1}$ and $\alpha_{\text{iron}} = 12 \times 10^{-6}\,(^\circ\text{C})^{-1}$, then the lengths of brass and iron taken so that the difference between them is always 0.1 m for any variation in temperature are respectively

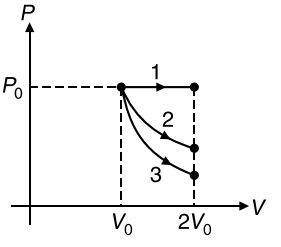
(A) 0.3 m, 0.4 m (B) 0.4 m, 0.3 m
(C) 0.2 m, 0.3 m (D) 0.3 m, 0.2 m

247. The molar specific heat in a process carried on a diatomic gas if the gas does a work of $\frac{Q}{4}$ when a heat Q is supplied to it is

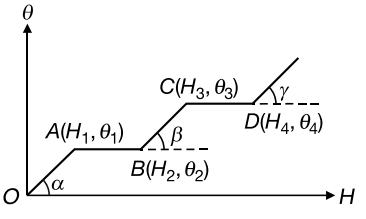
(A) $\frac{2R}{5}$ (B) $\frac{5R}{2}$
(C) $\frac{10R}{3}$ (D) $\frac{6R}{7}$

248. A gas is expanded from volume V_0 to $2V_0$ under three different processes. Process 1 is isobaric, process 2 is isothermal and process 3 is adiabatic. Let ΔU_1, ΔU_2 and ΔU_3 be the change in internal energy of the gas in these three processes. Then

(A) $\Delta U_1 > \Delta U_2 > \Delta U_3$
(B) $\Delta U_1 < \Delta U_2 < \Delta U_3$
(C) $\Delta U_2 < \Delta U_1 < \Delta U_3$
(D) $\Delta U_2 < \Delta U_3 < \Delta U_1$



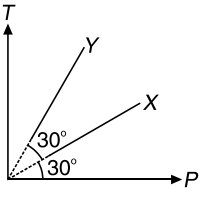
249. The graph shows the variation of temperature (θ) of one kilogram of a material with the heat (H) supplied to it. At O, the substance is in the solid state.
From the graph, we can conclude that



(A) θ_2 is the melting point of the solid
(B) BC represents the change of state from solid to liquid
(C) $(H_2 - H_1)$ represents the latent heat of fusion of the substance
(D) $(H_3 - H_1)$ represents the latent heat of vaporization of the liquid

250. In the following graph, X and Y are two curves corresponding to different masses of the same gas in two identical vessels. The ratio of mass in Y to that in X is

(A) $1 : \sqrt{3}$
(B) $1 : 3$
(C) $2 : 3$
(D) $2 : \sqrt{3}$

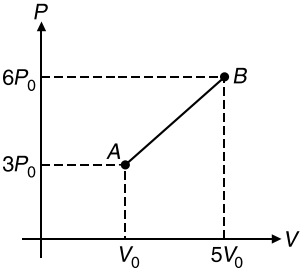


251. Two cylinders fitted with pistons contain equal amounts of an ideal diatomic gas at 300 K. The piston of A is free to move, while that of B is held fixed. The same amount of heat is given to the gas in each cylinder. If the rise in temperature of the gas in A is 30 K, then the rise in temperature of gas in B is

(A) 30 K (B) 18 K
(C) 50 K (D) 42 K

252. One mole of a monatomic ideal gas undergoes the process $A \to B$ in the given P-V diagram. The specific heat for this process is

(A) $\frac{3R}{2}$
(B) $\frac{13R}{6}$
(C) $\frac{5R}{2}$
(D) $2R$

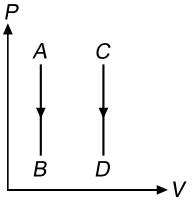


253. A gas undergoes a process in which its pressure P and volume V are related as $PV^n = \text{constant}$, where n is a constant. In this process it is observed that the work is done at the expense of internal energy for

(A) $n = \gamma - 1$ (B) $n = \gamma + 1$
(C) $n = \gamma$ (D) $n = 1 - \gamma$

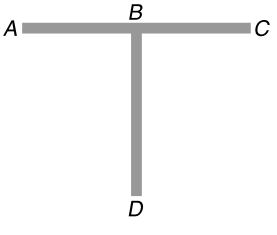
254. Two isochoric processes AB and CD are shown in P-V diagram for the same sample of gas. Heat exchange will be

(A) more in process AB
(B) same in both processes
(C) more in process CD
(D) zero in both processes



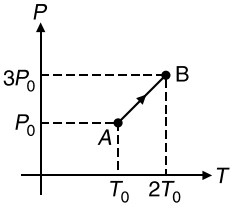
255. Three conducting rods of same material and cross-section are shown in figure. Temperature of A, D and C are maintained at 20 °C, 90 °C and 0 °C. The ratio of lengths of BD and BC if there is no heat flow in AB is

(A) $\frac{2}{7}$
(B) $\frac{7}{2}$
(C) $\frac{9}{2}$
(D) $\frac{2}{9}$



256. Pressure versus temperature graph of an ideal gas is as shown in figure. Density of the gas at point A is ρ_0. Density at B will be

(A) $\frac{3}{4}\rho_0$
(B) $\frac{3}{2}\rho_0$
(C) $\frac{4}{3}\rho_0$
(D) $2\rho_0$



257. Three identical vessels have gases at the same temperature and pressure. The first vessel contains neon (monatomic), the second contains chlorine (diatomic) and the third contains uranium hexafluoride (polyatomic). If n_1, n_2, n_3 be the number of moles and v_1, v_2, v_3 the rms velocity of gases in vessel 1, 2 and 3 respectively, then

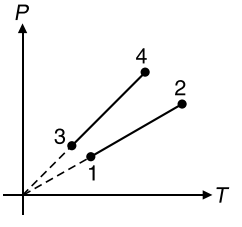
(A) $n_1 > n_2 > n_3$ and $v_1 > v_2 > v_3$
(B) $n_1 < n_2 < n_3$ and $v_1 < v_2 < v_3$
(C) $n_1 = n_2 = n_3$ and $v_1 = v_2 = v_3$
(D) $n_1 = n_2 = n_3$ and $v_1 > v_2 > v_3$

258. The coefficient of linear expansion of iron is $0.000011\ \text{K}^{-1}$. An iron rod is 10 m long at 27 °C. The length of the rod will be decreased by 1.1 mm when the temperature of the rod changes to

(A) 0 °C (B) 10 °C
(C) 17 °C (D) 20 °C

259. Pressure versus temperature graph of an ideal gas of equal number of moles of different volumes are plotted as shown in figure. Choose the correct alternative

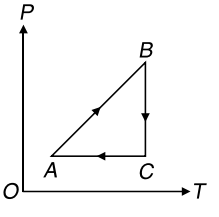
(A) $V_1 = V_2$, $V_3 = V_4$ and $V_2 > V_3$
(B) $V_1 = V_2$, $V_3 = V_4$ and $V_2 < V_3$
(C) $V_1 = V_2 = V_3 = V_4$
(D) $V_4 > V_3 > V_2 > V_1$

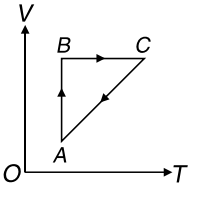


260. Resistance of a platinum resistance thermometer is 2 ohm at 20 °C, 4 ohm at 100 °C and 2.2 ohm at some temperature of T °C. Then T is

(A) 7 (B) 14
(C) 21 (D) 28

261. A cyclic process is shown in the p-T diagram. which of the curves show the same process on a V-T diagram?



(A)


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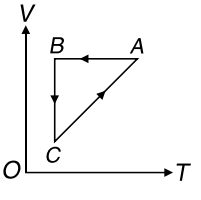

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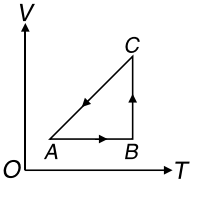

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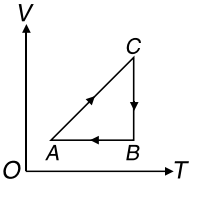


262. A system S receives heat continuously from an electrical heater of power 10 W. The temperature of S becomes constant at 50 °C when the surrounding temperature is 20 °C.

After the heater is switched off, S cools from 35.1 °C to 34.9 °C in 1 minute. The heat capacity of S is

(A) $750\ \text{J}(^\circ\text{C})^{-1}$ (B) $1500\ \text{J}(^\circ\text{C})^{-1}$

(C) $3000\ \text{J}(^\circ\text{C})^{-1}$ (D) $6000\ \text{J}(^\circ\text{C})^{-1}$

263. A box contains N molecules of a perfect gas at temperature T_1 and pressure P_1. The number of molecules in the box is doubled keeping the total kinetic energy of the gas same as before. If the new pressure is P_2 and temperature T_2, then

(A) $P_2 = P_1, T_2 = T_1$ (B) $P_2 = P_1, 2T_2 = T_1$

(C) $P_2 = 2P_1, T_2 = T_1$ (D) $P_2 = 2P_1, 2T_2 = T_1$

264. n mole of an ideal gas is supplied heat such that its temperature changes from T_1 to T_2 and pressure from P_1 to P_2. If ΔS is the change in entropy of the system then

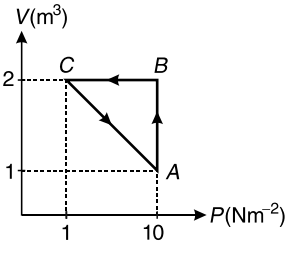
(A) $\Delta S = nC_V \ln\left(\frac{T_2}{T_1}\right) + nR\ln\left(\frac{V_2}{V_1}\right)$

(B) $\Delta S = nC_V \ln\left(\frac{T_2}{T_1}\right)$

(C) $\Delta S = nR\ln\left(\frac{V_2}{V_1}\right)$

(D) Data insufficient to arrive at conclusion

265. An ideal gas is taken through $A \to B \to C \to A$, as shown in figure. If the net heat supplied to the gas in the cycle is 5 J, the work done by the gas in the process $C \to A$ is



(A) –5 J (B) –10 J

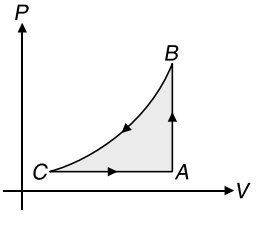
(C) –15 J (D) –20 J

266. If on heating a liquid through 80 °C in a vessel, the mass overflown from the vessel is $\left(\frac{1}{100}\right)^{\text{th}}$ of mass still remaining, the coefficient of apparent expansion of liquid is about

(A) $1.25\times10^{-4}\ ^\circ\text{C}^{-1}$ (B) $2.5\times10^{-4}\ ^\circ\text{C}^{-1}$

(C) $1.25\times10^{-5}\ ^\circ\text{C}^{-1}$ (D) $2.5\times10^{-5}\ ^\circ\text{C}^{-1}$

267. A sample of an ideal gas is taken through a cycle as shown in figure. It absorbs 50 J of energy during the process AB, no heat during BC, rejects 70 J during CA. If 40 J of work is done on the gas during BC, internal energy of gas at A is 1500 J, then the internal energy at C would be



(A) 1590 J (B) 1620 J

(C) 1540 J (D) 1570 J

268. If a 60 kg man is having a temperature of 103 °F. Assuming the specific heat of human body to be $1\ \text{calg}^{-1}(^\circ\text{C})^{-1}$, the heat required to raise the temperature of the body to this level from the normal body temperature of 98.4 °F is

(A) 122 cal (B) 154 kcal

(C) 66 kcal (D) 184 kcal

269. n_1 mole of a monatomic gas is mixed with n_2 mole of diatomic gas such that $\gamma_{\text{mixture}} = 1.5$

(A) $n_1 = 2n_2$ (B) $n_2 = 2n_1$

(C) $n_1 = n_2$ (D) $n_1 = 3n_2$

270. Heat is supplied to a diatomic gas at constant pressure. The ratio of $\Delta Q : \Delta U : \Delta W$ is

(A) 5 : 3 : 2 (B) 5 : 2 : 3

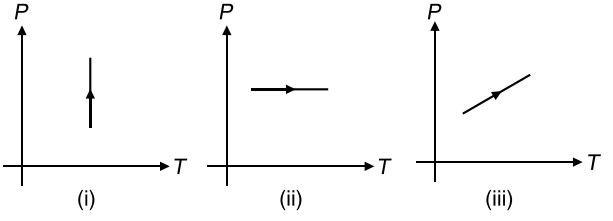
(C) 7 : 5 : 2 (D) 7 : 2 : 5

271. A perfect gas goes from state A to another state B by absorbing 8×10^5 J of heat and doing 6.5×10^5 J of external work. It is now transferred between the same two states in another process in which it absorbs 10^5 J of heat. Then in the second process, work done

(A) on gas is 0.5×10^5 J (B) by gas is 0.5×10^5 J

(C) on gas is 10^5 J (D) by gas is 10^5 J

272. Pressure versus temperature graphs of an ideal gas are as shown in figure. Select the incorrect statement.



(A) Density of gas is increasing in graph (i)

(B) Density of gas is decreasing in graph (ii)

(C) Density of gas is constant in graph (iii)

(D) None of these

273. For a gas $\frac{R}{C_P} = \frac{2}{3}$. Possibly the gas is

(A) diatomic

(B) monatomic

(C) a mixture of diatomic and monatomic

(D) polyatomic

274. The dimensional representation of thermal resistance is

(A) $ML^{-2}T^{-2}K^{-1}$ (B) $M^{-1}L^{-2}T^{3}K$

(C) $ML^{2}T^{-3}K^{-2}$ (D) $ML^{2}T^{-2}K^{-2}$

275. An ideal heat engine exhausting heat at 27 °C is to have a 60% efficiency. It must take heat at

(A) 327 °C (B) 373 °C

(C) 477 °C (D) 573 °C

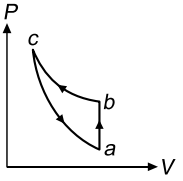
276. Pressure P, volume V and temperature T of a certain material are related by $P = \frac{\alpha T^2}{V}$. Here α is a constant. The work done by the material when temperature changes from T_0 to $2T_0$ while pressure remains constant is

(A) $6\alpha T_0^3$ (B) $\frac{3}{2}\alpha T_0^2$

(C) $2\alpha T_0^2$ (D) $3\alpha T_0^2$

277. The relation between U, P and V for an ideal gas is $U = 2 + 3PV$. The gas is
(A) monatomic
(B) diatomic
(C) polyatomic
(D) either a monatomic or diatomic

278. An ideal gas taken through the cyclic process $a \to b \to c \to a$ absorbs 100 J of heat during the part ab, no heat during $b \to c$, 50 J of work is done on the gas during the part $b \to c$. If internal energy of the gas at a is 1200 J, then change in internal energy in $c \to a$ process is



(A) 100 J
(B) –150 J
(C) 150 J
(D) –1050 J

279. The temperature of a gas contained in a closed vessel increases by 1 °C when pressure of the gas is increased by 1%. The initial temperature of the gas is
(A) 100 K (B) 100 °C
(C) 250 K (D) 250 °C

280. A certain gas is filled in two identical containers at pressures P_1 and P_2 and absolute temperatures T_1 and T_2 respectively. When the vessels are joined, the gases intermix and reach a common pressure P and common temperature T. Then $\frac{P}{T}$ equals
(A) $\frac{1}{2}\left(\frac{P_1}{T_1}+\frac{P_2}{T_2}\right)$ (B) $\frac{P_1T_2+P_2T_1}{(T_1+T_2)^2}$
(C) $\frac{P_1T_1+P_2T_2}{(T_1+T_2)^2}$ (D) $\left(\frac{P_1}{T_1}+\frac{P_2}{T_2}\right)$

281. A gas, is heated at constant pressure. The fraction of heat supplied used for external work is
(A) $\frac{1}{\gamma}$ (B) $\left(1-\frac{1}{\gamma}\right)$
(C) $\gamma - 1$ (D) $\left(1-\frac{1}{\gamma^2}\right)$

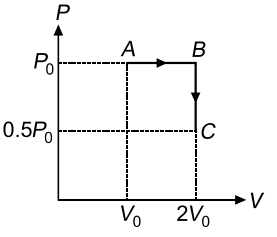
282. A gas at pressure P_0 is contained in a vessel. If the masses of all the molecules are halved and their speeds doubled, the resulting pressure would be
(A) $4P_0$ (B) $2P_0$
(C) P_0 (D) $\frac{P_0}{2}$

283. The rms speed of molecules of a gas contained in a vessel moving with a speed v is $\bar{c}$ as seen by a person moving with the vessel. Then pressure exerted by the gas is
(A) $\frac{1}{3}\rho(\bar{c})^2$ (B) $\frac{1}{3}\rho(\bar{c}+v)^2$
(C) $\frac{1}{3}\rho(\bar{c}-v)^2$ (D) $\frac{1}{3}\rho\left[(\bar{c})^2-v^2\right]$

284. The internal energy of an ideal gas is
(A) ZERO (B) PV
(C) $\frac{PV}{\gamma-1}$ (D) $\frac{\gamma PV}{\gamma-1}$

285. A body having a surface area of 5.0 cm^2, radiates 300 J of energy per minute at a temperature of 727 °C. The emissivity of the body is
(Stefan's constant is 5.67×10^{-8} $Wm^{-2}K^{-4}$)
(A) 0.09 (B) 0.18
(C) 0.36 (D) 0.54

286. One mole of an ideal monatomic gas is taken from point A to C along the path ABC. The initial temperature at A is T_0. For the process $A \to B \to C$,



(A) heat absorbed is $0.5RT_0$
(B) heat liberated is $2RT_0$
(C) change in internal energy is zero
(D) work done is zero

287. The plot of isotherms will not be a straight line when a plot is drawn between
(A) PV and V (B) S (entropy) and T
(C) V and P (D) P and T

288. The following are the observations made by five students in measuring C_P and C_V in cal $mol^{-1}(°C)^{-1}$ respectively. Which would you consider as the most reliable observation?
(A) 5 and 3 (B) 4 and 6
(C) 5 and 7 (D) 4 and 3

289. For an ideal monatomic gas, the universal gas constant R is n times the molar heat capacity at constant pressure C_p. Here n is
(A) 0.67 (B) 1.4
(C) 0.4 (D) 1.67

290. If most probable speed of the molecules of certain diatomic gas at room temperature is found to be about 1.6 kms^{-1}, then the gas is
(A) O_2 (B) N_2
(C) H_2 (D) CO_2

291. n moles of a gas expands from volume V_1 to V_2 such that external work done equals the heat supplied. The work done by the gas is
(A) $nRT\left(\frac{V_2}{V_1}\right)$ (B) $nRT\left(\frac{V_2}{V_1}-1\right)$
(C) $nRT\ln\left(\frac{V_2}{V_1}\right)$ (D) $nRT\ln\left(\frac{V_2}{V_1}+1\right)$

292. A metallic rod of unit length and unit area of cross section is heated through 1 °C. If the Young's modulus of elasticity is E and the linear expansivity is $\alpha(°C)^{-1}$, then the compressional force required to prevent the rod from expanding is
(A) $E\alpha$ (B) $\frac{\alpha}{E}$
(C) $\frac{E}{\alpha}$ (D) $3E\alpha$

293. Let Q_1 and Q_2 be the respective heats supplied to the gas in processes shown in Figure-I and Figure-II, then

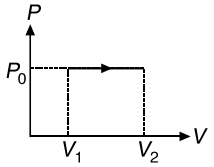


FIGURE-I

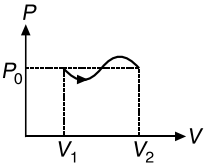


FIGURE-II

(A) $Q_1 = Q_2$ (B) $Q_1 > Q_2$

(C) $Q_1 < Q_2$ (D) $Q_1 \gg Q_2$

294. A point source of heat of power P is placed at the centre of a spherical shell of mean radius R. The material of the shell has thermal conductivity k. If the temperature difference between the outer and the inner surface of the shell is not to exceed T, then the thickness of the shell should not be less than

(A) $\frac{2\pi R^2 kT}{P}$ (B) $\frac{4\pi R^2 kT}{P}$

(C) $\frac{\pi R^2 kT}{P}$ (D) $\frac{\pi R^2 kT}{4P}$

295. A sealed container with negligible coefficient of expansion contained helium. When it is heated from 27 °C to 327 °C, the average kinetic energy of helium becomes

(A) 12 times (B) 8 times

(C) 4 times (D) 2 times

MULTIPLE CORRECT CHOICE TYPE QUESTIONS

This section contains Multiple Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

1. Internal energy of an ideal diatomic gas at 300 K is 100 J. Out of this 100 J

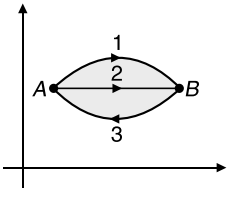
(A) potential energy is zero

(B) rotational kinetic energy is 40 J

(C) translational kinetic energy is 60 J

(D) translational kinetic energy is 100 J

2. A gas undergoes the change in its state from position A to position B via three different paths as shown in figure. Select the correct alternative(s).



(A) Change in internal energy in all the three paths is equal

(B) In all the three paths heat is absorbed by the gas

(C) Heat absorbed/released by the gas is maximum in path 1

(D) Temperature of the gas first increases and then decreases continuously in path 1

3. A steel drill making 180 rpm is used to drill a hole in a block of steel. The mass of the steel block and the drill is 180 g each. The entire mechanical work is used up in producing heat such that the rate of rise of temperature of the block is 0.5 °Cs^{-1}. P is the power rating of the drill and τ is the couple required to drive the drill. (Given specific heat of steel $c_{steel} = 0.10$ calg^{-1}(°C)$^{-1}$)

(A) $P = 37.8$ W (B) $P = 9$ W

(C) $\tau = 2$ Nm (D) $\tau = 6.3$ Nm

4. The molar specific heat for a gas may have a value given by

(A) $C_V = \frac{dU}{dT}$ (B) $C_P = \left(\frac{đQ}{dT}\right)_P$

(C) $C = \frac{dU}{dT} + P\frac{dV}{dT}$ (D) Data Insufficient

5. Which of the following quantities is independent of the nature of the gas at same temperature

(A) the number of molecules in 1 mole

(B) the number of molecules in equal volume

(C) the translational kinetic energy of 1 mole

(D) the kinetic energy of unit mass

6. Two spheres A and B have same radius but the heat capacity of A is greater than that of B. The surfaces of both are painted black. They are heated to the same temperature and allowed to cool. Then

(A) A cools faster than B

(B) both A and B cool at the same rate

(C) at any temperature the ratio of their rates of cooling is a constant

(D) B cools faster than A

7. Number of collisions per unit area of molecules of a gas on the wall of a container will increase when the

(A) temperature and volume both are doubled

(B) temperature and volume both are halved

(C) pressure and temperature both are doubled

(D) None of above

8. One end of the metal rod of area A and length l is kept in steam. A steady state is reached after some time in which the amount of heat passing through any cross-section of rod per second is Q. Q will increase if

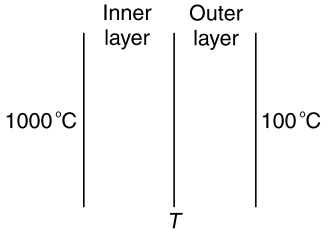
(A) A is increased

(B) l is increased

(C) the room temperature is increased

(D) the room temperature is decreased

9. The temperature drop through a two layer furnace wall is 900 °C. Each layer is of equal area of cross section. Which of the following actions will result in lowering the temperature T of the interface?



(A) by increasing the thermal conductivity of outer layer
(B) by increasing thermal conductivity of inner layer
(C) by increasing thickness of outer layer
(D) by increasing thickness of inner layer

10. 1 kg of ice at 0 °C is mixed with 1.5 kg of water at 45 °C. If the latent heat of fusion of ice is 80 $calg^{-1}$, then

(A) the temperature of the mixture is 0 °C
(B) mixture contains 156.25 g of ice
(C) mixture contains 843.75 g of ice
(D) the temperature of the mixture is 15 °C

11. Two rods of length L_1 and L_2 are made of materials of coefficients of linear expansions α_1 and α_2 respectively such that $L_1\alpha_1 = L_2\alpha_2$. The temperature of the rods is increased by ΔT and correspondingly the change in their respective lengths be ΔL_1 and ΔL_2.

(A) $\Delta L_1 \neq \Delta L_2$
(B) $\Delta L_1 = \Delta L_2$
(C) Difference in length $(L_1 - L_2)$ is a constant and is independent of rise of temperature
(D) Data is insufficient to arrive at a conclusion

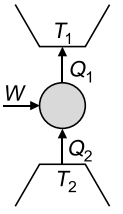
12. Two identical containers each of volume V_0 are joined by a small pipe. The containers contain identical gases at temperature T_0 and pressure P_0. One container is heated to temperature $2T_0$ while maintaining the other at the same temperature. The common pressure of the gas is P and n is the number of moles of gas in container at temperature $2T_0$.

(A) $P = 2P_0$ (B) $P = \frac{4}{3}P_0$
(C) $n = \frac{2}{3}\frac{P_0V_0}{RT_0}$ (D) $n = \frac{3}{2}\frac{P_0V_0}{RT_0}$

13. Two gases, initially having the same pressure, volume and temperature expand to the same volume, the first isothermally and the second adiabatically.
(A) Greater work is done by the gas for isothermal process.
(B) The isothermal process has greater final pressure.
(C) The isothermal process has greater final temperature (same as initial value) in comparison to adiabatic process.
(D) Data is insufficient to arrive at a conclusion.

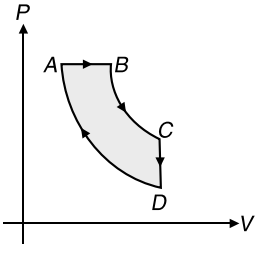
14. In PROBLEM 13, if compression is carried instead of expansion then
(A) greater work is done on the gas for an adiabatic process.
(B) the adiabatic process has greater final pressure.
(C) the adiabatic process has greater final temperature due to the fact that adiabatic compression leads to heating.
(D) data is insufficient to arrive at a conclusion.

15. Consider a heat pump as shown in Figure. If $W > 0$, then the correct possibilities are



(A) $Q_1 > Q_2 > 0$
(B) $Q_2 > Q_1 > 0$
(C) $Q_2 < Q_1 < 0$
(D) $Q_1 < 0, Q_2 > 0$

16. A monatomic gas of n moles undergoes a cyclic process $ABCDA$ as shown in figure. Process AB is isobaric, BC is adiabatic, CD is isochoric and DA is isothermal. The maximum and minimum temperature in the cycle are $4T_0$ and T_0 respectively. Then



(A) $T_B > T_C > T_D$
(B) heat is released by the gas in the process CD
(C) heat is supplied to the gas in the process AB
(D) total heat supplied to the gas is $2nRT_0\log_e(2)$

17. A planet having surface temperature T K has a solar constant S. An angle θ is subtended by the sun at the planet.

(A) $S \propto T^2$ (B) $S \propto T^4$
(C) $S \propto \theta^0$ (D) $S \propto \theta^2$

18. Two identical vessels contain helium and hydrogen at same temperature, then
(A) average kinetic energy per mole of hydrogen is equal to the average kinetic energy per mole of helium
(B) average translational kinetic energy per mole of hydrogen is equal to the average translational kinetic energy per mole of helium
(C) average kinetic energy per mole of hydrogen is $\frac{3}{5}$ times the average kinetic energy per mole of helium
(D) average kinetic energy per mole of hydrogen is $\frac{5}{3}$ times the average kinetic energy per mole of helium

19. Select the incorrect statement(s).
(A) A refrigerator is reverse of a heat engine.
(B) A heat pump is the same as a refrigerator.
(C) The coefficient of performance of a refrigerator can be infinite.
(D) In a heat engine, heat cannot be fully converted to work.

20. A body of mass m has gram specific heat c.
(A) Heat capacity of the body is mc
(B) Water equivalent of the body is m
(C) Water equivalent of the body is mc
(D) Heat capacity of the body is c

21. A gas is found to obey the law $p^2V = \text{constant}$. The initial temperature and volume are T_0 and V_0. If the gas expands to a volume $3V_0$, then

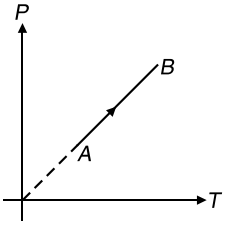
(A) final temperature becomes $\sqrt{3}T_0$

(B) internal energy of the gas will increase

(C) final temperature becomes $\frac{T_0}{\sqrt{3}}$

(D) internal energy of the gas decreases

22. During the process A-B of an ideal gas



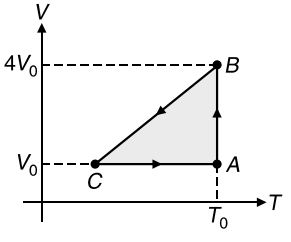
(A) work done on the gas is zero

(B) density of the gas is constant

(C) slope of line AB from the T-axis is inversely proportional to the number of moles of the gas

(D) slope of line AB from the T-axis is directly proportional to the number of moles of the gas

23. One mole of an ideal gas in initial state A undergoes a cyclic process $ABCA$, as shown in the figure. Its pressure at A is P_0. Choose the correct option(s) from the following.



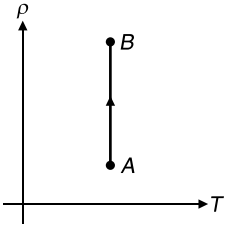
(A) Internal energies at A and B are the same

(B) Work done by the gas in process AB is $P_0V_0 \log_e 4$

(C) Pressure at C is $\frac{P_0}{4}$

(D) Temperature at C is $\frac{T_0}{4}$

24. The density (ρ) of an ideal gas varies with temperature T as shown in figure. Then



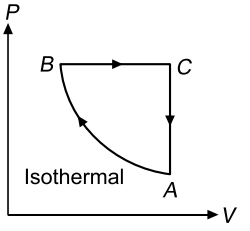
(A) the product of P and V at A is equal to the product of P and V at B

(B) pressure at B is greater than the pressure at A

(C) work done by the gas during the process AB is negative

(D) the change in internal energy from A to B is zero

25. A cyclic process of an ideal monatomic gas is shown in Figure. Select the correct statement(s)



(A) Work done by gas in process AB is more than that of the process BC.

(B) Net heat energy has been supplied to system.

(C) Temperature of gas is maximum in state B.

(D) In process CA, heat energy is rejected out by system.

26. At ordinary temperatures, the molecules of an ideal gas have only translational and rotational kinetic energies. At high temperatures they may also have vibrational energy. As a result of this at higher temperatures (C_V =molar heat capacity at constant volume)

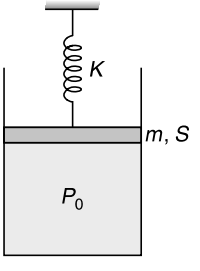
(A) $C_V = \frac{3}{2}R$ for a monatomic gas

(B) $C_V > \frac{3}{2}R$ for a monatomic gas

(C) $C_V < \frac{5}{2}R$ for a diatomic gas

(D) $C_V > \frac{5}{2}R$ for a diatomic gas

27. In the arrangement shown in figure. Gas is thermally insulated. An ideal gas is filled in the cylinder having pressure P_0 (> atmospheric pressure P_a). Spring of force constant k is initially unstretched. Piston of mass m and area S is frictionless. In equilibrium piston rises up a distance x_0, then



(A) final pressure of the gas is $P_a + \frac{kx_0}{S} + \frac{mg}{S}$

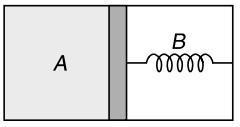
(B) work done by the gas is $\frac{1}{2}kx_0^2 + mgx_0$

(C) decrease in internal energy of the gas is $\frac{1}{2}kx_0^2 + mgx_0 + P_aSx_0$

(D) All of the above

28. A thermally insulated chamber of volume $2V_0$ is divided by a frictionless piston of area S into two equal parts A and B. Part A has an ideal gas at pressure P_0 and temperature T_0 and in part B is vacuum. A massless spring of force constant

k is connected with piston and the wall of the container as shown. Initially spring is unstretched. Gas in chamber A is allowed to expand. Let in equilibrium spring is compressed by x_0. Then



(A) final pressure of the gas is $\frac{kx_0}{S}$

(B) work done by the gas is $\frac{1}{2}kx_0^2$

(C) change in internal energy of the gas is $\frac{1}{2}kx_0^2$

(D) temperature of the gas is decreased

29. In a thermodynamic process helium gas obeys the law $TP^{-2/5} = \text{constant}$. If temperature of 2 moles of the gas is raised from T to $3T$, then

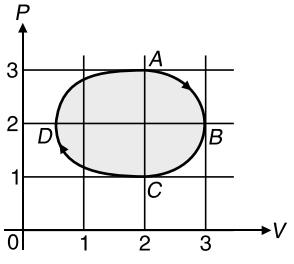
(A) heat given to the gas is $9RT$

(B) heat given to the gas is zero

(C) increase in internal energy is $6RT$

(D) work done by the gas is $-6RT$

30. The figure shows the P-V plot of an ideal gas taken through a cycle $ABCDA$. The part ABC is a semi-circle and CDA is half of an ellipse. Then,



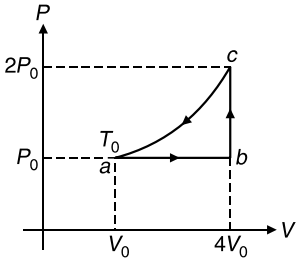
(A) The process during the path $A \to B$ is isothermal.

(B) Heat flows out of the gas during the path $B \to C \to D$.

(C) Work done during the path $A \to B \to C$ is zero.

(D) Positive work is done by the gas in the cycle $ABCDA$.

31. One mole of an ideal monatomic gas (initial temperature T_0) is made to go through the cycle $abca$ shown in the figure. If U denotes the internal energy, then choose the correct alternatives



(A) $U_c - U_a = 10.5RT_0$ (B) $U_b - U_a = 4.5RT_0$

(C) $U_c > U_b > U_a$ (D) $U_c - U_b = 6RT_0$

32. Which of the following quantities depend on temperature only for a given ideal gas

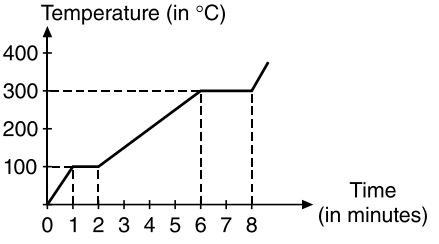
(A) internal energy of the gas

(B) product PV of the gas

(C) the ratio of pressure and density of the gas

(D) root mean square speed of the gas

33. Heat is supplied to 100 g of a solid (specific heat 0.5 kcal kg °C^{-1}) till it boils. The graph between temperature and time is as follows. Then select the correct statement(s).



(A) Heat supplied per minute is 5 kcal

(B) Boiling point is 300 °C

(C) Specific heat in liquid state is 1 kcalkg^{-1}°C^{-1}

(D) None of these is correct

34. For a gas if C_P is the specific heat at constant pressure, C_V is the specific heat at constant volume, R is the Universal Gas constant and f be the degrees of freedom then

(A) $f = \frac{2C_V}{R}$ (B) $f = 2\left(\frac{C_P}{R} - 1\right)$

(C) $f = \frac{C_V}{R}$ (D) $f = \frac{C_P}{R} - 1$

35. An ideal gas is heated from temperature T_1 to T_2 under various conditions. The correct statement(s) is/are

(A) $\Delta U = nC_V(T_2 - T_1)$ for isobaric, isochoric and adiabatic processes

(B) Work is done at expense of internal energy in an adiabatic process and both have equal values

(C) $\Delta U = 0$for an isothermal process

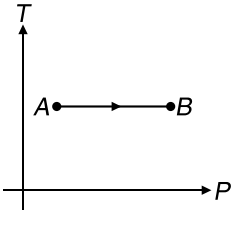
(D) $C = 0$for an adiabatic process

36. n mole of an ideal gas is heated such that its temperature, pressure and volume, all three change simultaneously from T_1, P_1, V_1 to T_2, P_2, V_2. Further in this process the specific heat equals zero. If W is the work done then

(A) $W = \frac{nR}{1-\gamma}(T_2 - T_1)$ (B) $W = \frac{1}{1-\gamma}(P_2V_2 - P_1V_1)$

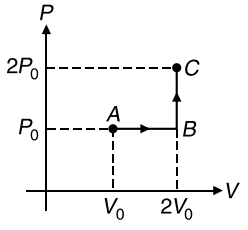
(C) $W = \frac{nR}{1+\gamma}(T_2 - T_1)$ (D) $W = \frac{1}{1+\gamma}(P_2V_2 - P_1V_1)$

37. Temperature versus pressure graph of an ideal gas is shown in figure. During the process AB



(A) internal energy of the gas remains constant
(B) volume of the gas is increased
(C) work done by the atmosphere on the gas is positive
(D) pressure is inversely proportional to volume

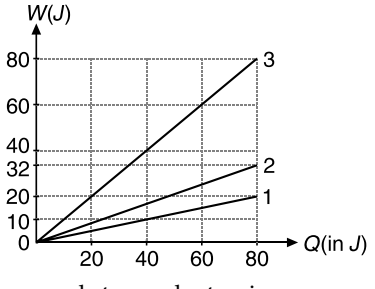
38. One mole of an ideal monatomic gas is taken from A to C along the path ABC. The temperature of the gas at A is T_0. For the process ABC (R =universal gas constant)



(A) work done by the gas is RT_0
(B) change in internal energy of the gas is $\frac{11}{2}RT_0$
(C) heat absorbed by the gas is $\frac{11}{2}RT_0$
(D) heat absorbed by the gas is $\frac{13}{2}RT_0$

39. C_P is always greater than C_V due to the fact that
(A) no work is being done on heating the gas at constant volume.
(B) when a gas absorbs heat at constant pressure its volume must change so as to do some external work.
(C) the internal energy is a function of temperature only for an ideal gas.
(D) for the same rise of temperature, the internal energy of a gas changes by a smaller amount at constant volume than at constant pressure.

40. In the Figure shown, the amount of heat supplied to one mole of an ideal gas is plotted on the horizontal axis and the amount work performed by the gas is drawn on the vertical axis. The graphs are isobars for different gases. The initial states of both gases are same. The curve



(A) 1 corresponds to a polyatomic gas
(B) 2 corresponds to a monatomic gas
(C) 3 is not possible
(D) All of these

41. A spherical black body of radius r, radiates a power P at temperature T when placed in surroundings at temperature $T_0 (\ll T)$. If R is the rate of cooling then

(A) $P \propto (T - T_0)$ (B) $P \propto T^4$
(C) $P \propto r^2$ (D) $R \propto \frac{1}{r}$

42. In PROBLEM 41, if temperature of body is comparable to the temperature of surroundings then

(A) $R \propto (T - T_0)$ (B) $R \propto \frac{1}{r^2}$
(C) $R \propto \frac{1}{r}$ (D) $R \propto (T^4 - T_0^4)$

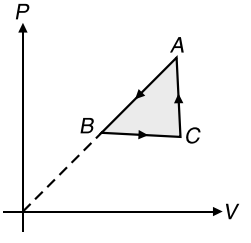
43. Two ends of area A of a uniform rod of thermal conductivity k are maintained at different but constant temperatures. At any point on the rod, the temperature gradient is $\frac{dT}{dl}$. If I be the thermal current in the rod, then

(A) $I \propto A$ (B) $I \propto \frac{dT}{dl}$
(C) $I \propto A^0$ (D) $I \propto \frac{1}{\left(\frac{dT}{dl}\right)}$

44. A metal rod of length L_0, made of material of Young's modulus Y, area A is fixed between two rigid supports. The coefficient of linear expansion of the rod is α. The rod is heated such that the tension in the rod is T

(A) $T \propto L_0$ (B) $T \propto A_0^0$
(C) $T \propto A$ (D) $T \propto L_0^0$

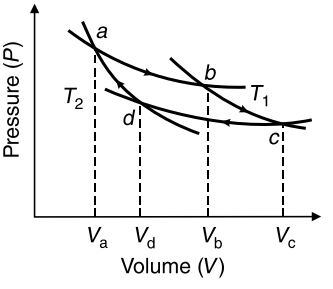
45. P-V diagram of a cyclic process $ABCA$ is as shown in figure. Select the correct statement(s).



(A) $Q_{A\to B}$ = negative (B) $\Delta U_{B\to C}$ = positive
(C) $\Delta U_{C\to A}$ = negative (D) W_{CAB} = negative

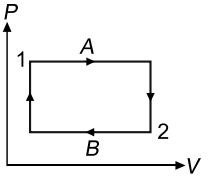
46. The quantity $\frac{mkT}{V}$ of an ideal gas depends on (m is the mass of the gas, k is the Boltzmann's Constant)
(A) temperature of the gas
(B) volume of the gas
(C) pressure of the gas
(D) nature of the gas

47. An ideal gas undergoes the cyclic process shown in an isotherm below.



(A) $T_1 = T_2$ (B) $T_1 > T_2$
(C) $V_a V_c = V_b V_d$ (D) $V_a V_b = V_c V_d$

48. Let ΔU_1 and ΔU_2 be change in internal energy in process A and B respectively, Q and W be the net heat given and net work done by the system in the cyclic process $A+B$ as shown in Figure, then select the correct statement(s).



(A) $\Delta U_1 + \Delta U_2 = 0$

(B) $\Delta U_1 - \Delta U_2 = 0$

(C) $Q - W = 0$

(D) $Q + W = 0$

49. A heat engine absorbs heat at 327 °C and exhausts heat at127 °C. The efficiency of engine is η and the maximum amount of work performed by the engine per kilocalorie of heat input is W.

(A) $\eta = 0.38$ (B) $\eta = 0.33$

(C) $W = 1596$ J (D) $W = 1400$ J

50. A vessel X contains 1 mole of O_2 gas (molar mass 32) at a temperature T and pressure P. Another identical vessel Y contains one mole of He gas (molar mass 4) at temperature $2T$, then

(A) pressure in the container Y is $\frac{P}{8}$

(B) kinetic energy of O_2 molecule = kinetic energy of He molecule

(C) pressure in the container Y is $2P$

(D) kinetic energy of He molecule $= \frac{6}{5}$ (kinetic energy of O_2 molecule)

51. The internal energy of a system remains constant when it undergoes

(A) a cyclic process

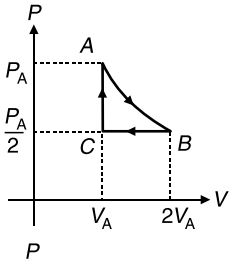
(B) an isothermal process

(C) an adiabatic process

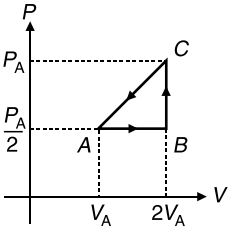
(D) any process in which the heat given out by the system equals the work done on the system

52. Three moles of an ideal gas $\left(C_P = \frac{7}{2}R\right)$ at pressure P_A and temperature T_A is isothermally expanded to twice its initial volume. It is then compressed at constant pressure to its original volume. Finally, the gas is compressed at constant volume to its original pressure P_A. The correct P-V and P-T diagrams indicating the process are

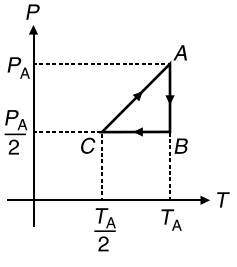
(A)



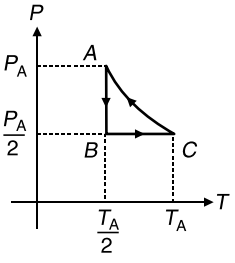
(B)



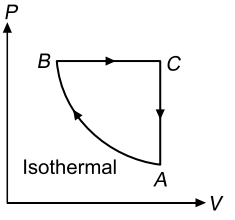
(C)



(D)



53. A cyclic process of an ideal monatomic gas is shown in Figure. Select the correct statement(s)



(A) Work done by gas in process AB is more than that of the process BC.

(B) Net heat energy has been supplied to system.

(C) Temperature of gas is maximum in state B.

(D) In process CA, heat energy is rejected out by system.

54. A bimetallic strip is made up of two metals with different α.

(A) On heating, it bends towards the metal with high α.

(B) On heating, it bends towards the metal with low α.

(C) On cooling, it bends towards the metal with high α.

(D) On cooling, it bends towards the metal with low α.

55. An ideal gas is allowed to expand against a vacuum in a rigid insulator container. Select the correct alternative(s).

(A) Work done by the gas is zero

(B) Pressure of the gas is inversely proportional to volume of the gas

(C) Change in internal energy of the gas is zero

(D) Temperature of the gas remains constant during expansion

56. n moles of an ideal monatomic gas undergoes a process in which the temperature changes with volume as $T = KV^2$. If the temperature of the gas changes from T_0 to $4T_0$ then

(A) work done by the gas is $3nRT_0$

(B) heat supplied of the gas is $6nRT_0$

(C) work done by the gas is $\frac{3}{2}nRT_0$

(D) heat supplied to the gas is $\frac{3}{2}nRT_0$

57. A uniform cylinder of steel of mass M, radius R is placed on frictionless bearings and set to rotate about its vertical axis with angular velocity ω_0. After the cylinder has reached the specified state of rotation it is heated without any mechanical contact from temperature T_0 to $T_0 + \Delta T$. If $\frac{\Delta I}{I}$ is the fractional change in moment of inertia of the cylinder and $\frac{\Delta\omega}{\omega_0}$ be the fractional change in the angular velocity of the cylinder and α be the coefficient of linear expansion, then

(A) $\frac{\Delta I}{I} = \frac{2\Delta R}{R}$ (B) $\frac{\Delta I}{I} = -\frac{\Delta\omega}{\omega_0}$

(C) $\frac{\Delta\omega}{\omega_0} = -2\alpha\Delta T$ (D) $\frac{\Delta I}{I} = -\frac{2\Delta R}{R}$

58. In an isothermal expansion of a gas

(A) pressure remains constant

(B) temperature remains constant

(C) rms velocity of gas molecules remains constant

(D) density remains constant

59. For free expansion of an ideal gas in an insulated container, there is
(A) no heat exchanged
(B) work done
(C) no change of internal energy
(D) no change in pressure

60. n moles of an ideal gas undergoes a cyclic process in which it absorbs a heat Q_1 at constant temperature T_1 such that the expansion ratio is α. It rejects a heat Q_2 at constant temperature $T_2 (< T_1)$. The efficiency of the cyclic process is η, the total work done in the cyclic process is W and dU is the change in internal energy of the process.

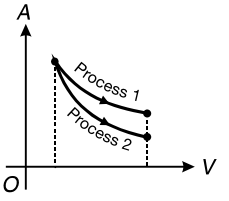
(A) $\eta = 1 - \left(\frac{T_2}{T_1}\right)$

(B) $W = Q_1 - Q_2 = nR(T_1 - T_2)\log_e \alpha$

(C) $dU = 0$

(D) $dU = nRT_1 \log_e \alpha$

61. The indicator diagram for two processes 1 and 2 carried on an ideal gas is shown in figure. If m_1 and m_2 be the slopes $\left(\frac{dP}{dV}\right)$ for Process 1 and Process 2 respectively, then



(A) $m_1 = m_2$ (B) $m_1 > m_2$

(C) $m_1 < m_2$ (D) $m_2 C_V = m_1 C_P$

REASONING BASED QUESTIONS

This section contains Reasoning type questions, each having four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Each question contains STATEMENT 1 and STATEMENT 2. You have to mark your answer as

Bubble (A) If both statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1.
Bubble (B) If both statements are TRUE but STATEMENT 2 is not the correct explanation of STATEMENT 1.
Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE.
Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 is TRUE.

1. **Statement-1:** On a T-V graph (T on y-axis), the curve for adiabatic expansion would be a monotonically decreasing curve.
Statement-2: The slope of an adiabatic process represented on T-V graph is always positive.

2. **Statement-1:** The specific heat of a gas in an adiabatic process is zero and in an isothermal process is infinite.
Statement-2: Specific heat of a gas is directly proportional to change in heat and inversely proportional to change in temperature.

3. **Statement-1:** We can change the temperature of a body without giving (or taking) heat to (or from) it.
Statement-2: According to the Law of Conservation of Energy, total energy of a system should remain conserved.

4. **Statement-1:** When temperature difference across the two sides of a wall is increased, its coefficient of thermal conductivity increases.
Statement-2: Coefficient of thermal conductivity depends on nature of material of the wall.

5. **Statement-1:** V-T graph in a process is rectangular hyperbola. Then P-T graph in the same process will be a parabola.
Statement-2: If V-T graph is rectangular hyperbola, then V decrease when pressure increase.

6. **Statement-1:** A gas possesses a unique value of specific heat capacity.
Statement-2: Specific heat capacity is the heat required to raise the temperature of unit mass of the substance by one unit.

7. **Statement-1:** The isothermal curves intersect each other at a certain point.
Statement-2: The isothermal change takes place slowly, so, the isothermal curves have very little slope.

8. **Statement-1:** Specific heat of a gas at constant pressure is greater than its specific heat at constant volume.
Statement-2: At constant pressure, some heat is spent in expansion of the gas.

9. **Statement-1:** During an adiabatic expansion temperature of gas must decrease.
Statement-2: During adiabatic process $TV^{\gamma-1} = \text{constant}$, where γ is the adiabatic exponent of gas.

10. **Statement-1:** The coefficient of volume expansion at constant pressure for diatomic gas is more than that for monatomic gas.
Statement-2: The coefficient of volume expansion of gas at constant pressure is independent of the degrees of freedom of gas.

11. **Statement-1:** Two bodies at different temperatures, if brought in contact do not necessary settle to the mean temperature.
Statement-2: The two bodies may have different thermal capacities.

12. **Statement-1:** Melting point of ice decreases upon increase in pressure.
Statement-2: Volume of water is smaller than volume of ice.

13. **Statement-1:** Melting of solid causes no change in internal energy
Statement-2: Specific latent heat is the heat required to melt a unit mass of solid.

LINKED COMPREHENSION TYPE QUESTIONS

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct. (For the sake of competitiveness there may be a few questions that may have more than one correct options)

Comprehension 1

An amount Q of heat is given to a monatomic ideal gas in a process in which the gas performs a work $\frac{Q}{2}$ on its surroundings. Based on the above facts, answer the following questions.

1. The equation of the process discussed is given by
   (A) $PV^2 = \text{constant}$
   (B) $PV^3 = \text{constant}$
   (C) $P^3V = \text{constant}$
   (D) $P^2V = \text{constant}$

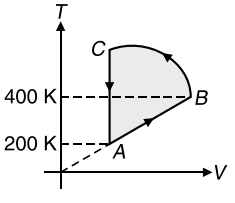
2. Molar heat capacity of gas for the process is
   (A) R (B) $2R$
   (C) $3R$ (D) $\frac{R}{2}$

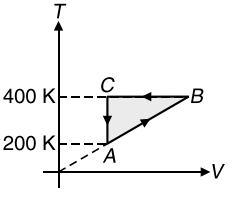
3. If at a temperature T_0, the volume of gas is V_0, then at temperature $T = 4T_0$, the volume of gas becomes
   (A) $4V_0$ (B) $8V_0$
   (C) $\frac{V_0}{4}$ (D) $16V_0$

Comprehension 2

Four moles of an ideal gas is initially in a state A having pressure 2×10^5 Nm^{-2} and temperature 200 K. Keeping the pressure constant, the gas is taken to state B at temperature of 400 K. The gas is then taken to a state C in such a way that its temperature increases and volume decreases. Also, from B to C, the magnitude of $\frac{dT}{dV}$ increases. The volume of gas at state C is equal to its volume at state A. Now gas is taken to initial state A keeping volume constant. A total of 1000 J of heat is withdrawn from the sample in the cyclic process. Taking $R = 8.3$ Jmol^{-1} K^{-1}. Based on the above facts, answer the following questions.

4. The correct graph between temperature T and volume V for the cyclic process is

(A)


(B)


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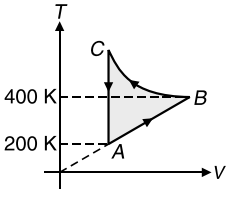
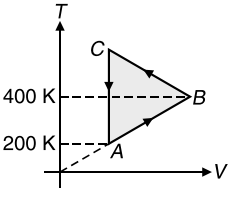


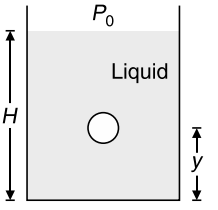
(D)


5. The work done in path B to C is
   (A) 1000 J (B) −1000 J
   (C) −7640 J (D) 5640 J

6. The volume of gas at state C is
   (A) 0.0332 m^3 (B) 0.22 m^3
   (C) 0.332 m^3 (D) 3.32 m^3

Comprehension 3

A small spherical monatomic ideal gas bubble $\left(\gamma = \frac{5}{3}\right)$ is trapped inside a liquid of density ρ_l (shown in figure). Assume that the bubble does not exchange any heat with the liquid. The bubble contains n moles of gas. The temperature of the gas when the bubble is at the bottom is T_0, the height of the liquid is H and the atmospheric pressure is P_0 (Neglect surface tension). Based on the above facts, answer the following questions.



7. As the bubble moves upwards, besides the buoyancy force the following forces are acting on it
   (A) Only the force of gravity
   (B) The force due to gravity and the force due to the pressure of the liquid
   (C) The force due to gravity, the force due to the pressure of the liquid and the force due to viscosity of the liquid
   (D) The force due to gravity and the force due to viscosity of the liquid

8. When the gas bubble is at a height y from the bottom, its temperature is
   (A) $T_0\left(\frac{P_0+\rho_l g H}{P_0+\rho_l g y}\right)^{\frac{2}{5}}$ (B) $T_0\left(\frac{P_0+\rho_l g (H-y)}{P_0+\rho_l g H}\right)^{\frac{2}{5}}$
   (C) $T_0\left(\frac{P_0+\rho_l g H}{P_0+\rho_l g y}\right)^{\frac{3}{5}}$ (D) $T_0\left(\frac{P_0+\rho_l g (H-y)}{P_0+\rho_l g H}\right)^{\frac{3}{5}}$

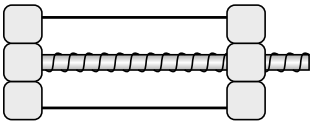
9. The buoyancy force acting on the gas bubble is (assume R is the universal gas constant).
   (A) $\rho_l n R g T_0 \frac{(P_0+\rho_l g H)^{\frac{2}{5}}}{(P_0+\rho_l g y)^{\frac{7}{5}}}$
   (B) $\frac{\rho_l n R g T_0}{(P_0+\rho_l g H)^{\frac{2}{5}}(P_0+\rho_l g (H-y))^{\frac{3}{5}}}$
   (C) $\rho_l n R g T_0 \frac{(P_0+\rho_l g H)^{\frac{3}{5}}}{(P_0+\rho_l g y)^{\frac{8}{5}}}$
   (D) $\frac{\rho_l n R g T_0}{(P_0+\rho_l g H)^{\frac{3}{5}}(P_0+\rho_l g (H-y))^{\frac{2}{5}}}$

Comprehension 4

A steel bolt of cross-sectional area $A_b = 5\times10^{-5}$ m^2 is passed through a cylindrical tube made of aluminium. The cross-sectional area of the tube material is $A_t = 10^{-4}$ m^2 and its length is $l = 50$ cm. The bolt is just taut so that there is no stress in the bolt and the temperature of the assembly is increased through $\Delta T = 10$ °C. Given, coefficient of linear thermal expansion of steel, $\alpha_b = 10^{-5}$ °C^{-1}.

Young's modulus of steel $Y_b = 2\times10^{11}$ Nm^{-2}.

Young's modulus of Al, $Y_t = 10^{11}$ Nm^{-2}, coefficient of linear thermal expansion of Al, $\alpha_t = 2\times10^{-5}$ °C^{-1}. Based on the above facts, answer the following questions.



10. The compressive strain in tube is

(A) 10^{-4} (B) 5×10^{-5}

(C) 2×10^{-3} (D) 10^{-6}

11. The compressive stress in tube is

(A) 5×10^{6} Nm^{-2} (B) 10^{5} Nm^{-2}

(C) 10^{8} Nm^{-2} (D) 10^{3} Nm^{-2}

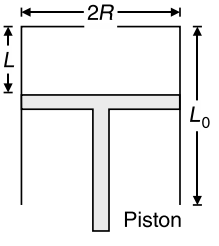
12. The tensile stress in bolt is

(A) 10^{4} Nm^{-2} (B) 10^{7} Nm^{-2}

(C) 2×10^{8} Nm^{-2} (D) 10^{10} Nm^{-2}

Comprehension 5

A fixed thermally conducting cylinder has a radius R and height L_0. The cylinder is open at its bottom and has a small hole at its top. A piston of mass M is held at a distance L from the top surface, as shown in the figure. The atmospheric pressure is P_0. Based on the above facts, answer the following questions.



13. The piston is now pulled out slowly and held at a distance $2L$ from the top. The pressure in the cylinder between its top and piston will then be

(A) P_0 (B) $\dfrac{P_0}{2}$

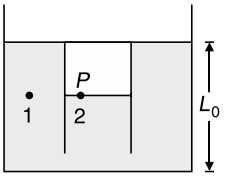
(C) $\dfrac{P_0}{2}+\dfrac{Mg}{\pi R^2}$ (D) $\dfrac{P_0}{2}-\dfrac{Mg}{\pi R^2}$

14. While the piston is at a distance $2L$ from the top, the hole at the top is sealed. The piston is then released, to a position where it can stay in equilibrium. In this condition, the distance of the piston from the top is

(A) $\left(\dfrac{2P_0\pi R^2}{\pi R^2P_0+Mg}\right)(2L)$ (B) $\left(\dfrac{P_0\pi R^2-Mg}{\pi R^2P_0}\right)(2L)$

(C) $\left(\dfrac{P_0\pi R^2+Mg}{\pi R^2P_0}\right)(2L)$ (D) $\left(\dfrac{P_0\pi R^2}{\pi R^2P_0-Mg}\right)(2L)$

15. The piston is taken completely out of the cylinder. The hole at the top is sealed. A water tank is brought below the cylinder and put in a position so that the water surface in the tank is at the same level as the top of the cylinder as shown in the figure. The density of the water is ρ. In equilibrium, the height H of the water column in the cylinder satisfies



(A) $\rho g(L_0-H)^2+P_0(L_0-H)+L_0P_0=0$

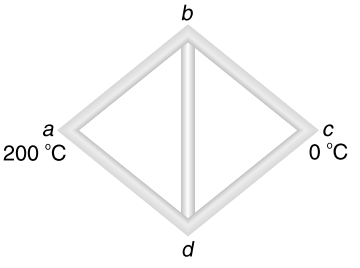
(B) $\rho g(L_0-H)^2-P_0(L_0-H)-L_0P_0=0$

(C) $\rho g(L_0-H)^2+P_0(L_0-H)-L_0P_0=0$

(D) $\rho g(L_0-H)^2-P_0(L_0-H)+L_0P_0=0$

Comprehension 6

Five rods of same material and same cross section are joined as shown. Lengths of rods ab, ad and bc are l, $2l$ and $3l$ respectively. Ends a and c are maintained at temperatures 200 °C and 0 °C respectively.



Based on the above facts, answer the following questions.

16. The length x of rod dc for which there will be no heat flow through rod bd is

(A) $4l$ (B) $2l$

(C) $6l$ (D) $9l$

17. Then temperature of junction b or d is

(A) 120 °C (B) 160 °C

(C) 90 °C (D) 150 °C

Comprehension 7

A certain amount of ice is supplied heat at a constant rate for 7 min. For the first one minute the temperature rises uniformly with time. Then it remains constant for the next 4 min and again the temperature rises at uniform rate for the last 2 min. Given that $c_{ice} = 0.5$ calg^{-1} °C, $L_{ice} = 80$ calg^{-1}. Based on the above facts, answer the following questions.

18. The initial temperature of ice is

(A) –10 °C (B) –20 °C

(C) –30 °C (D) –40 °C

19. The final temperature of the ice, at the end of 7 min is

(A) 20 °C (B) 30 °C

(C) 40 °C (D) 50 °C

Comprehension 8

A gaseous mixture enclosed in a vessel of volume V consists of one mole of a gas A with $\gamma = \left(\dfrac{C_P}{C_V}\right) = \dfrac{5}{3}$ and another gas B with $\gamma = \dfrac{7}{5}$ at a certain temperature T. The relative molar masses of the gases A and B are 4 and 32, respectively. The gases A and B do not react with each other and are assumed to be ideal. The

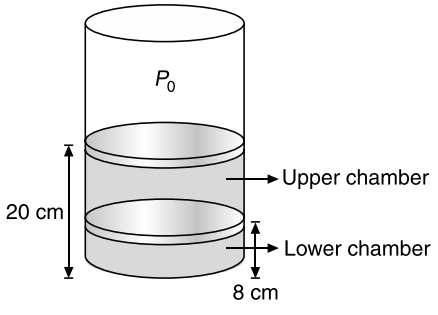
gaseous mixture follows the equation $PV^{\frac{19}{13}} = \text{constant}$, in an adiabatic process. Based on the above facts, answer the following questions.

20. Number of moles of the gas B in the gaseous mixture is
(A) 1 mole (B) 3 mole
(C) 4 mole (D) 2 mole

21. The mixture is compressed adiabatically to $\frac{1}{5}$ of its initial volume V. The change in its adiabatic compressibility in terms of the given quantities is (Given that $5^{\frac{19}{13}} = 10.48$).
(A) $-0.025\frac{V}{T}$ (B) $-0.050\frac{V}{T}$
(C) $-0.075\frac{V}{T}$ (D) $-0.100\frac{V}{T}$

Comprehension 9

In a cylindrical container of sufficiently large height, two easily moving pistons enclose certain amount of same ideal gas in two chambers as shown in Figure.



The upper piston is at a height 20 cm from the bottom and lower piston is at a height 8 cm from the bottom. The mass of each piston is m kg and cross sectional area of each piston is A m^2, where $\frac{mg}{A} = P_0$ and P_0 is the atmospheric pressure given by 1×10^5 Nm^{-2}. The cylindrical container and pistons are made of conducting material. Initially the temperature of gas is 27 °C and whole system is in equilibrium. Now if the upper piston is slowly lifted by 16 cm and held in that position with the help of some external force. As a result, the lower piston rises slowly by l cm. Based on the information give, answer the following questions.

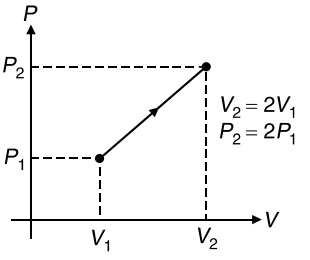
22. The value of l is
(A) 2 cm (B) 4 cm
(C) 8 cm (D) 6 cm

23. The ratio of volume of gas in upper chamber to that of in lower chamber in final state is
(A) 2 : 1 (B) 1 : 2
(C) 4 : 1 (D) 1 : 4

24. The pressure of gas in lower chamber in final state is
(A) 1.0×10^5 Nm^{-2} (B) 2.0×10^5 Nm^{-2}
(C) 3.0×10^5 Nm^{-2} (D) 4.0×10^5 Nm^{-2}

Comprehension 10

A quantity of an ideal monatomic gas consists of n moles initially at temperature T_1. The pressure and volume are then slowly doubled in such a manner so as to trace out a straight line passing through origin on a P-V diagram as shown. Based on the above facts, answer the following questions.



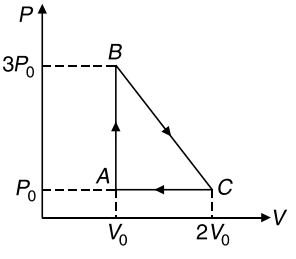
25. For this process, the ratio $\frac{W}{nRT_1}$ is equal to (where W is work done by the gas) is
(A) 1.5 (B) 3
(C) 4.5 (D) 6

26. For the same process, the ratio $\frac{Q}{nRT_1}$ is equal to (where Q is heat supplied to the gas) is
(A) 1.5 (B) 3
(C) 4.5 (D) 6

27. If C is defined as the average molar specific heat for the process, then $\frac{C}{R}$ has value
(A) 1.5 (B) 2
(C) 3 (D) 6

Comprehension 11

One mole of an ideal monatomic gas is taken round the cyclic process $ABCA$ as shown in figure.



Based on the above facts, answer the following questions.

28. The work done in the complete cycle is
(A) zero (B) P_0V_0, by the system
(C) $\frac{P_0V_0}{2}$, by the system (D) P_0V_0, on the system

29. If dQ_{CA} is the heat absorbed/rejected by the gas in the path CA, then
(A) $dQ_{CA} = \frac{5}{2}P_0V_0$, absorbed (B) $dQ_{CA} = \frac{3}{2}P_0V_0$, absorbed
(C) $dQ_{CA} = \frac{5}{2}P_0V_0$, rejected (D) $dQ_{CA} = \frac{3}{2}P_0V_0$, rejected

30. If dQ_{AB} is the heat absorbed/rejected by the gas in the path AB, then
(A) $dQ_{AB} = \frac{3}{2}P_0V_0$, absorbed (B) $dQ_{AB} = 3P_0V_0$, absorbed
(C) $dQ_{AB} = \frac{3}{2}P_0V_0$, rejected (D) $dQ_{AB} = 3P_0V_0$, rejected

31. If d_{BC} is the heat absorbed/rejected by the gas in the path BC, then

(A) $dQ_{BC}=\frac{3}{2}P_0V_0$, absorbed (B) $dQ_{BC}=\frac{1}{2}P_0V_0$, rejected

(C) $dQ_{BC}=\frac{1}{2}P_0V_0$, absorbed (D) $dQ_{BC}=\frac{3}{2}P_0V_0$, rejected

32. The maximum temperature attained by the gas during the cycle is

(A) $\frac{25}{8}\frac{P_0V_0}{R}$ (B) $\frac{3P_0V_0}{R}$

(C) $\frac{8}{25}\frac{P_0V_0}{R}$ (D) $\frac{1}{3}\frac{P_0V_0}{R}$

Comprehension 12

Consider one mole of an ideal gas having $\gamma=\frac{C_P}{C_V}$ which expands according to the law $P=\alpha V$ where P is the pressure, V is the volume and α is a constant. Assuming the initial volume of the gas to be V_0 and when expanded to a volume nV_0, answer the following questions.

33. The work done by the gas is

(A) $\frac{\alpha V_0^2}{2}(n^2-1)$ (B) $\alpha V_0^2(n^2-1)$

(C) $\alpha V_0^2(1-n^2)$ (D) $\frac{\alpha V_0^2}{2}(1-n^2)$

34. The change in internal energy of gas is

(A) $\frac{\alpha}{(1-\gamma)}V_0^2(n^2-1)$ (B) $\frac{\alpha}{(\gamma-1)}V_0^2(n^2-1)$

(C) $\frac{\alpha}{2(1-\gamma)}V_0^2(n^2-1)$ (D) $\frac{\alpha}{2(\gamma-1)}V_0^2(n^2-1)$

35. The specific heat for the process is

(A) $\left(\frac{\gamma-1}{\gamma+1}\right)\frac{R}{2}$ (B) $\left(\frac{\gamma+1}{\gamma-1}\right)\frac{R}{2}$

(C) $\left(\frac{\gamma+1}{\gamma-1}\right)R$ (D) $\left(\frac{\gamma-1}{\gamma+1}\right)R$

Comprehension 13

In a thermodynamic process, it is observed that the internal energy of an ideal gas varies with pressure P and volume V as $U=3PV$. If the gas expands from V_0 to $2V_0$ at a constant pressure P_0. Based on the above facts, answer the following questions.

36. The heat absorbed by the gas in the process is

(A) P_0V_0 (B) $2P_0V_0$

(C) $3P_0V_0$ (D) $4P_0V_0$

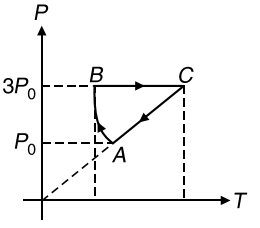
37. The value of $\gamma\left(=\frac{C_p}{C_v}\right)$ for the given gas is

(A) 1.33 (B) 1.4

(C) 1.67 (D) 1.5

Comprehension 14

One mole of monatomic gas is taken through a cyclic process shown. If $T_A=300$ K and the process AB is defined as $PT=$ constant, then based on the above facts, answer the following questions.



38. Work done in process AB is

(A) $400R$ (B) $-400R$

(C) $200R$ (D) $-300R$

39. The change in internal energy in process CA

(A) $900R$ (B) $300R$

(C) $-1200R$ (D) $-900R$

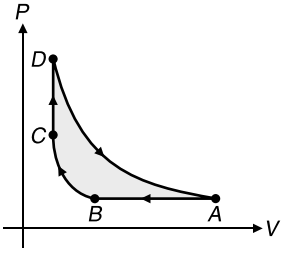
40. Heat transferred in the process BC is

(A) $1000R$ (B) $500R$

(C) $2000R$ (D) $1500R$

Comprehension 15

One mole of an ideal monatomic gas is taken around a cyclic process as shown. The processes AB, BC, CD and DA are isobaric, adiabatic, isochoric and isothermal respectively. It is known that $T_A=4T_B$ and $V_A=(8\sqrt{2})V_D$. Based on the above facts, answer the following questions.



41. The temperature at state C is T_C, given by

(A) $T_C=T_B$ (B) $T_C=2T_B$

(C) $T_C=3T_C$ (D) None of these

42. The work done by gas during the cyclic process in one cycle is

(A) $5.2RT_B$ (B) $4RT_B$

(C) $3.2RT_B$ (D) None of these

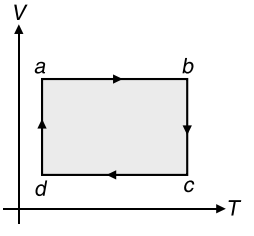
43. Efficiency of cycle of the cyclic process is

(A) 21% (B) 41%

(C) 51% (D) None of these

Comprehension 16

The V-T graph for an ideal gas is shown in figure. Based on the information provided and a careful observation of the graph, answer the following questions.



44. Work done by the gas in complete cyclic process $a\to b\to c\to d\to a$ is

(A) ZERO (B) positive

(C) negative (D) data is insufficient

45. Heat is supplied to the gas in process/es

(A) da, ab and bc (B) da and ab only

(C) da only (D) ab and bc only

Comprehension 17

An ideal gas undergoes a process in which $T = T_0 + aV^3$, where T_0 and a are positive constants and V is molar volume. Based on the information given, answer the following questions.

46. The volume at which the pressure attains a minimum value is

(A) $\left(\frac{T_0}{2a}\right)^{\frac{1}{3}}$ (B) $\left(\frac{T_0}{3a}\right)^{\frac{1}{3}}$

(C) $\left(\frac{a}{2T_0}\right)^{\frac{2}{3}}$ (D) $\left(\frac{a}{3T_0}\right)^{\frac{2}{3}}$

47. The minimum pressure attainable per mole of gas is

(A) $\frac{3}{4}\left(a^{\frac{5}{3}}R^{\frac{2}{3}}T_0^{\frac{2}{3}}\right)2^{\frac{1}{3}}$ (B) $\frac{3R}{2}\left(3a^2T_0^2\right)^{1/3}$

(C) $\frac{3}{2}\left(a^{\frac{1}{2}}R^{\frac{2}{3}}T_0^{\frac{3}{4}}\right)4^{\frac{1}{3}}$ (D) $\frac{3R}{2}\left(2aT_0^2\right)^{1/3}$

Comprehension 18

An ideal gas having initial pressure P, volume V and temperature T is allowed to expand adiabatically until its volume becomes 5.66 V while its temperature falls to $\frac{T}{2}$.

Based on the above facts, answer the following questions.

48. The degrees of freedom possessed by the gas molecules is/are

(A) 1 (B) 2

(C) 3 (D) 5

49. The work done by the gas during expansion as a function of initial pressure P and volume V is

(A) 2.31 PV

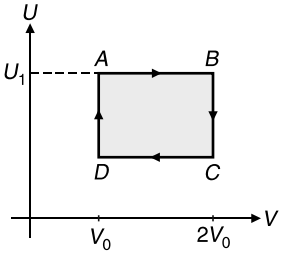
(B) 1.23 PV

(C) 3.21 PV

(D) None of these seems to be correct

Comprehension 19

One mole of an ideal gas has an internal energy given by $U = U_0 + 2PV$, where P is the pressure and V the volume of the gas and U_0 is a constant. This gas undergoes the quasi-static cyclic process $ABCD$ as shown in the U-V diagram. Based on the above facts, answer the following questions.



50. The molar heat capacity of the gas at constant pressure is

(A) $2R$ (B) $3R$

(C) $\frac{5}{2}R$ (D) $4R$

51. The work done by the ideal gas in the process AB is

(A) zero (B) $\frac{U_1 - U_0}{2}$

(C) $\frac{U_0 - U_1}{2}$ (D) $\frac{U_1 - U_0}{2}\log_e(2)$

52. Assuming that the gas consists of a mixture of two gases, the gas is

(A) Monatomic

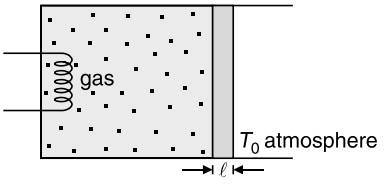
(B) Diatomic

(C) A mixture of mono and diatomic gases

(D) A mixture of di- and tri-atomic gases

Comprehension 20

A non-conducting vessel containing n moles of a diatomic gas is fitted with a conducting piston. The cross-sectional area, thickness and thermal conductivity of piston are A, l and K respectively. The right side of piston is open to atmosphere at temperature T_0. Heat is supplied to the gas by means of an electric heater at a constant rate q. Based on the above facts, answer the following questions.



53. The temperature of gas as a function of time is

(A) $T = T_0 + \frac{ql}{KA}\left(1 - e^{-\frac{2KAt}{7nRl}}\right)$

(B) $T = T_0 + \frac{ql}{KA}\left(1 - e^{-\frac{7nRl}{2KAt}}\right)$

(C) $T = T_0 + \frac{ql}{KA}\left(e^{-\frac{2KAt}{7nRl}}\right)$

(D) None of these

54. The maximum temperature of gas is

(A) $T_{max} = T_0 + \frac{KA}{ql}$ (B) $T_{max} = T_0 + \frac{ql}{KA}$

(C) $T_{max} = \frac{ql}{KA}$ (D) $T_{max} = T_0 + \frac{ql}{KA}(1 - e)$

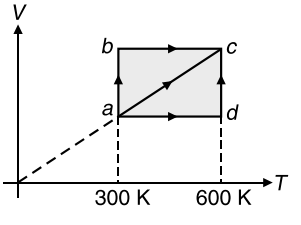
55. The ratio of the maximum volume to the minimum volume is

(A) $1 + \frac{KAT_0}{ql}$ (B) $\frac{KAT_0}{ql}$

(C) $\frac{ql}{KAT_0}$ (D) $1 + \frac{ql}{KAT_0}$

Comprehension 21

Two moles of an ideal monatomic gas is taken from a to c, via three paths $a \to b \to c$, $a \to c$ and $a \to d \to c$ as shown. Based on the above facts, answer the following questions.



56. Work done by the gas in process $a \to c$ is

(A) $1000R$ (B) $900R$

(C) $600R$ (D) $1500R$

57. If work done by the gas in $a \to b \to c$ is W_1, in $a \to c$ work done is W_2 and in $a \to d \to c$ work done is W_3, then

(A) $W_2 > W_3 > W_1$ (B) $W_1 > W_2 > W_3$

(C) $W_2 > W_1 > W_3$ (D) $W_3 > W_2 > W_1$

Comprehension 22

Two moles of helium gas $\left(\gamma = \frac{5}{3}\right)$ is initially at temperature 27 °C and occupies a volume of 20 litre. The gas is first expanded at constant pressure until the volume is doubled. Then it undergoes an adiabatic change until the temperature returns to its initial value. Based on the above facts, answer the following questions.

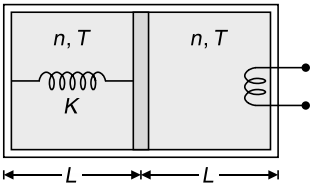
58. The final volume of the gas is
(A) 100 litre (B) 110 litre
(C) 113 litre (D) 120 litre

59. The final pressure of the gas is
(A) 3.4×10^6 Nm^{-2} (B) 4.4×10^6 Nm^{-2}
(C) 5.4×10^6 Nm^{-2} (D) 6.4×10^6 Nm^{-2}

60. The work done by the gas is
(A) 14259 J (B) 19452 J
(C) 12954 J (D) 12459 J

Comprehension 23

In the situation shown in figure the spring of spring constant K is at its natural length. In both the chambers of the non-conducting vessel separated by a non-conducting piston, n moles of monatomic gas is filled at temperature T. Heat is now supplied to the right chamber. It is observed that the temperature of the left chamber does not change and the piston is displaced by $\frac{L}{4}$. Based on the above facts, answer the following questions.



61. The change in temperature of the right chamber is
(A) $2T + \frac{5}{16}\frac{KL^2}{nR}$ (B) $\frac{T}{3} + \frac{KL^2}{16nR}$
(C) $\frac{2T}{3} + \frac{5KL^2}{16nR}$ (D) $\frac{2T}{3} + \frac{5KL^2}{nR}$

62. The change in the internal energy of the right chamber is
(A) $nRT + \frac{15}{32}KL^2$ (B) $nRT + \frac{KL^2}{32}$
(C) $2nRT + 15KL^2$ (D) $3nRT + \frac{15}{32}KL^2$

63. The heat transferred into the system is
(A) $2nRT + \frac{KL^2}{2}$ (B) $nRT + \frac{KL^2}{2}$
(C) $3nRT + \frac{KL^2}{32}$ (D) $nRT + \frac{KL^2}{32}$

Comprehension 24

In a thermodynamic process, the temperature T varies with volume V as $T = T_0 + \alpha V^3$, where T_0 and α are constants. Based on the above facts, answer the following questions.

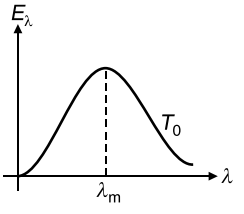
64. The volume at which the pressure is the minimum is
(A) $\sqrt{\frac{T_0}{2\alpha}}$ (B) $\left(\frac{T_0}{3\alpha}\right)^{\frac{1}{3}}$
(C) $\left(\frac{T_0}{2\alpha}\right)^{\frac{1}{3}}$ (D) $\left(\frac{\alpha}{2T_0}\right)^{\frac{2}{3}}$

65. Molar heat capacity of gas in the above process
(A) $C_V + \frac{R}{3} + \frac{RT_0}{3\alpha V^3}$ (B) $C_V + \frac{RT_0}{3\alpha V^3}$
(C) $C_V - \frac{R}{3} + \frac{RT_0}{3\alpha V^3}$ (D) None of these

66. The ratio of the minimum attainable pressure during process to $nRT_0^{\frac{2}{3}}\alpha^{\frac{1}{3}}$ is
(A) $2^{\frac{1}{3}}$ (B) $\left(2^{\frac{1}{3}} + 2^{\frac{2}{3}}\right)$
(C) $\left(2^{\frac{2}{3}} + 2^{\frac{4}{3}}\right)$ (D) $2^{\frac{2}{3}}$

Comprehension 25

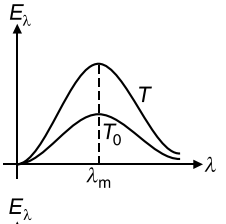
Figure shows a radiant energy spectrum graph for a black body at a temperature T_0. Based on the spectrum graph, answer the following questions.

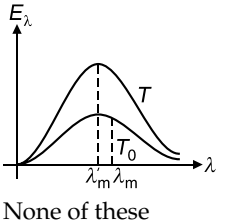


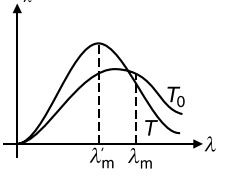
67. Select the correct statement.
(A) The radiant energy is equally distributed among all the possible wavelengths
(B) For a particular wavelength the spectral intensity is minimum
(C) The area under the curve is equal to the total intensity (energy per unit area per second) radiated by the body at that temperature
(D) None of these

68. If the temperature of the body is raised to a higher temperature T, then select the correct statement.
(A) The intensity of radiation for every wavelength decreases
(B) The maximum intensity occurs at a greater wavelength
(C) The area under the graph decreases
(D) The area under the graph is proportional to the fourth power of temperature

69. Identify the graph which correctly represents the spectral intensity versus wavelength graph at two temperatures T and $T_0 (< T)$.

(A)


(B)


(C)


(D) None of these

Comprehension 26

Two perfect gases at absolute temperatures T_1 and T_2 are mixed. There is no loss of energy. The masses of the molecules are m_1 and m_2. The number of molecules in the two gases are n_1 and n_2. Based on the above facts, answer the following questions.

70. What is the average kinetic energy of all the molecules of two gases before mixing?

(A) $\frac{1}{2}k(n_1T_1+n_2T_2)$ (B) $\frac{3}{2}k(n_1T_1+n_2T_2)$

(C) $\frac{5}{2}k(n_1T_1+n_2T_2)$ (D) $\frac{7}{2}k(n_1T_1+n_2T_2)$

71. If T is the temperature of the mixture, then the average kinetic energy of the two gases after mixing is

(A) $\frac{1}{2}k(n_1+n_2)T$ (B) $\frac{3}{2}k(n_1+n_2)T$

(C) $\frac{5}{2}k(n_1+n_2)T$ (D) $\frac{7}{2}k(n_1+n_2)T$

72. The temperature T of the mixture is

(A) $n_1T_1+n_2T_2$ (B) $\frac{n_1T_1}{n_1+n_2}$

(C) $\frac{n_2T_2}{n_1+n_2}$ (D) $\frac{n_1T_1+n_2T_2}{n_1+n_2}$

Comprehension 27

An ideal diatomic gas having $C_V=\frac{5R}{2}$ occupies a volume V_0 at a pressure P_0. The gas undergoes a process in which the pressure is directly proportional to the volume. At the end of the process, the rms speed of the gas molecules gets doubled from its initial value. Based on the above facts, answer the following questions.

73. The molar heat capacity of the gas in the given process is

(A) $3R$ (B) $3.5R$

(C) $4R$ (D) $2.5R$

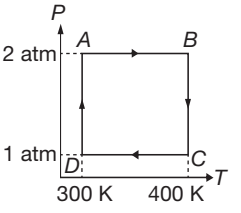
74. The heat supplied to the gas in the given process is

(A) $7P_0V_0$ (B) $8P_0V_0$

(C) $9P_0V_0$ (D) $10P_0V_0$

Comprehension 28

Two moles of helium gas undergo a cyclic process as shown in figure. Assuming the gas to be ideal, calculate the following quantities in this process. Based on the above facts, answer the following questions.



75. The net change in internal energy for the complete process is

(A) 4608 J (B) 6408 J

(C) zero (D) None of these

76. If dQ and dW indicate the total work done in the complete cyclic process, then

(A) $dQ=8064$ J, $dW=1152$ J (B) $dQ=dW=8064$ J

(C) $dQ=1152$ J, $dW=8064$ J (D) $dQ=dW=1152$ J

77. If dS denotes the total entropy in the complete cyclic process, then

(A) $dS=3.84\ \text{JK}^{-1}$ (B) $dS=2.88\ \text{JK}^{-1}$

(C) $dS=3.29\ \text{JK}^{-1}$ (D) $dS=$ zero

MATRIX MATCH/COLUMN MATCH TYPE QUESTIONS

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

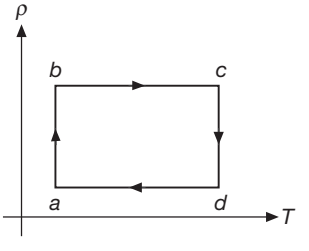
If the correct matches are A → p, s and t; B → q and r; C → p and q; and D → s and t; then the correct darkening of bubbles will look like the following:

	p	q	r	s	t
A	●	○	○	●	●
B	○	●	●	○	○
C	●	●	○	○	○
D	○	○	○	●	●

1. Match the processes in **COLUMN-I** to their property(ies) in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Cyclic process	(p) $\Delta U<0$
(B) Isobaric process	(q) $Q=W$
(C) Isochoric process	(r) $W=nR\Delta T$
(D) Adiabatic expansion	(s) $Q=\Delta U$
	(t) $\Delta U=0$

2. The density (ρ) vs temperature (T) graph for a cyclic process is shown in figure, match the quantities in **COLUMN-I** to their matches in **COLUMN-II**.



COLUMN-I	COLUMN-II
(A) Process $a \to b$	(p) P increasing
(B) Process $b \to c$	(q) P decreasing
(C) Process $c \to d$	(r) Isochoric process
(D) Process $d \to a$	(s) $\Delta U = 0$

3. Match the following.

COLUMN-I	COLUMN-II
(A) Work done by an ideal gas in a closed system	(p) Path dependent
(B) Internal energy of an ideal gas in a closed system	(q) State function
(C) Molar heat capacity of an ideal gas in a closed system	(r) May be constant (other than zero)
(D) Heat absorbed or rejected by an ideal gas in a closed system.	(s) May be zero

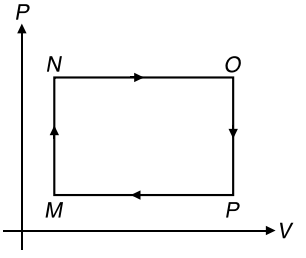
4. For one mole of a monatomic gas, match the following.

COLUMN-I	COLUMN-II
(A) Isothermal Bulk modulus	(p) $-\frac{RT}{V^2}$
(B) Adiabatic Bulk modulus	(q) $-\frac{5P}{3V}$
(C) Slope of P-V graph in an Isothermal process	(r) $\frac{RT}{V}$
(D) Slope of P-V graph in an Adiabatic process	(s) $\frac{5RT}{3V}$
	(t) $\frac{4RT}{V}$

5. Match the physical quantities in **COLUMN-I** to their dimensional formula in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Specific heat	(p) $MLT^{-3}K^{-1}$
(B) Coefficient of thermal conductivity	(q) $MT^{-3}K^{-4}$
(C) Boltzmann constant	(r) $L^2T^{-2}K^{-1}$
(D) Stefan's constant	(s) $ML^2T^{-2}K^{-1}$

6. With reference to the following P-V diagram, match the following quantities.



COLUMN-I	COLUMN-II
(A) Process MN	(p) $W = 0$
(B) Process NO	(q) $W < 0$
(C) Process OP	(r) $W > 0$
(D) Process PM	(s) $\Delta U > 0$
	(t) $\Delta U < 0$

7. Match the following.

COLUMN-I	COLUMN-II
(A) $\frac{R}{1-\gamma}(T_f - T_i)$	(p) Change in internal energy of an Isothermal process.
(B) ZERO	(q) Molar specific heat of an Adiabatic process.
(C) $R(T_f - T_i)$	(r) Internal energy of n mole of gas sample.
(D) $\frac{PV}{\gamma - 1}$	(s) Work done in an Isobaric process carried on one mole of gas.
	(t) Work done in an Isochoric process.
	(u) Work done in an Adiabatic process carried on one mole of gas.

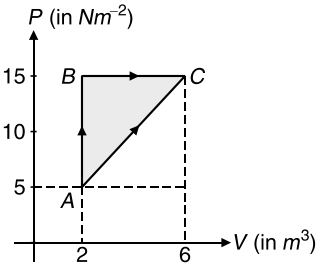
8. Match the following

COLUMN-I	COLUMN-II
(A) Temperature of a gas	(p) Internal energy increases
(B) Work done by the gas	(q) Intermolecular force decreases
(C) Thermal expansion	(r) Path function
(D) Mechanical compression.	(s) State function

9. An ideal monatomic gas is taken through one of the following reversible processes expressed by the equation in **COLUMN-I**. Match the molar heat capacity of the gas, expressed in multiples of R, in **COLUMN-II** with the appropriate process.

COLUMN-I	COLUMN-II
(A) $PV^{5/3}$ =constant	(p) $\frac{5R}{2}$
(B) TV^2 =constant	(q) $\frac{3R}{2}$
(C) V =constant	(r) R
(D) P =constant	(s) ZERO

10. An ideal gas is taken along the reversible processes as shown in the Figure.



COLUMN-I	COLUMN-II
(A) For process $B \to C$	(p) $\Delta Q > 0$
(B) For process $A \to B$	(q) $\Delta W > 0$
(C) For process $A \to B \to C$	(r) $\Delta U > 0$
(D) For process $A \to C$	(s) $\Delta W = 0$

11. Three liquids A, B and C having masses m, $2m$ and $3m$, have same gram specific heat and have temperatures 20 °C, 40 °C and 60 °C respectively. Temperature of the mixture when

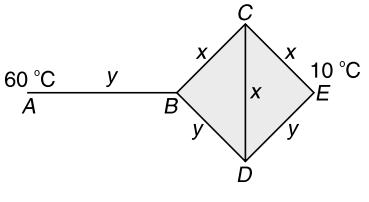
COLUMN-I	COLUMN-II
(A) A and B are mixed	(p) 35 °C
(B) A and C are mixed	(q) 52 °C
(C) B and C are mixed	(r) 50 °C
(D) A, B and C all three are mixed	(s) 33.3 °C
	(t) 46.67 °C

INTEGER/NUMERICAL ANSWER TYPE QUESTIONS

In this section, the answer to each question is a numerical value obtained after doing series of calculations based on the data given in the question(s).

1. An ice cube of mass 0.1 kg at 0 °C is placed in an isolated container which is at 227 °C. The specific heat c of the container varies with temperature T according to the empirical relation $c = A + BT$, where $A = 100$ calkg^{-1}K^{-1} and $B = 2 \times 10^{-2}$ calkg^{-1}K^{-2}. If the final temperature of the container is 27 °C, determine the mass of the container, in gram.
(Latent heat of fusion for water $= 8 \times 10^4$ calkg^{-1}, specific heat of water $= 10^3$ calkg^{-1}K^{-1}).

2. Three rods of material x and three rods of material y are connected as shown in figure. All the rods are of identical length and cross sectional area. If the end A is maintained at 60 °C and the junction E at 10 °C, calculate temperature of junctions B, C and D, in °C. The thermal conductivity of x is 0.92 calcm^{-1}s^{-1}(°C)$^{-1}$ and that of y is 0.46 calcm^{-1}s^{-1}(°C)$^{-1}$.



3. Calculate the work done on the gas or by the gas, in joule, when one mole of a perfect gas is compressed adiabatically. The initial pressure and volume of the gas are 10^5 Nm^{-2} and 6 litres respectively. The final volume of the gas is 2 litres molar specific heat of the gas at constant volume is $\frac{3R}{2}$.

4. An ideal monatomic gas is confined by a spring loaded massless piston of cross-section 8×10^{-3} m^2. Initially, the gas is at 300 K and occupies a volume of 2.4×10^{-3} m^3 and the spring is in its relaxed state. The gas is heated by an electric heater until the piston moves out slowly without friction by 0.1 m. Calculate

(a) the final temperature of the gas, in kelvin.

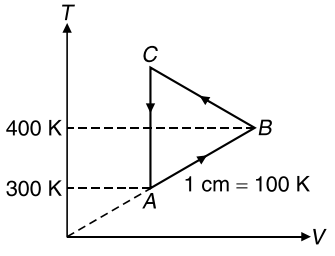
(b) the heat supplied by the heater, in joule.

The force constant of the spring is 8000 Nm^{-1}, atmospheric pressure is 1×10^5 Nm^{-2}. The cylinder and the piston are thermally insulated.

5. A spherical shell A emits like black body. It is kept in vacuum and isolated from all radiators. Its temperature in steady state is maintained 1000 K by some internal mechanism which gives it a constant power. Now this shell is enveloped by another shell B of almost same radius and emits like black body. Find the temperature of both shells, in kelvin, in the new steady state. Assume that internal mechanism of shell A produces the same power in the later case also. Given that, $(1.19)^4 = 2$.

6. In the construction of a flyover, steel girders of length 12 m are used to place one after another. How much gap, in mm, should be left between the two at their junction so that in summers when peak temperature is about 48 °C, there should not be any compression. Given that the construction is done in peak winters when temperature is 18 °C. Given that the coefficient of linear expansion of steel is 1.1×10^{-5} °C^{-1}.

7. Figure shows a cyclic process performed on one mole of an ideal gas. A total of 1000 J of heat is withdrawn from the gas in a complete cycle. Find the work done on/by the gas, in joule, during the process $B\to C$. Given $R = 8.3$ J mol^{-1} K^{-1}.



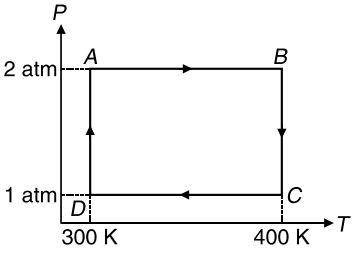
8. A pendulum gives correct time at 32 °C at a place where $x = 9.8$ ms^{-2}. The pendulum consists of a light steel rod connected to a heavy ball. If it is taken to a different place where $g = 9.788$ ms^{-2}. At what temperature, in °C, will it give correct time? The coefficient of linear expansion of steel is 12×10^{6} °C^{-1}.

9. Two moles of helium gas undergo a cyclic process as shown in figure. Assuming the gas to be ideal, calculate the following quantities in this process, in calorie.

(a) The net change in the heat energy.

(b) The net work done.

(c) The net change in internal energy.



Given that $R = 2$ calmol^{-1}K^{-1} and $\log_e(2) = 0.7$.

10. A steel rod is clamped between two rigid supports at 20 °C. If the temperature of rod increases to 50 °C, the strain developed in the rod due to this is $x\times10^{-5}$. Calculate x. Given that the coefficient of linear expansion of steel is 1.2×10^{-5} °C^{-1}.

11. A hollow sphere of glass whose external and internal radii are 11 cm and 9 cm respectively is completely filled with ice at 0 °C and placed in a bath of boiling water. How long, in seconds, will it take for the ice to melt completely? Given that density of ice is 0.9 gcm^{-3}, latent heat of fusion of ice is 80 calg^{-1} and thermal conductivity of glass is 0.002 calcm^{-1}s^{-1}(°C)$^{-1}$.

12. One mole of an ideal monatomic gas undergoes the process $P = \alpha\sqrt{T}$, where α is a positive constant.

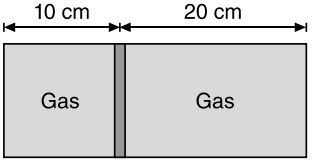
(a) The work done by the gas if its temperature increases by 50 K is $*R$.

(b) The molar specific heat of the gas is $*R$.

where $*$ in both (a) and (b) is not readable. Find the value of $*$ for both (a) and (b).

13. A bar with a crack at its centre buckles as a result of temperature rise of 32 °C. If the fixed distance L_0 is 4 m and the coefficient of linear expansion of the bar is 25×10^{-6}(°C)$^{-1}$ find the rise x of the centre, in mm, to the nearest two digit integer.

14. A horizontal cylindrical container of length 30 cm is partitioned by a tight-fitting separator. The separator is diathermic but conducts heat very slowly. Initially the separator is in the state shown in Figure.

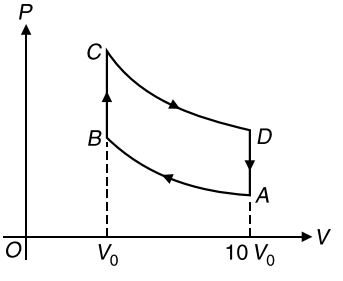


The temperature of left part of cylinder is 100 K and that on right part is 400 K. Initially the separator is in equilibrium. It is observed that when heat is conducted from right to left part, separator displaces to the right. Calculate the displacement of separator, in centimetre, after a long time when gases in the parts of cylinder are in thermal equilibrium.

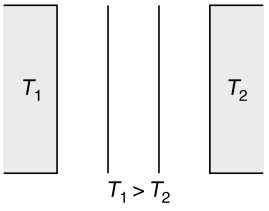
15. A cube of coefficient of linear expansion α_s is floating in a bath containing a liquid of coefficient of volume expansion γ_l. When the temperature is raised by ΔT, the depth upto which the cube is submerged in the liquid remains the same. Find the ratio $\dfrac{\gamma_l}{\alpha_s}$.

16. Calculate the temperature (in kelvin) of the helium gas so that the rms speed of helium atoms becomes equal to escape speed from the earth's surface.

17. One mole of oxygen undergoes a cyclic process in which volume of the gas change 10 times within the cycle, as shown in the figure. Process: AB and CD are adiabatic. Find the efficiency of the process, in percent. Take $(10)^{0.4} = 2.5$.



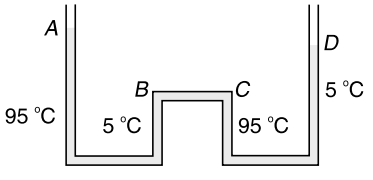
18. A black plane surface at a constant high temperature T_1 is parallel to another black plane surface at a constant lower temperature T_2. Between the plates is vacuum.



In order to reduce the heat flow due to radiation, a heat shield consisting of two thin black plates, thermally isolated from each other is placed between the warm and the cold surfaces and parallel to these. After sometime stationary conditions are obtained. The stationary heat flow reduces, due to the presence of the heat shield by a factor of $f = \frac{1}{x}$ where $1 \le x \le 11$. Find x. Neglect the effects due to finite size of the surfaces.

19. Two cylinders A and B fitted with pistons contain equal amounts of an ideal diatomic gas at 300 K. The piston of A is free to move while that of B is held fixed. The same amount of heat is given to the gas in each cylinder. If the rise in temperature of the gas in A is 30 K, then calculate the rise in temperature of the gas in B in kelvin.

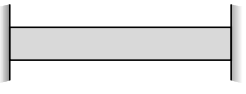
20. The apparatus shown in figure consists of four glass columns connected by horizontal sections. The height of two central columns B and C are 49 cm each. The two outer columns A and D are open to the atmosphere. A and C are maintained at a temperature of 95 °C while the columns B and D are maintained at 5 °C. The height of the liquid in A and D measured from the base line are 52.8 cm and 51 cm respectively. The coefficient of volume expansion of the liquid is $x \times 10^{-4}\,(°C)^{-1}$. Find x.



21. An ideal gas is enclosed in a cylinder with a movable piston on top. The piston has mass of 8000 g and an area of 5 cm^2 and is free to slide up and down, keeping the pressure of the gas constant. How much work is done, in joule, as the temperature of 0.2 mol gas is raised from 200 °C to 300 °C. Given $R = 8.3\ \text{Jmol}^{-1}\text{K}^{-1}$.

22. One end of a copper rod of length 1 m and area of cross section 4×10^{-4} m^2 is maintained at 100 °C. At the other end of the rod ice is kept at 0 °C. Neglecting the loss of heat from the surroundings find the mass of ice, in gram, melted in 1 hr. Given $K_{Cu} = 401\ \text{Wm}^{-1}\text{K}^{-1}$ and $L_f = 3.35 \times 10^5\ \text{Jkg}^{-1}$.

23. A steel rod of cross-sectional area 2×10^{-6} m^2 is fixed between two vertical walls as shown in Figure.

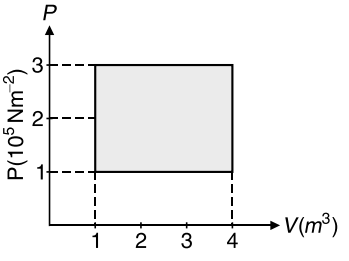


Initially at 20 °C, there is no force between the ends of the rod and the walls. Find the force which the rod will exert on walls at 100 °C. Given that the coefficient of linear expansion of steel is 1.2×10^{-5} $°C^{-1}$ and its Young's modulus is $2 \times 10^{11}\ \text{Nm}^{-2}$.

24. A solid body floats in a liquid at a temperature $t = 50$ °C being completely submerged in it. What percentage of the volume of the body is submerged in the liquid after it is cooled to $t_0 = 0$ °C, if the coefficient of cubic expansion for the solid is $\gamma_s = 0.3 \times 10^{-5}$ $°C^{-1}$ and of the liquid $\gamma_l = 8 \times 10^{-5}$ $°C^{-1}$.

25. An ideal gas having initial pressure P, volume V and temperature T is allowed to expand adiabatically until its volume becomes 5.66 V while its temperature falls to $\frac{T}{2}$. Calculate the degrees of freedom of the gas molecules. Given $\log_e(5.66) = 1.73$ $\log_e(2) = 0.70$

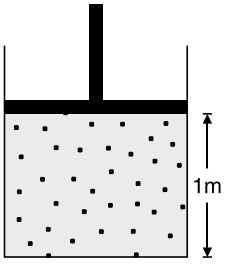
26. Calculate the work done by one kilomole of an ideal gas, in mega joule, in reversibly traversing the cycle shown in figure, ten times. Indicate the direction of traversal around the cycle if the net work is done by the gas.



27. The temperature of 100 g of water is to be raised from 24 °C to 90 °C by adding steam to it. Calculate the mass of the steam, in gram, required for this purpose.

28. One gram of water (volume 1 cc) becomes 1671 cc of steam when boiled at a pressure of one atm. The latent heat of vaporisation of water is 540 calg^{-1}. Compute the external work done and increase in internal energy, in calorie. $(1\ \text{atm} = 10^5\ \text{Nm}^{-2})$.

29. The piston cylinder arrangement shown contains a diatomic gas at temperature 300 K. The cross-sectional area of the cylinder is 1 m^2. Initially the height of the piston above the base of the cylinder is 1 m. The temperature is now raised to 400 K at constant pressure. Find the new height of the piston above the base of the cylinder. If the piston is now brought back to its original height without any heat loss, find the new equilibrium temperature of the gas, in kelvin. Given that $\left(\frac{4}{3}\right)^{0.4} = 1.12$.



30. A pitcher contains 10 kg of water at 20 °C. Water comes to its outer surface through its porous walls and gets evaporated. The latent heat required for evaporation is taken from the water inside the pitcher and hence the water inside the pitcher gets cooled down. If the rate of evaporation of water is 0.2 gs^{-1}, calculate the approximate time (in second) in which temperature of water inside the pitcher drops to 15 °C. Given that the specific heat of water is 4200 $Jkg^{-1}°C^{-1}$ and latent heat of vaporisation of water is 2.27×10^6 Jkg^{-1}.

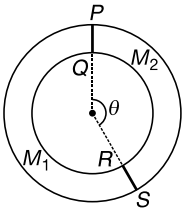
31. An ideal diatomic gas undergoes a process in which its internal energy relates to the volume as $U=\alpha\sqrt{V}$, where α is a constant.

(a) The molar specific heat of the gas is $\frac{*R}{2}$, where $*$ is not readable. Find $*$.

(b) Find the work performed by the gas, in joule, to increase its internal energy by 100 J.

32. A body cools in 10 minutes from 60 °C to 40 °C. What will be its temperature, in °C, after next 10 minutes? The temperature of the surroundings is 10 °C.

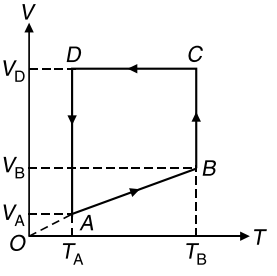
33. A ring shaped tube contains equal masses of two ideal gases of molar masses $M_1=32$ and $M_2=28$ respectively separated by one fixed partition PQ and another freely movable stopper RS as shown in Figure.



If temperature in the ring is same everywhere, then calculate the angle θ in degree.

34. A metal ball at 40 °C is dropped from a height of 6 km. The ball is heated due to the air resistance and it completely melts before reaching the ground. The molten substance falls slowly on the ground. Calculate the latent heat of fusion of metal, in $kJkg^{-1}$. Specific heat of lead is 125 $Jkg^{-1}(°C)^{-1}$, melting point of metal is 200 °C and $g=10$ ms^{-2}.

35. A monatomic ideal gas of two moles is taken through a cyclic process starting from A as shown in the figure. The volume ratios are $\frac{V_B}{V_A}=2$ and $\frac{V_D}{V_A}=4$. If the temperature T_A at A is 27 °C. Calculate



(a) the temperature of the gas at point B, in kelvin.

(b) heat absorbed or released by the gas in each process, in calorie.

(c) the total work done by the gas during the complete cycle, in calorie.

Given that $\log_e(2)=0.7$ and $R=2$ $calmol^{-1}K^{-1}$.

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1. **[Online September 2020]**

A gas mixture consists of 3 mole of oxygen and 5 mole of argon at temperature T. Assuming the gases to be ideal and the oxygen bond to be rigid, the total internal energy (in units of RT) of the mixture is

(A) 11 (B) 15

(C) 20 (D) 13

2. **[Online September 2020]**

An ideal gas in a closed container is slowly heated. As its temperature increases, which of the following statements are true?

(1) the mean free path of the molecules decreases.

(2) the mean collision time between the molecules decreases.

(3) the mean free path remains unchanged.

(4) the mean collision time remains unchanged

(A) (3) and (4) (B) (1) and (2)

(C) (1) and (4) (D) (2) and (3)

3. **[Online September 2020]**

When the temperature of a metal wire is increased from 0 °C to 10 °C, its length increases by 0.02%. The percentage change in its mass density will be closest to

(A) 0.008 (B) 0.06

(C) 0.8 (D) 2.3

4. **[Online September 2020]**

A bakelite beaker has volume capacity of 500 cc at 30 °C. When it is partially filled with V_m volume (at 30°) of mercury, it is found that the unfilled volume of the beaker remains constant as temperature is varied. If $\gamma_{(\text{beaker})}=6\times10^{-6}$ C^{-1} and $\gamma_{(\text{mercury})}=1.5\times10^{-4}$ $°C^{-1}$, where γ is the coefficient of volume expansion, then V_m (in cc) is close to_____.

5. **[Online September 2020]**

To raise the temperature of a certain mass of gas by 50 °C at a constant pressure, 160 calorie of heat is required. When the same mass of gas is cooled by 100 °C at constant volume, 240 calorie of heat is released. How many degrees of freedom does each molecule of this gas have (assume gas to be ideal)?

(A) 5 (B) 3

(C) 6 (D) 7

6. **[Online September 2020]**

A calorimeter of water equivalent 20 g contains 180 g of water at 25 °C. m gram of steam at 100 °C is mixed in it will the temperature of the mixture is 31 °C. The value of m is

close to (Latent heat of water is 540 $calg^{-1}$, specific heat of water 1 $calg^{-1}°C^{-1}$)

(A) 2.6 (B) 2

(C) 4 (D) 3.2

7. [Online September 2020]

Match the C_P/C_V ratio for ideal gases with different type of molecules.

Molecular Type	C_P/C_V
(A) Monoatomic	(p) 7/5
(B) Diatomic rigid molecules	(q) 9/7
(C) Diatomic non-rigid molecules	(r) 4/3
(D) Triatomic rigid molecules	(s) 5/3

(A) A → (s), B → (p), C → (q), D → (r)

(B) A → (s), B → (q), C → (p), D → (r)

(C) A → (r), B → (s), C → (q), D → (p)

(D) A → (q), B → (r), C → (p), D → (s)

8. [Online September 2020]

The specific heat of water is 4200 $Jkg^{-1}K^{-1}$ and the latent heat of ice is 3.4×10^5 Jkg^{-1}. 100 g of ice at 0 °C is placed in 200 g of water at 25 °C. The amount of ice that will melt as the temperature of water reaches 0 °C is close to (in gram)

(A) 61.7 (B) 63.8

(C) 69.3 (D) 64.6

9. [Online September 2020]

A closed vessel contains 0.1 mole of a monoatomic ideal gas at 200 K. If 0.05 mole of the same gas at 400 K is added to it, the final equilibrium temperature (in K) of the gas in the vessel will be close to_____.

10. [Online September 2020]

The change in the magnitude of the volume of an ideal gas when a small additional pressure ΔP is applied at a constant temperature, is the same as the change when the temperature is reduced by a small quantity ΔT at constant pressure. The initial temperature and pressure of the gas were $300K$ and 2 atm respectively. If $|\Delta T| = C|\Delta P|$ then value of C in K/atm is_____.

11. [Online September 2020]

A bullet of mass 5 g, travelling with a speed of 210 ms^{-1}, strikes a fixed wooden target. One half of its kinetic energy is converted into heat in the bullet while the other half is converted into heat in the wood. The rise of temperature of bullet if specific heat of its material is 0.030 $calg^{-1}°C^{-1}$ and 1 cal = 4.2×10^7 erg is close to

(A) 83.3 °C (B) 87.5 °C

(C) 119.2 °C (D) 38.4 °C

12. [Online September 2020]

Number of molecules in a volume of 4 cm^3 of a perfect monatomic gas at some temperature T and at a pressure of 2 cm of mercury is close to

(Given, mean kinetic energy of a molecule at T is 4×10^{-14} erg, $g = 980$ cms^{-2}, density of mercury is 13.6 gcm^{-3})

(A) 5.8×10^{18} (B) 5.8×10^{16}

(C) 4.0×10^{18} (D) 4.0×10^{16}

13. [Online September 2020]

Two different wires having lengths L_1 and L_2, and respective temperature coefficient of linear expansion α_1 and α_2, are joined end-to-end. Then the effective temperature coefficient of linear expansion is

(A) $\frac{4\alpha_1\alpha_2}{\alpha_1+\alpha_2}\frac{L_2L_1}{(L_2+L_1)^2}$ (B) $2\sqrt{\alpha_1\alpha_2}$

(C) $\frac{\alpha_1+\alpha_2}{2}$ (D) $\frac{\alpha_1+L_1+\alpha_2L_2}{L_1+L_2}$

14. [Online September 2020]

Nitrogen gas is at 300 °C temperature. The temperature (in K) at which the rms speed of a H_2 molecule would be equal to the rms speed of a nitrogen molecule, is_____. (Molar mass of N_2 gas 28 g)

15. [Online September 2020]

Molecules of an ideal gas are known to have three translational degrees of freedom and two rotational degrees of freedom. The gas is maintained at a temperature of T. The total internal energy, U of a mole of this gas, and the value of $\gamma\left(=\frac{C_P}{C_V}\right)$ given, respectively, by

(A) $U=\frac{5}{2}RT$ and $\gamma=\frac{6}{5}$ (B) $U=5RT$ and $\gamma=\frac{7}{5}$

(C) $U=5RT$ and $\gamma=\frac{6}{5}$ (D) $U=\frac{5}{2}RT$ and $=\frac{7}{5}$

16. [Online September 2020]

Initially a gas of diatomic molecules is contained in a cylinder of volume V_1 at a pressure P_1 and temperature 250 K. Assuming that 25% of the molecules get dissociated causing a change in number of moles. The pressure of the resulting gas at temperature 2000 K, when contained in a volume $2V_1$ is given by P_2. The ratio $\frac{P_2}{P_1}$ is

17. [Online September 2020]

In a dilute gas at pressure P and temperature T, the mean time between successive collisions of a molecule varies with T as

(A) $\sqrt{T}$ (B) $\frac{1}{T}$

(C) $\frac{1}{\sqrt{T}}$ (D) T

18. [Online September 2020]

A heat engine is involved with exchange of heat of 1915 J, –40 J, +125 J and Q J, during one cycle achieving an efficiency of 50.0%. The value of Q is

(A) 640 J (B) 400 J

(C) 980 J (D) 40 J

19. [Online September 2020]

A balloon filled with helium (32 °C and 1.7 atm.) bursts. Immediately afterwards the expansion of helium can be considered as

(A) irreversible isothermal (B) irreversible adiabatic
(C) reversible adiabatic (D) reversible isothermal

20. [Online September 2020]

If minimum possible work is done by a refrigerator in converting 100 g of water at 0 °C to ice, how much heat (in calories) is released to the surrounding at temperature 27 °C (Latent heat of ice is 80 $calg^{-1}$) to the nearest integer is_____.

21. [Online September 2020]

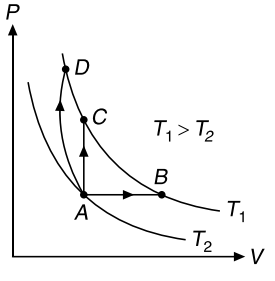
Match the thermodynamic processes taking place in a system with the correct conditions. In the table ΔQ is the heat supplied, ΔW is the work done and ΔU is change in internal energy of the system

Process	Condition
(A) Adiabatic	(p) $\Delta W = 0$
(B) Isothermal	(q) $\Delta Q = 0$
(C) Isochoric	(r) $\Delta U \neq 0, \Delta W \neq 0, \Delta Q \neq 0$
(D) Isobaric	(s) $\Delta U = 0$

(A) A → (q), B → (s), C → (p), D → (r)
(B) A → (q), B → (p), C → (s), D → (r)
(C) A → (p), B → (p), C → (q), D → (r)
(D) A → (p), B → (q), C → (s), D → (s)

22. [Online September 2020]

Three different processes that can occur in an ideal monatomic gas are shown in the P vs V diagram. The paths are labelled as $A \to B$, $A \to C$ and $A \to D$. The change in internal energies during these process are taken as E_{AB}, E_{AC}, E_{AD} and the work done as W_{AB}, W_{AC} and W_{AD}. The correct relation between these parameters are



(A) $E_{AB} = E_{AC} = E_{AD}, W_{AB} > 0, W_{AC} = 0, W_{AD} > 0$
(B) $E_{AB} < E_{AC} < E_{AD}, W_{AB} > 0, W_{AC} > W_{AD}$
(C) $E_{AB} = E_{AC} < E_{AD}, W_{AB} > 0, W_{AC} = 0, W_{AD} > 0$
(D) $E_{AB} > E_{AC} > E_{AD}, W_{AB} < W_{AC} < W_{AD}$

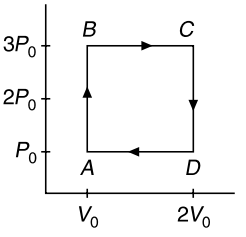
23. [Online September 2020]

In an adiabatic process, the density of a diatomic gas becomes 32 times its initial value. The final pressure of the gas is found to be n times the initial pressure. The value of n is

(A) 326 (B) $\frac{1}{32}$
(C) 32 (D) 128

24. [Online September 2020]

An engine operates by taking a monatomic ideal gas through the cycle shown in Figure.



The percentage efficiency of the engine is close to_____.

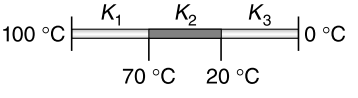
25. [Online September 2020]

A metallic sphere cools from 50 °C to 40 °C in 300 s. If atmospheric temperature around is 20 °C, then the sphere's temperature after the next 5 minutes will be close to

(A) 33 °C (B) 35 °C
(C) 31 °C (D) 28 °C

26. [Online September 2020]

Three rods of identical cross-section and lengths are made of three different materials of thermal conductivity K_1, K_2, and K_3, respectively. They are joined together at their ends to make a long rod (see figure). One end of the long rod is maintained at 100 °C and the other at 0 °C (see figure). If the joints of the rod are at 70 °C and 20 °C in steady state the there is no loss of energy from the surface of the rod, the correct relationship between K_1, K_2 and K_3 is



(A) $K_1 : K_3 = 2 : 3; K_2 : K_3 = 2 : 5$
(B) $K_1 < K_2 < K_3$
(C) $K_1 : K_2 = 5 : 2; K_1 : K_3 = 3 : 5$
(D) $K_1 > K_2 > K_3$

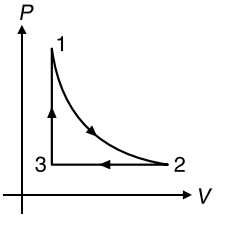
27. [Online January 2020]

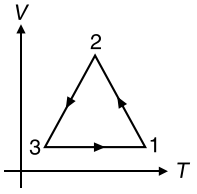
A Carnot engine having an efficiency of $\frac{1}{10}$ is being used as a refrigerator. If the work done on the refrigerator is 10 J, the amount of heat absorbed from the reservoir at lower temperature is

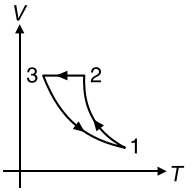
(A) 99 J (B) 100 J
(C) 90 J (D) 1 J

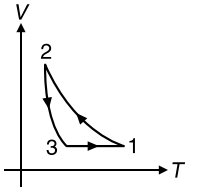
28. [Online January 2020]

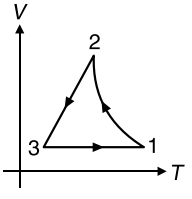
Which of the following is an equivalent cyclic process corresponding to the thermodynamic cyclic given in Figure? Where, $1 \to 2$ is adiabatic. (Graphs are schematic and are not to scale)



(A) 

(B) 

(C) 

(D) 

29. [Online January 2020]

Starling at temperature 300 K, one mole of an ideal diatomic gas $(\gamma = 1.4)$ is first compressed adiabatically from volume V_1 to $V_2 = \frac{V_1}{16}$. It is then allowed to expand isobarically to volume $2V_2$. If all the processes are the quasi-static then the final temperature of the gas (in °K) is (to the nearest integer) _____.

30. [Online January 2020]

An engine takes in 5 moles of air at 20 °C and 1 atm, and compresses it adiabatically to $\frac{1}{10^{\text{th}}}$ of the original volume. Assuming air to be a diatomic ideal gas made up of rigid molecules, the change in its internal energy during this process comes out to be X kJ. The value of X to the nearest integer is_____.

31. [Online January 2020]

Two moles of an ideal gas with $\frac{C_P}{C_V} = \frac{5}{3}$ are mixed with 3 moles of another ideal gas with $\frac{C_P}{C_V} = \frac{4}{3}$. The value of $\frac{C_P}{C_V}$ for the mixture is

(A) 1.50 (B) 1.42

(C) 1.45 (D) 1.47

32. [Online January 2020]

A non-isotropic solid metal cube has coefficients of linear expansion as 5×10^{-5} °C^{-1} along the x-axis and 5×10^{-6} °C^{-1} along the y and the z-axis. If the coefficient of volume expansion of the solid is $C \times 10^{-16}$ °C^{-1} then the value of C is_____.

33. [Online January 2020]

M gram of steam at 100 °C is mixed with 200 g of ice at its melting point in a thermally insulated container. If it produces liquid water at 40 °C (heat of vaporization of water is 540 calg^{-1} and heat of fusion of ice is 80 calg^{-1}), the value of M is_____.

34. [Online January 2020]

Consider a mixture of n moles of helium gas and $2n$ moles of oxygen gas (molecules taken to be rigid) as an ideal gas. Its $\frac{C_P}{C_V}$ value will be

(A) $\frac{67}{45}$ (B) $\frac{19}{13}$

(C) $\frac{23}{15}$ (D) $\frac{40}{27}$

35. [Online January 2020]

Three containers C_1, C_2 and C_3 have water at different temperatures. The table below shows the final temperature T when different amounts of water (given in liters) are taken from each containers and mixed (assume no loss of heat during the process)

C_1	C_2	C_3	T
1 lt	2 lt	–	60 °C
–	1 lt	2 lt	30 °C
2 lt	–	1 lt	60 °C
1 lt	1 lt	1 lt	θ

The value of θ (in °C to the nearest integer) is……

36. [Online January 2020]

Consider two ideal diatomic gases A and B at some temperature T. Molecules of the gas A are rigid, and have a mass m. Molecules of the gas B have an additional vibrational mode, and have a mass $\frac{m}{4}$. The ratio of the specific heats (C_V^A and C_V^B) of gas A and B, respectively is

(A) 7 : 9 (B) 5 : 7

(C) 3 : 5 (D) 5 : 9

37. [Online January 2020]

Two gases argon (atomic radius 0.07 nm, atomic weight 40) and xenon (atomic radius 0.1 nm, atomic weight 140) have the same number density and are at the same temperature. The ratio of their respective mean free times is closest to

(A) 3.67 (B) 4.67

(C) 1.83 (D) 2.3

38. [Online April 2019]

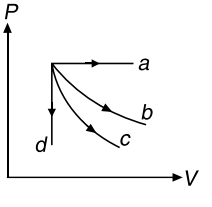
A thermally insulated vessel contains 150 g of water at 0 °C. Then the air from the vessel is pumped out adiabatically. A fraction of water turns into ice and the rest evaporates at 0 °C itself. The mass of evaporated water will be closest to (Latent heat of vaporization of water = 2.10×10^6 Jkg^{-1} and Latent heat of Fusion of water = 3.36×10^5 Jkg^{-1})

(A) 130 g (B) 150 g

(C) 20 g (D) 35 g

39. [Online April 2019]

The given diagram shows four processes i.e., isochoric, isobaric, isothermal and adiabatic. The correct assignment of the processes, in the same order is given by



(A) *adcb* (B) *adbc*

(C) *dabc* (D) *dacb*

40. [Online April 2019]

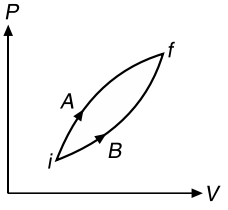
For a given gas at 1 atm pressure, rms speed of the molecules is 200 ms^{-1} at 127 °C. At 2 atm pressure and at 227 °C, the rms speed of the molecules will be

(A) $100\sqrt{5}$ ms^{-1} (B) 100 ms^{-1}

(C) $80\sqrt{5}$ ms^{-1} (D) 80 ms^{-1}

41. [Online April 2019]

Following figure shows two processes A and B for a gas. If ΔQ_A and ΔQ_B are the amount of heat absorbed by the system in two cases, and ΔU_A and ΔU_B are changes in internal energies, respectively, then



(A) $\Delta Q_A > \Delta Q_B$, $\Delta U_A = \Delta U_B$

(B) $\Delta Q_A = \Delta Q_B$; $\Delta U_A = \Delta U_B$

(C) $\Delta Q_A > \Delta Q_B$, $\Delta U_A > \Delta U_B$

(D) $\Delta Q_A < \Delta Q_B$, $\Delta U_A < \Delta U_B$

42. [Online April 2019]

An HCl molecule has rotational, translational and vibrational motions. If the rms velocity of HCl molecules in its gaseous phase is $\bar{v}$, m is its mass and k_B is Boltzmann constant, then its temperature will be

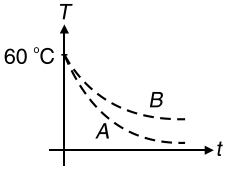
(A) $\frac{m\bar{v}^2}{5k_B}$ (B) $\frac{m\bar{v}^2}{6k_B}$

(C) $\frac{m\bar{v}^2}{7k_B}$ (D) $\frac{m\bar{v}^2}{3k_B}$

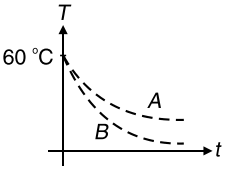
43. [Online April 2019]

Two identical beakers A and B contain equal volumes of two different liquids at 60 °C each and left to cool down. Liquid in A has density of 8×10^2 kgm^{-3} and specific heat of 2000 $Jkg^{-1}K^{-1}$ while liquid in B has density of 10^3 kgm^{-3} and specific heat of 4000 $Jkg^{-1}K^{-1}$. Which of the following best describes their temperature versus time graph schematically? (assume the emissivity of both the beakers to be the same)

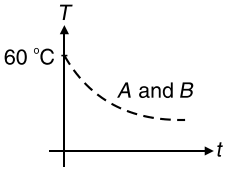
(A)



(B)



(C)



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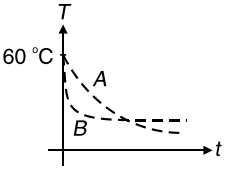


44. [Online April 2019]

The specific heats, C_P and C_V of a gas of diatomic molecules, A, are given (in units of $Jmol^{-1}K^{-1}$) by 29 and 22, respectively. Another gas of diatomic molecules, B, has the corresponding values 30 and 21. If they are treated as ideal gases, then

(A) A is rigid but B has a vibrational mode.

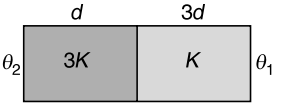
(B) A has a vibrational mode but B has none.

(C) Both A and B have a vibrational mode each.

(D) A has one vibrational mode and B has two.

45. [Online April 2019]

Two materials having coefficients of thermal conductivity $3K$ and K and thickness d and $3d$ respectively, are joined to form a slab as shown in the figure. The temperatures of the outer surfaces are θ_2 and θ_1 respectively, $(\theta_2 > \theta_1)$. The temperature at the interface is



(A) $\frac{\theta_1}{6}+\frac{5\theta_2}{6}$ (B) $\frac{\theta_1}{10}+\frac{9\theta_2}{10}$

(C) $\frac{\theta_1}{3}+\frac{2\theta_2}{3}$ (D) $\frac{\theta_2+\theta_1}{2}$

46. [Online April 2019]

n moles of an ideal gas with constant volume heat capacity C_V undergoes an isobaric expansion by certain volume. The ratio of the work done in the process, to the heat supplied is

(A) $\frac{4nR}{C_V+nR}$ (B) $\frac{nR}{C_V+nR}$

(C) $\frac{4nR}{C_V-nR}$ (D) $\frac{nR}{C_V-nR}$

47. [Online April 2019]

A cylinder with fixed capacity of 67.2 lt contains helium gas at STP. The amount of heat needed to raise the temperature of the gas by 20 °C is (Given that $R = 8.31$ $Jmol^{-1}K^{-1}$)

(A) 700 J (B) 350 J

(C) 374 J (D) 748 J

48. [Online April 2019]

One mole of an ideal gas passes through a process where pressure and volume obey the relation $P = P_0\left[1-\frac{1}{2}\left(\frac{V_0}{V}\right)^2\right]$. Here P_0 and V_0 are constants. Calculate the change in the temperature of the gas if its volume changes from V_0 to $2V_0$.

(A) $\frac{1}{4}\frac{P_0V_0}{R}$ (B) $\frac{5}{4}\frac{P_0V_0}{R}$

(C) $\frac{1}{2}\frac{P_0V_0}{R}$ (D) $\frac{3}{4}\frac{P_0V_0}{R}$

49. [Online April 2019]

When heat Q is supplied to a diatomic gas of rigid molecules, at constant volume its temperature increases by ΔT. The heat required to produce the same change in temperature, at a constant pressure is

(A) $\frac{3}{2}Q$ (B) $\frac{2}{3}Q$

(C) $\frac{7}{5}Q$ (D) $\frac{5}{3}Q$

50. [Online April 2019]

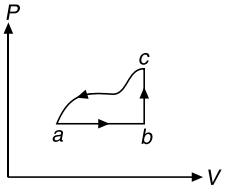
A sample of an ideal gas is taken through the cyclic process $abca$ as shown in the figure. The change in the internal energy of the gas along the path ca is –180 J. The gas absorbs 250 J of heat along the path ab and 60 J along the path bc. The work done by the gas along the path abc is

(A) 130 J

(B) 100 J

(C) 140 J

(D) 120 J



51. [Online April 2019]

When M_1 gram of ice at –10 °C (specific heat 0.5 $calg^{-1}°C^{-1}$) is added to M_2 gram of water at 50 °C, finally no ice is left and the water is at 0 °C. The value of latent heat of ice, in $calg^{-1}$ is

(A) $\frac{50M_2}{M_1}-5$ (B) $\frac{5M_1}{M_2}-50$

(C) $\frac{50M_2}{M_1}$ (D) $\frac{5M_2}{M_1}-5$

52. [Online April 2019]

Two moles of helium gas is mixed with three moles of hydrogen molecules (taken to be rigid). What is the molar specific heat of mixture at constant volume?

$\left(R=8.3\ \text{Jmol}^{-1}\text{K}^{-1}\right)$

(A) 19.7 $\text{Jmol}^{-1}\text{K}^{-1}$ (B) 21.6 $\text{Jmol}^{-1}\text{K}^{-1}$

(C) 15.7 $\text{Jmol}^{-1}\text{K}^{-1}$ (D) 17.4 $\text{Jmol}^{-1}\text{K}^{-1}$

53. [Online April 2019]

A Carnot engine has an efficiency of $\frac{1}{6}$. When the temperature of the sink is reduced by 62 °C, its efficiency is doubled. The temperatures of the source and the sink are, respectively,

(A) 99 °C, 37 °C (B) 37 °C, 99 °C

(C) 124 °C, 62 °C (D) 62 °C, 124 °C

54. [Online April 2019]

A diatomic gas with rigid molecules does 10 J of work when expanded at constant pressure. What would be the heat energy absorbed by the gas, in this process?

(A) 30 J (B) 35 J

(C) 25 J (D) 40 J

55. [Online April 2019]

The temperature, at which the root mean square velocity of hydrogen molecules equals their escape velocity from the earth, is closest to

[Boltzmann constant $k_B=1.38\times10^{-23}\ \text{JK}^{-1}$

Avogadro Number $N_A=6.02\times10^{26}\ \text{kg}^{-1}$

Radius of Earth 6.4×10^6 m

Gravitational acceleration on Earth $g=10\ \text{ms}^{-2}$]

(A) 10^4 K (B) 650 K

(C) 800 K (D) 3×10^5 K

56. [Online April 2019]

A 25×10^{-3} m^3 volume cylinder is filled with 1 mol of O_2 gas at room temperature 300 K. The molecular diameter of O_2 and its root mean square speed, are found to be 0.3 nm and 200 ms^{-1}, respectively. What is the average collision rate (per second) for an O_2 molecule?

(A) $\sim10^{12}$ (B) $\sim10^{10}$

(C) $\sim10^{11}$ (D) $\sim10^{13}$

57. [Online April 2019]

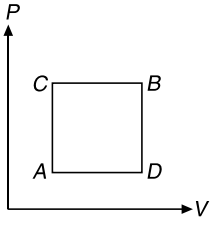
The number density of molecules of a gas depends on their distance r from the origin as $n(r)=n_0e^{-\alpha r^4}$. Then the total number of molecules is proportional to

(A) $n_0\alpha^{-\frac{3}{4}}$ (B) $\sqrt{n_0}\alpha^{\frac{1}{2}}$

(C) $n_0\alpha^{\frac{1}{4}}$ (D) $n_0\alpha^{-3}$

58. [Online January 2019]

A gas can be taken from A and B via two different processes ACB and ADB. When path ACB is used 60 J of heat flows into the system and 30 J of work is done by the system. If path ADB is used work done by the system is 10 J. The heat Flow into the system in path ADB is

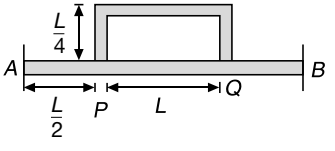


(A) 100 J (B) 80 J

(C) 20 J (D) 40 J

59. [Online January 2019]

Temperature difference of 120 °C is maintained between two ends of a uniform rod AB of length $2L$. Another bent rod PQ, of same cross-section as AB and length $\frac{3L}{2}$, is connected across AB (see figure). In steady state, temperature difference between P and Q will be close to



(A) 35 °C (B) 45 °C

(C) 60 °C (D) 75 °C

60. [Online January 2019]

A mixture of 2 moles of helium gas atomic mass 4 u and 1 mole of argon gas atomic mass 40 u is kept at 300 K in a container. The ratio of their rms speeds $\frac{v_{\text{rms}}(\text{helium})}{v_{\text{rms}}(\text{argon})}$, is close to

(A) 2.24 (B) 0.45

(C) 3.16 (D) 0.32

61. [Online January 2019]

A rod, length L at room temperature and uniform area of cross section A, is made of a metal having coefficient of linear expansion $\alpha\ °C^{-1}$. It is observed that an external compressive force F, is applied on each of its ends, prevents any change in the length of the rod, when its temperature rises by ΔT kelvin. The Young's modulus Y for this metal is

(A) $\frac{F}{A\alpha(\Delta T-273)}$ (B) $\frac{F}{A\alpha\Delta T}$

(C) $\frac{2F}{A\alpha\Delta T}$ (D) $\frac{F}{2A\alpha\Delta T}$

62. [Online January 2019]

Two Carnot engines A and B are operated in series. The first one, A, receives heat at $T_1 (= 600\ \text{K})$ and rejects to a reservoir at temperature T_2. The second engine B receives heat rejected by the first engine and, in turn, rejects to a heat reservoir at $T_3 (= 400\ \text{K})$. Calculate the temperature T_2 if the work outputs of the two engines are equal

(A) 300 K (B) 400 K

(C) 600 K (D) 500 K

63. [Online January 2019]

A 15 g mass of nitrogen gas is enclosed in a vessel at a temperature 27 °C. Amount of heat transferred to the gas, so that rms velocity of molecules is doubled, is about (Take $R = 8.3\ \text{JK}^{-1}\text{mol}^{-1}$)

(A) 3 kJ (B) 14 kJ

(C) 10 kJ (D) 0.9 kJ

64. [Online January 2019]

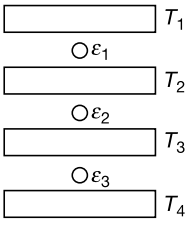
Three Carnot engines operate in series between a heat source at a temperature T_1 and a heat sink at temperature T_4 (see figure). There are two other reservoirs at temperature T_2 and T_3, as shown, with $T_1 > T_2 > T_3 > T_4$. The three engines are equally efficient if

(A) $T_2 = \left(T_1 T_4^2\right)^{\frac{1}{3}};\ T_3 = \left(T_1^2 T_4\right)^{\frac{1}{3}}$

(B) $T_2 = \left(T_1^3 T_4\right)^{\frac{1}{4}};\ T_3 = \left(T_1 T_4^3\right)^{\frac{1}{4}}$

(C) $T_2 = \left(T_1 T_4\right)^{\frac{1}{2}};\ T_3 = \left(T_1^2 T_4\right)^{\frac{1}{3}}$

(D) $T_2 = \left(T_1^2 T_4\right)^{\frac{1}{3}};\ T_3 = \left(T_1 T_4^2\right)^{\frac{1}{3}}$



65. [Online January 2019]

Half mole of an ideal monoatomic gas is heated at constant pressure of 1 atm from 20 °C to 90 °C. Work done by gas is close to

(Gas constant $R = 8.31\ \text{Jmol}^{-1}\text{K}^{-1}$)

(A) 291 J (B) 581 J

(C) 146 J (D) 73 J

66. [Online January 2019]

An unknown metal of mass 192 g heated to a temperature of 100 °C was immersed into a brass calorimeter of mass 128 g containing 240 g of water at a temperature of 8.4 °C. Calculate the specific heat of the unknown metal if water temperature stabilizes at 21.5 °C. (Specific heat of brass is $394\ \text{Jkg}^{-1}\text{K}^{-1}$)

(A) $916\ \text{Jkg}^{-1}\text{K}^{-1}$ (B) $1232\ \text{Jkg}^{-1}\text{K}^{-1}$

(C) $654\ \text{Jkg}^{-1}\text{K}^{-1}$ (D) $458\ \text{Jkg}^{-1}\text{K}^{-1}$

67. [Online January 2019]

Two kg of a monatomic gas is at a pressure of $4 \times 10^4\ \text{Nm}^{-2}$. The density of the gas is $8\ \text{kgm}^{-3}$. What is the order of energy of the gas due to its thermal motion?

(A) 10^3 J (B) 10^5 J

(C) 10^4 J (D) 10^6 J

68. [Online January 2019]

A heat source at $T = 10^3$ K is connected to another heat reservoir at $T = 10^2$ K by a copper slab which is 1 m thick. Given that the thermal conductivity of copper is $0.1\ \text{WK}^{-1}\text{m}^{-1}$, the energy flux through it in the steady state is

(A) $200\ \text{Wm}^{-2}$ (B) $65\ \text{Wm}^{-2}$

(C) $120\ \text{Wm}^{-2}$ (D) $90\ \text{Wm}^{-2}$

69. [Online January 2019]

A gas mixture consists 3 mole oxygen and 5 mole argon at temperature T. Considering only translational and rotational modes, the total internal energy of the system is

(A) $4RT$ (B) $12RT$

(C) $15RT$ (D) $20RT$

70. [Online January 2019]

A rigid diatomic ideal gas undergoes an adiabatic process at room temperature. The relation between temperature and volume for this process is $TV^x = \text{constant}$, then x is

(A) $\frac{2}{5}$ (B) $\frac{2}{3}$

(C) $\frac{5}{3}$ (D) $\frac{3}{5}$

71. [Online January 2019]

Ice at –20 °C is added to 50 g of water at 40 °C. When the temperature of the mixture reaches 0 °C, it is found that 20 g of ice is still unmelted. The amount off ice added to the water was close to

(Specific heat of water = $4.2\ \text{Jg}^{-1}\ ^\circ\text{C}^{-1}$

Specific heat of ice = $2.1\ \text{Jg}^{-1}\ ^\circ\text{C}^{-1}$

Heat of fusion of water at 0 °C = $334\ \text{Jg}^{-1}$)

(A) 100 g (B) 40 g

(C) 50 g (D) 60 g

72. [Online January 2019]

A metal ball of mass 0.1 kg is heated upto 500 °C and dropped into a vessel of heat capacity $800\ \text{JK}^{-1}$ and containing 0.5 kg water. The initial temperature of water and vessel is 30 °C. What is the approximate percentage increment in the temperature of the water? [Specific Heat Capacities of water and metal are, respectively, $4200\ \text{Jkg}^{-1}\text{K}^{-1}$ and $400\ \text{Jkg}^{-1}\text{K}^{-1}$]

(A) 25% (B) 20%

(C) 30% (D) 15%

73. [Online January 2019]

When 100 g of a liquid A at 100 °C is added to 50 g of a liquid B at temperature 75 °C, the temperature of the mixture becomes 90 °C. The temperature of the mixture, if 100 g of liquid A at 100 °C is added to 50 g of liquid B at 50 °C, will be

(A) 85 °C (B) 80 °C

(C) 70 °C (D) 60 °C

74. [Online January 2019]
Two rods A and B of identical dimensions are at temperature 30 °C. If A is heated upto 180 °C and B upto T °C, then the new lengths are the same. If the ratio of the coefficients of linear expansion of A and B is 4 : 3, then the value of T is

(A) 270 °C (B) 230 °C
(C) 250 °C (D) 200 °C

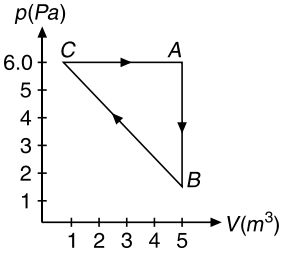
75. [Online January 2019]
In a process, temperature and volume of one mole of an ideal monoatomic gas are varied according to the relation $VT = K$, where K is a constant. In this process, the temperature of the gas is increased by ΔT. The amount of heat absorbed by gas is (R is gas constant)

(A) $\frac{3}{2}R\Delta T$ (B) $\frac{1}{2}R\Delta T$
(C) $\frac{2K}{3}\Delta T$ (D) $\frac{1}{2}KR\Delta T$

76. [Online January 2019]
A thermometer graduated according to a linear scale reads a value x_0 when in contact with boiling water, and $\frac{x_0}{3}$ when in contact with ice. What is the temperature of an object in °C, if this thermometer in the contact with the object reads $\frac{x_0}{2}$?

(A) 40 (B) 60
(C) 35 (D) 25

77. [Online January 2019]
For the given cyclic process CAB as shown for a gas, the work done is



(A) 30 J (B) 10 J
(C) 5 J (D) 1 J

78. [Online January 2019]
A cylinder of radius R is surrounded by a cylindrical shell of inner radius R and outer radius $2R$. The thermal conductivity of the material of the inner cylinder is K_1 and that of the outer cylinder is K_2. Assuming no loss of heat, the effective thermal conductivity of the system for heat flowing along the length of the cylinder is

(A) $\frac{K_1 + K_2}{2}$ (B) $\frac{K_1 + 3K_2}{4}$
(C) $K_1 + K_2$ (D) $\frac{2K_1 + 3K_2}{5}$

79. [Online January 2019]
An ideal gas occupies a volume of 2 m^3 at a pressure of 3×10^6 Pa. The energy of the gas is

(A) 10^8 J (B) 9×10^6 J
(C) 6×10^4 J (D) 3×10^2 J

80. [Online January 2019]
A vertical closed cylinder is separated into two parts by a frictionless piston of mass m and of negligible thickness. The piston is free to move along the length of the cylinder. The length of the cylinder above the piston is l_1, and that below the piston is l_2, such that $l_1 > l_2$. Each part of the cylinder contains n moles of an ideal gas at equal temperature T. If the piston is stationary, its mass, m will be given by: (R is universal gas constant and g is the acceleration due to gravity)

(A) $\frac{nRT}{g}\left(\frac{l_1 - l_2}{l_1 l_2}\right)$ (B) $\frac{nRT}{g}\left(\frac{1}{l_2} + \frac{1}{l_1}\right)$
(C) $\frac{RT}{g}\left(\frac{2l_1 + l_2}{l_1 l_2}\right)$ (D) $\frac{RT}{ng}\left(\frac{l_1 - 3l_2}{l_1 l_2}\right)$

81. [Online January 2019]
An ideal gas is enclosed in a cylinder at pressure of 2 atm and temperature, 300 K. The mean time between two successive collisions is 6×10^{-8} s. If the pressure is doubled and temperature is increased to 500 K, the mean time between two successive collisions will be close to

(A) 2×10^{-7} s (B) 3×10^{-6} s
(C) 0.5×10^{-8} s (D) 4×10^{-8} s

82. [2018]
The mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} hydrogen molecules strike, second, a fixed wall of area 2 cm^2 at an angle of 45° to the normal, and rebound elastically with a speed of 10^3 ms^{-1}, then the pressure on the wall is nearly

(A) 2.35×10^3 Nm^{-2} (B) 4.70×10^3 Nm^{-2}
(C) 2.35×10^2 Nm^{-2} (D) 4.70×10^2 Nm^{-2}

83. [Online 2018]
The value closest to the thermal velocity of a Helium atom at room temperature (300 K) in ms^{-1} is $\left[k_B = 1.4 \times 10^{-23}\ JK^{-1};\ m_{He} = 7 \times 10^{-27}\ kg\right]$

(A) 1.3×10^3 (B) 1.3×10^5
(C) 1.3×10^2 (D) 1.3×10^4

84. [Online 2018]
Two moles of helium are mixed with n moles of hydrogen. If $\frac{C_P}{C_V} = \frac{3}{2}$ for the mixture, then the value of n is

(A) 1 (B) 2
(C) 3 (D) $\frac{3}{2}$

85. [2018]
Two moles of an idea monatomic gas occupies a volume V at 27 °C. The gas expands adiabatically to a volume $2V$. Calculate (1) the final temperature of the gas and (2) change in its internal energy.

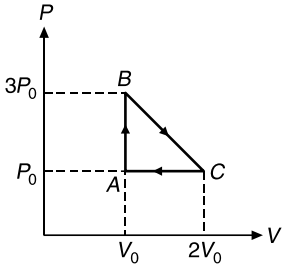
(A) (1) 198 K (2) 2.7 kJ
(B) (1) 195 K (2) –2.7 kJ
(C) (1) 189 K (2) –2.7 kJ
(D) (1) 195 K (2) 2.7 kJ

86. [Online 2018]
One mole of an ideal monoatomic gas is compressed isothermally in a rigid vessel to double its pressure at room temperature, 27 °C. The work done on the gas will be
(A) $300R$ (B) $300R\ln 2$
(C) $300R\ln 6$ (D) $300R\ln 7$

87. [Online 2018]
A Carnot engine works as a refrigerator between 250 K and 300 K. It receives 500 cal heat from the reservoir at the lower temperature. The amount of work done in each cycle to operate the refrigerator is
(A) 2520 J (B) 772 J
(C) 2100 J (D) 420 J

88. [Online 2018]
Two Carnot engines A and B are operated in series. Engine A receives heat from a reservoir at 60 K and rejects heat to a reservoir at temperature T. Engine B receives heat rejected by engine A and in turn rejects it to a reservoir at 100 K. If the efficiencies of the two engines A and B are represented by η_A and η_B, respectively, then what is the value of $\frac{\eta_B}{\eta_A}$?
(A) $\frac{7}{12}$ (B) $\frac{5}{12}$
(C) $\frac{12}{7}$ (D) $\frac{12}{5}$

89. [Online 2018]
One mole of an ideal monoatomic gas is taken along the path $ABCA$ as shown in the PV diagram. The maximum temperature attained by the gas along the path BC is given by
(A) $\frac{5}{8}\frac{P_0V_0}{R}$
(B) $\frac{25}{8}\frac{P_0V_0}{R}$
(C) $\frac{25}{4}\frac{P_0V_0}{R}$
(D) $\frac{25}{16}\frac{P_0V_0}{R}$



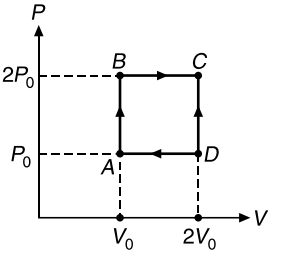
90. [2017]
C_p and C_v are specific heats at constant pressure and constant volume respectively. It is observed that
$C_p - C_v = a$ for hydrogen gas
$C_p - C_v = b$ for nitrogen gas
The correct relation between a and b is
(A) $a = \frac{1}{14}b$ (B) $a = b$
(C) $a = 14b$ (D) $a = 28b$

91. [Online 2017]
An ideal gas has molecules with 5 degrees of freedom. The ratio of specific heats at constant pressure (C_P) and at constant volume (C_V) is
(A) 6 (B) $\frac{7}{2}$
(C) $\frac{5}{2}$ (D) $\frac{7}{5}$

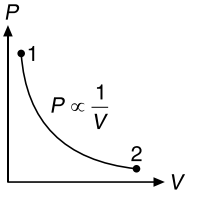
92. [Online 2017]
N moles of a diatomic gas in a cylinder are at a temperature T. Heat is supplied to the cylinder such that the temperature remains constant but n moles of the diatomic gas get converted into monoatomic gas. What is the change in the total kinetic energy of the gas?
(A) 0 (B) $\frac{5}{2}nRT$
(C) $\frac{1}{2}nRT$ (D) $\frac{3}{2}nRT$

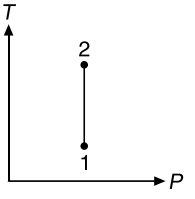
93. [2017]
The temperature of an open room of volume 30 m^3 increases from 17 °C to 27 °C due to the sunshine. The atmospheric pressure in the room remains 1×10^5 Pa. If n_i and n_f are the number of molecules in the room before and after heating, then $n_f - n_i$ will be
(A) -1.61×10^{23} (B) 1.38×10^{23}
(C) 2.5×10^{25} (D) -2.5×10^{25}

94. [2017]
An engine operates by taking n moles of an ideal gas through the cycle $ABCDA$ shown in figure. The thermal efficiency of the figure. The thermal efficiency of the engine is (Take $C_V = 1.5R$, where R is gas constant)
(A) 0.15 (B) 0.32
(C) 0.24 (D) 0.08



95. [Online 2017]
For the P-V diagram given for an ideal gas, out of the following which one correctly represents the T-P diagram?



(A)


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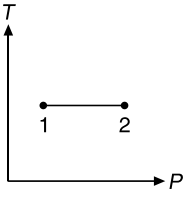

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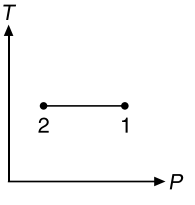
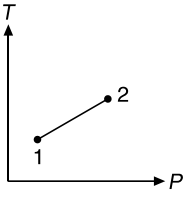
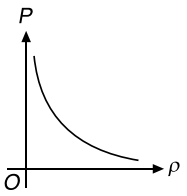
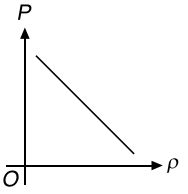
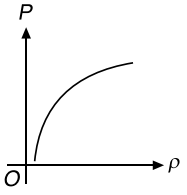


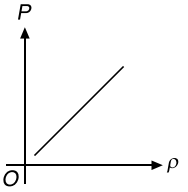
(D)


96. [Online 2016]
Which of the following shows the correct relationship between the pressure P and density ρ of an ideal gas at constant temperature?

(A) 

(B) 

(C) 

(D) 

97. [2016]

An ideal gas undergoes a quasi-static, reversible process in which its molar heat capacity C remains constant. If during this process the relation of pressure P and volume V is given by $PV^n = \text{constant}$, then n is given by (Here C_P and C_V are molar specific heat at constant pressure and constant volume, respectively)

(A) $n = \dfrac{C - C_V}{C - C_P}$ (B) $n = \dfrac{C_P}{C_V}$

(C) $n = \dfrac{C - C_P}{C - C_V}$ (D) $n = \dfrac{C_P - C}{C - C_V}$

98. [2016]

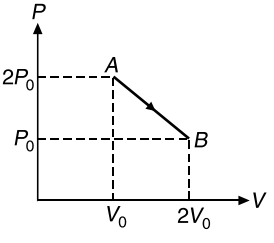
n moles of an ideal gas undergoes a process $A \to B$ as shown in the figure. The maximum temperature of the gas during the process will be

(A) $\dfrac{9P_0V_0}{nR}$

(B) $\dfrac{9P_0V_0}{4nR}$

(C) $\dfrac{3P_0V_0}{2nR}$

(D) $\dfrac{9P_0V_0}{2nR}$



99. [Online 2016]

The ratio of work done by an ideal monoatomic gas to the heat supplied to it in an isobaric process is

(A) $\dfrac{2}{5}$ (B) $\dfrac{3}{2}$

(C) $\dfrac{3}{5}$ (D) $\dfrac{2}{3}$

100. [Online 2016]

200 g water is heated from 40 °C to 60 °C. Ignoring the slight expansion of water, the change in its internal energy is close to (Given specific heat of water is 4184 $\text{Jkg}^{-1}\text{K}^{-1}$)

(A) 167.4 kJ (B) 8.4 kJ

(C) 4.2 kJ (D) 16.7 kJ

101. [Online 2016]

A Carnot freezer takes heat from water at 0 °C inside it and rejects it to the room at a temperature of 27 °C. The latent heat of ice is 336×10^3 Jkg^{-1}. If 5 kg of water at 0 °C is converted into ice at 0 °C by the freezer, then the energy consumed by the freezer is close to

(A) 1.51×10^5 J (B) 1.68×10^6 J

(C) 1.71×10^7 J (D) 1.67×10^5 J

102. [2015]

Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increases as V^q, where V is the volume of the gas. The value of q is $\left(\gamma = \dfrac{C_P}{C_V}\right)$

(A) $\dfrac{3\gamma + 5}{6}$ (B) $\dfrac{3\gamma - 5}{6}$

(C) $\dfrac{\gamma + 1}{2}$ (D) $\dfrac{\gamma - 1}{2}$

103. [Online 2015]

In an ideal gas at temperature T, the average force that a molecule applies on the walls of a closed container depends on T as T^q. A good estimate for q is

(A) 2 (B) 1

(C) $\dfrac{1}{2}$ (D) $\dfrac{1}{4}$

104. [Online 2015]

Using equipartition of energy, the specific heat (in $\text{Jkg}^{-1}\text{K}^{-1}$) of aluminium at room temperature can be estimated to be (atomic weight of aluminium = 27)

(A) 25 (B) 410

(C) 925 (D) 1850

105. [2015]

A solid body of constant heat capacity 1 J/°C is being heated by keeping it in contact with reservoirs in two ways

(i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.

(ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat.

In both the cases body is brought from initial temperature 127 °C to final temperature 527 °C. Entropy change of the body in the two cases respectively is

(A) $\ln 2,\ 4\ln 2$ (B) $\ln 2,\ \ln 2$

(C) $\ln 2,\ 2\ln 2$ (D) $2\ln 2,\ 8\ln 2$

106. [2015]

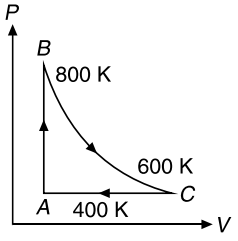
Consider a spherical shell of radius R at temperature T. The black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume $u = \dfrac{U}{V} \propto T^4$ and pressure $p = \dfrac{1}{3}\left(\dfrac{U}{V}\right)$. If the shell now undergoes an adiabatic expansion the relation between T and R is

(A) $T \propto e^{-R}$ (B) $T \propto e^{-3R}$

(C) $T \propto \dfrac{1}{R}$ (D) $T \propto \dfrac{1}{R^3}$

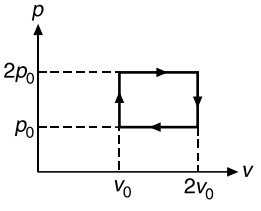
107. [2014]

One mole of diatomic ideal gas undergoes a cyclic process ABC as shown in figure. The process BC is adiabatic. The temperatures at A, B and C are 400 K, 800 K and 600 K respectively. Choose the correct statement



(A) The change in internal energy in whole cyclic process is $250R$.
(B) The change in internal energy in the process CA is $700R$.
(C) The change in internal energy in the process AB is $-350R$.
(D) The change in internal energy in the process BC is $-500R$.

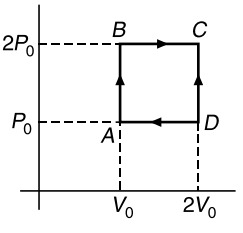
108. **[2013]**
The below pressure volume diagram represents the thermodynamic cycle of an engine, operating with an ideal monoatomic gas. The amount of heat, extracted from the source in a single cycle is



(A) p_0v_0
(B) $\left(\frac{13}{2}\right)p_0v_0$
(C) $\left(\frac{11}{2}\right)p_0v_0$
(D) $4p_0v_0$

109. **[2012]**
A Carnot engine, whose efficiency is 40%, takes in heat from a source maintained at a temperature of 500 K. It is desired to have an engine of efficiency 60%. Then, the intake temperature for the same exhaust (sink) temperature must be
(A) 1200 K
(B) 750 K
(C) 600 K
(D) efficiency of Carnot engine cannot be made larger than 50%

110. **[2012]**
Helium gas goes through a cycle $ABCDA$ (consisting of two isochoric and two isobaric lines) as shown in figure. Efficiency of this cycle is nearly (Assume the gas to be close to ideal gas)



(A) 9.1%
(B) 10.5%
(C) 12.5%
(D) 15.4%

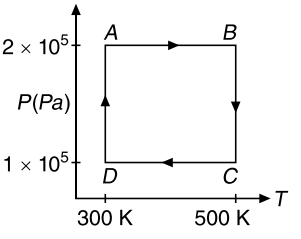
111. **[2011]**
A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats γ. It is moving with speed v and is suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases in kelvin by

(A) $\frac{(\gamma-1)}{2(\gamma+1)R}Mv^2$ (B) $\frac{(\gamma-1)}{2\gamma R}Mv^2$
(C) $\frac{\gamma Mv^2}{2R}$ (D) $\frac{(\gamma-1)}{2R}Mv^2$

112. **[2011]**
A Carnot engine operating between temperatures T_1 and T_2 has efficiency $\frac{1}{6}$. When T_2 is lowered by 62 K, its effi+ciency increases to $\frac{1}{3}$. Then T_1 and T_2 are, respectively
(A) 372 K and 310 K (B) 372 K and 330 K
(C) 330 K and 268 K (D) 310 K and 248 K

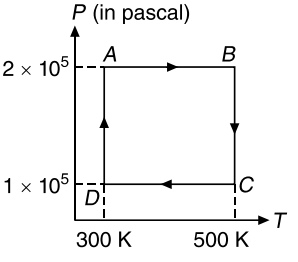
113. **[2011]**
100 g of water is heated from 30 °C to 50 °C. Ignoring the slight expansion of the water, the change in its internal energy is (specific heat of water is 4184 $Jkg^{-1}K^{-1}$)
(A) 4.2 kJ (B) 8.4 kJ
(C) 84 kJ (D) 2.1 kJ

114. **[2010]**
A diatomic ideal gas is used in a Carnot engine as the working substance. If during the adiabatic expansion part of the cycle the volume of the gas increases from V to $32V$, the efficiency of the engine is



(A) 0.25 (B) 0.5
(C) 0.75 (D) 0.99

Directions: Question number 115, 116 and 117 are based on the following paragraph. Two moles of helium gas are taken over the cycle $ABCDA$, as shown in the P-T diagram. Take $\ln 2 \approx 0.69$



115. **[2009]**
Assuming the gas to be ideal, the work done on the gas in taking if from A to B is
(A) $200R$ (B) $300R$
(C) $400R$ (D) $500R$

116. **[2009]**
The work done on the gas in taking it from D to A is
(A) $-414R$ (B) $+414R$
(C) $-R$ (D) $+690R$

117. **[2009]**
The net work done on the gas in the cycle $ABCDA$ is
(A) ZERO (B) $276R$
(C) $1076R$ (D) $1904R$

118. [2009]

One kg of a diatomic gas is at a pressure of 8×10^4 Nm^{-2}. The density of the gas is 4 kgm^{-3}. What is the energy of the gas due to its thermal motion?

(A) 3×10^4 J (B) 5×10^4 J

(C) 6×10^4 J (D) 7×10^4 J

ARCHIVE: JEE ADVANCED

Single Correct Choice Type Questions

(In this section each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct)

1. [JEE (Advanced) 2019]

A current carrying wire heats a metal rod. The wire provides a constant power (P) to the rod. The metal rod is enclosed in an insulated container. It is observed that the temperature (T) in the metal rod changes with time (t) as

$$T(t)=T_0\left(1+\beta t^{\frac{1}{4}}\right)$$

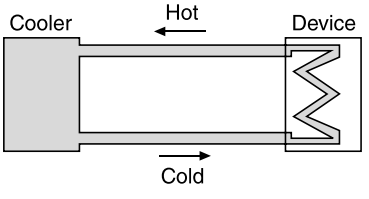
where β is a constant with appropriate dimension while T_0 is a constant with dimension of temperature. The heat capacity of the metal is

(A) $\dfrac{4P\left(T(t)-T_0\right)^2}{\beta^4T_0^3}$ (B) $\dfrac{4P\left(T(t)-T_0\right)^4}{\beta^4T_0^5}$

(C) $\dfrac{4P\left(T(t)-T_0\right)^3}{\beta^4T_0^4}$ (D) $\dfrac{4P\left(T(t)-T_0\right)}{\beta^4T_0^2}$

2. [JEE (Advanced) 2016]

A water cooler of storage capacity 120 litre can cool water at a constant rate of P watts. In a closed circulation system (as shown schematically in Figure), the water from the cooler is used to cool an external device that generates constantly 3 kW of heat (thermal load). The temperature of water fed into the device cannot exceed 30 °C and the entire stored 120 litres of water is initially cooled to 10 °C. The entire system is thermally insulated. The minimum value of P (in watts) for which the device can be operated for 3 hours is



(Specific heat of water is 4.2 kJkg^{-1}K^{-1} and the density of water is 1000 kgm^{-3})

(A) 1600 (B) 2067

(C) 2533 (D) 3933

3. [JEE (Advanced) 2016]

The ends Q and R of two thin wires, PQ and RS, are soldered (joined) together. Initially, each of the wire has a length of 1 m 10 °C. Now, the end P is maintained at 10 °C, while the end S is heated and maintained at 400 °C. The system is thermally insulted from its surroundings. If the thermal conductivity of wire PQ is twice that of the wire RS and the coefficient of linear thermal expansion of PQ is 1.2×10^{-5} K^{-1}, the change in length of the wire PQ is

(A) 0.78 mm (B) 0.90 mm

(C) 1.56 mm (D) 2.34 mm

4. [JEE (Advanced) 2016]

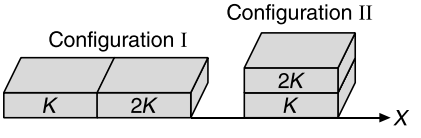
A gas is enclosed in a cylinder with a movable frictionless piston. Its initial thermodynamic state at pressure $p_i=10^5$ Pa and volume $V_1=10^{-3}$ m^3 changes to a final state at $p_f=\left(\dfrac{1}{32}\right)\times10^5$ Pa and $V_f=8\times10^{-3}$ m^3 in an adiabatic quasi-static process, such that $p^3V^5=$ constant. Consider another thermodynamic process that brings the system from the same initial state to the same final state in two steps: an (isovolumetric) process at volume V_f. The amount of heat supplied to the system in the two-step process is approximately

(A) 112 J (B) 294 J

(C) 588 J (D) 813 J

5. [JEE (Advanced) 2013]

Two rectangular blocks, having identical dimensions, can be arranged either in configuration I or in configuration II as shown in Figure.



One of the blocks has thermal conductivity K and the other $2K$. The temperature difference between the ends along the X-axis is the same in both the configurations. It takes 9 s to transport a certain amount of heat from the hot end to the cold end in the configuration I. The time to transport the same amount of heat in the configuration II is

(A) 2.0 s (B) 3.0 s

(C) 4.5 s (D) 6.0 s

6. [JEE (Advanced) 2013]

Two non-reactive monoatomic ideal gases have their atomic masses in the ratio 2 : 3. The ratio of their partial pressures, when enclosed in a vessel kept at a constant temperature, is 4 : 3. The ratio of their densities is

(A) 1 : 4 (B) 1 : 2

(C) 6 : 9 (D) 8 : 9

7. [IIT-JEE 2012]

Two moles of ideal helium gas are in a rubber balloon at 30 °C. The balloon is fully expandable and can be assumed to require no energy in its expansion. The temperature of the gas in the balloon is slowly changed to 35 °C. The amount of heat required in raising the temperature is nearly (take $R=8.31$ Jmol^{-1}K^{-1})

(A) 62 J (B) 104 J
(C) 124 J (D) 208 J

8. [IIT-JEE 2012]
Three very large plates of same area are kept parallel and close to each other. They are considered as ideal black surfaces and have very high thermal conductivity. The first and third plates are maintained at temperatures $2T$ and $3T$ respectively. The temperature of the middle (i.e., second) plate under steady state condition is

(A) $\left(\frac{65}{2}\right)^{\frac{1}{4}}T$ (B) $\left(\frac{97}{4}\right)^{\frac{1}{4}}T$
(C) $\left(\frac{97}{2}\right)^{\frac{1}{4}}T$ (D) $(97)^{\frac{1}{4}}T$

9. [IIT-JEE 2012]
A mixture of 2 moles of helium gas (atomic mass = 4 amu) and 1 mole of argon gas (atomic mass = 40 amu) is kept at 300 K in a container. The ratio of the rms speeds $\left(\frac{v_{\text{rms}}(\text{helium})}{v_{\text{rms}}(\text{argon})}\right)$ is

(A) 0.32 (B) 0.45
(C) 2.24 (D) 3.16

10. [IIT-JEE 2011]
5.6 litre of helium gas at STP is adiabatically compressed to 0.7 litre. Taking the initial temperature to be T_1, the work done in the process is

(A) $\frac{9}{8}RT_1$ (B) $\frac{3}{2}RT_1$
(C) $\frac{15}{8}RT_1$ (D) $\frac{9}{2}RT_1$

11. [IIT-JEE 2010]
A substance of mass M kg requires an input power P watt to remain in the molten state at its melting point. When the power source is turned off, the sample completely solidifies in time t second. The specific latent heat of fusion of the substance is

(A) $\frac{2Pt}{M}$ (B) $\frac{Pt}{2M}$
(C) $\frac{Pt}{M}$ (D) $\frac{PM}{t}$

12. [IIT-JEE 2010]
A real gas behaves like an ideal gas if its
(A) pressure and temperature are both high
(B) pressure and temperature are both low
(C) pressure is high and temperature is low
(D) pressure is low and temperature is high

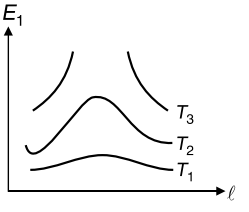
13. [IIT-JEE 2009]
A piece of glass is heated to a high temperature and then allowed to cool. If it cracks, a possible reason for this is the following property of the glass
(A) low thermal conductivity
(B) high thermal conductivity
(C) high specific heat
(D) low specific heat

14. [IIT-JEE 2008]
An ideal gas is expanding such that PT^2 =constant. The coefficient of volume expansion of the gas is

(A) $\frac{1}{T}$ (B) $\frac{2}{T}$
(C) $\frac{3}{T}$ (D) $\frac{4}{T}$

15. [IIT-JEE 2005]
Calorie is defined as the amount of heat required to raise temperature of 1 g of water by 1 °C and it is defined under which of the following conditions?
(A) From 14.5 °C to 15.5 °C at 760 mm of Hg
(B) From 98.5 °C to 99.5 °C at 760 mm of Hg
(C) From 13.5 °C to 14.5 °C at 76 mm of Hg
(D) From 3.5 °C to 4.5 °C at 76 mm of Hg

16. [IIT-JEE 2005]
Variation of radiant energy emitted by sun, filament of tungsten lamp and welding arc as a function of its wavelength is shown in figure. Which of the following option is the correct match?



(A) Sun-T_1, tungsten filament-T_2, welding arc-T_3
(B) Sun-T_2, tungsten filament-T_1, welding arc-T_3
(C) Sun-T_3, tungsten filament-T_2, welding arc-T_1
(D) Sun-T_1 tungsten filament-T_3, welding arc-T_2

17. [IIT-JEE 2005]
In which of the following process, convection does not take place primarily?
(A) Sea and land breeze
(B) Boiling of water
(C) Warming of glass of bulb due to filament
(D) Heating air around a furnace

18. [IIT-JEE 2005]
Water of volume 2 litre in a container is heated with a coil of 1 kW at 27 °C. The lid of the container is open and energy dissipates at rate of 160 Js^{-1}. In how much time temperature will rise from 27 °C to 77 °C?
[Given specific heat of water is 4.2 $kJkg^{-1}$]
(A) 8 min 20 s (B) 6 min 2 s
(C) 7 min (D) 14 min

19. [IIT-JEE 2004]
Three discs, A, B and C having radii 2 m, 4 m and 6 m respectively are coated with carbon black on their outer surfaces. The wavelengths corresponding to maximum intensity are 300 nm, 400 nm and 500 nm, respectively. The power radiated by them are Q_A, Q_B and Q_C respectively
(A) Q_A is maximum (B) Q_B is maximum
(C) Q_C is maximum (D) $Q_A = Q_B = Q_C$

20. [IIT-JEE 2004]

Two identical conducting rods are first connected independently to two vessels, one containing water at 100 °C and the other containing ice at 0 °C. In the second case, the rods are joined end to end and connected to the same vessels. Let q_1 and q_2 gs^{-1} be the rate of melting of ice in the two cases respectively. The ratio $\frac{q_1}{q_2}$ is

(A) $\frac{1}{2}$ (B) $\frac{2}{1}$

(C) $\frac{4}{1}$ (D) $\frac{1}{4}$

21. [IIT-JEE 2004]

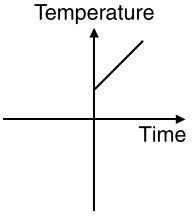
An ideal gas expands isothermally from a volume V_1 and V_2 and then compressed to original volume V_1 adiabatically. Initial pressure is P_1 and final pressure is P_3. The total work done is W. Then

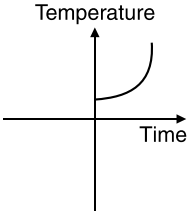
(A) $P_3 > P_1, W > 0$ (B) $P_3 < P_1, W < 0$

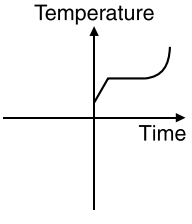
(C) $P_3 > P_1, W < 0$ (D) $P_3 = P_1, W = 0$

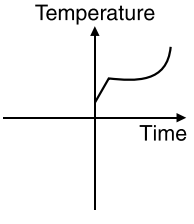
22. [IIT-JEE 2004]

Liquid oxygen at 50 K is heated to 300 K at constant pressure of 1 atm. The rate of heating is constant. Which of the following graphs represents the variation of temperature with time?

(A) 

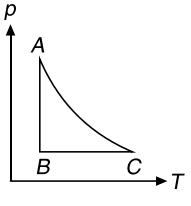
(B) 

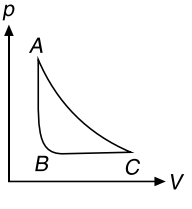
(C) 

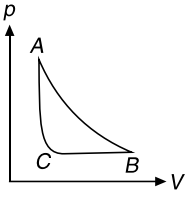
(D) 

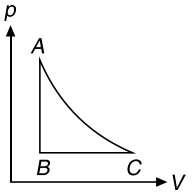
23. [IIT-JEE 2003]

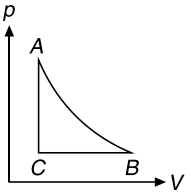
The p-T diagram for an ideal gas is shown in Figure, where AC is an adiabatic process, find the corresponding p-V diagram



(A) 

(B) 

(C) 

(D) 

24. [IIT-JEE 2003]

2 kg of ice at –20 °C is mixed with 5 kg of water at 20 °C in an insulating vessel having a negligible heat capacity. Calculate the final mass of water remaining in the container. It is given that the specific heats of water and ice are 1 $kcalkg^{-1}$ $°C^{-1}$ and 0.5 $kcalkg^{-1}$ $°C^{-1}$ while the latent heat of fusion of ice is 80 $kcalkg^{-1}$.

(A) 7 kg (B) 6 kg

(C) 4 kg (D) 2 kg

25. [IIT-JEE 2003]

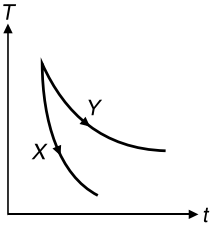
Two rods, one of aluminium and the other made of steel, having initial length l_1 and l_2 are connected together to form a single rod of length $l_1 + l_2$. The coefficients of linear expansion for aluminium and steel are α_a and α_s respectively. If the length of each rod increases by the same amount when their temperature are raised by t °C, then find the ratio $\frac{l_1}{l_1 + l_2}$

(A) $\frac{\alpha_s}{\alpha_a}$ (B) $\frac{\alpha_a}{\alpha_s}$

(C) $\frac{\alpha_s}{(\alpha_a + \alpha_s)}$ (D) $\frac{\alpha_a}{(\alpha_a + \alpha_s)}$

26. [IIT-JEE 2003]

The graph, shown in the diagram, represents the variation of temperature (T) of the bodies, x and y having same surface area, with time (t) due to the emission of radiation. Find the correct relation between the emissivity and absorptivity power of the two bodies



(A) $E_x > E_y$ and $a_x < a_y$

(B) $E_x < E_y$ and $a_x > a_y$

(C) $E_x > E_y$ and $a_x > a_y$

(D) $E_x < E_y$ and $a_x < a_y$

27. [IIT-JEE 2002]

An ideal black-body at room temperature is thrown into a furnace. It is observed that

(A) initially it is the darkest body and at later times the brightest

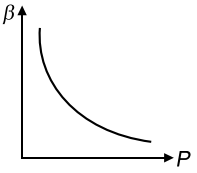
(B) it is the darkest body at all times

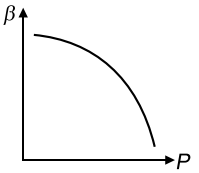
(C) it cannot be distinguished at all times

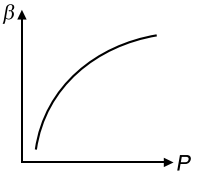
(D) initially it is the darkest body and at later times it cannot be distinguished

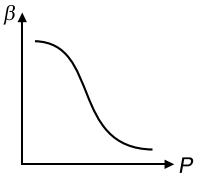
28. [IIT-JEE 2002]

Which of the following graphs correctly represent the variation of $\beta = -\frac{dV/dP}{V}$ with P for an ideal gas at constant temperature?

(A) 

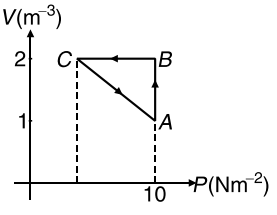
(B) 

(C) 

(D) 

29. [IIT-JEE 2002]

An ideal gas is taken through the cycle $A \to B \to C \to A$, as shown in the figure. If the net heat supplied to the gas in the cycle is 5 J, the work done by the gas in the process is



(A) –5 J

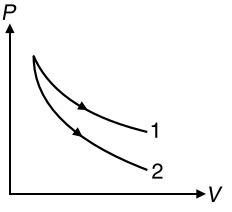
(B) –10 J

(C) –15 J

(D) –20 J

30. [IIT-JEE 2001]

P-V plots for two gases during adiabatic processes are shown in the figure. Plots 1 and 2 should correspond respectively to



(A) He and O_2

(B) O_2 and He

(C) He and Ar

(D) O_2 and N_2

31. [IIT-JEE 2001]

In a given process of an ideal gas, $dW = 0$ and $dQ < 0$. Then for the gas

(A) the temperature will decrease

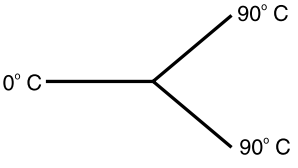
(B) the volume will increase

(C) the pressure will remain constant

(D) the temperature will increase

32. [IIT-JEE 2001]

Three rods made of the same material and having the same cross-section have been joined as shown in the figure. Each rod is of the same length. The left and right ends are kept at 0 °C and 90 °C respectively. The temperature of junction of the three rods will be



(A) 45 °C

(B) 60 °C

(C) 30 °C

(D) 20 °C

33. [IIT-JEE 2000]

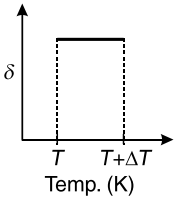
Two monatomic ideal gases 1 and 2 of molecular masses m_1 and m_2 respectively are enclosed in separate containers kept at same temperature. The ratio of the speed of sound in gas 1 to that in gas 2 is given by

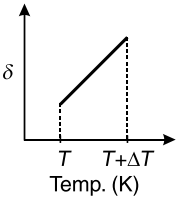
(A) $\sqrt{\frac{m_1}{m_2}}$ (B) $\sqrt{\frac{m_2}{m_1}}$

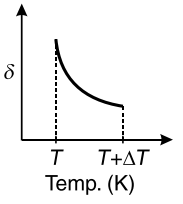
(C) $\frac{m_1}{m_2}$ (D) $\frac{m_2}{m_1}$

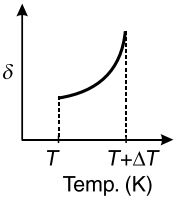
34. [IIT-JEE 2000]

An ideal gas is initially at a temperature T and volume V. Its volume is increased by ΔV due to an increase in temperature ΔT, pressure remaining constant. The quantity $\delta = \frac{\Delta V}{V\Delta T}$ varies with temperature as

(A) 

(B) 

(C) 

(D) 

35. [IIT-JEE 2000]

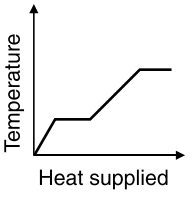
Starting with the same initial conditions, an ideal gas expands from volume V_1 to V_2 in three different ways. The work done by the gas is W_1 if the process is purely isothermal, W_2 if purely isobaric and W_3 if purely adiabatic. Then

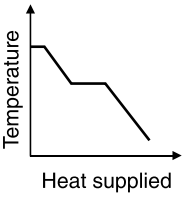
(A) $W_2 > W_1 > W_3$ (B) $W_2 > W_3 > W_1$

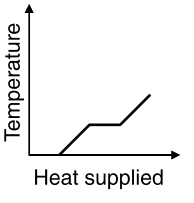
(C) $W_1 > W_2 > W_3$ (D) $W_1 > W_3 > W_2$

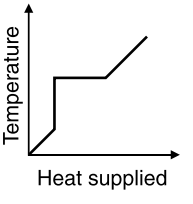
36. [IIT-JEE 2000]

A block of ice at –10 °C is slowly heated and converted to steam at100 °C. Which of the following curves represents the phenomenon qualitatively?

(A) 

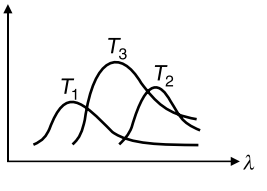
(B) 

(C) 

(D) 

37. [IIT-JEE 2000]

The plots of intensity verses wavelength for three black bodies at temperature T_1, T_2 and T_3 respectively are as shown. Their temperatures are such that



(A) $T_1 > T_2 > T_3$

(B) $T_1 > T_3 > T_2$

(C) $T_2 > T_3 > T_1$

(D) $T_3 > T_2 > T_1$

38. [IIT-JEE 2000]
A monatomic ideal gas, initially at temperature T_1, is enclosed in a cylinder fitted with a frictionless piston. The gas is allowed to expand to a temperature T_2 by releasing the piston suddenly. If L_1 and L_2 be the lengths of the gas column before and after the expansion respectively, then $\frac{T_1}{T_2}$ is given by

(A) $\left(\frac{L_1}{L_2}\right)^{\frac{2}{3}}$ (B) $\frac{L_1}{L_2}$

(C) $\frac{L_2}{L_1}$ (D) $\left(\frac{L_2}{L_1}\right)^{\frac{2}{3}}$

39. [IIT-JEE 1999]
A gas mixture consists of 2 mole of oxygen and 4 mole of argon at temperature T. Neglecting all vibrational modes, the total internal energy of the system is

(A) $4RT$ (B) $15RT$

(C) $9RT$ (D) $11RT$

40. [IIT-JEE 1999]
The ratio of the speed of sound in nitrogen gas to that in helium gas, at 300 K is

(A) $\sqrt{\frac{2}{7}}$ (B) $\sqrt{\frac{1}{7}}$

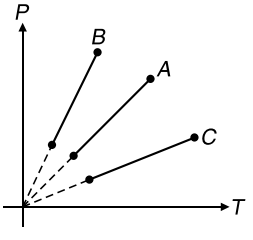
(C) $\frac{\sqrt{3}}{5}$ (D) $\frac{\sqrt{6}}{5}$

41. [IIT-JEE 1998]
A black body is at a temperature of 2880 K. The energy of radiation emitted by this object with wavelength between 499 nm and 500 nm is U_1, between 999 nm and 1000 nm is U_2 and between 1499 nm and 1500 nm is U_3. The Wein constant, $b = 2.88 \times 10^6$ nm-K. Then

(A) $U_1 = 0$ (B) $U_3 = 0$

(C) $U_1 > U_2$ (D) $U_2 > U_1$

42. [IIT-JEE 1998]
Pressure versus temperature graph of an ideal gas at constant volume V of an ideal gas is shown by the straight line A. Now mass of the gas is doubled and the volume is halved, then the corresponding pressure versus temperature graph will be shown by the line



(A) A (B) B

(C) C (D) None of these

43. [IIT-JEE 1998]
Two identical containers A and B have frictionless pistons. They contain the same volume of an ideal gas at the same temperature. The mass of the gas in A is m_A and that in B is m_B. The gas in each cylinder is now allowed to expand isothermally to double the initial volume. The change in the pressure in A and B respectively is Δp and $1.5\,\Delta p$. Then

(A) $4m_A = 9m_B$ (B) $2m_A = 3m_B$

(C) $3m_A = 2m_B$ (D) $9m_A = 4m_B$

44. [IIT-JEE 1998]
A vessel contains a mixture of one mole of oxygen and two moles of nitrogen at 300 K. The ratio of the average rotational kinetic energy per oxygen molecule to per nitrogen molecule is

(A) 1 : 1

(B) 1 : 2

(C) 2 : 1

(D) depends on the moment of inertia of the two molecules

45. [IIT-JEE 1997]
A spherical black body with a radius of 12 cm radiates 450 W power at 500 K. If the radius were halved and the temperature doubled, the power radiated in watt would be

(A) 225 (B) 450

(C) 900 (D) 1800

46. [IIT-JEE 1997]
A vessel contains 1 mole of O_2 gas (relative molar mass 32) at a temperature T. The pressure of the gas is P. An identical vessel containing 1 mole of He gas (relative molar mass 4) at a temperature $2T$ has a pressure of

(A) $\frac{P}{8}$ (B) P

(C) $2P$ (D) $8P$

47. [IIT-JEE 1997]
The average translational kinetic energy of O_2 (molar mass 32) molecules at a particular temperature is 0.048 eV. The translational kinetic energy of N_2 (molar mass 28) molecules in eV at the same temperature is

(A) 0.0015 (B) 0.003

(C) 0.048 (D) 0.768

48. [IIT-JEE 1997]
The intensity of radiation emitted by the sun has its maximum value at a wavelength of 510 nm and that emitted by the north star has the maximum value at 350 nm. If these stars behave like blackbodies, then the ratio of the surface temperature of the sun and the north star is :

(A) 1.46 (B) 0.69

(C) 1.21 (D) 0.83

49. [IIT-JEE 1997]
The average translational energy and the rms speed of molecules in a sample of oxygen gas at 300 K are 6.21×10^{-21} J and 484 ms^{-1} respectively. The corresponding values at 600 K are nearly (assuming ideal gas behaviour)

(A) 12.42×10^{-21} J, 968 ms^{-1} (B) 8.78×10^{-21} J, 684 ms^{-1}

(C) 6.21×10^{-21} J, 968 ms^{-1} (D) 12.42×10^{-21} J, 684 ms^{-1}

50. [IIT-JEE 1996]
The temperature of an ideal gas is increased from 120 K to 480 K. If at 120 K the root mean square velocity of the gas molecules is v, at 480 K it becomes

(A) $4v$ (B) $2v$

(C) $\frac{v}{2}$ (D) $\frac{v}{4}$

51. [IIT-JEE 1995]

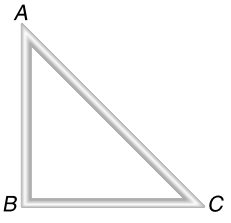
Two metallic spheres S_1 and S_2 are made of the same material and have got identical surface finish. The mass of S_1 is thrice that of S_2. Both the spheres are heated to the same high temperature and placed in the same room having lower temperature but are thermally insulated from each other. The ratio of the initial rate of cooling of S_1 to that S_2 is

(A) $\frac{1}{3}$ (B) $\frac{1}{\sqrt{3}}$

(C) $\frac{\sqrt{3}}{1}$ (D) $\left(\frac{1}{3}\right)^{1/3}$

52. [IIT-JEE 1995]

Three rods of identical cross-sectional area and made from the same metal form the sides of an isosceles triangle ABC right angled at B. The points A and B are maintained at temperatures T and $\sqrt{2}T$ respectively in the steady state. Assuming the only heat conduction takes place temperature of point C will be



(A) $\frac{3T}{\sqrt{2}+1}$ (B) $\frac{T}{\sqrt{2}+1}$

(C) $\frac{T}{\sqrt{3}(\sqrt{2}-1)}$ (D) $\frac{T}{\sqrt{2}-1}$

53. [IIT-JEE 1992]

Three closed vessels A, B and C are at the same temperature and contain gases which obey the Maxwellian distribution of velocities. Vessel A contains only O_2, B only N_2 and C a mixture of equal quantities of O_2 and N_2. If the average speed of O_2 molecules in vessel A is v_1, that of N_2 molecules in vessel B is v_2, then the average speed of O_2 molecules in vessel C is ($M$is the mass of an oxygen molecule)

(A) $\frac{v_1+v_2}{2}$ (B) v_1

(C) $\sqrt{v_1 v_2}$ (D) $\sqrt{\frac{3k_BT}{M}}$

54. [IIT-JEE 1990]

When an ideal diatomic gas is heated at constant pressure the fraction of the heat energy supplied which increases the internal energy of the gas is

(A) $\frac{2}{5}$ (B) $\frac{3}{5}$

(C) $\frac{3}{7}$ (D) $\frac{5}{7}$

55. [IIT-JEE 1988]

One mole of an ideal monatomic gas at temperature T_0 expands slowly according to the law $\frac{P}{V}=$ constant. If the final temperature is $2T_0$, heat supplied to the gas is

(A) $2RT_0$ (B) $\frac{3}{2}RT_0$

(C) RT_0 (D) $\frac{1}{2}RT_0$

56. [IIT-JEE 1988]

One mole of monatomic gas is mixed with one mole of diatomic gas. Then $\gamma\left(=\frac{C_P}{C_V}\right)$ for the mixture is

(A) 1.40 (B) 1.50

(C) 1.53 (D) 3.07

57. [IIT-JEE 1986]

Steam at 100 °C is passed into 1.1 kg of water contained in a calorimeter of water equivalent 0.02 kg at 15 °C till the temperature of the calorimeter and its contents rises to 80 °C. The mass of the steam condensed in kg is

(A) 0.130 (B) 0.065

(C) 0.260 (D) 0.135

58. [IIT-JEE 1985]

70 calorie of heat is required to raise the temperature of 2 mole of an ideal gas at constant pressure from 40 °C to 45 °C. The amount of heat required to raise the temperature of the same gas through the same range at constant volume is

(A) 90 calorie (B) 70 calorie

(C) 50 calorie (D) 30 calorie

59. [IIT-JEE 1984]

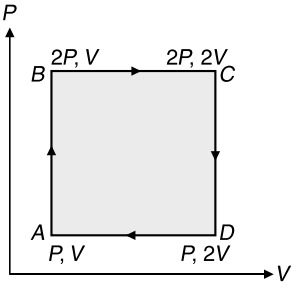
At room temperature a diatomic gas is found to have an rms speed of 1930 ms^{-1}. The gas is

(A) H_2 (B) Cl_2

(C) O_2 (D) F_2

60. [IIT-JEE 1983]

An ideal monatomic gas is taken round the cycle $ABCDA$ as shown in the P-V diagram (shown in figure). The work done during the cycle is



(A) PV

(B) $2PV$

(C) $\frac{1}{2}PV$

(D) zero

Multiple Correct Choice Type Questions

(In this section each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct)

1. [JEE (Advanced) 2020]

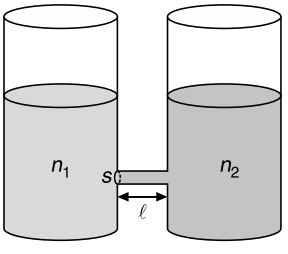
The filament of a light bulb has surface area 64 mm^2. The filament can be considered as a black body at temperature 2500 K emitting radiation like a point source when viewed from far. At night the light bulb is observed from a distance of 100 m. Assume the pupil of the eyes of the observer to be circular with radius 3 mm. Then

(Take Stefan-Boltzmann constant $=5.67\times10^{-8}$ $Wm^{-2}K^{-4}$, Wien's displacement constant $=2.90\times10^{-3}$ mK, Planck's constant $=6.63\times10^{-34}$ Js, speed of light in vacuum $=3.00\times10^{8}$ ms^{-1})

(A) power radiated by the filament is in the range 642 W to 645 W
(B) radiated power entering into one eye of the observer is in the range 3.15×10^{-8} W to 3.25×10^{-8} W
(C) the wavelength corresponding to the maximum intensity of light is 1160 nm
(D) taking the average wavelength of emitted radiation to be 1740 nm, the total number of photons entering per second into one eye of the observer is in the range 2.75×10^{11} to 2.85×10^{11}

2. [JEE (Advanced) 2020]

As shown schematically in Figure, two vessels contain water solutions (at temperature T) of potassium permanganate $(KMnO_4)$ of different concentrations n_1 and n_2 $(n_1 > n_2)$ molecules per unit volume with $\Delta n = (n_1 - n_2) \ll n_1$. When they are connected by a tube of small length l and cross-sectional area S, $KMnO_4$ starts to diffuse from the left to the right vessel through the tube. Consider the collection of molecules to behave as dilute ideal gases and the difference in their partial pressure in the two vessels causing the diffusion. The speed v of the molecules is limited by the viscous force $-\beta v$ on each molecule, where β is a constant. Neglecting all terms of the order $(\Delta n)^2$, which of the following is/are correct? (k_B is the Boltzmann constant)



(A) The force causing the molecules to move across the tube is $\Delta n k_B T S$
(B) Force balance implies $n_1 \beta v l = \Delta n k_B T$
(C) Total number of molecules going across the tube per sec is $\left(\frac{\Delta n}{l}\right)\left(\frac{k_B T}{\beta}\right)S$
(D) Rate of molecules getting transferred through the tube does not change with time

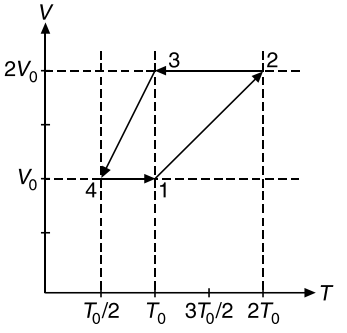
3. [JEE (Advanced) 2019]

A mixture of ideal gas containing 5 moles of monatomic gas and 1 mole of rigid diatomic gas is initially at pressure P_0, volume V_0, and temperature T_0. If the gas mixture is adiabatically compressed to a volume $\frac{V_0}{4}$, then the correct statement(s) is/are, (Given $2^{1.2} = 2.3$; $2^{3.2} = 9.2$; R is gas constant)

(A) The work $|W|$ done during the process is $13RT_0$
(B) The final pressure of the gas mixture after compression is in between $9P_0$ and $10P_0$
(C) Adiabatic constant of the gas mixture is 1.6
(D) The average kinetic energy of the gas mixture after compression is in between $18RT_0$ and $19RT_0$

4. [JEE (Advanced) 2019]

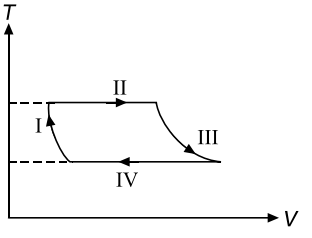
One mole of a monatomic ideal gas goes through a thermodynamic cycle, as shown in the volume versus temperature VT diagram. The correct statement(s) is/are [R is the gas constant]



(A) The above thermodynamic cycle exhibits only isochoric and adiabatic processes
(B) The ratio of heat transfer during processes $1\to2$ and $3\to4$ is $\left|\frac{Q_{1\to2}}{Q_{3\to4}}\right| = \frac{1}{2}$
(C) The ratio of heat transfer during processes $1\to2$ and $2\to3$ is $\left|\frac{Q_{1\to2}}{Q_{2\to3}}\right| = \frac{5}{3}$
(D) Work done in this thermodynamic cycle $(1\to2\to3\to4\to1)$ is $|W| = \frac{1}{2}RT_0$

5. [JEE (Advanced) 2018]

One mole of a monatomic ideal gas undergoes a cyclic process as shown in the figure (where V is the volume and T is the temperature). Which of the statements below is (are) true?



(A) Process I is an isochoric process
(B) In process II, gas absorbs heat
(C) In process IV, gas releases heat
(D) Processes I and III are not isobaric

6. [JEE (Advanced) 2017]

A human body has a surface area of approximately 1 m^2. The normal body temperature is 10 K above the surrounding room temperature T_0. Take the room temperature to be $T_0 = 300$ K. For $T_0 = 300$ K, the value of $\sigma T_0^4 = 460$ Wm^{-2} (where σ is the Stefan Boltzmann constant). Which of the following options is/are correct?

(A) If the body temperature rises significantly, then the peak in the spectrum of electromagnetic radiation emitted by the body would shift to longer wavelengths
(B) If the surrounding temperature reduces by a small amount $\Delta T_0 \ll T_0$, then to maintain the same body temperature the same (living) human being needs to radiate $\Delta W = 4\sigma T_0^3 \Delta T_0$ more energy per unit time

(C) The amount of energy radiated by the body in 1 s is close to 60 J

(D) Reducing the exposed surface area of the body (e.g. by curling up) allows humans to maintain the same body temperature while reducing the energy lost by radiation

7. [JEE (Advanced) 2015]

A container of fixed volume has a mixture of one mole of hydrogen and one mole of helium in equilibrium at temperature T. Assuming the gases are ideal, the correct statements is/are

(A) The average energy per mole of the gas mixture is $2RT$

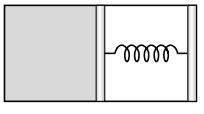
(B) The ratio of speed of sound in the gas mixture to that in helium gas is $\sqrt{\frac{6}{5}}$

(C) The ratio of the rms speed of helium atoms to that of hydrogen molecules is $\frac{1}{2}$

(D) The ratio of the rms speed of helium atoms to that of hydrogen molecules is $\frac{1}{\sqrt{2}}$

8. [JEE (Advanced) 2015]

An ideal monoatomic gas is confined in a horizontal cylinder by a spring piston (as shown in Figure). Initially the gas is at temperature T_1, pressure p_1 and volume V_1 and the spring is in its relaxed state. The gas is then heated very slowly to temperature T_2, pressure p_2 and volume V_2. During this process the piston moves out by a distance x. Ignoring the friction between the piston and the cylinder, the correct statements is/are



(A) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the energy stored in the spring is $\frac{1}{4}p_1V_1$

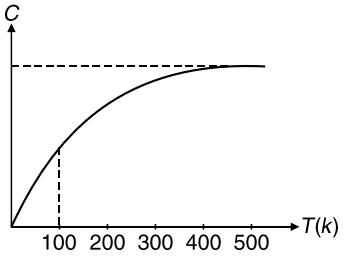
(B) If $V_2 = 2V_1$ and $T_2 = 3T_1$ then the change in internal energy is $3p_1V_1$

(C) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the work done by the gas is $\frac{7}{3}p_1V_1$

(D) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the heat supplied to the gas is

9. [JEE (Advanced) 2013]

The figure below shows the variation of specific heat capacity (C) of a solid as a function of temperature (T). The temperature is increased continuously from 0 to 500 K at a constant rate. Ignoring any volume change, the following statement(s) is(are) correct to a reasonable approximation.



(A) The rate at which heat is absorbed in the range 0-100 K approximately varies linearly with temperature T.

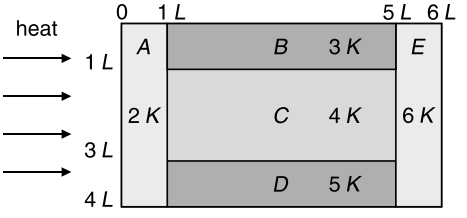
(B) Heat absorbed in increasing the temperature from 0-100 K is less than the heat required for increasing the temperature from 400-500 K.

(C) There is no change in the rate of heat absorption in the range 400-500 K.

(D) The rate of heat absorption increases in the range 200-300 K.

10. [IIT-JEE 2011]

A composite block is made of slabs A, B, C, D and E of different thermal conductivities (given in terms of a constant K) and sizes (given in terms of length, L) as shown in the figure. All slabs are of same width. Heat Q flows only from left to right through the blocks. Then in steady state



(A) heat flow through A and E slabs are same

(B) heat flow through slab E is maximum

(C) temperature difference across slab E is smallest

(D) $\begin{pmatrix}\text{Heat flow}\\ \text{through } C\end{pmatrix} = \begin{pmatrix}\text{Heat flow}\\ \text{through } B\end{pmatrix} + \begin{pmatrix}\text{Heat flow}\\ \text{through } D\end{pmatrix}$

11. [IIT-JEE 2009]

C_V and C_P denote the molar specific heat capacities of a gas at constant volume and constant pressure, respectively. Then

(A) $C_P - C_V$ is larger for a diatomic ideal gas than for a monatomic ideal gas

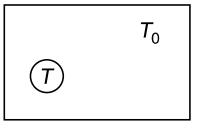
(B) $C_P + C_V$ is larger for a diatomic ideal gas than for a monatomic ideal gas

(C) $\frac{C_P}{C_V}$ is larger for a diatomic ideal gas than for a monatomic ideal gas

(D) $(C_P)(C_V)$ is larger for a diatomic ideal gas than for a monatomic ideal gas

12. [IIT-JEE 2006]

A black body of temperature T is inside a chamber of temperature T_0. Now the closed chamber is slightly opened to sun such that temperature of black body (T) and chamber (T_0) remains constant



(A) black body will absorb more radiation

(B) black body will absorb less radiation

(C) black body emit more energy

(D) black body emit energy equal to energy absorbed by it

13. [IIT-JEE 1999]

A bimetallic strip is formed out of two identical strips one of copper and the other of brass. The coefficients of linear expansion of the two metals are α_C and α_B. On heating, the temperature of the strip goes up by ΔT and the strip bends to form an arc of radius of curvature R. Then R is

(A) proportional to ΔT

(B) inversely proportional to ΔT

(C) proportional to $|\alpha_B - \alpha_C|$

(D) inversely proportional to $|\alpha_B - \alpha_C|$

14. [IIT-JEE 1998]

During the melting of a slab of ice at 273 K at atmospheric pressure

(A) positive work is done by the ice-water system on the atmosphere

(B) positive work is done on the ice-water system by the atmosphere

(C) the internal energy of the ice-water increases

(D) the internal energy of the ice-water system decreases

15. [IIT-JEE 1998]

Let $\bar{v}$, v_{rms} and v_p represent respectively the mean speed, root mean square speed and the most probable speed of the molecules in an ideal monatomic gas at absolute temperature T. The mass of the molecule is m. Then

(A) no molecule can have energy greater than $\sqrt{2}v_{rms}$

(B) no molecule can have speed less than $\frac{v_p}{\sqrt{2}}$

(C) $v_p < \bar{v} < v_{rms}$

(D) the average kinetic energy of a molecule is $\frac{3}{4}mv_p^2$

16. [IIT-JEE 1995]

From the following statements concerning ideal gas at any given temperature T, select the correct one(s)

(A) The coefficient of volume expansion at constant pressure is the same for all ideal gases

(B) The average translational kinetic energy per molecule of oxygen gas is $3kT$, k being Boltzmann constant

(C) The mean-free path of molecules increases with increase in the pressure

(D) In a gaseous mixture, the average translational kinetic energy of the molecules of each component is different

17. [IIT-JEE 1994]

Two bodies A and B have thermal emissivities of 0.01 and 0.81, respectively. The outer surface areas of two bodies are the same. The two bodies emit total radiant power at the same rate. The wavelength λ_B corresponding to maximum spherical spectral radiance in the radiation from B is shifted from the wavelength corresponding to the maximum spectral radiance in from A by 1.00 μm. If the temperature of A is 5802 K, then

(A) the temperature of B is 1934 K

(B) $\lambda_B = 1.5$ μm

(C) the temperature of B is 11604 K

(D) the temperature of B is 2901 K

18. [IIT-JEE 1993]

An ideal gas is taken from the state A (pressure P, volume V) to the state B (pressure $P/2$, volume $2V$) along a straight line path in the P-V diagram. Select the correct statements from the following

(A) The work done by the gas in the process A to B exceeds the work that would be done by it if the system were taken from A to B along an isotherm

(B) In the T-V diagram, the path AB becomes a part of a parabola

(C) In the P-T diagram, the path AB becomes a part of a hyperbola

(D) In going from A to B, the temperature T of the gas first increases to a maximum value and then decreases

19. [IIT-JEE 1989]

For an ideal gas

(A) the change in internal energy in a constant pressure process from temperature T_1 and T_2 is equal to $nC_V(T_2 - T_1)$, where C_V is the molar heat capacity at constant volume and n the number of moles of the gas.

(B) the change in internal energy of the gas and the work done by the gas are equal in magnitude in a adiabatic process

(C) the internal energy does not change in an isothermal process

(D) no heat is added or removed in an adiabatic process

Reasoning Based Questions

This section contains Reasoning type questions, each having four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Each question contains STATEMENT 1 and STATEMENT 2. You have to mark your answer as

Bubble (A) If both statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1.

Bubble (B) If both statements are TRUE but STATEMENT 2 is not the correct explanation of STATEMENT 1.

Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE.

Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 is TRUE.

1. [IIT-JEE 2007]

Statement-1: The total translational kinetic energy of all the molecules of a given mass of an ideal gas is 1.5 times the product of its pressure and its volume.

Statement-2: The molecules of a gas collide with each other and the velocities of the molecules change due to the collision.

Matrix Match/Column Match Type Questions

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

If the correct matches are A → p, s and t; B → q and r; C → p and q; and D → s and t; then the correct darkening of bubbles will look like the following:

	p	q	r	s	t
A	●	○	○	●	●
B	○	●	●	○	○
C	●	●	○	○	○
D	○	○	○	●	●

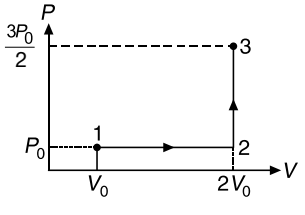
1. [JEE (Advance) 2019]

Answer the following by appropriately matching the lists based on the information given in the paragraph In a thermodynamic process on an ideal monatomic gas, the infinitesimal heat absorbed by the gas is given by $T\Delta X$, where T is temperature of the system and ΔX is the infinitesimal change in a thermodynamic quantity X of the system. For a mole of monatomic ideal gas $X = \frac{3}{2}R\ell n\left(\frac{T}{T_A}\right) + R\ell n\left(\frac{V}{V_A}\right)$.

Here, R is gas constant, V is volume of gas, T_A and V_A are constants. The **COLUMN-I** below gives some quantities involved in a process and **COLUMN-II** gives some possible values of these quantities.

COLUMN-I	COLUMN-II
A. Work done by the system in process $1 \to 2 \to 3$	p. $\frac{1}{3}RT_0 \ln 2$
B. Change in internal energy in process $1 \to 2 \to 3$	q. $\frac{1}{3}RT_0$
C. Heat absorbed by the system in process $1 \to 2 \to 3$	r. RT_0
D. Heat absorbed by the system in process $1 \to 2$	s. $\frac{4}{3}RT_0$
	t. $\frac{1}{3}RT_0(3+\ln 2)$
	u. $\frac{5}{6}RT_0$

If the process carried out on one mole of monatomic ideal gas is as shown in figure in the PV diagram with $P_0V_0 = \frac{1}{3}RT_0$, then give the correct match for COLUMN-I in COLUMN-II.



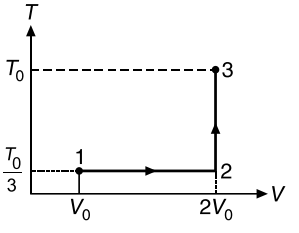
2. [JEE (Advance) 2019]

In a thermodynamic process on an ideal monatomic gas, the infinitesimal heat absorbed by the gas is given by $T\Delta X$, where T is temperature of the system and ΔX is the infinitesimal change in a thermodynamic quantity X of the system. For a mole of monatomic ideal gas $X = \frac{3}{2}R\ln\left(\frac{T}{T_A}\right) + R\ln\left(\frac{V}{V_A}\right)$.

Here, R is gas constant, V is volume of gas, T_A and V_A are constants. The **LIST-I** below gives some quantities involved in a process and **LIST-II** gives some possible values of these quantities.

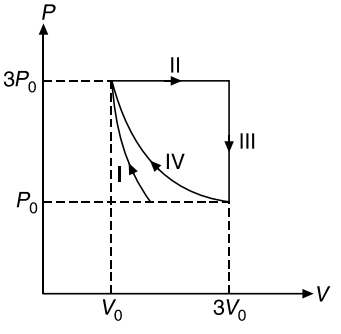
LIST-I	LIST-II
A. Work done by the system in process $1 \to 2 \to 3$	p. $\frac{1}{3}RT_0 \ln 2$
B. Change in internal energy in process $1 \to 2 \to 3$	q. $\frac{1}{3}RT_0$
C. Heat absorbed by the system in process $1 \to 2 \to 3$	r. RT_0
D. Heat absorbed by the system in process $1 \to 2$	s. $\frac{4}{3}RT_0$
	t. $\frac{1}{3}RT_0(3+\ln 2)$
	u. $\frac{5}{6}RT_0$

If the process on one mole of monatomic ideal gas is as shown in the TV diagram with $P_0V_0 = \frac{1}{3}RT_0$, the correct match is,



3. [JEE (Advance) 2018]

One mole of a monatomic ideal gas undergoes four thermodynamic processes as shown schematically in the PV-diagram below. Among these four processes, one is isobaric, one is isochoric, one is isothermal and one is adiabatic. Match the processes mentioned in **COLUMN-I** with the corresponding statements in **COLUMN-II**.



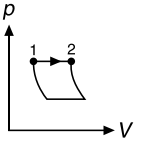
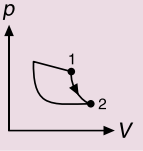
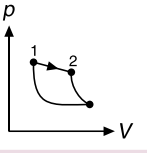
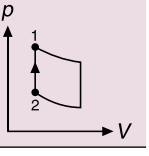
COLUMN-I	COLUMN-II
(A) In process I	(p) Work done by the gas is zero.
(B) In process II	(q) Temperature of the gas remains unchanged.

(*Continued*)

COLUMN-I	COLUMN-II
(C) In process III	(r) No heat is exchanged between the gas and its surroundings.
(D) In process IV	(s) work done by the gas is $6P_0V_0$.

Directions (Q. Nos. 4–6) Matching the information given in the three columns of the following table.

An ideal gas is undergoing a cyclic thermodynamic process in different ways as shown in the corresponding p-V diagrams in column 3 of the table. Consider only the path from state 1 to state 2. W denotes the corresponding work done on the system. The equations and plots in the table have standards notations and used in thermodynamic process. Here γ is the ratio of heat capacities at constant pressure and constant volume. The number of moles in the gas is n.

COLUMN-I	COLUMN-II	COLUMN-III
(1) $W_{1\to2}=\frac{1}{\gamma-1}(p_2V_2-p_1V_1)$	(A) Isothermal	(p) 
(2) $W_{1\to2}=-pV_2+pV_1$	(B) Isochoric	(q) 
(3) $W_{1\to2}=0$	(C) Isobaric	(r) 
(4) $W_{1\to2}=-nRT\ln\left(\frac{V_2}{V_1}\right)$	(D) Adiabatic	(s) 

4. [JEE (Advanced) 2017]

Which one of the following options correctly represents a thermodynamic process that is used as a correction in the determination of the speed of sound in an ideal gas?

(A) (4) (B) (r) (B) (1) (B) (Q)
(C) (1), (D) (q) (D) (3) (D) (R)

5. [JEE (Advanced) 2017]

Which of the following options is the only correct representation of a process in which $\Delta U=\Delta Q-p\Delta V$?

(A) (2) (C) (s) (B) (2) (C) (p)
(C) (3) (C) (p) (D) (2) (D) (r)

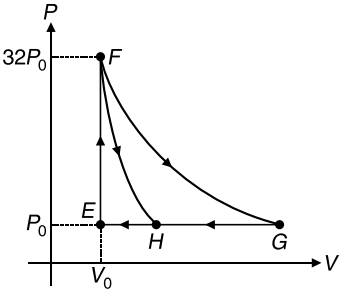
6. [JEE (Advanced) 2017]

Which one of the following options is the correct combination?

(A) (2) (D) (p) (B) (3) (B) (s)
(C) (2) (D) (r) (D) (4) (B) (s)

7. [JEE (Advanced) 2013]

One mole of a monatomic ideal gas is taken along two cyclic processes $E\to F\to G\to E$ and $E\to F\to H\to E$ as shown in the PV diagram. The processes involved are purely isochoric, isobaric, isothermal or adiabatic.

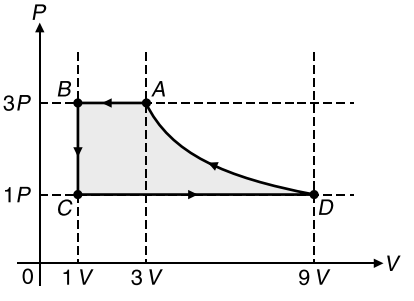


Match the paths in **COLUMN-I** with the magnitudes of the work done in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) $G\to E$	(p) $160P_0V_0\log_e 2$
(B) $G\to H$	(q) $36P_0V_0$
(C) $F\to H$	(r) $24P_0V_0$
(D) $F\to G$	(s) $31P_0V_0$

8. [JEE (Advanced) 2011]

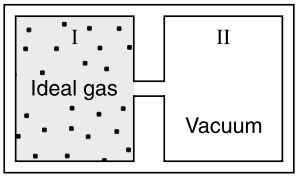
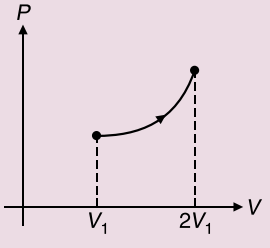
One mole of a monatomic ideal gas is taken through a cycle $ABCDA$ as shown in the P-V diagram. **COLUMN-II** gives the characteristics involved in the cycle. Match them with each of the processes given in **COLUMN-I.**



COLUMN-I	COLUMN-II
(A) Process $A\to B$	(p) Internal energy decreases
(B) Process $B\to C$	(q) Internal energy increases
(C) Process $C\to D$	(r) Heat is lost
(D) process $D\to A$	(s) Heat is gained
	(t) Work is done on the gas

9. [IIT-JEE 2008]

COLUMN-I contains a list of processes involving expansion of an ideal gas. Match this with **COLUMN-II** describing the thermodynamic change during this process.

COLUMN-I	COLUMN-II
(A) An insulated container has two chambers separated by a valve. Chamber I contain an ideal gas and the Chamber II has vacuum. The valve is opened. 	(p) The temperature of the gas decreases.
(B) An ideal monatomic gas expands to twice its original volume such that its pressure $P \propto \frac{1}{V^2}$, where V is the volume of the gas.	(q) The temperature of the gas increases or remains constant.
(C) An ideal monatomic gas expands to twice its original volume such that its pressure $P \propto \frac{1}{V^{4/3}}$, where V is its volume.	(r) The gas loses heat.
(D) An ideal monatomic gas expands such that its pressure P and volume V follows the behaviour shown in the graph. 	(s) The gas gains heat.

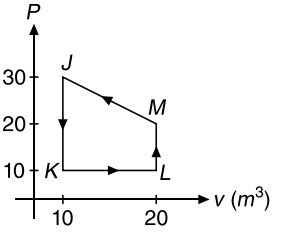
10. [IIT-JEE 2007]

COLUMN I gives some devices and **COLUMN II** gives some processes on which the functioning of these devices depends. Match the devices in **COLUMN I** with the processes in **COLUMN II**.

COLUMN-I	COLUMN-II
(A) Bimetallic strip	(p) Radiation from a hot body
(B) Steam engine	(q) Energy conversion
(C) Incandescent lamp	(r) Melting
(D) Electric fuse	(s) Thermal expansion of solids

11. [IIT-JEE 2006]

Match the following for the given process

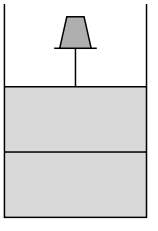


COLUMN-I	COLUMN-II
(A) Process $J \to K$	(p) $Q > 0$
(B) Process $K \to L$	(q) $W < 0$
(C) Process $L \to M$	(r) $W > 0$
(D) Process $M \to J$	(s) $Q < 0$

Linked Comprehension Type Questions

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct. (For the sake of competitiveness there may be a few questions that may have more than one correct options)

Comprehension I



In the Figure a container is shown to have a movable (without friction) piston on top. The container and the piston are all made of perfectly insulating material allowing no heat transfer between outside and inside the container. The container is divided into two compartments by a rigid partition made of a thermally conducting material that allows slow transfer of heat.

The lower compartment of the container is filled with 2 moles of an ideal monoatomic gas at 700 K and the upper compartment is filled with 2 moles of an ideal diatomic gas at 400 K. The heat capacities per mole of an ideal monoatomic gas are $C_V = \frac{3}{2}R$, $C_p = \frac{5}{2}R$, and those for an ideal diatomic gas are $C_V = \frac{5}{2}R$, $C_p = \frac{7}{2}R$

1. [JEE (Advanced) 2014]

Consider the partition to be rigidly fixed so that it does not move. When equilibrium is achieved, the final temperature of the gases will be

(A) 550 K (B) 525 K

(C) 513 K (D) 490 K

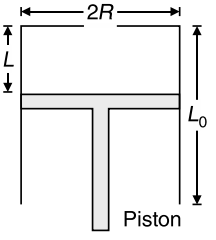
2. **[JEE (Advanced) 2014]**
Now consider the partition to be free to move without friction so that the pressure of gases in both compartments is the same. Then total work done by the gases till the time they achieve equilibrium will be

(A) $250R$ (B) $200R$

(C) $100R$ (D) $-100R$

Comprehension 2

A fixed thermally conducting cylinder has a radius R and height L_0. The cylinder is open at its bottom and has a small hole at its top. A piston of mass M is held at a distance L from the top surface, as shown in the figure. The atmospheric pressure is P_0. Based on the above facts, answer the following questions.



3. **[IIT-JEE 2007]**
The piston is now pulled out slowly and held at a distance $2L$ from the top. The pressure in the cylinder between its top and piston will then be

(A) P_0 (B) $\frac{P_0}{2}$

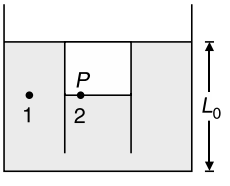
(C) $\frac{P_0}{2}+\frac{Mg}{\pi R^2}$ (D) $\frac{P_0}{2}-\frac{Mg}{\pi R^2}$

4. While the piston is at a distance $2L$ from the top, the hole at the top is sealed. The piston is then released, to a position where it can stay in equilibrium. In this condition, the distance of the piston from the top is

(A) $\left(\frac{2P_0\pi R^2}{\pi R^2P_0+Mg}\right)(2L)$ (B) $\left(\frac{P_0\pi R^2-Mg}{\pi R^2P_0}\right)(2L)$

(C) $\left(\frac{P_0\pi R^2+Mg}{\pi R^2P_0}\right)(2L)$ (D) $\left(\frac{P_0\pi R^2}{\pi R^2P_0-Mg}\right)(2L)$

5. The piston is taken completely out of the cylinder. The hole at the top is sealed. A water tank is brought below the cylinder and put in a position so that the water surface in the tank is at the same level as the top of the cylinder as shown in the figure. The density of the water is ρ. In equilibrium, the height H of the water column in the cylinder satisfies



(A) $\rho g(L_0-H)^2+P_0(L_0-H)+L_0P_0=0$

(B) $\rho g(L_0-H)^2-P_0(L_0-H)-L_0P_0=0$

(C) $\rho g(L_0-H)^2+P_0(L_0-H)-L_0P_0=0$

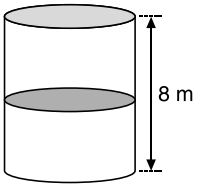
(D) $\rho g(L_0-H)^2-P_0(L_0-H)+L_0P_0=0$

Integer/Numerical Answer Type Questions

In this section, the answer to each question is a numerical value obtained after series of calculations based on the data provided in the question(s).

1. **[JEE (Advanced) 2020]**
Consider one mole of helium gas enclosed in a container at initial pressure P_1 and volume V_1. It expands isothermally to volume $4V_1$. After this, the gas expands adiabatically and its volume becomes $32V_1$. The work done by the gas during isothermal and adiabatic expansion processes are W_{iso} and W_{adia}, respectively. If the ratio $\frac{W_{iso}}{W_{adia}}=f\ln 2$, then f is……

2. **[JEE (Advanced) 2020]**
A thermally isolated cylindrical closed vessel of height 8 m is kept vertically. It is divided into two equal parts by a diathermic (perfect thermal conductor) frictionless partition of mass 8.3 kg. Thus, the partition is held initially at a distance of 4 m from the top, as shown in the schematic figure below. Each of the two parts of the vessel contains 0.1 mole of an ideal gas at temperature 300 K. The partition is now released and moves without any gas leaking from one part of the vessel to the other. When equilibrium is reached, the distance of the partition from the top (in m) will be …… (take the acceleration due to gravity = 10 ms^{-2} and the universal gas constant = 8.3 $Jmol^{-1}K^{-1}$)



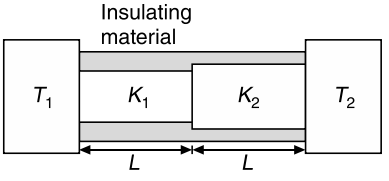
3. **[JEE (Advanced) 2020]**
A spherical bubble inside water has radius R. Take the pressure inside the bubble and the water pressure to be p_0. The bubble now gets compressed radially in an adiabatic manner so that its radius becomes $(R-a)$. For $a \ll R$ the magnitude of the work done in the process is given by $\left(4\pi p_0Ra^2\right)X$, where X is a constant and $\gamma=\frac{C_p}{C_V}=\frac{41}{30}$. The value of X is……

4. **[JEE (Advanced) 2020]**
A container with 1 kg of water in it is kept in sunlight, which causes the water to get warmer than the surroundings. The average energy per unit time per unit area received due to the sunlight is 700 Wm^{-2} and it is absorbed by the water over an effective area of 0.05 m^2. Assuming that the heat loss from the water to the surroundings is governed by Newton's law of cooling, the difference (in °C) in the temperature of water and the surroundings after a long time will be…… (Ignore effect of the container, and take constant for Newton's law of cooling = 0.001 s^{-1}, Heat capacity of water = 4200 $Jkg^{-1}K^{-1}$)

5. **[JEE (Advanced) 2019]**
A liquid at 30 °C is poured very slowly into a Calorimeter that is at temperature of 110 °C. The boiling temperature of the liquid is 80 °C. It is found that the first 5 g of the liquid completely evaporates. After pouring another 80 g of the liquid the equilibrium temperature is found to be 50 °C. The ratio of the latent heat of the liquid to its specific heat will be_____°C. [Neglect the heat exchange with surrounding]

6. **[JEE (Advance) 2018]**
Two conducting cylinders of equal length but different radii are connected in series between two heat baths kept at temperatures $T_1=300$ K and $T_2=100$ K, as shown in the figure.

The radius of the bigger cylinder is twice that of the smaller one and the thermal conductivities of the materials of the smaller and the larger cylinders are K_1 and K_2 respectively. If the temperature at the junction of the two cylinders is the steady state is 200 K, then $\frac{K_1}{K_2}$ =_____.



7. [JEE (Advance) 2018]

One mole of a monatomic ideal gas undergoes an adiabatic expansion in which its volume becomes eight times its initial value. If the initial temperature of the gas is 100 K and the universal gas constant $R = 8.0\ \text{Jmol}^{-1}\text{K}^{-1}$, the decrease in its internal energy, in Joule, is_____.

8. [JEE (Advanced) 2016]

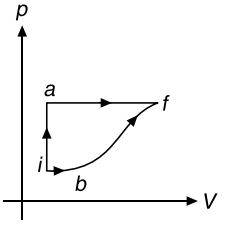
A metal is heated in a furnace where a sensor is kept above the metal surface to read the power radiated (P) by the metal. The sensor has a scale that displays $\log_2\left(\frac{P}{P_0}\right)$, where P_0 is a constant. When the metal surface is at a temperature of 487 C, then sensor shows a value 1. Assume that the emissivity of the metallic surface remains constant. What is the value displayed by the sensor when the temperature of the metal surface is raised to 2767 °C?

9. [JEE (Advanced) 2015]

Two spherical stars A and B emit black body radiation. The radius of A is 400 times that of B and A emits 10^4 times the power emitted from B. The ratio $\left(\frac{\lambda_A}{\lambda_B}\right)$ of their wavelengths λ_A and λ_B at which the peaks occur in their respective radiation curves is

10. [JEE (Advanced) 2014]

A thermodynamics system is taken from an initial state i with internal state i with internal energy $U_i = 100$ J to the final state f along two different paths iaf and ibf, as schematically shown in Figure.



The work done by the system along the paths af, ib and bf are $W_{af} = 200$ J, $W_{ib} = 50$ J and $W_{bf} = 100$ J respectively. The heat supplied to the system along the path iaf, ib and bf are Q_{iaf}, Q_{bf} respectively. If the internal energy of the system in the state b is $U_b = 200$ J and $Q_{iaf} = 500$ J, the ratio $\frac{Q_{bf}}{Q_{ib}}$ is

11. [IIT-JEE 2011]

Steel wire of length L at 40 °C is suspended from the ceiling and then a mass m is hung from its free end. The wire is cooled down from 40 °C to 30 °C to regain its original length L. The coefficient of linear thermal expansion of the steel is $10^{-5}\ °\text{C}^{-1}$, Young's modulus of steel is $10^{11}\ \text{Nm}^{-2}$ and radius of the wire is 1 mm. Assume that $L \gg$ diameter of the wire. Then the value of m in kg is nearly.

12. [IIT-JEE 2010]

A piece of ice (heat capacity $= 2100\ \text{Jkg}^{-1}\ °\text{C}^{-1}$ and latent heat $= 3.36 \times 10^5\ \text{J kg}^{-1}$) of mass m gram is at –5 °C at atmospheric pressure. It is given 420 J of heat so that the ice starts melting. Finally when the ice-water mixture is in equilibrium, it is found that 1 g of ice has melted. Assuming there is no other heat exchange in the process, find the value of m.

13. [IIT-JEE 2010]

Two spherical bodies A (radius 6 cm) and B (radius 18 cm) are at temperature T_1 and T_2 respectively. The maximum intensity in the emission spectrum of A is at 500 nm and in that of B is at 1500 nm. Considering them to be black bodies, what will be the ratio of the rate of total energy radiated by A to that of B?

14. [IIT-JEE 2010]

A diatomic ideal gas is compressed adiabatically to $\frac{1}{32}$ of its initial volume. If the initial temperature of the gas is T_i (in Kelvin) and the final temperature is aT_i, then find the value of a.

15. [IIT-JEE 2009]

A meal rod AB of length $10x$ has its one end A in ice at 0 °C and the other end B in water at 100 °C. If a point P on the rod is maintained at 400 °C, then it is found that equal amounts of water and ice evaporate and melt per unit time. The latent heat of evaporation of water is $540\ \text{calg}^{-1}$ and latent heat of melting of ice is $80\ \text{calg}^{-1}$. If the point P is at a distance of λx from the ice end A, find the value of λ. (Neglect any heat loss to the surrounding)..

ANSWER KEYS–TEST YOUR CONCEPTS AND PRACTICE EXERCISES

Test Your Concepts-I (Based on Thermometry and Thermal Expansion)

1. (a) too long, (b) 20.0048 cm
2. (a) 0.068 cm, (b) 88.49 cm
3. 90.01 cm
4. (i) $\gamma = 3\alpha$ (ii) $\gamma = 2\alpha$ (iii) $\gamma = 3\alpha$
5. 0.00864 cm
6. $\frac{V_0(\gamma - 3\alpha)t}{A_0(1+2\alpha t)}$
7. 36.28 s
8. 2.31×10^{-6} dyne
9. 1.6×10^{3} N
10. 4273 K, $(1.5\times10^{7}+273)$ K, 6.825%, 1.82×10^{-3}%
11. $\frac{\sqrt{3}}{200}\theta$
12. $\frac{1}{T}$
13. –40 °F
14. (a) 0.0135 kgm^2s^{-1}, (b) 3.2×10^{-3}
15. (a) 29.94 °C, (b) 118.64 °C
17. $\ell_s = 14.16$ cm and $\ell_c = 9.16$ cm
18. $2\times10^{-5}(°C)^{-1}$
19. 7 cm
20. 2.5458 cm
21. For Neon –248.58 °C, –415.44 °F
    For Carbon dioxide –56.60 °C, –69.88 °F
22. $\ell_1 = \frac{\ell\alpha_2}{\alpha_2 - \alpha_1}$ and $\ell_2 = \frac{\ell\alpha_1}{\alpha_2 - \alpha_1}$

Test Your Concepts-II (Based on Calorimetry)

1. 100 °C, $m_{water} = 222.26$ g, $m_{steam} = 7.74$ g
2. 8.6×10^{-3} °C
3. 200 s
4. 23
5. 20 °C
6. 50 g
7. 200 $Jkg^{-1}(°C)^{-1}$
8. 100 °C, 667 g steam, 1333 g water
9. 36.67 °C
10. 0.481 $calg^{-1}°C^{-1}$,16.55 g
11. 20.25 °C
12. 410 ms^{-1}
13. 0.805 kg
14. (a) 37.8 W, (b) 2 Nm
15. 178 K
16. 27.08 °C
17. 4200 $Jkg^{-1}K^{-1}$
18. 5820 cal
19. 100 °C
20. 0.8

Test Your Concepts-III (Based on Kinetic Theory of Gases and Ideal Gas Equation)

1. 87.76 cm of Hg
2. 600 ms^{-1}
3. $10^{21}m_0v$ and 0.073 Nm^{-2}
4. $\frac{T}{2}\left(\frac{p_1}{T_1}+\frac{p_2}{T_2}\right)$
5. 15.35 cm
6. 24 cm
7. 2.32×10^{-21} J
9. 2.095×10^{5} Nm^{-2}
10. 76.57 cm of Hg
11. 0.026 N
12. 115.2 g
13. $P_{left} = 12.471\times10^{5}$ Pa
    $P_{middle} = 28.06\times10^{5}$ Pa and
    $P_{right} = 15.589\times10^{5}$ Pa]
14. 337.5 K
15. (a) 8.26 km, (b) 0.09 km
16. RT/Mg
17. 5.62×10^{-21} J
18. 1.494 kgm^{-3}
19. 4.1×10^{5} Nm^{-2}
20. 13.32 mg
22. $p_0 e^{\frac{M\omega^2r^2}{2RT}}$
23. $-\frac{Mg}{R}$
24. 8.5×10^{-2} N
25. (a) $p_0(1-ah)^{Mg/aRT_0}$, (b) $\frac{p_0}{(1+ah)^{Mg/aRT_0}}$

Test Your Concepts-IV (Based on Internal Energy, Degrees of Freedom and Molar Specific Heats for Ideal Gases)

1. $\frac{n_1T_1+n_2T_2}{n_1+n_2}$
2. 2075 J
3. (a) 7.3×10^{3} J, (b) 6.07×10^{-21} J
4. (a) 6.21×10^{-21} $Jmole^{-1}$ (b) 10.35×10^{-21} $Jmole^{-1}$
   (c) 6235.5 $Jmole^{-1}$

5. 74.8 J
6. $\frac{T_1T_2(p_1+p_2)}{p_1T_2+p_2T_1}$, $\frac{p_2T_1(p_1+p_2)}{p_1T_2+p_2T_1}$ and $\frac{3p_1p_2V(T_2-T_1)}{2(p_1T_2+p_2T_1)}$
7. 1.4
8. 2×10^9 rads^{-1}
9. $\frac{2R\Delta T}{\gamma-1}$

Test Your Concepts-V (Based on Work Done and First Law of Thermodynamics)

1. 1
2. $3\alpha T_0^2$
3. 3.14 J
4. 421 kJ, 553 kJ
5. Process 1: -60, 50,
   Process 2: 5, 80 and
   Process 3: 20, 120
6. (a) 5 J, (b) –45 J, (c) 40 J, (d) 15 J
7. [6.6×10^5 J,1.8×10^5 J,8.4×10^5 J]
8. [7500 J]
9. $\frac{\pi}{2}$ atmlt
10. [2400 J, 0 J, -600 J, 0 J, 1800 J]

Test Your Concepts-VI (Based on Isochoric, Isobaric and Isothermal Processes)

1. (a) $300R$, (b) $500R$, (c) $200R$
3. 320 J
4. 2490 J
5. 1.4
6. 7 J
7. (b) $3RT_0\log_e(2)-\frac{3}{2}RT_0$
   (c) $3RT_0\log_e(2)-\frac{21}{4}RT_0$
8. $2\pi\sqrt{\frac{M\ell_0}{2P_0A+k\ell_0}}$
9. 50 J
10. $\frac{\log_e(x)}{\log_e\left(1+\frac{\Delta V}{V}\right)}$
12. 24.92 J
13. (a) $-2P_0V_0$ (b) 0
14. $RT\left((n-1)-\log_e(n)\right)$
15. (a) 20 cal, (b) 50 cal

Test Your Concepts-VII (Based on Adiabatic Process)

1. $2\pi\sqrt{\frac{mV_0}{2\gamma A^2P_0}}$
2. –125 J
3. (b) 113.1×10^{-3} m^3, 0.44×10^5 Nm^{-2}
   (c) 12470 J]
4. 423 K,$W_{adia}=-82$ J,$W_{isob}=-41$ J, $W_{isot}=105$ J, $W_{total}=-18$ J
5. 369.3 K, 2.46×10^5 Nm^{-2}
6. 588 K, –5976 J
7. (a) Adiabatic
8. 100 J
9. (a) $\frac{(P_1+P_2)T_1T_2}{P_1T_2+P_2T_1}$, $\frac{P_2(P_1+P_2)T_1}{P_1T_2+P_2T_1}$
   (b) $V_{left}=V\left(\frac{P_1T_2}{P_2T_1+P_1T_2}\right)$, $V_{right}=V\left(\frac{P_2T_1}{P_2T_1+P_1T_2}\right)$
10. 1.533, 1.153
11. $C_P=\left(1-\frac{T_0}{V_0}\tan\theta\right)R$, $C_V=\frac{RT_0\tan\theta}{V_0}$

Test Your Concepts-VIII (Based on Polytropic Process)

1. $\frac{3}{2}RT$
2. (a) $W=\alpha\ln(\eta)-\frac{RT_0(\eta-1)}{(\gamma-1)}$
   (b) $PV^\gamma e^{\frac{\alpha(\gamma-1)}{PV}}=$ constant
3. $\frac{2P_0}{3R}\sqrt{\frac{P_0}{3\alpha}}$
4. $C=C_V+\frac{R}{\alpha V}$, $C=C_V+\frac{R}{1+\alpha V}$
5. $\frac{\gamma R\Delta T}{1-\gamma}$
6. $3R$
7. $\Delta U=\frac{\alpha V^2}{\gamma-1}(\eta^2-1)$, $W=\frac{\alpha V^2}{2}(\eta^2-1)$, $C=C_V+\frac{R}{2}$
8. $\alpha=-\gamma$
9. (a) 500 J, (b) $C=2R$

Test Your Concepts-IX (Based on Cyclic Process, Heat Engine and Refrigerator)

1. $-1200R$, $Q_{AB}=-2100R$, $Q_{BC}=1500R$ and $Q_{CA}=831.6R$
2. $\eta=1-\frac{T_2}{T_1}$.
3. $W=\frac{1}{2}nR\left(\frac{T_3}{T_2}+\frac{T_2}{T_1}-2\right)(T_2-T_1)$
5. –4525.6 J
6. 2304.64 J
7. 2075 J
8. $\eta=1-\gamma\left(\frac{n-1}{n^\gamma-1}\right)$

9. $U_C = 140$ J, $Q_{AB} = 70$ J, $Q_{CA} = -180$ J
   $W_{ABCA} = -20$ J
10. $\eta = \left(\frac{\gamma - 1}{1 + \gamma n}\right)(n - 1)$

Test Your Concepts-X (Based on Conduction)

1. (a) 1.6×10^3 watt, (b) 1.75
2. (a) 84 °C, (b) 1.28 cals^{-1}
3. $\frac{mL}{100\pi k\ell}\log_e\left(\frac{r_2}{r_1}\right)$
4. 10.34 cm from water at 100 °C
5. 145.5 °C, 118.2 °C
6. $T = (\Delta T)_0 e^{-\alpha t}$, where $\alpha = \frac{kA(C_1 + C_2)}{\ell C_1 C_2}$
7. 0.87 (°C)cm^{-1}
8. 166 s
9. (a) $T = T_S + (T_S - T_0)e^{-\alpha t}$, where $\alpha = \frac{2T_0 kA}{5P_0 V_0 a}$ (b) $h = \left(\frac{T}{T_0}\right)h_0$
10. $t = \frac{15nR\ell}{16kA}\log_e(2)$

Test Your Concepts-XI (Based on Radiation)

1. 9 minute
2. 969.3 K
3. 36.6 Wm^{-1}K^{-1}
4. 1.01×10^5 s
5. $\frac{\rho dc(\eta^3 - 1)}{18\sigma e T_0^3}$
6. 0.206 °Cs^{-1}
7. $T = 2000\left[\log_e\left(\frac{10}{5 + x}\right) + 1\right]$
8. 0.115 °Cs^{-1}
9. 0.3
10. 76.2 min
11. 800 kgm^{-3}
12. 3000 K

Single Correct Choice Type Questions

1. B	2. B	3. B	4. C	5. C	6. B	7. B	8. B	9. C	10. C
11. D	12. D	13. A	14. C	15. B	16. D	17. D	18. B	19. C	20. D
21. C	22. B	23. C	24. A	25. C	26. C	27. B	28. B	29. A	30. A
31. C	32. A	33. C	34. A	35. D	36. B	37. A	38. C	39. B	40. C
41. A	42. C	43. C	44. D	45. C	46. B	47. C	48. D	49. B	50. B
51. D	52. C	53. A	54. D	55. C	56. C	57. C	58. C	59. A	60. C
61. A	62. D	63. B	64. B	65. D	66. C	67. D	68. B	69. C	70. A
71. C	72. B	73. A	74. C	75. A	76. A	77. B	78. A	79. C	80. B
81. B	82. B	83. A	84. D	85. A	86. C	87. C	88. A	89. A	90. D
91. D	92. B	93. C	94. B	95. D	96. D	97. D	98. A	99. D	100. B
101. A	102. D	103. C	104. C	105. B	106. C	107. A	108. B	109. A	110. C
111. D	112. A	113. B	114. A	115. B	116. D	117. C	118. C	119. A	120. C
121. A	122. C	123. B	124. A	125. D	126. C	127. C	128. C	129. C	130. B
131. D	132. C	133. C	134. A	135. A	136. C	137. A	138. B	139. D	140. B
141. D	142. C	143. B	144. B	145. B	146. D	147. D	148. B	149. A	150. A
151. D	152. D	153. C	154. B	155. A	156. C	157. B	158. B	159. B	160. C
161. B	162. A	163. A	164. C	165. A	166. A	167. A	168. C	169. B	170. B
171. D	172. A	173. D	174. B	175. D	176. B	177. D	178. B	179. A	180. B
181. C	182. C	183. B	184. A	185. B	186. B	187. A	188. C	189. D	190. B
191. D	192. C	193. B	194. D	195. A	196. C	197. D	198. B	199. D	200. B
201. A	202. B	203. C	204. A	205. B	206. B	207. C	208. D	209. A	210. A
211. C	212. B	213. C	214. A	215. B	216. C	217. A	218. B	219. B	220. D
221. B	222. C	223. A	224. B	225. A	226. A	227. D	228. A	229. D	230. D

231. D	**232.** B	**233.** C	**234.** C	**235.** D	**236.** A	**237.** B	**238.** B	**239.** C	**240.** C
241. D	**242.** B	**243.** C	**244.** C	**245.** C	**246.** C	**247.** C	**248.** A	**249.** C	**250.** B
251. D	**252.** B	**253.** C	**254.** C	**255.** B	**256.** B	**257.** D	**258.** C	**259.** A	**260.** D
261. B	**262.** B	**263.** B	**264.** A	**265.** A	**266.** A	**267.** A	**268.** B	**269.** C	**270.** C
271. A	**272.** C	**273.** B	**274.** B	**275.** C	**276.** D	**277.** C	**278.** B	**279.** A	**280.** A
281. B	**282.** B	**283.** A	**284.** C	**285.** B	**286.** C	**287.** C	**288.** A	**289.** C	**290.** C
291. C	**292.** A	**293.** A	**294.** B	**295.** D					

Multiple Correct Choice Type Questions

1. A, B, C	**2.** A, B, C	**3.** A, C	**4.** A, B, C	**5.** A, C
6. C, D	**7.** B, C	**8.** A, D	**9.** A, D	**10.** A, B
11. B, C	**12.** B, C	**13.** A, B, C	**14.** A, B, C	**15.** A, C
16. A, B, C	**17.** B, D	**18.** B, D	**19.** C, D	**20.** A, C
21. A, B	**22.** A, B, D	**23.** A, B	**24.** A, B, C, D	**25.** B, D
26. A, D	**27.** A, C	**28.** A, B, C, D	**29.** B, C, D	**30.** B, D
31. A, B, C, D	**32.** C, D	**33.** A, B, C	**34.** A, B	**35.** A, B, C, D
36. A, B	**37.** A, C, D	**38.** A, C	**39.** A, B	**40.** D
41. B, C, D	**42.** A, C	**43.** A, B	**44.** C, D	**45.** A, B, D
46. C, D	**47.** B, C	**48.** A, C	**49.** B, D	**50.** C, D
51. A, B, D	**52.** A, C	**53.** B, D	**54.** B, C	**55.** A, B, C, D
56. B, C	**57.** A, B, C	**58.** B, C	**59.** A, C	**60.** A, B, C
61. C, D				

Reasoning Based Questions

1. C	**2.** A	**3.** C	**4.** D	**5.** B	**6.** D	**7.** D	**8.** A	**9.** A	**10.** D
11. A	**12.** A	**13.** D							

Linked Comprehension Type Questions

1. C	**2.** C	**3.** B	**4.** C	**5.** C	**6.** A	**7.** D	**8.** B	**9.** B	**10.** B
11. A	**12.** B	**13.** A	**14.** D	**15.** C	**16.** C	**17.** D	**18.** D	**19.** C	**20.** D
21. A	**22.** B	**23.** A	**24.** B	**25.** A	**26.** D	**27.** B	**28.** B	**29.** C	**30.** B
31. C	**32.** A	**33.** A	**34.** B	**35.** B	**36.** D	**37.** A	**38.** B	**39.** D	**40.** C
41. B	**42.** A	**43.** B	**44.** C	**45.** B	**46.** A	**47.** D	**48.** D	**49.** B	**50.** B
51. D	**52.** C	**53.** A	**54.** B	**55.** D	**56.** C	**57.** D	**58.** C	**59.** B	**60.** D
61. C	**62.** A	**63.** B	**64.** C	**65.** A	**66.** B	**67.** C	**68.** D	**69.** B	**70.** B
71. B	**72.** D	**73.** A	**74.** C	**75.** C	**76.** D	**77.** D			

Matrix Match/Column Match Type Questions

1. A → (q, t)	B → (r)	C → (s)	D → (p)
2. A → (p, s)	B → (p, r)	C → (q, s)	D → (q, r)
3. A → (p, s)	B → (q, r)	C → (p, r, s)	D → (p, s)
4. A → (r)	B → (s)	C → (p)	D → (q)
5. A → (r)	B → (p)	C → (s)	D → (q)
6. A → (p, s)	B → (r, s)	C → (p, t)	D → (q, t)
7. A → (u)	B → (p, q, t)	C → (s)	D → (r)
8. A → (s)	B → (r)	C → (q)	D → (p)
9. A → (s)	B → (r)	C → (q)	D → (p)
10. A → (p, q, r)	B → (p, r, s)	C → (p, q, r)	D → (p, q, r)
11. A → (s)	B → (r)	C → (q)	D → (t)

Integer/Numerical Answer Type Questions

1. 495	**2.** 30, 20, 20	**3.** 972	**4.** (a) 800, (b) 720	**5.** 1190	**6.** 3.96
7. 1830	**8.** 70	**9.** 280, 280, 0	**10.** 36	**11.** 589	**12.** (a) 25, (b) 2
13. 80	**14.** 10	**15.** 2	**16.** 20000	**17.** 60	**18.** 3
19. 42	**20.** 2	**21.** 166	**22.** 172	**23.** 384	**24.** 96
25. 5	**26.** 6, clockwise	**27.** 12	**28.** 40, 500	**29.** 448	**30.** 462
31. (a) 9, (b) 80	**32.** 28	**33.** 192	**34.** 40		

35. (a) 600; (b) $Q_{AB} = 3000$, absorbed, $Q_{BC} = 1680$, absorbed, $Q_{CD} = 1800$, released, $Q_{DA} = 1680$, released; (c) 1200

ARCHIVE: JEE MAIN

1. B	**2.** D	**3.** B	**4.** 20	**5.** C	**6.** B	**7.** A	**8.** A	**9.** 266.67	**10.** 150
11. B	**12.** C	**13.** D	**14.** 41	**15.** D	**16.** 5	**17.** C	**18.** C	**19.** B	**20.** 8791
21. A	**22.** A	**23.** D	**24.** 19.00	**25.** A	**26.** A	**27.** C	**28.** D	**29.** 1818.00	**30.** 46
31. B	**32.** 60	**33.** 40	**34.** B	**35.** 50	**36.** B	**37.** C	**38.** C	**39.** C	**40.** A
41. A	**42.** D	**43.** B	**44.** B	**45.** B	**46.** Bonus	**47.** D	**48.** B	**49.** C	**50.** A
51. A	**52.** D	**53.** A	**54.** B	**55.** A	**56.** B	**57.** A	**58.** D	**59.** B	**60.** C
61. B	**62.** D	**63.** C	**64.** D	**65.** A	**66.** A	**67.** C	**68.** D	**69.** C	**70.** A
71. B	**72.** B	**73.** B	**74.** B	**75.** B	**76.** D	**77.** B	**78.** B	**79.** B	**80.** A
81. D	**82.** A	**83.** A	**84.** C	**85.** C	**86.** B	**87.** D	**88.** C	**89.** B	**90.** C
91. D	**92.** C	**93.** D	**94.** A	**95.** C	**96.** D	**97.** C	**98.** B	**99.** A	**100.** D
101. D	**102.** C	**103.** B	**104.** C	**105.** B	**106.** C	**107.** D	**108.** B	**109.** B	**110.** D
111. D	**112.** A	**113.** B	**114.** C	**115.** C	**116.** B	**117.** B	**118.** B		

ARCHIVE: JEE ADVANCED

Single Correct Choice Type Questions

1. C	**2.** B	**3.** A	**4.** C	**5.** A	**6.** D	**7.** D	**8.** C	**9.** D	**10.** A
11. C	**12.** D	**13.** A	**14.** C	**15.** A	**16.** C	**17.** C	**18.** A	**19.** B	**20.** C
21. C	**22.** C	**23.** None	**24.** B	**25.** C	**26.** C	**27.** A	**28.** A	**29.** A	**30.** B
31. A	**32.** B	**33.** B	**34.** C	**35.** A	**36.** A	**37.** B	**38.** D	**39.** D	**40.** C
41. D	**42.** B	**43.** C	**44.** A	**45.** D	**46.** C	**47.** C	**48.** B	**49.** D	**50.** B
51. D	**52.** A	**53.** B	**54.** D	**55.** A	**56.** B	**57.** D	**58.** C	**59.** A	**60.** A

Multiple Correct Choice Type Questions

1. B, C, D	**2.** A, B, C	**3.** A, B, C	**4.** C, D	**5.** B, C, D
6. B, C, D	**7.** A, B, D	**8.** A, B, C	**9.** A, B, C, D	**10.** A, B, C, D
11. B, D	**12.** D	**13.** B, D	**14.** B, C	**15.** C, D
16. A, C	**17.** A, B	**18.** A, B, D	**19.** A, B, C, D	

Reasoning Based Questions

1. B

Matrix Match/Column Match Type Questions

1. A → (q, r, s, u)	B → (q, s, r, u)	C → (s, r, q, t)	D → (q, r, p, u)
2. A → (p, r, t, s)	B → (p, t, q, t)	C → (p, r, t, p)	D → (s, t, q, u)
3. A → (r)	B → (s)	C → (p)	D → (q)
4. (C)	**5.** (B)	**6.** (B)	
7. A → (s)	B → (r)	C → (q)	D → (p)
8. A → (p, r, t)	B → (p, r)	C → (q, s)	D → (r, t)
9. A → (q)	B → (p, r)	C → (p, s)	D → (q, s)
10. A → (s)	B → (q)	C → (p, q)	D → (q, r)
11. A → (s)	B → (p, r)	C → (p)	D → (q, s)

Linked Comprehension Type Questions

1. D	**2.** D	**3.** A	**4.** D	**5.** C

Integer/Numerical Answer Type Questions

1. 1.78	**2.** 6	**3.** 2.05	**4.** 8.33	**5.** 270	**6.** 4.00	**7.** 900.00	**8.** 9	**9.** 2	**10.** 2
11. 3	**12.** 8	**13.** 9	**14.** 4	**15.** 9					

CHAPTER

3 Simple Harmonic Motion

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Dynamics of SHM, Phase Difference
(b) Differential Equation for SHM, Condition for Motion to be SHM
(c) Energy in SHM
(d) Spring Mass Systems
(e) Rotational SHM
(f) Simple Pendulum
(g) Compound Pendulum
(h) SHM in other Physical Systems
(i) Composition of SHM
(j) Damped Oscillations
(k) Forced Oscillations & Resonance

All this is followed by a variety of Exercise Sets (fully solved) which contain questions as per the latest JEE pattern. At the end of Exercise Sets, a collection of problems asked previously in JEE (Main and Advanced) are also given.

INTRODUCTION

In this chapter, we shall be studying a special type of periodic motion called Simple Harmonic Motion (SHM). This is a repeating motion of an object in which the object continues to observe to and fro motion about a mean position at fixed time interval (under ideal situations). However, if the time interval is not fixed, then the motion may be called as Oscillatory. The back and forth movements of such an object are called oscillations. We will focus our attention on a special case of periodic motion called simple harmonic motion. It is observed that all periodic motions can be modelled as combinations of simple harmonic motions and hence SHM forms a basic building block for more complicated periodic motion.

PERIODIC MOTION

When a body or a moving particle repeats its motion along a definite path after regular intervals of time, its motion is said to be **Periodic Motion** and interval of time is called **time period** or harmonic motion period T and its reciprocal is called the frequency ν i.e., $\nu = \frac{1}{T}$. The path of periodic motion may be linear, circular, elliptical or any other curve. For example, revolution of earth about the sun, rotation of earth about its own axis.

Mathematically, if any function of time $f(t)$ can be expressed as $f(t+T) = f(t)$, then that function can be regarded as periodic with period T.

OSCILLATORY MOTION

An oscillatory motion need not be periodic and need not have fixed extreme positions. For example, motion of pendulum of a wall clock (because the battery of the wall clock wears out with time).

The oscillatory motions in which energy is conserved can also be called as periodic. Oscillations in which energy is consumed due to some resistive forces and hence total mechanical energy decreases are called as **Damped oscillations.**

The force/torque (directed towards equilibrium point) acting in oscillatory motion is called restoring force/torque.

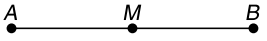
SIMPLE HARMONIC MOTION (SHM)

SHM is a special type of oscillatory motion in which the restoring force is proportional to the displacement from the mean position (for small displacement from mean position). It is the simplest (easy to analyse) form of oscillatory motion. The amplitude of oscillations of the particle is very small. At any instant the displacement of a particle

in oscillatory motion can be expressed in terms of sinusoidal functions (sine and cosine functions). These functions are called **harmonic functions**. That is why an oscillatory motion is also called a **harmonic motion**.

A simple harmonic motion can be expressed in terms of one single sine or cosine function or a linear combination of sine and cosine functions.

If a particle is moving to and fro about an equilibrium point M (called as the mean position), along a straight line as shown in Figure, then we can call its motion to be SHM, where A and B are extreme positions and $AM = MB =$ Amplitude (a)



However, when a body or a particle is free to rotate about a given axis executing angular oscillations, then this type of SHM can be regarded as angular SHM.

Problem Solving Technique(s)

(a) All Simple Harmonic Motions (SHMs) are periodic but all periodic motions may or may not be an SHM.

(b) All SHMs are oscillatory motions but all oscillatory motions may or may not be SHM.

EQUILIBRIUM POSITION OR MEAN POSITION

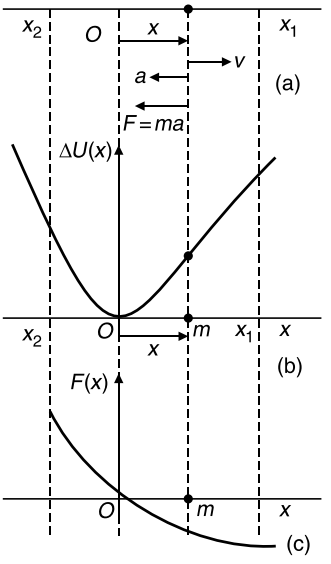
In mechanical oscillations a body oscillates about its mean position which is also its **equilibrium position**. At the equilibrium position no net force (or torque) acts on the oscillating body. The displacement (linear or angular) of an oscillating particle is its distance (linear or angular) from the equilibrium position at any instant.

When a body oscillates along a straight line within two fixed limits, its displacement x changes periodically in both magnitude and direction, its velocity v and acceleration $a = \ddot{x}$ also varies periodically in magnitude and direction. Since $F = ma = m\ddot{x}$, therefore, the force acting on the body also varies in magnitude and direction with time.

In terms of energy, we can say that a particle executing harmonic motion moves back and forth about a point at which the potential energy is minimum (equilibrium position). The force acting on the body at any position is given by

$$F = -\frac{dU}{dx}$$

When a body is displaced from its equilibrium it is acted upon by a restoring force (or torque) which always acts to accelerate the body in the direction of its equilibrium position as shown in Figure.



(a) A body of mass m oscillates harmonically between points x_1 and x_2 about the equilibrium position O.

(b) The potential energy of the body as a function of position. The force acting on the body is

$$F = -\frac{dU}{dx}$$

(c) The force acting on the body as a function of position x. Note that the force is always directed toward the equilibrium position.

DYNAMICS OF SIMPLE HARMONIC MOTION

The restoring force for small displacement x is given by $F = -kx$, where k is called the **force constant of the system**. Force constant is defined as the restoring force per unit displacement. Its SI unit is Nm^{-1}. If m is mass of the body and a is its acceleration, then for simple harmonic motion

$$F = ma = -k\ddot{x}$$

$$\Rightarrow \quad m\frac{d^2x}{dt^2} + kx = 0$$

$$\Rightarrow \quad \frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

$$\Rightarrow \quad \frac{d^2x}{dt^2} + \omega^2 x = 0$$

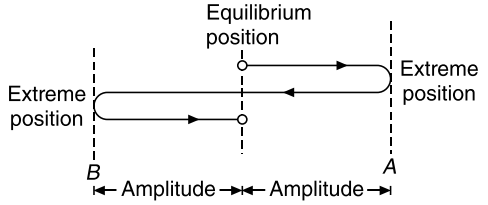
$$\Rightarrow \quad \ddot{x} + \omega^2 x = 0, \text{ where } \omega = \sqrt{k/m}$$

This is the differential equation of SHM.

The general solution to this equation is

$$x = a\sin(\omega t + \phi)$$

where a is the amplitude, $\omega t+\phi$ is the phase (also called as instantaneous phase), ϕ is phase constant or initial phase angle (in radian) also called as epoch.



Consider a particle to execute SHM and assume that the path of the particle is on a straight line, then x is the displacenment of particle from the mean position at that instant. The displacement of a particle executing SHM at time t is given by

$$x = a\sin(\omega t+\phi)$$

Also, note that the displacement of a particle executing SHM at time t can be given by

$$y = a\sin(\omega t+\phi)$$

Amplitude, a or A, is the maximum value of displacement of the particle from its equilibrium position i.e., mean position. So, amplitude of SHM is half the separation between the extreme points of SHM. It depends upon the energy of the system.

Angular frequency, ω of SHM is $\omega = \dfrac{2\pi}{T} = 2\pi f$ and its SI unit is rads^{-1}.

Frequency, f or ν is the number of oscillations completed in unit time interval, so we have $f = \nu = \dfrac{1}{T} = \dfrac{\omega}{2\pi}$. Its unit is sec^{-1} or Hz.

Time period, T is the smallest time interval after which oscillatory motion gets repeated, so time period T given by

$$T = \frac{1}{\nu} = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

Mathematically at $t = T$, the displacement x of SHM must satisfy $x(t+T) = x(t)$. For example,

$$\sin\left[\omega\left(t+\frac{2\pi}{\omega}\right)\right] = \sin(\omega t) \text{ and}$$

$$\cos\left[\omega\left(t+\frac{2\pi}{\omega}\right)\right] = \cos(\omega t)$$

ILLUSTRATION 1

Find the period of the function, $y = \sin(\omega t)+\sin(2\omega t)+\sin(3\omega t)$.

SOLUTION

The given function can be written as

$$y = y_1 + y_2 + y_3$$

where, $y_1 = \sin(\omega t)$, $T_1 = \dfrac{2\pi}{\omega} = T$

$y_2 = \sin(2\omega t)$, $T_2 = \dfrac{2\pi}{2\omega} = \dfrac{\pi}{\omega} = \dfrac{T}{2}$ and

$y_3 = \sin(3\omega t)$, $T_3 = \dfrac{2\pi}{3\omega} = \dfrac{T}{3}$

We see that, $T_1 = 2T_2$ and $T_1 = 3T_3$

So, time period of the given function is $T_1 = T = \dfrac{2\pi}{\omega}$, because in time $T = \dfrac{2\pi}{\omega}$, first function completes one oscillation, the second function two oscillations and the third function completes three oscillations.

ILLUSTRATION 2

Calculate the amplitude and initial phase of a particle in SHM, whose motion equation is given as

$$x = a\sin\omega t + b\cos\omega t$$

SOLUTION

Since $x = a\sin(\omega t)+b\cos(\omega t)$ …(1)

In this given equation, we can write

$$a = A\cos\phi \quad \ldots(2)$$

and $b = A\sin\phi$ …(3)

So, the given equation (1) transforms to

$$x = A\sin(\omega t+\phi) \quad \ldots(4)$$

Equation (4) is a general equation of SHM having amplitude A and ϕ is the initial phase of the oscillating particle at $t = 0$.

Squaring and adding equations (2) and (3), we get amplitude as

$$A = \sqrt{a^2+b^2}$$

Dividing (3) by (2), we get initial phase as

$$\tan\phi = \frac{b}{a}$$

$$\Rightarrow \quad \phi = \tan^{-1}\left(\frac{b}{a}\right)$$

PHASE AND PHASE DIFFERENCE

Phase is the physical quantity which represents the state of motion of particle (e.g., its position and direction of motion at any instant).

The argument $(\omega t+\phi)$ of sinusoidal function is called instantaneous phase of the motion. It gives the position and direction of motion at any instant.

The constant ϕ in equation of SHM is called phase constant or initial phase or epoch. It depends on initial position and direction of velocity.

If $\phi = 0$, then the body is initially at the mean position or starts from the mean position, then we have $x = a\sin\omega t$, because at $t = 0$, $x = 0$.

Similarly, if $\phi = \frac{\pi}{2}$, then the body is initially at the positive extreme position or starts from the positive extreme, then we have $x = a\cos\omega t$, because at $t = 0$, $x = a$.

Also, note that there is a phase difference of $\frac{\pi}{2}$ between the sine and the cosine forms.

If two particles perform SHM and their equations are

$$y_1 = a\sin(\omega t + \phi_1) \text{ and } y_2 = a\sin(\omega t + \phi_2)$$

then, phase difference is given by

$$\Delta\phi = (\omega t + \phi_2) - (\omega t + \phi_1) = \phi_2 - \phi_1$$

The phase changes with time as

$$\Delta\phi = \omega\Delta t = 2\pi f\Delta t = \frac{2\pi}{T}\Delta t$$

We may also define the period T as that time in which the phase changes by 2π.

CASE-I: Same phase or In Phase

Two vibrating particles are said to be in same phase, if the phase difference between them is an even multiple of π or time interval is an even multiple of $\frac{T}{2}$ because 1 time period is equivalent to 2π rad.

CASE-II: Opposite phase or Out of Phase

When the two vibrating particles cross their respective mean positions at the same time moving in opposite directions, then the phase difference between the two vibrating particles is 180°.

Opposite phase means the phase difference between the particle is an odd multiple of π (i.e., $\pi, 3\pi, 5\pi, 7\pi\ldots$) or the time interval is an odd multiple of $\frac{T}{2}$.

ILLUSTRATION 3

The position of a particle moving along x–axis is given by $x = 0.8\sin(12t + 0.3)$ m, where t is in second. Calculate the amplitude and the period of the motion. Also, find the phase, position, velocity and acceleration at $t = 0.6$ s.

SOLUTION

On comparing the given equation with the standard equation of SHM i.e., $x = A\sin(\omega t + \phi)$, we see that the amplitude is $A = 0.8$ m and the angular frequency $\omega = 12$ rads^{-1}.

$$\Rightarrow \quad T = \frac{2\pi}{\omega} = 0.524 \text{ s}.$$

Velocity and acceleration at any time are given by

$$v = \frac{dx}{dt} = 0.96\cos(12t + 0.3) \text{ ms}^{-1}$$

$$a = \frac{dv}{dt} = -11.5\sin(12t + 0.3) \text{ ms}^{-2}$$

At $t = 0.6$ s, the phase of motion is

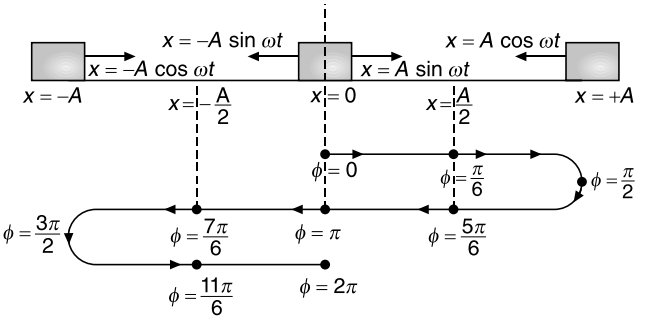
$$\omega t + \phi = (12)(0.6) + 0.3 = 7.5 \text{ rad}.$$

When this is used in the above expressions, we get

$$x = 0.75 \text{ m}, v = 0.333 \text{ ms}^{-1} \text{ and } a = -10.8 \text{ ms}^{-2}$$

EQUATION OF SHM AND PHASE

Consider the equation, $x = A\sin(\omega t + \phi)$, where ϕ is initial phase.



For example, phase difference between two particles at $\frac{A}{2}$ *from mean position moving opposite to each other is* $\Delta\phi = \frac{5\pi}{6} - \frac{\pi}{6} = \frac{2\pi}{3}$.

Also, we note that

(a) a particle executing SHM takes $\frac{T}{6}$ s to move from extreme position to half way between extreme and mean position.

(b) a particle executing SHM takes $\frac{T}{12}$ s to move from mean position to midway between mean and extreme position.

(c) when two particles executing SHM with time periods T_1 and $T_2\,(< T_1)$ start at the same time, then the particles will be in phase after n oscillations of T_2 and $(n-1)$ oscillations of T_1, if

$$nT_2 = (n-1)T_1$$

ILLUSTRATION 4

Two particles move parallel to the x-axis about the origin with the same frequency and amplitude a. At a certain instant they are found at distances $\frac{a}{3}$ from the origin on opposite sides but their velocity is found to be in the same direction. Calculate the phase different between the two particles.

SOLUTION

Let $x_1 = a\sin\omega t$ and $x_2 = a\sin(\omega t + \phi)$ be two SHMs.

$$\Rightarrow \quad \frac{a}{3} = a\sin\omega t \text{ and } -\frac{a}{3} = a\sin(\omega t + \phi)$$

$$\Rightarrow \quad \sin\omega t = \frac{1}{3} \text{ and } \sin(\omega t + \phi) = -\frac{1}{3}$$

$$\Rightarrow \quad \frac{1}{3}\cos\phi + \sqrt{1 - \frac{1}{9}}\sin\phi = -\frac{1}{3}$$

$$\Rightarrow \quad 9\cos^2\phi + 2\cos\phi - 7 = 0$$

$$\Rightarrow \quad \cos\phi = \frac{-2 \pm \sqrt{4 + 4\times 7\times 9}}{18} = -1,\ \frac{7}{9}$$

$$\Rightarrow \quad \delta = 180^\circ \text{ or } \delta = \cos^{-1}\left(\frac{7}{9}\right)$$

Now $v_1 = a\omega\cos\omega t$, $v_2 = a\omega\cos(\omega + \phi)$. When we substitute $\phi = 180^\circ$ we find that v_1 and v_2 are of opposite signs. Hence $\phi = 180^\circ$ is not acceptable.

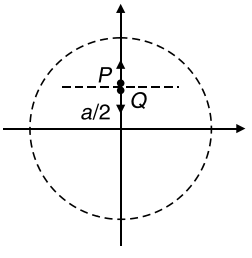
$$\Rightarrow \quad \phi = \cos^{-1}\left(\frac{7}{9}\right)$$

ILLUSTRATION 5

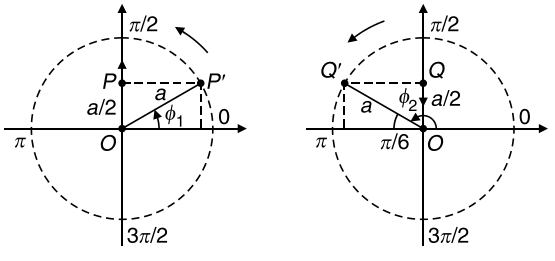
Two particles execute SHM with same amplitude a and same angular frequency ω on same straight line with same mean position. During oscillation, if they cross each while going in opposite direction when at a distance $a/2$ from mean position, then calculate the phase difference between the two SHMs.

SOLUTION

Let particles P (moving up) and Q (moving down) cross each other at $\frac{a}{2}$ from mean position as shown in Figure.



These two respective particles P and Q in SHM along with their corresponding particles P' and Q' in circular motion are shown separately in the two Figures below.



At this instant, phase of P is

$$\phi_1 = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

Similarly, for particle Q, phase is

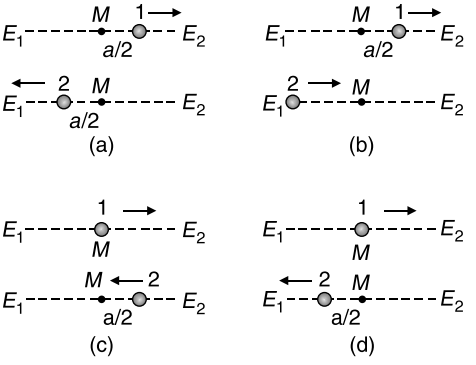
$$\phi_2 = \pi - \sin^{-1}\left(\frac{1}{2}\right) = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

Since both are oscillating at same angular frequency, so their phase difference remains constant and is given by

$$\Delta\phi = \phi_2 - \phi_1 = \frac{5\pi}{6} - \frac{\pi}{6} = \frac{2\pi}{3}$$

ILLUSTRATION 6

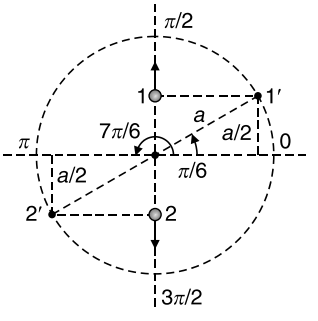
Calculate the phase difference between two particles 1 and 2 executing simple harmonic motion with the same frequency if they are found in the states shown in Figure at four different points of time.



SOLUTION

(a) For particle 1, $\phi_1 = \sin^{-1}\left(\frac{a/2}{a}\right) = \frac{\pi}{6}$

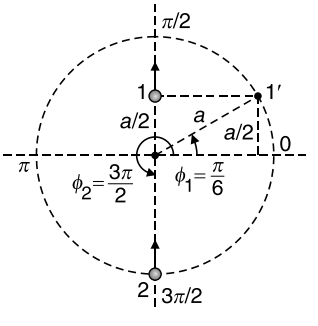
For particle 2, $\phi_2 = \pi + \sin^{-1}\left(\frac{a/2}{a}\right) = \frac{7\pi}{6}$



$$\Rightarrow \quad \Delta\phi_1 = \phi_2 - \phi_1 = \frac{7\pi}{6} - \frac{\pi}{6} = \pi \text{ radian}$$

(b) For particle 1, $\phi_1 = \sin^{-1}\left(\frac{a/2}{a}\right) = \frac{\pi}{6}$

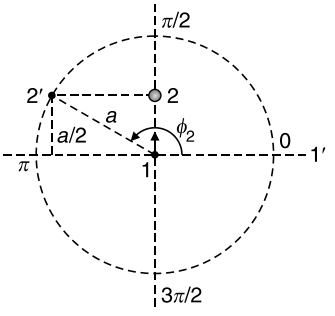
For particle 2, $\phi_2 = \sin^{-1}\left(\frac{-a}{a}\right) = \frac{3\pi}{2}$



$$\Rightarrow \quad \Delta\phi = \phi_2 - \phi_1 = \frac{3\pi}{2} - \frac{\pi}{6} = \frac{4\pi}{3}$$

(c) For particle 1, $\phi_1 = 0$

For particle 2, $\phi_2 = \pi - \sin^{-1}\left(\frac{a/2}{a}\right)$

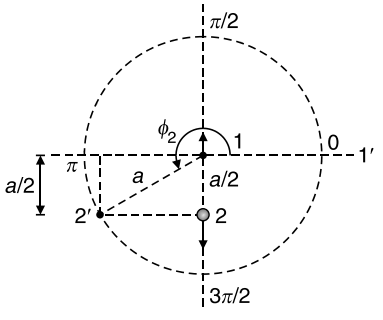


$$\Rightarrow \quad \phi_2 = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

$$\Rightarrow \quad \Delta\phi = \frac{5\pi}{6}$$

(d) For particle 1, $\phi_1 = 0$

For particle 2, $\phi_2 = \pi + \sin^{-1}\left(\frac{a/2}{a}\right) = \frac{7\pi}{6}$



From figure $\phi = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

ILLUSTRATION 7

Two particles are executing SHM of same amplitude and frequency along the same straight line. They pass one another when going in opposite directions, each time their displacement is half of their amplitude. Calculate the phase difference between the particles.

SOLUTION

Let two simple harmonic motions are $x = A\sin(\omega t)$ and $x = A\sin(\omega t + \phi)$

In the first case, we have $\frac{A}{2} = A\sin(\omega t)$

$$\Rightarrow \quad \sin(\omega t) = \frac{1}{2} \text{ OR } \cos(\omega t) = \frac{\sqrt{3}}{2}$$

In the second case, we have

$$\frac{A}{2} = A\sin(\omega t + \phi)$$

$$\Rightarrow \quad \frac{1}{2} = \sin(\omega t)\cos\phi + \cos(\omega t)\sin\phi$$

$$\Rightarrow \quad \frac{1}{2} = \frac{1}{2}\cos\phi + \frac{\sqrt{3}}{2}\sin\phi$$

$$\Rightarrow \quad 1 - \cos\phi = \sqrt{3}\sin\phi$$

$$\Rightarrow \quad (1-\cos\phi)^2 = 3\sin^2\phi$$

$$\Rightarrow \quad (1-\cos\phi)^2 = 3(1-\cos^2\phi)$$

$$\Rightarrow \quad (1-\cos\phi)^2 = 3(1+\cos\phi)(1-\cos\phi)$$

$$\Rightarrow \quad (1-\cos\phi) = 3(1+\cos\phi)$$

$$\Rightarrow \quad 4\cos\phi = -2$$

$$\Rightarrow \quad \cos\phi = -\frac{1}{2}$$

$$\Rightarrow \quad \phi = 120^\circ = \frac{2\pi}{3} \text{ radian}$$

DIFFERENTIAL EQUATION FOR SHM

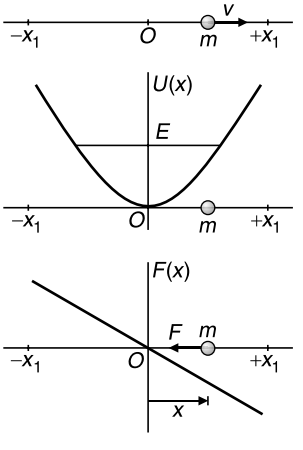
Let us consider a mass m attached to a spring of force constant k oscillating along a straight line. The potential energy of this mass varies with x as

$$U(x) = \frac{1}{2}kx^2 \qquad \ldots(1)$$

The force F acting on the particle is given by

$$F = -\frac{dU}{dx} = -\frac{k}{2}\frac{d}{dx}\left(x^2\right) = -kx \qquad \ldots(2)$$

Such an oscillatory motion in which restoring force acting on the particle is directly proportional to the small displacement x from the equilibrium position is called Simple Harmonic Motion. The potential energy function of such a particle is represented by a symmetric curve as shown in the Figure.



Note that the limits of oscillation are equally spaced about the equilibrium position. Applying Newton's Second Law in equation (2), we get

$$F = ma = -kx$$

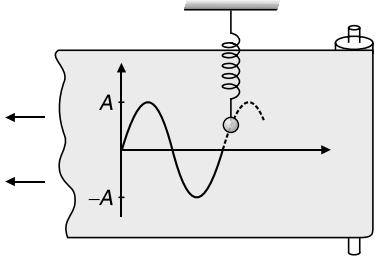
Since acceleration is $a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = \ddot{x}$

$$\Rightarrow \quad m\frac{d^2x}{dt^2} = -kx$$

$$\Rightarrow \quad \frac{d^2x}{dt^2} + \left(\frac{k}{m}\right)x = 0 \qquad \ldots(3)$$

This equation (3) is called the **characteristic differential equation of SHM**. It gives a relation between a function of the time $x(t)$ and its second derivative $\frac{d^2x}{dt^2}$ i.e., acceleration $(= \ddot{x})$. To find the position of the particle as a function of the time, we must find a function $x(t)$ which satisfies this relation.

It is obvious from a simple experiment in which an oscillating particle traces a sinusoidal curve on a moving strip of paper as shown in Figure.



In general, the equation of a simple harmonic motion may be represented by any of the following functions

$$x = A\sin(\omega t + \phi)$$

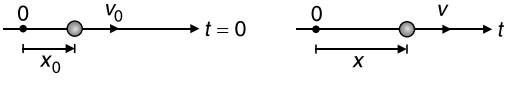
$$x = A\cos(\omega t + \phi)$$

$$x = A\sin(\omega t) + B\cos(\omega t)$$

All the above three equations are the solution to the differential equation $\ddot{x} + \omega^2 x = 0$

EQUATION OF MOTION OF A SIMPLE HARMONIC MOTION

Let us consider a particle of mass m (moving along the x direction) on which a force $F = -kx$ acts, where k is a positive constant and x is the displacement of the particle from the assumed origin as shown in Figure.



The particle then executes SHM with the centre of oscillation i.e., mean position at the origin. We wish to calculate the displacement x and the velocity v of the particle as function of time. Initially, let position of particle be x_0 and its velocity be v_0, so at $t = 0$, we have $x = x_0$ and $v = v_0$. The acceleration a of the particle at any instant is given by

$$a = \frac{F}{m} = -\left(\frac{k}{m}\right)x = -\omega^2 x, \text{ where } \omega = \sqrt{\frac{k}{m}}$$

$$\Rightarrow \quad \frac{dv}{dt} = v\frac{dv}{dx} = -\omega^2 x \qquad \left\{\because \frac{dv}{dt} = v\frac{dv}{dx}\right\}$$

$$\Rightarrow \quad vdv = -\omega^2 x dx$$

Integrating within appropriate limits, we get

$$\int_{v_0}^{v} vdv = \int_{x_0}^{x} -\omega^2 x dx$$

$$\Rightarrow \quad \left(\frac{v^2}{2}\right)\Big|_{v_0}^{v} = -\omega^2\left(\frac{x^2}{2}\right)\Big|_{x_0}^{x}$$

$$\Rightarrow \quad v^2 - v_0^2 = -\omega^2\left(x^2 - x_0^2\right)$$

$$\Rightarrow \quad v^2 = v_0^2 + \omega^2 x_0^2 - \omega^2 x^2$$

$$\Rightarrow \quad v = \sqrt{\left(v_0^2 + \omega^2 x_0^2\right) - \omega^2 x^2}$$

$$\Rightarrow \quad v = \omega\sqrt{\left(\frac{v_0^2}{\omega^2} + x_0^2\right) - x^2}$$

Since v_0, x_0, ω are constants, so we can write

$$\left(\frac{v_0}{\omega}\right)^2 + x_0^2 = A^2 \qquad \ldots(1)$$

The above equation becomes

$$v = \omega\sqrt{A^2 - x^2} \qquad \ldots(2)$$

$$\Rightarrow \quad \frac{dx}{dt} = \omega\sqrt{A^2 - x^2} \qquad \left\{\because v = \frac{dx}{dt}\right\}$$

$$\Rightarrow \quad \frac{dx}{\sqrt{A^2 - x^2}} = \omega dt \qquad \ldots(3)$$

Since, the displacement is x_0 initially and at time t the displacement is x, so on integrating equation (3), we get

$$\int_{x_0}^{x} \frac{dx}{\sqrt{A^2 - x^2}} = \int_{0}^{t} \omega dt$$

Since $\int \frac{dx}{\sqrt{A^2 - x^2}} = \sin^{-1}\left(\frac{x}{A}\right)$

$$\Rightarrow \quad \left[\sin^{-1}\left(\frac{x}{A}\right)\right]\Big|_{x_0}^{x} = \omega t\Big|_{0}^{t}$$

$$\Rightarrow \quad \sin^{-1}\left(\frac{x}{A}\right) - \sin^{-1}\left(\frac{x_0}{A}\right) = \omega t \qquad \ldots(4)$$

Writing $\sin^{-1}\left(\frac{x_0}{A}\right) = \delta$, equation (4) becomes

$$\sin^{-1}\left(\frac{x}{A}\right) = \omega t + \delta$$

$$\Rightarrow \quad x = A\sin(\omega t + \delta) \qquad \ldots(5)$$

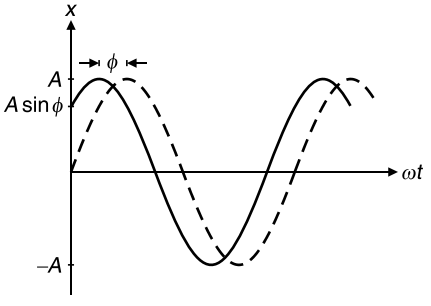
The velocity at time t is

$$v = \frac{dx}{dt} = A\omega\cos(\omega t + \delta) \qquad \ldots(6)$$

In equations (5) and (6), δ is the initial phase (can also be represented by ϕ_0) and $\omega t + \delta = \phi$ is called instantaneous phase.

CHARACTERISTICS OF SHM

In equation $x = A\sin(\omega t + \phi)$, the argument $(\omega t + \phi)$ is called the **phase**, where ϕ is called the phase constant. Both the phase and the phase constant are measured in radians. The value of ϕ depends on the position wherefrom we start measuring time.



The characteristic differential equation i.e., $\ddot{x} + \omega^2 x = 0$ of SHM does not represent one single motion but a group or family of possible motions which have some features in common but differ in other ways. In this case ω is common to all the allowed motions, but A and ϕ may differ among them. The amplitude A and the phase constant ϕ of the oscillation are determined by the initial position and speed of the particle. These two initial conditions will specify A and ϕ exactly. One very important distinctive feature of SHM is the relation between the displacement, velocity and acceleration of oscillatory particle.

$$x = A\sin(\omega t)$$

$$v = \frac{dx}{dt} = \omega A\cos(\omega t) = \omega A\sin\left(\omega t + \frac{\pi}{2}\right)$$

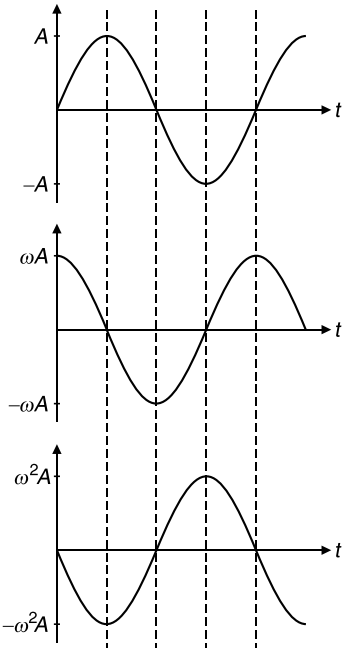
$$a = \frac{d^2x}{dt^2} = -\omega^2 A\sin\omega t = \omega^2 A\sin(\omega t + \pi)$$

Substituting value of x in above equation, we get

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

$$\Rightarrow \quad \frac{d^2x}{dt^2} + \omega^2 x = 0$$

This is the standard characteristic differential equation of SHM. Notice that the maximum displacement is A, the maximum speed is A and the maximum acceleration is $\omega^2 A$. The displacement, velocity and acceleration versus time graphs have been plotted in Figure.



The velocity function is $\frac{\pi}{2}$ ahead of the displacement function and the acceleration function is $\frac{\pi}{2}$ ahead of the velocity function.

CONDITION FOR MOTION TO BE SHM

For SHM is to occur, following three conditions must be satisfied.

(a) There must be a position of **stable equilibrium** also called as **MEAN POSITION.**
At the stable equilibrium potential energy is minimum i.e., $\frac{dU}{dx} = 0$ and $\frac{d^2U}{dx^2} > 0$

(b) There must be **no dissipation** of energy

(c) The acceleration is proportional to the displacement and opposite in direction, i.e.

$$a = -\omega^2 x$$

ILLUSTRATION 8

If a particle moves in a potential energy field $U = U_0 - ax + bx^2$, where a and b are positive constants, obtained an expression for the force acting on it as a function of position. At what point does the force vanish? Is this a point of stable equilibrium? Also Calculate the force constant and frequency of the particle.

SOLUTION

Since, the force and the potential energy are related to each other by the relation

$$F = -\frac{dU}{dx} = a - 2bx$$

$\Rightarrow$ $F = 0$, at $x = \frac{a}{2b}$

$\Rightarrow$ $\frac{d^2U}{dx^2} = 2b > 0$

i.e., $x = \frac{a}{2b}$ is a point of minimum potential energy. Hence, the equilibrium is stable. So, $k = 2b$ and

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{2b}{m}}$$

VELOCITY OF A PARTICLE IN SHM

Let, $x = A\sin(\omega t)$, then velocity of a particle is

$$v = \frac{dy}{dt} = \frac{d}{dt}[A\sin(\omega t)]$$

$\Rightarrow$ $v = A\omega\cos(\omega t)$

$\Rightarrow$ $v = v_0\cos(\omega t)$

where, the velocity amplitude is

$$v_0 = A\omega$$

Also, $v = A\omega\cos(\omega t)$

$\Rightarrow$ $v = A\omega\sqrt{1-\sin^2(\omega t)}$

$\Rightarrow$ $v = A\omega\sqrt{1-\frac{x^2}{A^2}} = \omega\sqrt{A^2 - x^2}$

$\Rightarrow$ $v^2 = \omega^2A^2 - \omega^2x^2$

$\Rightarrow$ $v^2 + \omega^2x^2 = \omega^2A^2$

$\Rightarrow$ $\frac{v^2}{\omega^2A^2} + \frac{x^2}{A^2} = 1$

which is the equation of an ellipse.

Conceptual Note(s)

Velocity vs Displacement curve must be an ellipse

ILLUSTRATION 9

A point moves along the x-axis according to the law $x = a\sin^2\left(\omega t - \frac{\pi}{4}\right)$. Calculate the amplitude and period of oscillations and draw the plot x vs t. Also calculate the velocity projection v_x as a function of the coordinate x and draw the plot v_x vs x.

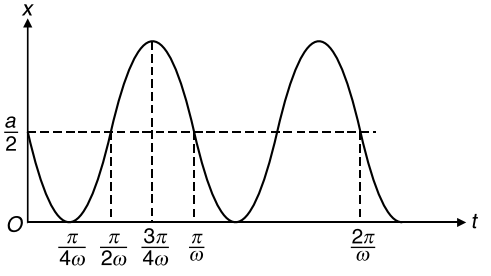
SOLUTION

Given equation is $x = a\sin^2\left(\omega t - \frac{\pi}{4}\right)$

Since $\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$\Rightarrow$ $x = \frac{a}{2} - \frac{a}{2}\cos\left(2\omega t - \frac{\pi}{2}\right)$ …(1)

From equation (1) the amplitude of SHM is $\frac{a}{2}$, time period is $\frac{\pi}{\omega}$ with mean position at $x = \frac{a}{2}$. Plot of x vs t is shown in Figure.



Differentiating equation (1), we get

$$v = \frac{a}{2}\times 2\omega\sin\left(2\omega t - \frac{\pi}{2}\right) = a\omega\sin\left(2\omega t - \frac{\pi}{2}\right)$$

$\Rightarrow$ $v = a\omega\sqrt{1-\cos^2\left(2\omega t - \frac{\pi}{2}\right)} = a\omega\sqrt{1-\frac{\left(\frac{a}{2}-x\right)^2}{(a/2)^2}}$

$\Rightarrow$ $v_x = 2\omega\sqrt{\left(\frac{a}{2}\right)^2 - \left(\frac{a}{2}-x\right)^2}$

$\Rightarrow$ $v_x^2 = 4\omega^2x(a-x)$

$\Rightarrow$ $\left(\frac{v_x}{2\omega}\right)^2 + \left(\frac{a}{2}-x\right)^2 = \left(\frac{a}{2}\right)^2$

$\Rightarrow$ $\frac{v_x^2}{(a\omega)^2} + \left(\frac{\frac{a}{2}-x}{a/2}\right)^2 = 1$ …(2)

Equation (2) is an equation of ellipse as shown in Figure.

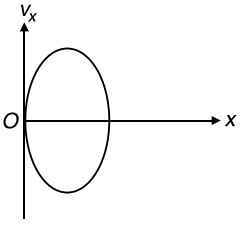


ILLUSTRATION 10

A particle executes SHM in a straight line. The maximum speed of the particle during its motion is v_m. Find the average speed of the particle during its SHM.

SOLUTION

Let $x = A\sin(\omega t)$, so $v = A\omega\cos(\omega t)$

$$\Rightarrow \quad v_{av} = \frac{\int_0^T v\,dt}{\int_0^T dt} = \frac{A\omega}{T}\int_0^T \cos(\omega t)\,dt$$

$$\Rightarrow \quad v_{av} = \frac{\omega A}{T}\left[\left|\int_0^{T/2} \cos(\omega t)\,dt\right| + \left|\int_{T/2}^{T} \cos(\omega t)\,dt\right|\right]$$

$$\Rightarrow \quad v_{av} = \frac{\omega A}{\omega T}(2+2) = \frac{4A}{T} = \frac{4A\omega}{2\pi} = \frac{2A\omega}{\pi}$$

Since, $v_{\max} = v_m = A\omega$

$$\Rightarrow \quad v_{av} = \frac{2A\omega}{\pi} = \frac{2v_m}{\pi}$$

POTENTIAL ENERGY OF A PARTICLE IN SHM

The potential energy U at a displacement x is the work done against the restoring force in moving the body from the mean position to this position.

$$PE = U = \int_0^y kx\,dx = \frac{1}{2}kx^2$$

$$\Rightarrow \quad U = \frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2A^2\sin^2(\omega t)$$

The maximum value of potential energy is $U_{\max} = \frac{1}{2}m\omega^2A^2$ at the extreme position i.e., at $x = \pm A$ and minimum value of potential energy is $U_{\min} = 0$ at the mean position i.e., at $x = 0$.

KINETIC ENERGY OF A PARTICLE IN SHM

If $x = A\sin(\omega t)$, then $v = A\omega\cos(\omega t)$, so kinetic energy K of a particle executing SHM is given by

$$KE = K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2A^2\cos^2(\omega t)$$

$$\Rightarrow \quad K = \frac{1}{2}m\omega^2\left(A^2 - x^2\right)$$

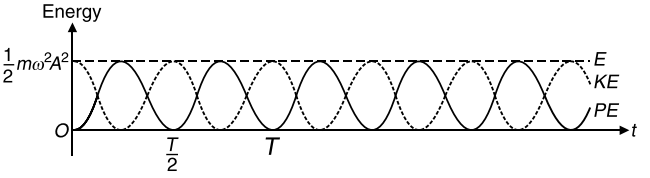
The maximum value of kinetic energy is $K_{\max} = \frac{1}{2}m\omega^2A^2$ at the mean position i.e., at $x = 0$ and minimum value of kinetic energy is $K_{\min} = 0$ at the extreme position i.e., at $x = \pm A$.

MECHANICAL ENERGY OF A PARTICLE IN SHM

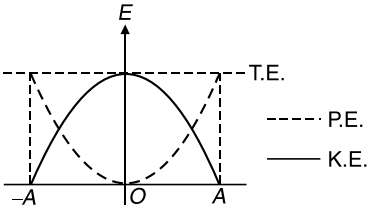
The mechanical energy E of a particle executing SHM is

$$E = K + U = \frac{1}{2}m\omega^2A^2 = \text{constant}$$

So, we observe that total mechanical energy is a constant. The variations of U, K and E with time are shown in Figure.



The variations of U, K and E with displacement are shown in Figure.



At $x = 0$, $U = 0$ and the energy is purely kinetic i.e., $E = K_{\max} = \frac{1}{2}m(\omega A)^2$. At extreme points or turning points of the SHM, kinetic energy is zero and the energy is purely potential i.e., $E = U_{\max} = \frac{1}{2}m(\omega A)^2$.

Note that the frequency of vibration of the kinetic and potential energy is twice that of the frequency of oscillation.

The instantaneous total mechanical energy of the spring-mass system may be written as

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{constant}$$

Differentiating it w.r.t. time, we get

$$\frac{dE}{dt} = \frac{1}{2}m\frac{d}{dt}\left(v^2\right) + \frac{1}{2}k\frac{d}{dx}\left(x^2\right)$$

$$\Rightarrow \quad 0 = mv\frac{dv}{dt} + kx\frac{dx}{dt}$$

Since, $v = \frac{dx}{dt}$ and $\frac{dv}{dt} = \frac{d^2x}{dt^2}$, therefore,

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

This is the differential equation of SHM.

ILLUSTRATION 11

In an SHM, at the initial moment of time, the particle's displacement is 4 cm and its velocity is –3 ms^{-1}. The particle's mass is 4 kg and its total energy 50 J. Write down the equation of the SHM and find the distance travelled by the particle in 0.314 s from the start.

SOLUTION

Total energy of the particle is 50 J, so

$$\frac{1}{2}m\omega^2A^2 = 50$$

$$\Rightarrow \quad A\omega = \sqrt{\frac{2\times 50}{4}} = 5$$

Since $v^2 = \omega^2 A^2 - \omega^2 x^2$

$$\Rightarrow \quad (-3)^2 = 25 - \omega^2 x^2$$

$$\Rightarrow \quad \omega^2 x^2 = 25 - 9$$

$$\Rightarrow \quad \omega x = \sqrt{16} = 4$$

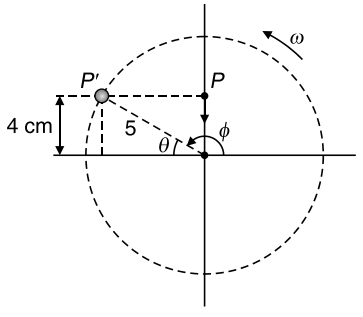
$$\Rightarrow \quad \omega(0.04) = 4$$

$$\Rightarrow \quad \omega = 100 \text{ rads}^{-1}$$

Since $A\omega = 5$

$$\Rightarrow \quad A = \frac{5}{100} = 0.05 \text{ m} = 5 \text{ cm}$$

To calculate the initial phase, we shall map the SHM on a circle as shown in Figure.



Since $\sin\theta = \frac{4}{5}$, i.e. $\theta = 53°$

So, initial phase is $\phi = 180° - 53° = 127°$

Also, $180° = \pi$ radian

$$\Rightarrow \quad \phi = 127 \times \frac{\pi}{180} \text{ radian}$$

$$\Rightarrow \quad \phi = 2.2 \text{ radian}$$

Equation of SHM is given by

$$y = 0.05\sin(100t + 2.2) \text{ metre}$$

Time period of SHM is $T = \frac{2\pi}{\omega} = 0.0628$ s

Number of oscillations in 0.5 s are

$$N = \frac{0.314}{0.0628} = 5$$

Hence, total distance travelled in 5 oscillations is

$$l = 5(4A) = 20A = 20(0.05) = 1 \text{ m}$$

ILLUSTRATION 12

A particle of mass m is oscillating in SHM along x-axis about its mean position O with angular frequency ω and amplitude A. At the instant when the particle is passing the position $x = \frac{\sqrt{3}A}{2}$ and going away from O, an impulsive blow is given to it in the direction of motion. The impulse of this blow is of magnitude $J = mA\omega$. Calculate the new amplitude of vibration in terms of A.

SOLUTION

Velocity of particle at $x = \frac{\sqrt{3}A}{2}$ is

$$v = \omega\sqrt{A^2 - x^2} = \frac{A\omega}{2}$$

Applying impulse momentum theorem, we get

$$J = mv_f - mv$$

$$\Rightarrow \quad mA\omega = mv_f - m\left(\frac{A\omega}{2}\right)$$

$$\Rightarrow \quad v_f = \frac{3}{2}A\omega$$

If A' be the new amplitude, then we have

$$v_f = \omega\sqrt{A'^2 - x^2}$$

$$\Rightarrow \quad \frac{3}{2}A\omega = \omega\sqrt{A'^2 - \frac{3}{4}A^2}$$

$$\Rightarrow \quad \frac{9}{4}A^2 = \left(A'^2 - \frac{3}{4}A^2\right)$$

$$\Rightarrow \quad A'^2 = \left(\frac{9}{4} + \frac{3}{4}\right)A^2 = 3A^2$$

$$\Rightarrow \quad A' = \sqrt{3}A$$

ILLUSTRATION 13

The potential energy of a particle oscillating on x-axis is given as

$$U = 20 + (x-2)^2$$

Here U is in joules and x in metres. Total mechanical energy of the particle is 36 J.

(a) State whether the motion of the particle is simple harmonic or not.
(b) Find the mean position.
(c) Find the maximum kinetic energy of the particle.

SOLUTION

(a) $F = -\frac{dU}{dx} = -2(x-2)$

Let us substitute $x - 2 = X$, then we get

$F = -2X$

Since, $F \propto -X$

The motion of the particle is simple harmonic

(b) The mean position of the particle is $X = 0$ or $x - 2 = 0$, which gives $x = 2$ m

(c) Maximum kinetic energy of the particle is,

$K_{max} = E - U_{min} = 36 - 20$

$K_{max} = 16$ J

U_{min} is 20 J at mean position or at $x = 2$ m

Problem Solving Technique(s)

CALCULATING ANGULAR FREQUENCY OF A PARTICLE IN SHM USING TAYLOR'S METHOD:

If mathematical function is given by $y = f(x)$, then by Taylor's theorem, we have

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \ldots \quad \ldots(1)$$

If x is taken as the displacement of a particle from its mean position and the restoring force F acting on the particle depends on this x, then this restoring force $F = f(x)$ acting on particle at a distance x from mean position can be written by using Taylor's expansion series as

$$F = f(x) = f(0) + xf(0) + \frac{x^2}{2!}f''(0) + \ldots \quad \ldots(2)$$

In this case, $f(0) = 0$ because at $x = 0$ or mean position restoring force is zero and for small displacements of particle higher powers of x can be neglected. Hence restoring force F is given by using equation (2).

$\Rightarrow \quad F = -xf'(0)$

$\Rightarrow \quad m\ddot{x} = -[f'(0)]x$

$\Rightarrow \quad \ddot{x} + \left(\frac{f'(0)}{m}\right)x = 0$

Comparing this equation with general differential equation of SHM i.e., $\ddot{x} + \omega^2 x = 0$, we get

$$\omega = \sqrt{\frac{f'(0)}{m}}$$

$$\Rightarrow \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{f'(0)}}$$

However, if mean position is at x_0 (instead of $x = 0$), then

$$\omega = \sqrt{\frac{f'(x_0)}{m}}$$

ILLUSTRATION 14

A particle of mass m is located in a uni-dimensional potential field for which the potential energy of the particle depends on the coordinate x as $U(x) = U_0(1 - \cos Cx)$; U_0 and C are constants. Using Taylor's method, calculate the period of small oscillations that the particle performs about the equilibrium position.

SOLUTION

The potential energy of the oscillating particle is given by

$$U = U_0(1 - \cos Cx)$$

Force on particle is given by

$$F = \left|\frac{dU}{dx}\right|$$

$\Rightarrow \quad F = U_0 C \sin(Cx)$

This force is given as a function of displacement of particle x from its mean position, so angular frequency of its SHM from Taylor's method is given by

$$\omega = \sqrt{\frac{f'(0)}{m}} \quad \ldots(1)$$

where, $f'(0) = \left.\frac{dF}{dx}\right|_{x=0} = \left.U_0 C^2 \cos(Cx)\right|_{x=0}$

$\Rightarrow \quad f'(0) = U_0 C^2$

Substituting in equation (1), we get

$$\omega = \sqrt{\frac{U_0 C^2}{m}}$$

$\Rightarrow \quad T = 2\pi\sqrt{\frac{m}{U_0 C^2}}$

ILLUSTRATION 15

The potential energy (U) of a particle varies as a function of x-coordinate in a given force field as $U = \frac{a}{x^2} - \frac{b}{x}$, where a and b are positive constants. Calculate the period of small oscillations of the particle about the mean position (in the field).

SOLUTION

Given that $U = \frac{a}{x^2} - \frac{b}{x}$

$\Rightarrow \quad F = \left|\frac{dU}{dx}\right| = -\frac{2a}{x^3} + \frac{b}{x^2}$

At mean position say $x = x_0$, $F = 0$

$\Rightarrow \quad -\frac{2a}{x_0^3} + \frac{b}{x_0^2} = 0$

$\Rightarrow \quad x_0 = \frac{2a}{b}$

According to Taylor's Theorem, we have

$$\omega = \sqrt{\frac{F'(x_0)}{m}} = \sqrt{\frac{F'(2a/b)}{m}}$$

Now, $F = -\frac{2a}{x^3} + \frac{b}{x^2}$

$\Rightarrow \quad F' = \frac{6a}{x^4} - \frac{2b}{x^3}$

$\Rightarrow \quad F'\left(\frac{2a}{b}\right) = \frac{6a}{(2a/b)^4} - \frac{2b}{(2a/b)^3}$

$$\Rightarrow \quad F'\left(\frac{2a}{b}\right)=\frac{3b^4}{8a^3}-\frac{b^4}{4a^3}=\frac{b^4}{8a^3}$$

$$\Rightarrow \quad \omega=\sqrt{\frac{F'(2a/b)}{m}}=\sqrt{\frac{b^4}{8ma^3}}$$

$$\Rightarrow \quad T=2\pi\sqrt{\frac{8ma^3}{b^4}}$$

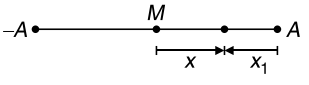
$$\Rightarrow \quad T=\frac{4\pi a}{b^2}\sqrt{2ma}$$

ILLUSTRATION 16

A particle moves with simple harmonic motion in a straight line. In the first second after starting from rest, it travels a distance x_1 and in the next second it travels a distance x_2 in the same direction. Find the amplitude of the motion.

SOLUTION

Starting from rest implies that the particle starts from extreme position as shown in Figure.



So, $x_1 = A - x_1$

$$\Rightarrow \quad x_1 = A - A\cos(\omega t)$$

So, at $t = 1$ s, $x_1 = A(1-\cos\omega)$ …(1)

Similarly, at $t = 2$ s, $x_1 + x_2 = A(1-\cos 2\omega)$ …(2)

From equation (1), we get

$$\cos\omega = \frac{A-x_1}{A}$$

From equation (2), we get

$$x_1 + x_2 = 2A\sin^2\omega$$

$$\Rightarrow \quad \sin^2\omega = \frac{x_1+x_2}{2A}$$

Since, $\sin^2\omega + \cos^2\omega = 1$

$$\Rightarrow \quad \left(\frac{x_1+x_2}{2A}\right)+\left(\frac{A-x_1}{A}\right)^2 = 1$$

$$\Rightarrow \quad Ax_1 + Ax_2 + 2A^2 + 2x_1^2 - 4Ax_1 = 2A^2$$

$$\Rightarrow \quad A = \frac{2x_1^2}{3x_1 - x_2}$$

ILLUSTRATION 17

A particle executes simple harmonic motion. If the velocities at distances of 4 cm and 5 cm from the equilibrium position are 13 cms^{-1} and 5 cms^{-1} respectively, calculate the period and amplitude.

SOLUTION

Since, $v = \omega\sqrt{A^2 - x^2}$

At $x = 4$ cm, $v = 13$ cms^{-1}

$$\Rightarrow \quad 13 = \omega\sqrt{A^2 - 16} \qquad \ldots(1)$$

At $x = 5$ cm, $v = 5$ cms^{-1}

$$\Rightarrow \quad 5 = \omega\sqrt{A^2 - 25} \qquad \ldots(2)$$

Divide equation (1) by (2), we get

$$\left(\frac{13}{5}\right)^2 = \frac{A^2-16}{A^2-25}$$

$$\Rightarrow \quad 169A^2 - 4225 = 5A^2 - 80$$

$$\Rightarrow \quad 164A^2 = 4145$$

$$\Rightarrow \quad A^2 = 25.2$$

$$\Rightarrow \quad A = 5.02 \text{ cm}$$

Substituting value of A in equation (1), we get

$$13 = \omega\sqrt{A^2 - 16}$$

$$\Rightarrow \quad 13 = \omega\sqrt{25.274 - 16}$$

$$\Rightarrow \quad \omega = \frac{13}{3.312} = 3.925 \text{ rads}^{-1}$$

$$\Rightarrow \quad T = \frac{2\pi}{\omega} = \frac{6.28}{3.925} = 1.6 \text{ s}$$

ILLUSTRATION 18

At the moment $t = 0$, a particle starts moving along the x axis so that its velocity projection v_x varies as $v_x = 35\cos(\pi t)$ cms^{-1}, where t is expressed in seconds. Calculate the distance covered by this particle during 2.80 s from the start.

SOLUTION

Time period of particle is

$$T = \frac{2\pi}{\omega} = 2 \text{ s}$$

So, in 2.5 s i.e., $\frac{5T}{4}$, particle covers distance $5A$

Since, maximum speed of the particle is

$$35 = A\omega$$

$$\Rightarrow \quad A = \frac{35}{\omega} = \frac{35}{3.14} = 11.14 \text{ cm}$$

In remaining 0.3 sec, particle covers a distance s given by

$$s = 5A + x_0 \qquad \ldots(1)$$

Since $v^2 = \omega^2(A^2 - x^2)$ …(2)

where v at 2.8 s is

$$v = 35\cos(3.14 \times 2.8) = -28.22 \text{ cms}^{-1}$$

Substituting in (2), we get

$$(28.22)^2 = (3.14)^2\left[(11.14)^2 - x^2\right]$$

$$\Rightarrow \quad 80.77 = (11.14)^2 - x^2$$

$$\Rightarrow \quad x^2 = 43.33$$

$$\Rightarrow \quad x = 6.59 \text{ cm}$$

Hence distance covered is

$$x_0 = A - x = 11.14 - 6.59$$

$$\Rightarrow \quad x_0 = 4.55 \text{ cm}$$

So, from (1), we get

$$s = 5A + x_0$$

$$\Rightarrow \quad s = 5(11.14) + 4.55 = 60.25 \text{ cm}$$

ILLUSTRATION 19

In SHM, the distances of a particle from the mean position of its path at three consecutive seconds are observed to be x, y and z. Calculate the period of oscillation.

SOLUTION

Let $x = A\sin(\omega t)$, so we get

$$x = A\sin\omega \quad (\text{at } t = 1 \text{ s})$$

$$y = A\sin 2\omega \quad (\text{at } t = 2 \text{ s})$$

$$z = A\sin 3\omega \quad (\text{at } t = 3 \text{ s})$$

Now, $x + z = A(\sin\omega + \sin 3\omega)$

$$\Rightarrow \quad x + z = 2[A\sin(2\omega)]\cos\omega$$

$$\Rightarrow \quad x + z = 2y\cos\omega$$

$$\Rightarrow \quad \omega = \cos^{-1}\left(\frac{x+z}{2y}\right)$$

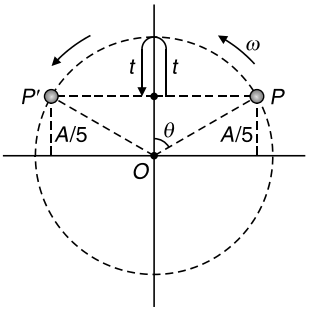
$$\Rightarrow \quad T = \frac{2\pi}{\omega} = \frac{2\pi}{\cos^{-1}\left(\frac{x+z}{2y}\right)}$$

ILLUSTRATION 20

A particle executes SHM with amplitude A and angular frequency ω. At an instant when particle is at a distance $A/5$ from mean position and moving away from it. Find the time after which it will come back to this position again and also find the time after which it will pass through mean position.

SOLUTION

The SHM is drawn on circular mapping as shown in Figure.



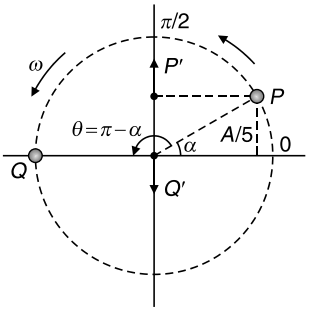
For calculating time taken by particle to return to same position, we see that

$$\theta = \cos^{-1}\left(\frac{A/5}{A}\right) = \cos^{-1}\left(\frac{1}{5}\right)$$

$$\Rightarrow \quad t = \frac{\theta}{\omega} = \frac{\cos^{-1}(1/5)}{\omega}$$

$$\Rightarrow \quad t_{P\to P'} = 2t = \frac{2\cos^{-1}(1/5)}{\omega}$$

For the particle to cross the mean position, we have again drawn SHM on circular mapping as shown in Figure.



So, $t = \frac{\theta}{\omega} = \frac{\pi - \alpha}{\omega} = \frac{\pi - \sin^{-1}(1/5)}{\omega}$

ILLUSTRATION 21

A particle performing SHM is in equilibrium at $t = 1$ second and has a speed of 0.25 ms^{-1} at $t = 2$ second. If the period of oscillation is 6 second, calculate the amplitude of oscillation, initial phase and velocity of particle at 6 second.

SOLUTION

Method-I

Let $y = A\sin(\omega t + \phi)$ be the equation of SHM, then at $t = 1$ s, $y = 0$ (because particle at mean or equilibrium)

$$\Rightarrow \quad 0 = A\sin[\omega(1) + \phi]$$

$$\Rightarrow \quad \sin\left(\frac{\pi}{3} + \phi\right) = 0$$

$$\Rightarrow \quad \phi = -\frac{\pi}{3}$$

Hence initial phase is $\phi = -\frac{\pi}{3}$ radian

Now $v = \frac{dy}{dt} = A\omega\cos(\omega t + \phi)$

At $t = 2$ s, $v = 0.25$ ms^{-1}

$$\Rightarrow \quad 0.25 = A\left(\frac{\pi}{3}\right)\cos\left(\frac{2\pi}{3} - \frac{\pi}{3}\right)$$

$$\Rightarrow \quad 0.25 = A\left(\frac{\pi}{3}\right)\left(\frac{1}{2}\right)$$

$$\Rightarrow \quad A = \frac{3}{2\pi}\text{ m}$$

At $t = 6$ s, we get

$$v = A\omega\cos(6\omega + \phi)$$

$$\Rightarrow \quad v = \left(\frac{3}{2\pi}\right)\left(\frac{\pi}{3}\right)\cos\left(\frac{5\pi}{3}\right)$$

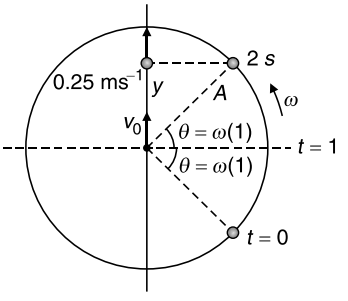
$$\Rightarrow \quad v = (0.5)(0.5) = 0.25 \text{ ms}^{-1}$$

Method-II

Given that, $\omega = \frac{2\pi}{T} = \frac{2\pi}{6} = \frac{\pi}{3}$ rads^{-1}

At $t = 1$ s, $v_0 = A\omega$

Since $\theta = \omega t = \omega(1)$, so from Figure, we get



$$0.25 = A\omega\cos\theta$$

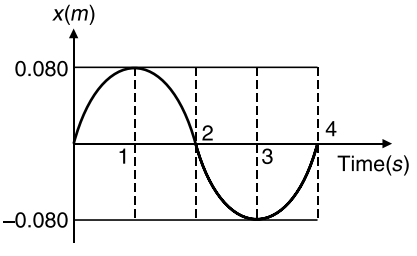
and solve further.

Test Your Concepts-I

Based on SHM Properties

(Solutions on page H.178)

1. An object of mass 0.8 kg is attached to one end of a spring and the system is set into simple harmonic motion. The displacement x of the object as a function of time is shown in the figure.



With the help of given data, calculate the
(a) amplitude A of the motion,
(b) angular frequency ω,
(c) spring constant k,
(d) speed of the object at $t = 1.0$ s and
(e) magnitude of acceleration at $t = 1.0$ s.

2. Prove that $x = Ae^{i\omega t}$ is an equation of SHM, where $i = \sqrt{-1}$.

3. Describe the motion of a particle acted upon by a force
(a) $F = -2(x-2)^3$ (b) $F = -2(x-2)^2$
(c) $F = -2(x-2)$

4. A particle in simple harmonic motion is at rest a distance of 6 cm from its equilibrium position at time $t = 0$. Its period is 2 s. Write expressions for its position x, its velocity v, and its acceleration a as functions of time.

5. When a particle is executing SHM. Calculate the ratio of mean velocity (during motion from one end of the path to the other) and the maximum velocity. Also calculate the ratio of average acceleration (during motion from one extreme to the centre) to the maximum acceleration.

6. (a) The potential energy of a harmonic oscillator of mass 2 kg in its resting position is 5 J, its total energy is 9 J and its amplitude is 1 cm. Calculate its time-period.
(b) A particle of mass 5 g lies in a potential field $V = (8x^2 + 200)$ erg per gram. Calculate its time period.

7. Two particles describe SHM of the same period and same amplitude along the same line about the same equilibrium position O. At a moment when they are at the same displacements their velocities are 1.6 ms^{-1} in opposite directions. At another moment when their displacements are equal in magnitude but on either side of O their velocities are 1.2 ms^{-1} in the same direction. Find the maximum speed of the particles and the phase difference between them.

8. A particle starts its SHM from mean position at $t = 0$. If its time period is T and amplitude A. Calculate the distance travelled by the particle in the time from $t = 0$ to $t = \frac{5T}{4}$.

9. A block is executing simple harmonic motion on a frictionless horizontal surface with a amplitude of 0.100 m. At a point 0.060 m away from equilibrium, the speed of the block is 0.360 ms^{-1}.
(a) What is the period?
(b) What is the displacement when the speed is 0.120 ms^{-1}?
(c) A small object whose mass is much less than the mass of the block is placed on the oscillating block. If the small object is just on the verge of slipping at the end point of the path, then find the coefficient of static friction between the small object and the block.

10. Two particles are in SHM with the same amplitude and frequency along the same line and about the same point. If the maximum separation between them is $\sqrt{3}$ times their amplitude, calculate the phase difference between them.

11. A particle of mass m free to move in the x-y plane is subjected to a force whose components are $F_x = -kx$ and $F_y = -ky$ where k is a constant. The particle is released when $t = 0$ at the point $(2, 3)$. Prove that the subsequent motion is simple harmonic along the straight line $2y - 3x = 0$.

12. The position of a particle is given by $x = 4\sin(2t)$, where x is in metres and t is in seconds.
(a) What is the maximum value of x and at what time first after $t = 0$ is this maximum?
(b) Find an expression for the velocity of the particle as a function of time. Also find the initial velocity of the particle.
(c) Find an expression for the acceleration of the particle as a function of time. Also find the initial and the maximum value of the acceleration of the particle.

13. Two particles executing SHM with same angular frequency and amplitudes A and $2A$ on same straight line with same mean position cross each other in opposite direction at a distance $A/3$ from mean position. Find the phase difference in the two SHMs.

14. A 0.2 kg object hangs from an ideal spring of negligible mass. When the object is pulled down 0.1 m below its equilibrium position and released, it vibrates with a period of 1.80 s.
(a) Find its speed as it passes through the equilibrium position.
(b) Find its acceleration when it is 0.050 m above the equilibrium position.
(c) Find the time required by the particle (when moving upwards) to move from a point 0.05 m below its equilibrium position to a point 0.05 m above it.
(d) If the motion of the object is stopped at the mean position and the object is removed from the spring, then find the length by which the spring shortens.

15. A linear harmonic oscillator has a total mechanical energy of 200 J Potential energy of it at mean position is 50 J. Find,
(a) the maximum kinetic energy
(b) the minimum potential energy
(c) the potential energy at extreme positions.

16. A point particle of mass 0.1 kg is executing SHM of amplitude of 0.1 m. When the particle passes through the mean position, its kinetic energy is 8×10^{-3} J. Obtain the equation of motion of this particle if this initial phase of oscillation is 45°.

17. A particle of mass m is located in a unidimensional potential field where the potential energy of the particle depends on the coordinate x as $U(x)(1-\cos bx)$; U_0 and b are constants. Find the period of small oscillations that the particle performs about the equilibrium position.

18. A tray is moved horizontally back and forth in simple harmonic motion at a frequency of $f = 2.00$ Hz. On this tray is an empty cup. Obtained the coefficient of static friction between the tray and the cup, given that the cup begins slipping when the amplitude of the motion is 5.00×10^{-2} m.

19. A 1×10^{-2} kg block is resting on a horizontal frictionless surface and is attached to a horizontal spring whose spring constant is 124 Nm^{-1}. The block is given an initial speed of 8 ms^{-1} parallel to the spring axis, while the spring is initially unstrained. What is the amplitude of the resulting simple harmonic motion?

20. The period of an oscillating particle of amplitude A is 8 s. At $t = 0$ it is in its equilibrium position.
(a) Calculate the distance it travels in the first 4 s. Compare with the distance it travels in the next 4 s?
(b) The distance travelled in the first 2 s and the next 2 s?
(c) The first second and the next second?

21. When the displacement of a body oscillating on a spring is half its amplitude, what fraction of its total energy is its kinetic energy? At what displacement are its kinetic and potential energies equal?

22. A plank with a body of mass m placed on it starts moving straight up according to the law $y = a(1-\cos\omega t)$, where y is the displacement from the initial position, $\omega = 11$ $rads^{-1}$. Find:
(a) the time dependence of the force that the body exerts on the plank.
(b) the minimum amplitude of oscillation of the plank at which the body starts falling behind the plank.

23. A particle has displacement x given by $x = 3\cos(5\pi t + \pi)$, where x is in metres and t in seconds. Find the
(a) frequency f and the period T of the motion.
(b) greatest distance the particle travels from equilibrium.
(c) position of the particle at time $t = 0$ and at time $t = \frac{1}{2}$ s.

UNDERSTANDING SHM FOR PHYSICAL SYSTEMS

Whenever we have to prove that a particular physical system follows SHM, then we proceed through the following series of steps.

STEP-1 Locate the mean position of the system.

STEP-2 Give the system a small displacement {linear or angular} from the mean position.

STEP-3 For that extremely small displacement find the restoring force (or restoring torque).

STEP-4 This restoring force (or torque) must be directed towards the mean position.

STEP-5 This restoring force (or torque) must be directly proportional to the displacement (or angular displacement) from the mean position.

On positive confirmation of all the five steps we can say that the system follows simple harmonic motion and then time period (T) is given by STEP-6.

STEP-6 $T = 2\pi\sqrt{\dfrac{x}{|\ddot{x}|}}$ OR $T = 2\pi\sqrt{\dfrac{\theta}{|\ddot{\theta}|}}$

MASS-SPRING SYSTEM

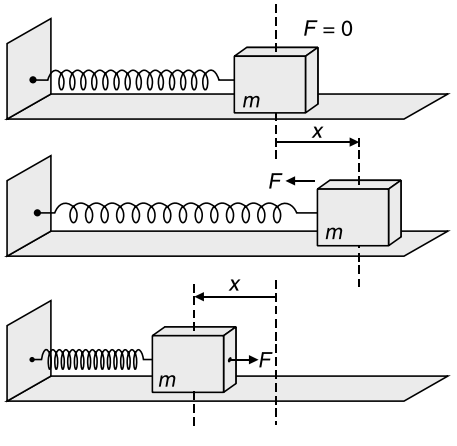
When a spring is compressed or stretched by a small amount, a restoring force is produced in it, which is proportional to the displacement x. So, we have

$$F = -kx$$

The constant k is called the **spring constant or the force constant of the spring**.

Horizontal Oscillations

Consider a light spring of constant k, one end of which is fixed rigidly to a wall and the other end is attached to a body of mass m, which is free to move on a frictionless horizontal surface. The equilibrium position of the system is shown in Figure.



When the body is pulled to the right by a deforming force (not shown in Figure), the restoring force exerted by the spring on the body is directed to the left. When the body is pushed to the left, the restoring force is directed to the right.

$$\Rightarrow \quad F = -kx$$

$$\Rightarrow \quad m\ddot{x} = -kx$$

$$\Rightarrow \quad \ddot{x} + \frac{k}{m}x = 0$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{x}{|\ddot{x}|}} = 2\pi\sqrt{\frac{m}{k}}$$

When the body is released it executes SHM with time period $T = 2\pi\sqrt{\dfrac{m}{k}}$.

Also, we know that the instantaneous total mechanical energy of the spring-mass system may be written as

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{constant}$$

Differentiating it w.r.t. time, we get

$$\frac{dE}{dt} = \frac{1}{2}m\frac{d}{dt}\left(v^2\right) + \frac{1}{2}k\frac{d}{dx}\left(x^2\right)$$

$$\Rightarrow \quad 0 = mv\frac{dv}{dt} + kx\frac{dx}{dt}$$

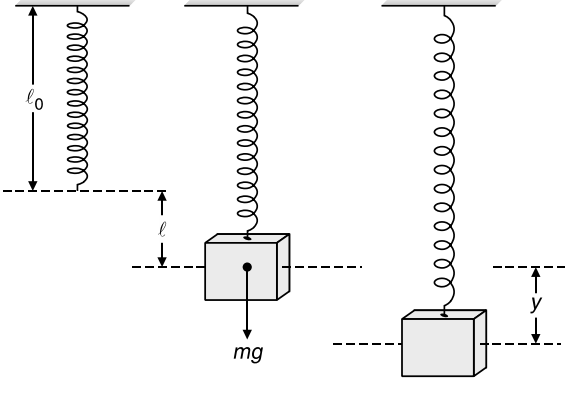
Since, $v = \dfrac{dx}{dt}$ and $\dfrac{dv}{dt} = \dfrac{d^2x}{dt^2}$, therefore,

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

This is the differential equation of SHM.

Vertical Oscillations

Consider a light spring suspended vertically from a fixed support, having a mass m connected to its lower end as shown in Figure.



In this case the equilibrium position of the spring is that position in which the spring is stretched by a length l such that the restoring force balances the weight mg. Hence,

$$kl = mg$$

$$\Rightarrow \quad k = \frac{mg}{l}$$

When the body is pulled further from this position through a distance y, it executes SHM. If F be the restoring force, then

$$F = k(l + y) - Mg$$

$$\Rightarrow \quad F = kl + ky - Mg$$

$$\Rightarrow \quad F = Mg + ky - Mg$$

$$\Rightarrow \quad F = -ky$$

Negative sign indicates the restoring nature of force.

$$\Rightarrow \quad m\ddot{y} = -ky$$

$$\Rightarrow \quad \ddot{y} + \frac{k}{m}y = 0$$

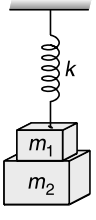
$$\Rightarrow \quad T = 2\pi\sqrt{\frac{y}{|\ddot{y}|}} = 2\pi\sqrt{\frac{m}{k}}$$

Conceptual Note(s)

It should be noted that the time period in vertical oscillations is same as that in horizontal oscillations. It does not depend on g.

ILLUSTRATION 22

Two masses m_1 and m_2 are suspended together by a massless spring of spring constant k. When the masses are in equilibrium m_1 is removed without disturbing the system. Find the angular frequency and amplitude of oscillation of m_2.



SOLUTION

As m_1 is removed, the mass m_2 will oscillate and so

$$T = 2\pi\sqrt{\frac{m_2}{k}}, \text{ i.e., } \omega = \frac{2\pi}{T} = \sqrt{\frac{k}{m_2}}$$

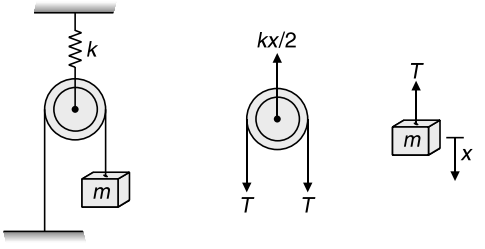
Furthermore, the stretch produced by $m_1 g$ will set an amplitude,
i.e., $m_1 g = kA$ i.e., $A = \left(\frac{m_1 g}{k}\right)$

ILLUSTRATION 23

For the arrangement shown in the figure, find the period of oscillation.

SOLUTION

Obviously, when the block is displaced down by x, the spring will stretch by $\frac{x}{2}$.



From the free body diagram of the pulley,

$$\frac{kx}{2} = 2T$$

$$\Rightarrow \quad T = \frac{kx}{4}$$

The net restoring force on the block is T.
Using the Second Law of motion, we get

$$m\frac{d^2x}{dt^2} = -T = -\frac{kx}{4}$$

$$\Rightarrow \quad \frac{d^2x}{dt^2} + \left(\frac{k}{4m}\right)x = 0$$

Thus, the period of SHM is given by

$$T = 2\pi\sqrt{\frac{4m}{k}}$$

Note that we have not taken gravity into account as it does not affect the time period.

ILLUSTRATION 24

A horizontal spring block system of (force constant k) and mass Mexecutes SHM with amplitude A. When the block is passing through its equilibrium position an object of mass m is put on it and the two moves together. Find the new amplitude and frequency of vibration.

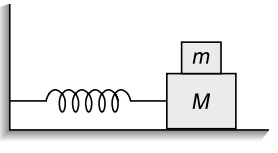
SOLUTION

Since initially mass M and finally $(m+M)$ is oscillating, so we have

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{M}} \text{ and } f' = \frac{1}{2\pi}\sqrt{\frac{k}{(m+M)}}$$

$$\Rightarrow \quad \frac{f'}{f} = \sqrt{\frac{M}{(m+M)}}$$

$$\Rightarrow \quad f' = f\sqrt{\frac{M}{(m+M)}} \qquad \ldots(1)$$



Now by Conservation of Linear Momentum

$$Mv = (m+M)V'$$

At equilibrium, $v = A\omega = 2\pi Af$ $\{\because \omega = 2\pi f\}$

$$M(2\pi fA) = (m+M)2\pi f'A'$$

$$\Rightarrow \quad \frac{A'}{A} = \left(\frac{M}{m+M}\right)\left(\frac{f}{f'}\right)$$

$$\Rightarrow \quad A' = A\sqrt{\frac{M}{(m+M)}} \qquad \{\text{from equation (1)}\}$$

ILLUSTRATION 25

A 2 kgmass is attached to a spring of force constant 600 Nm^{-1} and rests on a smooth horizontal surface. A second mass of 1 kg slides along the surface toward the first at 6 ms^{-1}.

(a) Find the amplitude of oscillation if the masses make a perfectly inelastic collision and remain together on the spring. What is the period of oscillation?

(b) Find the amplitude and period of oscillation if the collision is perfectly elastic.

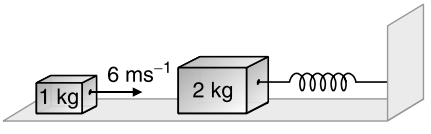
(c) For each case, write down the position x as a function of time t for the mass attached to the spring, assuming that the collision occurs at time $t = 0$. What is the impulse given to the 2 kg mass in each case?

SOLUTION

(a) From Conservation of Linear Momentum,

$$1\times 6 = (1+2)v$$

$$\Rightarrow \quad v = 2 \text{ ms}^{-1}$$



By Law of Conservation of Mechanical Energy, we have

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{1}{2}kA^2$$

$$\Rightarrow \quad A = \left(\sqrt{\frac{m}{k}}\right)v = \left(\sqrt{\frac{3}{600}}\right)\times 2$$

$$\Rightarrow \quad A = 0.141 \text{ m} = 14.1 \text{ cm}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{3}{600}} = 0.44 \text{ s}$$

(b) For perfectly elastic collision, we have

$$v_2' = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_2 + \left(\frac{2m_1}{m_1 + m_2}\right)v_1$$

$$v_2' = 0 + \left(\frac{2\times 1}{1+2}\right)\times 6 = 4 \text{ ms}^{-1}$$

$$\Rightarrow \quad A = \left(\sqrt{\frac{m}{k}}\right)v_2' = 4\sqrt{\frac{2}{600}}$$

$$\Rightarrow \quad A = 0.23 \text{ m} = 23 \text{ cm}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{2}{600}} = 0.36 \text{ s}$$

(c) In the first case, $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.44} = 14.28 \text{ rads}^{-1}$

and amplitude $A = 14.1$ cm

$$\Rightarrow \quad x = A\sin\omega t = (14.1 \text{ cm})\sin(14.28t)$$

In the second case, $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.36} = 17.45 \text{ rads}^{-1}$

and amplitude $A = 23$ cm

$$\Rightarrow \quad x = 23\sin(17.45t) \text{ cm}$$

Impulse, $J_1 = \Delta P = 2\times 2 = 4$ Ns in the first case

and $J_2 = 4\times 2 = 8$ Ns in the second case.

ILLUSTRATION 26

A 2 kg block is attached to a spring for which $k = 200 \text{ Nm}^{-1}$. It is held at an extension of 5 cm and then released at $t = 0$. Find

(a) the displacement as a function of time

(b) the velocity when $x = +\frac{A}{2}$

(c) the acceleration when $x = +\frac{A}{2}$

SOLUTION

(a) We need to find a, ω, and ϕ in equation. The amplitude is the maximum extension; that is, $A = 0.05$ m

We know the angular frequency of the spring-mass system is given by $\omega = \sqrt{\frac{k}{m}} = 10 \text{ rads}^{-1}$

To find ϕ we note that at $t = 0$ we are given $x = +A$ and $v = 0$.

Thus, from the equation of displacement and velocity, we get

$$x = A\sin(\omega t + \phi)$$

$$\Rightarrow \quad A\sin(0+\phi)$$

$$v = \omega A(\omega t + \phi)$$

$$\Rightarrow \quad 0 = 10A\cos(0+\phi)$$

Since $\sin\phi = 1$ and $\cos\phi = 0$, it follows that $\phi = \frac{\pi}{2}$ rad.

Thus, $x = 0.05\sin\left(10t + \frac{\pi}{2}\right) m$ …(1)

(b) In order to find the velocity, we have to find the time t, when $x = \frac{A}{2}$. Equation (1) yields $\frac{1}{2} = \sin\left(10t + \frac{\pi}{2}\right)$, from which we infer that

$$\left(10t+\frac{\pi}{2}\right)=\frac{\pi}{6} \quad \text{OR} \quad \left(10t+\frac{\pi}{2}\right)=\frac{5\pi}{6}$$

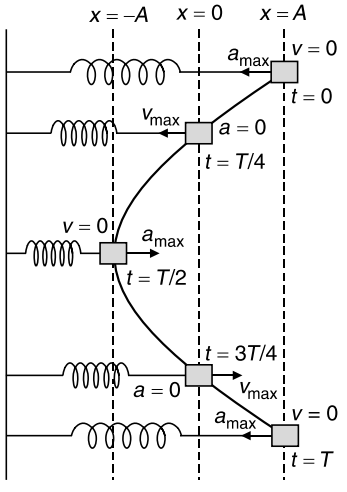
The velocity is given by

$$v=\frac{dx}{dt}=0.5\cos\left(10t+\frac{\pi}{2}\right)$$

$$\Rightarrow \quad v=0.5\cos\left(\frac{\pi}{6}\right)=+0.43 \text{ ms}^{-1} \quad \text{OR}$$

$$v=0.5\cos\left(\frac{5\pi}{6}\right)=-0.43 \text{ ms}^{-1}$$

At a given position, there are two velocities of equal magnitude but opposite directions.



(c) The acceleration at $x=\frac{A}{2}$ may be found from the equation,

$$a=-\frac{k}{m}x=-\omega^2 x=-(10 \text{ rads}^{-1})^2\left(\frac{0.05}{2} \text{ m}\right)$$

$$a=-2.5 \text{ ms}^{-2}$$

SHM OF FREE BODIES IN ABSENCE OF EXTERNAL FORCES

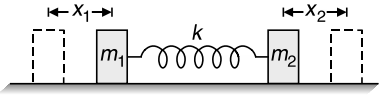
We know for oscillations of a body restoring force must be there due to which the body oscillates about its mean position. In some special cases, it is possible that a system oscillates due to only internal forces and internal forces of system provide the required centripetal force for oscillations. We take an illustrative example to explain such situation.

ILLUSTRATION 27

Two masses m_1 and m_2 are connected by a spring of force constant k and are placed on a frictionless horizontal surface. Show that if the masses are displaced slightly in opposite directions and released, the system will execute simple harmonic motion. Calculate the frequency of oscillation.

SOLUTION

Let masses m_1 and m_2 be displaced by x_1 and x_2 respectively from their equilibrium position in opposite direction so that the total extension in the spring will be $x=x_1+x_2$. Due to this stretch a restoring force kx will act on each mass and so equation of mass m_1 will be



$$m_1\frac{d^2x_1}{dt^2}=-kx, \text{ i.e., } \frac{d^2x_1}{dt^2}=-\frac{k}{m_1}x \qquad \ldots(1)$$

while that for m_2 will be

$$m_2\frac{d^2x_2}{dt^2}=-kx, \text{ i.e., } \frac{d^2x_2}{dt^2}=-\frac{k}{m_2}x \qquad \ldots(2)$$

But as $x=x_1+x_2$, i.e., $\frac{d^2x}{dt^2}=\frac{d^2x_1}{dt^2}+\frac{d^2x_2}{dt^2}$ …(3)

So, substituting equation (1) and (2) in (3)

$$\frac{d^2x}{dt^2}=-\left(\frac{1}{m_1}+\frac{1}{m_2}\right)kx$$

$$\Rightarrow \quad \frac{d^2x}{dt^2}=-\frac{k}{m}x \text{ where } \frac{1}{m}=\frac{1}{m_1}+\frac{1}{m_2}$$

$$\Rightarrow \quad \frac{d^2x}{dt^2}=-\omega^2 x \text{ where } \omega^2=\frac{k}{m}$$

$$f=\frac{1}{2\pi}\sqrt{\frac{k}{m}}=\frac{1}{2\pi}\sqrt{\frac{k(m_1+m_2)}{m_1m_2}}$$

ILLUSTRATION 28

A block with a mass of 2 kg hangs without vibrating at the end of a spring of spring constant 500 Nm^{-1}, which is attached to the ceiling of an elevator. The elevator is moving upwards with an acceleration $\frac{g}{3}$. At time $t=0$, the acceleration suddenly ceases.

(a) What is the angular frequency of oscillation of the block after the acceleration ceases?

(b) By what amount is the spring stretched during the time when the elevator is accelerating?

(c) What is the amplitude of oscillation and initial phase angle observed by a rider in the elevator?

Take the upward direction to be positive. take $g=10 \text{ ms}^{-2}$.

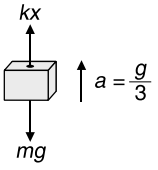
SOLUTION

(a) Angular frequency $\omega=\sqrt{\frac{k}{m}}$

$$\Rightarrow \quad \omega=\sqrt{\frac{500}{2}}$$

$$\Rightarrow \quad \omega=15.81 \text{ rads}^{-1}$$

(b) Equation of motion of the block (white elevator is accelerating) is,

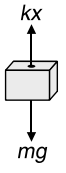


$$kx - mg = ma = m\frac{g}{3}$$

$$\Rightarrow \quad x = \frac{4mg}{3k} = \frac{(4)(2)(10)}{(3)(500)} = 0.053 \text{ m}$$

$$\Rightarrow \quad x = 5.3 \text{ cm}$$

(c) (i) In equilibrium when the elevator has zero acceleration, the equation of motion is,



$$kx_0 = mg$$

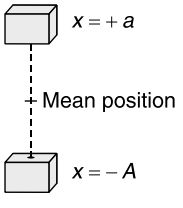
$$\Rightarrow \quad x_0 = \frac{mg}{k} = \frac{(2)(10)}{500} = 0.04 \text{ m}$$

$$\Rightarrow \quad x_0 = 4 \text{ cm}$$

So, amplitude $A = x - x_0 = 5.3 - 4.0$

$$\Rightarrow \quad A = 1.3 \text{ cm}$$

(ii) At time $t = 0$, block is at $x = -A$. Therefore, substituting $x = -A$ and $t = 0$ in equation, we get



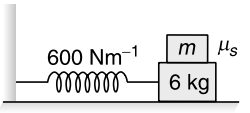
$$x = A\sin(\omega t + \phi)$$

So, the initial phase is

$$\phi = \frac{3\pi}{2}$$

ILLUSTRATION 29

With the assumption of no slipping, determine the mass m of the block which must be placed on the top of a 6 kg cart in order that the system period is 0.75 s. What is the minimum coefficient of static friction μ_s for which the block will not slip relative to the cart if the cart is displaced 50 mm from the equilibrium position and released. Take $g = 9.8 \text{ ms}^{-2}$.



SOLUTION

Since, $T = 2\pi\sqrt{\frac{m+6}{600}}$ $\left\{\because\ T = 2\pi\sqrt{\frac{m}{k}}\right\}$

$$\Rightarrow \quad 0.75 = 2\pi\sqrt{\frac{m+6}{600}}$$

$$\Rightarrow \quad m = \frac{(0.75)^2 \times 600}{(2\pi)^2} - 6 = 2.55 \text{ kg}$$

Maximum acceleration of SHM of amplitude A is

$$a_{\max} = \omega^2 A$$

So, maximum force on mass m is $m\omega^2 A$ which is being provided by the force of friction between the mass and cart. Therefore,

$$\mu_s mg \geq m\omega^2 A$$

$$\Rightarrow \quad \mu_s \geq \frac{\omega^2 A}{g}$$

$$\Rightarrow \quad \mu_s \geq \left(\frac{2\pi}{T}\right)^2 \frac{A}{g}$$

$$\Rightarrow \quad \mu_s \geq \left(\frac{2\pi}{0.75}\right)^2 \left(\frac{0.05}{9.8}\right) \qquad \{\because\ A = 50 \text{ mm}\}$$

$$\Rightarrow \quad \mu_s \geq 0.358$$

Thus, the minimum value of μ_s should be 0.358

COUPLED SPRING SYSTEM

Springs in Series

Suppose two springs of force constants k_1 and k_2 are connected in series to a mass m. Let m be displaced to the right through a distance y. If the extensions of the two springs are y_1 and y_2 respectively, then

$$y = y_1 + y_2$$

If F is the restoring force, then

$$F = -k_1 y_1 = -k_2 y_2$$

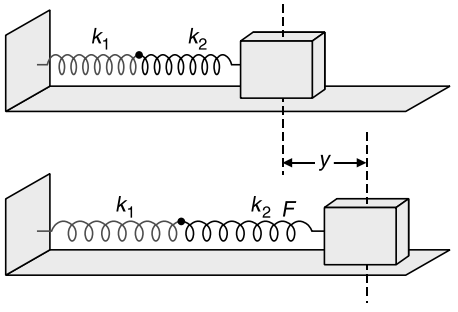
$$\Rightarrow \quad y_1 = -\frac{F}{k_1} \text{ and } y_2 = -\frac{F}{k_2}$$

$$\Rightarrow \quad y = -F\left(\frac{1}{k_1} + \frac{1}{k_2}\right) = -F\left(\frac{k_1 + k_2}{k_1 k_2}\right)$$

$$\Rightarrow \quad F = -\left(\frac{k_1 k_2}{k_1 + k_2}\right) y$$

This shows that the effective force constant of the two springs is given by

$$k_{\text{series}} = k_S = \frac{k_1 k_2}{k_1 + k_2} \text{ or } \frac{1}{k_{\text{series}}} = \frac{1}{k_1} + \frac{1}{k_2}$$



such that k_1, $2k_S$, k_2 are in **Harmonic Progression**
The time period of oscillation is

$$T = 2\pi\sqrt{\frac{m}{k_S}} = 2\pi\sqrt{m\left(\frac{k_1 + k_2}{k_1 k_2}\right)}$$

If $k_1 = k_2 = k$ (say), then $k_S = \frac{k}{2}$

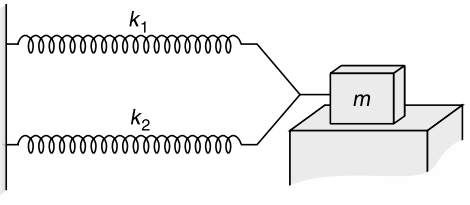
Springs in Parallel

In this case, if $m$is displaced to the right through a distance y, the extension produced in each spring is the same. If F_1 and F_2 are the restoring forces produced in $k_1$and k_2, respectively, then

$$F_1 = -k_1 y \text{ and } F_2 = -k_2 y$$

The total restoring force F is

$$F = F_1 + F_2 = -(k_1 + k_2)y$$



This shows that the effective force constant is

$$k_{\text{parallel}} = k_p = k_1 + k_2$$

The time period of oscillation is

$$T = 2\pi\sqrt{\frac{m}{k_P}} = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$$

If $k_1 = k_2 = k$ (say), then $k_P = 2k$.

Conceptual Note(s)

(a) If n springs are connected in series, then

$$\frac{1}{k_S} = \frac{1}{k_1} + \frac{1}{k_2} + \ldots + \frac{1}{k_n}$$

(b) If n springs are connected in parallel, then

$$k_P = k_1 + k_2 + \ldots + k_n$$

(c) If a spring of spring constant k, natural length ℓ, is divided into parts, then

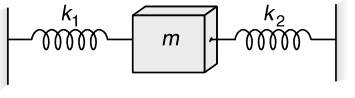
$$k\ell = \text{constant}$$

$$\Rightarrow k_1\ell_1 = k_2\ell_2 = \ldots = k_n\ell_n = k\ell$$

where $\ell_1 + \ell_2 + \ldots + \ell_n = \ell$

Mass Connected Between Two Springs

If the body is displaced to one side, one of the springs gets extended and the other gets compressed. The restoring forces due to both, say F_1 and F_2, are in the same direction. The total restoring force F is $F_1 + F_2$. Now, if y is the displacement of the body, then



$$F_1 = -k_1 y \text{ and } F_2 = -k_2 y$$

$$\Rightarrow \quad F = -(k_1 + k_2)y$$

Thus, showing that the effective force constant is

$$k_{\text{eff}} = k_1 + k_2$$

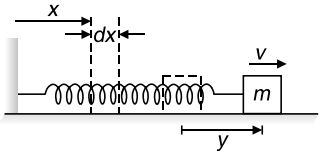
ILLUSTRATION 30

A mass m is connected to a spring of mass m_s and oscillates in SHM on a smooth horizontal surface. The force constant of the spring is k. Find the time period of oscillation.

SOLUTION

Let l be the length of the spring. Let V be the speed of mass m in its displaced position y. Since the spring is not massless, so it will also have some kinetic energy. To find this kinetic energy consider a segment of spring of length dx at a distance x from the fixed end. As the velocity of different segments will be different in an oscillating spring, we assume that velocity of the segment is directly proportional to its distance from the fixed end, so for this segment, we have

$$dm = \frac{m_s}{l}dx \text{ and } v = \left(\frac{x}{l}\right)V$$



So, the kinetic energy of this segment is given by

$$dK_s = \frac{1}{2}(dm)(v^2) = \frac{1}{2}\left(\frac{m_s}{l}dx\right)\left(\frac{x}{l}V\right)^2$$

$$\Rightarrow \quad K_s = \int_0^l dK_s$$

Integrating we get,

$$K_s = \frac{1}{6}m_s V^2$$

The mechanical energy of the system in displaced position of the block will be,

$$E = \begin{pmatrix}\text{Kinetic}\\ \text{Energy}\\ \text{of Mass}\end{pmatrix} + \begin{pmatrix}\text{Kinetic}\\ \text{Energy}\\ \text{of Spring}\end{pmatrix} + \begin{pmatrix}\text{Elastic}\\ \text{Potential}\\ \text{Energy}\\ \text{of Spring}\end{pmatrix}$$

$$\Rightarrow \quad E = \frac{1}{2}mV^2 + \frac{1}{6}m_s V^2 + \frac{1}{2}ky^2$$

Since, E =constant

$$\Rightarrow \quad \frac{dE}{dt} = 0$$

$$\Rightarrow \quad mV\left(\frac{dV}{dt}\right) + \frac{1}{3}m_s V\frac{dV}{dt} + ky\frac{dy}{dt} = 0$$

Substituting $\frac{dV}{dt} = a$ and $\frac{dy}{dt} = V$, we get

$$\left(m + \frac{m_y}{3}\right)a = -ky$$

$$\Rightarrow \quad a \propto -y$$

Therefore, motion is simple harmonic in nature, with time period

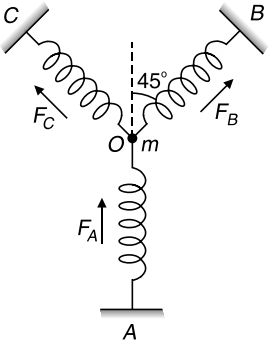
$$T = 2\pi\sqrt{\left|\frac{y}{a}\right|} = 2\pi\sqrt{\frac{m + \frac{m_s}{3}}{k}}$$

ILLUSTRATION 31

A particle of mass m is attached to three identical springs A, B and C each of force constant k as shown in figure. If the particle of mass k is pushed slightly against the spring A and released, find the time period of oscillations.

SOLUTION

When the particle of mass m at O is pushed by y in the direction of A, spring A will be compressed by y while B and C will be stretched by $y' = y\cos 45°$, so the total restoring force on the mass m along OA is given by



$$F = \left(F_A + F_B\cos 45° + F_C\cos 45°\right)$$

$$\Rightarrow \quad F = -\left(ky + 2\left(ky'\right)\cos 45\right)$$

$$\Rightarrow \quad F = -\left(ky + 2k\left(y\cos 45\right)\cos 45\right)$$

$$\Rightarrow \quad F = -k'y, \text{ where } k' = 2k$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{m}{k'}} = 2\pi\sqrt{\frac{m}{2k}}$$

ILLUSTRATION 32

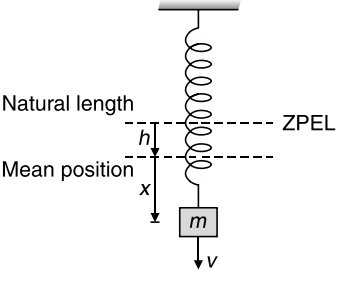
Calculate the angular frequency of vertical oscillations of a block of mass m attached to a spring of spring constant k. (Assume spring to be light).

SOLUTION

Let the extension in the spring at equilibrium be h. Then,

$$mg = kh \qquad \ldots(1)$$

At any instant, let further extension in the spring be x and v be the velocity of block as shown in Figure.



Total mechanical energy of block at this instant is

$$E = \frac{1}{2}mv^2 + \frac{1}{2}k(x+h)^2 - mgx$$

Since total mechanical energy (E) is constant, so

$$\frac{dE}{dt} = 0$$

$$\Rightarrow \quad \frac{d}{dt}\left(\frac{1}{2}mv^2+\frac{1}{2}k(h+x)^2-mgx\right)=0$$

$$\Rightarrow \quad \frac{m}{2}\left(2v\frac{dv}{dt}\right)+\frac{k}{2}\left(2(h+x)\frac{dx}{dt}\right)-mg\left(\frac{dx}{dt}\right)=0$$

$$\Rightarrow \quad mv\frac{d^2x}{dt^2}+k(h+x)v-mgv=0$$

$$\Rightarrow \quad m\frac{d^2x}{dt^2}+kh+kx-mg=0 \qquad \ldots(2)$$

Substituting equation (1) in (2), we get

$$m\frac{d^2x}{dt^2}+kx=0$$

$$\Rightarrow \quad \frac{d^2x}{dt^2}+\frac{k}{m}x=0$$

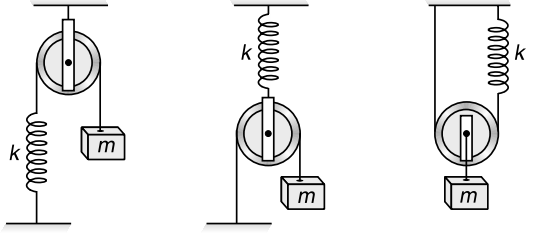
Comparing with standard equation of SHM i.e., $\ddot{x}+\omega^2x=0$, we get

$$\omega=\sqrt{\frac{k}{m}}$$

Since block is given an initial velocity v_0 at mean position, so we have $v_0 = A\omega$, where A is amplitude of oscillations given by $A=\frac{v_0}{\omega}=v_0\sqrt{\frac{m}{k}}$.

ILLUSTRATION 33

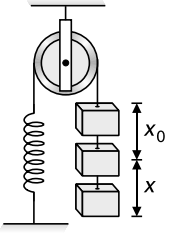
Figure shows a system consisting of a massless pulley, a spring of force constant k and a block of mass m. If the block is slightly displaced vertically down from its equilibrium position and released, find the period of its vertical oscillation in cases (a), (b) and (c).



SOLUTION

(a) In equilibrium $kx_0 = mg$ …(1)

When further depressed by an amount x, net restoring force (upwards) is,



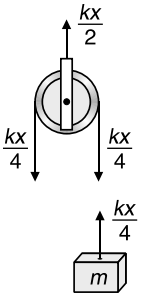
$$F=-\left[k(x+x_0)-mg\right]$$

$$\Rightarrow \quad F=-kx \qquad \{\because\ kx_0=mg\}$$

$$\Rightarrow \quad a=-\frac{k}{m}x$$

$$\Rightarrow \quad T=2\pi\sqrt{\left|\frac{x}{a}\right|}=2\pi\sqrt{\frac{m}{k}}$$

(b) In this case if the mass m moves down a distance x from its equilibrium position, then pulley will move down by $\frac{x}{2}$. So, the extra force in spring will be $k\left(\frac{x}{2}\right)$.



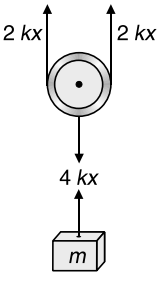
Now, as the pulley is massless, this force $\frac{kx}{2}$ is equal to extra $2T$ or $T=\frac{kx}{4}$. This is also the restoring force of the mass. Hence,

$$F=-\frac{kx}{4}$$

$$\Rightarrow \quad a=-\frac{k}{4m}x$$

$$\Rightarrow \quad T=2\pi\sqrt{\left|\frac{x}{a}\right|}=2\pi\sqrt{\frac{4m}{k}}$$

(c) In this situation if the mass m moves down a distance x from its equilibrium position, the pulley will also move by x and so the spring will stretch by $2x$.



Therefore, the spring force will be $2kx$. The restoring force on the block will be $4kx$. Hence,

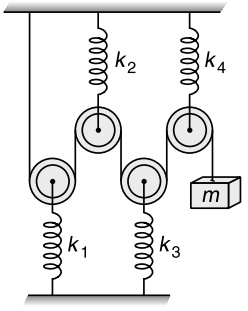
$$F=-4kx$$

$$\Rightarrow \quad a=-\left(\frac{4k}{m}\right)x$$

$$\Rightarrow \quad T=2\pi\sqrt{\left|\frac{x}{a}\right|}=2\pi\sqrt{\frac{m}{4k}}$$

ILLUSTRATION 34

In the arrangement shown in figure, pulleys are small light and springs are ideal. k_1, k_2, k_3 and k_4 are force constants of the springs. Calculate period of small vertical oscillations of block of mass m.



SOLUTION

When the mass m is displaced from its mean position by a distance x, let F be the restoring (extra tension) force produced in the string. By this extra tension further elongation in the springs are $\frac{2F}{k_1}$, $\frac{2F}{k_2}$, $\frac{2F}{k_3}$ and $\frac{2F}{k_4}$ respectively. Then,

$$x = 2\left(\frac{2F}{k_1}\right)+2\left(\frac{2F}{k_2}\right)+2\left(\frac{2F}{k_3}\right)+2\left(\frac{2F}{k_4}\right)$$

$$\Rightarrow \quad F\left(\frac{4}{k_1}+\frac{4}{k_2}+\frac{4}{k_3}+\frac{4}{k_4}\right) = -x$$

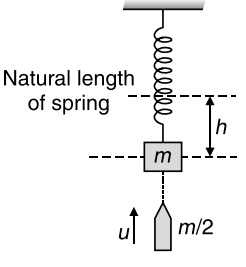
Here negative sign shows the restoring nature of force

$$\Rightarrow \quad a = -\frac{x}{m\left(\frac{4}{k_1}+\frac{4}{k_2}+\frac{4}{k_3}+\frac{4}{k_4}\right)}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\left|\frac{x}{a}\right|} = 4\pi\sqrt{m\left(\frac{1}{k_1}+\frac{1}{k_2}+\frac{1}{k_3}+\frac{1}{k_4}\right)}$$

ILLUSTRATION 35

Consider spring block pendulum system hanging in equilibrium. A bullet of mass $m/2$ moving at a speed u hits the block of mass m from downward direction and gets embedded in it as shown in Figure.



Calculate the amplitude of oscillation of the block. Also find the time taken by the block to reach its upper extreme position after being hit by bullet.

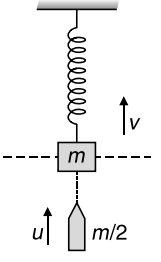
SOLUTION

When the block is in equilibrium, then let the spring have an extension h, such that

$$mg = kh \qquad \ldots(1)$$

When the bullet of mass $m/2$ gets embedded in the block, then the new mass of block becomes $3m/2$. The new mean position of the block will be at a depth h_1 from the old mean position, such that

$$\frac{3}{2}mg = k(h+h_1) \qquad \ldots(2)$$



From equations (1) and (2), we get

$$h_1 = \frac{mg}{2k}$$

Just after impact, due to inelastic collision, the velocity of block becomes v, then by law of conservation of momentum, we get

$$\left(\frac{m}{2}\right)u = \left(\frac{3m}{2}\right)v$$

$$\Rightarrow \quad v = \frac{u}{3}$$

Since the block now executes SHM and at $t = 0$, the block is at a distance $h_1 = \frac{mg}{2k}$ above its mean position having a velocity $\frac{u}{3}$. If amplitude of oscillation is A, then we have

$$\frac{u}{3} = \omega\sqrt{A^2-\left(\frac{mg}{2k}\right)^2} \qquad \left\{\because\ v = \omega\sqrt{A^2-x^2}\right\}$$

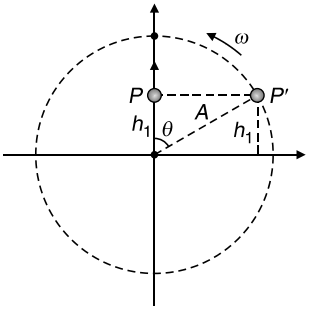
Also, for this new spring block system, we have

$$\omega = \sqrt{\frac{k}{3m/2}} = \sqrt{\frac{2k}{3m}}$$

$$\Rightarrow \quad \frac{u^2}{9} = \frac{2k}{3m}\left(A^2-\frac{m^2g^2}{4k^2}\right)$$

$$\Rightarrow \quad A = \sqrt{\frac{mu^2}{6k}+\left(\frac{mg}{2k}\right)^2} \qquad \ldots(3)$$

The time taken by particle to reach the topmost point can be obtained by drawing the circular motion representation of SHM as shown in Figure.



This figure shows the position of block P and its corresponding circular motion particle P' at $t = 0$. Block P will reach its upper extreme position when particle P' will traverse the angle θ and reach the topmost point. As P' moves at constant angular velocity ω, it will take a time given by

$$t = \frac{\phi}{\omega} \text{ where, } \phi = \cos^{-1}\left(\frac{h_1}{A}\right) \text{ and } \omega = \sqrt{\frac{2k}{3m}}$$

$$\Rightarrow \quad t = \sqrt{\frac{3m}{2k}}\cos^{-1}\left(\frac{mg}{2kA}\right)$$

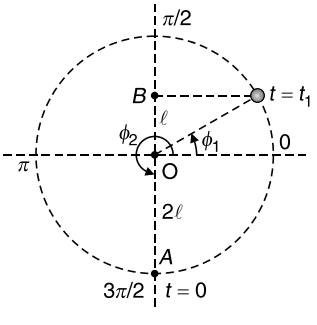
ILLUSTRATION 36

A heavy particle is attached to one end of an elastic string, the other end of which is fixed. The modulus of elasticity of the string is such that in equilibrium the string length is double its natural length. The string is drawn vertically down till it is four times its natural length and then let go. Show that the particle will return to this point in time $\sqrt{\frac{l}{g}}\left(\frac{4\pi}{3}+2\sqrt{3}\right)$, where l is the natural length of the string.

SOLUTION

Let k be the equivalent force constant of string, then at equilibrium we have $Mg = k(2l - l)$.

Further extension by $2l$, so that total length becomes $4l$, will make the particle shoot upwards (on being released) upto natural length of spring. Time taken by particle is obtained by drawing the phase diagram as shown in Figure.



Here $\Delta\phi = \frac{3\pi}{2} - \frac{\pi}{6} = \frac{4\pi}{3}$

Time for particle to go from A to B is

$$t_1 = \frac{\Delta\phi}{\omega}, \text{ where } \omega = \sqrt{\frac{g}{l}}$$

$$\Rightarrow \quad t_1 = \frac{4\pi}{3}\sqrt{\frac{l}{g}}$$

At position B, speed of particle is

$$v = \omega\sqrt{(2l)^2 - (l)^2}$$

$$\Rightarrow \quad v = \sqrt{3}\omega l = \sqrt{3gl}$$

After point B, the string will slack and particle will be in free fall motion, so time taken by it to go up and come back to point B is given as

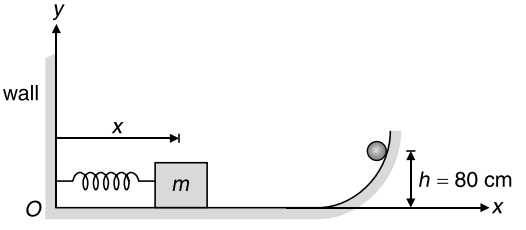
$$t_2 = \frac{2\sqrt{3gl}}{g} = 2\sqrt{3}\sqrt{\frac{l}{g}}$$

Thus, total time after which particle will come back to point A is

$$T = t_1 + t_2 = \sqrt{\frac{l}{g}}\left(\frac{4\pi}{3} + 2\sqrt{3}\right)$$

ILLUSTRATION 37

A block of mass $m = 1$ kg is attached to a free end of a spring whose other end is fixed with a wall performing simple harmonic motion as shown in Figure.



The position of the block from O at any instant is $x = 2 + \frac{1}{\sqrt{2}}\sin(2t)$ where x in meter and t is in second. A shell of same mass is released from smooth the circular path at a height $h = 80$ cm. The shell collides elastically with the block performing SHM and finally reaches up to height 5 cm along circular path. Neglecting friction, find where the collision takes place.

SOLUTION

Initial speed of shell before collision is

$$v_1 = \sqrt{2gh_1} = \sqrt{2\times10\times0.8} = 4 \text{ ms}^{-1}$$

Find speed of shell after collision

$$v_2 = \sqrt{2gh_2} = \sqrt{2\times10\times0.05} = 1 \text{ ms}^{-1}$$

Since, masses of block and shell are equal and they collide elastically, so v_2 is the block velocity just before collision which is given as

$$v = \frac{dx}{dt} = \sqrt{2}\cos(2t) = 1$$

$$\Rightarrow \quad \cos(2t) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad 2t = \frac{\pi}{4} \text{ or } \frac{7\pi}{4}$$

$$\Rightarrow \quad t = \frac{\pi}{8} \text{ or } \frac{7\pi}{8}$$

Position of block at $t = \frac{\pi}{8}$ is

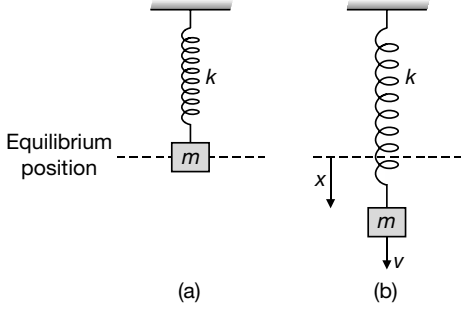
$$x_1 = 2 + \frac{1}{\sqrt{2}} \sin\left(\frac{\pi}{4}\right) = 2 + \frac{1}{2} = 2.5 \text{ m}$$

Position of block at $t = \frac{7\pi}{8}$ is

$$x_2 = 2 + \frac{1}{\sqrt{2}} \sin\left(\frac{7\pi}{8}\right) = 2 - \frac{1}{2} = 1.5 \text{ m}$$

ILLUSTRATION 38

A spring block system is hanging in equilibrium. The block of system is pulled down by a distance x and imparted a velocity v in downward direction as shown in Figure. Calculate the time it will take to reach its mean position.



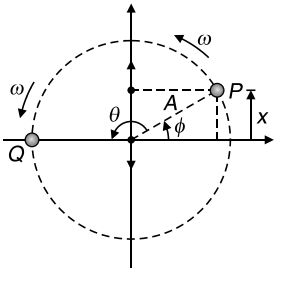
SOLUTION

When the block is pulled down by x (from the mean position) and is given a velocity v downwards, then it executes SHM of amplitude A (say) and $\omega = \sqrt{k/m}$. The velocity of block at distance x from the mean position is

$$v^2 = \omega^2 (A^2 - x^2) = \frac{k}{m}(A^2 - x^2)$$

$$\Rightarrow \quad A = \sqrt{\frac{mv^2}{k} + x^2}$$

Taking downward direction as positive, the mapping of this SHM on circular motion is shown in Figure. (The concept behind this mapping is that initially the particle is at distance x from the mean position).



Time taken by the particle to go from start to mean position is

$$t = \frac{\theta}{\omega} = \frac{\pi - \phi}{\omega}$$

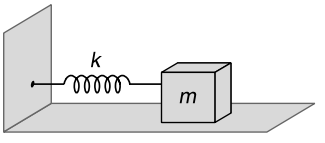
where $\phi = \sin^{-1}\left(\frac{x}{A}\right)$

$$\Rightarrow \quad t = \frac{\pi - \sin^{-1}(x/A)}{\omega} = \sqrt{\frac{m}{k}}\left[\pi - \sin^{-1}\left(\frac{x}{a}\right)\right]$$

$$\Rightarrow \quad t = \sqrt{\frac{m}{k}}\left[\pi - \sin^{-1}\left(\frac{\sqrt{k}x}{\sqrt{mv^2 + kx^2}}\right)\right]$$

ILLUSTRATION 39

A block of mass m lying on a smooth horizontal surface is attached to one end of a spring of force constant k.



The block is now pulled towards right by a distance x_0 and released. When the block passes through a point at a displacement $x_0/2$ from mean position, another block of same mass is gently placed on it which sticks to it due to friction. Calculate the new amplitude of oscillation, the time taken by it to reach its mean position and extreme position on the left side.

SOLUTION

Velocity of block at $\frac{x_0}{2}$ from mean position is

$$v = \omega\sqrt{A^2 - x^2}, \text{ where } \omega = \sqrt{\frac{k}{m}}$$

$$\Rightarrow \quad v = \omega\sqrt{x_0^2 - \frac{x_0^2}{4}} = \frac{\sqrt{3}}{2}x_0\omega \qquad \{\because A = x_0\}$$

$$\Rightarrow \quad v = \frac{\sqrt{3}}{2}x_0\sqrt{\frac{k}{m}} \qquad \ldots(1)$$

Now, when another block of same mass is placed on this block, then the new angular frequency of system is

$$\omega' = \sqrt{\frac{k}{2m}}$$

If A' be the new amplitude of oscillations of combined system of blocks, then we have

$$v' = \omega'\sqrt{A'^2 - \left(\frac{x_0}{2}\right)^2} \qquad \ldots(2)$$

where, by law of conservation of linear momentum we get

$$(m + m)v' = mv$$

$$\Rightarrow \quad v' = \frac{v}{2} \qquad \ldots(3)$$

Substituting (3) in equation (2), we get

$$\left(\frac{v}{2}\right)^2=\left(\frac{k}{2m}\right)\left(A'^2-\frac{x_0^2}{4}\right)$$

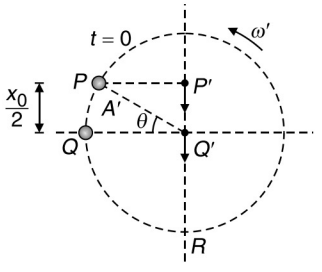
Using equation (1), we get

$$\frac{3}{16}x_0^2\left(\frac{k}{m}\right)=\frac{k}{2m}\left(A'^2-\frac{x_0^2}{4}\right)$$

$$\Rightarrow \quad \frac{3}{8}x_0^2+\frac{x_0^2}{4}=A'^2$$

$$\Rightarrow \quad A'=\left(\frac{\sqrt{5}}{2\sqrt{2}}\right)x_0$$

Initially the combined blocks are at a distance $\frac{x_0}{2}$ from the mean position moving towards the mean position. So, mapping this situation on a circle shown in Figure, we get



$$\theta=\sin^{-1}\left(\frac{x_0/2}{A'}\right)=\sin^{-1}\left(\sqrt{\frac{2}{5}}\right)$$

$$t_{P\to Q}=\frac{\theta}{\omega'}=\frac{\sin^{-1}\left(\sqrt{2/5}\right)}{\omega}$$

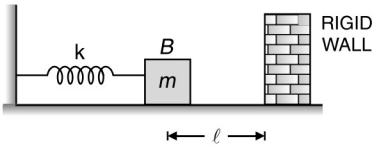
$$\Rightarrow \quad t_{P\to Q}=\sqrt{\frac{2m}{k}}\sin^{-1}\left(\sqrt{\frac{2}{5}}\right)$$

Similarly, time taken by P to reach the other extreme is

$$t_{P\to R}=\frac{\pi/2+\theta}{\omega'}=\sqrt{\frac{2m}{k}}\left[\frac{\pi}{2}+\sin^{-1}\sqrt{\frac{2}{5}}\right]$$

ILLUSTRATION 40

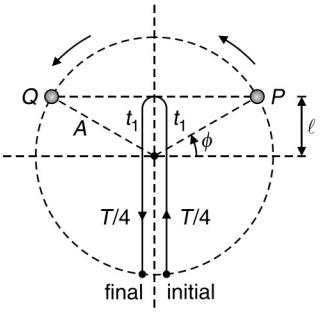
A block B of mass m resting on a smooth horizontal surface, attached to a spring of force constant k which is rigidly fixed on the wall on left side is shown in Figure.



At a distance l to the right of block there is a rigid wall. If block is pushed toward left so that spring is compressed by a distance $5l/3$ and released, it will start its oscillations. If collision of block with the wall is considered to be perfectly elastic. Find the time period of oscillations of the block. (Assume size of block to be small compared to l).

SOLUTION

Since the block is released from rest at a distance $5l/3$ from its mean position, so amplitude of oscillation is $A=\frac{5l}{3}$ and $\omega=\sqrt{\frac{k}{m}}$. However, on the other side the distance available is $l\left(<5l/3\right)$ due to presence of rigid wall. So, block will move a distance l (away from mean) collide elastically with wall to return to mean position. This situation is mapped on the circle shown in Figure.



Time taken to go from initial to final is

$$t=\frac{T}{4}+t_1+t_1+\frac{T}{4}=2\left(\frac{T}{4}+t_1\right)$$

where $t_1=\frac{\phi}{\omega}=\frac{\sin^{-1}\left(l/A\right)}{\omega}=\frac{\sin^{-1}\left(3/5\right)}{\omega}$

$$\Rightarrow \quad t_1=\sqrt{\frac{m}{k}}\sin^{-1}\left(\frac{3}{5}\right)$$

Hence, $t=2\left(\frac{T}{4}+t_1\right)$

$$\Rightarrow \quad t=2\left[\frac{1}{4}\left(2\pi\sqrt{\frac{m}{k}}\right)+\sqrt{\frac{m}{k}}\sin^{-1}\left(\frac{3}{5}\right)\right]$$

$$\Rightarrow \quad t=\sqrt{\frac{m}{k}}\left[\pi+2\sin^{-1}\left(\frac{3}{5}\right)\right] \qquad \ldots(1)$$

Please note that, we can also calculate the time period of oscillation by subtracting time to go from P to Q from T.

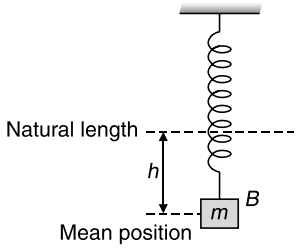
$$\Rightarrow \quad t=T-t_{P\to Q}, \text{ where } t_{P\to Q}=\frac{2\theta}{\omega}=\frac{2\cos^{-1}\left(3/5\right)}{\omega}$$

$$\Rightarrow \quad t=\sqrt{\frac{m}{k}}\left[2\pi-2\cos^{-1}\left(\frac{3}{5}\right)\right] \qquad \ldots(2)$$

(Both these equations (1) and (2) will give same numerical value).

ILLUSTRATION 41

A spring block system is hanging in equilibrium. If a velocity v_0 is imparted to the block of mass m in downward direction, calculate the amplitude of SHM of block and the time after which it will reach a point at half of the amplitude.



SOLUTION

Initially in equilibrium, we have $mg = kh$

The velocity v_0 imparted to the block at the mean position simply implies that this velocity is the maximum velocity, hence $v_0 = A\omega$

where A is amplitude of SHM and $\omega = \sqrt{\frac{k}{m}}$

$$\Rightarrow \quad A = \frac{v_0}{\omega} = v_0\sqrt{\frac{m}{k}}$$

Since the particle starts from the mean position, so we have

$$x = A\sin(\omega t)$$

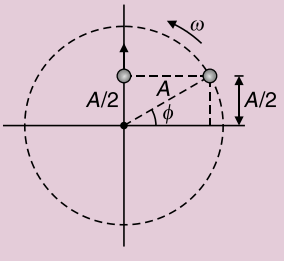
At $x = \frac{A}{2}$, we have $\frac{A}{2} = A\sin(\omega t)$

$$\Rightarrow \quad \sin(\omega t) = \frac{1}{2}$$

$$\Rightarrow \quad \omega t = \frac{\pi}{6}$$

$$\Rightarrow \quad t = \frac{\pi}{6\omega} = \frac{\pi}{6}\sqrt{\frac{m}{k}}$$

We can also calculate this result by mapping the SHM on a circle as shown in Figure.

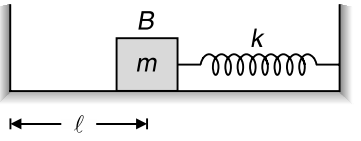


$$\phi = \sin^{-1}\left(\frac{A/2}{A}\right) = \frac{\pi}{6}$$

$$\Rightarrow \quad t = \frac{\phi}{\omega} = \frac{\pi}{6\omega}$$

ILLUSTRATION 42

A block B of mass m is resting on a horizontal smooth floor at a distance l from a rigid wall. Block is pushed to the right by a distance $3l/2$ and then released. When the block passes from its mean position, another block of mass M is placed gently on it such that both stick to each other due to friction. Calculate the value of M/m so that the combined block just collides with the left wall. (Ignore size of block compared to l).



SOLUTION

Amplitude is $A = \frac{3l}{2}$ but available space on left is $l\left(< \frac{3l}{2}\right)$.

When block B executes SHM, then $\omega = \sqrt{\frac{k}{m}}$. Applying conservation of linear momentum (at the mean position), we get

$$m(A\omega) = (M+m)A'\omega'$$

Since the combined block just collides with the left wall so, $A' = l$

$$\Rightarrow \quad m\left(\frac{3l}{2}\right)\sqrt{\frac{k}{m}} = (M+m)l\sqrt{\frac{k}{M+m}}$$

$$\Rightarrow \quad \frac{3}{2}\sqrt{m} = \sqrt{M+m}$$

$$\Rightarrow \quad 9m = 4(M+m)$$

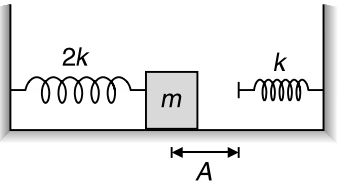
$$\Rightarrow \quad 9m = 4M + 4m$$

$$\Rightarrow \quad 4M = 5m$$

$$\Rightarrow \quad \frac{M}{m} = \frac{5}{4}$$

ILLUSTRATION 43

Initially the springs are unstretched as shown in Figure.



The left spring is now compressed by $2A$ by moving the mass m (which is always attached to it) and then released calculate the time to touch right spring, maximum compression in right spring and equilibrium position of mass m.

SOLUTION

When left spring is compressed by $2A$, then amplitude of oscillations will also be $2A$. On being released, the block will hit the right spring (in time t) when its displacement from the mean position is A i.e., $x = A$.

Since, $x = 2A\sin\left(\omega t + \frac{\pi}{2}\right) = 2A\cos(\omega t)$

$\Rightarrow \quad A = 2A\cos(\omega t)$

$\Rightarrow \quad \cos(\omega t) = \frac{1}{2}$

$\Rightarrow \quad \omega t = \frac{\pi}{3}$

$\Rightarrow \quad t = \frac{\pi}{3\omega} = \frac{\pi}{3}\sqrt{\frac{m}{2k}}$

Now when the mass m strikes the right spring, then it moves under combined influence of both the springs. Let the mass m be in equilibrium when the right spring is compressed by say x_0. Then

$$2k(A - x_0) = kx_0$$

$\Rightarrow \quad x_0 = \frac{2A}{3}$

If x_m be the maximum compression in the right spring, then by Work-Energy Theorem, we have

$$\frac{1}{2}(2k)(2A)^2 = \frac{1}{2}(2k)(A - x_m)^2 + \frac{1}{2}kx_m^2$$

$\Rightarrow \quad 8kA^2 = 2kA^2 + 2kx_m^2 - 4kAx_m + kx_m^2$

$\Rightarrow \quad 3x_m^2 - 4Ax_m - 6A^2 = 0$

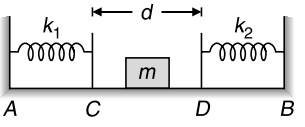
$\Rightarrow \quad x_m = \frac{4A \pm \sqrt{16A^2 - 4(3)(-6A^2)}}{2(3)}$

$\Rightarrow \quad x_m = \frac{4A + \sqrt{88A^2}}{6} = \left(\frac{4 + 2\sqrt{22}}{6}\right)A$

$\Rightarrow \quad x_m = \left(\frac{2 + \sqrt{22}}{3}\right)A$

ILLUSTRATION 44

Two light springs of force constant k_1, k_2 and a block of mass m are in one line AB on a smooth horizontal table such that one end of each spring is fixed on rigid supports and the other end is free as shown in Figure.



The distance between the free ends of the springs is d. If the block moves along AB with a velocity v in between springs, calculate the period of oscillation of the block.

SOLUTION

During motion, the block oscillates as a spring block system while in contact with both left and right spring for half the oscillation. So, the time it is in contact with springs is the sum of half time period of each spring, hence we have

$$\Delta t_1 = \frac{T_1}{2} + \frac{T_2}{2} = \pi\sqrt{\frac{m}{k_1}} + \pi\sqrt{\frac{m}{k_2}}$$

Between points C and D it moves uniformly, so

$$\Delta t_2 = \frac{2d}{v}$$

Total period is $T = \Delta t_1 + \Delta t_2$

$\Rightarrow \quad T = \pi\left(\sqrt{\frac{m}{k_1}} + \sqrt{\frac{m}{k_2}}\right) + \frac{2d}{v}$

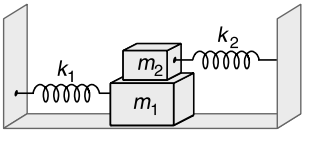
Test Your Concepts-II

Based on Spring Mass Systems

(Solutions on page H.181)

1. A block with mass M attached to a horizontal spring with force constant k is moving with simple harmonic motion having amplitude A_1. At the instant when the block passes through its equilibrium position a lump of putty with mass m is dropped vertically on the block from a very small height and sticks to it.
   (a) Find the new amplitude and period.
   (b) Repeat part (a) for the case in which the putty is dropped on the block when it is at one end of its path.

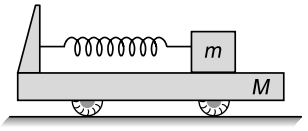
2. In the shown arrangement, both the springs are in their natural lengths. The coefficient of friction between m_2 and m_1 is μ. There is no friction between m_1 and the surface. If the blocks are displaced slightly, they together perform simple harmonic motion. Obtain



   (a) Frequency of such oscillations.
   (b) The condition if the frictional force on block m_2 is to act in the direction of its displacement from mean position.
   (c) If the condition obtained in (b) is met, what can be maximum amplitude of their oscillations?

3. A mass M attached to a spring oscillates with a period of 2 s. If the mass is increased by 2 kg, the period increases by one second. Calculate the initial mass M.

4. A mass m is attached to a cart of mass M by a spring of force constant k as shown in Figure.



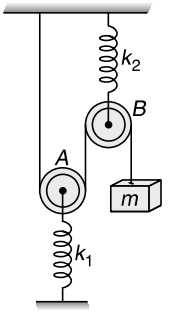
If the friction between m and surface of cart as well as between cart and floor is neglected, calculate the time period of small oscillation of cart-block system.

5. A 0.2 kg object hangs from an ideal spring of negligible mass. When the object is pulled down 0.1 m below its equilibrium position and released, it vibrates with a period of 1.80 s.
(a) Find its speed as it passes through the equilibrium position.
(b) Find its acceleration when it is 0.050 m above the equilibrium position.
(c) Find the time required by the particle (when moving upwards) to move from a point 0.05 m below its equilibrium position to a point 0.05 m above it.
(d) If the motion of the object is stopped at the mean position and the object is removed from the spring, then find the length by which the spring shortens.

6. A 1×10^{-2} kg block is resting on a horizontal frictionless surface and is attached to a horizontal spring whose spring constant is 124 Nm^{-1}. The block is given an initial speed of 8 ms^{-1} parallel to the spring axis, while the spring is initially unstrained. What is the amplitude of the resulting simple harmonic motion?

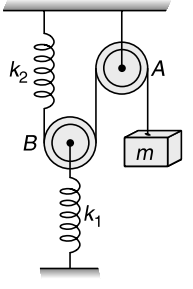
7. A mass m_1 sliding on a frictionless horizontal surface is attached to a spring of force constant k. It oscillates with amplitude A. When the spring is at its greatest extension and the mass is instantaneously at rest, a second mass m_2 is placed on top of m_1.
(a) What is the smallest value of coefficient of static friction μ_s between the two masses so that m_2 does not slip over m_1.
(b) Explain how the total energy E, the amplitude A, the angular frequency ω, and the period T are changed by placing m_2 on m_1 in this way, assuming no slipping. Check your results with the expression for the total energy, $E=\frac{1}{2}m\omega^2A^2$.

8. A block of mass m is attached to one end of a light inextensible string passing over a smooth light pulley B and under another smooth light pulley A as shown in the figure. The other end of a string is fixed to a ceiling. A and B are held by springs of spring constants k_1 and k_2. Find angular frequency of small oscillation of the system.

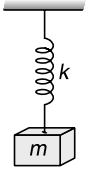


9. An object of mass m is supported by a vertical spring with force constant 1800 Nm^{-1}. When pulled down 2.5 cm and released, it oscillates at 5.5 Hz. Find the
(a) mass m.
(b) equilibrium stretching of the spring.
(c) expressions for $x(t)$, $v(t)$, and $a(t)$.

10. A block of mass m is tied to one end of a string which passes over a smooth fixed pulley A and under a light smooth movable pulley B. The other end of the string is attached to the lower and of a spring of spring constant k_2. Find the period of small oscillations of mass m about its equilibrium positions.



11. A spring mass system is hanging from the ceiling of an elevator in equilibrium. The elevator suddenly starts accelerating upwards with acceleration a, find



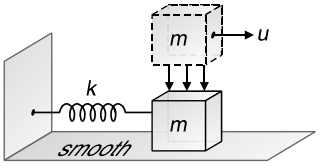
(a) the frequency
(b) the amplitude of the resulting SHM

12. A spring with force constant $k=150$ Nm^{-1} is suspended from a rigid support. A 1 kg mass is attached at the bottom end and released from rest when the spring is unstretched.
(a) How far down does the mass move before it starts up again?
(b) How far below the starting point is the equilibrium position for the 1 kg mass?

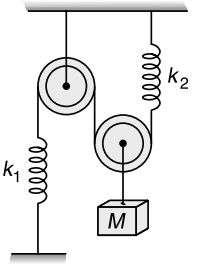
(c) What is the period of the oscillation?
(d) Relative to this equilibrium position, what is the total energy of the mass-spring system?
(e) What is the speed of the mass when it first reaches the equilibrium position? How long after starting does the mass have this speed?
(f) If instead of being attached to the spring the mass had been dropped, when would it have reached the (previous) equilibrium level and what would its speed be? $\left(g = 9.8\ \text{ms}^{-2}\right)$.

13. A spring stretches by 0.018 m when a 2.8 kg object is suspended from its end. How much mass should be attached to this spring so that its frequency of vibration is $f = 3.0$ Hz?

14. A block of mass m resting on a smooth horizontal ground is attached to one end of a spring of force constant k in natural length. Another block of same mass moving with a horizontal velocity u towards right is gently placed on the block such that it sticks to it due to friction. Calculate the time it will take to reach its extreme position. Also find the amplitude of oscillations of the combined mass $2m$.



15. Find the time period of oscillation of M in the arrangement shown in the figure. The pulleys are smooth and massless.



ROTATIONAL SYSTEMS OR ANGULAR SHM

In rotational systems executing SHM, we first give a small angular displacement θ to the system about the mean position and then calculate the restoring torque τ of the system about the mean position as a function of θ. If τ is proportional to this small θ and is directed towards the mean position, then we can say the motion is SHM. Mathematically

$$\tau = -C\theta$$

where, C is a constant of proportionality. If I be the moment of inertia of the system about the specified axis of rotation, then $\tau = I\dfrac{d^2\theta}{dt^2}$.

$$\Rightarrow \quad I\frac{d^2\theta}{dt^2} = -C\theta$$

$$\Rightarrow \quad \frac{d^2\theta}{dt^2} + \left(\frac{C}{I}\right)\theta = 0$$

$$\Rightarrow \quad \frac{d^2\theta}{dt^2} + \omega^2\theta = 0, \text{ where } \omega = \sqrt{\frac{C}{I}}$$

Also, we can calculate the total mechanical energy E of the system at some intermediate instant and since for SHM

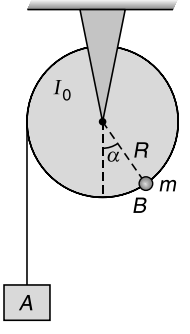
$$E = \text{constant}$$

$$\Rightarrow \quad \frac{dE}{dt} = 0$$

Following illustrations demonstrate the above written methods to calculate the time period of SHM executed by rotational systems.

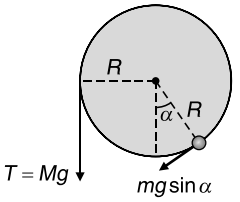
ILLUSTRATION 45

A pulley block system in which a block A is hanging on one side of pulley and on other side a small bead B of mass m is welded on pulley as shown in Figure. The moment of inertia of pulley is I_0 and the system is in equilibrium when bead is at an angle α from the vertical. If the system is slightly disturbed from its equilibrium position, find the time period of its oscillations.



SOLUTION

Let M be the mass of hanging block. In equilibrium, torque due to tension is balanced by torque due to m as shown in Figure. So, we have

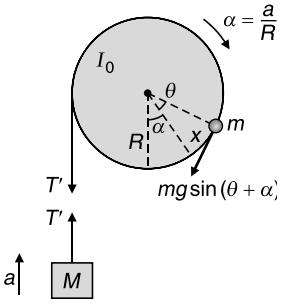


$$TR = (mg\sin\alpha)R$$

$$\Rightarrow \quad Mg = mg\sin\alpha$$

$$\Rightarrow \quad M = m\sin\alpha \qquad \ldots(1)$$

Let the bead be given a small angular displacement θ in the counter clockwise sense. On being released, the pulley rotates in clockwise sense and block moves upwards with an acceleration a as shown in Figure.



Applying Newton's Second Law for translational motion, we get

$$T' - Mg = Ma \qquad \ldots(2)$$

Applying Newton's Second Law for rotational motion, we get

$$mg\sin(\theta+\alpha)R - T'R = I_0\alpha$$

$$\Rightarrow \quad mg\sin(\theta+\alpha) - T' = \frac{I_0 a}{R^2} \qquad \ldots(3)$$

where, I_0 is moment of inertia of bead and pulley about the axis of rotation i.e., $I = mR^2 + I_0$

Substituting (2) in (3), we get

$$\left(M + \frac{I}{R^2}\right)a = mg\sin(\theta+\alpha) - Mg \qquad \ldots(4)$$

Since θ is small, so $\sin\theta \approx \theta$ and $\cos\theta \approx 1$

$$\sin(\theta+\alpha) = \sin\theta\cos\alpha + \cos\theta\sin\alpha$$

$$\Rightarrow \quad \sin(\theta+\alpha) = \theta\cos\alpha + \sin\alpha \qquad \ldots(5)$$

Substituting value of (5) in (4), we get

$$\left(M + \frac{I_0}{R^2}\right)a = (mg\cos\alpha)\theta + mg\sin\alpha - Mg$$

$$\Rightarrow \quad \left(M + \frac{I_0}{R^2}\right)a = (mg\cos\alpha)\theta \qquad \{\because\ M = m\sin\alpha\}$$

$$\Rightarrow \quad a = -\frac{mg\cos\alpha}{\left(M + \frac{I}{R^2}\right)}\frac{x}{R}$$

Negative sign indicates that acceleration (and hence restoring force is directed towards mean position).

$$\Rightarrow \quad \ddot{x} = -\left[\frac{mg\cos\alpha}{R\left(M + m + \frac{I_0}{R^2}\right)}\right]x$$

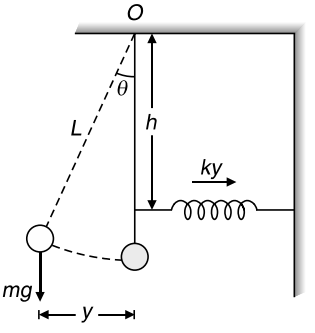
$$\Rightarrow \quad T = 2\pi\sqrt{\frac{x}{|\ddot{x}|}} = 2\pi\sqrt{\frac{R\left(m + m\sin\alpha + \frac{I_0}{R^2}\right)}{mg\cos\alpha}}$$

ILLUSTRATION 46

A simple pendulum of length L and mass m has as spring of force constant k connected to it at a distance h below its point of suspension. Find the frequency of vibrations of the system for small values of amplitude.

SOLUTION

Let the pendulum be given a small angular displacement θ about the mean position as shown in Figure.



Then, the spring will stretch by $y = h\tan\theta$.

So there restoring torque about O will be due to both force of gravity and elastic force of the spring.

$$\Rightarrow \quad \tau = -\left(mg(L\sin\theta) + k(h\tan\theta)h\right)$$

Now for small θ, $\tan\theta \approx \sin\theta \approx \theta$

$$\Rightarrow \quad \tau = -\left(mgL + kh^2\right)\theta \qquad \ldots(1)$$

Since restoring torque is linear, so motion is angular SHM.

Also, $\tau = I\alpha = mL^2\frac{d^2\theta}{dt^2}$, where $I = mL^2$

$$\Rightarrow \quad mL^2\frac{d^2\theta}{dt^2} + \left(mgL + kh^2\right)\theta = 0 \qquad \{\text{from (1)}\}$$

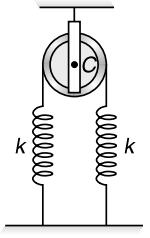
$$\Rightarrow \quad \frac{d^2\theta}{dt^2} + \omega^2\theta = 0, \text{ where } \omega^2 = \left(\frac{mgL + kh^2}{mL^2}\right)$$

$$\Rightarrow \quad f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{mgL + kh^2}{mL^2}}$$

ILLUSTRATION 47

A 7 kg disk is free to rotate about a horizontal axis passing through its centre C. Determine the period of oscillation of the disk if the springs have sufficient tension in them to

prevent the string from slipping on the disk as it oscillates. The radius of the disk is 10 cm and spring constant of both the springs is 600 Nm^{-1}.



SOLUTION

Let the angular displacement of the disk be θ. Angular velocity of disk at this instant is ω and compression and elongation in the springs is,

$$x = R\theta$$

Mechanical energy of the system in this position is,

$$E = \frac{1}{2}I\omega^2 + \frac{1}{2}kx^2 + \frac{1}{2}kx^2$$

$$\Rightarrow \quad E = \frac{1}{2}\left(\frac{1}{2}mR^2\right)\omega^2 + kx^2$$

$$\Rightarrow \quad E = \frac{1}{4}mR^2\omega^2 + kR^2\theta^2$$

Since, E is constant, so $\frac{dE}{dt} = 0$

$$\Rightarrow \quad \frac{1}{2}mR^2\omega\left(\frac{d\omega}{dt}\right) + 2kR^2\theta\left(\frac{d\theta}{dt}\right) = 0$$

Substituting $\frac{d\theta}{dt} = \omega$ and $\frac{d\omega}{dt} = \alpha = \ddot{\theta}$, we get

$$m\ddot{\theta} + 4k\theta = 0 \qquad \ldots(1)$$

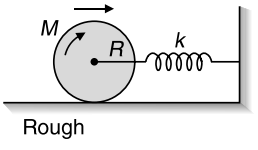
Since, α or $\ddot{\theta}$ is proportional to $-\theta$, the motion is simple harmonic, the time period of which is,

$$T = 2\pi\sqrt{\left|\frac{\theta}{\alpha}\right|} = 2\pi\sqrt{\frac{\theta}{|\ddot{\theta}|}} = 2\pi\sqrt{\frac{m}{4k}} \qquad \{\text{from (1)}\}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{7}{4\times 600}} = 0.34 \text{ s}$$

ILLUSTRATION 48

A solid cylinder of mass M and radius R is attached to a spring of stiffness k as shown in the figure. The cylinder can roll without slipping on a rough horizontal surface. Show that the centre of mass of the cylinder executes SHM and determine its time period.



SOLUTION

Consider the situation when the spring is extended by x. The total energy of the cylinder plus spring system is

$$E = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}kx^2 = \text{constant}$$

where, $I = \frac{1}{2}MR^2$ and $\omega = \frac{v}{R}$

$$\Rightarrow \quad \frac{3}{4}Mv^2 + \frac{1}{2}kx^2 = \text{constant}$$

Since, E is constant, so $\frac{dE}{dt} = 0$

$$\Rightarrow \quad \frac{3}{4}M(2v)\frac{dv}{dt} + \frac{1}{2}k(2x)\frac{dx}{dt} = 0$$

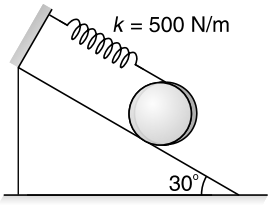
Noting that $v = \frac{dx}{dt}$ and $\frac{dv}{dt} = \frac{d^2x}{dt^2}$

$$\Rightarrow \quad \frac{d^2x}{dt^2} + \frac{2k}{3M}x = 0$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{3M}{2k}}$$

ILLUSTRATION 49

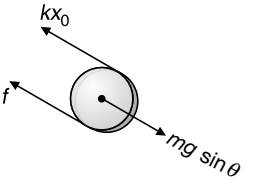
A 14 kg uniform cylinder can roll without sliding on a 30° incline. A belt is attached to the rim of the cylinder and a spring holds the cylinder at rest in the position shown in Figure.



If the cylinder is moved 50 mm down the incline and released, calculate the of small oscillations and the maximum acceleration of centre of cylinder.

SOLUTION

Let x_0 be the extension in the spring in equilibrium position and f the force of friction in the direction shown in figure.



Then for the equilibrium of cylinder

$$kx_0 + f = mg\sin\theta \qquad \ldots(1)$$

and $(kx_0)R = fR \qquad \ldots(2)$

From these two equations, we get

$$2kx_0 = mg\sin\theta \qquad \ldots(3)$$

Now, suppose the centre of the cylinder is now pulled down by a distance x, this will further stretch the spring by $2x$. Let v be the linear speed of the centre of the cylinder at that instant, then the angular speed of the cylinder at this instant will be $\omega = \frac{v}{R}$. Total mechanical energy of the system at this instant will be,

$$E = \frac{1}{2}k(x_0 + 2x)^2 - mg(x\sin\theta) + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$\Rightarrow \quad E = \frac{1}{2}k(x_0 + 2x)^2 - mgx\sin\theta + \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v^2}{R^2}\right)$$

$$\Rightarrow \quad E = \frac{1}{2}k(x_0 + 2x)^2 - mgx\sin\theta + \frac{3}{4}mv^2$$

Since, E is constant, so $\frac{dE}{dt} = 0$

$$\Rightarrow \quad 2k(x_0 + 2x)\frac{dx}{dt} - mg\sin\theta\frac{dx}{dt} + \frac{3}{2}mv\frac{dv}{dt} = 0$$

Since, $\frac{dx}{dt} = v$, $\frac{dv}{dt} = a$ and $2kx_0 = mg\sin\theta$

$$\Rightarrow \quad \frac{3}{2}ma = -4kx$$

Since, $a \propto -x$, motion is simple harmonic

$$\Rightarrow \quad T = 2\pi\sqrt{\left|\frac{x}{a}\right|} = 2\pi\sqrt{\frac{3m}{8k}} = 2\pi\sqrt{\frac{3\times14}{8\times5000}} = 0.203 \text{ s}$$

Maximum acceleration of centre of cylinder is

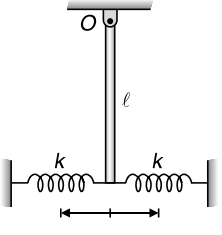
$$a_{max} = \omega^2 A$$

$$\Rightarrow \quad a_{max} = \left(\frac{2\pi}{T}\right)^2 A, \text{ where } A = 50 \text{ mm} = 0.05 \text{ m}$$

$$\Rightarrow \quad a_{max} = \left(\frac{2\pi}{0.203}\right)^2 (0.05) = 47.9 \text{ ms}^{-2}$$

ILLUSTRATION 50

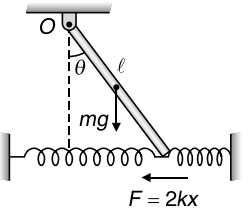
Calculate the frequency of small oscillations of a thin uniform vertical rod of mass m and length l hinged at the point O as shown in Figure.



The combined stiffness of each of the spring is equal to k. The mass of the spring is negligible.

SOLUTION

When the rod is displaced by a small angle θ as shown in Figure, then both the springs are deformed by a distance $x = l\theta$.



We observe that the left spring is stretched and the right spring is compressed, so that both the springs will exert a torque on rod in same sense as the restoring torque. The net restoring torque on rod is given by

$$\tau = -\left[(2kx)(l\sin\theta) + (mg)\left(\frac{l}{2}\sin\theta\right)\right]$$

Since θ is small, so $\sin\theta \approx \theta = \frac{x}{l}$

$$\Rightarrow \quad \tau = -\left(2kxl\theta + \frac{mgl\theta}{2}\right)$$

$$\Rightarrow \quad \tau = -\left(2kl^2\theta + \frac{mgl\theta}{2}\right) \qquad \{\because x = l\theta\}$$

$$\Rightarrow \quad \tau = -\left(2kl^2 + \frac{mgl}{2}\right)\theta$$

Negative sign indicates restoring nature of force. If rod has an angular acceleration α, then, restoring torque is

$$\tau = I_0\alpha$$

$$\Rightarrow \quad \tau = \left(\frac{ml}{3}\right)^2\alpha$$

where, $I_0 = \frac{ml^2}{3}$ is the moment of inertia of rod about an axis passing through O

$$\Rightarrow \quad \frac{ml^2}{3}\alpha = -\left(2kl^2 + \frac{mgl}{2}\right)\theta$$

$$\Rightarrow \quad \ddot{\theta} = -\left(\frac{2kl^2 + (mgl/2)}{ml^2/3}\right)\theta \qquad \left\{\because \alpha = \frac{d^2\theta}{dt^2} = \ddot{\theta}\right\}$$

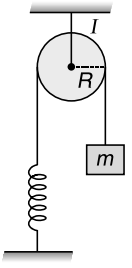
$$\Rightarrow \quad \ddot{\theta} = -\left(\frac{12kl + 3mg}{2ml}\right)\theta$$

$$\Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\left|\frac{\ddot{\theta}}{\theta}\right|} = \frac{1}{2\pi}\sqrt{\frac{12kl + 3mg}{2ml}}$$

ILLUSTRATION 51

A pulley block system is initially in equilibrium. If the block is displaced down slightly from its equilibrium position and released, calculate the time period of oscillation of the

system if the pulley has radius R and moment of inertia I. Assume there is sufficient friction present between pulley and string so that string will not slip over pulley surface.

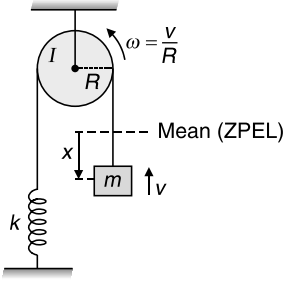


SOLUTION

Initially, in equilibrium let extension in the spring be h. Then, we have

$$mg = kh \qquad \ldots(1)$$

When the block is released, then at any instant let it be at a distance x below the mean position and moving with a velocity v towards it as shown in Figure.



Total mechanical energy of block at this instant is

$$E = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}k(x+h)^2 - mgx \qquad \ldots(2)$$

For no slipping of string on pulley surface, we have

$$v = R\omega \qquad \ldots(3)$$

So, equation (2) becomes

$$E = \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v^2}{R^2}\right) + \frac{1}{2}k(x+h)^2 - mgx$$

$$\Rightarrow \quad E = \frac{1}{2}\left(m + \frac{I}{R^2}\right)v^2 + \frac{1}{2}k(x+h)^2 - mgx$$

Since E is constant, so we have $\frac{dE}{dt} = 0$

$$\Rightarrow \quad \left(m + \frac{I}{R^2}\right)\frac{vdv}{dt} + kv(x+h) - mgv = 0$$

$$\Rightarrow \quad \left(m + \frac{I}{R^2}\right)\frac{d^2x}{dt^2} + kx + kh - mg = 0 \qquad \ldots(4)$$

Substituting equation (1) in (4), we get

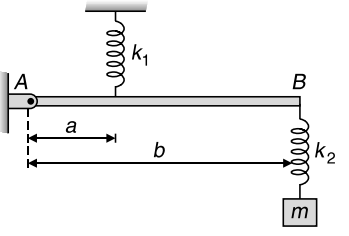
$$\left(m + \frac{I}{R^2}\right)\frac{d^2x}{dt^2} + kx = 0$$

$$\Rightarrow \quad \ddot{x} + \left[\frac{k}{\left(m + \frac{I}{R^2}\right)}\right]x = 0$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{x}{|\ddot{x}|}} = 2\pi\sqrt{\frac{m + \frac{I}{R^2}}{k}}$$

ILLUSTRATION 52

Calculate the time period of oscillations of the system shown in Figure. The bar is rigid and light. The springs are also light. Initially in equilibrium bar is horizontal.



SOLUTION

Initially in equilibrium, let h_1 and h_2 be extensions in springs, then

$$(k_1h_1)a = (k_2h_2)b \qquad \ldots(1)$$

Also, $k_2h_2 = mg$ $\qquad \ldots(2)$

When mass m is displaced slightly by x (downwards), then let further extensions in the springs be x_1 and x_2. Given that the bar is light, so we have

$$k_1(x_1 + h_1)a \approx k_2(x_2 + h_2)b$$

$$\Rightarrow \quad k_1x_1a = k_2x_2b \qquad \ldots(3)$$

Also, when left spring extends by x_1, then the end B moves down by $x_1\left(\frac{b}{a}\right)$, due to which total extension x is given by

$$x = x_2 + x_1\left(\frac{b}{a}\right) \qquad \ldots(4)$$

$$\Rightarrow \quad x = x_2 + \left(\frac{k_2x_2b}{k_1a}\right)\frac{b}{a} = x_2\left(1 + \frac{k_2b^2}{k_1a^2}\right)$$

$$\Rightarrow \quad x_2 = \frac{x}{1 + \left(k_2b^2/k_1a^2\right)}$$

Restoring force (F) acting on mass m is

$$F = -(k_2x_2 - mg)$$

Negative sign indicates that F is directed towards the mean position.

$$\Rightarrow \quad F = -\left[\frac{k_2}{1 + \left(k_2b^2/k_1a^2\right)}x - mg\right]$$

$$\Rightarrow \quad \ddot{x} = -\left[\frac{k_1k_2a^2}{m\left(k_1a^2+k_2b^2\right)}\right]x + g$$

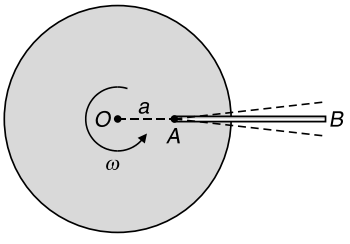
$$\Rightarrow \quad \ddot{x} = -\omega^2 x + g$$

$$\Rightarrow \quad \omega = \frac{2\pi}{T} = \sqrt{\frac{k_1k_2a^2}{m\left(k_1a^2+k_2b^2\right)}}$$

$$\Rightarrow \quad T = \frac{2\pi}{a}\sqrt{\frac{m\left(k_1a^2+k_2b^2\right)}{k_1k_2}}$$

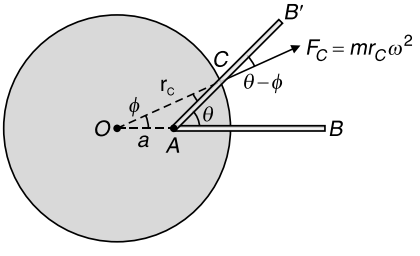
ILLUSTRATION 53

A smooth horizontal disc rotates about the vertical axis O with a constant angular velocity ω. A thin uniform rod AB of length l performs small oscillations about the vertical axis A fitted to the disc at a distance a from the axis of the disc as shown in Figure. Calculate the angular frequency of these oscillations.



SOLUTION

In plane of disc, let the rod be tilted slightly by an angle θ as shown in Figure.



The restoring torque (τ) on the rod is

$$\tau = -\left[\left(mr_C\omega^2\right)\sin\left(\theta-\phi\right)\right]\frac{l}{2}$$

Since, $a\phi = \frac{l}{2}(\theta-\phi)$

$$\Rightarrow \quad \phi = \frac{l\theta/2}{a+(l/2)} \qquad \ldots(1)$$

Also, $r_C \approx a + \frac{l}{2}$ $\quad \{\because \theta \text{ is small}\}$

$$\Rightarrow \quad \tau = -\frac{mr_C\omega^2 l}{2}(\theta-\phi) \qquad \ldots(2)$$

Substituting equation (2) in equation (1), we get

$$\tau = -\frac{mr_C\omega^2 l}{2}\left(1-\frac{l/2}{a+(l/2)}\right)\theta$$

$$\Rightarrow \quad \tau = -\left(\frac{ml\omega^2 a}{2}\right)\theta$$

$$\Rightarrow \quad \ddot{\theta} + \left(\frac{ml\omega^2 a}{2I}\right)\theta = 0 \qquad \{\because \tau = I\ddot{\theta}\}$$

where, I is the moment of inertia of the rod about A i.e., $I = \frac{ml^2}{3}$

$$\Rightarrow \quad \ddot{\theta} + \left[\frac{ml\omega^2 a}{2\left(ml^2/3\right)}\right]\theta = 0$$

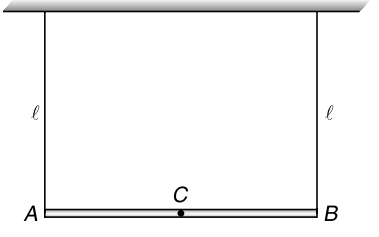
$$\Rightarrow \quad \ddot{\theta} + \left(\frac{3\omega^2 a}{2l}\right)\theta = 0$$

If ω_0 is the angular frequency of oscillations, then

$$\omega_0 = \sqrt{\frac{|\ddot{\theta}|}{\theta}} = \sqrt{\frac{3\omega^2 a}{2l}}$$

ILLUSTRATION 54

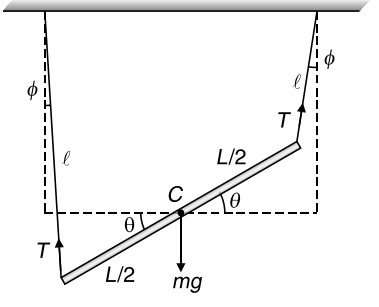
A uniform rod AB of mass m is suspended by two identical strings of length l as shown in Figure.



The rod is turned through a small angle in horizontal plane about and the vertical axis is passing through its centre C. In this process the strings are deviated by a small angle θ_0 from vertical. The rod is then released so that it starts performing angular SHM. Calculate the angular frequency of oscillations and the rod's oscillation energy.

SOLUTION

When the rod is twisted through an angle θ, then let the string deviate through an angle ϕ as shown in Figure.



Assuming length of rod to be L, we have

$$\frac{L}{2}\theta = l\phi \qquad \ldots(1)$$

Since the rod is in vertical equilibrium, so

$$2T\cos\phi = mg$$

where, T is tension in each string holding the rod.
For small ϕ, $\cos\phi \approx 1$, so $2T = mg$

$$\Rightarrow \quad T = \frac{mg}{2} \qquad \ldots(2)$$

The component of tension $T\sin\phi$ acting on each end of rod produces a restoring torque given by

$$\tau = -(2T\sin\phi)\left(\frac{L}{2}\right)$$

Negative sign shows that restoring torque is directed towards mean position. For small ϕ, $\sin\phi \approx \phi$

$$\Rightarrow \quad \tau = I\ddot{\theta} = -TL\phi$$

Using equations (1) and (2), we get

$$\tau = -\left(\frac{mgL}{2}\right)\left(\frac{L}{2l}\right)\theta = -\left(\frac{mgL^2}{4l}\right)\theta$$

$$\Rightarrow \quad I\ddot{\theta} + \frac{mgL^2}{4l}\theta = 0, \text{ where } I = \frac{mL^2}{12}$$

$$\Rightarrow \quad \ddot{\theta} + \left(\frac{mgL^2}{4lI}\right)\theta = 0$$

$$\Rightarrow \quad \omega = \sqrt{\frac{mgL^2}{4lI}} = \sqrt{\frac{mgL^2}{4l\left(mL^2/12\right)}} = \sqrt{\frac{3g}{l}}$$

Since it is given that initially strings are deflected by an angle θ_0, so if β is the angular amplitude of oscillations, then

$$\frac{L}{2}\beta = l\theta_0 \qquad \{\text{from equation (1)}\}$$

$$\Rightarrow \quad \beta = \left(\frac{2l}{L}\right)\theta_0$$

The total energy of oscillation of rod is $E = \frac{1}{2}I\omega^2\beta^2$

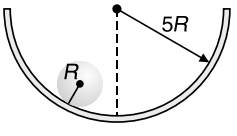
$$\Rightarrow \quad E = \frac{1}{2}\left(\frac{mL^2}{12}\right)\left(\frac{3g}{l}\right)\left(\frac{4l^2}{L^2}\theta_0^2\right) = \frac{1}{2}mgl\theta_0^2$$

Test Your Concepts-III

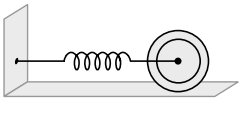
Based on Rotational SHM

(Solutions on page H.183)

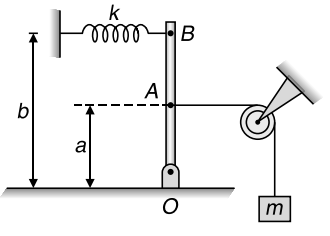
1. A solid sphere of radius R rolls without slipping in a cylindrical trough of radius $5R$. Find the time period of small oscillations.



2. The disk has a weight of 100 N and rolls without slipping on the horizontal surface as it oscillates about its equilibrium position. If the disk is displaced, by rolling it counter clockwise 0.4 rad, determine the equation which describes its oscillatory motion when it is released.

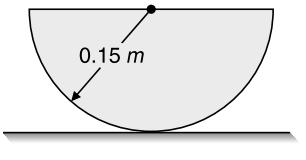


3. A massless rigid rod is hinged at O. A string carrying a mass m at one end is attached to point A on the rod. At another point B of the rod, a horizontal spring of force constant k is attached as shown in Figure.

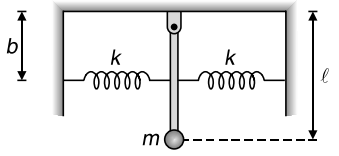


Calculate the period of small vertical oscillations of mass m around its equilibrium position. Consider that the rod is initially vertical and is in equilibrium at that instant.

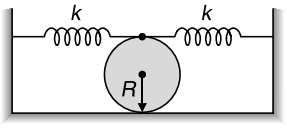
4. Determine the natural period of vibration of the 50 N semi-circular disk. Given that the centre of mass of a semi-circular disk lies at a distance of $\frac{4r}{3\pi}$ from the centre.



5. Calculate the time period of small oscillations of the spring loaded pendulum. The equilibrium position is vertical as shown in Figure. The mass of the rod is negligible and treat the mass as a particle.

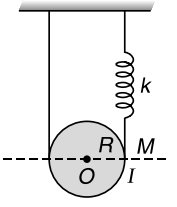


6. A solid uniform cylinder of mass M performs small oscillations in horizontal plane if slightly displaced from its mean position shown in Figure.

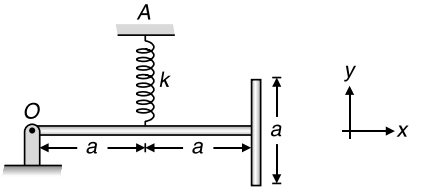


Initially springs are in natural lengths and cylinder does not slip on ground during oscillations due to friction between ground and cylinder. If force constant of each spring is k, then calculate the time period of small oscillations of cylinder.

7. The pulley shown in Figure has a moment of inertia I about its axis and mass m. Calculate the time period of vertical oscillation of its centre of mass. The spring is light, has spring constant k and the string does not slip over the pulley.



8. A T bar of uniform cross section and mass M is supported in a vertical plane by a hinge O and a spring of force constant k at A. Calculate the period for small amplitude rotational oscillations in the xy plane.

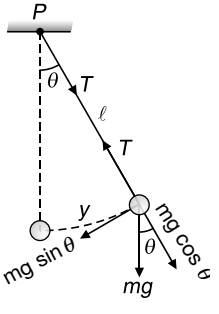


SIMPLE PENDULUM

A simple pendulum consists of a point mass suspended from a fixed point by a light inextensible string. In equilibrium, the mass lies vertically below the point of suspension P. If it is displaced to one side and then released, it oscillates about the equilibrium position.

Suppose at any instant the pendulum makes an angle θ with the vertical. The forces acting on the mass m are its weight mg and tension T in the string. The weight mg may be resolved into two components, the radial component $mg\cos\theta$ and the tangential component $mg\sin\theta$ is the restoring force. Hence

$$F = -mg\sin\theta$$



Note that the restoring force is not proportional to θ but to $\sin\theta$. The motion is, therefore, not simple harmonic. However, if θ is small, then $\sin\theta \approx \theta$

$$\Rightarrow \quad F = -mg\theta$$

The displacement along the arc is $y = \theta l$ and for small θ this is nearly straight-line motion. Hence

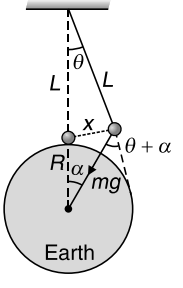
$$\Rightarrow \quad F = m\ddot{y} = -\left(\frac{mg}{l}\right)y$$

$$\Rightarrow \quad \ddot{y} + \frac{g}{l}y = 0$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{y}{|\ddot{y}|}} = 2\pi\sqrt{\frac{l}{g}}$$

SHM OF A PENDULUM OF LARGE LENGTH

In the derivation of a simple pendulum, we had assumed the length of the pendulum to be much less than the radius (R) of earth so that the force mg is always directed vertically downwards. However, if the length (L) of the pendulum is large, then the force mg will be directed towards the centre of the earth as shown in Figure.



When bob is displaced by amount x, restoring force towards mean position will be $mg\sin(\theta+\alpha)$. From free body diagram (FBD), we have

$$F = ma = m\ddot{x} = -mg\sin(\theta+\alpha)$$

For small displacement, θ is small, hence α is also small, i.e., $(\theta+\alpha)$ is small, so $\sin(\theta+\alpha) \approx \theta+\alpha$

$$\Rightarrow \quad ma = m\ddot{x} = -mg(\theta+\alpha)$$

$$\Rightarrow \quad \ddot{x} + g\left(\frac{1}{L}+\frac{1}{R}\right)x = 0$$

$$\Rightarrow \quad \omega = \sqrt{\frac{|\ddot{x}|}{x}} = \sqrt{g\left(\frac{1}{L}+\frac{1}{R}\right)}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{L}+\frac{1}{R}\right)}}$$

CASE-I:

When $L \ll R$, when length of pendulum is small compared to radius of the earth, then

$$g\left(\frac{1}{L}+\frac{1}{R}\right) \approx \frac{g}{L}$$

$$\Rightarrow \quad T \approx 2\pi\sqrt{\frac{L}{g}}$$

CASE-II:

When $L \gg R$, as in the case of infinite pendulum, then

$$g\left(\frac{1}{L}+\frac{1}{R}\right) \approx \frac{g}{R}$$

$$\Rightarrow \quad T \approx 2\pi\sqrt{\frac{R}{g}}$$

ILLUSTRATION 55

A ball is suspended by a thread of length L at the point O on the wall PQ which is inclined to the vertical by an angle α. The thread with the ball is now displaced through a small angle β away from the vertical and also from the wall. If the ball is released, find the period of oscillation of the pendulum when (a) $\beta < \alpha$ (b) $\beta > \alpha$.

Assume the collision on the wall to be perfectly elastic.

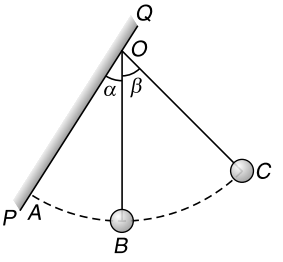
SOLUTION

The motion of simple pendulum is angular SHM; so, its equation of motion will be

$$\theta = \theta_0 \sin\omega t \text{ with } \omega = \sqrt{\frac{g}{L}}$$

(a) When $\beta < \alpha$, i.e., when angular amplitude β is lesser than α, the pendulum will oscillate with its natural frequency, so that

$$T_1 = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}} \qquad \ldots(1)$$



(b) When $\beta > \alpha$, time taken by pendulum to move from B to C and back to B,

$$t_1 = \frac{T}{2} = \frac{1}{2}\left(2\pi\sqrt{\frac{L}{g}}\right) = \pi\sqrt{\frac{L}{g}} \qquad \ldots(2)$$

Now as in case of simple harmonic motion, $\theta = \theta_0 \sin\omega t$

So, time taken by the pendulum to move from equilibrium position B to A,

i.e., for $\theta = \alpha$ when $\theta_0 = \beta$, will be given by $\alpha = \beta\sin\omega t$,

$$\text{i.e., } t = \frac{1}{\omega}\sin^{-1}\left(\frac{\alpha}{\beta}\right)$$

So, time taken by pendulum to move from B to A and back to B,

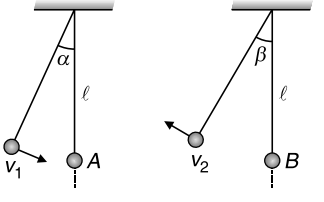
$$t_2 = 2t = 2\sqrt{\frac{L}{g}}\sin^{-1}\left(\frac{\alpha}{\beta}\right) \qquad \left\{\text{since } \omega = \sqrt{\frac{g}{L}}\right\}$$

So, time period of motion,

$$T_2 = t_1 + t_2 = \sqrt{\frac{L}{g}}\left(\pi + 2\sin^{-1}\left(\frac{\alpha}{\beta}\right)\right)$$

ILLUSTRATION 56

Figure shows two identical simple pendulums of length l. One is tilted at an angle α and imparted an initial velocity v_1 toward mean position and at the same time other is projected away from mean position with a velocity v_2 at an initial angular displacement β. Calculate the phase difference in oscillations of these two pendulums.



SOLUTION

Given that first pendulum bob is given a velocity v_1 at a displacement $l\alpha$ from mean position.

Since, $v^2 = \omega^2(A^2 - x^2)$

$$\Rightarrow \quad v_1 = \omega\sqrt{A_1^2 - (l\alpha)^2}$$

where, A_1 is the amplitude of SHM of this bob

For simple pendulum, $\omega = \sqrt{\frac{g}{l}}$

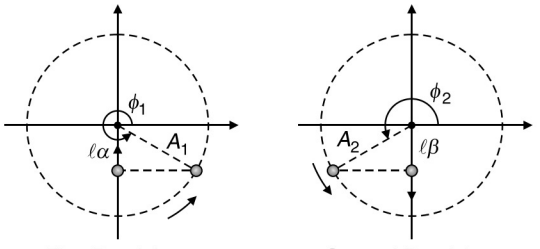
$$\Rightarrow \quad v_1^2 = \frac{g}{l}\left(A_1^2 - l^2\alpha^2\right)$$

$$\Rightarrow \quad A_1 = \sqrt{l^2\alpha^2 + \frac{v_1^2 l}{g}} \qquad \ldots(1)$$

Similarly, if A_2 is the amplitude of SHM of second pendulum, then we have

$$v_2 = \omega\sqrt{A_2^2 - (l\beta)^2}$$

$$\Rightarrow \quad A_2 = \sqrt{l^2\beta^2 + \frac{v_2^2 l}{g}} \qquad \ldots(2)$$



$$\phi_1 = 2\pi - \sin^{-1}\left(\frac{l\alpha}{A_1}\right)$$

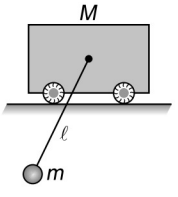
$$\phi_2 = \pi + \sin^{-1}\left(\frac{l\beta}{A_2}\right)$$

$$\Rightarrow \quad \Delta\phi = \pi + \sin^{-1}\left(\frac{l\beta}{A_2}\right) - 2\pi + \sin^{-1}\left(\frac{l\alpha}{A_1}\right)$$

$$\Rightarrow \quad \Delta\phi = \sin^{-1}\left(\frac{l\alpha}{A_1}\right) + \sin^{-1}\left(\frac{l\beta}{A_2}\right) - \pi$$

ILLUSTRATION 57

A trolley of mass M that can slide on frictionless rails has a pendulum bob of mass m connected to it through a massless inextensible string of length l. The pendulum can swing in the vertical plane. Calculate the time period of oscillation of the pendulum if it is left free after giving a small displacement.



SOLUTION

Since there is no external force in the horizontal direction, the centre of mass (CM) will not move along the horizontal direction.

Since the CM is at a distance $l' = \dfrac{Ml}{M+m}$ from m.

The pendulum can be considered to be oscillating from the point of suspension i.e., CM and having string of length l'. So, its time period is

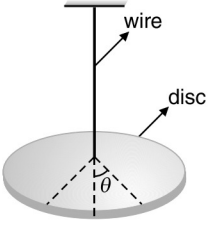
$$T = 2\pi\sqrt{\frac{l'}{g}} = 2\pi\sqrt{\frac{Ml}{(M+m)g}}$$

TORSIONAL PENDULUM

On rotating a body from its position of equilibrium, a restoring torque proportional to angle of rotation comes into play, the body executes angular (or rotational) SHM If τ is torque when the angle of rotation is θ, then

$$\tau \propto \theta$$

$$\Rightarrow \quad \tau = -C\theta, \quad \text{where } C \text{ is called } \textbf{torsional constant}.$$



If I is the moment of inertia of the body about the specified axis of rotation and α is the angular acceleration, then

$$\tau = I\alpha = I\frac{d^2\theta}{dt^2} = -C\theta$$

$$\Rightarrow \quad \frac{d^2\theta}{dt^2} + \frac{C}{I}\theta = 0$$

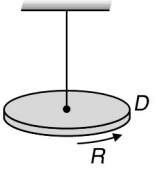
This is the differential equation of angular SHM the time period is clearly

$$T = 2\pi\sqrt{\frac{\theta}{|\ddot{\theta}|}} = 2\pi\sqrt{\frac{I}{C}}$$

A typical torsional pendulum is a disc suspended by a wire attached to the centre of mass of the disc. When the disc is rotated, the wire gets twisted and a restoring torque is produced in it. The disc, therefore, executes angular oscillations on being released.

ILLUSTRATION 58

A torsional pendulum consists of a uniform disc D of mass M and radius R attached to a thin rod of torsional constant C as shown in Figure.



Calculate the amplitude and the energy of small torsional oscillations of the disc, if initially the disc was imparted angular speed ω_0.

SOLUTION

For a torsional pendulum, we have $\omega = \sqrt{\frac{C}{I}}$, where $I = \frac{1}{2}MR^2$. So, $\omega = \sqrt{\frac{2C}{MR^2}}$

Since the disc is imparted an angular speed ω_0 (at mean position), so if β is the angular amplitude, then

$$\omega_0 = \beta\omega = \beta\sqrt{\frac{2C}{MR^2}}$$

$$\Rightarrow \quad \beta = \sqrt{\frac{MR^2}{2C}}\omega_0$$

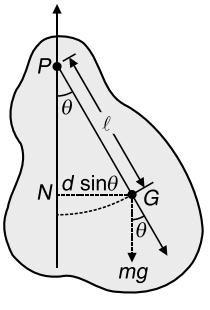
For angular SHM, assuming $U_{\min} = 0$, the total energy of oscillation is given by

$$E = \frac{1}{2}I\omega^2\beta^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{2C}{MR^2}\right)\left(\frac{MR^2}{2C}\omega_0^2\right)$$

$$\Rightarrow \quad E = \frac{1}{4}MR^2\omega_0^2$$

PHYSICAL PENDULUM OR COMPOUND PENDULUM

Any rigid body mounted so that it is capable of swinging in a vertical plane about some axis passing through it is called **a physical pendulum**. It is an example of angular SHM



Consider a body of irregular shape pivoted about a horizontal frictionless axis passing through P and displaced from equilibrium position by angle θ. The equilibrium position is that in which the centre of gravity G lies vertically below P. If M is mass of body and I, the moment of inertia of the body about an axis passing through pivot P, then

$$\tau = -(Mg)(l\sin\theta)$$

$$\Rightarrow \quad I\alpha = -(Mgl)\theta \qquad \{\theta \text{ small, so } \sin\theta \approx \theta\}$$

$$\Rightarrow \quad I\ddot{\theta} + (Mgl)\theta = 0$$

$$\Rightarrow \quad \ddot{\theta} + \left(\frac{Mgl}{I}\right)\theta = 0$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{\theta}{|\ddot{\theta}|}} = 2\pi\sqrt{\frac{I}{Mgl}}$$

Further by Parallel Axis Theorem,

$$I = I_G + Ml^2$$

$$\Rightarrow \quad I = MK^2 + Ml^2 \qquad \{\because\ I_G = MK^2\}$$

where K is the radius of gyration of body.

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{MK^2 + Ml^2}{Mgl}}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{1}{g}\left(l + \frac{K^2}{l}\right)} = 2\pi\sqrt{\frac{l_{\text{effective}}}{g}}$$

$l_{\text{eff}} = l + \frac{K^2}{l}$ is also called **the effective length of the compound pendulum**. When $l = 0$, then $T \to \infty$.

The period of oscillation T of the physical pendulum is minimum when T^2 is minimum i.e., $\frac{d}{dl}(T^2) = 0$

$$\Rightarrow \quad \frac{4\pi^2}{g}\frac{d}{dl}\left(l + \frac{K^2}{l}\right) = 0$$

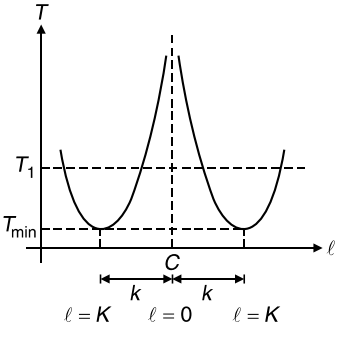
$$\Rightarrow \quad 1 - \frac{K^2}{l^2} = 0$$

$$\Rightarrow \quad l = \pm K$$

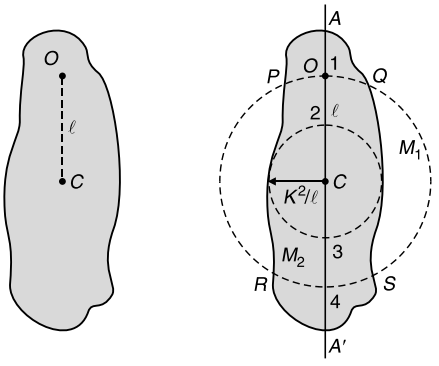
The period of oscillation T of the physical pendulum is minimum when the distance of the point of suspension from the centre of mass of the pendulum is equal to the radius of gyration K of the body.

$$T_{\min} = 2\pi\sqrt{\frac{1}{g}\left(K + \frac{K^2}{K}\right)} = 2\pi\sqrt{\frac{2K}{g}}$$

Experimentally, the plot of T vs l is shown in Figure.



We observe that in a compound pendulum when a straight line AA' is drawn passing through the centre of gravity of the body as shown in Figure, then there exist four points 1, 2, 3, 4 on this line about which if a body is suspended, then the time period of small oscillation of body remains same.



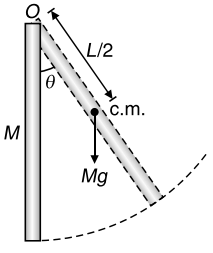
This can also be seen from the plot of T vs l discussed earlier.

ILLUSTRATION 59

A rod of mass M and length L is pivoted about its end O as shown in the figure. Find the period of SHM.

SOLUTION

Restoring torque is $\tau_0 = -Mg\frac{L}{2}\sin\theta$



Using Newton's Second Law and the small angle approximation, we get

$$I_0\frac{d^2\theta}{dt^2} + Mg\frac{L}{2}\theta = 0$$

where, $I_0 = \frac{ML^2}{12} + \frac{ML^2}{4} = \frac{ML^2}{3}$

$$\Rightarrow \quad \frac{d^2\theta}{dt^2} + \left(\frac{3g}{2L}\right)\theta = 0$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{2L}{3g}}$$

ILLUSTRATION 60

A uniform rod of mass m and length L performs small oscillations about a horizontal axis passing through its upper end. Find the mean kinetic energy of the rod during its oscillation period if at $t = 0$ it is deflected from vertical by an angle θ_0 and imparted an angular velocity ω_0.

SOLUTION

This rod oscillates like a physical pendulum whose time period is given by $T = 2\pi\sqrt{\frac{I}{Mgd}}$, where I is moment of inertia of rod about axis passing through point of suspension, d is separation between centre of mass of rod and the point of suspension.

In the given case

$$I = \frac{ML^2}{2} \text{ and } d = \frac{L}{2}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{2L}{3g}} \text{ i.e., } \omega = \sqrt{\frac{3g}{2L}}$$

Initially angular displacement of rod is θ_0 and it is imparted an angular velocity ω_0. If angular amplitude be β, then we have

$$\omega_0 = \omega\sqrt{\beta^2 - \theta_0^2} = \sqrt{\frac{3g}{2L}}\sqrt{\beta^2 - \theta_0^2}$$

$$\Rightarrow \quad \beta = \sqrt{\frac{2L\omega_0^2}{3g} + \theta_0^2}$$

Mean kinetic energy of rod during oscillation is

$$\langle K\rangle = \frac{1}{4}I\omega^2\beta^2$$

$$\Rightarrow \quad \langle K\rangle = \frac{1}{4}\left(\frac{ML^2}{3}\right)\left(\frac{3g}{2L}\right)\left(\frac{2L\omega_0^2}{3g} + \theta_0^2\right)$$

$$\Rightarrow \quad \langle K\rangle = \frac{MgL}{8}\left(\frac{2L\omega_0^2}{3g} + \theta_0^2\right)$$

ILLUSTRATION 61

A uniform rod of mass m and length l performs small oscillations about the horizontal axis passing through its upper end. Find the mean kinetic energy of the rod averaged over one oscillation period if at the initial moment it was deflected from the vertical by an angle θ_0 and then imparted an angular velocity Ω.

SOLUTION

Angular frequency ω of a physical pendulum is

$$\omega = \sqrt{\frac{mgd}{I}}$$

where $d = \frac{l}{2}$, $I = \frac{ml^2}{3}$

$$\Rightarrow \quad \omega = \sqrt{\frac{3g}{2l}}$$

Let β be angular amplitude of rod, then

$$\Omega = \omega\sqrt{\beta^2 - \theta_0^2}$$

$$\Rightarrow \quad \beta^2 = \frac{\Omega^2}{\omega^2} + \theta_0^2$$

Mean kinetic energy of rod is

$$\langle K\rangle = \frac{1}{4}I\omega^2\beta^2 = \frac{1}{4}\left(\frac{ml^2}{3}\right)\left(\frac{3g}{2l}\right)\left(\frac{\Omega^2(2l)}{3g} + \theta_0^2\right)$$

$$\Rightarrow \quad \langle K\rangle = \frac{1}{8}mgl\theta_0^2 + \frac{1}{12}ml^2\Omega^2$$

Test Your Concepts-IV

Based on Pendulum Systems

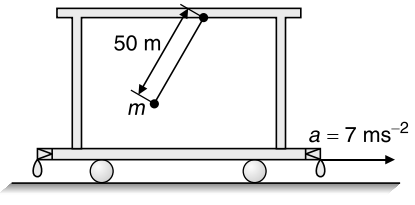
(Solutions on page H.185)

1. A pendulum is secured on a cart rolling without friction down an inclined plane of inclination α. The period of the pendulum on an immobile cart is T_0. How will the period of the pendulum change when the cart rolls down the slope?

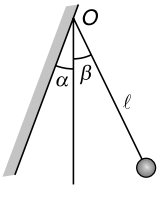
2. Pendulum A is a physical pendulum made from a thin, rigid and uniform rod whose length is d. One end of this rod is attached to the ceiling by a frictionless hinge, so the rod is free to swing back and forth. Pendulum B is a simple pendulum whose length is also d. Obtained the ratio $\frac{T_A}{T_B}$ of their periods for small-angle oscillations.

3. Determine the period of small oscillations of a mathematical pendulum, that is ball suspended by a thread $\ell = 20$ cm in length, if it is located in a liquid whose density is $\eta = 3.0$ times less than that of the ball. The resistance of the liquid is to be neglected.

4. A simple pendulum 50 cm long is suspended from the roof of a cart accelerating in the horizontal direction with $a = 7\ \text{ms}^{-2}$ (figure). Find the period of small oscillations of the pendulum about its equilibrium angle.



5. A ball is suspended by a thread of length l at the point O on the wall, forming a small angle α with the vertical as shown in Figure.

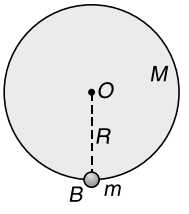


Then the thread with the ball was deviated through a small angle $\beta(\beta > \alpha)$ and set free. Assuming the collision of the ball against the wall to be perfectly elastic, find the oscillation period of such a pendulum.

6. A simple pendulum of length ℓ is suspended from the ceiling of a cart which is sliding without friction on an inclined plane of inclination θ. What will be the time period of the pendulum?

7. Two pendula begin to swing simultaneously. During the first fifteen oscillations of the first pendulum the other pendulum makes only ten swings. Determine the ratio between the lengths of these pendula.

8. A uniform disc of mass M and radius R is hanging vertically with the help of an axle passing through its centre. A small amount (mass m) of mud is stuck at the bottom end B near rim of the disc. If the disc is now given small angular displacement, find the period of its oscillations. What is the equivalent length of simple pendulum?



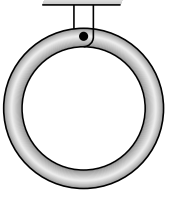
9. The angular displacement of simple pendulum is given by

$$\theta = 0.1\pi \sin\left(2\pi t + \frac{\pi}{6}\right) \text{rad}$$

The mass of the bob is 0.4 kg. Calculate

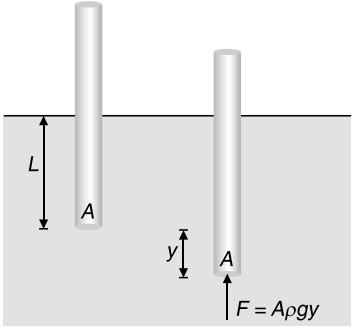
(a) the length of the simple pendulum; and
(b) the velocity of the bob at $t = 0.25$ s

10. A ring radius r is suspended from a point on its circumference. Determine its angular frequency of small oscillations.



OSCILLATIONS OF A FLOATING POLE

Consider a pole of cross-sectional area A and mass M floating in a liquid of density ρ. If the length immersed in the liquid in the equilibrium position is L, then using Archimedes' Principle and Laws of Floatation, the weight of the body is balanced by the upthrust acting on the body, so we have



$$Mg = AL\rho g$$

$$\Rightarrow \quad M = AL\rho$$

If the pole is now pushed down slightly by a distance y, it experiences an additional upthrust equal to $yA\rho g$. This provides the restoring force, tending to bring the pole back to the equilibrium position. So,

$$F = -yA\rho g = -(A\rho g)y$$

$$\Rightarrow \quad M\ddot{y} = -(A\rho g)y$$

$$\Rightarrow \quad \ddot{y} + \left(\frac{A\rho g}{M}\right)y = 0$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{y}{|\ddot{y}|}} = 2\pi\sqrt{\frac{M}{A\rho g}} = 2\pi\sqrt{\frac{L}{g}}$$

where, L is the equilibrium length of the pole immersed in liquid.

ILLUSTRATION 62

A cylinder of mass M, radius r and height h is suspended by a spring whose upper end is fixed. The cylinder is submerged in water such that in equilibrium, the cylinder sinks to half its height. At a certain moment, the cylinder was submerged to $\frac{2}{3}$ of the height and then with no initial velocity started to move vertically. If the spring constant is k and density of water is ρ, then calculate the period of oscillations.

SOLUTION

In equilibrium, the cylinder (of area A) is submerged to a depth $\frac{h}{2}$ and if spring elongation is x_0, then we have

$$kx_0 + \left(A\frac{h}{2}\right)\rho g = Mg \qquad \ldots(1)$$

If cylinder is displaced down further by x and released, then its equation of motion is

$$-\left[k(x_0 + x) + \left(\frac{h}{2} + x\right)A\rho g - Mg\right] = Ma$$

$$\Rightarrow \quad a = \ddot{x} = -\left(\frac{k + A\rho g}{M}\right)x$$

The negative sign indicates the restoring nature of force, so we get

$$\omega = \sqrt{\frac{|\ddot{x}|}{x}}$$

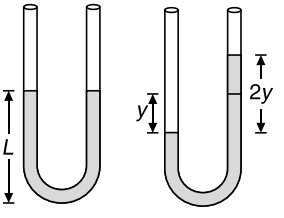
Since $A = \pi r^2$, so we get

$$\omega = \sqrt{\frac{k + (\pi r^2)\rho g}{M}}$$

$$\Rightarrow \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{M}{k + \pi r^2\rho g}}$$

LIQUID OSCILLATING IN A U-TUBE

Consider a U-tube of cross-sectional area A containing a liquid. Let L be the length of the liquid in each limb of tube. In equilibrium the level of liquid in the two limbs is same. Let the liquid be depressed by a distance y in limb 1. Then it rises through the same distance in limb 2.



Therefore, the liquid level in limb 2 becomes higher by $2y$ than that in limb 1. The restoring force, which tends to bring the liquid back to the original level in limb 2, is given by

$$F = -2yA\rho g = -(2A\rho g)y$$

$$\Rightarrow \quad M\ddot{y} = -(2A\rho g)y$$

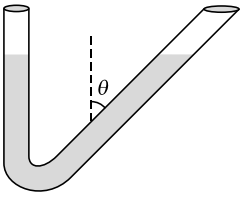
$$\Rightarrow \quad (2L)A\rho\ddot{y} = -(2A\rho g)y$$

$$\Rightarrow \quad \ddot{y} + \frac{g}{L}y = 0$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{y}{|\ddot{y}|}} = 2\pi\sqrt{\frac{L}{g}}$$

ILLUSTRATION 63

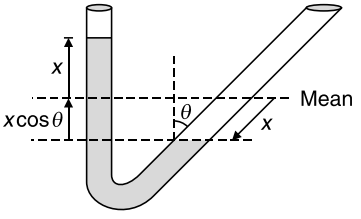
Calculate the period of small oscillations of mercury of mass m poured into a bent tube whose right arm forms an angle θ with the vertical as shown in Figure.



The cross-sectional area of the tube is A and viscosity of mercury is to be neglected.

SOLUTION

Let the mercury be displaced in tube by a distance x as shown in Figure.



The excess pressure due to level difference is

$$\Delta P = (x + x\cos\theta)\rho g$$

Thus, restoring force acting on mercury is

$$F = ma = -A(\Delta P)$$

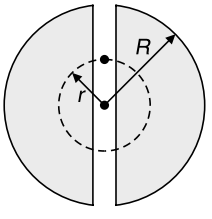
Negative sign tells that the restoring force is directed towards the mean position.

$$\Rightarrow \quad a = \ddot{x} = -\left[\frac{\rho g A}{m}(1+\cos\theta)\right]x$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{x}{|\ddot{x}|}} = 2\pi\sqrt{\frac{m}{A\rho g(1+\cos\theta)}}$$

ILLUSTRATION 64

Suppose a tunnel could be dug through the earth from one side to the other along a diameter, as shown in figure. Show that the motion of the particle is simple harmonic and determine its time period.



SOLUTION

The gravitational force on the particle at a distance r from the centre of the earth arises entirely from that portion of matter of the earth in shells internal to the position of the particle. The external shells exert no force on the particle. The value of gravity at a distance r is given by

$$g' = g\left(\frac{r}{R}\right)$$

where $g = \dfrac{GM}{R^2}$ is gravity at the surface of earth.

The net force on the particle is

$$F = -mg' = -mg\left(\frac{r}{R}\right)$$

Using Newton's Law,

$$F = m\frac{d^2r}{dt^2} = -\left(\frac{mg}{R}\right)r$$

$$\Rightarrow \quad \frac{d^2r}{dt^2} + \left(\frac{g}{R}\right)r = 0$$

This is the differential equation of SHM and period of oscillation is

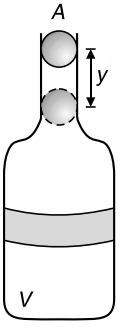
$$T = 2\pi\sqrt{\frac{R}{g}}$$

Since $R = 6.4\times10^6$ m and $g = 9.81$ ms^{-2}

$$\Rightarrow \quad T = 84.2 \text{ min}$$

BALL OSCILLATING IN THE NECK OF AN AIR CHAMBER

Consider an air chamber of volume V, having a neck of cross-sectional area A and a ball of mass m fitted smoothly in the neck. If the ball is pressed down slightly by y, the volume of air decreases by Ay.



If B is the Bulk's modulus of air the excess pressure dP produced is given by

$$dP = -B\left(\frac{yA}{V}\right) \qquad \left\{\because B = -\frac{dP}{dV/V}\right\}$$

A restoring force F having magnitude AdP, comes into play and is directed upwards. Thus

$$F = -\left(\frac{BA^2}{V}\right)y$$

$$\Rightarrow \quad m\ddot{y} = -\left(\frac{BA^2}{V}\right)y$$

$$\Rightarrow \quad \ddot{y} + \left(\frac{BA^2}{mV}\right)y = 0$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{y}{|\ddot{y}|}} = 2\pi\sqrt{\frac{mV}{BA^2}}$$

Conceptual Note(s)

(a) If the pressure volume relation is isothermal, then

$$B_{iso} = P_{atm} = P$$

$$\Rightarrow\ T = 2\pi\sqrt{\frac{mV}{PA^2}}$$

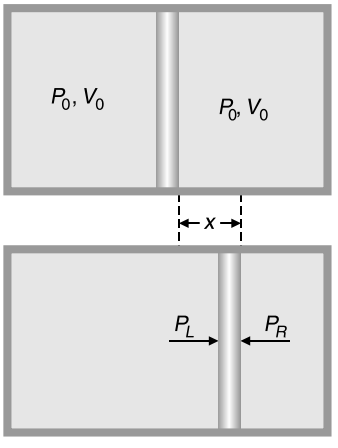
(b) If the pressure volume relation is adiabatic, then

$$B_{ad} = \gamma P_{atm} = \gamma P$$

$$\Rightarrow\ T = 2\pi\sqrt{\frac{mV}{\gamma PA^2}}$$

ILLUSTRATION 65

A closed and isolated cylinder contains ideal gas. An adiabatic separator of mass m, cross-sectional area A divides the cylinder into two equal parts, each with volume V_0 and pressure P_0 in equilibrium. Assuming that the separator can move without friction, find the oscillation frequency when the separator is slightly displaced.

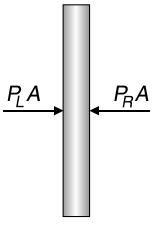


SOLUTION

Let the piston be displaced by a small distance x towards right.

Volume of the right part is $V_R = V_0 - Ax$

Volume of the left part is $V_L = V_0 + Ax$



As the cylinder is isolated from surroundings and separator is adiabatic (non-conducting), the process is adiabatic.

For Right Part

$$P_0V_0^{\gamma} = P_RV_R^{\gamma}$$

$$\Rightarrow\ P_0V_0^{\gamma} = P(V_0 - Ax)^{\gamma}$$

$$\Rightarrow\ P_R = \frac{P_0V_0^{\gamma}}{V_0^{\gamma}\left(1-\frac{Ax}{V_0}\right)^{\gamma}}$$

$$\Rightarrow\ P_R = P_0\left(1-\frac{Ax}{V_0}\right)^{-\gamma}$$

$$\Rightarrow\ P_R = P_0\left(1+\frac{\gamma Ax}{V_0}\right) \qquad \left\{\because \frac{Ax}{V_0} \text{ is small quantity}\right\}$$

For Left Part

$$P_0V_0^{\gamma} = P_LV_L^{\gamma} = P_L(V_0 + Ax)^{\gamma}$$

$$\Rightarrow\ P_L = \frac{P_0V_0^{\gamma}}{V_0^{\gamma}\left(1+\frac{Ax}{V_0}\right)^{\gamma}}$$

$$\Rightarrow\ P_L \approx P_0\left(1-\frac{\gamma Ax}{V_0}\right)$$

Net restoring force acting on the piston is

$$F = -(P_RA - P_LA)$$

$$\Rightarrow\ F = -\left[P_0\left(1+\frac{\gamma Ax}{V_0}\right)A - P_0\left(1+\frac{\gamma Ax}{V_0}\right)A\right]$$

$$\Rightarrow\ F = m\ddot{x} = -\left(\frac{2P_0\gamma A^2}{V_0}\right)x$$

$$\Rightarrow\ \ddot{x} + \left(\frac{2P_0\gamma A^2}{V_0m}\right)x = 0$$

Comparing with $\ddot{x} + \omega^2 x = 0$, we get

$$\omega = \sqrt{\frac{2P_0\gamma A^2}{V_0m}} = \frac{2\pi}{T} = 2\pi f$$

$$\Rightarrow\ f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{2P_0\gamma A^2}{V_0m}}$$

ILLUSTRATION 66

A simple pendulum consists of a small sphere of mass m carrying a charge $+q$ suspended by a thread of length l. The pendulum is placed in a uniform electric field of strength E directed vertically upwards. Calculate the period of small oscillations of the pendulum, if the electrostatic force acting on the sphere is less than the gravitational force.

SOLUTION

A simple pendulum with bob of mass m having charge $+q$ with electric field E in the region directed vertically upwards is shown in Figure (a).

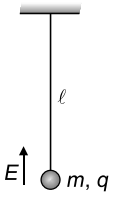


Figure (a)

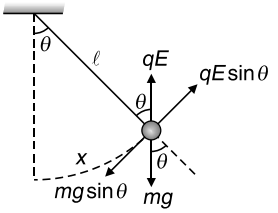


Figure (b)

When bob is in equilibrium, tension in string is

$$T = mg - qE$$

When the bob is given a small displacement x from its mean position as shown in Figure (b), then the restoring force on it is given by

$$F = -(mg\sin\theta - qE\sin\theta)$$

where, θ is the angular displacement of bob from mean position.

For small oscillations, $\sin\theta \approx \theta \approx x/l$

$$\Rightarrow \quad F = -(mg - qE)\theta$$

$$\Rightarrow \quad F = (mg - qE)\frac{x}{l}$$

If a is the acceleration of bob, then

$$a = \frac{F}{m} = -\left(\frac{mg - qE}{ml}\right)x$$

$$\Rightarrow \quad \omega = \sqrt{\frac{|\ddot{x}|}{x}} = \sqrt{\frac{mg - qE}{ml}}$$

$$\Rightarrow \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{ml}{mg - qE}} = 2\pi\sqrt{\frac{l}{(mg - qE)/m}}$$

This result can be written as $T = 2\pi\sqrt{\dfrac{l}{g_{\text{eff}}}}$

where, $g_{\text{eff}} = g - \dfrac{qE}{m}$ is the effective value of acceleration due to gravity under the influence of electric field.

Similarly, if in this problem electric field E is reversed in direction i.e., acts in downward direction, then net effective force on bob in downward direction is increased and effective gravity can be written as

$$g_{\text{eff}} = g + \frac{qE}{m}$$

So, in this case, the time period of a pendulum is written as

$$T = 2\pi\sqrt{\frac{l}{g_{\text{eff}}}} = 2\pi\sqrt{\frac{l}{g + \dfrac{qE}{m}}}$$

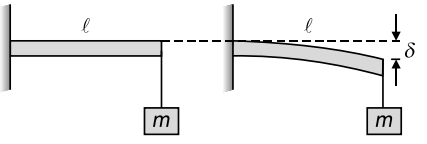
ILLUSTRATION 67

A light wooden rod fixed at one end is kept horizontal. A load of mass 0.4 kg tied to the free end of the rod causes that end to be depressed by $\delta = 2.5$ cm. If this load is disturbed slightly (in vertical direction) from mean position, then calculate the period of oscillations. Take $g = 10\ \text{ms}^{-2}$.

SOLUTION

Due to elasticity in rod it sags by weight of load. If its shear modulus of elasticity is η, then for equilibrium of system we have

$$\eta = \frac{F/A}{\delta/l}$$



where, A is area of cross-section of rod.

$$\frac{Mg}{A} = \frac{\eta\delta}{l} \qquad \ldots(1)$$

When the load is depressed by x, then a restoring force F is developed in rod such that

$$F - Mg = Ma \qquad \ldots(2)$$

where, $\eta = \dfrac{F/A}{(\delta + x)/l}$

$$\Rightarrow \quad F = \frac{\eta A}{l}(\delta + x) \qquad \ldots(3)$$

Substituting (3) in (2), we get

$$\frac{\eta A}{l}(\delta + x) - Mg = Ma$$

$$\Rightarrow \quad a = \ddot{x} = \left(\frac{\eta A}{Ml}\right)x$$

Since this acceleration a (i.e., $\ddot{x}$) is directed towards the mean position, so we have

$$\ddot{x} = -\left(\frac{\eta A}{Ml}\right)x = -\left(\frac{g}{\delta}\right)x \qquad \{\because \text{of equation (1)}\}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{x}{|\ddot{x}|}} = 2\pi\sqrt{\frac{\delta}{g}}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{2.5}{10}} = 3.14\ \text{s}$$

COMPOSITION OF TWO SHM OF THE SAME PERIOD ALONG THE SAME LINE

Let the two SHM's be

$$y_1 = A_1\sin\omega t \text{ and } y_2 = A_2\sin(\omega t + \phi)$$

The resultant displacement

$$y = y_1 + y_2 = A_1\sin\omega t + A_2\sin(\omega t + \phi)$$

$$\Rightarrow \quad y = A_1\sin\omega t + A_2\sin\omega t\cos\phi + A_2\cos\omega t\sin\phi$$

$$\Rightarrow \quad y = (A_1 + A_2\cos\phi)\sin\omega t + A_2\sin\phi\cos\omega t \qquad \ldots(1)$$

Let $A_1 + A_2\cos\phi = R\cos\theta$ and $A_2\sin\phi = R\sin\theta$.

Substituting in (1), we get

$$y = R[\cos\theta\sin(\omega t) + \sin\theta\cos(\omega t)]$$

$$\Rightarrow \quad y = R\sin(\omega t + \theta)$$

Thus, the resultant motion is also simple harmonic along the same line and has the same time period. Its amplitude R is

$$R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

and it is phase θ ahead of the first motion, where

$$\tan\theta = \frac{A_2\sin\phi}{A_1 + A_2\cos\phi}$$

Problem Solving Technique(s)

If $y_1 = a\sin(\omega t)$ and $y_2 = b\cos(\omega t)$ are two SHM then by the superimposition of these two SHM we get

$$y = y_1 + y_2$$

$$y = a\sin(\omega t) + b\cos(\omega t)$$

$$\Rightarrow \quad y = A\sin(\omega t + \phi)$$

This is also the equation of SHM, having

Amplitude $A = \sqrt{a^2 + b^2}$ and Phase $\phi = \tan^{-1}\left(\frac{b}{a}\right)$

COMPOSITION OF TWO SHM OF SAME PERIOD AT RIGHT ANGLES TO EACH OTHER

Let the two motions at right angles be

$$x = A\sin(\omega t) \qquad \ldots(1)$$

$$y = B\sin(\omega t + \phi) \qquad \ldots(2)$$

along the x and y-axis respectively

Equation (1) gives

$$\sin(\omega t) = \frac{x}{A}$$

$$\Rightarrow \quad \cos(\omega t) = \sqrt{1 - \frac{x^2}{A^2}}$$

Equation (2) gives

$$\frac{y}{B} = \sin(\omega t)\cos\phi + \cos(\omega t)\sin\phi$$

$$\Rightarrow \quad \frac{y}{B} = \left(\frac{x}{A}\right)\cos\phi + \left[\sqrt{1 - \frac{x^2}{A^2}}\right]\sin\phi$$

$$\Rightarrow \quad \frac{y}{B} - \frac{x}{A}\cos\phi = \sqrt{1 - \frac{x^2}{A^2}}\sin\phi$$

Squaring, we get

$$\frac{y^2}{B^2} + \frac{x^2}{A^2}\cos^2\phi - \frac{2xy}{AB}\cos\phi = \left(1 - \frac{x^2}{A^2}\right)\sin^2\phi$$

$$\Rightarrow \quad \frac{y^2}{B^2} + \frac{x^2}{A^2}(\cos^2\phi + \sin^2\phi) - \frac{2xy}{AB}\cos\phi = \sin^2\phi$$

$$\Rightarrow \quad \frac{x^2}{A^2} + \frac{y^2}{B^2} - \frac{2xy}{AB}\cos\phi = \sin^2\phi$$

This is the equation of an ellipse.

CASE-I: For $\phi = 0$

The equation becomes $\frac{x^2}{A^2} + \frac{y^2}{B^2} - \frac{2xy}{AB} = 0$

$$\Rightarrow \quad \frac{x}{A} - \frac{y}{B} = 0$$

$$\Rightarrow \quad y = \frac{B}{A}x$$

This is the equation of a straight line. Thus, the resultant motion is a SHM along a straight line, passing through the origin, inclined at an angle $\tan^{-1}\left(\frac{B}{A}\right)$ to the x-axis.

CASE-II: For $\phi = \pi$

The equation becomes

$$\Rightarrow \quad \frac{x^2}{A^2} + \frac{y^2}{B^2} + \frac{2xy}{AB} = 0$$

$$\Rightarrow \quad \frac{x}{A} + \frac{y}{B} = 0$$

$$\Rightarrow \quad y = -\left(\frac{B}{A}\right)x$$

The resultant SHM is along a straight line inclined at $\tan^{-1}\left(\frac{-B}{A}\right)$ to the x axis.

CASE-III: For $\phi = \pi/2$

The equation becomes $\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$, which is an ellipse

If $A = B$, the equation is $x^2 + y^2 = A^2$, which is a circle.

Problem Solving Technique(s)

A uniform circular motion may thus be regarded as a combination of two similar simple harmonic motions at right angles to each other and differing in phase by $\frac{\pi}{2}$.

CASE-IV: For $\phi = \pi/4$

The equation becomes

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} - \frac{2xy}{AB}\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{2}$$

$$\Rightarrow \quad \frac{x^2}{A^2} + \frac{y^2}{B^2} - \frac{\sqrt{2}xy}{AB} = \frac{1}{2}$$

which is the equation of an oblique ellipse.

Problem Solving Technique(s)

If two oscillations of different frequencies at right angles are combined, then the resulting motion is more complicated. It is not even periodic unless the two frequencies are in the ratio of integers. The resulting curves are called **Lissajous Figures (not in syllabus).**

ILLUSTRATION 68

Two linear simple harmonic motions of equal amplitudes a and frequencies ω and 2ω are impressed on a particle along x and y axis respectively. If the initial phase difference between them is $\frac{\pi}{2}$, calculate the resultant trajectory equation of the particle.

SOLUTION

Let $x = a\cos(\omega t)$, then according to problem, we have

$$y = a\cos\left(2\omega t + \frac{\pi}{2}\right)$$

$$\Rightarrow \quad y = -a\sin(2\omega t)$$

$$\Rightarrow \quad y = -2a\sin(\omega t)\cos(\omega t)$$

$$\Rightarrow \quad y = -2a\left(\sqrt{1-\frac{x^2}{a^2}}\right)\left(\frac{x}{a}\right)$$

$$\Rightarrow \quad a^2y^2 = 4x^2(a^2 - x^2)$$

ILLUSTRATION 69

Three simple harmonic motions in the same direction having the same amplitude a and same period are superposed. If each differs in phase from the next by $45°$, then find the resultant equation, resultant amplitude, phase of the resultant motion relative to the first, ratio of the energy of the resultant motion to that possessed by any single motion.

SOLUTION

Let these three simple harmonic motions be represented by

$$y_1 = a\sin\left(\omega t - \frac{\pi}{4}\right)$$

$$y_2 = a\sin\omega t \text{ and}$$

$$y_3 = a\sin\left(\omega t + \frac{\pi}{4}\right)$$

On superimposing, resultant SHM will be

$$y = a\left[\sin\left(\omega t - \frac{\pi}{4}\right) + \sin\omega t + \sin\left(\omega t + \frac{\pi}{4}\right)\right]$$

$$\Rightarrow \quad y = a\left(2\sin\omega t\cos\frac{\pi}{4} + \sin\omega t\right)$$

$$\Rightarrow \quad y = a(1+\sqrt{2})\sin\omega t$$

So, resultant amplitude is $A = (1+\sqrt{2})a$

Since, energy in SHM is proportional to square of the amplitude, so we have

$$\frac{E_{\text{Resultant}}}{E_{\text{Single}}} = \left(\frac{A}{a}\right)^2 = (\sqrt{2}+1)^2 = 3+2\sqrt{2}$$

DAMPED HARMONIC OSCILLATION

A body set into oscillation continues to oscillate for ever with the same amplitude if no dissipative forces are present. Such oscillations are called **free oscillations**. In free oscillations, as there is no dissipation of energy, so the total mechanical energy remains conserved.

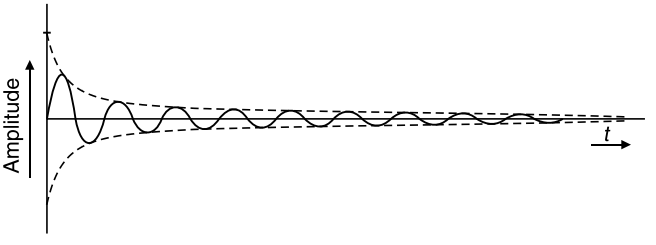
If frictional forces are present, the amplitude of oscillation gradually decreases because energy is dissipated. Such oscillations are **called damped oscillations.** In most cases the damping force is directly proportional to the speed, e.g., viscous drag due to air. The equation of motion of a damped harmonic oscillator can, therefore, be written as

$$F = -ky - bv, \text{ where } b \text{ is damping constant}$$

$$\Rightarrow \quad m\frac{d^2y}{dt} + b\frac{dy}{dt} + ky = 0$$

The solution of this equation for small b is

$$y = Ae^{\frac{-bt}{2m}}\sin(\omega' t + \phi_0), \text{ where } \omega' = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$



We obtain the following results

(a) The amplitude of oscillations decreases exponentially with time. Exponential fall means that if amplitude becomes $\frac{1}{n}$ of initial value in t time, then in next t time, it will become $\frac{1}{n^2}$ of its initial value and so on.

(b) The angular frequency of oscillation is

$$\omega' = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$

So, linear frequency is given by

$$f = \frac{\omega'}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

The frequency is slightly smaller than the frequency of free oscillations, i.e., damping slows down the motion.

(c) If $b \ll \sqrt{km}$, then $\omega' \approx \omega = \sqrt{\dfrac{k}{m}}$

(d) If we assume $b = 0$, then all the equations of the damped oscillator will reduce to the corresponding equations of an undamped oscillator.

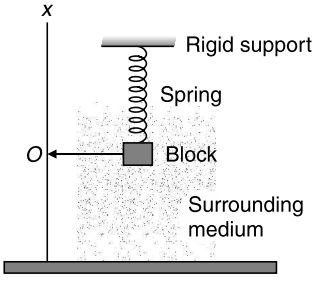
The mechanical energy of the undamped oscillator is $E = \dfrac{1}{2}kA^2$. However, for a damped oscillator, the amplitude is not constant and varies with time as $A' = Ae^{-\left(\frac{b}{2m}\right)t}$. So, we can say that the mechanical energy will have a value given by

$$E' = \frac{1}{2}k(A')^2 = \frac{1}{2}kA^2\left(e^{-\frac{bt}{2m}}\right)^2 = \frac{1}{2}kA^2e^{-\frac{bt}{m}}$$

This expression makes us conclude that the total energy of the system decreases exponentially with time.

ILLUSTRATION 70

Consider the damped oscillator shown in Figure.



If the mass m of the block is 400 g, $k = 45\ \text{Nm}^{-1}$ and the damping constant b is $80\ \text{gs}^{-1}$. Calculate the period of oscillation, time taken for its amplitude of vibrations to drop to half of its initial value and the time taken for its mechanical energy to drop to half its initial value.

SOLUTION

Since, $km = 45 \times 0.4 = 18\ \text{kgNm}^{-1} = 18\ \text{kg}^2\text{s}^{-2}$

$\Rightarrow \quad \sqrt{km} = 4.243\ \text{kgs}^{-1}$

Also, we are given that $b = 0.08\ \text{kgs}^{-1}$

Since $b \ll \sqrt{km}$, so $\omega' \approx \omega = \sqrt{\dfrac{k}{m}}$

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.4}{45}} = 0.6\ \text{s}$$

For $A' = \dfrac{A}{2}$, we have

$$Ae^{-\frac{bt}{2m}} = \frac{A}{2}, \text{ where } \frac{b}{2m} = \frac{0.08}{2(0.4)} = 0.1$$

$$\Rightarrow \quad t = \frac{\log(2)}{b/2m} = \frac{0.693}{0.1} = 6.93\ \text{s}$$

For $E' = \dfrac{E}{2}$, we have

$$\frac{1}{2}kA^2e^{-\frac{bt}{m}} = \frac{1}{2}\left(\frac{1}{2}kA^2\right)$$

$$\Rightarrow \quad \frac{1}{2} = e^{-\frac{bt}{m}}, \text{ where } \frac{b}{m} = \frac{0.08}{0.4} = 0.2$$

$$\Rightarrow \quad t = \frac{\log(2)}{b/m} = \frac{0.693}{0.2} = 3.46\ \text{s}$$

This time is just the half of the time in which amplitude decays to half its initial value.

FORCED OSCILLATIONS AND RESONANCE

Suppose a system is made to oscillate by subjecting it to an external periodic force. This is done by linking it in some way with another oscillating system, which is generally called **the driver**. When a periodic force is applied on an oscillating body, then the body begins to vibrate with the frequency of driving force and the oscillating body is called **driven harmonic oscillation**. The resulting oscillations are called **forced oscillations**. The general equation of a damped oscillator, oscillating under influence of an external harmonic force $F_0 \sin(\omega'' t)$ is written as

$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = F_0 \sin(\omega'' t)$$

The displacement of body from mean position is

$$y = A'' \sin(\omega'' t - \phi)$$

where A'' is amplitude, given by

$$A'' = \frac{F_0/m}{\sqrt{(\omega^2 - \omega''^2)^2 + \dfrac{\omega''^2 b^2}{m^2}}}$$

$$\phi = \tan^{-1}\left[\frac{(\omega'' b/m)}{\omega^2 - \omega''^2}\right]$$

and $\omega = \sqrt{\dfrac{k}{m}}$ is natural frequency of the oscillator.

Problem Solving Technique(s)

(a) The system oscillates with frequency of driver ω'' rather than with its natural frequency ω.

(b) The amplitude of oscillation is small if ω'' is very different from ω. As $\omega'' \to \omega$, the amplitude goes on increasing. When $\omega'' = \omega$, the amplitude is maximum. This situation is called **Resonance**. If in addition, the damping force is absent, then the amplitude tends to become infinite and the system will break down. However, if some damping is present, the amplitude becomes large but remains finite.

Test Your Concepts-V

Based on SHM in Other Physical Systems, Composition of SHM, Damped Oscillations, Forced Oscillations & Resonance

(Solutions on page H.186)

1. A simple pendulum consists of a small sphere of mass m suspended by a thread of length ℓ. The sphere carries a positive charge q. The pendulum is placed in a uniform electric field of strength E directed vertically upwards. With what period will pendulum oscillate if the electrostatic force acting on the sphere is less than the gravitational force?

2. A particle is constrained to move on a smooth circular wire frame of radius r which rotates uniformly about a diameter which is vertical. If in the position of relative rest, the radius drawn to the particle makes an angle α with the vertical, find the period of small oscillations about this position.

3. Calculate the period of small oscillations in a horizontal plane performed by a ball of mass 40 g fixed at the middle of a horizontally stretched string 1 m in length. The tension of the string is assumed to be constant and equal to 10 N.

4. A point mass m is suspended at the end of a massless wire of length l and cross-section A. If Y is the young's modulus of elasticity for the wire, obtain the frequency of oscillation for the simple harmonic motion along the vertical line.

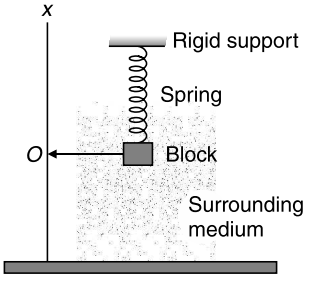
5. Consider the earth as a uniform sphere of mass M and radius R. Imagine a straight smooth tunnel made through the earth which connects any two points on its surface. Show that the motion of a particle of mass m along this tunnel under the action of gravitation would be simple harmonic. Hence, determine the time that a particle would take to go from one end to the other through the tunnel.

6. An ideal gas whose adiabatic exponents is γ, is enclosed in a vertical cylindrical container and supports a freely moving piston of mass M. The piston and the cylinder have equal cross-sectional area A. Atmospheric pressure is P_0 and when the piston is in equilibrium, the volume of the gas is V_0. The piston is now displaced slightly from the equilibrium position. Assuming that the system is completely isolated from its surroundings, show that the piston executes simple harmonic motion and calculate the period of small oscillations.

7. Find the displacement equation of the simple harmonic motion obtained by combining the motions $x_1 = 2\sin\omega t$, $x_2 = 4\sin\left(\omega t + \frac{\pi}{6}\right)$ and $x_3 = 6\sin\left(\omega t + \frac{\pi}{3}\right)$.

8. In damped oscillations, the amplitude of oscillations is reduced to half of its initial value of 5 cm at the end of 25 oscillations. What will be its amplitude when the oscillator completes 50 oscillations?

9. For the damped oscillator shown in figure, $k = 200\ \text{Nm}^{-1}$ and damping constant $b = 40\ \text{gs}^{-1}$.



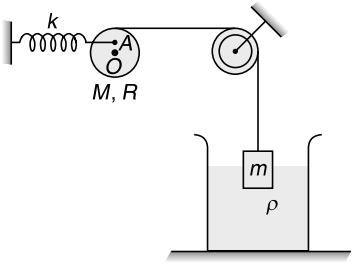
If period of oscillation in the absence of damping is 0.5 s, then calculate mass of block if $b \ll \sqrt{km}$.

10. The amplitude of a damped oscillator becomes half in one minute. The amplitude after 3 minutes will be $1/x$ times of the original. Determine the value of x.

SOLVED PROBLEMS

PROBLEM 1

Figure shows a solid uniform cylinder of radius R and mass M, which is free to rotate about a fixed horizontal axis O and passes through centre of the cylinder. One end of an ideal spring of force constant k is fixed and the other end is hinged to the cylinder at A. Distance OA is equal to $\frac{R}{2}$. An inextensible thread is wrapped round the cylinder and passes over a smooth, small pulley. A block of equal mass M and having cross sectional area A is suspended from free end of the thread. The block is partially immersed in a non-viscous liquid of density ρ.



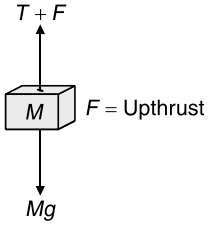
If in equilibrium, spring is horizontal and line OA is vertical, calculate frequency of small oscillations of the system.

SOLUTION

In equilibrium, we have

$$T + F = Mg \qquad \ldots(1)$$

When the block is further depressed by x, weight Mg remains unchanged, upthrust F increases by ρAxg and let ΔT be the increase in tension.



If a is the acceleration of block then,

$$\Delta T + \rho Axg = Ma \qquad \ldots(2)$$

Restoring torque on the cylinder is given by

$$\tau = \left(\frac{kx}{2}\frac{R}{2} - \Delta TR\right) = \left(\frac{kxR}{4} - (Ma - \rho Axg)R\right)$$

$$\Rightarrow \quad \frac{1}{2}MR^2\alpha = \left[\frac{kR^2\theta}{4} - (MR\alpha - \rho AgR\theta)R\right]$$

$$\Rightarrow \quad \frac{3}{2}MR^2\alpha = \left(\frac{kR^2}{4} + \rho AgR^2\right)\theta$$

$$\Rightarrow \quad \alpha = \frac{-\left(\frac{k}{4} + \rho Ag\right)}{\frac{3}{2}M}\theta$$

Here negative sign has been used for restoring nature of torque. So, we get

$$f = \frac{1}{2\pi}\sqrt{\left|\frac{\alpha}{\theta}\right|}$$

$$\Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\frac{k + 4\rho Ag}{6M}}$$

PROBLEM 2

A pendulum clock is mounted in an elevator which starts going up with a constant acceleration $a(< g)$. At a height h the acceleration of the car reverses, its magnitude remaining the same. How soon after the start of the motion will the clock show the right time again?

SOLUTION

Time of ascent $t_1 = \sqrt{\frac{2h}{a}}$ $\qquad \left\{\because\ h = \frac{1}{2}at^2\right\}$

$$\Rightarrow \quad T \propto \frac{1}{\sqrt{g}}$$

$$\Rightarrow \quad \frac{T'}{T} = \sqrt{\frac{g}{g+a}}$$

$$\Rightarrow \quad T' = T\sqrt{\frac{g}{g+a}}$$

$$\Rightarrow \quad \Delta T = (T - T') = T\left(1 - \sqrt{\frac{g}{g+a}}\right)$$

Time gained in time t_1 is

$$\Delta t_1 = \left(\frac{\Delta T}{T'}\right)t_1$$

$$\Rightarrow \quad \Delta t_1 = \sqrt{\frac{2h}{a}}\left(\sqrt{\frac{g+a}{g}} - 1\right) \qquad \ldots(1)$$

If t_2 be the time of descent, then in this case, $\frac{T'}{T} = \sqrt{\frac{g}{g-a}}$

$$\Rightarrow \quad T' = T\sqrt{\frac{g}{g-a}}$$

$$\Rightarrow \quad \Delta T = T' - T = T\left(\sqrt{\frac{g}{g-a}} - 1\right)$$

Time lost in time t_2 is

$$\Delta t_2 = \left(\frac{\Delta T}{T'}\right) t_2$$

$$\Rightarrow \quad \Delta t_2 = t_2\left(1-\sqrt{\frac{g-a}{g}}\right) \qquad \ldots(2)$$

The clock will show the right time again if,

$$\Delta t_1 = \Delta t_2$$

$$\Rightarrow \quad \sqrt{\frac{2h}{a}}\left(\sqrt{\frac{g+a}{g}}-1\right) = t_2\left(1-\sqrt{\frac{g-a}{g}}\right)$$

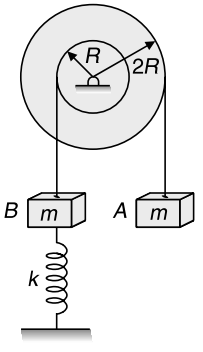
$$\Rightarrow \quad t_2 = \sqrt{\frac{2h}{a}}\left(\frac{\sqrt{g+a}-\sqrt{g}}{\sqrt{g}-\sqrt{g-a}}\right)$$

So, total time $t = t_1 + t_2$

$$\Rightarrow \quad t = \sqrt{\frac{2h}{a}}\left[\frac{\sqrt{g+a}-\sqrt{g-a}}{\sqrt{g}-\sqrt{g-a}}\right]$$

PROBLEM 3

In the figure shown pulley in massless. Initially the blocks are held at a height such that spring is in relaxed position. The block A is released. Find the



(a) amplitude and velocity amplitude of A.

(b) frequency of the oscillation of block B. There is no slipping anywhere.

SOLUTION

(a) (i) Applying Law of Conservation of Mechanical Energy, we get

$$\begin{pmatrix}\text{Decrease in}\\ \text{Gravitational}\\ \text{P.E. of } A\end{pmatrix} = \begin{pmatrix}\text{Increase in}\\ \text{Gravitational}\\ \text{P.E. of } B\end{pmatrix} + \begin{pmatrix}\text{Increase in}\\ \text{Elastic P.E.}\\ \text{of Spring}\end{pmatrix}$$

$$\Rightarrow \quad mg(2x_0) = mgx_0 + \frac{1}{2}kx_0^2 \qquad \ldots(1)$$

where, x_0 =displacement amplitude of A

Solving equation (1), we get

$$x_0 = \frac{2mg}{k}$$

(ii) Again, Applying Law of Conservation of Mechanical Energy at the mean position, we get

$$mgx_0 = mg\frac{x_0}{2} + \frac{1}{2}k\left(\frac{x_0}{2}\right)^2 + \frac{1}{2}mv_m^2 + \frac{1}{2}m\left(\frac{v_m}{2}\right)^2$$

where, v_m =velocity amplitude of A

$$\Rightarrow \quad v_m = 2g\sqrt{\frac{m}{5k}}$$

(b) Maximum velocity of $B = \omega$ (maximum displacement of B)

$$\Rightarrow \quad \frac{v_m}{2} = \omega\left(\frac{x_0}{2}\right)$$

$$\Rightarrow \quad f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{5m}}$$

Conceptual Note(s)

A and B both oscillate simple harmonically with same angular frequency ω but different displacement and velocity amplitudes. Frequency or time period can also be obtained by energy method as under.

$$E = \frac{1}{2}m_Av_A^2 + \frac{1}{2}m_Bv_B^2 - m_Agx_A + m_Bgx_B + \frac{1}{2}k\left(\frac{x_0}{2}+x_B\right)^2$$

$$\Rightarrow \quad E = \frac{1}{2}mv^2 + \frac{1}{2}m\left(\frac{v}{2}\right)^2 - mgx + mg\frac{x}{2} + \frac{1}{2}k\left(\frac{x_0}{2}+\frac{x}{2}\right)^2$$

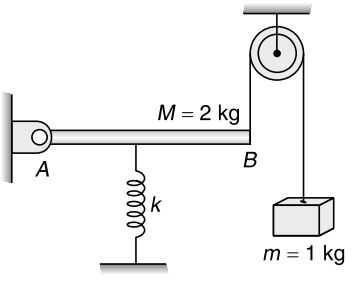
where $x_0 = \dfrac{2mg}{k}$ and x is the displacement (downwards) of A from its mean position x_0. Since E =constant

$$\Rightarrow \quad \frac{dE}{dt} = 0$$

So, we can calculate the desired frequency.

PROBLEM 4

In the arrangement shown in figure, AB is a uniform rod of length $l = 90$ cm and mass $M = 2$ kg. The rod is free to rotate about a horizontal axis passing through end A. A thread passes over a light, smooth and small pulley. One end of the thread is attached with end B of the rod and the other end carries a block of mass $m = 1$ kg. To keep the system in equilibrium one end of an ideal spring of force constant $K = 7500\ \text{Nm}^{-1}$ is attached with mid-point of the rod and the other end is fixed such that in equilibrium, the spring is vertical and the rod is horizontal. If in equilibrium, part of the thread between end B and pulley is vertical, calculate frequency of small oscillations of the system.



Also, calculate the maximum possible angular amplitude of the rod so that block remains oscillating with the rod without blocking of thread. $\left(g = 10\ \text{ms}^{-2}\right)$.

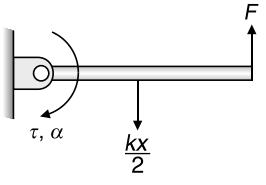
SOLUTION

In equilibrium, the spring is relaxed. When mass m is displaced (downwards) by x from its equilibrium position, then let F be the extra tension in the string. The spring will stretch by $\frac{x}{2}$.

Net restoring torque is given by

$$\tau = \left(\frac{kx}{2}\right)\frac{l}{2} - Fl \qquad \ldots(1)$$

$$\Rightarrow \quad \left(\frac{Ml^2}{3} + ml^2\right)\alpha = -\frac{kl^2\theta}{4} \qquad \{\because\ x = l\theta\}$$



Here negative sign has been introduced due to restoring nature of torque. Since, α is proportional to $-\theta$, motion is simple harmonic in nature. So,

$$f = \frac{1}{2\pi}\sqrt{\left|\frac{\alpha}{\theta}\right|} = \frac{1}{2\pi}\sqrt{\frac{k}{4\left(\frac{M}{3} + m\right)}} = \frac{1}{2\pi}\sqrt{\frac{7500}{4\left(\frac{2}{3} + 1\right)}}$$

$$\Rightarrow \quad f = \frac{15\sqrt{5}}{2\pi}\ \text{Hz}$$

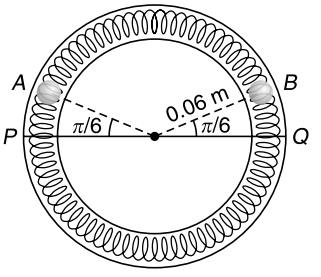
Further, $\omega^2 l\theta_{\max} = g$

$$\Rightarrow \quad \theta_{\max} = \frac{g}{\omega^2 l} = \frac{10}{\left(15\sqrt{5}\right)^2(0.9)}$$

$$\Rightarrow \quad \theta_{\max} = 9.88 \times 10^{-3}\ \text{rad}$$

PROBLEM 5

Two identical balls A and B, each of mass 0.1 kg, are attached to two identical massless springs. The spring-mass system is constrained to move inside a rigid smooth pipe bent in the form of a circle as shown in figure. The pipe is fixed in a horizontal plane. The centres of the ball can move in a circle of radius 0.06 metre. Each spring has a natural length of 0.06π metre and spring constant $0.1\ \text{Nm}^{-1}$. Initially, both the balls are displaced by an angle $\theta = \frac{\pi}{6}$ radian with respect to the diameter PQ of the circle (as shown in figure) and released from rest.



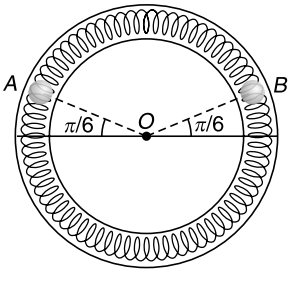
(a) Calculate the frequency of oscillation of ball A.
(b) Find the speed of ball A when A and B are at the two ends of the diameter PQ.
(c) What is the total energy of the system?

SOLUTION

(a) Given: Mass of each ball A and B, $m = 0.1$ kg
Radius of circle, $R = 0.06$ m
Natural length of spring,

$$l_0 = 0.06\pi = \pi R \qquad \{\text{Half circle}\}$$

and spring constant, $k = 0.1\ \text{Nm}^{-1}$
In the stretched position elongation in each spring $(x = R\theta)$
Spring in lower side is stretched by $2x$ and on upper side compressed by $2x$.



Therefore, each spring will exert a force $2kx$ on each block.
Hence, a restoring force, $F = 4kx$ will act on A in the direction shown in figure.
Restoring torque of this force about origin,

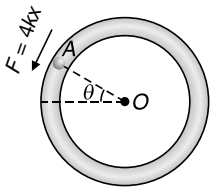
$$\tau = -F \cdot R = -(4kx)R = -(4kR\theta)R$$

$$\Rightarrow \quad \tau = -4kR^2 \cdot \theta \qquad \ldots(1)$$

Since, $\tau \propto -\theta$, each ball executes angular SHM about origin O.
Equation (1) can be rewritten as

$$I\alpha = -4kR^2\theta \text{ or } \left(mR^2\right)\alpha = -4kR^2\theta$$

$$\Rightarrow \quad \alpha = -\left(\frac{4k}{m}\right)\theta$$



So, frequency of oscillation is given by

$$f = \frac{1}{2\pi}\sqrt{\left|\frac{\text{acceleration}}{\text{displacement}}\right|}$$

$$\Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\left|\frac{\alpha}{\theta}\right|} = \frac{1}{2\alpha}\sqrt{\left|\frac{4k}{m}\right|}$$

Substituting the values, we get

$$f = \frac{1}{2\pi}\sqrt{\frac{4\times 0.1}{0.1}} = \frac{1}{\pi}\ \text{Hz}$$

(b) In stretched position, potential energy of the system is

$$\text{P.E.} = 2\left(\frac{1}{2}k\right)(2x)^2 = 4kx^2$$

and in mean position, both the balls have only kinetic energy. Hence

$$\text{K.E.} = 2\left(\frac{1}{2}mv^2\right) = mv^2$$

By Law of Conservation of Energy, we get

$$(\text{Loss in KE}) = (\text{Gain in PE})$$

$$\Rightarrow \quad 4kx^2 = mv^2$$

$$\Rightarrow \quad v = 2x\sqrt{\frac{k}{m}} = 2R\theta\sqrt{\frac{k}{m}}$$

Substituting the values

$$v = 2(0.06)\left(\frac{\pi}{6}\right)\sqrt{\frac{0.1}{0.1}}$$

$$\Rightarrow \quad v = 0.0628\ \text{ms}^{-1}$$

(c) Total energy of the system,

E =P.E. in stretched position

E =K.E. in mean position

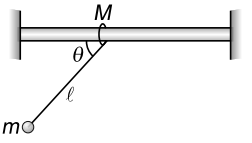
$$\Rightarrow \quad E = mv^2 = (0.1)(0.0628)^2\ \text{J}$$

$$\Rightarrow \quad E = 3.9\times 10^{-4}\ \text{J}$$

PROBLEM 6

A bead of mass M can slide on a smooth straight horizontal wire and a particle of mass m is attached to the body by a light string of length l. The particle is held in contact with the wire with the string taut and is then left fall. When the string is inclined to the wire at an angle θ, find the

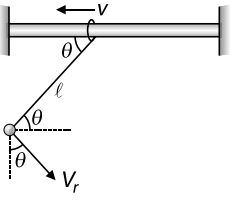
(a) distance x slipped along the wire by the bead and
(b) angular velocity ω of the string.



SOLUTION

(a) Displacement of m relative to M is

$$x_r = l(1-\cos\theta)$$



Let x be the displacement of M (towards left). Absolute displacement of m will be $x_r - x$ (towards right). For the centre of mass to remain stationary, we have

$$m(x_r - x) = Mx$$

$$\Rightarrow \quad x = \left(\frac{m}{M+m}\right)l(1-\cos\theta)$$

(b) By Law of Conservation of Mechanical Energy, we get

$$mgl\sin\theta = \frac{1}{2}Mv^2 + \frac{1}{2}m\left[(v_r\sin\theta - v)^2 + v_r^2\cos^2\theta\right] \ldots(1)$$

Applying Law of Conservation of Linear Momentum, we get

$$m(v_r\sin\theta - v) = Mv$$

$$\Rightarrow \quad v = \frac{mv_r\sin\theta}{M+m} \qquad \ldots(2)$$

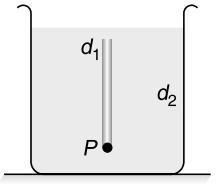
From equations (1) and (2), we get

$$v_r = \sqrt{\frac{2gl\sin\theta(M+m)}{M+m\cos^2\theta}}$$

$$\Rightarrow \quad \omega = \frac{v_r}{l} = \sqrt{\frac{2g(M+m)\sin\theta}{(M+m\cos^2\theta)l}}$$

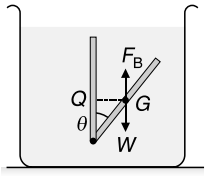
PROBLEM 7

A thin rod of length L and uniform cross-section is pivoted at its lowest point P inside a stationary homogeneous and non-viscous liquid (as shown in figure). The rod is free to rotate in a vertical plane about a horizontal axis passing through P. The density d_1 of the material of the rod is smaller than the density d_2 of the liquid. The rod is displaced by small angle θ from its equilibrium position and then released. Show that the motion of the rod is simple harmonic and determine its angular frequency in terms of the given parameters.



SOLUTION

Let S be the area of cross-section of the rod. In the displaced position, as shown in figure,



The weight (W) and upthrust (F_B) both pass through its centre of gravity G.

Here, $W = (\text{Volume}) \times (\text{Density of rod}) \times g$

$\Rightarrow \quad W = (SL)(d_1)g$

The buoyant force F_B is given by

$$F_B = (\text{Volume}) \times (\text{Density of liquid}) \times g$$

$\Rightarrow \quad F_B = (SL)(d_2)g$

Given that $d_1 < d_2$. Therefore, $W < F_B$

Therefore, net force acting at G is given by

$$F = F_B - W = (SLg)(d_2 - d_1)\text{upwards}$$

Restoring torque of this force about point P is

$$\tau = F \times \tau_\perp = (SLg)(d_2 - d_1)(QG)$$

$$\Rightarrow \quad \tau = -(SLg)(d_2 - d_1)\left(\frac{L}{2}\sin\theta\right)$$

Here, negative sign shows the restoring nature of torque. Since θ is small, so $\sin\theta \approx \theta$

$$\Rightarrow \quad \tau = -\left(\frac{SL^2 g(d_2 - d_1)}{2}\right)\theta \qquad \ldots(1)$$

From equation (1), we get

$$\tau \propto -\theta$$

Hence, motion of the rod is simple harmonic.

Further, we know that $\tau = I\dfrac{d^2\theta}{dt^2} = I\alpha$

Rewriting equation (1) as

$$\Rightarrow \quad I\frac{d^2\theta}{dt^2} = -\left(\frac{SL^2 g(d_2 - d_1)}{2}\right)\theta \qquad \ldots(2)$$

where, I =moment of inertia of rod about an axis passing through P given by

$$I = \frac{ML^2}{3} = \frac{(SLd_1)L^2}{3}$$

Substituting this value of I in equation (2), we get

$$\frac{d^2\theta}{dt^2} = -\left(\frac{3}{2}\frac{g(d_2 - d_1)}{d_1 L}\right)\theta$$

Comparing this equation with standard differential equation of SHM, i.e.,

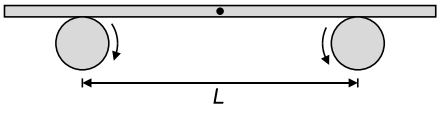
$$\frac{d^2\theta}{dt^2} = -\omega^2\theta$$

The angular frequency of oscillation is given by

$$\omega = \sqrt{\frac{3g(d_2 - d_1)}{2d_1 L}}$$

PROBLEM 8

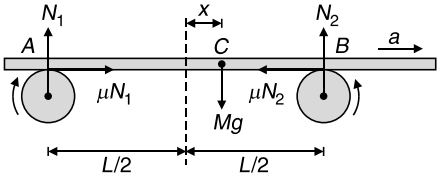
A uniform plate of mass M stays horizontally and symmetrically on two wheels rotating in opposite directions as shown in Figure.



The separation between the wheels is L. The friction coefficient between each wheel and the plate is μ. Find the time period of oscillation of the plate if it is slightly displaced along its length and released.

SOLUTION

In equilibrium the normal reactions due to two wheels are equal. Hence, the frictional forces are also equal and balance each other. When the plate is displaced by x towards right, the normal N_2 due to right wheel increases and N_1 due to left wheel decreases. The resultant friction $\mu(N_2 - N_1)$ is towards left resulting in oscillatory motion. In displaced position, $N_1 + N_2 = Mg$



Balancing moments about A, we get

$$mg\left(\frac{L}{2} + x\right) = N_2 L$$

$$\Rightarrow \quad N_2 = Mg\left(\frac{1}{2} + \frac{x}{L}\right)$$

Balancing moments about B, we get

$$N_1 = Mg\left(\frac{1}{2} - \frac{x}{L}\right)$$

So, restoring force F is given by

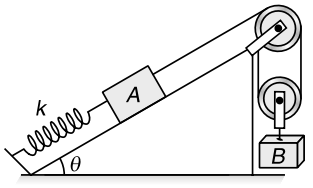
$$F = M\ddot{x} = -\mu(N_1 - N_2) = -\frac{2\mu Mg}{L}x$$

$$\Rightarrow \quad \omega^2 = \frac{2\mu g}{L}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{L}{2\mu g}}$$

PROBLEM 9

Calculate the angular frequency of the system shown in figure. Friction is absent everywhere and the threads, spring and pulleys are massless. Given that $m_A = m_B = m$.



SOLUTION

Let x_0 be the extension in the spring in equilibrium. Then for equilibrium of block A and B, we have

$$T = kx_0 + mg\sin\theta \qquad \ldots(1)$$

and $\quad 2T = mg \qquad \ldots(2)$

where, T is the tension in the string.

Now, suppose A is further displaced by a distance x from its mean position and v be its speed at this moment. Then B lowers by $\frac{x}{2}$ and speed of B at this instant will be $\frac{v}{2}$. Total energy of the system in this position is given by

$$E = \frac{1}{2}k(x + x_0)^2 + \frac{1}{2}m_A v^2 + \frac{1}{2}m_B\left(\frac{v}{2}\right)^2 + m_A g h_A - m_B g h_B$$

$$\Rightarrow \quad E = \frac{1}{2}k(x + x_0)^2 + \frac{1}{2}mv^2 + \frac{1}{8}mv^2 + mgx\sin\theta - mg\frac{x}{2}$$

$$\Rightarrow \quad E = \frac{1}{2}k(x + x_0)^2 + \frac{5}{8}mv^2 + mgx\sin\theta - mg\frac{x}{2}$$

Since, E is constant, so

$$\frac{dE}{dt} = 0$$

$$\Rightarrow \quad 0 = k(x + x_0)\frac{dx}{dt} + \frac{5}{4}mv\left(\frac{dv}{dt}\right) + mg(\sin\theta)\left(\frac{dx}{dt}\right) - \frac{mg}{2}\left(\frac{dx}{dt}\right)$$

Substituting, $\frac{dx}{dt} = v = \dot{x}$, $\frac{dv}{dt} = a$ and from equations (1) and (2), using $kx_0 + mg\sin\theta = \frac{mg}{2}$, we get

$$\frac{5}{4}ma = -kx$$

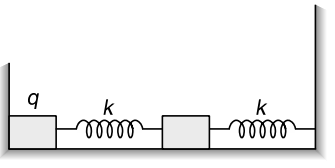
Since, $a \propto -x$, so, motion is simple harmonic, time period of which is

$$T = 2\pi\sqrt{\left|\frac{x}{a}\right|} = 2\pi\sqrt{\frac{5m}{4k}}$$

$$\Rightarrow \quad \omega = \frac{2\pi}{T} = \sqrt{\frac{4k}{5m}}$$

PROBLEM 10

One end of each of two identical springs, natural length 9 cm and force constant $k = 45\ \text{Nm}^{-1}$ is attached with a small particle of mass $m = 30$ g. Other end of right spring is fixed with a wall and other end of left spring is attached with a fixed block having a positive charge $q = 1\ \mu\text{C}$ as shown in figure. The particle rests over a smooth horizontal plane and springs are non-deformed.



Calculate deformation of springs when positive charge $q = 1\ \mu\text{C}$ is given to the particle and equilibrium is attained. Also calculate the frequency of small oscillations of the particle.

SOLUTION

In equilibrium

$$(\text{Net Repulsive Force}) = (\text{Total Spring Force})$$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(0.09 + x)^2} = 2kx$$

$$\Rightarrow \quad x = 0.01\ \text{m} = 1\ \text{cm}$$

At a distance r, the electrostatic repulsion between the particles is

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}$$

$$\Rightarrow \quad dF_e = -\frac{2q^2}{4\pi\varepsilon_0 r^3}dr$$

In equilibrium $r = 9 + 1 = 10\ \text{cm} = 0.1\ \text{m}$

When the particle is displaced by a distance y from equilibrium position, we have

$$\text{Net restoring force} = -\left[2ky + \frac{2q^2}{(4\pi\varepsilon_0)r^3}y\right]$$

$$\Rightarrow \quad ma = -\left[2k + \frac{2q^2}{(4\pi\varepsilon_0)r^3}\right]y$$

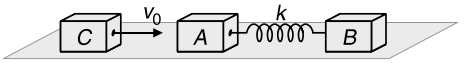
$$\Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\left|\frac{a}{y}\right|}$$

Substituting the values, we get

$$f = \frac{30}{\pi}\ \text{Hz}$$

PROBLEM 11

Two blocks A and B of masses $m_1 = 3$ kg and $m_2 = 6$ kg respectively are connected with each other by a spring of force constant $k = 200\ \text{Nm}^{-1}$ as shown in figure. Blocks are pulled away from each other by $x_0 = 3$ cm and then released. When spring is in its natural length and blocks are moving towards each other, another block of mass $m = 3$ kg moving with velocity $v_0 = 0.4\ \text{ms}^{-1}$ (towards right) collides with A and gets stuck to it. Neglecting friction, calculate



(a) velocities v_1 and v_2 of the blocks A and B respectively just before collision and their angular frequency.
(b) velocity of centre of mass of the system, after collision,
(c) amplitude of oscillations of combined body,
(d) loss of energy during collision.

SOLUTION

(a) $\frac{1}{2}kx_0^2 = \frac{1}{2}\mu v_r^2$

where, μ = reduced mass $= \frac{6\times 3}{6+3} = 2$ kg

$$\Rightarrow \quad v_{\text{relative}} = \sqrt{\frac{k}{\mu}}x_0 = \left(\sqrt{\frac{200}{2}}\right)(3\times 10^{-2})$$

$$\Rightarrow \quad v_{\text{relative}} = 0.3\ \text{ms}^{-1} = 2v + v$$

$$\Rightarrow \quad v = 0.1\ \text{ms}^{-1}$$

$$\Rightarrow \quad v_1 = 0.2\ \text{ms}^{-1} \text{ and } v_2 = 0.1\ \text{ms}^{-1}$$

Angular frequency $\omega = \sqrt{\frac{k}{\mu}} = \sqrt{\frac{200}{2}} = 10\ \text{rads}^{-1}$

(b) $v_{\text{cm}} = \frac{m_1v_1 + m_2v_2 + m_3v_3}{m_1 + m_2 + m_3}$

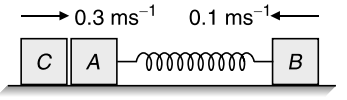
$$\Rightarrow \quad v_{\text{cm}} = \frac{(3)(0.2) - (6)(0.1) + 3(0.4)}{3+6+3}$$

$$\Rightarrow \quad v_{\text{cm}} = 0.1\ \text{ms}^{-1} \text{ (towards right)}$$

(c) After collision velocity of block A and C is

$$v_0 = \frac{(3\times 0.2) + (3)(0.4)}{3+3} = 0.3\ \text{ms}^{-1}$$

and velocity of block B is $v_2 = 0.1\ \text{ms}^{-1}$



The spring will compress till velocity of all the blocks become equal to the centre of mass. Applying Law of Conservation of Mechanical Energy, we get

$$\frac{1}{2}(3+3)(0.3)^2 + \frac{1}{2}(6)(0.1)^2 = \frac{1}{2}(3+3+6)(0.1)^2 + \frac{1}{2}kA^2$$

$$\Rightarrow \quad A = 0.048\ \text{m}$$

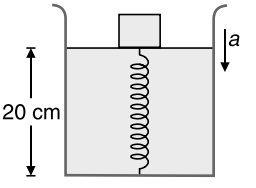
$$\Rightarrow \quad A = 4.8\ \text{cm}$$

(d) $\Delta E = \frac{1}{2}(3)(0.4)^2 + \frac{1}{2}(3)(0.2)^2 - \frac{1}{2}(3+3)(0.3)^2$

$$\Rightarrow \quad \Delta E = 0.24 + 0.06 - 0.27 = 0.03\ \text{J}$$

PROBLEM 12

A rectangular tank having base $15\ \text{cm} \times 20\ \text{cm}$ is filled with water (density $\rho = 1000\ \text{kgm}^{-3}$) upto 20 cm height. One end of an ideal spring of natural length 20 cm and force constant $k = 280\ \text{Nm}^{-1}$ is fixed to the bottom of a tank so that spring, remains vertical. This system is in an elevator moving downwards with acceleration $a = 2\ \text{ms}^{-2}$. Cubical block of side $l = 10$ cm and mass $m = 2$ kg is gently placed over in figure.



(a) Calculate compression of the spring in equilibrium position.
(b) If block is attached to spring and slightly pushed down from equilibrium position and released, calculate frequency of its vertical oscillations. $\left(g = 10\ \text{ms}^{-2}\right)$.

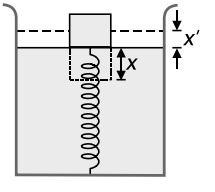
SOLUTION

(a) Let in equilibrium compression of spring is x. Water of volume l^2x is displaced from its original position and level of liquid in the tank rises by x', then,

$$l^2x = \left(A - l^2\right)x'$$

Substituting the values, we get

$$x' = 0.5x$$



Equation of motion for the block is,

$$mg - \text{upthrust} - \text{spring force} = ma$$

$$\Rightarrow \quad mg - (x + x')l^2\rho g - kx = ma$$

Substituting the values, we get

$$x = 0.04 \text{ m}$$

$$\Rightarrow \quad x = 4 \text{ cm}$$

(b) If the block is slightly pushed downwards by the amount y, both upthrust and spring force increase.

$$\begin{pmatrix}\text{Net}\\ \text{Restoring}\\ \text{Force}\end{pmatrix} = \begin{pmatrix}\text{Increase}\\ \text{in}\\ \text{Upthrust}\end{pmatrix} + \begin{pmatrix}\text{Increase}\\ \text{in Spring}\\ \text{Force}\end{pmatrix}$$

$$\Rightarrow \quad F = -\left[(y + 0.5y)l^2\rho g + ky\right]$$

Substituting the values, we get

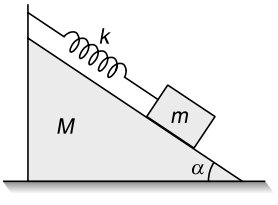
$$F = -400y$$

$$\Rightarrow \quad a = -\frac{400}{2}y = -200y$$

$$\Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\left|\frac{a}{y}\right|} = \frac{5\sqrt{2}}{\pi} \text{ Hz}$$

PROBLEM 13

Consider a block of mass m to be placed on a wedge of mass M, having wedge angle α. Assuming all the contacts to the frictionless, find the angular frequency of wedge and block system.



SOLUTION

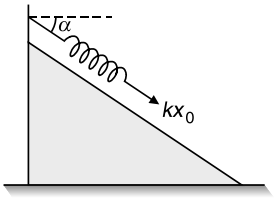
In equilibrium, we have

$$kx_0 \cos\alpha = N \sin\alpha \quad \ldots(1)$$

$$N = mg\cos\alpha \quad \ldots(2)$$

From these two equations, we get

$$mg\sin\alpha = kx_0 \quad \ldots(3)$$



When further displaced by x (downwards). Let v_r be the velocity of m w.r.t. M and v be the velocity of M at this instant. Applying Law of Conservation of Linear Momentum in horizontal direction, we get

$$Mv = m(v_r\cos\alpha - v)$$

$$\Rightarrow \quad v = \frac{mv_r\cos\alpha}{M+m}$$

Further total energy E of system is given by

$$E = \frac{1}{2}k(x + x_0)^2 - mgx\sin\alpha + \frac{1}{2}Mv^2 + \frac{1}{2}m\left[(v_r\cos\alpha - v)^2 + (v_r\sin\alpha)^2\right]$$

$$\Rightarrow \quad E = \frac{1}{2}k(x + x_0)^2 - mgx\sin\alpha + \frac{1}{2}\frac{Mm^2}{(M+m)^2}v_r^2\cos^2\alpha + \frac{1}{2}m\left[v_r^2 + v^2 - 2vv_r\cos\alpha\right]$$

$$\Rightarrow \quad E = \frac{1}{2}k(x + x_0)^2 - mgx\sin\alpha + \frac{1}{2}\frac{m^2}{(M+m)}v_r^2\cos^2\alpha + \frac{1}{2}mv_r^2 - \frac{m^2v_r^2\cos^2\alpha}{(M+m)}$$

$$\Rightarrow \quad E = \frac{1}{2}k(x + x_0)^2 - mgx\sin\alpha + \frac{1}{2}m\left[1 - \frac{m\cos^2\alpha}{M+m}\right]v_r^2$$

$$\Rightarrow \quad E = \frac{1}{2}k(x + x_0)^2 - mgx\sin\alpha + \frac{1}{2}m\left[\frac{M + m\sin^2\alpha}{M+m}\right]v_r^2$$

Since, E =constant

$$\Rightarrow \quad \frac{dE}{dt} = 0$$

$$\Rightarrow \quad 0 = k(x + x_0)\frac{dx}{dt} + mv_r\left(\frac{dv_r}{dt}\right)\left(\frac{M + m\sin^2\alpha}{M+m}\right) - mg\sin\alpha\left(\frac{dx}{dt}\right)$$

Substituting, $mg\sin\alpha = kx_0$, $\frac{dx}{dt} = v_r$ and $\frac{dv_r}{dt} = a_r$, we get

$$m\left(\frac{M + m\sin^2\alpha}{M+m}\right)a_r = -kx$$

$$\Rightarrow \quad a_r = \frac{-k(M+m)}{m(M + m\sin^2\alpha)}x$$

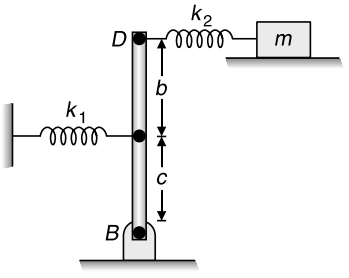
Comparing with $a_r = -\omega^2 x$, we get

$$\omega = \sqrt{\frac{k(M+m)}{m(M + m\sin^2\alpha)}}$$

Please note that, here x is the displacement of block relative to the wedge.

PROBLEM 14

Find the angular frequency of motion of block m for small motion of rod BD when we neglect the inertial effects of the rod BD. Spring constants are k_1 and k_2. Neglect friction forces.



SOLUTION

Let extension in springs be x_1 and x_2. Then $x_1 = c\theta$ but $x_2 \neq (b+c)\theta$, because the block will also move towards the right.

Suppose the block moves towards right by x, then

$$x = x_2 + (b+c)\theta \qquad \ldots(1)$$

Since, $\Sigma\tau_B = 0$

$$\Rightarrow \quad k_1 x_1 c = k_2 x_2 (b+c)$$

$$\Rightarrow \quad k_1 (c\theta) c = k_2 [x - (b+c)\theta](b+c)$$

$$\Rightarrow \quad \left[k_1 c^2 + k_2 (b+c)^2\right]\theta = k_2 x (b+c)$$

$$\Rightarrow \quad \theta = \frac{k_2 (b+c) x}{k_1 c^2 + k_2 (b+c)^2} \qquad \ldots(2)$$

If F is the restoring force on the block, then

$$F = k_2 x_2$$

$$\Rightarrow \quad x_2 = \frac{F}{k_2} \qquad \ldots(3)$$

From equation (1), (2) and (3), we get

$$x = \frac{F}{k_2} + (b+c)\theta$$

$$\Rightarrow \quad x = \frac{F}{k_2} + \frac{(b+c) k_2 x_2 (b+c)}{k_1 c^2}$$

$$\Rightarrow \quad x = \frac{F}{k_2} = \frac{(b+c) F (b+c)}{k_1 c^2}$$

$$F = \frac{k_1 k_2 c^2}{k_1 c^2 + k_2 (b+c)^2} x$$

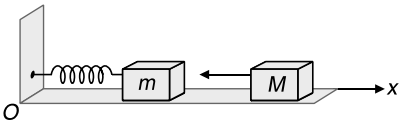
So, the effective spring constant is given by

$$k_{\text{eff}} = \frac{k_1 k_2 c^2}{k_1 c^2 + k_2 (b+c)^2}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{k_{\text{eff}}}{m}} = \sqrt{\frac{k_1 k_2 c^2}{m\left[k_1 c^2 + k_2 (b+c)^2\right]}}$$

PROBLEM 15

One end of an ideal spring is fixed to a wall at origin O and axis of spring is parallel to x-axis. A block of mass $m = 1$ kg is attached to free end of the spring and it is performing S.H.M. Equation of position of the block in coordinate system shown in figure is $x = 10 + 3\sin(10 \cdot t)$. Here t is in second and x in cm. Another block of mass $M = 3$ kg, moving towards the origin with velocity 30 cmsec^{-1} collides with the block performing S.H.M. at $t = 0$ and gets stuck to it. Calculate the



(a) new amplitude of oscillations,
(b) new equation for position of the combined body,
(c) loss of energy during collision. Neglect friction.

SOLUTION

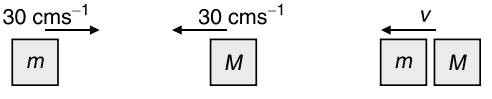
(a) Since, $\omega^2 = \dfrac{k}{m}$ $\qquad \left\{\because\ \omega = \sqrt{\dfrac{k}{m}}\right\}$

$$\Rightarrow \quad k = m\omega^2 = (1)(10)^2 = 100 \text{ Nm}^{-1}$$

At $t = 0$, block of mass m is at mean position ($x = 10$ cm) and moving towards positive x-direction with velocity $A\omega$ or 30 cms^{-1}

By Law of Conservation of Linear Momentum, we get

$$(M+m)v = M(30) - m(30)$$



Substituting the values, we have

$$v = 15 \text{ cms}^{-1}$$

$$\Rightarrow \quad v = 0.15 \text{ ms}^{-1}$$

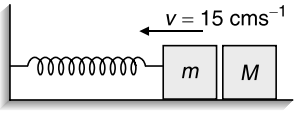
By Law of Conservation of Mechanical Energy, we get

$$\frac{1}{2}(M+m)v^2 = \frac{1}{2}kA^2$$

$$\Rightarrow \quad A = \left(\sqrt{\frac{M+m}{k}}\right)v = \left(\frac{4}{100}\right)^{\frac{1}{2}}(0.15)$$

$$\Rightarrow \quad A = 0.03 \text{ m}$$

$$\Rightarrow \quad A = 3 \text{ cm}$$



(b) $\omega' = \sqrt{\dfrac{k}{M+m}} = \sqrt{\dfrac{100}{4}} = 5 \text{ radsec}^{-1}$

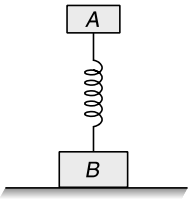
$$\Rightarrow \quad x' = 10 - 3\sin 5t$$

(c) $\Delta E = \frac{1}{2}(1)(0.3)^2 + \frac{1}{2}(3)(0.3)^2 - \frac{1}{2}(4)(0.15)^2$

$\Rightarrow \quad \Delta E = 0.135 \text{ J}$

PROBLEM 16

A body A of mass $m_1 = 1.00$ kg and a body B of mass $m_2 = 4.10$ kg are interconnected by a spring as shown in figure. The body A performs free vertical harmonic oscillations with amplitude $a = 1.6$ cm and frequency $\omega = 25$ rads^{-1}. Neglecting the mass of the spring, find the maximum and minimum values of force that this system exerts on the bearing surface.



SOLUTION

Maximum Force

$$k = \omega^2 m_1 = (25)^2 (1) = 625 \text{ Nm}^{-1}$$

Acceleration of centre of mass in extreme position,

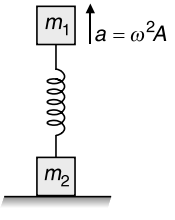
$$a_{CM} = \frac{m_1 a}{m_1 + m_2}(\text{upwards}) = \frac{m_1 \omega^2 A}{m_1 + m_2}$$

Now, $N_{\max} - (m_1 + m_2)g = \frac{m_1 \omega^2 A}{(m_1 + m_2)} \times (m_1 + m_2)$

$\Rightarrow \quad N_{\max} = (m_1 + m_2)g + m_1 \omega^2 A$

$\Rightarrow \quad N_{\max} = (1 + 4.10)9.8 + (1)(25)^2 (1.6 \times 10^{-2})$

$\Rightarrow \quad N_{\max} = 59.98$ newton



Minimum Force

$$a_{\text{cm}} = \frac{m_1 a}{m_1 + m_2} \qquad \{\text{downwards}\}$$

Since, $(m_1 + m_2)g - N_{\min} = (m_1 + m_2)a_{\text{cm}}$

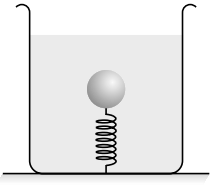
$\Rightarrow \quad N_{\min} = (m_1 + m_2)g - m_1 \omega^2 A$

$\Rightarrow \quad N_{\min} = (1 + 4.10)9.8 - (1)(25)^2 (1.6 \times 10^{-2})$

$\Rightarrow \quad N_{\min} = 39.98$ newton

PROBLEM 17

As a submerged body moves through a fluid, the particles of the fluid, flow around the body and thus acquire kinetic energy. In the case of a sphere moving in an ideal fluid, the total kinetic energy acquired by the fluid is $\frac{1}{4}\rho V v^2$ where ρ is the mass density of fluid, V the volume of sphere and v is the velocity of the sphere. Consider a 0.5 kg hollow spherical shell of radius 8 cm which is held submerged in a tank of water by a spring of force constant 500 Nm^{-1}.

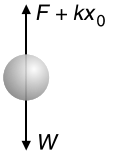


(a) Neglecting fluid friction, determine the period of vibration of the shell when it is displaced vertically and then released.

(b) Solve part (a) assuming that the tank is accelerated upward at the constant rate of 8 ms^{-2}. Density of water is 10^3 kgm^{-3}.

SOLUTION

(a) Let F be the upthrust and W the weight of the sphere. In equilibrium let x_0 be the compression of the spring, then



$$F = kx_0 = W$$

$\Rightarrow \quad kx_0 = W - F \qquad \ldots(1)$

If the sphere is further compressed by x, then total energy of the system is given by

$$E = -(W - F)x + \frac{1}{2}k(x + x_0)^2 + \frac{1}{2}mv^2 + \frac{1}{4}\rho V v^2$$

Since, friction is absent, total energy remains constant, hence

$$\frac{dE}{dt} = 0$$

$$\Rightarrow \quad 0 = -(W - F)\cdot\frac{dx}{dt} + k(x + x_0)\frac{dx}{dt} + mv\left(\frac{dv}{dt}\right) + \frac{1}{2}\rho V v\left(\frac{dv}{dt}\right) \qquad \ldots(2)$$

From equations (1) and (2), with substitutions $\frac{dx}{dt} = v$ and $\frac{dv}{dt} = \ddot{x}$, we get

$$\ddot{x}+\frac{k}{\left(m+\frac{\rho V}{2}\right)}x=0$$

$\Rightarrow \ddot{x}\propto -x$

Oscillations are simple harmonic, time period of which is given by

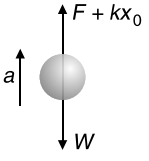
$$T=2\pi\sqrt{\left|\frac{x}{\ddot{x}}\right|}=2\pi\sqrt{\frac{m+\frac{1}{2}\rho V}{k}}$$

$$\Rightarrow T=2\pi\sqrt{\frac{0.5+\frac{1}{2}\times 10^3\times\frac{4}{3}\times\pi\times(0.08)^3}{500}}$$

$\Rightarrow T=0.352$ s

(b) When it is accelerated upwards with an acceleration a, then

$$F'=\frac{F(g+a)}{g}$$



Now, $F'+kx_0-W=\left(\frac{W}{g}\right)a$

$$\Rightarrow kx_0=\frac{W}{g}\cdot a+W-F\left(1+\frac{a}{g}\right)$$

$$\Rightarrow kx_0=(W-F)+\frac{a}{g}(W-F)$$

$$\Rightarrow kx_0=(W-F)\left(1+\frac{a}{g}\right) \quad \ldots(3)$$

When displaced downwards, total energy is given by

$$E=-(W-F)\frac{(g+a)}{g}x+\frac{1}{2}k(x+x_0)^2+\frac{1}{2}mv^2+\frac{1}{4}\rho Vv^2$$

Substituting $\frac{dE}{dt}=0$

$$\Rightarrow 0=-(W-F)\left(1+\frac{a}{g}\right)\frac{dx}{dt}+k(x+x_0)\frac{dx}{dt}+mv\left(\frac{dv}{dt}\right)+\frac{1}{2}\rho vV\left(\frac{dv}{dt}\right) \quad \ldots(4)$$

From equations (4) and (3) we get the same result as was obtained in part (a), i.e.,

$T=0.352$ s

PROBLEM 18

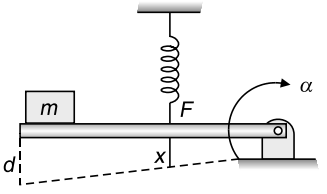
A rod AB of mass M is attached as shown in figure to a spring of constant k. A small block of mass m is placed on the rod at its free end A. In equilibrium rod is horizontal, i.e., spring is stretched.

(a) If end A is moved down through a small distance d and released, determine the period of vibration.

(b) Determine the largest allowable value of d if the block m is to remain at all times in contact with the rod.

SOLUTION

(a) Since, $\frac{d}{l}=\frac{x}{b}$



$$\Rightarrow x=\left(\frac{b}{l}\right)d$$

Unbalanced force (extra force) is given by

$$F=kx=k\left(\frac{b}{l}\right)d$$

Restoring torque, $\tau=F\cdot b=\left(\frac{kb^2}{l}\right)d$

$$\Rightarrow \alpha\left(\frac{Ml^2}{3}+ml^2\right)=-(kb^2)\theta \text{ as } d=l\theta$$

$$\Rightarrow T=2\pi\sqrt{\left|\frac{\theta}{\alpha}\right|}=2\pi\sqrt{\frac{\frac{Ml^2}{3}+ml^2}{kb^2}}$$

(b) Maximum acceleration of block $\le g$

$$a_{\max}=\omega^2 d_{\max}=\frac{kb^2}{\frac{Ml^2}{3}+ml^2}d_{\max}$$

For m to remain in contact with the rod at all the times, we have

$$a_{\max}\le g$$

$$\Rightarrow \frac{kb^2 d_{\max}}{\left(\frac{Ml^2}{3}+ml^2\right)}\le g$$

$$\Rightarrow d_{\max}\le\frac{g\left(\frac{Ml^2}{3}+ml^2\right)}{kb^2}$$

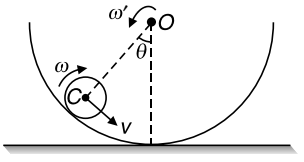
PROBLEM 19

Consider a solid homogeneous cylinder of radius r rolling inside a fixed hollow cylinder of radius R. Find the frequency of small oscillations of the inner cylinder about the stable equilibrium position.

SOLUTION

For pure rolling to take place, we have

$$v = \omega r \qquad \ldots(1)$$



Let ω' be the angular velocity of C w.r.t. O, then

$$\omega' = \frac{v}{R-r} = \omega\left(\frac{r}{R-r}\right)$$

$$\Rightarrow \quad \frac{d\omega'}{dt} = \left(\frac{r}{R-r}\right)\frac{d\omega}{dt}$$

$$\Rightarrow \quad \alpha' = \left(\frac{r}{R-r}\right)\alpha \qquad \ldots(2)$$

Since, $\alpha = \frac{a}{r}$ $\qquad \ldots(3)$

and $a = \dfrac{g\sin\theta}{1+\dfrac{I}{mr^2}}$ were $I = \frac{1}{2}mr^2$ $\qquad \ldots(4)$

So, from equations (2), (3) and (4), we get

$$\alpha' = \frac{2g\sin\theta}{3(R-r)}$$

For small oscillations $\sin\theta \approx \theta$ and being restoring in nature, so

$$\alpha' = -\frac{2g\theta}{3(R-r)}$$

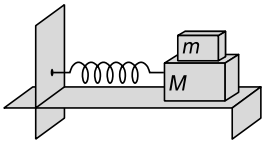
$$\Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\left|\frac{\alpha'}{\theta}\right|} = \frac{1}{2\pi}\sqrt{\frac{2g}{3(R-r)}}$$

PRACTICE EXERCISES

SINGLE CORRECT CHOICE TYPE QUESTIONS

This section contains Single Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. A mass M is attached to a horizontal spring of force constant K fixed on one side to a rigid support. The mass oscillates on a frictionless surface with time period T and amplitude A. When the mass M is in equilibrium position, another mass m is gently placed on it. Then the new time period and amplitude of oscillation will be



(A) $2\pi\sqrt{\frac{M+m}{K}}, \sqrt{\frac{M}{M-m}}A$ (B) $2\pi\sqrt{\frac{M+m}{K}}, \sqrt{\frac{M}{M+m}}A$

(C) $2\pi\sqrt{\frac{M-m}{K}}, \sqrt{\frac{M-m}{K}}A$ (D) $2\pi\sqrt{\frac{M}{K}}, \frac{M}{M+m}A$

2. Two blocks with masses $m_1 = 1$ kg and $m_2 = 2$ kg are connected by a spring of spring constant 24 Nm^{-1} and placed on a frictionless horizontal surface. The block m_1 is imparted an initial velocity $v_0 = 12$ cms^{-1} to the right. The amplitude of oscillation is

(A) 1 cm (B) 2 cm

(C) 3 cm (D) 4 cm

3. The function $x = x_0 \sin^2(\omega t)$ represents

(A) an SHM of amplitude x_0 with a period $\frac{2\pi}{\omega}$

(B) an SHM of amplitude $\frac{x_0}{2}$ with a period $\frac{\pi}{\omega}$

(C) a periodic motion with a period $\frac{2\pi}{\omega}$ but not SHM

(D) a periodic motion with a period $\frac{\pi}{\omega}$ but not SHM

4. Vertical displacement of a plank with a body of mass m on it is varying according to law $y = \sin(\omega t) + \sqrt{3}\cos(\omega t)$. The minimum value of ω for which the mass just breaks off the plank and the moment it occurs first after $t = 0$, are given by Assume y to be positive vertically upwards.

(A) $\sqrt{\frac{g}{2}}, \frac{\sqrt{2}}{6}\frac{\pi}{\sqrt{g}}$ (B) $\frac{g}{\sqrt{2}}, \frac{2}{3}\sqrt{\frac{\pi}{g}}$

(C) $\sqrt{\frac{g}{2}}, \frac{\pi}{3}\sqrt{\frac{2}{g}}$ (D) $\sqrt{2g}, \sqrt{\frac{2\pi}{3g}}$

5. A charged particle is deflected by two mutually perpendicular oscillating electric fields such that the displacement of the particle due to each one of them is given by $x = a\cos(\omega t)$ and $y = a\cos\left(\omega t + \frac{\pi}{6}\right)$ respectively. The trajectory followed by the charged particle is

(A) a circle with equation $x^2 + y^2 = a^2$.

(B) a straight line with equation $y = \sqrt{3}x$.

(C) an ellipse with equation $x^2 + y^2 - xy = \frac{3a^2}{4}$.

(D) an ellipse with equation $x^2 + y^2 - \sqrt{3}xy = \frac{a^2}{4}$.

6. If potential energy of a particle varies with position x according to the relation $U = x^2 - 4x + 4$, then motion of particle will be

(A) periodic but not SHM with mean at $x = 0$

(B) periodic but not SHM with mean position $x = 2$

(C) SHM with mean position $x = 2$

(D) uniformly accelerated motion starting at $x = 2$

7. Time period of a simple pendulum of length L is T_1 and time period of a uniform rod of the same length L pivoted about one end and oscillating in a vertical plane is T_2. Amplitude of oscillations in both the cases is small. Then $\frac{T_1}{T_2}$ is

(A) $\sqrt{\frac{4}{3}}$ (B) 1

(C) $\sqrt{\frac{3}{2}}$ (D) $\sqrt{\frac{1}{3}}$

8. A particle is executing SHM according to the equation $x = A\sin(\omega t)$. The average speed of the particle over the interval $0 \le t \le \frac{\pi}{6\omega}$ is

(A) $\frac{\sqrt{3}A\omega}{2\pi}$ (B) $\frac{\sqrt{3}A\omega}{4}$

(C) $\frac{3A\omega}{\pi}$ (D) $\left(2-\sqrt{3}\right)A\omega$

9. The potential energy of a particle of mass 1 kg is, $U = 10 + (x-2)^2$, where, U is in joule and x in metre. On the positive x-axis particle travels upto $x = +6$ m. Choose the incorrect statement.

(A) On negative x-axis particle travels upto $x = -2$ m

(B) The maximum kinetic energy of the particle is 16 J

(C) The period of oscillation of the particle is $\sqrt{2}\pi$ seconds

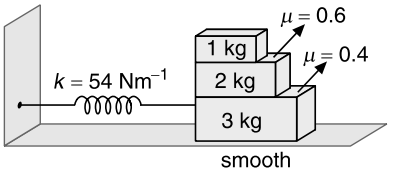
(D) None of the above

10. A particle executing SHM is described by the equation $y = 2\sin(100t) + 1$. The amplitude and mean position of SHM respectively are

(A) 3, 2 (B) 1, 2

(C) 2, 1 (D) $\sqrt{5}$, 1

11. In the figure shown, what can be the maximum amplitude of the system so that there is no slipping between any of the blocks?



(A) $\frac{2}{7}$ m (B) $\frac{3}{4}$ m

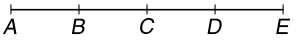
(C) $\frac{4}{9}$ m (D) $\frac{10}{3}$ m

12. In PROBLEM 11, if the value of force constant k is increased then the maximum amplitude calculated will

(A) remain same (B) increase

(C) decrease (D) data in insufficient

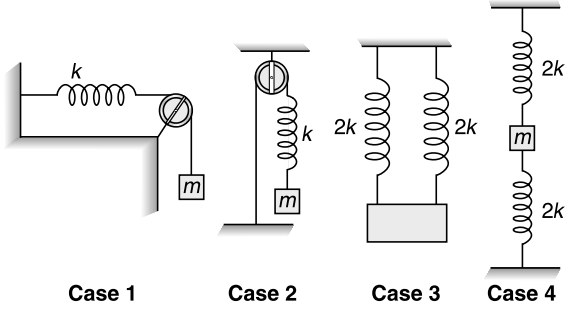
13. A body performs SHM along the straight line $ABCDE$ with C as the mid-point of AE. Its kinetic energies at B and D are each one fourth of its maximum value. If $AE = 2R$, the distance between B and D is



(A) $\frac{\sqrt{3}}{2}R$ (B) R

(C) $\sqrt{3}R$ (D) $\sqrt{2}R$

14. A block of mass m is suspended by different springs of force constant shown in figure. Let time period of oscillation in these four positions be T_1, T_2, T_3 and T_4. Then



(A) $T_1 = T_2 = T_4$ (B) $T_1 = T_2$ and $T_3 = T_4$

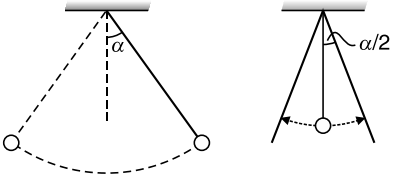
(C) $T_1 = T_2 = T_3$ (D) $T_1 = T_3$ and $T_2 = T_4$

15. Springs of constants k, $2k$, $4k$, $8k$, ..., $2048k$ are connected in series. A mass m is attached to one end and the system is allowed to oscillate. The time period is approximately

(A) $2\pi\sqrt{\frac{m}{2k}}$ (B) $2\pi\sqrt{\frac{2m}{k}}$

(C) $2\pi\sqrt{\frac{m}{4k}}$ (D) $2\pi\sqrt{\frac{4m}{k}}$

16. A simple pendulum swings with an initial angular amplitude α. It is now made to oscillate between two walls making an angle α in a symmetrical way as shown in Figure.



Assuming collisions with the walls to be elastic, the ratio of initial time period to new time period is

(A) 2 : 1 (B) 3 : 1

(C) 3 : 2 (D) 4 : 3

17. Two springs fixed at one end are stretched by 5 cm and 10 cm, respectively, when masses 0.5 kg and 1 kg are suspended at their lower ends. When displaced slightly from their mean positions and released, they will oscillate with time periods in the ratio

(A) $1 : \sqrt{2}$ (B) 1 : 2

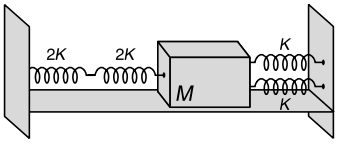
(C) $\sqrt{2} : 1$ (D) 2 : 1

18. Two particles are executing SHM in a straight line. Amplitude A and time period T of both the particles are equal. At time $t = 0$, one particle is at displacement $x_1 = +A$ and the other at $x_2 = -\frac{A}{2}$ and they are approaching towards each other. The time after which they cross each other is equal to

(A) $\frac{T}{3}$ (B) $\frac{T}{4}$

(C) $\frac{5T}{6}$ (D) $\frac{T}{6}$

19. A mass M is attached to four springs of spring constants $2K$, $2K$, K, K as shown in figure. The mass is capable of oscillating on a frictionless horizontal floor. If it is displaced slightly and released, the frequency of resulting S.H.M. would be



(A) $\frac{1}{2\pi}\sqrt{\frac{3K}{M}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{4K}{M}}$

(C) $\frac{1}{2\pi}\sqrt{\frac{11K}{2M}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{2K}{3M}}$

20. A particle performs harmonic oscillations along a straight line with a period T and amplitude a. Mean velocity of the particle averaged over the time interval, during which it travels a distance $\frac{a}{2}$ starting from the extreme position, is

(A) $\frac{a}{T}$ (B) $\frac{2a}{T}$

(C) $\frac{3a}{T}$ (D) $\frac{a}{2T}$

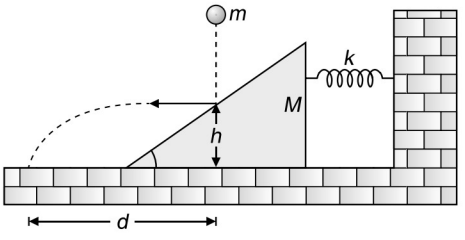
21. A small ball of density ρ_0 is released from rest from the surface of a liquid whose density varies with depth h as $\rho = \frac{\rho_0}{2}(\alpha + \beta h)$, where m is the mass of the ball. Select the most appropriate one option.

(A) The particle will execute SHM
(B) The maximum speed of the ball is $\left(\frac{2-\alpha}{\sqrt{2\beta}}\right)\sqrt{g}$
(C) Both (A) and (B) are correct
(D) Both (A) and (B) are wrong
Here, α and β are positive constants of proper dimensions with $\alpha < 2$.

22. Two particles are in SHM along same line with same amplitude A and same time period T. At time $t = 0$, particle 1 is at $+A/2$ and moving towards positive x-axis. At the same time particle 2 is at $-A/2$ and moving towards negative x-axis. Find the time when they will collide
(A) $2T/3$ (B) $5T/12$
(C) $4T/3$ (D) $2T/5$

23. The force constant of a linear harmonic oscillator is 3×10^4 Nm^{-1}, amplitude is 0.1 m and total energy is 200 joules. Then its
(A) minimum potential energy is zero
(B) maximum kinetic energy is 200 joules
(C) maximum potential energy is 150 joules
(D) maximum kinetic energy is 150 joules

24. A ball of mass m when dropped from certain height as shown in diagram, strikes a wedge kept on smooth horizontal surface and move horizontally just after impact.



If the ball strikes the ground at a distance d from its initial line of fall, then the amplitude of oscillation of wedge after being hit by the ball will be
(A) $\frac{md}{M}\sqrt{\frac{Mg}{2kh}}$ (B) $\frac{d}{m}\sqrt{\frac{kMg}{2h}}$
(C) $d\sqrt{\frac{kg}{2Mh}}$ (D) $d\sqrt{\frac{kg}{2mh}}$

25. The table shows the values of the acceleration a of a particle executing SHM at the displacement x. The time period of SHM is

a(ms^{-2})	27	9	0	–18	–36
x(m)	–3	–1	0	2	4

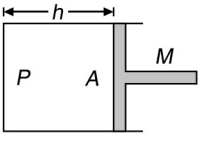
(A) $\frac{\pi}{3}$ s (B) π s
(C) $\frac{2\pi}{3}$ s (D) 1 s

26. A particle moves such that its acceleration is given by $a = -\beta(x-2)$, where β is a positive constant and x the position from origin. The time period of oscillations is
(A) $2\pi\sqrt{\beta}$
(B) $2\pi\sqrt{1/\beta}$
(C) $2\pi\sqrt{\beta+2}$
(D) $2\pi\sqrt{1/(\beta+2)}$

27. Two springs of force constant $\frac{k}{5}$ and $\frac{k}{4}$ are joined in series and then connected to a mass m. The frequency of oscillation of the mass will be
(A) $\frac{1}{6\pi}\sqrt{\frac{k}{m}}$ (B) $\frac{3}{2\pi}\sqrt{\frac{k}{m}}$
(C) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ (D) $\frac{2}{\pi\sqrt{20}}\sqrt{\frac{k}{m}}$

28. In PROBLEM 27, if the springs are connected in parallel, the frequency of oscillation will be
(A) $\frac{3}{2\pi}\sqrt{\frac{k}{m}}$ (B) $\frac{3}{2\pi\sqrt{20}}\sqrt{\frac{k}{m}}$
(C) $\frac{2}{\pi\sqrt{20}}\sqrt{\frac{k}{m}}$ (D) $\frac{3}{\pi}\sqrt{\frac{k}{m}}$

29. A cylindrical piston of mass M and cross-sectional area A slide smoothly inside a long cylinder closed at one end, enclosing a certain mass of gas. The cylinder is kept with its axis horizontal. If the piston is distributed from its equilibrium position, it oscillates simple harmonically. The period of oscillation is

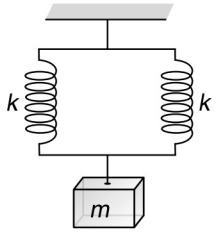


(A) $2\pi\sqrt{\frac{Mh}{PA}}$ (B) $2\pi\sqrt{\frac{MA}{Ph}}$
(C) $2\pi\sqrt{\frac{M}{PAh}}$ (D) $2\pi\sqrt{MPhA}$

30. A particle is executing SHM along a straight line and has mean position at $x = 0$. The time period of the SHM is 20 s and its amplitude 5 cm. The time taken by the particle to go from $x = 4$ cm to $x = -3$ cm can be
(A) 10 s (B) 8 s
(C) 5 s (D) 20 s

31. Two simple pendulums have time periods T and $\frac{5T}{4}$. They start vibrating at the same instant from the mean position in the same phase. The phase difference between them when the bigger pendulum completes one oscillation will be
(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$
(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

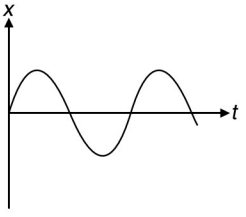
32. The two spring mass system, shown in the figure, oscillates with a period T. If only one spring is used, the same time period will be



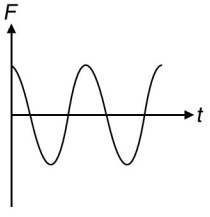
(A) $T/\sqrt{2}$ (B) $T/2$

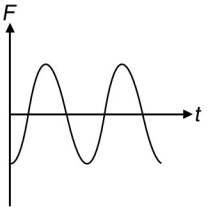
(C) $\sqrt{2}T$ (D) $2T$

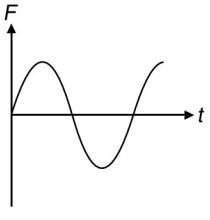
33. Displacement-time graph of a particle executing SHM is shown in figure.

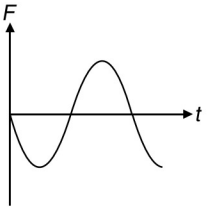


The corresponding force-time graph of the particle is

(A) 

(B) 

(C) 

(D) 

34. Two simple harmonic motions $y_1 = A\sin(\omega t)$ and $y_2 = A\cos(\omega t)$ are superimposed on a particle of mass m. The total mechanical energy of the particle is

(A) $\frac{1}{2}m\omega^2A^2$ (B) $m\omega^2A^2$

(C) $\frac{1}{4}m\omega^2A^2$ (D) ZERO

35. A particle of mass 10 g moves on x-axis in a field where potential energy per unit mass is given by expression $V = 8\times10^4x^2$ erg/g where x is in cm. If the total energy of the particle is 8×10^7 erg, then relation between x and time t is

(A) $x = 10\sin(400t+\phi)$ cm

(B) $x = \sin(400t+\phi)$ m

(C) $x = 10\sin(40t+\phi)$ cm

(D) $x = 100\sin(4t+\phi)$ m

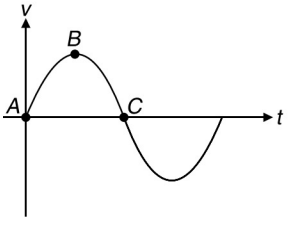
(For all the given four options, ϕ is constant)

36. A particle moves according to the law $x = a\cos\left(\frac{\pi t}{2}\right)$. The distance covered by it in the time interval between $t = 0$ to $t = 3$ s is

(A) $4a$ (B) $3a$

(C) $2a$ (D) a

37. The v-t graph of a particle in SHM is as shown in figure. Select the incorrect statement.



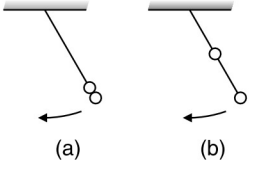
(A) At A particle is at mean position and moving towards positive direction

(B) At B acceleration of particle is zero

(C) At C acceleration of particle is maximum and in positive direction

(D) None of the above

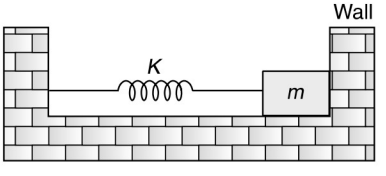
38. A long, straight massless rod is pivoted about one end in a vertical plane. In configuration (a), two small identical masses are attached to the free end and in configuration (b), one mass is moved to the center of the rod. The ratio of frequency of small oscillations in configuration (b) to that in configuration (a) is



(A) $\sqrt{6}:\sqrt{5}$ (B) $\sqrt{3}:\sqrt{2}$

(C) $6:5$ (D) $5:3$

39. In the figure, the block of mass m, attached to the spring of stiffness k is in contact with the completely elastic wall and the compression in the spring is l. The spring is compressed further by l by displacing the block towards left and is then released. If the collision between the block and the wall is completely elastic then the time period of oscillations of the block will be



(A) $\frac{2\pi}{3}\sqrt{\frac{m}{k}}$ (B) $2\pi\sqrt{\frac{m}{k}}$

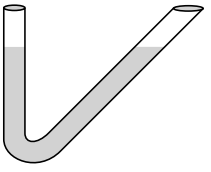
(C) $\frac{\pi}{3}\sqrt{\frac{m}{k}}$ (D) $\frac{\pi}{6}\sqrt{\frac{m}{k}}$

40. A particle performs SHM in a straight line. In the first second, starting from rest, it travels a distance a and in the next second it travels a distance b in the same direction. The amplitude of the SHM is

(A) $a-b$ (B) $\dfrac{2a-b}{3}$

(C) $\dfrac{2a^2}{3a-b}$ (D) $a+b$

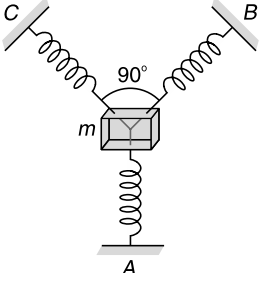
41. The period of oscillation of mercury of mass m and density ρ poured into a bent tube of cross sectional area S whose right arm forms an angle $60°$ with the vertical is



(A) $2\pi\sqrt{\dfrac{m}{\rho g S}}$ (B) $2\pi\sqrt{\dfrac{2m}{3\rho g S}}$

(C) $2\pi\sqrt{\dfrac{2m}{(2+\sqrt{3})\rho g S}}$ (D) $2\pi\sqrt{\dfrac{m}{2\rho g S}}$

42. A particle of mass m is attached to three identical springs A, B, C each of force constant K. If mass is slightly pushed against the spring A and released the time period of mass m would be



(A) $2\pi\sqrt{m/K}$ (B) $2\pi\sqrt{m/2K}$

(C) $2\pi\sqrt{m/3K}$ (D) $2\pi\sqrt{3K/m}$

43. Two pendulums of lengths 1 m and 16 m are in phase at mean position at a certain instant of time. If T is time period of shorter pendulum, the minimum time after which they will again be in phase is

(A) $T/3$ (B) $2T/3$

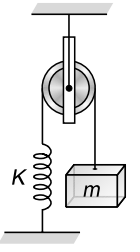
(C) $4T/3$ (D) $8T/3$

44. In case of a particle, executing simple harmonic motion, potential energy is E_1 at a displacement x and E_2 at a displacement y. Its potential energy at displacement $(x+y)$ is

(A) $\sqrt{E_1+E_2}$ (B) $\sqrt{E_1}+\sqrt{E_2}$

(C) E_1+E_2 (D) $\left(\sqrt{E_1}+\sqrt{E_2}\right)^2$

45. Figure shows a mass m suspended with a massless inextensible string passing over a frictionless pulley. The other end of string is connected a spring of force constant K, whose other end is connected to a rigid floor. If mass is stretched a little and allowed to vibrate; the time period of oscillations of mass is



(A) $2\pi\sqrt{K/m}$ (B) $2\pi\sqrt{m/K}$

(C) $2\pi\sqrt{2m/K}$ (D) $2\pi\sqrt{m/2K}$

46. The maximum tension in the string of a pendulum is two times the minimum tension. Let θ_0 be the angular amplitude, then

(A) $\cos\theta_0=\dfrac{1}{2}$ (B) $\cos\theta_0=\dfrac{3}{4}$

(C) $\cos\theta_0=\dfrac{2}{3}$ (D) $\cos\theta_0=\dfrac{3}{5}$

47. If the displacement x and velocity v of a particle executing SHM are related to each other by the equation $4v^2=25-x^2$, then the time period of SHM is

(A) π (B) 2π

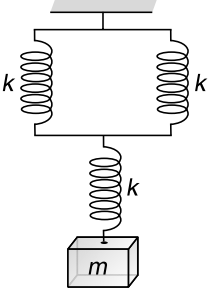
(C) 4π (D) 6π

48. An accurate pendulum clock is mounted on the ground floor of a high building. The time that it will loose or gain in one day when transferred to top storey of a building which is $h=200$ m higher than the ground floor is (Radius of earth is 6.4×10^6 m)

(A) it will lose 6.2 s (B) it will lose 2.7 s

(C) it will gain 5.2 s (D) it will gain 1.6 s

49. The frequency of a vertical oscillations of the three spring mass system, shown in the figure, is



(A) $\dfrac{1}{2\pi}\sqrt{\dfrac{3k}{2m}}$ (B) $\dfrac{1}{2\pi}\sqrt{\dfrac{2k}{3m}}$

(C) $\dfrac{1}{2\pi}\sqrt{\dfrac{3k}{m}}$ (D) $\dfrac{1}{2\pi}\sqrt{\dfrac{k}{3m}}$

50. The initial position and velocity of a particle executing SHM described by the equation $y=a\sin(\omega t+\theta)$ are 3 cm and (1.5π) cms^{-1} respectively. If angular frequency of particle is 0.5π rads^{-1}, then its amplitude is

(A) 3 cm (B) $2\sqrt{2}$ cm

(C) $3\sqrt{2}$ cm (D) $2\sqrt{3}$ cm

51. The potential energy of a particle of mass 1 kg in motion along the x-axis is given by $U = 4(1-\cos 2x)$ J, where x is in metre. The period of small oscillations (in second) is

(A) 2π (B) π

(C) $\frac{\pi}{2}$ (D) $\sqrt{2}\pi$

52. A body of mass 5 g is executing S.H.M. with amplitude 10 cm. Its maximum velocity is 100 cms^{-1}. Its velocity will be 50 cms^{-1} at a displacement from the mean position equal to

(A) 5 cm (B) $5\sqrt{3}$

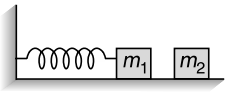
(C) 10 cm (D) $10\sqrt{3}$

53. A particle performs SHM of amplitude A along a straight line. When it is at a distance $\frac{\sqrt{3}A}{2}$ from mean position, its kinetic energy suddenly increases by an amount $\frac{1}{2}m\omega^2A^2$ due to an impulsive force. The new amplitude of oscillation will be

(A) $\frac{\sqrt{5}A}{2}$ (B) $\frac{\sqrt{3}A}{2}$

(C) $\sqrt{2}A$ (D) $\sqrt{5}A$

54. The two blocks of mass m_1 and m_2 are kept on a smooth horizontal table as shown in figure. Block of mass m_1 but not m_2 is fastened to the spring. If now both the blocks are pushed to the left so that the spring is compressed a distance d. The amplitude of oscillation of block of mass m_1, after the system is released is



(A) $d\sqrt{\frac{m_1}{m_1+m_2}}$ (B) $d\sqrt{\frac{m_2}{m_1+m_2}}$

(C) $d\sqrt{\frac{2m_2}{m_1+m_2}}$ (D) $d\sqrt{\frac{2m_1}{m_1+m_2}}$

55. The displacement of the motion of a particle is represented (in metre) by the equation

$$y = 0\cdot 4\left\{\cos^2\left(\frac{\pi t}{2}\right) - \sin^2\left(\frac{\pi t}{2}\right)\right\}$$

The motion of the particle is

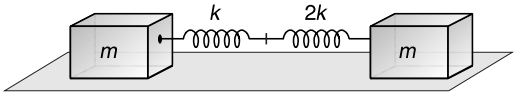
(A) oscillatory but not SHM

(B) SHM of amplitude 0.4 m, frequency 0.5 Hz

(C) SHM of amplitude $0.4\sqrt{2}$ m, frequency 2 Hz

(D) SHM of amplitude 0.8 m, frequency 2 Hz

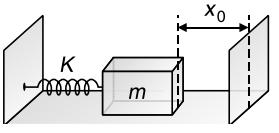
56. The time period for small oscillations of the two blocks connected with the springs as shown in Figure is



(A) $2\pi\sqrt{\frac{3m}{k}}$ (B) $2\pi\sqrt{\frac{3m}{4k}}$

(C) $2\pi\sqrt{\frac{3m}{8k}}$ (D) $2\pi\sqrt{\frac{3m}{2k}}$

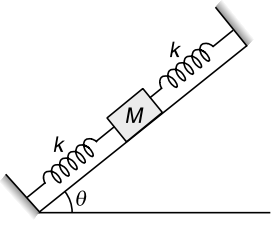
57. One end of a spring of force constant K is fixed to a vertical wall and other to a body of mass m resting on a smooth horizontal surface. There is another wall at a distance x_0 from the body. The spring is then compressed by $2x_0$ and released. The time taken to strike the wall from its compressed position is



(A) $\frac{4}{3}\pi\sqrt{\frac{m}{K}}$ (B) $\frac{5}{6}\pi\sqrt{\frac{m}{K}}$

(C) $\frac{1}{6}\pi\sqrt{\frac{m}{K}}$ (D) $\frac{2}{3}\pi\sqrt{\frac{m}{K}}$

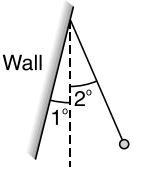
58. On a smooth inclined plane a body of mass M is attached between two springs. The other ends of the springs are fixed to firm supports. If each spring has a force constant k, the period of oscillation of the body is (assuming the spring as massless)



(A) $2\pi\sqrt{\frac{M}{2k}}$ (B) $2\pi\sqrt{\frac{2M}{k}}$

(C) $2\pi\sqrt{\frac{M\sin\theta}{2k}}$ (D) $2\pi\sqrt{\frac{2M\sin\theta}{k}}$

59. A simple pendulum of length 1 m, hanging from an inclined wall, is made to oscillate from an angle 2° with the vertical as shown in Figure.



If collisions with the wall are elastic, time period of oscillation will be (use $g = \pi^2$)

(A) $\frac{2}{3}$ s (B) $\frac{4}{3}$ s

(C) 2 s (D) $\frac{1}{3}$ s

60. Two particles execute simple harmonic motion of the same amplitude and frequency along close parallel lines. They pass each other moving in opposite directions each time their displacement is half their amplitude. Their phase difference is

(A) $\frac{5\pi}{6}$ (B) $\frac{4\pi}{3}$

(C) $\frac{\pi}{6}$ (D) $\frac{2\pi}{3}$

61. Maximum speed of a particle in simple harmonic motion is v_{max}. Then average speed of a particle in SHM is equal to

(A) $\frac{v_{max}}{2}$ (B) $\frac{v_{max}}{\pi}$

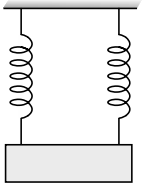
(C) $\frac{\pi v_{max}}{2}$ (D) $\frac{2v_{max}}{\pi}$

62. A ring of radius R and mass M is suspended from a point very close to its periphery. If it is displaced in its plane by a small angle and released, the period of oscillations will be

(A) $2\pi\sqrt{\frac{R}{g}}$ (B) $2\pi\sqrt{\frac{2R}{g}}$

(C) $2\pi\sqrt{\frac{3R}{g}}$ (D) $2\pi\sqrt{\frac{R}{2g}}$

63. An object suspended from a spring exhibits oscillation of period T. Now the spring is cut in two halves and the same object is suspended with two halves as shown in figure. The new time period of oscillation will become



(A) $T/\sqrt{2}$ (B) $2T$

(C) $T/2$ (D) $T/2\sqrt{2}$

64. The maximum acceleration of a particle in SHM is made two times keeping the maximum speed to be constant. It is possible when

(A) amplitude of oscillation is doubled while frequency remains constant
(B) amplitude is doubled while frequency is halved
(C) frequency is doubled while amplitude is halved
(D) frequency of oscillation is doubled while amplitude remains constant

65. A particle is moving in a circle of radius $R = 1$ m with constant speed $v = 4$ ms^{-1}. The ratio of displacement to acceleration of the foot of the perpendicular drawn from the particle on the diameter of the circle is

(A) $\frac{1}{16}(\text{second})^2$ (B) $\frac{1}{2}(\text{second})^2$

(C) $2(\text{second})^2$ (D) $16(\text{second})^2$

66. A light spring of force constant 100 Nm^{-1} is kept straight and unstretched on a smooth horizontal table with its ends fixed. A small bead of mass 40 g is now attached at the middle of the spring. If the bead is slightly displaced along the length of the spring and released, time period of oscillations will be

(A) $\frac{\pi}{50}$ second (B) $\frac{\pi}{100}$ second

(C) $\frac{\pi}{25}$ second (D) $\frac{2\pi}{25}$ second

67. A mass M is suspended from a spring of negligible mass. The spring is pulled a little and then released so that it executes simple harmonic oscillations with a period T. If the mass is increased by m, the period becomes $\frac{5T}{4}$. The ratio $\left(\frac{m}{M}\right)$ is

(A) $\frac{4}{5}$ (B) $\frac{5}{4}$

(C) $\frac{9}{16}$ (D) $\frac{25}{16}$

68. A simple pendulum is suspended from the roof of a trolley that moves freely down a plane of inclination α. The time period of oscillation is

(A) $2\pi\sqrt{\frac{L}{g}}$ (B) $2\pi\sqrt{\frac{L}{g\cos\alpha}}$

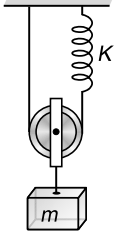
(C) $2\pi\sqrt{\frac{L}{g\sin\alpha}}$ (D) $2\pi\sqrt{\frac{L}{g\tan\alpha}}$

69. A particle is in linear simple harmonic motion between two points, A and B, 10 cm apart. Taking direction from A to B as the positive direction, respective signs of velocity of the particle at the following positions is

(a) at the mid-point of AB going towards A
(b) at 2 cm away from B going towards A
(c) at 3 cm away from A going towards B
(d) at 4 cm away from B going towards A

(A) $-, -, +, +$
(B) $-, -, +, -$
(C) $+, +, -, -$
(D) $+, -, +, -$

70. A mass m is suspended from a massless pulley, which itself is suspended by a massless string and a spring as shown in figure. What will be the time period of oscillations of the mass? The force constant of spring is K



(A) $2\pi\sqrt{\frac{m}{2K}}$ (B) $2\pi\sqrt{\frac{m}{K}}$

(C) $\pi\sqrt{\frac{m}{K}}$ (D) $\pi\sqrt{\frac{m}{2K}}$

71. Two simple pendulums having lengths 1 m and 16 m are both given small displacements in the same direction at the same instant. They will again be in phase at the mean position after the shorter pendulum has completed n oscillations where n is

(A) $\frac{1}{4}$ (B) 4

(C) 5 (D) 16

72. A sphere of radius R is floating half submerged in a liquid of density ρ. If the sphere is slightly pushed down and released, the frequency of vertical oscillations is

(A) $\frac{1}{2\pi}\sqrt{\frac{g}{R}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{3g}{2R}}$

(C) $\frac{1}{2\pi}\sqrt{\frac{g}{2R}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{3g}{R}}$

73. A small sphere is placed on a concave mirror of radius of curvature 5 m a little away from its centre. When the sphere is released it oscillates. Assuming that the oscillation is simple harmonic, the time period is $\left(g = 10\ \text{ms}^{-2}\right)$

(A) $\frac{\pi}{\sqrt{2}}$ s (B) $\frac{\pi}{2}$ s

(C) π s (D) $\pi\sqrt{2}$ s

74. If R denotes the radius of the earth, then the time period of a simple pendulum of infinite length is

(A) infinite (B) $2\pi\sqrt{\frac{R}{g}}$

(C) $2\pi\sqrt{\frac{2R}{g}}$ (D) $2\pi\sqrt{\frac{R}{2g}}$

75. The period of a particle executing S.H.M. is 8 s. At $t = 0$ it is at the mean position. The ratio of the distances covered by the particle in the 1st second to the 2nd second is

(A) $\frac{1}{\sqrt{2}+1}$ (B) $\sqrt{2}$

(C) $\frac{1}{\sqrt{2}}$ (D) $\sqrt{2}+1$

76. A tunnel is dug across earth along one of its diameters. Two masses m and $2m$ are dropped from two opposite ends of the tunnel. The masses collide and stick to each other. If R be the radius of earth, then amplitude of subsequent SHM will be

(A) R (B) $\frac{R}{2}$

(C) $\frac{R}{3}$ (D) $\frac{2R}{3}$

77. When the particle executing S.H.M. passes through mean position, it has

(A) Minimum kinetic energy and minimum potential energy

(B) Minimum kinetic energy and maximum potential energy

(C) Maximum kinetic energy and minimum potential energy

(D) Maximum kinetic energy and maximum potential energy

78. A particle of mass 2 kg is executing S.H.M. given by $y = 6\cos\left(100t + \frac{\pi}{4}\right)$ cm. Its maximum kinetic energy is

(A) 18 J (B) 36 J

(C) 18×10^4 J (D) 36×10^4 J

79. A particle is executing S.H.M. Then the graph of velocity as a function of displacement is

(A) straight line (B) circle

(C) ellipse (D) hyperbola

80. Time period of a simple pendulum is T. If its length increased by 2%, the new time period becomes

(A) $0.98T$ (B) $1.02T$

(C) $0.99T$ (D) $1.01T$

81. The mass and diameter of a planet are twice those of the earth. The time period of a simple pendulum on this planet, if it is a second's pendulum on earth, is

(A) $\frac{1}{\sqrt{2}}$ s (B) 2 s

(C) $\frac{1}{2}$ s (D) $2\sqrt{2}$ s

82. The angular frequency of a spring block system is ω_0. This system is suspended from the ceiling of an elevator moving downwards with a constant speed v_0. The block is at rest relative to the elevator. Lift is suddenly stopped. Assuming the downward as a positive directions, choose the incorrect statement.

(A) Amplitude of block is $\frac{v_0}{\omega_0}$

(B) Initial phase of block is π

(C) Equation of motion for block is $\frac{v_0}{\omega_0}\sin\omega_0 t$

(D) Maximum speed of the block is v_0

83. A body, hung from a spring, executes vertical oscillations with period T. The body is now immersed in a non-viscous liquid having density one-tenth of that of the body. When set into two vertical oscillations with the body remaining fully immersed in the liquid all the time, the period will be

(A) $\sqrt{\frac{10}{9}}\,T$ (B) $\sqrt{\frac{9}{10}}\,T$

(C) T (D) $\frac{1}{10}\,T$

84. Four simple harmonic vibrations $x_1 = 8\sin(\omega t)$, $x_2 = 6\sin\left(\omega t + \frac{\pi}{2}\right)$, $x_3 = 4\sin(\omega t + \pi)$ and $x_4 = 2\sin\left(\omega t + \frac{3\pi}{2}\right)$ are superimposed on each other. The resulting amplitude and its phase difference with x_1 are respectively

(A) 20, $\tan^{-1}\left(\frac{1}{2}\right)$ (B) $4\sqrt{2}, \frac{\pi}{2}$

(C) 20, $\tan^{-1}(2)$ (D) $4\sqrt{2}, \frac{\pi}{4}$

85. Two particles P and Q describe SHM of same amplitude a, same frequency f along the same straight line. Maximum distance between the two particles is observed to be $a\sqrt{2}$. Phase difference between the particle is

(A) zero (B) $\frac{\pi}{2}$

(C) $\frac{\pi}{6}$ (D) $\frac{\pi}{3}$

86. Two masses M and m are suspended together by a light spring of force constant k. When the masses are in equilibrium, M is removed without disturbing the system. The amplitude of oscillation is

(A) $\frac{Mg}{k}$ (B) $\frac{mg}{k}$

(C) $\frac{(M+m)g}{k}$ (D) $\frac{(M-m)g}{k}$

87. A wire of length l, area of cross section A and Young's modulus of elasticity Y is suspended from the roof of a building. A block of mass m is attached at lower end of the wire. If the block is displaced from its mean position and then released the block starts oscillating. Time period of these oscillations will be

(A) $2\pi\sqrt{\frac{Al}{mY}}$ (B) $2\pi\sqrt{\frac{AY}{ml}}$

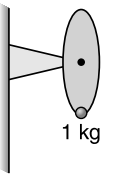
(C) $2\pi\sqrt{\frac{ml}{YA}}$ (D) $2\pi\sqrt{\frac{m}{YAl}}$

88. A pan with a set of weights is attached to a light spring. The period of vertical oscillations is 0.5 s. When some additional weights are put on the pan, the period of oscillations increases by 0.1 s. The extension caused by the additional weights is

(A) 1.3 cm (B) 2.7 cm

(C) 3.8 cm (D) 5.5 cm

89. A disc of mass 2 kg and radius 5 m is free to rotate about its axis kept horizontal. A particle of mass 1 kg is attached to the bottom most point of the disc as shown in Figure.



Time period (in seconds) of small oscillation of the system about the horizontal axis is

(A) 2π (B) π

(C) 4π (D) 0.5π

90. A metallic sphere is filled with water and hung by a long thread. It is made to oscillate. If there is a small hole in the bottom through which water slowly flows out, the time period will

(A) go on increasing till the sphere is empty

(B) go on decreasing till the sphere is empty

(C) remain unchanged throughout

(D) first increase then decrease till the sphere is empty and the period will now be the same as when the sphere was full of water

91. A simple pendulum 4 m long swings with an amplitude of 0.2 m. Acceleration of pendulum at the ends of its path is (Take $g = 10\ \text{ms}^{-2}$)

(A) zero (B) $10\ \text{ms}^{-2}$

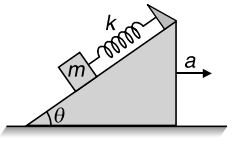
(C) $0.5\ \text{ms}^{-2}$ (D) $2.5\ \text{ms}^{-2}$

92. A uniform disc of radius R is pivoted at point O on its circumference. The time period of small oscillations will be

(A) $2\pi\sqrt{\frac{R}{g}}$ (B) $2\pi\sqrt{\frac{2R}{3g}}$

(C) $2\pi\sqrt{\frac{2R}{g}}$ (D) $2\pi\sqrt{\frac{3R}{2g}}$

93. A spring-block system is kept on a smooth wedge of inclination θ as shown in Figure.



The wedge is moving with constant acceleration a. Time period of small oscillation of block, assuming that at all times mass remains in contact with the wedge, is

(A) $2\pi\sqrt{\frac{m}{k\left(1+\frac{g^2}{a^2}\right)}}$ (B) $2\pi\sqrt{\frac{m\sin\theta}{k\left(1+\frac{g^2}{a^2}\right)}}$

(C) $2\pi\sqrt{\frac{m\sin\theta}{k}}$ (D) $2\pi\sqrt{\frac{m}{k}}$

94. A body falling freely on a planet covers 8 m in 2 s. The time period of one metre long simple pendulum on the planet will be

(A) 1.57 s (B) 3.14 s

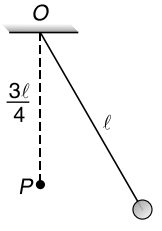
(C) 6.28 s (D) None of these

95. A particle of mass m is executing oscillations about the origin on the x-axis with amplitude A. Its potential energy is given as $U(x)=\alpha x^4$ where α is a positive constant. The x-coordinate of mass where potential energy is one third the kinetic energy of particle is

(A) $\pm\frac{1}{2}A$ (B) $\pm\frac{1}{\sqrt{2}}A$

(C) $\pm\frac{1}{3}A$ (D) $\pm\frac{1}{\sqrt{3}}A$

96. A pendulum has time period T for small oscillations. An obstacle P situated below the point of suspension O at a distance $\frac{3l}{4}$. The pendulum is released from rest. Throughout the motion the moving string makes small angle with vertical. Time after which the pendulum returns back to its initial position is



(A) T (B) $\frac{3}{4}T$

(C) $\frac{4}{3}T$ (D) $\frac{5}{4}T$

97. A particle performs SHM on x-axis from $x=-2$ cm to $x=6$ cm with time period of 0.5 s. It was located at $x=4$ cm and moving in positive x-direction at $t=0$, then

(a) equation of the SHM is $x=2+4\sin(4\pi t+30°)$

(b) equation of the SHM is $x=-2+2\cos 4\pi t$

(c) acceleration of the particle at $x=4$ cm is $32\pi^2$ cms^{-2} towards positive x-direction

(A) (a) only (B) (a) and (c)

(C) (b) and (c) (D) (c) only

98. A 4 kg particle is moving along the x-axis under the action of the force (in newton), $F=-\frac{\pi^2 x}{16}$ where x is the displacement from mean position in meters. At $t=2$ s, the particle passes through the origin and at $t=10$ s, its speed is $4\sqrt{2}$ ms^{-1}. The amplitude of the motion (in meters) is

(A) $\frac{32\sqrt{2}}{\pi}$ (B) $\frac{16}{\pi}$

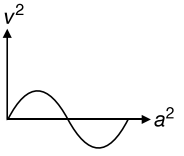
(C) $\frac{4}{\pi}$ (D) $\frac{16\sqrt{2}}{\pi}$

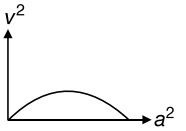
99. A particle starts oscillating simple harmonically from its equilibrium position. The ratio of kinetic and potential energy of the particle at time $\frac{T}{12}$, where T is the time period is

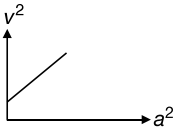
(A) 2 : 1 (B) 3 : 1

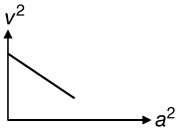
(C) 4 : 1 (D) 1 : 4

100. The graph between square of velocity and square of acceleration of a simple harmonic motion is

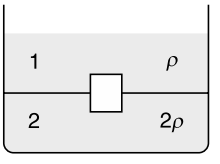
(A) 

(B) 

(C) 

(D) 

101. A plank of area of cross-section A is half immersed in liquid 1 of density ρ and half in liquid 2 of density 2ρ. The period of oscillation of the plank when it is slightly depressed downwards is



(A) $2\pi\sqrt{\frac{m}{\rho Ag}}$ (B) $\pi\sqrt{\frac{m}{\rho Ag}}$

(C) $2\pi\sqrt{\frac{3m}{2\rho Ag}}$ (D) $2\pi\sqrt{\frac{m}{3\rho Ag}}$

102. A particle of mass 5×10^{-5} kg is placed at the lowest point of a smooth parabola having the equation $x^2=40y$ where x, y are in centimetre. If it is displaced slightly and it moves such that it is constrained to move along the parabola, the angular frequency of oscillation will be, approximately

(A) 0.1 s^{-1} (B) 0.5 s^{-1}

(C) 0.7 s^{-1} (D) 9 s^{-1}

103. Displacement time equation of a particle executing SHM is given by $x=A\sin\left(\omega t+\frac{\pi}{6}\right)$. The minimum time taken by the particle to go from $-\frac{A}{2}$ to $+\frac{A}{2}$ is

(A) $\frac{\pi}{3\omega}$ (B) $\frac{\pi}{2\omega}$

(C) $\frac{2\pi}{\omega}$ (D) $\frac{\pi}{\omega}$

104. A U tube of uniform bore of cross-sectional area A is set up vertically with open ends up. A liquid of mass M and density d is poured into it. The liquid column will oscillate with a period

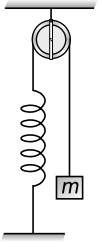
(A) $2\pi\sqrt{\frac{M}{g}}$ (B) $2\pi\sqrt{\frac{MA}{gd}}$

(C) $2\pi\sqrt{\frac{M}{Agd}}$ (D) $2\pi\sqrt{\frac{M}{2Agd}}$

105. An object of mass 0.2 kg executes simple harmonic oscillations along the x-axis with a frequency $\frac{25}{\pi}$ Hz. At the position $x = 0.04$ m, the object has kinetic energy 0.5 J and potential energy 0.4 J. Assuming the potential energy to be zero at the mean position, the amplitude of oscillation is

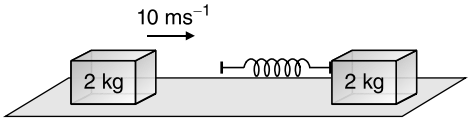
(A) 6 cm (B) 4 cm
(C) 8 cm (D) 2 cm

106. A mass $m = 8$ kg is attached to a spring as shown in figure and held in position so that the spring remains unstretched. The spring constant is 200 Nm^{-1}. The mass m is then released and begins to undergo small oscillations. If $g = 10$ ms^{-2}, then the maximum velocity of the mass is



(A) 1 ms^{-1} (B) 2 ms^{-1}
(C) 4 ms^{-1} (D) 5 ms^{-1}

107. A 2 kg block, moving with 10 ms^{-1}, strikes a spring of constant π^2 Nm^{-1} attached to 2 kg block at rest kept on a smooth floor as shown in Figure.



The time for which rear moving block remains in contact with the spring is

(A) $\sqrt{2}$ s (B) $\frac{1}{\sqrt{2}}$ s
(C) 1 s (D) $\frac{1}{2}$ s

108. A simple pendulum has a length L. The mass of the bob is m. The bob is given a charge $+q$. The pendulum is suspended between the plates of a charged parallel plate capacitor which are placed vertically. If E is electric field intensity between the plates, then the time period of oscillation will be

(A) $2\pi\sqrt{\frac{L}{g}}$ (B) $2\pi\sqrt{\frac{L}{\sqrt{g+\frac{qE}{m}}}}$
(C) $2\pi\sqrt{\frac{L}{\sqrt{g-\frac{qE}{m}}}}$ (D) $2\pi\sqrt{\frac{L}{\sqrt{g^2+\left(\frac{qE}{m}\right)^2}}}$

109. The potential energy of a harmonic oscillator of mass 2 kg in its mean position is 5 J. If its total energy is 9 J and its amplitude is 0.01 m, its time period is

(A) $\left(\frac{\pi}{10}\right)$ s (B) $\left(\frac{\pi}{20}\right)$ s
(C) $\left(\frac{\pi}{50}\right)$ s (D) $\frac{\pi}{100}$ s

110. A pendulum having a time period T in air, has a time period $2T$ when made to oscillate in water. Ignoring viscosity of water, the density of the pendulum bob is equal to

(A) 2 gcc^{-1} (B) 1.33 gcc^{-1}
(C) 1.5 gcc^{-1} (D) 1.67 gcc^{-1}

111. The displacement of two identical particles executing SHM are represented by equations

$$x_1 = 4\sin\left(10t + \frac{\pi}{6}\right) \text{ and } x_2 = 5\cos(\omega t).$$

The value of ω for which the energy of both the particles will be the same is

(A) 16 unit (B) 6 unit
(C) 4 unit (D) 8 unit

112. The displacement-time equation of a particle executing SHM is $x = A\sin(\omega t + \phi)$. At time $t = 0$ position of the particle is $x = \frac{A}{2}$ and it is moving along negative x-direction. Then

(A) $\phi = \frac{\pi}{6}$ (B) $\phi = \frac{\pi}{3}$
(C) $\phi = \frac{2\pi}{3}$ (D) $\phi = \frac{5\pi}{6}$

113. A simple pendulum has time period $T = 2$ s in air. If the whole arrangement is placed in a non-viscous liquid whose density is $\frac{1}{2}$ time the density of bob. The time period in the liquid will be

(A) $\frac{2}{\sqrt{2}}$ s (B) 4 s
(C) $2\sqrt{2}$ s (D) $4\sqrt{2}$ s

114. A simple pendulum suspended from the roof of an elevator at rest has a time period T_1. When the elevator moves up with an acceleration a, its time period becomes T_2. When the elevator moves down with an acceleration a, its time period becomes T_3. Then

(A) $T_1 = \sqrt{T_2 T_3}$ (B) $T_1 = T_2 T_3$
(C) $T_1 = \frac{\sqrt{2}T_2T_3}{\sqrt{T_2^2+T_3^2}}$ (D) $T_1 = \frac{T_2T_3}{\sqrt{T_2^2+T_3^2}}$

115. A simple pendulum has a time period T. The pendulum is completely immersed in a non-viscous liquid whose density is one-tenth of that of the material of the bob. The time period of the pendulum immersed in liquid is

(A) T (B) $\sqrt{\frac{9}{10}}T$
(C) $\sqrt{\frac{10}{9}}T$ (D) $\frac{T}{10}$

116. A particle executing SHM while moving from one extremity is found at distance x_1, x_2 and x_3 from the centre at the end of three successive second. The time period of oscillation in terms of $\theta = \cos^{-1}\left(\frac{x_1 + x_3}{2x_2}\right)$ is

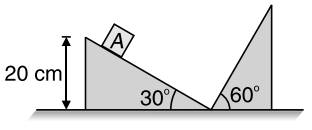
(A) $\frac{2\pi}{\theta}$ (B) $\frac{\pi}{\theta}$

(C) θ (D) $\frac{\pi}{2\theta}$

117. The displacement-time equation of a particle is given by $x = A\sin\left(\omega t + \frac{\pi}{6}\right) - A\cos\left(\omega t + \frac{\pi}{6}\right)$. Select the correct statement.

(A) The motion of the particle is not simple harmonic
(B) At $t = 0$, acceleration of particle is positive
(C) At $t = 0$, velocity of particle is negative
(D) None of these

118. The time period of motion of the block A shown in Figure (if bend at the bottom of fixed wedges, the friction between block and wedges are ignored) is



(A) 0.4 (B) $0.8\left(1 + \frac{1}{\sqrt{3}}\right)$

(C) 0.8 (D) $0.4\left(1 + \frac{1}{\sqrt{3}}\right)$

119. Two bodies M and N of equal masses are suspended from two separate massless springs of spring constants k_1 and k_2 respectively. If the two bodies oscillate vertically such that their maximum velocities are equal, the ratio of the amplitude of vibration of M to that of N is

(A) $\frac{k_1}{k_2}$ (B) $\sqrt{\frac{k_2}{k_1}}$

(C) $\frac{k_2}{k_1}$ (D) $\sqrt{\frac{k_1}{k_2}}$

120. The maximum displacement of the particle executing S.H.M. is 1 cm and the maximum acceleration is $(1.57)^2$ cms^{-2}. Its time period is

(A) 0.25 s (B) 4.0 s
(C) 1.57 s (D) 3.14 s

121. Two pendulums of length 100 cm and 121 cm start vibrating. At some instant the two are at the mean position in the same phase. After how many vibrations of the longer pendulum will the two be in the same phase at the mean position again

(A) 10 (B) 11
(C) 20 (D) 21

122. A bob of mass m is hanging from a point with the help of a light string of length L. A small impulse gives the bob a horizontal velocity v. The amplitude of resulting SHM is

(A) $v\sqrt{\frac{L}{g}}$ (B) $\frac{v}{2}\sqrt{\frac{L}{g}}$

(C) $\sqrt{\frac{v^2 L}{2g}}$ (D) $\sqrt{\frac{v^2 L}{8g}}$

123. The period of oscillation of a simple pendulum at a place where the acceleration due to gravity is g is T. The speed of oscillation at a place where the acceleration due to gravity is $1.02g$ will be

(A) T (B) $1.02T$
(C) $0.99T$ (D) $1.01T$

124. A weakly damped harmonic oscillator of frequency n_1 is driven by an external periodic force of frequency n_2. When the steady state is reached, the frequency of the oscillator will be

(A) n_1 (B) n_2

(C) $\frac{1}{2}(n_1 + n_2)$ (D) $\sqrt{n_1 + n_2}$

125. A simple pendulum of length L is inside a container filled with a non-viscous liquid of density σ. The density of material of the pendulum bob is ρ. If the container is placed in a lift which is moving down with retardation a_0, then time period of simple pendulum is

(A) $2\pi\sqrt{\frac{L}{g\left(1 - \frac{\sigma}{\rho}\right) - a_0}}$ (B) $2\pi\sqrt{\frac{L}{g\left(1 - \frac{\sigma}{\rho}\right) + a_0}}$

(C) $2\pi\sqrt{\frac{L}{(g + a_0)\left(1 - \frac{\sigma}{\rho}\right)}}$ (D) $2\pi\sqrt{\frac{L}{(g - a_0)\left(1 - \frac{\sigma}{\rho}\right)}}$

126. The displacement-time equation of a particle executing SHM is $x = 4\sin(\omega t) + 3\sin\left(\omega t + \frac{\pi}{3}\right)$, where x is in centimetre and t in second. The amplitude of oscillation of the particle is approximately

(A) 7 cm (B) 5 cm
(C) 6 cm (D) 9 cm

127. A block is kept on a rough horizontal plank. The coefficient of friction between the block and the plank is $\frac{1}{2}$. The plank is undergoing SHM of angular frequency 10 rads^{-1}. If $g = 10$ ms^{-2}, then the maximum amplitude of plank in which the block does not slip over the plank is

(A) 4 cm (B) 5 cm
(C) 10 cm (D) 16 cm

128. The equation of a particle of mass m executing SHM is given by $y = a\sin\omega t + a\cos\omega t$. The total mechanical energy of particle is

(A) $\frac{1}{2}m\omega^2 a^2$ (B) $m\omega^2 a^2$

(C) $\frac{1}{4}m\omega^2 a^2$ (D) ZERO

129. Three masses 0.1 kg, 0.3 kg and 0.4 kg are suspended at end of a spring. When the 0.4 kg mass is removed, the system oscillates with a period 2 s. When the 0.3 kg mass is also removed, the system will oscillate with a period

(A) 1 s (B) 2 s
(C) 3 s (D) 4 s

130. A pole is floating in a liquid with 80 cm of its length immersed. It is pushed down a certain distance and then released. Time period of vertical oscillation is

(A) $\frac{4\pi}{7}$ s (B) $\frac{3\pi}{7}$ s
(C) $\frac{2\pi}{7}$ s (D) $\frac{\pi}{7}$ s

131. Time period of a simple pendulum of length R suspended over earth is equal to (R is radius of earth and g is acceleration due to gravity on the surface of earth)

(A) ∞ (B) $2\pi\sqrt{\frac{R}{g}}$
(C) $2\pi\sqrt{\frac{2R}{g}}$ (D) $2\pi\sqrt{\frac{R}{2g}}$

132. A particle executing S.H.M. has an acceleration of 64 cms^{-2} when its displacement is 4 cm. Its time period, in second is

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$
(C) π (D) 2π

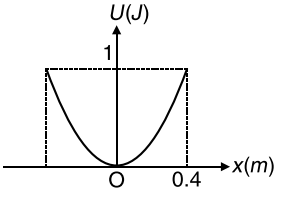
133. Two linear simple harmonic motions of equal amplitudes a and frequencies ω and 2ω are impressed on a particle along x and y axis respectively. If the initial phase difference between them is $\frac{\pi}{2}$, the resultant trajectory equation of the particle is

(A) $a^2y^2 = x^2(a^2 - x^2)$
(B) $a^2y^2 = 2x^2(a^2 - x^2)$
(C) $a^2y^2 = 4x^2(a^2 - x^2)$
(D) $a^2y^2 = 8x^2(a^2 - x^2)$

134. A man weighing 60 kg stands on the horizontal platform of a spring balance. The platform starts executing simple harmonic motion of amplitude 0.1 m and frequency $\left(\frac{2}{\pi}\right)$ hertz. Then the spring balance reading fluctuates approximately between

(A) 50 kg and 60 kg (B) 60 kg and 70 kg
(C) 40 kg and 50 kg (D) 50 kg and 70 kg

135. A particle of mass 2 kg performs simple harmonic motion and its potential energy U varies with position x as shown in Figure. The period of oscillation of the particle is



(A) $\frac{2\pi}{5}$ s (B) $\frac{2\sqrt{2}\pi}{5}$ s
(C) $\frac{\sqrt{2}\pi}{5}$ s (D) $\frac{4\pi}{5}$ s

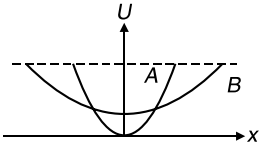
136. The minimum phase difference between the two simple harmonic oscillations $y_1 = \frac{1}{2}\sin(\omega t) + \frac{\sqrt{3}}{2}\cos(\omega t)$, and $y_2 = \sin(\omega t) + \cos(\omega t)$ is

(A) $\frac{\pi}{6}$ (B) $-\frac{\pi}{6}$
(C) $\frac{\pi}{12}$ (D) $\frac{7\pi}{12}$

137. A mass m attached to a spring oscillates with a period of 3 s. If the mass is increased by 1 kg the period increases by 1 s. The initial mass m in kg is

(A) $\frac{7}{9}$ (B) $\frac{9}{7}$
(C) $\frac{14}{9}$ (D) $\frac{18}{7}$

138. Potential energy (U) versus displacement (x) curves for two particles, of same mass, executing SHM about same mean position $(x = 0)$ on x-axis are shown in Figure. Then relation between their angular frequencies is

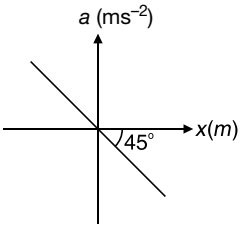


(A) $\omega_A > \omega_B$
(B) $\omega_A < \omega_B$
(C) $\omega_A = \omega_B$
(D) $\omega_B = 2\omega_A$

139. A flat horizontal board moves up and down in S.H.M. of amplitude A. Then the smallest permissible value of the time period, such that an object on the board may not lose contact with the board

(A) $2\pi\sqrt{\frac{g}{A}}$ (B) $\pi\sqrt{\frac{g}{A}}$
(C) $\pi\sqrt{\frac{A}{g}}$ (D) $2\pi\sqrt{\frac{A}{g}}$

140. The acceleration-displacement graph of a particle executing SHM is shown in given figure. The time period of its oscillation in seconds is



(A) $\frac{\pi}{2}$ (B) 2π

(C) π (D) $\frac{\pi}{4}$

141. A body executes S.H.M. with an amplitude A. Its energy is half kinetic and half potential when the displacement is

(A) $\frac{A}{3}$ (B) $\frac{A}{2}$

(C) $\frac{A}{\sqrt{2}}$ (D) $\frac{A}{2\sqrt{2}}$

142. A particle of mass m moving along x-axis has a potential energy given by $U(x) = a + bx^2$, where a and b are positive constants and x is displacement from mean position. The frequency of SHM executed by the particle depends on

(A) b alone (B) b and a alone

(C) b and m alone (D) b, a and m alone

143. The frequency of a particle executing SHM is 10 Hz. A particle is suspended from a vertical spring. At the highest point of its oscillation the spring is unstretched. The maximum speed of the particle is

Take $g = 10\ \text{ms}^{-2}$.

(A) $2\pi\ \text{ms}^{-1}$ (B) $\pi\ \text{ms}^{-1}$

(C) $\frac{1}{\pi}\ \text{ms}^{-1}$ (D) $\frac{1}{2\pi}\ \text{ms}^{-1}$

144. A particle is vibrating in S.H.M. Its velocities are v_1 and v_2 when the displacements from the mean position are y_1 and y_2, respectively, then its time period is

(A) $2\pi\sqrt{\frac{y_1^2 + y_2^2}{v_1^2 + v_2^2}}$ (B) $2\pi\sqrt{\frac{v_1^2 + v_2^2}{y_1^2 + y_2^2}}$

(C) $2\pi\sqrt{\frac{v_2^2 - v_1^2}{y_1^2 - y_2^2}}$ (D) $2\pi\sqrt{\frac{y_1^2 - y_2^2}{v_2^2 - v_1^2}}$

145. A ball of mass 2 kg hanging from a spring oscillates with a time period 2π second. If the ball is removed when it is in equilibrium position, then the spring shortens by

(A) g metre (B) $\frac{g}{2}$ metre

(C) $2g$ metre (D) 2π metre

146. The potential energy of a simple harmonic oscillator of mass 2 kg at its mean position is 5 J. If its total energy is 9 J and its amplitude is 0.01 m, then its time period in seconds will be

(A) $\frac{\pi}{10}$ (B) $\frac{\pi}{20}$

(C) $\frac{\pi}{50}$ (D) $\frac{\pi}{100}$

147. For the particle executing S.H.M. the kinetic energy E is given by $E = E_0 \cos^2(\omega t)$. The maximum value of potential energy is

(A) $\frac{E_0}{2}$ (B) $\frac{E_0}{\sqrt{2}}$

(C) E_0 (D) None of these

148. A particle of mass 0.1 kg executes SHM under a force $F = -(10\,x)$ N, such that the speed of the particle at mean position is 6 ms^{-1}. Then amplitude of oscillations is

(A) 0.6 m (B) 0.2 m

(C) 0.4 m (D) 0.1 m

149. Time period of a body executing SHM is π seconds. The numerical value of its velocity is equal to that of its acceleration when it is at a distance 10 cm from the mean position. Amplitude of oscillation is

(A) $10\sqrt{2}$ cm (B) $10\sqrt{3}$ cm

(C) 20 cm (D) $10\sqrt{5}$ cm

150. The period of the simple pendulum in a stationary lift is T. If the lift moves upwards with an acceleration g, the period will be

(A) infinite (B) $\sqrt{\frac{3}{5}}T$

(C) $\sqrt{\frac{5}{3}}T$ (D) $\frac{T}{\sqrt{2}}$

151. The displacement of a simple harmonic oscillator is given by $y = 5\cos\left(2\pi t + \frac{\pi}{3}\right)$. The speed of the oscillator will be maximum at

(A) 1 s (B) 0 s

(C) $\frac{1}{12}$ s (D) $\frac{1}{2}$ s

152. The vertical extension in a light spring by weight of 1 kg, in equilibrium, is 9.8 cm. The period of oscillation of the spring, in second will be

(A) $\frac{2\pi}{10}$ (B) $\frac{2\pi}{100}$

(C) 20π (D) 200π

153. A girl swinging on a swing in the sitting position. If she stands up the period of the swing will be

(A) shorter

(B) longer

(C) remain unchanged

(D) None of these

MULTIPLE CORRECT CHOICE TYPE QUESTIONS

This section contains Multiple Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

1. A cylindrical log of wood is floating in a large pool of water with its length normal to water's surface. The log has a radius r, mass m, length l and has density σ. If the log is depressed below its equilibrium depth d (but not beneath the surface of water) and then released, it executes harmonic oscillations with time period T. (Assume density of water to be ρ)

(A) $T = 2\pi\sqrt{\dfrac{m}{2\pi\rho r l g}}$ (B) $T = \dfrac{2\pi}{r}\sqrt{\dfrac{m}{\pi\rho g}}$

(C) $d = \dfrac{m}{2\pi r\rho l}$ (D) $d = \dfrac{m}{\pi r^2\rho}$

2. The x-t equation of a particle moving along x-axis is given by $x = A + A(1-\cos\omega t)$, then the

(A) particle oscillates simple harmonically between points $x = 2A$ and $x = A$

(B) velocity of particle is maximum at $x = 2A$

(C) time taken by particle in travelling from $x = A$ to $x = 3A$ is $\dfrac{\pi}{\omega}$

(D) time taken by particle in travelling from $x = A$ to $x = 2A$ is $\dfrac{\pi}{2\omega}$

3. The displacement of a particle of mass 100 g from its mean position is given by $y = 0.05\sin[4\pi(5t+0.4)]$, where y in meter and t in second. Then, select the correct statement(s).

(A) Time period of motion is 0.1 s

(B) Maximum acceleration of particle is $10\pi^2$ ms^{-2}

(C) Total energy of SHM of particle is $0.05\pi^2$ J.

(D) Force on particle is zero when displacement is 0.05 m

4. A particle is executing SHM on a straight line. A and B are two points at which its velocity is zero. It passes through a certain point $P(AP < BP)$ at successive intervals of 0.5 sec and 1.5 sec with a speed of 3 ms^{-1}, then the

(A) maximum speed of particle is $3\sqrt{2}$ ms^{-1}

(B) maximum speed of particle is $\sqrt{2}$ ms^{-1}

(C) ratio $\dfrac{AP}{BP}$ is $\dfrac{\sqrt{2}-1}{\sqrt{2}+1}$

(D) ratio $\dfrac{AP}{BP}$ is $\dfrac{1}{\sqrt{2}}$

5. Time period of spring block system on surface of earth is T_1 and that of a simple pendulum is T_2. At height $h = R$, the radius of earth the corresponding values are T_1' and T_2' then

(A) $T_1' > T_1$ (B) $T_1' = T_1$

(C) $T_2' > T_2$ (D) $T_2' < T_2$

6. A student uses a simple pendulum of exactly 1 m length to determine g, the acceleration due to gravity. He uses a stop watch with the least count of 1 s for this and records 40 s for 20 oscillations. For this observation, which of the following statement(s) is (are) true?

(A) Error ΔT in measuring T, the time period, is 0.05 s

(B) Error ΔT in measuring T, the time period, is 1 s

(C) Percentage error in the determination of g is 5%

(D) Percentage error in the determination of g is 2.5%

7. In a spring block system, the force constant of spring is $k = 16$ Nm^{-1}, mass of the block is 1 kg and the maximum kinetic energy of the block is 8 J. Then

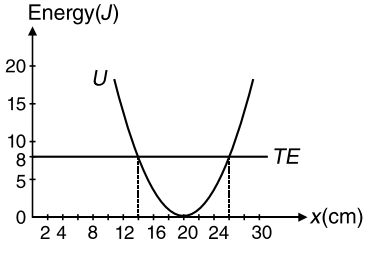
(A) amplitude of oscillation is 1 m

(B) at half the amplitude potential energy stored in the spring is 2 J

(C) at half the amplitude kinetic energy is 6 J

(D) angular frequency of oscillation is 16 $rads^{-1}$

8. The potential energy and the total energy line of a particle oscillating by means of a spring are shown in Figure. Then, select the correct statements.



(A) Amplitude of oscillation of particle is 6 cm

(B) Spring constant is 4.44 Nm^{-1}

(C) Kinetic energy of particle is maximum at $x = 20$ cm

(D) Spring constant is 4.44 kNm^{-1}

9. A particle moves along the x-axis according to the equation $x = 4 + 3\sin(2\pi t)$. Here x is in cm and t in second. Select the correct alternative(s).

(A) The motion of the particle is simple harmonic with mean position at $x = 4$ cm.

(B) The motion of the particle is simple harmonic with mean position at $x = -4$ cm.

(C) Amplitude of oscillation is 3 cm.

(D) Amplitude of oscillation is 7 cm.

10. Two particles are in SHM with same amplitude A and same angular frequency ω. At time $t = 0$, one is at $x = +\dfrac{A}{2}$ and the other is at $x = -\dfrac{A}{2}$. Both the particles start moving in the same direction, then

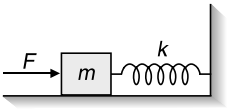
(A) phase difference between the two particles is $\dfrac{\pi}{3}$

(B) phase difference between the two particles is $\dfrac{2\pi}{3}$

(C) the particles will collide after time $t = \dfrac{\pi}{2\omega}$

(D) the particles will collide after time $t = \dfrac{3\pi}{4\omega}$

11. A constant force F is applied on a spring block system as shown in figure. The mass of the block is m and spring constant is k. Then block is placed over a smooth surface. Initially the spring was unstretched. Select the correct alternative(s).



(A) The block will execute SHM

(B) The amplitude of oscillation is $\frac{F}{2k}$

(C) The time period of oscillation is $2\pi\sqrt{\frac{m}{k}}$

(D) The maximum speed of block is $\sqrt{\frac{2Fx-kx^2}{m}}$

12. In a linear SHM, if $\vec{v}$ is velocity, $\vec{r}$ is displacement from mean position and $\vec{F}$ is the restoring force on the body, then select the correct statement(s).

(A) $\vec{v}\cdot\vec{r}$ is always negative

(B) $\vec{F}\cdot\vec{r}$ is always negative

(C) $\vec{v}\times\vec{r}$ is always zero

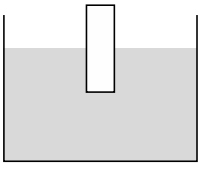
(D) $\vec{F}\times\vec{r}$ is always zero

13. A particle starts SHM at time $t=0$. Its amplitude is A and angular frequency is ω. At time $t=0$ its kinetic energy is $\frac{E}{4}$. Assuming potential energy to be zero at mean position, the displacement-time equation of the particle can be written as

(A) $x=A\cos\left(\omega t-\frac{\pi}{6}\right)$ (B) $x=A\sin\left(\omega t-\frac{2\pi}{3}\right)$

(C) $x=A\sin\left(\omega t+\frac{\pi}{3}\right)$ (D) $x=A\cos\left(\omega t+\frac{\pi}{6}\right)$

14. A plank is floating in a non-viscous liquid as shown in Figure. Select the correct statement(s).



(A) Plank is in stable vertical equilibrium.

(B) For small vertical displacement of the plank, motion is periodic but not simple harmonic.

(C) Even if oscillations are large, motion is simple harmonic till the plank gets fully immersed.

(D) For small oscillations of plank in vertical direction, motion is simple harmonic.

15. The density of a liquid ρ, varies with the depth h as $\rho=\alpha h$, where α is a positive constant. A small ball of density ρ_0 is released from the free surface of the liquid. Then the ball will

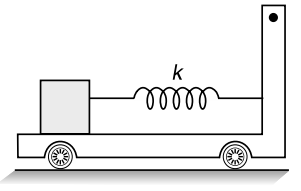
(A) execute SHM of amplitude $\frac{\rho_0}{\alpha}$.

(B) have its mean position at a depth $\frac{\rho_0}{2\alpha}$ below the free surface.

(C) sink to a maximum depth of $\frac{2\rho_0}{\alpha}$.

(D) all of the above are true.

16. A block of mass m is attached to a massless spring of force constant k, the other end of which is fixed from the wall of a truck as shown in figure. The block is placed over a smooth surface and initially the spring is unstretched. Suddenly the truck starts moving towards right with a constant acceleration a_0. As seen from the truck



(A) the particle will execute SHM

(B) the time period of oscillations will be $2\pi\sqrt{\frac{m}{k}}$

(C) the amplitude of oscillations will be $\frac{ma_0}{k}$

(D) the energy of oscillations will be $\frac{m^2a_0^2}{k}$

17. A rectangular block of mass m and area of cross-section A floats in a liquid of density ρ. If it is given a small vertical displacement from equilibrium, it undergoes oscillation with a time period T, then select the correct alternative(s)

(A) $T^2\propto m$ (B) $T^2\propto g$

(C) $T^2\propto\frac{1}{A}$ (D) $T^2\propto\frac{1}{\rho}$

18. A simple pendulum of length 1 m with a bob of mass m swings with an angular amplitude 30°. If $g=9.8\ \text{ms}^{-2}$, then

(A) time period of pendulum is 2 s

(B) tension in the string is greater than $mg\cos(15°)$ at angular displacement 15°

(C) rate of change of speed at angular displacement 15° is $g\sin(15°)$

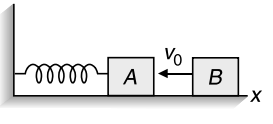
(D) tension in the string is $mg\cos(15°)$ at angular displacement 15°

19. Let E, K and U represent total energy, average kinetic energy over one period and average potential energy over one period respectively in case of a particle executing SHM. Then select the correct relation(s).

(A) $E=K=U$ (B) $K=U$

(C) $E=2K$ (D) $E=0.5U$

20. A block A of mass m connected with a spring of force constant k is executing SHM. The displacement-time equation of the block is $x=x_0+a\sin\omega t$. An identical block B moving towards negative x-axis with velocity v_0 collides elastically with block A at time $t=0$. Then



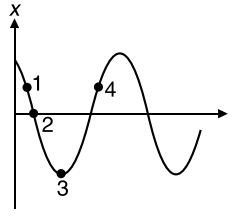
(A) displacement time equation of A after collision will be $x = x_0 - v_0\sqrt{\frac{m}{k}}\sin(\omega t)$

(B) displacement time equation of A after collision will be $x = x_0 + v_0\sqrt{\frac{m}{k}}\sin(\omega t)$

(C) velocity of B just after collision will be a ω towards positive x-direction

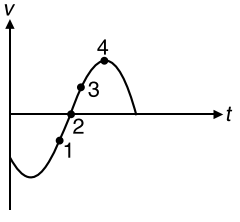
(D) velocity of B just after collision will be v_0 towards positive x-direction

21. The acceleration-time graph of a particle executing SHM is as shown in figure. Select the correct alternative(s).



(A) Displacement of particle at 1 is negative.
(B) Velocity of particle at 2 is positive.
(C) Potential energy of particle at 3 is maximum.
(D) Speed of particle at 4 is decreasing.

22. The velocity-time graph of a particle executing SHM is shown in figure. Select the correct alternative(s).



(A) At position 1, displacement of particle may be positive or negative.
(B) At position 2, displacement of particle is negative.
(C) At position 3, acceleration of particle is positive.
(D) At position 4, acceleration of particle is positive.

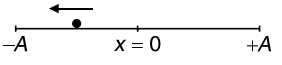
23. For a particle executing simple harmonic motion, the
(A) potential energy and kinetic energy may not be equal in mean position
(B) potential energy and kinetic energy may be equal in extreme position
(C) potential energy may be zero at extreme position
(D) kinetic energy plus potential energy oscillates simple harmonically

24. A person normally weighing 60 kg stands on a platform which oscillates up and down simple harmonically with a frequency 2 Hz and an amplitude 5 cm. If a machine on the platform gives the person's weight, then the $\left(g = 10\ \text{ms}^{-2}, \pi^2 = 10\right)$
(A) maximum reading of the machine is 108 kg
(B) maximum reading of the machine is 90 kg
(C) minimum reading of the machine is 12 kg
(D) minimum reading of the machine is zero

25. Potential energy (in joule) of a particle of mass 0.1 kg, moving along the x-axis, is given by $U = 5x(x-4)$ J, where x is in metre, then the
(A) particle is acted upon by a constant force
(B) speed of the particle is maximum at $x = 2$ m
(C) particle executes simple harmonic motion
(D) particle is acted upon by a force that varies linearly with x

26. The speed v of a particle moving along a straight line, when it is at a distance x from a fixed point on the line is given by $v^2 = 144 - 9x^2$. Select the correct alternative(s).
(A) The motion of the particle is SHM with time period $T = \frac{2\pi}{3}$ unit.
(B) The maximum displacement of the particle from the fixed point is 4 unit.
(C) The magnitude of acceleration at a distance 3 unit from the fixed points is 27 unit.
(D) The motion of the particle is periodic but not simple harmonic.

27. Two particles undergo SHM along the same line with the same time period (T) and equal amplitudes (A). At a particular instant on particle is at $x = -A$ and the other is at $x = 0$. They move in the same direction. They will cross each other at



(A) $t = \frac{4T}{3}$ (B) $t = \frac{3T}{8}$

(C) $x = \frac{A}{2}$ (D) $x = \frac{A}{\sqrt{2}}$

28. Two small particles P and Q each of mass m are fixed along x-axis at points $(a,\ 0)$ and $(-a,\ 0)$. A third particle R is kept at origin. Then
(A) if particle R is displaced along x-axis it will start oscillating
(B) oscillations of R along x-axis are not simple harmonic in nature
(C) if R is displaced along y-axis are not simple harmonic in nature
(D) oscillations along y-axis are not simple harmonic in nature

29. A body of mass m is suspended from two light springs of force constants k_1 and k_2 separately. The period of vertical oscillations are T_1 and T_2 respectively. Now the same body is suspended from the same two springs which are first connected in series and then in parallel. The period of vertical oscillations are T_s and T_p respectively

(A) $T_p < T_1 < T_2 < T_s$ and $k_1 > k_2$

(B) $\frac{1}{T_p^2} = \frac{1}{T_1^2} + \frac{1}{T_2^2}$

(C) $T_s^2 = T_1^2 + T_2^2$

(D) $\sqrt{T_s} = \sqrt{T_1} + \sqrt{T_2}$

30. A particle of mass m is moving in a potential well, for which the potential energy is given by $U(x) = U_0(1 - \cos ax)$ where U_0 and a are constants. Then (for the small oscillations)
    (A) the time period of small oscillations is $T = 2\pi\sqrt{\frac{m}{aU_0}}$
    (B) the speed of the particle is maximum at $x = 0$
    (C) the amplitude of oscillations is $\frac{\pi}{2a}$
    (D) the time period of small oscillations is $T = 2\pi\sqrt{\frac{m}{a^2U_0}}$

31. In simple harmonic motion of a particle maximum kinetic energy is 40 J and maximum potential energy is 60 J. Then
    (A) minimum potential energy will be 20 J
    (B) potential energy at half the displacement will be 30 J
    (C) kinetic energy at half the displacement is 40 J
    (D) potential energy or kinetic energy at some intermediate position cannot be found from the given data

REASONING BASED QUESTIONS

This section contains Reasoning type questions, each having four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Each question contains STATEMENT 1 and STATEMENT 2. You have to mark your answer as

Bubble (A) If both statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1.
Bubble (B) If both statements are TRUE but STATEMENT 2 is not the correct explanation of STATEMENT 1.
Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE.
Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 is TRUE.

1. **Statement-1:** The simple harmonic motion is to and fro and periodic.
   Statement-2: The motion of the earth is periodic.

2. **Statement-1:** The bob of a simple pendulum is a hollow ball full of water. If a fine hole is made at the bottom of the ball, then the time period will vary till the ball gets empty and then becomes constant.
   Statement-2: The time period of simple pendulum does not depend upon mass.

3. **Statement-1:** The periodic time of hard spring is less as compared to that of a soft spring.
   Statement-2: The periodic time is inversely proportional to the square root of the spring constant.

4. **Statement-1:** In a simple harmonic motion the kinetic and potential energies become equal when the displacement is $\frac{1}{\sqrt{2}}$ times the amplitude.
   Statement-2: In SHM kinetic energy is zero when potential energy is maximum.

5. **Statement-1:** In simple harmonic motion the velocity is maximum when the acceleration is maximum.
   Statement-2: Displacement and velocity of SHM differ in phase by $\frac{\pi}{2}$.

6. **Statement-1:** The time period of a pendulum, on a satellite orbiting the earth is infinity.
   Statement-2: Time period of a pendulum is inversely proportional to square root of acceleration due to gravity.

7. **Statement-1:** In SHM, acceleration is always directed towards the mean position.
   Statement-2: The body stops momentarily at the extreme positions and then moves back to mean position.

8. **Statement-1:** The length of a simple pendulum is increased by 4%, the corresponding decrease in time period will be 2%.
   Statement-2: $T \propto \sqrt{l}$

9. **Statement-1:** The time period of a simple pendulum of infinite length is infinite.
   Statement-2: The time period of a simple pendulum is directly proportional to the square root of length.

10. **Statement-1:** Simple harmonic motion is not a uniform motion.
    Statement-2: It is the projection of uniform circular motion along two mutually perpendicular diameters.

11. **Statement-1:** The graph of PE and KE of a particle in SHM with respect to position is a parabola.
    Statement-2: This is because PE and KE do not vary linearly with position.

12. **Statement-1:** Water in a *U*-tube executes SHM, the time period for mercury filled up to the same height in the *U*-tube be greater than that in case of water.
    Statement-2: The amplitude of an oscillating pendulum goes on decreasing.

13. **Statement-1:** Damped vibrations indicate loss of energy.
    Statement-2: The loss may be due to friction, air resistance.

14. **Statement-1:** If the earth suddenly contracts, then duration of day will decrease.
    Statement-2: The angular velocity of the earth's rotation will decrease.

15. **Statement-1:** A hole were drilled through the centre of earth and a ball is dropped into the hole at the end, it will not get out of other end of the hole.
    Statement-2: It will execute SHM and will be seen at the other end after 43.2 minutes from the time it is dropped.

16. **Statement-1:** The graph between velocity and displacement for a harmonic oscillator is a parabola.
Statement-2: Velocity does not change uniformly with displacement in simple harmonic motion.

17. **Statement-1:** If the amplitude of a simple harmonic oscillator is doubled, its total energy also becomes doubled.
Statement-2: The total energy is directly proportional to the square amplitude of vibration of the harmonic oscillator.

18. **Statement-1:** The height of a liquid column in a U-tube is 0.3 m. If the liquid in one of the limbs is depressed and then released the time period of a liquid column will be 1.1 s.
Statement-2: This follows from, the relation $t = 2\pi\sqrt{\frac{L}{g}}$.

19. **Statement-1:** For a simple pendulum the graph between g and T^2 is a rectangular hyperbola.
Statement-2: $T = 2\pi\sqrt{\frac{g}{l}}$

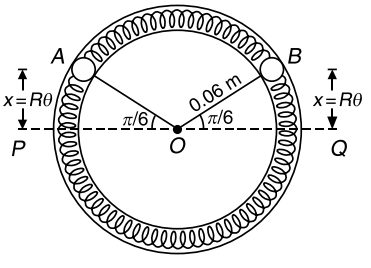
20. **Statement-1:** A man with a watch (spring wound) on his hand falls from the top of a tower. The watch will show the correct time.
Statement-2: The acceleration due to gravity will have no effect on time period of watch at the time of falling.

LINKED COMPREHENSION TYPE QUESTIONS

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct. (For the sake of competitiveness there may be a few questions that may have more than one correct options)

Comprehension 1

Two identical balls A and B, each of mass 0.1 kg, are attached to two identical massless springs. The springs-mass system is constrained to move inside a rigid smooth pipe bent in the form of a circle as shown in figure. The pipe is fixed in a horizontal plane. The centres of the balls can move in a circle of radius 0.06 m. Each spring has a natural length of 0.06π metre and spring constant 0.1 Nm^{-1}. Initially, both the balls are displaced by an angle $\theta = \frac{\pi}{6}$ radian with respect to the diameter PQ of the circle (as shown in figure) and released from rest.



Based on the above facts, answer the following questions.

1. The frequency of oscillation of ball B is
(A) $\frac{3}{\pi}$ Hz (B) $\frac{2}{\pi}$ Hz
(C) $\frac{1}{\pi}$ Hz (D) None of these is correct

2. Assume the balls A and B to be at the two ends of the diameter. The speed of the ball A at this instant is
(A) 0.0268 ms^{-1} (B) 0.063 ms^{-1}
(C) 0.63 ms^{-1} (D) 6.3 ms^{-1}

3. Total energy of the system is denoted by E. Then
(A) $E = 19\times10^{-4}$ J (B) $E = 29\times10^{-4}$ J
(C) $E = 39\times10^{-4}$ J (D) $E = 49\times10^{-4}$ J

Comprehension 2

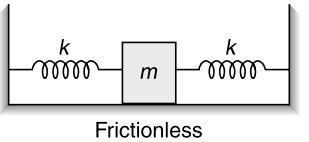
A force acting on a block is $F = (-4x + 8)$, where F is in Newton and x the position of block on x-axis in metres. The energy of oscillation is 18 J. Based on the information given, answer the following questions.

4. The motion of the block is
(A) periodic but not simple harmonic.
(B) not periodic.
(C) simple harmonic about origin, $x = 0$.
(D) simple harmonic about $x = 2$ m.

5. The block will oscillate between the points
(A) $x = 0$ and $x = 4$ m
(B) $x = -1$ m and $x = 5$ m
(C) $x = -2$ m and $x = 6$ m
(D) $x = 1$ m and $x = 3$ m

Comprehension 3

A block of mass m is attached to two springs, each of force constant k as shown. In the equilibrium position, the springs are in their natural length. The mass oscillates along the line of springs with amplitude A_0. At $t = 0$ the mass is at $+\frac{A_0}{2}$ from the equilibrium position and is moving to the right. At this very instant the right spring is removed without changing the velocity of the block. Based on the information given, answer the following questions.



6. The new time period of oscillation in terms of its original period T_0 is
(A) $2T_0$ (B) $\sqrt{2}T_0$
(C) $\frac{T_0}{2}$ (D) $\frac{T_0}{\sqrt{2}}$

7. The new amplitude A, of mass is

(A) A_0 (B) $\frac{\sqrt{3}A_0}{2}$

(C) $\frac{\sqrt{7}A_0}{2}$ (D) $\frac{3A_0}{2}$

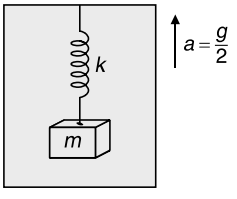
8. The velocity of the mass, when it passes through the equilibrium position is

(A) $\frac{\sqrt{19}\pi A_0}{T_0}$ (B) $\frac{\sqrt{2}\pi A_0}{T_0}$

(C) $\frac{\sqrt{14}\pi A_0}{T_0}$ (D) $\frac{\sqrt{14}\pi A_0}{2T_0}$

Comprehension 4

Consider a spring attached to the roof of the lift compartment. To the other end of the spring a block of mass m is attached. The lift is moving upwards with an acceleration $a = \frac{g}{2}$. Based on the information given, answer the following questions.



9. In mean position of the block's oscillations, the spring is

(A) compressed by $\frac{mg}{2k}$ (B) elongated by $\frac{mg}{k}$

(C) elongated by $\frac{mg}{2k}$ (D) elongated by $\frac{3mg}{2k}$

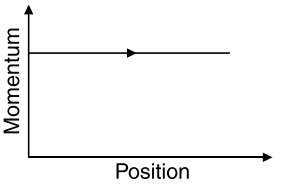
10. If maximum extension in the spring is $\frac{5mg}{2k}$, then maximum upward acceleration (a_1) and maximum downward acceleration (a_2) of the block are

(A) $a_1 = \frac{g}{2}$ (B) $a_1 = \frac{3g}{2}$

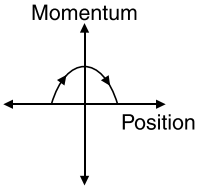
(C) $a_2 = \frac{g}{2}$ (D) $a_2 = \frac{3g}{2}$

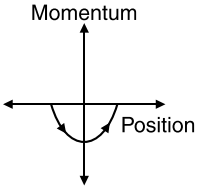
Comprehension 5

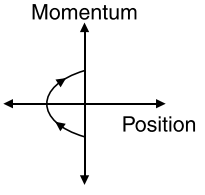
Phase space diagrams are useful tools in analysing all kinds of dynamical problems. They are especially useful in studying the changes in motion as initial position and momentum are changed. Here we consider some simple dynamical systems in one-dimension. For such systems, phase space is a plane in which position is plotted along horizontal axis and momentum is plotted along vertical axis. The phase space diagram is $x(t)$ vs $p(t)$ curve in this plane. The arrow on the curve indicates the time flow. For example, the phase space diagram for a particle moving with constant velocity is a straight line as shown in the figure. We use the sign convention in which position or momentum upwards (or to right) is positive and downwards (or to left) is negative. Based on the above facts, answer the following questions.

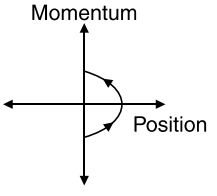


11. The phase space diagram for a ball thrown vertically up from ground is

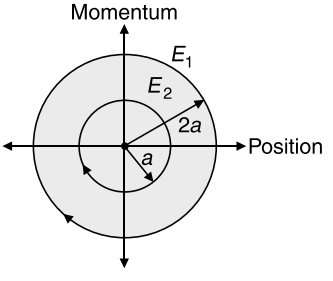
(A) 

(B) 

(C) 

(D) 

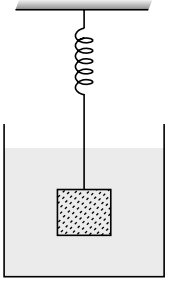
12. The phase space diagram for simple harmonic motion is a circle centred at the origin. In the figure, the two circles represent the same oscillator but for different initial conditions and E_1 and E_2 are the total mechanical energies respectively. Then

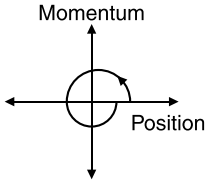


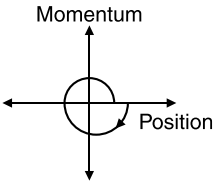
(A) $E_1 = \sqrt{2}E_2$ (B) $E_1 = 2E_2$

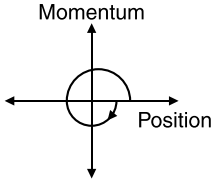
(C) $E_1 = 4E_2$ (D) $E_1 = 16E_2$

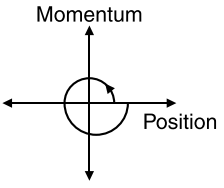
13. Consider the spring-mass system, with the mass submerged in water, as shown in the figure. The phase space diagram for one cycle of this system is



(A) 

(B) 

(C) 

(D) 

Comprehension 6

A point performs SHM along a straight line with a period $T = 0.60$ s and amplitude $a = 10$ cm. Based on the above facts, answer the following questions.

14. Starting from extreme position, in what time, the point travels a distance $\frac{a}{2}$?

(A) 10 s (B) 1.0 s
(C) 0.1 s (D) 0.01 s

15. In PROBLEM 14, the mean velocity of the point is

(A) 0.5 ms^{-1} (B) 1 ms^{-1}
(C) 1.5 ms^{-1} (D) 1.005 ms^{-1}

16. Starting from the stable equilibrium position, the point travels a distance $\frac{a}{2}$ in a time of

(A) 0.01 s (B) 0.03 s
(C) 0.04 s (D) 0.05 s

17. In PROBLEM 16, the mean velocity of the point is

(A) 0.5 ms^{-1} (B) 1 ms^{-1}
(C) 1.5 ms^{-1} (D) 1.005 ms^{-1}

Comprehension 7

A particle of mass 0.5 kg executes simple harmonic motion such that the force acting on the particle is given by $F = -4x$. The total mechanical energy of the particle is 10 J and amplitude of oscillations is 2 m. The initial acceleration of the particle is -16 ms^{-2}. Based on the information given, answer the following questions.

18. The kinetic energy of the particle at the mean position is

(A) 10 J (B) 8 J
(C) 6 J (D) 2 J

19. The displacement-time equation of the particle is

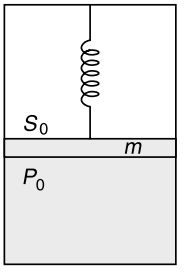
(A) $x = 2\sin(2t)$ (B) $x = 2\sin(4t)$
(C) $x = 2\cos(2t)$ (D) None of these

20. At $x = +1$ m, potential energy and kinetic energy of the particle are

(A) 2 J and 8 J
(B) 8 J and 2 J
(C) 6 J and 4 J
(D) 4 J and 6 J

Comprehension 8

A non-conducting piston of mass m and area S_0 divides a non-conducting, closed cylinder as shown in figure. Piston having mass m is connected with top wall of cylinder by a spring of force constant k. The upper part of the cylinder is evacuated and the lower part contains an ideal gas at pressure P_0 in equilibrium position. Assuming the adiabatic constant for the gas to be γ, the equilibrium length of each part to be l and neglecting friction. Based on the information given, answer the following questions.



21. Assuming $P_0S_0 > mg$, the compression in the spring at equilibrium position is

(A) ZERO (B) $\frac{2P_0S_0 - mg}{k}$
(C) $\frac{P_0S_0 - mg}{k}$ (D) $\frac{P_0S_0 - mg}{2k}$

22. Find angular frequency for small oscillation

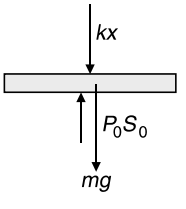
(A) $\sqrt{\frac{kl + \gamma P_0S_0}{2ml}}$ (B) $\sqrt{\frac{2kl + \gamma P_0S_0}{ml}}$
(C) $\sqrt{\frac{k}{m}}$ (D) $\sqrt{\frac{kl + \gamma P_0S_0}{ml}}$

23. If spring is removed and the upper part of the cylinder is opened, then the angular frequency for small oscillation (Assuming pressure of gas at equilibrium position is P_1 and length of gas column is ℓ_1) is

(A) $\sqrt{\frac{\gamma P_1S_0}{m\ell_1}}$ (B) $\sqrt{\frac{2\gamma P_1S_0}{m\ell_1}}$
(C) $\sqrt{\frac{\gamma P_1S_0}{4m\ell_1}}$ (D) $\sqrt{\frac{\gamma P_1S_0}{2m\ell_1}}$

Comprehension 9

A uniform thin cylindrical disk of mass M and radius R is attached to two identical massless springs of spring constant k which are fixed to the wall as shown in the figure.



The springs are attached to the axle of the disk symmetrically on either side at a distance d from its centre. The axle is massless and both the springs and the axle are in a horizontal plane. The unstretched length of each spring is L. The disk is initially at its

equilibrium position with its centre of mass (CM) at a distance L from the wall. The disk rolls without slipping with velocity $\vec{v}_0 = v_0\hat{i}$. The coefficient of friction is μ. Based on the above facts, answer the following questions.

24. The net external force acting on the disk when its centre of mass is at displacement x with respect to its equilibrium position is

(A) $-kx$ (B) $-2kx$

(C) $-\frac{2kx}{3}$ (D) $-\frac{4kx}{3}$

25. The centre of mass of the disk undergoes simple harmonic motion with angular frequency ω equal to

(A) $\sqrt{\frac{k}{M}}$ (B) $\sqrt{\frac{2k}{M}}$

(C) $\sqrt{\frac{2k}{3M}}$ (D) $\sqrt{\frac{4k}{3M}}$

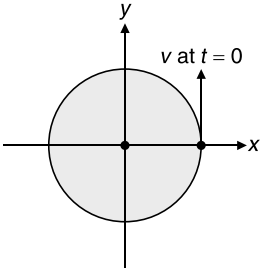
26. The maximum value of v_0 for which the disk will roll without slipping is

(A) $\mu g\sqrt{\frac{M}{k}}$ (B) $\mu g\sqrt{\frac{M}{2k}}$

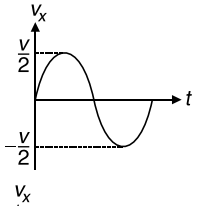
(C) $\mu g\sqrt{\frac{3M}{k}}$ (D) $\mu g\sqrt{\frac{5M}{2k}}$

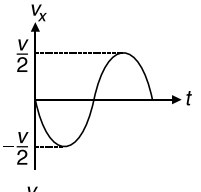
Comprehension 10

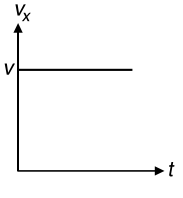
An object is fixed to the edge of a disk that is rotating with uniform circular motion. At time $t = 0$ the position and the velocity of the object are shown in the figure. The object travels around with the disk for a full rotation. Based on the information given, answer the following questions.

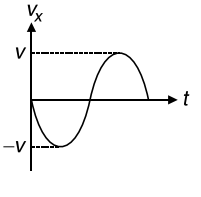


27. The x-component of the velocity, when plotted against time is represented by

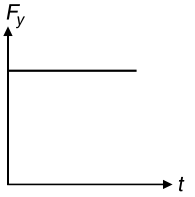
(A) 

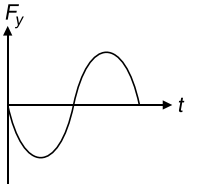
(B) 

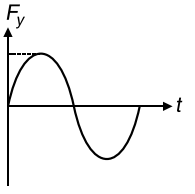
(C) 

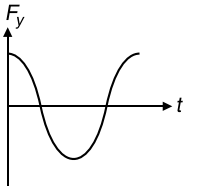
(D) 

28. The y-component of the force (plotted against time) that keeps the object moving in a circle is represented by

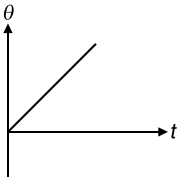
(A) 

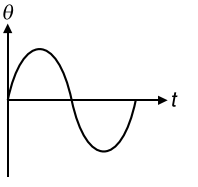
(B) 

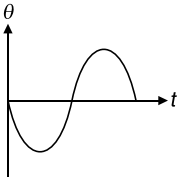
(C) 

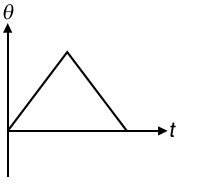
(D) 

29. The graph of the angle of the object's position (that it makes with the x-axis) plotted against the time t is best represented by

(A) 

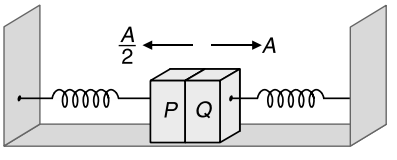
(B) 

(C) 

(D) 

Comprehension 11

Two identical blocks P and Q each has a mass m. They are attached to identical springs which are initially unstretched. Now the left spring (along with P) is compressed by A. Both the blocks are released simultaneously. They collide perfectly inelastically. Initially time period of each block was T.



Based on the information given, answer the following questions.

30. The time period of oscillation of combined mass is

(A) $\frac{T}{\sqrt{2}}$ (B) $\sqrt{2}T$

(C) T (D) $\frac{T}{2}$

31. The amplitude of combined mass is

(A) $\frac{A}{4}$ (B) $\frac{A}{2}$

(C) $\frac{2A}{3}$ (D) $\frac{3A}{4}$

32. The energy of oscillation of the combined mass is

(A) $\frac{1}{2}kA^2$ (B) $\frac{1}{4}kA^2$

(C) $\frac{1}{8}kA^2$ (D) $\frac{1}{16}kA^2$

Comprehension 12

A point performs SHM along a straight line with a period $T = 0.60$ s and amplitude $a = 10$ cm. Based on the information given, answer the following questions.

33. Starting from extreme position, the time taken by the point to travel a distance $\frac{a}{2}$ is

(A) 10 s (B) 1 s

(C) 0.1 s (D) 0.01 s

34. The average velocity of the point is

(A) 0.5 ms^{-1} (B) 1 ms^{-1}

(C) 1.5 ms^{-1} (D) 1.005 ms^{-1}

Comprehension 13

You know very well that a block attached to an elastic spring performs SHM. Consider such a spring block system lying on a smooth horizontal table with a block of mass $m = 2$ kg attached at one end of a spring $(k = 200\ Nm^{-1})$ whose other end is fixed. The block is pulled so that the spring is extended by 0.05 m. If at this moment $(t = 0)$, the block is projected with a speed of 1 ms^{-1} in the direction of increasing extension of the spring. Based on the above facts, answer the following questions.

35. The angular frequency (ω) of motion is

(A) 100 sec^{-1} (B) 10 sec^{-1}

(C) 20 sec^{-1} (D) 0

36. If the displacement (x) of the block is measured from the equilibrium position, it can be written as a function of time as $x = A\sin(\omega t + \delta)$. The constants A and δ have values

(A) 0.112 m, $\cos^{-1}(0.446)$

(B) 0.0025 m, $\sin^{-1}(0.446)$

(C) 0.112 m, $\sin^{-1}(0.446)$

(D) 0.1 m, $\sin^{-1}(0.446)$

37. First time (t) when spring attains maximum extension is $\left(\text{given, } \sin^{-1}(0.446) = 26.5^\circ\right)$

(A) 0.11 s (B) 0.22 s

(C) 0.33 s (D) 0.44 s

Comprehension 14

A particle of mass m is attached between two identical springs on a frictionless rectangular horizontal table of sides l and b. Each spring has a spring constant k and is initially unstretched as shown in Figure (A). The particle is pulled a distance y in a direction perpendicular to the initial length of the springs along as shown in Figure (B). Assume the initial lengths of the springs to be along X-axis such that the displacement of the particle is along Y-axis. Based on the information given, answer the following questions.

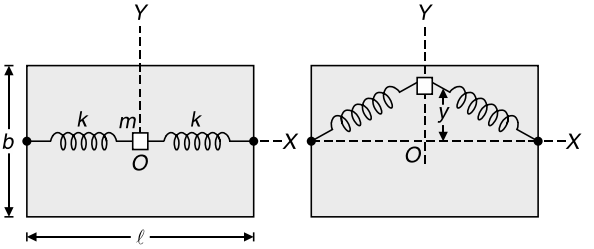


Figure (A) Figure (B)

38. Force exerted by the springs on the particle when it is at distance y from its initial position O is

(A) ZERO (B) $(2ky)\hat{j}$

(C) $2ky\left(\frac{\frac{l}{2}}{\sqrt{\frac{l^2}{4}+y^2}}-1\right)\hat{j}$ (D) $-2ky\left(\frac{\frac{l}{2}}{\sqrt{\frac{l^2}{4}+y^2}}+1\right)\hat{j}$

39. Assuming position O of the particle to be the position of zero potential energy, the potential energy of the system at a displacement y from O, as shown in Figure (B) is

(A) ZERO

(B) $k\left(y^2 + l\left(\frac{l}{2} - \sqrt{\frac{l^2}{4}+y^2}\right)\right)$

(C) $-k\left(y^2 + l\left(\frac{l}{2} + \sqrt{\frac{l^2}{4}+y^2}\right)\right)$

(D) $\frac{1}{2}ky^2$

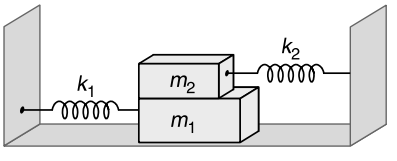
40. If $k = 100\ Nm^{-1}$, $l = 10$ m, $m = 2$ kg and the particle is released from a displacement $y = \sqrt{11}$ m from O, then the speed of the particle as it reaches the point O is

(A) 4.5 ms^{-1} (B) 12 ms^{-1}

(C) 10 ms^{-1} (D) 3.2 ms^{-1}

Comprehension 15

In the arrangement shown, both the springs are relaxed. The coefficient of friction between m_2 and m_1 is μ. There is no friction between m_1 and surface. When the blocks are displaced slightly from their mean position, they perform SHM together. Based on the information given, answer the following questions.



41. The frequency of oscillation of the system is

(A) $2\pi\sqrt{\frac{m_1+m_2}{k_1+k_2}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m_1+m_2}}$

(C) $2\pi\sqrt{\frac{k_1+k_2}{m_1+m_2}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{m_1+m_2}{k_1+k_2}}$

42. If the small displacement of blocks is x then acceleration of m_2 is

(A) $\frac{k_2 x}{m_2}$ (B) $\frac{(k_1+k_2)x}{m_2}$

(C) $\left(\frac{k_1+k_2}{m_1+m_2}\right)x$ (D) None of these

43. The condition in which frictional force on m_2 acts in the direction of its displacement from mean position is

(A) $\frac{m_1}{m_2} > \frac{k_1}{k_2}$ (B) $\frac{m_2}{m_1} > \frac{k_1}{k_2}$

(C) $\frac{m_1}{m_2} = \frac{k_1}{k_2}$ (D) None of these

44. If frictional force on m_2 acts in the direction of its displacement from mean position, then the maximum amplitude of oscillation is

(A) $\frac{\mu m_2 g(m_1+m_2)}{m_1k_2 - m_2k_1}$ (B) $\frac{\mu m_2 g(m_1+m_2)}{m_1k_1 - m_2k_2}$

(C) $\frac{\mu m_2 g(m_1+m_2)}{m_1k_2 + m_2k_1}$ (D) None of these

Comprehension 16

The displacement-time equation for two particles moving along x-axis are given by

$$x_1 = (8 + 3\sin\omega t)\,m \text{ and } x_2 = (4\cos\omega t)\,m$$

where $\omega = \pi$ rads^{-1}. Based on the information given, answer the following questions.

45. The two particles will collide after time

(A) $t = 1$ s (B) $t = 2$ s

(C) $t = 4$ s (D) $t \to \infty$

46. The two particles are at minimum distance after time

(A) $t = 1$ s (B) $t = 2$ s

(C) $t = 3$ s (D) $t = 4$ s

Comprehension 17

There are three type of equilibriums

(a) unstable equilibrium when potential energy is maximum
(b) stable equilibrium when potential energy is minimum
(c) Neutral equilibrium when potential energy is constant

Whenever the particle is displaced from its position of stable equilibrium it performs an oscillatory motion. A single conservative force $F(x)$ acts on a 1 kg particle that moves along the x-axis. The potential energy $U(x)$ is given by:

$$U(x) = 20 + (x-2)^2$$

where x is in meter. At $x = 5$ m the particle has a kinetic energy of 20 J. Based on the above facts, answer the following questions.

47. The maximum kinetic energy of a particle is

(A) 49 J (B) 20 J

(C) 29 J (D) 19 J

48. The position of equilibrium is at $x =$

(A) 2 m (B) 1 m

(C) 0.5 m (D) ZERO

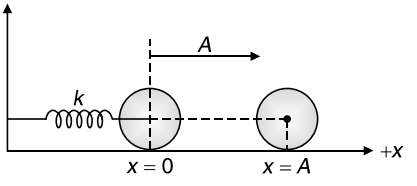
49. The angular frequency of the particle is

(A) $\sqrt{2}$ (B) 2

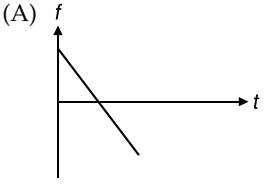
(C) $\sqrt{3}$ (D) $\sqrt{2}-2$

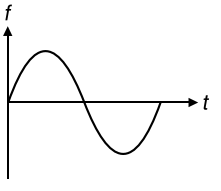
Comprehension 18

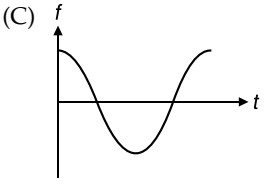
A disc of mass m and radius R is attached with a spring of force constant k at its centre as shown in figure. At $x = 0$, the spring is unstretched. The disc is moved to $x = A$ and then released. There is no slipping between disc and ground. Let f be the force of friction on the disc from the ground. Based on the information given, answer the following questions.

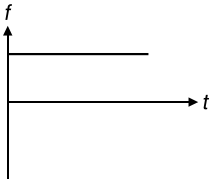


50. The f versus t (time) graph will be as

(A) 

(B) 

(C) 

(D) 

51. If $k = 10$ Nm^{-1}, $m = 2$ kg, $R = 1$ m and $A = 2$ m, then the linear speed of the disc at mean position is

(A) $\sqrt{\frac{40}{3}}$ ms^{-1} (B) $\sqrt{20}$ ms^{-1}

(C) $\sqrt{\frac{10}{3}}$ ms^{-1} (D) $\sqrt{\frac{50}{3}}$ ms^{-1}

Comprehension 19

A point moves along the x-axis according to the law:

$$x = a\sin^2\left(\omega t - \frac{\pi}{4}\right)$$

Based on the above facts, answer the following questions.

52. The amplitude of the particle is

(A) a (B) $\frac{a}{2}$

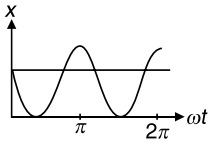
(C) $\frac{a}{4}$ (D) $\frac{a}{5}$

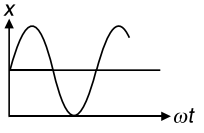
53. The time period of oscillations is

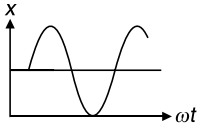
(A) $\frac{\pi}{\omega}$ (B) $\frac{2\pi}{\omega}$

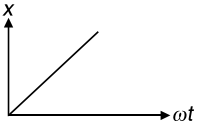
(C) $\frac{3\pi}{\omega}$ (D) $\frac{4\pi}{\omega}$

54. The plot of x as a function of ωt is

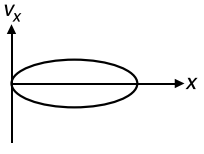
(A) 

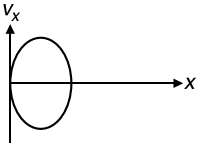
(B) 

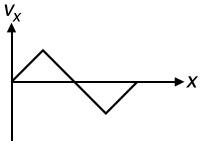
(C) 

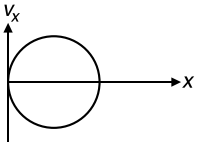
(D) 

55. The plot of velocity projection v_x as a function of x is

(A) 

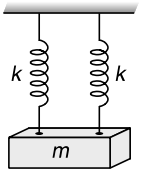
(B) 

(C) 

(D) 

Comprehension 20

A block of mass $m = \frac{1}{2}$ kg is attached with two springs each of force constant $k = 10\ \text{Nm}^{-1}$ as shown. The block is oscillation vertically up and down with amplitude $A = 25$ cm. When the block is at its mean position one of the spring breaks without changing momentum of block. Based on the information given, answer the following questions.



56. The new amplitude of oscillation is

(A) 10.6 cm
(B) 25 cm
(C) 43.3 cm
(D) 62.2 cm

57. If instead of breaking one spring, the $\frac{m}{2}$ mass would have fallen from the block, then the new amplitude is

(A) 15.8 cm
(B) 16.9 cm
(C) 21.6 cm
(D) 26.2 cm

MATRIX MATCH/COLUMN MATCH TYPE QUESTIONS

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

If the correct matches are A → p, s and t; B → q and r; C → p and q; and D → s and t; then the correct darkening of bubbles will look like the following:

	p	q	r	s	t
A	Ⓟ	Ⓠ	ⓡ	Ⓢ	Ⓣ
B	Ⓟ	Ⓠ	Ⓡ	Ⓢ	Ⓣ
C	Ⓟ	Ⓠ	ⓡ	Ⓢ	Ⓣ
D	Ⓟ	Ⓠ	ⓡ	Ⓢ	Ⓣ

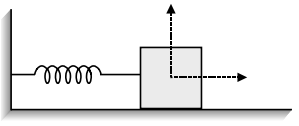
1. In $y = A\sin(\omega t) + A\sin\left(\omega t + \frac{2\pi}{3}\right)$ match the following table

COLUMN-I	COLUMN-II
(A) Motion	(p) is periodic but not SHM
(B) Amplitude	(q) is SHM
(C) Initial phase	(r) A
(D) Maximum velocity	(s) $\frac{\pi}{3}$
	(t) ωA

2. The displacement vs time function for a particle of 250 g in simple harmonic motion is $x = 1\sin(12\pi t)$. Match the **COLUMN-I** with **COLUMN-II**.

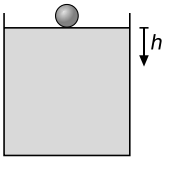
COLUMN-I	COLUMN-II
(A) Frequency with which kinetic energy oscillates is	(p) $\frac{1}{12}$
(B) Speed of particle is maximum at time given by	(q) $18\pi^2$
(C) Maximum potential energy is	(r) 12
(D) Force constant k is	(s) $36\pi^2$

3. A block of mass $m = 2$ kg is connected with spring of spring constant $k = 3200\ \text{Nm}^{-1}$ and placed on a frictionless horizontal surface as shown in figure. Initially it is compressed by a distance 10 cm. It is then released at $t = 0$. Match the times in **COLUMN-I** with their respective values (in second) in **COLUMN-II**.



COLUMN-I	COLUMN-II
(A) Time taken by the mass to move by the first 5 cm is	(p) $\frac{\pi}{160}$
(B) Time taken by the mass to move by the next 5 cm is	(q) $\frac{\pi}{240}$
(C) Time at which kinetic energy and potential energy become equal for the first time is	(r) $\frac{\pi}{120}$
(D) Time at which kinetic energy becomes one-fourth of its maximum value is	(s) $\frac{\pi}{40}$

4. In the arrangement shown, a ball of density ρ is released from the surface of a liquid whose density varies with depth h as $\rho_l = \alpha h$, where α is a positive constant. Assuming the liquid to be ideal, match the contents of **COLUMN-I** with those given in **COLUMN-II**.

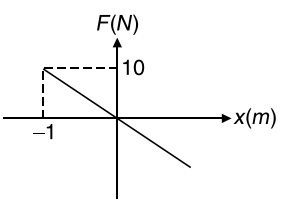


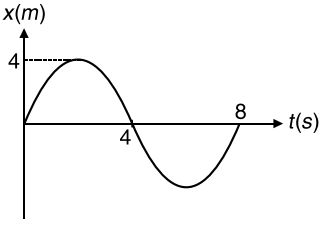
COLUMN-I	COLUMN-II
(A) Upthrust on ball	(p) will continuously decrease
(B) Speed of ball	(q) will continuously increase
(C) Net force on ball	(r) first increase then decrease
(D) Gravitational potential energy of ball	(s) first decrease then increase

5. A particle of mass 2 kg, released from $x = 7$ m is moving on a straight line under the action of force $F = (8 - 2x)$ N. For the subsequent motion match the following (all values in **COLUMN-II** are in S.I. units).

COLUMN-I	COLUMN-II
(A) Amplitude of SHM is	(p) 0.5
(B) Time taken to move from $x = 2.5$ m to $x = 4$ m (approx.)	(q) 3
(C) Total energy of SHM system	(r) 6
(D) Velocity of particle at $x = 4$ m	(s) 9

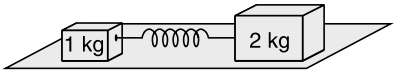
6. F-x and x-t graph of a particle of mass m attached to a spring of force constant k, in SHM are as shown in figure. Match the **COLUMN-I** with **COLUMN-II** (having all quantities in SI units).





COLUMN-I	COLUMN-II
(A) Mass of the particle	(p) $\frac{\pi}{4}$
(B) Maximum kinetic energy of particle	(q) $\left(\frac{160}{\pi^2}\right)$
(C) Angular frequency of particle	(r) 10
(D) Spring constant	(s) 80
	(t) None of these

7. In the two block spring system, force constant of spring is $k = 6\ \text{Nm}^{-1}$. Spring is stretched by 12 cm and then left. Match the following

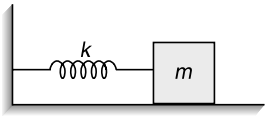


COLUMN-I	COLUMN-II
(A) Angular frequency of oscillation	(p) 14.4×10^{-3}
(B) Maximum kinetic energy of 1 kg	(q) 3
(C) Maximum kinetic energy of 2 kg	(r) $\frac{2}{3}$
(D) Reduced mass of the system	(s) 28.8

8. In SHM match the following

COLUMN-I	COLUMN-II
(A) Displacement and velocity	(p) Phase difference of zero
(B) Displacement and acceleration	(q) Phase difference of $\frac{\pi}{2}$
(C) Velocity and acceleration	(r) Phase difference of π
(D) Momentum and force	(s) None of these

9. For a mass attached to a spring placed on a horizontal frictionless table, match the following



COLUMN-I	COLUMN-II
(A) If mass of the block is doubled	(p) Energy of oscillation becomes 4 times
(B) If amplitude of oscillation is doubled	(q) Speed of particle becomes 2 times
(C) If force constant is doubled	(r) Time period becomes $\sqrt{2}$ times
(D) If angular frequency is doubled	(s) Potential energy becomes 2 times

10. For a particle executing SHM match the **COLUMN-I** with **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Acceleration-displacement graph	(p) Parabola
(B) Velocity-acceleration graph	(q) Straight line
(C) Acceleration-time graph	(r) Circle
(D) Velocity-time graph	(s) None of these

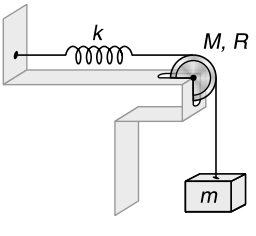
11. In case of second's pendulum, match the following (consider shape of each also)

COLUMN-I	COLUMN-II
(A) At pole	(p) $T > 2$ s
(B) On a satellite	(q) $T < 2$ s
(C) At mountain	(r) $T = 2$ s
(D) At centre of earth	(s) $T = 0$
	(t) $T \to \infty$

INTEGER/NUMERICAL ANSWER TYPE QUESTIONS

In this section, the answer to each question is a numerical value obtained after doing series of calculations based on the data given in the question(s).

1. In the Figure shown, a cord is attached between a 2 kg block and a spring with force constant $k = 20$ Nm^{-1}. The other end of the spring is attached to the wall, and the cord is placed over a pulley $(I = 0.60MR^2)$ of mass 5 kg and radius 0.5 m. Assuming no slipping occurs, what is the frequency of the oscillations when the body is set into motion?

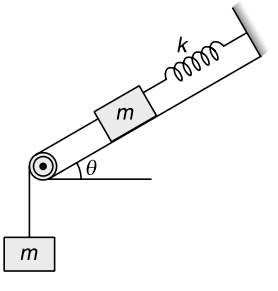


2. A 1.1 kg object is suspended from a vertical spring whose spring constant is 120 Nm^{-1}.

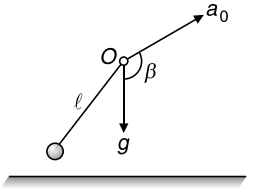
(a) Find the amount, in cm, by which the spring is stretched from its unstrained length.

(b) If the object is pulled straight down by an additional distance of 0.20 m and released from rest, then find the speed, to the nearest integer in ms^{-1}, with which the object passes through its original position on its way up.

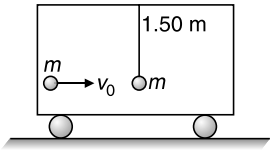
3. Calculate the natural angular frequency ω, in rads^{-1}, of the system shown in Figure. The mass and friction of the pulleys are negligible. Given $m = 2.5$ kg, $k = 20$ Nm^{-1}



4. Find the period of small oscillations of a mathematical pendulum of length l if its points of suspension O moves relative to the Earth's surface in an arbitrary direction with a constant acceleration a_0. The period if $l = 21$ cm, $a_0 = \frac{g}{2}$, and the angle between the vectors a_0 and g equals $\beta = 120°$ is T second. Find $10T$.



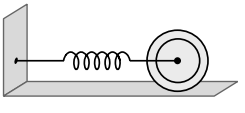
5. A ball of mass $m = 1$ kg is hung vertically by a thread of length $l = 1.50$ m. Upper end of the thread is attached to the ceiling of a trolley of mass $M = 4$ kg. Initially, trolley is stationary and it is free to move along horizontal rails without friction.



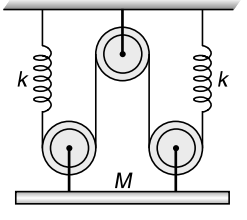
A shell of mass $m = 1$ kg, moving horizontally with velocity $v_0 = 6$ ms^{-1}, collides with the ball and gets stuck with it. As a result, thread starts to deflect towards right. If θ be the maximum deflection with the vertical, $g = 10$ ms^{-2}, then $\cos\theta = \frac{x}{y}$, where x and y are single digit integral values. Find x and y.

6. The period of small oscillations in a vertical plane performed by a ball of mass $m = 40$ g fixed at the middle of a horizontally stretched string $l = 1$ m in length is T second. The tension of the string is assumed to be constant and equal to $F = 10$ N. Find $5T$.

7. A solid cylinder of mass $m = 4$ kg is attached to a horizontal spring with force constant $k = 3$ Nm^{-1}. The cylinder can roll without slipping along the horizontal plane as shown in Figure. The centre of mass of the cylinder executes simple harmonic motion with a period $T = 2\pi\sqrt{*}$, where $*$ is not readable. Find $*$.



8. The natural frequency of the system shown in Figure is $f = \frac{1}{\pi}\sqrt{\frac{xk}{M}}$. The pulleys are smooth and massless. Find x.



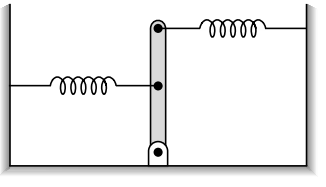
9. A spring of spring constant 1200 Nm^{-1} is hanging from the ceiling of an elevator, and a 5 kg object is attached to the lower end. By how much does the spring stretch (relative to its unstrained length), in cm, when the elevator is accelerating upward at $a = 2.2$ ms^{-2}? Take $g = 9.8$ ms^{-2}.

10. A spring is hanging vertically. An object of unknown mass is hung on the end of the unstretched spring and released from rest and it falls 4.9 cm before first coming to rest. If $g = 9.8$ ms^{-2}, then the period of motion is $\frac{\pi}{*}$, where $*$ is not readable. Find $*$.

11. A point of the surface of a solid sphere of radius 5 cm is attached directly to a pivot on the ceiling. The sphere swings back and forth as a physical pendulum with a small amplitude. What is the length of a simple pendulum, in cm, that has the same period as this physical pendulum?

12. A block with a mass of 3 kg is suspended from an ideal spring having negligible mass and stretches the spring 0.2 m.
(a) What is the force constant, in Nm^{-1}, of the spring.
(b) What is the angular frequency, in rads^{-1}, of oscillation of the block if it is pulled down and released?

13. A rod of length l and mass m, pivoted at one end, is held by a spring at its mid-point a spring at far end, both pulling in opposite directions. The springs have spring constant k, and at equilibrium their pull is perpendicular to the rod. The frequency of small oscillations about the equilibrium position is $f = \frac{1}{2\pi}\sqrt{\frac{*k}{2ml}}$, where $*$ is not readable. Find $*$.

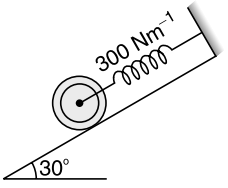


14. A 100 g object executes simple harmonic motion with a frequency of 20 Hz and amplitude of 0.5 cm. If $\pi^2 = 10$, then find the
(a) spring constant k, in Nm^{-1}, for the force acting on it.
(b) maximum acceleration, in ms^{-2}.
(c) total energy of the motion in mJ.

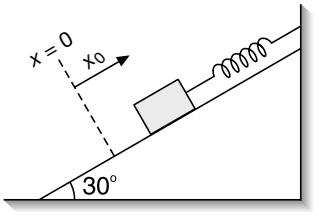
15. Find the angular frequency, in rads^{-1}, of small oscillations of a uniform rod with length $l = 14.7$ cm, pivoted at one end. Given: $g = 9.8$ ms^{-2}.

16. Suppose a block of mass 1 kg is resting on the smooth floor of a truck, attached to its front by a spring of force constant 100 Nm^{-1}. At $t = 0$, the truck begins to move with constant acceleration 2 ms^{-2}. Find the amplitude, in cm, and the angular frequency, in rads^{-1}, of oscillations of the block relative to floor of the truck.

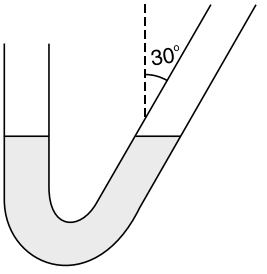
17. Determine the angular frequency of vibration, in rads^{-1}, of the 500 g disk attached to a support with a spring of force constant 300 Nm^{-1}, as shown in Figure. Assume the disk does not slip on the inclined surface.



18. The drawing shows a block $(m = 1.7\ \text{kg})$ and a spring $(k = 310\ \text{Nm}^{-1})$ on a frictionless incline. The spring is compressed by $x_0 = 0.31$ m relative to its unstrained position of $x = 0$ m and then released. What is the speed of the block, in ms^{-1} to the nearest integer, when the spring is still compressed by $x_f = 0.14$ m?



19. The period of oscillations of mercury of mass $m = 200$ g poured into a bent tube, as shown in Figure, whose right arm forms an angle $\theta = 30°$ with the vertical and whose left arm is vertical is T seconds. The cross-sectional area of the tube is $S = 0.50\ \text{cm}^2$. The viscosity of mercury is to be neglected. Find $10T$.



20. A pendulum beats seconds at the surface of the earth. Calculate how much it loses or gains per day, in second to the nearest integer, if it is taken to

(a) a mine 8 km below

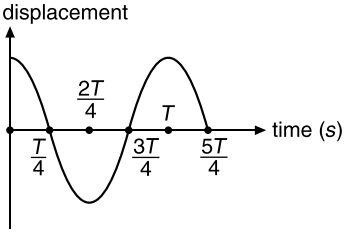
(b) a point 8 km above the surface.

Given: Radius of the earth is $R = 6.4 \times 10^6$ m.

ARCHIVE: JEE MAIN

1. **[Online September 2020]**

The displacement time graph of a particle executing S.H.M. is given in Figure (sketch is schematic and not to scale)



Which of the following statements is/are true for this motion?

(1) The force is zero $t = \frac{3T}{4}$

(2) The acceleration is maximum at $t = T$

(3) The speed is maximum at $t = \frac{T}{4}$

(4) The P.E. is equal to K.E. of the oscillation at $t = \frac{T}{2}$

(A) (1), (2) and (4)
(B) (2), (3) and (4)
(C) (1) and (4)
(D) (1), (2) and (3)

2. **[Online September 2020]**

A ring is hung on a nail. It can oscillate, without slipping or sliding (1) in its plane with a time period T_1 and (2) back and forth in a direction perpendicular to its plane, with a period T_2 the ratio $\frac{T_1}{T_2}$ will be

(A) $\frac{2}{\sqrt{3}}$ (B) $\frac{\sqrt{2}}{3}$

(C) $\frac{2}{3}$ (D) $\frac{3}{\sqrt{2}}$

3. **[Online September 2020]**

An object of mass m is suspended at the end of a massless wire of length L and area of cross-section, A. Young modulus of the material of the wire is Y. If the mass is pulled down slightly its frequency of oscillation along the vertical direction is

(A) $f = \frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$ (B) $f = \frac{1}{2\pi}\sqrt{\frac{YL}{mA}}$

(C) $f = \frac{1}{2\pi}\sqrt{\frac{mA}{YL}}$ (D) $f = \frac{1}{2\pi}\sqrt{\frac{mL}{YA}}$

4. **[Online September 2020]**

When a particle of mass m is attached to a vertical spring of spring constant k and released, its motion is described by $y(t) = y_0 \sin^2 \omega t$, where y is measured from the lower end of unstretched spring. Then ω is

(A) $\sqrt{\frac{g}{y_0}}$ (B) $\sqrt{\frac{g}{2y_0}}$

(C) $\frac{1}{2}\sqrt{\frac{g}{y_0}}$ (D) $\sqrt{\frac{2g}{y_0}}$

5. **[Online April 2019]**

A damped harmonic oscillator has a frequency of 5 oscillations per second. The amplitude drops to half its value for every 10 oscillations. The time it will take to drop to $\frac{1}{1000}$ of the original amplitude is close to

(A) 100 s (B) 10 s
(C) 50 s (D) 20 s

6. [Online April 2019]

A simple pendulum oscillating in air has period T. The bob of the pendulum is completely immersed in a non-viscous liquid. The density of the liquid is $\frac{1}{16}$th of the material of the bob. If the bob is inside liquid all the time, its period of oscillation in this liquid is

(A) $2T\sqrt{\frac{1}{14}}$ (B) $4T\sqrt{\frac{1}{15}}$
(C) $4T\sqrt{\frac{1}{14}}$ (D) $2T\sqrt{\frac{1}{10}}$

7. [Online April 2019]

The displacement of a damped harmonic oscillator is given by $x(t)=e^{-0.1t}\cos(10\pi t+\phi)$. Here t is in seconds. The time taken for its amplitude of vibration to drop to half of its initial value is close to

(A) 7 s (B) 27 s
(C) 13 s (D) 4 s

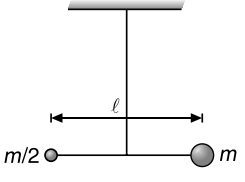
8. [Online April 2019]

A person of mass M is, sitting on a swing of length L and swinging with an angular amplitude θ_0. If the person stands up when the swing passes through its lowest point, the work done by him, assuming that his centre of mass moves by a distance $l(l \ll L)$, is close is

(A) Mgl (B) $Mgl\left(1+\theta_0^2\right)$
(C) $Mgl\left(1+\frac{\theta_0^2}{2}\right)$ (D) $Mgl\left(1-\theta_0^2\right)$

9. [Online January 2019]

Two masses m and $\frac{m}{2}$ are connected at the two ends of a massless rigid rod of length l. The rod is suspended by a thin wire of torsional constant k at the centre of mass of the rod-mass system (see figure). Because of torsional constant k, the restoring torque is $\tau=k\theta$ for angular displacement θ. If the rod is rotated by θ_0 and released, the tension in it when it passes through its mean position will be



(A) $\frac{k\theta_0^2}{2l}$ (B) $\frac{k\theta_0^2}{l}$
(C) $\frac{2k\theta_0^2}{l}$ (D) $\frac{3k\theta_0^2}{l}$

10. [Online January 2019]

A rod of mass M and length $2L$ is suspended at its middle by a wire. It exhibits torsional oscillations. If two masses each of m are attached at distance $\frac{L}{2}$ from its centre on both sides, it reduces the oscillation frequency by 20%. The value of ratio $\frac{m}{M}$ is close to

(A) 0.77 (B) 0.17
(C) 0.37 (D) 0.57

11. [Online January 2019]

A particle is executing simple harmonic motion (SHM) of amplitude A, along the x-axis, about $x=0$. When its potential energy (PE) equals kinetic energy (KE), the position of the particle will be

(A) $\frac{A}{\sqrt{2}}$ (B) A
(C) $\frac{A}{2\sqrt{2}}$ (D) $\frac{A}{2}$

12. [Online January 2019]

A particle executes simple harmonic motion with an amplitude of 5 cm. When the particle is at 4 cm from the mean position, the magnitude of its velocity in SI units is equal to that of its acceleration. Then, its periodic time in seconds is

(A) $\frac{4\pi}{3}$ (B) $\frac{3}{8}\pi$
(C) $\frac{8\pi}{3}$ (D) $\frac{7}{3}\pi$

13. [Online January 2019]

A particle undergoing simple harmonic motion has time dependent displacement given by $x(t)=A\sin\left(\frac{\pi t}{90}\right)$. The ratio of kinetic to potential energy of this particle at $t=210$ s will be

(A) 1 (B) 3
(C) 2 (D) $\frac{1}{9}$

14. [Online January 2019]

A simple pendulum of length 1 m is oscillating with an angular frequency 10 rads^{-1}. The support of the pendulum starts oscillating up and down with a small angular frequency of 1 rads^{-1} and an amplitude of 10^{-2} m. The relative change in the angular frequency of the pendulum is best given by

(A) 10^{-3} rads^{-1} (B) 10^{-1} rads^{-1}
(C) 10^{-5} rads^{-1} (D) 1 rads^{-1}

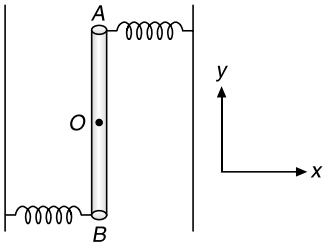
15. [Online January 2019]

The mass and the diameter of a planet are three times the respective values for the Earth. The period of oscillation of a simple pendulum on the Earth is 2 s. The period of oscillation of the same pendulum on the planet would be

(A) $\frac{2}{\sqrt{3}}$ s (B) $\frac{3}{2}$ s
(C) $\frac{\sqrt{3}}{2}$ s (D) $2\sqrt{3}$ s

16. [Online January 2019]

Two light identical springs of spring constant k are attached horizontally at the two ends of a uniform horizontal rod AB of length ℓ and mass m. The rod is pivoted at its centre O and can rotate freely in horizontal plane. The other ends of the two springs are fixed to rigid supports as shown in Figure. The rod is gently pushed through a small angle and released. The frequency of resulting oscillation is



(A) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$

(C) $\frac{1}{2\pi}\sqrt{\frac{3k}{m}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{6k}{m}}$

17. [Online January 2019]

A simple harmonic motion is represented by $y = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$ cm

The amplitude and time period of the motion are

(A) 10 cm, $\frac{2}{3}$ s (B) 5 cm, $\frac{2}{3}$ s

(C) 10 cm, $\frac{3}{2}$ s (D) 5 cm, $\frac{3}{2}$ s

18. [Online January 2019]

A cylindrical plastic bottle of negligible mass is filled with 310 ml of water and left floating in a pond with still water. If pressed downward slightly and released, it starts performing simple harmonic motion at angular frequency ω. If the radius of the bottle is 2.5 cm then ω is close to: (density of water $= 10^3$ kgm^{-3})

(A) 2.50 rads^{-1} (B) 3.75 rads^{-1}

(C) 5.00 rads^{-1} (D) 1.25 rads^{-1}

19. [2018]

A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12} s^{-1}. What is the force constant of the bonds connecting one atom with the other? (Mole wt. of silver $= 108$ and Avogadro number $= 6.02 \times 10^{23}$ gmmole^{-1})

(A) 6.4 Nm^{-1} (B) 7.1 Nm^{-1}

(C) 2.2 Nm^{-1} (D) 5.5 Nm^{-1}

20. [Online 2018]

Two simple harmonic motions, as shown here, are at right angles. They are combined to form Lissajous figures.
If $x(t) = A\sin(at + \delta)$ and $y(t) = B\sin(bt)$
Identify the correct match below.

Parameters	Curve
(A) $A = B$, $a = b$, $\delta = \frac{\pi}{2}$	Line
(B) $A \neq B$, $a = b$, $\delta = 0$	Parabola
(C) $A = B$, $a = 2b$, $\delta = \frac{\pi}{2}$	Circle
(D) $A \neq B$, $a = b$, $\delta = \frac{\pi}{2}$	Ellipse

21. [Online 2018]

An oscillator of mass M is at rest in its equilibrium position in a potential $V = \frac{1}{2}k(x - X)^2$. A particle of mass m comes from right with speed u and collides completely inelastically with M and sticks to it. This process repeats every time the oscillator crosses its equilibrium position. The amplitude of oscillations after 13 collisions is $(M = 10,\ m = 5,\ u = 1,\ k = 1)$

(A) $\frac{1}{\sqrt{3}}$ (B) $\frac{2}{3}$

(C) $\sqrt{\frac{3}{5}}$ (D) $\frac{1}{2}$

22. [Online 2018]

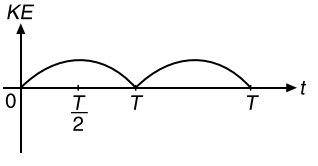
A particle executes simple harmonic motion and is located at $x = a$, b and c at times t_0, $2t_0$ and $3t_0$ respectively. The frequency of the oscillation is

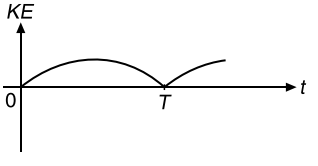
(A) $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+b}{2c}\right)$ (B) $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{2a+3c}{b}\right)$

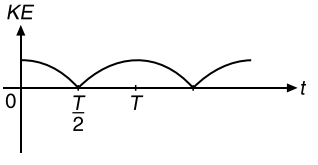
(C) $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+2b}{3c}\right)$ (D) $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+c}{2b}\right)$

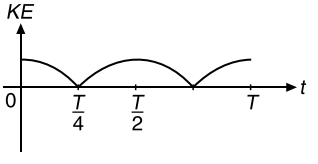
23. [2017]

A particle is executing simple harmonic motion with a time period T. At time $t = 0$. It is at its position of equilibrium. The kinetic energy-time graph of the particle will look like

(A)


(B)


(C)


(D)


24. [Online 2017]

A 1 kg block attached to a spring vibrates with a frequency of 1 Hz on a frictionless horizontal table. Two springs identical to the original spring are attached in parallel to an 8 kg block placed on the same table. So, the frequency of vibration of the 8 kg block is

(A) $\frac{1}{2\sqrt{2}}$ Hz (B) $\frac{1}{2}$ Hz

(C) 2 Hz (D) $\frac{1}{4}$ Hz

25. [Online 2017]

The ratio of maximum acceleration to maximum velocity in a simple harmonic motion is $10\ s^{-1}$. At, $t = 0$, the displacement is 5 m. What is the maximum acceleration? The initial phase is $\frac{\pi}{4}$.

(A) $500\sqrt{2}\ ms^{-2}$ (B) $500\ ms^{-2}$

(C) $750\sqrt{2}\ ms^{-2}$ (D) $750\ ms^{-2}$

26. [Online 2017]

A block of mass 0.1 kg is connected to an elastic spring of spring constant $640\ Nm^{-1}$ and oscillates in a damping medium of damping constant $10^{-2}\ kgs^{-1}$. The system dissipates its energy gradually. The time taken for its mechanical energy of vibration to drop to half of its initial value, is closest to

(A) 2 s (B) 3.5 s

(C) 7 s (D) 5 s

27. [2016]

A particle performs simple harmonic motion with amplitude A. Its speed is trebled at the instant that it is at a distance $\frac{2A}{3}$ from equilibrium position. The new amplitude of the motion is

(A) $\frac{7A}{3}$ (B) $\frac{A}{3}\sqrt{41}$

(C) $3A$ (D) $A\sqrt{3}$

28. [2015]

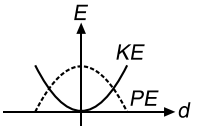
A pendulum made of a uniform wire of cross-sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to T_M. If the Young's modulus of the material of the wire is Y then $\frac{1}{Y}$ is equal to (g = gravitational acceleration)

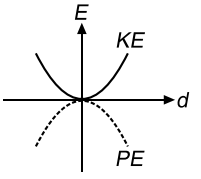
(A) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{Mg}$ (B) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{Mg}{A}$

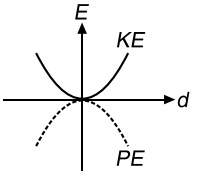
(C) $\left[1 - \left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$ (D) $\left[1 - \left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$

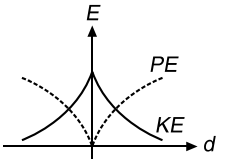
29. [2015]

For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly? (graphs are schematic and not drawn to scale)

(A) 

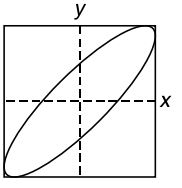
(B) 

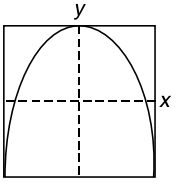
(C) 

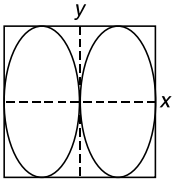
(D) 

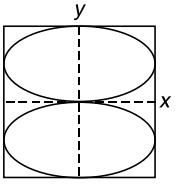
30. [Online 2015]

x and y displacements of a particle are given as $x(t) = a\sin\omega t$ and $y(t) = a\sin 2\omega t$. Its trajectory will look like

(A) 

(B) 

(C) 

(D) 

31. [Online 2015]

A simple harmonic oscillator of angular frequency $2\ rads^{-1}$ is acted upon by external force $F = (\sin t)$ N. If the oscillator is at rest in its equilibrium position at $t = 0$, its position at later times is proportional to

(A) $\sin t + \frac{1}{2}\sin 2t$ (B) $\sin t + \frac{1}{2}\cos 2t$

(C) $\cos t - \frac{1}{2}\sin 2t$ (D) $\sin t - \frac{1}{2}\sin 2t$

32. [Online 2015]

A cylindrical block of wood (density $= 650\ kgm^{-3}$), of base area $30\ cm^2$ and height 54 cm, floats in a liquid of density $900\ kgm^{-3}$. The block is depressed slightly and then released. The time period of the resulting oscillations of the block would be equal to that of a simple pendulum of length (nearly)

(A) 65 cm (B) 52 cm

(C) 39 cm (D) 26 cm

33. [Online 2015]

A pendulum with time period of 1 s is losing energy due to damping. At certain time its energy is 45 J. If after completing 15 oscillations, its energy has become 15 J, its damping constant $(\text{in } s^{-1})$ is

(A) $\frac{1}{30}\ln 3$ (B) $\frac{1}{15}\ln 3$

(C) 2 (D) $\frac{1}{2}$

34. [2014]
A particle moves with simple harmonic motion in a straight line. In first τ s, after starting from rest it travels a distance a and in next τ s it travels $2a$, in same direction, then
(A) amplitude of motion is $3a$
(B) time period of oscillations is 8τ
(C) amplitude of motion is $4a$
(D) time period of oscillations is 6τ

35. [2013]
The amplitude of a damped oscillator decreases to 0.9 times its original magnitude in 5 s. In another 10 s it will decrease to α times its original magnitude where α equals
(A) 0.6 (B) 0.7
(C) 0.81 (D) 0.729

36. [2013]
An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M. The piston and the cylinder have equal cross-sectional area A. When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is P_0. The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion with frequency
(A) $\frac{1}{2\pi}\frac{A\gamma P_0}{V_0 M}$ (B) $\frac{1}{2\pi}\frac{V_0 M P_0}{A^2\gamma}$
(C) $\frac{1}{2\pi}\sqrt{\frac{A^2\gamma P_0}{MV_0}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{MV_0}{A\gamma P_0}}$

37. [2012]
If a simple pendulum has significant amplitude (up to a factor of $\frac{1}{e}$ of original) only in the period between $t = 0$ s to $t = \tau$ s, then τ may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity, with b as the constant of proportionality, the average life time of the pendulum is (assuming damping is small) in seconds
(A) b (B) $\frac{1}{b}$
(C) $\frac{2}{b}$ (D) $\frac{0.693}{b}$

38. [2011]
Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the x-axis. Their mean position is separated by distance $X_0 (X_0 > A)$. If the maximum separation between them is $(X_0 + A)$, the phase difference between their motion is
(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$
(C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

39. [2011]
A mass M, attached to a horizontal spring, executes SHM with an amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of $\left(\frac{A_1}{A_2}\right)$ is
(A) $\frac{M}{M+m}$ (B) $\frac{M+m}{M}$
(C) $\left(\frac{M}{M+m}\right)^{\frac{1}{2}}$ (D) $\left(\frac{M+m}{M}\right)^{\frac{1}{2}}$

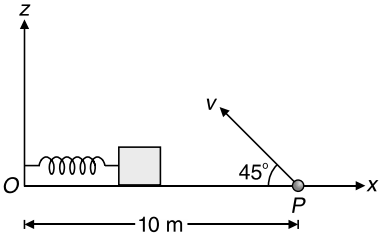
40. [2009]
If x, v and a denote the displacement, the velocity and the acceleration of a particle executing simple harmonic motion of time period T, then, which of the following does not change with time?
(A) $a^2T^2 + 4\pi^2 v^2$ (B) $\frac{aT}{x}$
(C) $aT + 2\pi v$ (D) $\frac{aT}{v}$

ARCHIVE: JEE ADVANCED

Single Correct Choice Type Problems

(In this section each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct)

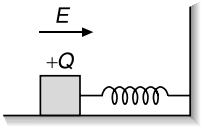
1. [IIT-JEE 2012]
A small block is connected to one end of a massless spring of un-stretched length 4.9 m. The other end of the spring (shown in Figure) is fixed. The system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and released from rest at $t = 0$. It then executes simple harmonic motion with angular frequency $\omega = \frac{\pi}{3}$ rads^{-1}. Simultaneously at $t = 0$, a small pebble is projected with speed v from point P at an angle of $45°$ as shown in Figure. Point P is at a horizontal distance of 10 m from O. If the pebble hits the block at $t = 1$ s, the value of v is $(\text{Take } g = 10 \text{ ms}^{-2})$



(A) $\sqrt{50}$ ms^{-1} (B) $\sqrt{51}$ ms^{-1}
(C) $\sqrt{52}$ ms^{-1} (D) $\sqrt{53}$ ms^{-1}

2. [IIT-JEE 2011]

A wooden block performs SHM on a frictionless surface with frequency ν_0. The block carries a charge $+Q$ on its surface. If now a uniform electric field E is switched-on as shown, then the SHM of the block will be



(A) of the same frequency and with shifted mean position
(B) of the same frequency and with the same mean position
(C) of changed frequency and with shifted mean position
(D) of charged frequency and with the same mean position

3. [IIT-JEE 2011]

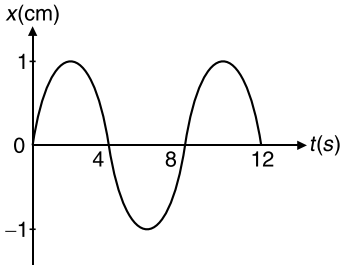
A point mass is subjected to two simultaneous sinusoidal displacements in x-direction, $x_1(t) = A\sin(\omega t)$ and $x_2(t) = A\sin\left(\omega t + \frac{2\pi}{3}\right)$. Adding a third sinusoidal displacement $x_3(t) = B\sin(\omega t + \phi)$ brings the mass to a complete rest. The values of B and ϕ are

(A) $\sqrt{2}A, \frac{3\pi}{4}$ (B) $A, \frac{4\pi}{3}$

(C) $\sqrt{3}A, \frac{5\pi}{6}$ (D) $A, \frac{\pi}{3}$

4. [IIT-JEE 2009]

The x-t graph of a particle undergoing simple harmonic motion is shown below. The acceleration of the particle at $t = \frac{4}{3}$ s is

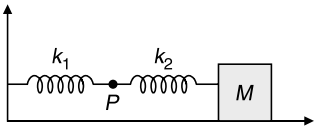


(A) $\frac{\sqrt{3}}{32}\pi^2$ cms^{-2} (B) $-\frac{\pi^2}{32}$ cms^{-2}

(C) $\frac{\pi^2}{32}$ cms^{-2} (D) $-\frac{\sqrt{3}}{32}\pi^2$ cms^{-2}

5. [IIT-JEE 2009]

The mass M shown in Figure oscillates in simple harmonic motion with amplitude A. The amplitude of the point P is

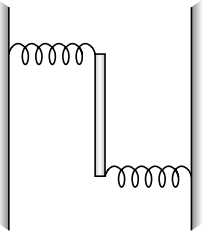


(A) $\frac{k_1 A}{k_2}$ (B) $\frac{k_2 A}{k_1}$

(C) $\frac{k_1 A}{k_1 + k_2}$ (D) $\frac{k_2 A}{k_1 + k_2}$

6. [IIT-JEE 2009]

A uniform rod of length L and mass M is pivoted at the centre. Its two ends are attached to two springs of equal spring constants k. The springs are fixed to rigid supports as shown in Figure and the rod is free to oscillate in the horizontal plane. The rod is gently pushed through a small angle θ in one direction and released. The frequency of oscillation is

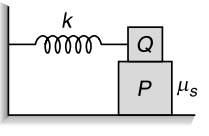


(A) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$

(C) $\frac{1}{2\pi}\sqrt{\frac{6k}{M}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{24k}{M}}$

7. [IIT-JEE 2004]

A block P of mass m is placed on a horizontal frictionless plane. A second block of same mass m is placed on it and is connected to a spring of spring constant k, the two blocks are pulled by a distance A. Block Q oscillates without slipping. What is the maximum value of frictional force between the two blocks?



(A) $\frac{kA}{2}$ (B) kA

(C) $\mu_s mg$ (D) ZERO

8. [IIT-JEE 2005]

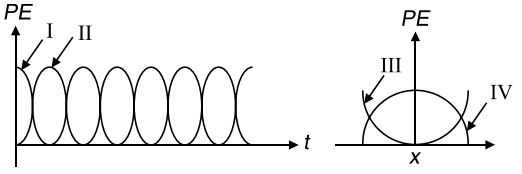
A simple pendulum has time period T_1. The point of suspension is now moved upward according to the relation $y = Kt^2$, $\left(K = 1\ \text{ms}^{-2}\right)$ where y is the vertical displacement. The time period now becomes T_2. The ratio of $\frac{T_1^2}{T_2^2}$ is $\left(g = 10\ \text{ms}^{-2}\right)$

(A) $\frac{6}{5}$ (B) $\frac{5}{6}$

(C) 1 (D) $\frac{4}{5}$

9. [IIT-JEE 2003]

For a particle executing SHM the displacement x given by $x = A\cos\omega t$. Identify the graph which represents the variation of potential energy (PE) as a function of time t and displacement x.



(A) I, III (B) II, IV
(C) II, III (D) I, IV

10. [IIT-JEE 2001]

A particle executes simple harmonic motion between $x = -A$ and $x = +A$. The time taken for it to go from 0 to $\frac{A}{2}$ is T_1 and to go from $\frac{A}{2}$ to A is T_2, then

(A) $T_1 < T_2$ (B) $T_1 > T_2$
(C) $T_1 = T_2$ (D) $T_1 = 2T_2$

11. [IIT-JEE 2000]

The period of oscillation of simple pendulum of length L suspended from the roof of the vehicle which moves without friction, down an inclined plane of inclination α, is given by

(A) $2\pi\sqrt{\frac{L}{g\cos\alpha}}$ (B) $2\pi\sqrt{\frac{L}{g\sin\alpha}}$
(C) $2\pi\sqrt{\frac{L}{g}}$ (D) $2\pi\sqrt{\frac{L}{g\tan\alpha}}$

12. [IIT-JEE 1999]

A particle free to move along the x-axis has potential energy given by $U(x) = k\left[1 - \exp(-x^2)\right]$ for $-\infty \le x \le +\infty$, where k is a positive constant of appropriate dimensions. Then

(A) at points away from the origin, the particle is in unstable equilibrium
(B) for any finite non-zero value of x, there is a force directed away from the origin
(C) if its total mechanical energy is $\frac{k}{2}$, it has its minimum kinetic energy at the origin
(D) for small displacements from $x = 0$, the motion is simple harmonic

13. [IIT-JEE 1999]

A spring of force constant k is cut into two pieces such that one piece is double the length of the other. Then the long piece will have a force constant of

(A) $\frac{2}{3}k$ (B) $\frac{3}{2}k$
(C) $3k$ (D) $6k$

14. [IIT-JEE 1998]

A particle of mass m is executing oscillations about the origin on the x-axis. Its potential energy is $U(x) = k|x|^3$, where k is a positive constant. If the amplitude of oscillation is a, then its time period T is

(A) proportional to $\frac{1}{\sqrt{a}}$ (B) independent of a
(C) proportional to $\sqrt{a}$ (D) proportional to $a^{3/2}$

15. [IIT-JEE 1993]

One end of a long metallic wire of length L is tied to the celling. The other end is tied to a massless spring of spring constant K. A mass m hangs freely from the free end of the spring. The area of cross-section and the Young's modulus of the wire are A and Y respectively. If the mass is slightly pulled down and released, it will oscillate with a time period T equal to

(A) $2\pi\left(\frac{m}{k}\right)^{1/2}$ (B) $2\pi\sqrt{\frac{m(YA+KL)}{YAK}}$
(C) $2\pi\left(\frac{mYA}{kL}\right)^{1/2}$ (D) $2\pi\left(\frac{mL}{YA}\right)^{1/2}$

16. [IIT-JEE 1992]

A highly rigid cubical block A of small mass M and side L is fixed rigidly on to another cubical block B of the same dimensions and of low modulus of rigidity η such that the lower face of A completely covers the upper face of B. The lower face of B is rigidly held on a horizontal surface. A small force F is applied perpendicular to one of the sides faces of A. After the force is withdrawn, block A executes small oscillations, the time period of which is given by

(A) $2\pi\sqrt{M\eta L}$ (B) $2\pi\sqrt{\frac{M\eta}{L}}$
(C) $2\pi\sqrt{\frac{ML}{\eta}}$ (D) $2\pi\sqrt{\frac{M}{\eta L}}$

17. [IIT-JEE 1990]

A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical, from a fixed point by a massless spring, such that it is half-submerged in a liquid of density ρ at equilibrium position. When the cylinder is given a small downward push and released it starts oscillating vertically with a small amplitude. If the force constant of the spring is k, the frequency of oscillation of the cylinder is

(A) $\frac{1}{2\pi}\left(\frac{k - A\rho g}{M}\right)^{1/2}$ (B) $\frac{1}{2\pi}\left(\frac{k + A\rho g}{M}\right)^{1/2}$
(C) $\frac{1}{2\pi}\left(\frac{k + \rho g L^2}{M}\right)^{1/2}$ (D) $\frac{1}{2\pi}\left(\frac{k + A\rho g}{A\rho g}\right)^{1/2}$

18. [IIT-JEE 1988]

The bodies A and B of equal masses are suspended from two separate massless springs of constants K_1 and K_2, respectively. If the two oscillate vertically such that their maximum velocities are equal. The ratio of amplitude A to that of B is

(A) $\frac{K_1}{K_2}$ (B) $\frac{K_2}{K_1}$
(C) $\sqrt{\frac{K_1}{K_2}}$ (D) $\sqrt{\frac{K_2}{K_1}}$

19. [IIT-JEE 1987]

A particle executes S.H.M. with frequency f. The frequency with which its kinetic energy oscillates is

(A) $\frac{f}{2}$ (B) f

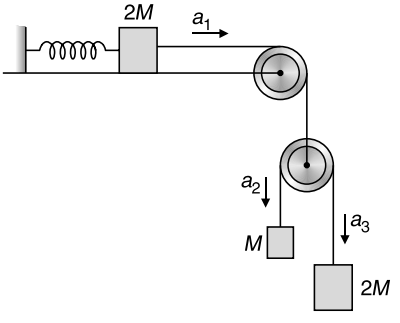
(C) $2f$ (D) $4f$

Multiple Correct Choice Type Problems

(In this section each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct)

1. [JEE (Advanced) 2019]

A block of mass $2M$ is attached to a massless spring with spring-constant k. This block is connected to two other blocks of masses M and $2M$ using two massless pulleys and strings. The accelerations of the blocks are a_1, a_2 and a_3 as shown in Figure. The system is released from rest with the spring in its unstretched state. The maximum extension of the spring is x_0. Which of the following option(s) is/are correct? (g is the acceleration due to gravity. Neglect friction)



(A) At an extension of $\frac{x_0}{4}$ of the spring, the magnitude of acceleration of the block connected to the spring is $\frac{3g}{10}$

(B) $x_0 = \frac{4Mg}{k}$

(C) $a_2 - a_1 = a_1 - a_3$

(D) When spring achieves an extension of $\frac{x_0}{2}$ for the first time, the speed of the block connected to the spring is $3g\sqrt{\frac{M}{5k}}$

2. [JEE (Advanced) 2016]

A block with mass M is connected by a massless spring with stiffness constant k to a rigid wall and moves without friction on a horizontal surface. The block oscillates with small amplitude A about an equilibrium position x_0. Consider two cases: (1) when the block is at x_0 and (2) when the block is at $x = x_0 + A$. If both the cases, a particle with mass $m(< M)$ is softly placed on the block after which they stick to each other. Which of the following statement(s) is (are) true about the motion after the mass m is placed on the mass M?

(A) The amplitude of oscillation in the first case changes by a factor of $\sqrt{\frac{M}{m+M}}$, whereas in the second case it remains unchanged

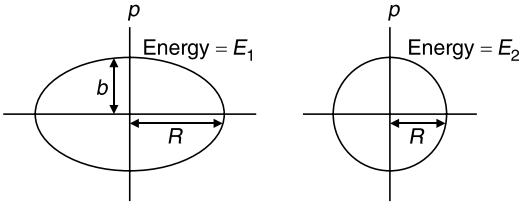
(B) The final time period of oscillation in both the cases is same

(C) The total energy decreases in both the cases

(D) The instantaneous speed at x_0 of the combined masses

3. [JEE (Advanced) 2015]

Two independent harmonic oscillators of equal masses are oscillating about the origin with angular frequencies ω_1 and ω_2 and have total energies E_1 and E_2, respectively. The variations of their momenta p with positions x are shown in Figures.



If $\frac{a}{b} = n^2$ and $\frac{a}{R} = n$, then the correct equations is/are

(A) $E_1\omega_1 = E_2\omega_2$ (B) $\frac{\omega_2}{\omega_1} = n^2$

(C) $\omega_1\omega_2 = n^2$ (D) $\frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$

4. [JEE (Advanced) 2013]

A particle of mass m is attached to one end of a mass-less spring of force constant k, lying on a frictionless horizontal plane. The other end of the spring is fixed. The particle starts moving horizontally from its equilibrium position at time $t = 0$ with an initial velocity u_0. When the speed of the particle is $0.5u_0$, it collides elastically with a rigid wall. After this collision,

(A) The speed of the particle when it returns to its equilibrium position is u_0

(B) The time at which the particle passes through the equilibrium position for the first time is $t = \pi\sqrt{\frac{m}{k}}$

(C) The time at which the maximum compression of the spring occurs is $t = \frac{4\pi}{3}\sqrt{\frac{m}{k}}$

(D) The time at which the particle passes through the equilibrium position for the second time is $t = \frac{5\pi}{3}\sqrt{\frac{m}{k}}$

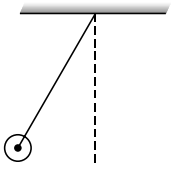
5. [IIT-JEE 2011]

A metal rod of length L and mass m is pivoted at one end. A thin disc of mass M and radius $R(< L)$ is attached at its centre to the free end of the rod. Consider two ways the disc is attached.

Case A: The disc is not free to rotate about its centre and

Case B: The disc is free to rotate about its centre.

The rod-disc system performs SHM in vertical plane after being released from the same displaced position. Which of the following statement(s) is/are true?



(A) Restoring torque in case A equals restoring torque in case B
(B) Restoring torque in case A is less than restoring torque in case B
(C) Angular frequency for case A is more than angular frequency for case B
(D) Angular frequency for case A is less than angular frequency for case B

6. [IIT-JEE 2006]

The function $x = A\sin^2\omega t + B\cos^2\omega t + C\sin\omega t\cos\omega t$ represents SHM

(A) for any value of A, B and C (except $C = 0$)
(B) if $A = -B$, $C = 2B$, amplitude $= |B\sqrt{2}|$
(C) if $A = B$; $C = 0$
(D) if $A = B$; $C = 2B$, amplitude $= |B|$

7. [IIT-JEE 1999]

Three simple harmonic motions in the same direction having the same amplitude and same period are superposed. If each differ in phase from the next by $45°$, then

(A) the resultant amplitude is $(1+\sqrt{2})$
(B) the phase of the resultant motion relative to the first is $90°$
(C) the energy associated with the resulting motion is $(3+2\sqrt{2})$ times the energy associated with any single motion
(D) the resulting motion is not simple harmonic

8. [IIT-JEE 1989]

A linear harmonic oscillator of force constant 2×10^6 Nm^{-1} and amplitude 0.01 m has a total mechanical energy of 160 J. Its

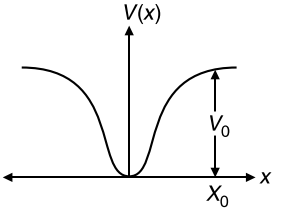
(A) maximum potential energy is 100 J
(B) maximum kinetic energy is 100 J
(C) maximum potential energy is 160 J
(D) maximum potential energy is zero

Linked Comprehension Type Questions

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct. (For the sake of competitiveness there may be a few questions that may have more than one correct options)

Comprehension I

When a particle of mass m moves on the x-axis in a potential of the form $V(x) = kx^2$, it performs simple harmonic motion. The corresponding time period is proportional to $\sqrt{\frac{m}{k}}$, as can be seen easily using dimensional analysis. However, the motion of a particle can be periodic even when its potential energy increases on both sides of $x = 0$ in a way different from kx^2 and its total energy is such that the particle does not escape to infinity. Consider a particle of mass m moving on the x-axis. Its potential energy is $V(x) = \alpha x^4 (\alpha > 0)$ for $|x|$ near the origin and becomes a constant equal to V_0 for $|x| \geq X_0$ (shown in Figure).



Based on the above facts, answer the following questions.

1. [IIT-JEE 2010]

If the total energy of the particle is E, it will perform periodic motion only if

(A) $E < 0$ (B) $E > 0$
(C) $V_0 > E > 0$ (D) $E > V_0$

2. [IIT-JEE 2010]

For periodic motion of small amplitude A, the time period T of this particle is proportional to

(A) $A\sqrt{\frac{m}{\alpha}}$ (B) $\frac{1}{A}\sqrt{\frac{m}{\alpha}}$
(C) $A\sqrt{\frac{\alpha}{m}}$ (D) $\frac{1}{A}\sqrt{\frac{\alpha}{m}}$

3. [IIT-JEE 2010]

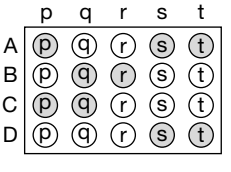
The acceleration of this particle for $|x| > X_0$ is

(A) proportional to V_0 (B) proportional to $\frac{V_0}{mX_0}$
(C) proportional to $\sqrt{\frac{V_0}{mX_0}}$ (D) ZERO

Matrix Match/Column Match Type Questions

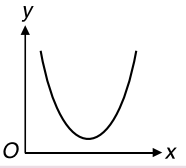
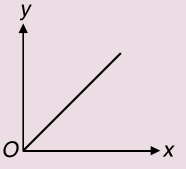
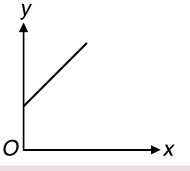
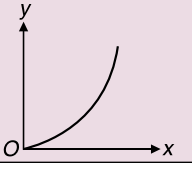
Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

If the correct matches are A → p, s and t; B → q and r; C → p and q; and D → s and t; then the correct darkening of bubbles will look like the following:



1. **[IIT-JEE 2008]**

COLUMN-I gives a list of possible set of parameters measured in some experiments. The variations of the parameters in the form of graphs are shown in **COLUMN-II**. Match the set of parameters given in **COLUMN-I** with the graphs given in **COLUMN-II**. Indicate your answer by darkening the appropriate bubbles of the 4×4 matrix given in the ORS.

COLUMN-I	COLUMN-II
(A) Potential energy of a simple pendulum (y-axis) as a function of displacement (x-axis)	(p) 
(B) Displacement (y-axis) as a function of time (x-axis) for a one dimensional motion at zero or constant acceleration when the body is moving along the positive x-direction	(q) 
(C) Range of a projectile (y-axis) as a function of its velocity (x-axis) when projected at a fixed angle	(r) 
(D) The square of the time period (y-axis) of a simple pendulum as a function of its length (x-axis)	(s) 

2. **[IIT-JEE 2007]**

COLUMN I describes some situations in which a small object moves. **COLUMN II** describes some characteristics of these motions. Match the situations in **COLUMN I** with the characteristics in **COLUMN II**.

COLUMN-I	COLUMN-II
(A) The object moves on the x-axis under a conservative force in such a way that its speed and position satisfy $v = c_1\sqrt{c_2 - x^2}$, where c_1 and c_2 are positive constants.	(p) The object executes a simple harmonic motion.
(B) The object moves on the x-axis in such a way that its velocity and its displacement from the origin satisfy $v = -kx$, where k is a positive constant.	(q) The object does not change its direction.
(C) The object is attached to one end of a mass-less spring of a given spring constant. The other end of the spring is attached to the ceiling of an elevator. Initially everything is at rest. The elevator starts going upwards with a constant acceleration a. The motion of the object is observed from the elevator during the period it maintains this acceleration.	(r) The kinetic energy of the object keeps on decreasing
(D) The object is projected from the earth's surface vertically upwards with a speed $2\sqrt{\frac{GM_e}{R_e}}$, where M_e is the mass of the earth and R_e is the radius of the earth. Neglect forces from object other than the earth.	(s) The object can change its direction only once.

Integer/Numerical Answer Type Questions

In this section, the answer to each question is a numerical value obtained after series of calculations based on the data provided in the question(s).

1. **[IIT-JEE 2010]**

A 0.1 kg mass is suspended from a wire of negligible mass. The length of the wire is 1 m and its cross-sectional area is 4.9×10^{-7} m^2. If the mass is pulled a little in the vertically downward direction and released, it performs simple harmonic motion of angular frequency 140 rads^{-1}. If the Young's modulus of the material of the wire is $n \times 10^9$ Nm^{-2}, the value of n is

ANSWER KEYS–TEST YOUR CONCEPTS AND PRACTICE EXERCISES

Test Your Concepts-I (Based on SHM Properties)

1. (a) 8 cm (b) 1.57 rads^{-1}
(c) 1.97 Nm^{-1} (d) zero
(e) 0.197 ms^{-2}

2. No Solution required

3. (a) Oscillatory but not harmonic
(b) Rectilinear along negative x direction
(c) SHM about $x = 2$

4. $6\cos(\pi t)$, $-6\pi\sin(\pi t)$, $-6\pi^2\cos(\pi t)$

5. $\frac{2}{\pi}, \frac{2}{\pi}$

6. (a) 3.14×10^{-2} s
(b) 1.571 s

7. 2 ms^{-1}, $2\sin^{-1}\left(\frac{4}{5}\right)$

8. $5A$

9. (a) 1.4 s (b) 0.0964 m
(c) 0.207

10. $\frac{2\pi}{3}$

11. No Answer

12. (a) $\frac{\pi}{4}$ s (b) 8 ms^{-1}
(c) 0

13. $\pi-\sin^{-1}\left(\frac{1}{6}\right)-\sin^{-1}\left(\frac{1}{3}\right)$

14. (a) 0.35 ms^{-1} (b) 0.61 ms^{-2}
(c) 0.3 s (d) 0.80 m

15. (a) 150 J (b) 50 J (c) 200 J

16. $0.1\sin\left(4t+\frac{\pi}{4}\right)$ metre

17. $2\pi\sqrt{\frac{m}{b^2U_0}}$

18. 0.806

19. 7.2 cm

20. (a) $2A$, $2A$, 1 (b) A, A, 1
(c) $\frac{A}{\sqrt{2}}, A\left(1-\frac{1}{\sqrt{2}}\right)$, 2.4

21. $\frac{3}{4}, \frac{A}{\sqrt{2}}$

22. (a) $m\left(g+a\omega^2\cos\omega t\right)$ (b) 8.01 cm

23. (a) 2.5 Hz, 0.4 s (b) 3 m (c) –3 m, 0

Test Your Concepts-II (Based on Spring Mass Systems)

1. (a) $A_1\sqrt{\frac{M}{M+m}}, 2\pi\sqrt{\frac{M+m}{k}}$ (b) $A_1, 2\pi\sqrt{\frac{M+m}{k}}$

2. (a) $\frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m_1+m_2}}$ (b) $\frac{k_1}{k_2}<\frac{m_1}{m_2}$
(c) $\frac{\mu(m_1+m_2)m_2g}{m_1k_2-m_2k_1}$

3. 1.6 kg

4. $2\pi\sqrt{\frac{mM}{k(M+m)}}$

5. (a) 0.35 ms^{-1} (b) 0.61 ms^{-2}
(c) 0.3 s (d) 0.80 m

6. 7.2 cm

7. (a) $\frac{kA}{(m_1+m_2)g}$
(b) A and E remain unchanged, $\omega_f=\left(\sqrt{\frac{m_1}{m_1+m_2}}\right)\omega_i$
(c) T increases, $T_f=\left(\sqrt{\frac{m_1+m_2}{m_1}}\right)T_i$

8. $\sqrt{\frac{k_1k_2}{4m(k_1+k_2)}}$

9. (a) 1.5 kg (b) 0.82 cm
(c) $x=2.5\cos(34.6t)$ cm, $v=-86.5\sin(34.6t)$ cms^{-1}
$a=-29.9\cos(34.6t)$ ms^{-2}

10. $2\pi\sqrt{\frac{m(k_1+4k_2)}{k_1k_2}}$

11. (a) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ (b) $\frac{ma}{k}$

12. (a) 13 cm (b) 6.5 cm
(c) 0.51 s (d) 0.32 J
(e) 0.8 ms^{-1}, 0.13 s (f) 0.12 s, 1.13 ms^{-1}

13. 4.3 kg

14. $\frac{\pi}{2}\sqrt{\frac{k}{2m}}$

15. $2\pi\sqrt{\frac{(k_1+k_2)m}{4k_1k_2}}$

Test Your Concepts-III (Based on Rotational SHM)

1. $2\pi\sqrt{\left|\frac{\theta}{\alpha'}\right|}=2\pi\sqrt{\frac{28R}{5g}}$

2. $0.4\cos(16.16t)$
3. $\frac{2\pi a}{b}\sqrt{\frac{m}{k}}$
4. 0.963 s
5. $2\pi\sqrt{\frac{ml^2}{mgl+2kb^2}}$
6. $2\pi\sqrt{\frac{3M}{16k}}$
7. $2\pi\sqrt{\frac{M+\frac{I}{R^2}}{4k}}$
8. $2\pi\sqrt{\frac{9M}{4k}}$

Test Your Concepts-IV (Based on Pendulum Systems)

1. $\frac{T_0}{\sqrt{\cos\alpha}}$
2. 0.816
3. 1.1 s
4. 12.8 s
5. $T=\sqrt{\frac{l}{g}}\left[\pi+2\sin^{-1}\left(\frac{\alpha}{\beta}\right)\right]$
6. $2\pi\sqrt{\frac{\ell}{g\cos\theta}}$
7. $\frac{4}{9}$
8. $\frac{\left(m+\frac{M}{2}\right)R}{m}$
9. (a) 0.25 m, (b) $-0.123\ \text{ms}^{-1}$
10. $\sqrt{\frac{g}{2r}}$

Test Your Concepts-V (Based on SHM in Other Physical Systems, Composition of SHM, Damped Oscillations, Forced Oscillations & Resonance)

1. $2\pi\sqrt{\frac{\ell}{g-\frac{qE}{m}}}$
2. $2\pi\sqrt{\frac{r\cos\alpha}{g\left(1+\cos^2\alpha\right)^{1/2}}}$
3. 0.2 s
4. $\frac{1}{2\pi}\sqrt{\frac{YA}{ml}}$
5. $\pi\sqrt{\frac{R^3}{GM}}$
6. $2\pi\sqrt{\frac{MV_0}{\gamma A\left(P_0A+Mg\right)}}$
7. $11.25\sin(\omega t+\phi),\ \phi=40.4^\circ$
8. 1.25 cm
9. 1.25 kg
10. 8

Single Correct Choice Type Questions

1. B	2. B	3. B	4. A	5. D	6. C	7. C	8. C	9. D	10. C
11. C	12. C	13. C	14. B	15. B	16. B	17. A	18. D	19. A	20. C
21. C	22. B	23. D	24. A	25. C	26. B	27. A	28. B	29. A	30. C
31. D	32. A	33. D	34. B	35. A	36. B	37. C	38. A	39. A	40. C
41. B	42. B	43. C	44. D	45. B	46. B	47. C	48. B	49. B	50. C
51. C	52. B	53. C	54. A	55. B	56. B	57. D	58. A	59. B	60. D
61. D	62. B	63. C	64. C	65. A	66. A	67. C	68. B	69. B	70. C
71. B	72. B	73. D	74. B	75. D	76. C	77. C	78. B	79. C	80. D
81. D	82. B	83. C	84. D	85. B	86. A	87. C	88. B	89. A	90. D
91. C	92. D	93. D	94. B	95. B	96. B	97. A	98. A	99. B	100. D
101. A	102. C	103. A	104. D	105. A	106. B	107. C	108. D	109. D	110. B
111. D	112. D	113. C	114. C	115. C	116. A	117. D	118. B	119. B	120. B
121. B	122. A	123. C	124. B	125. C	126. C	127. B	128. B	129. A	130. A
131. D	132. A	133. C	134. D	135. D	136. C	137. B	138. A	139. D	140. B
141. C	142. C	143. D	144. D	145. A	146. D	147. C	14. A	149. D	150. D
151. C	152. A	153. A							

Multiple Correct Choice Type Questions

1. B, D	**2.** B, C, D	**3.** A, C	**4.** A, C	**5.** B, C
6. A, C	**7.** A, B, C	**8.** A, B, D	**9.** A, C	**10.** A, C
11. A, C, D	**12.** B, C, D	**13.** A, B, C, D	**14.** A, C, D	**15.** A, C
16. A, B, C	**17.** A, C, D	**18.** B, C	**19.** B, C	**20.** A, C
21. A, B, C, D	**22.** B, C	**23.** A, B, C	**24.** A, C	**25.** B, C, D
26. A, B, C	**27.** B, D	**28.** C, D	**29.** A, B, C	**30.** B, C, D
31. A, B				

Reasoning Based Questions

1. B	**2.** B	**3.** B	**4.** B	**5.** D	**6.** A	**7.** A	**8.** D	**9.** D	**10.** B
11. B	**12.** D	**13.** A	**14.** C	**15.** A	**16.** D	**17.** D	**18.** A	**19.** C	**20.** A

Linked Comprehension Type Questions

1. C	**2.** B	**3.** C	**4.** D	**5.** B	**6.** B	**7.** C	**8.** D	**9.** D	**10.** B, C
11. D	**12.** C	**13.** B	**14.** C	**15.** A	**16.** D	**17.** B	**18.** D	**19.** D	**20.** D
21. C	**22.** D	**23.** A	**24.** D	**25.** D	**26.** C	**27.** D	**28.** B	**29.** A	**30.** C
31. A	**32.** D	**33.** C	**34.** A	**35.** B	**36.** C	**37.** A	**38.** C	**39.** B	**40.** C
41. B	**42.** C	**43.** A	**44.** A	**45.** D	**46.** B	**47.** A	**48.** A	**49.** A	**50.** C
51. A	**52.** B	**53.** A	**54.** A	**55.** B	**56.** C	**57.** C			

Matrix Match/Column Match Type Questions

1. A → (q)	B → (r)	C → (s)	D → (t)
2. A → (r)	B → (p)	C → (q)	D → (s)
3. A → (r)	B → (q)	C → (p)	D → (q)
4. A → (r)	B → (r)	C → (s)	D → (s)
5. A → (q)	B → (p)	C → (s)	D → (q)
6. A → (q)	B → (s)	C → (p)	D → (r)
7. A → (q)	B → (s)	C → (p)	D → (r)
8. A → (q)	B → (r)	C → (q)	D → (q)
9. A → (r)	B → (p)	C → (s)	D → (p, q)
10. A → (q)	B → (s)	C → (s)	D → (s)
11. A → (q)	B → (t)	C → (p)	D → (t)

Integer/Numerical Answer Type Questions

1. 4	2. (a) 9 (b) 3	**3.** 2	**4.** 8	5. $x = 3, y = 5$
6. 1	**7.** 2	**8.** 2	**9.** 5	**10.** 10
11. 7	**12.** (a) 147 (b) 7	**13.** 9	**14.** (a) 1600 (b) 80 (c) 20	**15.** 10
16. 2, 10	**17.** 20	**18.** 4	**19.** 8	**20.** (a) 54 (b) 108

ARCHIVE: JEE MAIN

1. D	**2.** A	**3.** A	**4.** B	**5.** D	**6.** B	**7.** A	**8.** B	**9.** B	**10.** C
11. A	**12.** C	**13.** *	**14.** A	**15.** D	**16.** D	**17.** A	**18.** *	**19.** B	**20.** D
21. A	**22.** D	**23.** D	**24.** B	**25.** A	**26.** C	**27.** A	**28.** A	**29.** B	**30.** C
31. D	**32.** C	**33.** A	**34.** D	**35.** D	**36.** C	**37.** C	**38.** B	**39.** D	**40.** B

* No given option is correct.

ARCHIVE: JEE ADVANCED

Single Correct Choice Type Problems

1. A	**2.** A	**3.** B	**4.** D	**5.** D	**6.** C	**7.** A	**8.** A	**9.** A	**10.** A
11. A	**12.** D	**13.** B	**14.** A	**15.** B	**16.** D	**17.** B	**18.** D	**19.** C	

Multiple Correct Choice Type Problems

1. C	**2.** A, B, D	**3.** B, D	**4.** A, D	**5.** A, D
6. B, D	**7.** A, C	**8.** B, C	**9.** A, C	**10.** A, C

Linked Comprehension Type Questions

1. C	**2.** B	**3.** D

Matrix Match/Column Match Type Questions

1. A → (p, s)	B → (q, r, s)	C → (s)	D → (q)
2. A → (p)	B → (q, r)	C → (p)	D → (q, r)

Integer/Numerical Answer Type Questions

1. 4

CHAPTER

4 Mechanical Waves

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Characteristics of Wave Motion
(b) Equation of Harmonic Wave
(c) Characteristic Wave Equation
(d) Particle Velocity, Wave Slope, Particle Acceleration
(e) Transverse Wave in a String and Properties
(f) Sound Wave and Properties
(g) Superposition of Waves
(h) Concept of Interference
(i) Stationary Waves, Organ Pipes
(j) Beats
(k) Doppler Effect of Sound and Light

All this is followed by a variety of Exercise Sets (fully solved) which contain questions as per the latest JEE pattern. At the end of Exercise Sets, a collection of problems asked previously in JEE (Main and Advanced) are also given.

INTRODUCTION

A wave is a disturbance from an equilibrium condition that propagates from one region of space to another without the transfer of matter. We can also say that wave is a disturbance which transfers from one part of the medium to the other without actual transfer of matter as a whole.

Mechanical Waves

Such waves require a material medium for propagation and are governed by Newton's Laws. These are the most common to be observed.

EXAMPLE
sound, waves on the surface of water, waves in a stretched string, compressional waves in a spring, seismic waves etc.

Electromagnetic Waves

Such waves do not require a material medium for propagation as they can travel through vacuum as well as through certain media. These waves are less familiar but are used constantly. All electromagnetic waves travel through vacuum at a same speed of $3\times10^{8}\ \text{ms}^{-1}$.

EXAMPLE: Light, X-rays, heat, radio waves etc.

Matter Waves

These waves are associated with electrons, photons, other fundamental particles as well as atoms and molecules. Since these particles form fundamental matter, so these waves are called **Matter Waves.**

CHARACTERISTICS OF WAVE MOTION

(a) In wave motion, there is transfer of energy and momentum from one place to another without any bulk motion of the medium. The particles of the medium simply vibrate about their mean positions and the disturbance propagates due to elastic and inertial properties of the medium.

(b) All the particles of the medium have the same kind of motion. However, there is a systematic phase change from one particle to another. The movement of a particle begins a little later than its predecessor.

(c) The wave velocity depends only on the nature of the medium, i.e., its elastic and inertial properties. It does not depend on the nature of the source, i.e., on the shape and size of the disturbance to be transmitted.

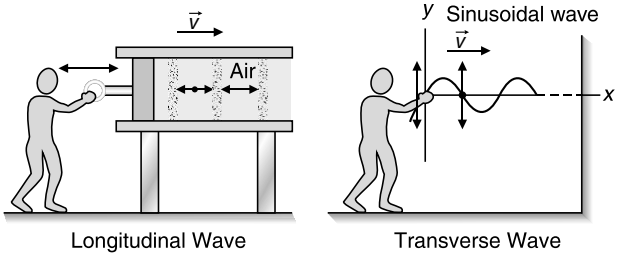
(d) On the other hand, the velocity of the particles (also called **particle velocity**) of the medium changes as they move from the mean position to the extreme position.

TYPES OF WAVE MOTION

Based on oscillations of particles and the direction of propagation, we are discussing two types of waves.

(a) Longitudinal Waves
(b) Transverse Waves

Longitudinal Waves are waves in which the particles of the medium oscillate along the direction of propagation. They travel in the form of compressions and rarefactions. A sound wave is set up in an air-filled pipe by moving a piston back and forth. Because the oscillations of an element of the air (represented by the dot) are parallel to the direction in which the wave travels, the wave is a longitudinal wave.



Transverse Waves are waves in which particles of the medium oscillate at right angles to the direction of propagation. They travel in the form of crests and troughs. A sinusoidal wave is sent along the string. A typical string element moves up and down continuously as the wave passes. This too is a transverse wave.

WAVE MOTION PARAMETERS

Wave Length (λ)

It is the spatial period of the wave at a given instant, i.e., it is the distance between two consecutive particles which vibrate in the same phase. Thus, in longitudinal waves, the distance between two successive compressions or rarefactions is equal to λ and in transverse waves, the distance between two successive crests or troughs is equal to λ.

Wave Number ($1/\lambda$)

The reciprocal of the wavelength is called Wave number and wave number is the measure of the number of waves present in a unit distance in the direction of the propagation of the wave. Mathematically, wave number $\bar{\nu} = 1/\lambda$

Time Period (T)

It is time in which the particle of a medium completes one vibration (to and fro) about its mean position.

Frequency (ν or f)

It is the number of vibrations performed by a source in unit time, so the number of crests (or troughs or compressions, or rarefactions) that pass a fixed point in unit time is called the **frequency** (ν or f). It is the reciprocal of the time period T and has SI unit hertz (Hz). If v is the speed of the wave, then $v = \nu\lambda = f\lambda$

Amplitude (A)

Amplitude is maximum displacement suffered by particles of a medium about their mean position.

Phase (ϕ)

Phase of a wave is position of a point in time on a wave form. It is expressed in radian or sometimes degree. Please keep in mind that $180° = \pi$ radian

Path Difference (Δx)

It is the difference in the path travelled by two waves. It is generally measured in terms of the wavelength of the associated waves. It is measured in metre.

Phase Difference ($\Delta\phi$)

It is difference in the phase travelled by two waves. It is generally measured in terms of path difference of the associated waves. It is measured in radian.

ILLUSTRATION 1

Calculate the velocity of sound in a gas, in which the difference in frequencies of two waves of wavelength 1 m and 1.01 m is 4 Hz.

SOLUTION

Let frequencies of two waves be f_1 and f_2. Then

$$f_1 - f_2 = 4.$$

Since, $v = f\lambda$, so we have

$$\frac{v}{\lambda_1} - \frac{v}{\lambda_2} = v\left(\frac{1}{1.0} - \frac{1}{1.01}\right) = 4$$

$$\Rightarrow \quad v = \frac{4 \times 1.01}{0.01} = 404 \text{ ms}^{-1}$$

RELATION BETWEEN PATH DIFFERENCE (Δx) AND PHASE DIFFERENCE ($\Delta\phi$)

In wave motion, the wave travels a distance λ in one period and there is a phase difference of 2π after one period. Also, we observe that a

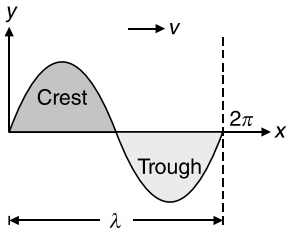
path difference of λ corresponds to phase of 2π

path difference of 1 corresponds to phase of $\frac{2\pi}{\lambda}$

path difference of Δx corresponds to phase of

$$\Delta\phi = \left(\frac{2\pi}{\lambda}\right)\Delta x \text{ OR } \Delta x = \left(\frac{\lambda}{2\pi}\right)\Delta\phi$$

where Δx is in meter and $\Delta\phi$ is in radian.



The term $\frac{2\pi}{\lambda} = k$ is called the Propagation constant.

$$\Rightarrow \quad \Delta\phi = \left(\frac{2\pi}{\lambda}\right)\Delta x = k\Delta x$$

ILLUSTRATION 2

A wave of frequency 500 Hz has a wave velocity of 350 ms^{-1}. Calculate the distance between two points which are 60° out of phase. Also calculate the phase difference between two points 10^{-3} s apart.

SOLUTION

Since, $\lambda = \frac{v}{f} = \frac{350}{500} = 0.7$ m

So, $\Delta\phi = \left(\frac{2\pi}{\lambda}\right)\Delta x$

$$\Rightarrow \quad \Delta x = \frac{\lambda(\Delta\phi)}{2\pi} = \frac{(0.7)(\pi/3)}{(2\pi)} = 0.116 \text{ m}$$

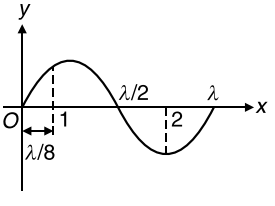
Also, $\Delta\phi = \omega\Delta t$

$$\Rightarrow \quad \Delta\phi = \left(\frac{2\pi}{T}\right)\Delta t = (2\pi f)\Delta t$$

$$\Rightarrow \quad \Delta\phi = (2\pi)(500)(10^{-3}) = \pi$$

ILLUSTRATION 3

Calculate the phase difference between the particle 1 and 2 located as shown in Figure.



SOLUTION

Path difference between the particles is

$$\Delta x = \left(\frac{\lambda}{2} - \frac{\lambda}{8}\right) + \frac{\lambda}{4} = \frac{5\lambda}{4}$$

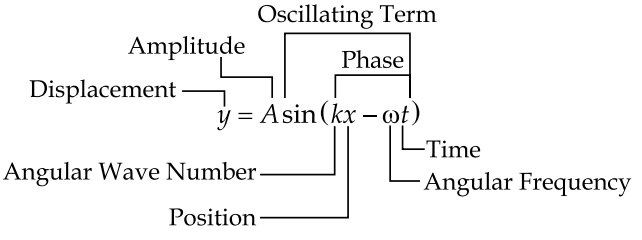
$$\Rightarrow \quad \Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}\left(\frac{5\lambda}{8}\right) = \frac{10\pi}{8} = \frac{2\pi}{5}$$

EQUATION OF A HARMONIC WAVE

When a wave travels through a medium, the particles of the medium oscillate about their mean positions. If oscillations are simple harmonic, the wave is called a **Harmonic Wave**. Such waves are generated by sources that execute S.H.M. Consider a one-dimensional wave travelling towards the positive direction of x-axis. The displacement y of a particle at path difference x at time t is given by

$$y = A\sin(\omega t - kx)$$

where, $\omega = 2\pi/T$ and $k = 2\pi/\lambda$ are respectively called as the Angular Frequency and **Angular Wave Number (also called as Propagation Constant)**.



Two more alternative forms of this equation are

$$y = A\sin\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)\right]$$

$$y = A\sin\left[\frac{2\pi}{\lambda}(vt - x)\right]$$

If a wave is travelling towards the negative x-axis then x is replaced by $-x$, i.e.,

$$y = A\sin(\omega t + kx)$$

$$y = A\sin\left[2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)\right]$$

$$y = A\sin\left[\frac{2\pi}{\lambda}(vt + x)\right]$$

ILLUSTRATION 4

The equation of a progressive wave is $y = 1.5\sin(328t - 1.27x)$, where y, x are in cm and t is in second. Calculate the amplitude, frequency, time period and wavelength of the wave.

SOLUTION

Comparing with standard equation of a progressive wave we get, amplitude $A = 1.5$ cm, angular frequency $\omega = 2\pi/T = 328$ $rads^{-1}$ and propagation constant $k = 2\pi/\lambda = 1.27$ $radcm^{-1}$.

$$\Rightarrow \quad T = \frac{2\pi}{328} = \frac{2\times 3.14}{328} = 0.019 \text{ s}$$

So, frequency $f = \frac{1}{T} = \frac{328}{2\pi} = \frac{328}{6.28} = 52.23$ Hz

$\Rightarrow \quad \lambda = \frac{2\pi}{1.27} = \frac{2\times 3.14}{1.27} = 4.94$ cm

CHARACTERISTIC WAVE EQUATION

The equation of a plane progressive wave is

$$y = A\sin(\omega t - kx)$$

The particle velocity v_P is given by

$$v_P = \frac{\partial y}{\partial t} = A\omega\cos(\omega t - kx) \quad \ldots(1)$$

Slope of displacement curve or strain is given by

$$\frac{\partial y}{\partial x} = -Ak\cos(\omega t - kx) \quad \ldots(2)$$

Differentiating (1) w.r.t. t,

$$\frac{\partial^2 y}{\partial t^2} = A\omega^2\sin(\omega t - kx) \quad \ldots(3)$$

Differentiating (2), w.r.t. x,

$$\frac{\partial^2 y}{\partial x^2} = -Ak^2\sin(\omega t - kx) \quad \ldots(4)$$

Dividing (3) by (4), we have

$$\frac{\partial^2 y/\partial t^2}{\partial^2 y/\partial x^2} = \frac{\omega^2}{k^2} = v^2$$

$$\Rightarrow \quad \frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \quad \ldots(5)$$

This is the differential equation of wave motion, travelling with speed v.

The general solution of this equation is of the form

$$y(x,\ t) = f(ax \pm bt) \quad \ldots(6)$$

Thus, any *finite function* of x and t which satisfies equation (5) or which can be written as equation (6) represents a wave. The only condition is that it should be finite everywhere and at all times. Further, if these conditions are satisfied, then speed of wave (v) is given by,

$$v = \frac{|\text{Coefficient of } t|}{|\text{Coefficient of } x|} = \frac{b}{a}$$

The plus $(+)$ sign between ax and bt implies that the wave is travelling along positive x-direction and minus $(-)$ sign shows that it is travelling along negative x-direction.

Problem Solving Technique(s)

Analytically, any finite function of space and time, i.e., any $y(x,t) = y$ which satisfies $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$ must represent the progressive wave equation.

Functions such as $y = A\sin(\omega t)$ or $y = A\sin(kx)$ do not satisfy above equation, hence do not represent waves.
On the other hand, functions such as

$A\sin(kx - \omega t)$, $A\sin(kx)\sin(\omega t)$,

$A\sin(kx - \omega t) + B\cos(kx + \omega t)$, $\sqrt{ax + bt}$,

$(ax - bt)^2$, $Ae^{-B(x-vt)^2}$ Or $A\cos^2(kx - \omega t)$

all satisfy the wave equation, and hence these are wave functions. Also note that, for a function to be wave function, the three quantities x, t and v must appear in the combinations $(x - vt)$ or $(x + vt)$. Thus, $(x - vt)^2$ is acceptable but $(x^2 - v^2t^2)$ is not. Negative sign between t and x implies that the wave is travelling along positive x-axis and vice-versa.

ILLUSTRATION 5

Which of the following four functions (out of 1, 2, 3 and 4) represent travelling waves?

1. $y = (x + 5t)^3$ **2.** $y = \tan(2x + 3t)$

3. $y = e^{-(4t+2x)^2}$ **4.** $y = \frac{1}{x + 3t}$

SOLUTION

All these functions are of type $y = f(ax + bt)$.
However, function 1 is infinite for large values of x and t, function 2 is infinite when $2x + 3t$ is close to $\frac{\pi}{2}$ and function 4 is infinite for very small negative value of $(4t + 2x)$.

Only function 3 i.e., $y = e^{-(4t+2x)}$ is finite for all x and t, hence only function 3 is an appropriate representation of a plane progressive wave.

ILLUSTRATION 6

Does the function $y = y_0 e^{-(x-vt)^2/k}$ represents a travelling wave? Also check mathematically.

SOLUTION

Yes, the function is of the form $f(x - vt)$ and is defined for every value of x and t. Since,

$$y = y_0 e^{-(x-vt)^2/k}$$

$$\Rightarrow \quad \frac{\partial y}{\partial t} = \frac{2y_0 v}{k}(x - vt)e^{-(x-vt)^2/k}$$

$$\Rightarrow \quad \frac{\partial^2 y}{\partial t^2} = \frac{2y_0 v^2}{k}e^{-(x-vt)^2/k}\left[-1 + \frac{2}{k}(x - vt)^2\right] \quad \ldots(1)$$

Also, $y = y_0 e^{-(x-vt)^2/k}$

$$\Rightarrow \quad \frac{\partial y}{\partial x} = -\frac{2y_0}{k}(x - vt)e^{-(x-vt)^2/k}$$

$$\Rightarrow \quad \frac{\partial^2 y}{\partial x^2} = \frac{2y_0}{k}e^{-(x-vt)^2/k}\left[-1 + \frac{2}{k}(x-vt)^2\right] \qquad \ldots(2)$$

Comparing (1) and (2), we get $\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$

Hence, the given function also satisfies the differential form of wave equation.

PARTICLE VELOCITY, WAVE SLOPE AND PARTICLE ACCELERATION IN A SINUSOIDAL WAVE

In a plane progressive harmonic wave the particles of the medium oscillate simple harmonically about their mean position. Therefore, all formulae that we have read in SHM also apply to particles here. For example, maximum particle velocity is $\pm A\omega$ at mean position and it is zero at extreme positions etc. Similarly, maximum particle acceleration is $\pm A\omega^2$ at extreme positions and zero at mean position. *However, wave velocity is different from the particle velocity. This depends on certain characteristics of the medium. Unlike the particle velocity which oscillates simple harmonically (between $+A\omega$ and $-A\omega$) the wave velocity is constant for given characteristics of the medium. The particle velocity is* $v_P = \frac{\partial y}{\partial t}$ *and wave slope (also called as strain) is* $\frac{\partial y}{\partial x}$.

Consider a sinusoidal wave

$$y = y(x, t) = A\sin(kx - \omega t) \qquad \ldots(1)$$

Then, $v_P = \frac{\partial y}{\partial t} = \frac{\partial y(x,t)}{\partial t} = -A\omega\cos(kx - \omega t)$ $\ldots(2)$

and $\frac{\partial y}{\partial x} = \frac{\partial y(x,t)}{\partial x} = Ak\cos(kx - \omega t)$ $\ldots(3)$

From (2) and (3), we get

$$\frac{\partial y}{\partial t} = -\left(\frac{\omega}{k}\right)\frac{\partial y}{\partial x} = -v\left(\frac{\partial y}{\partial x}\right) \qquad \left\{\because \frac{\omega}{k} = v\right\}$$

$$\Rightarrow \quad v_P = -v(\text{Wave Slope}) \qquad \ldots(4)$$

i.e., particle velocity at a given position and time is equal to negative of the product of wave velocity with slope of the wave at that point at that instant.

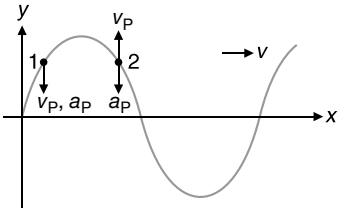
Particle acceleration is given by

$$a_P = \frac{\partial v}{\partial t} = \frac{\partial^2 y}{\partial t^2} = -\omega^2 A\sin(kx - \omega t) = -\omega^2 y(x, t)$$

So, the acceleration of the particle equals $-\omega^2$ times its displacement, similar to the result we obtained for SHM.

$$\Rightarrow \quad a_P = -\omega^2(\text{displacement}) \qquad \ldots(5)$$

Figure shows the velocity (v_P) and acceleration (a_P) given by Equations (4) and (5) for two points 1 and 2 on a string when a sinusoidal wave is travelling in it along the positive x-direction.



At 1, slope of the curve is positive. Hence from equation (4) particle velocity (v_P) is negative or downwards. Similarly, displacement of the particle is positive, so from equation (5) acceleration will be negative or downwards.
At 2, slope is negative while displacement is positive. Hence v_P will be positive (upwards) and a_P is negative (downwards). Also note that the direction of v_P will change if the wave travels along negative x-direction.

ILLUSTRATION 7

If $y(x, t) = 0.05\sin\left(\frac{\pi}{2}(10x - 40t) - \frac{\pi}{4}\right)$ m, then find the wavelength, frequency and wave velocity. Also calculate the particle velocity and acceleration at $x = 0.5$ m and $t = 0.05$ s

SOLUTION

The equation may be rewritten as,

$$y(x, t) = 0.05\sin\left(5\pi x - 20\pi t - \frac{\pi}{4}\right) \text{ m}$$

Comparing this with equation of plane progressive harmonic wave, $y(x, t) = A\sin(kx - \omega t + \phi)$ we get

Wave number, $k = \frac{2\pi}{\lambda} = 5\pi$ radm^{-1} i.e., $\lambda = 0.4$ m

Angular frequency, $\omega = 2\pi f = 20\pi$ rads^{-1}

$$\Rightarrow \quad f = 10 \text{ Hz}$$

Wave velocity, $v = f\lambda = \frac{\omega}{k} = 4$ ms^{-1} in $+x$ direction

The particle velocity and acceleration are given by

$$v_P = \frac{\partial y}{\partial t} = -(20\pi)(0.05)\cos\left(\frac{5\pi}{2} - \pi - \frac{\pi}{4}\right)$$

$$\Rightarrow \quad v_P = \frac{\partial y}{\partial t} = 2.22 \text{ ms}^{-1}$$

$$a_P = \frac{\partial^2 y}{\partial t^2} = -(20\pi)^2(0.05)\sin\left(\frac{5\pi}{2} - \pi - \frac{\pi}{4}\right)$$

$$\Rightarrow \quad a_P = 140 \text{ ms}^{-2}$$

ILLUSTRATION 8

A sinusoidal wave travelling in the positive direction on a stretched string has amplitude 2 cm, wavelength 1 m and wave velocity 5 ms^{-1}. At $x=0$ and $t=0$, it is given that $y=0$ and $\frac{\partial y}{\partial t}<0$. Find the wave function $y(x,\ t)$.

SOLUTION

We start with a general form for a wave, moving along $+x$ axis, i.e.,

$$y(x,\ t)=A\sin(\omega t-kx+\phi)$$

The amplitude given is $A=2$ cm $=0.02$ m

The wavelength is $\lambda=1$ m, so $k=\frac{2\pi}{\lambda}=2\pi$ $radm^{-1}$

Angular frequency $\omega=vk=10\pi$ $rads^{-1}$ $\{\because\ v=\omega/k\}$

$\Rightarrow$ $y(x,t)=(0.02)\sin[2\pi(5t-x)+\phi]$

For $x=0$ and $t=0$, we have $y=0$ and $\partial y/\partial t<0$

So, $0.02\sin\phi=0$ $\{\because\ y=0\}$

and $-0.2\pi\cos\phi<0$ $\{\because\ \partial y/\partial t<0\}$

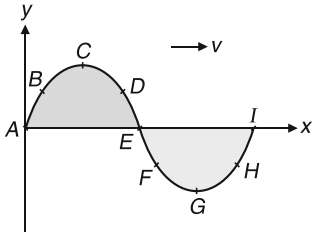
From these conditions, we may conclude that

$\phi=2n\pi$ where $n=0, 2, 4, 6,\ldots..$

$\Rightarrow$ $y(x,\ t)=0.02\sin(10\pi t-2\pi x)$

ILLUSTRATION 9

A transverse wave is travelling along a string from left to right. Figure represents shape of string (snap shot) at a given instant. At this instant, which points have an upward velocity, have downward velocity, have zero velocity and have maximum magnitude of velocity?



SOLUTION

For a travelling wave, the particle velocity v_P and the wave velocity v are related by

$$v_P=-v(\text{Wave Slope})$$

That is, the particle velocity is proportional to the negative of the slope of y-x curve.

Points having upward velocity, means v_P must be positive. It implies the slope must be negative, which is at points D, E and F.

Points having downward velocity, means v_P must be negative. It implies the slope must be positive, which is at points A, B, H and I.

Points having zero velocity, means the slope must be zero, which is at points C and G.

Points having maximum velocity, means the magnitude of slope must be maximum, which is points A, E and I.

The shape of the string after a small time Δt is shown by dotted line. The distances moved by different points in this time are indicated by arrows. Thus, the length of an arrow is proportional to magnitude of the velocity of that point. This diagram confirms the above conclusions.

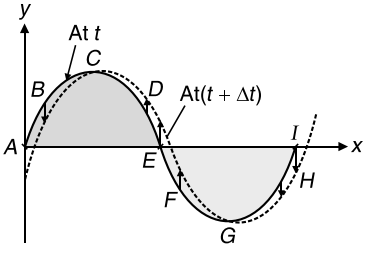


ILLUSTRATION 10

For a wave given by $y=a\sin(\omega t-kx)$, four points A at $x=0$, B at $x=\pi/4k$, C at $x=\pi/2k$ and D at $x=3\pi/4k$ are taken. For a particle at each of these points at $t=0$, describe whether the particle is moving or not and in what direction and describe whether the particle is speeding up, slowing down or instantaneously not accelerating.

SOLUTION

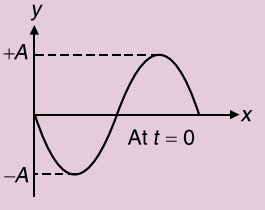
Since $y=a\sin(\omega t-kx)$, so particle velocity (v_P) and particle acceleration (a_P) are given by

$$v_P(x,\ t)=\frac{\partial y}{\partial t}=a\omega\cos(\omega t-kx)$$

$$a_P(x,\ t)=\frac{\partial^2 y}{\partial t^2}=-a\omega^2\sin(\omega t-kx)$$

For point *A*, at $t=0$ and $x=0$, we get $v_P=+a\omega$ and $a_P=0$ i.e., particle is moving up but its acceleration is zero.

Direction of velocity can be obtained by an alternate method. At $t=0$, $y=A\sin(-kx)=-A\sin kx$ i.e., y-x graph is as shown in Figure.



At $x=0$, slope is negative. Therefore, particle velocity is positive $(v_P=-v\times\text{slope})$ as the wave is travelling along positive x-direction.

For point *B*, at $t = 0$ and $x = \pi/4k$, we get

$$v_P = a\omega\cos(-\pi/4) = +a\omega/\sqrt{2} \qquad \{\because kx = \pi/4\}$$

$$a_P = -a\omega^2\sin(-\pi/4) = +a\omega^2/\sqrt{2}$$

Velocity of particle is positive, i.e., the particle is moving upwards (along positive y-direction). Since v_P and a_P are in the same direction (both are positive), so the particle is speeding up.

For point *C*, at $t = 0$ and $x = \pi/2k$, we get

$$v_P = a\omega\cos(-\pi/2) = 0 \qquad \{\because kx = \pi/2\}$$

$$a_P = -a\omega^2\sin(-\pi/2) = a\omega^2$$

i.e., particle is stationary or at its extreme position $(y = -a)$. So, it is speeding up at this instant.

For point *D*, at $t = 0$ and $x = 3\pi/4k$, we get

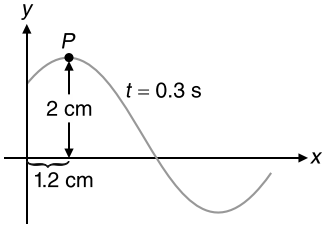
$$v_P = a\omega\cos(-3\pi/4) = -a\omega/\sqrt{2} \qquad \{\because kx = 3\pi/4\}$$

$$a_P = -a\omega^2\sin(-3\pi/4) = +a\omega^2/\sqrt{2}$$

Velocity of particle is negative i.e., the particle is moving downwards. Since v_P and a_P are in opposite directions, so, the particle is slowing down.

ILLUSTRATION 11

Figure shows a snapshot of a sinusoidal travelling wave taken at $t = 0.3$ s. The wavelength is 7.5 cm and the amplitude is 2 cm. If the crest P was at $x = 0$ at $t = 0$, write the equation of travelling wave.



SOLUTION

Given $\lambda = 7.5$ cm, so $k = \dfrac{2\pi}{\lambda} = 0.84\ \text{cm}^{-1}$

Since wave has travelled 1.2 cm in 0.3 s. So

Wave Speed $v = \dfrac{1.2}{0.3} = 4\ \text{cms}^{-1}$

$$\Rightarrow \quad \omega = vk = 3.36\ \text{rads}^{-1} \qquad \{\because v = \omega/k\}$$

Since the wave is travelling along positive x-direction and crest (maximum displacement) is at $x = 0$ at $t = 0$, we can write the wave equation as,

$$y(x, t) = A\cos(kx - \omega t)$$

$$\Rightarrow \quad y(x, t) = A\cos(\omega t - kx) \qquad \{\because \cos(-\theta) = \cos\theta\}$$

So, the required equation of the wave is

$$y(x,\ t) = 2\cos(0.84x - 3.36t)\ \text{cm}$$

GROUP VELOCITY

Speed of a single wave travelling in a medium is

$$v = v\lambda = \frac{\omega}{k}$$

and is called the **wave velocity** or **phase velocity**.
But if a number of waves having slightly different wavelengths travel in same medium, then medium is said to be dispersive and energy is not transmitted in medium with wave velocity or phase velocity, but it travels with **group velocity** v_g given by

$$v_g = \frac{d\omega}{dk} = v - \lambda\frac{dv}{d\lambda}$$

ILLUSTRATION 12

Waves passing through a certain medium have a dispersion relation, $\omega(k) = \sqrt{\omega_0^2 + \alpha^2k^2}$. Here α and ω_0 are constants. Find phase velocity and group velocity in this medium.

SOLUTION

The phase velocity is,

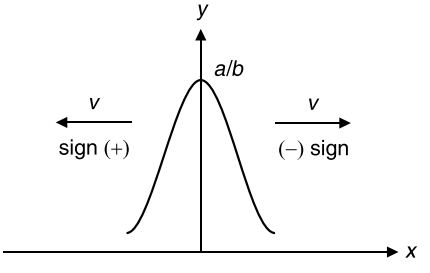
$$v = \frac{\omega}{k} = \frac{1}{k}\sqrt{\omega_0^2 + \alpha^2k^2}$$

The group velocity is given by,

$$v_g = \frac{d\omega}{dk} = \frac{\alpha^2k}{\sqrt{\omega_0^2 + \alpha^2k^2}} = \left(\frac{k}{\omega}\right)\alpha^2 = \frac{\alpha^2}{v}$$

A SYMMETRICAL WAVE PULSE

A wave pulse is a disturbance localised only in a small part of space at a given instant and its shape does not change during propagation. Pulse will be symmetric if at $t = 0$, $y(x) = y(-x)$



A symmetrical wave pulse shown has an equation

$$y = \frac{a}{b + (x \mp vt)^2}$$

where, a and b are constants. Speed of pulse is

$$v = \frac{|\text{Coefficient of } t|}{|\text{Coefficient of } x|}$$

Please note that, here the coefficient of x must be 1 and then the amplitude of the wave pulse is a/b. Negative sign is for pulse propagating along positive x-axis, positive sign for a pulse propagating along negative x-axis and v is the pulse speed.

ILLUSTRATION 13

A wave pulse on a horizontal string is represented by the function $y(x,\ t) = \frac{5}{1 + (x - 2t)^2}$ (cgs units). Plot this function at $t = 0$, 2.5 s and 5 s.

SOLUTION

At the given times, the function representing the wave pulse is given by

$$y(x,\ 0) = \frac{5}{1 + x^2}$$

$$\Rightarrow \quad y(x,\ 2.5\text{ s}) = \frac{5}{1 + (x - 5)^2}$$

$$\Rightarrow \quad y(x,\ 5\text{ s}) = \frac{5}{1 + (x - 10)^2}$$

The maximum of $y(x,\ 0)$ is 5 cm which it is located at $x = 0$ and the pulse is also centred at $x = 0$. At $t = 2.5$ s and 5 s, the centre of the pulse has moved to $x = 5$ cm and 10 cm, respectively. So, in each 2.5 s time interval, the pulse moves 5 cm in the positive x-direction. Its velocity is therefore +2 cms^{-1} a value that is also evident from the given function shown in Figure.

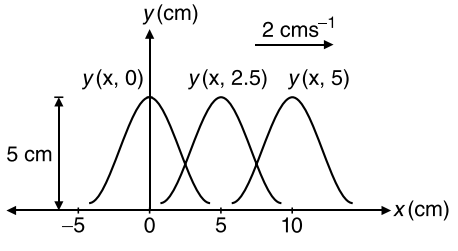


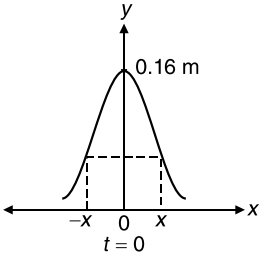
ILLUSTRATION 14

If $y(x,\ t) = \frac{0.8}{\left[(4x + 5t)^2 + 5\right]}$ represents a moving pulse where x and y are in metre and t in second. Then select the correct alternative(s).

(A) Pulse is moving in positive x-direction.
(B) In 2 s it will travel a distance of 2.5 m.
(C) Its maximum displacement is 0.16 m.
(D) It is a symmetric pulse.

SOLUTION

Shape of pulse at $t = 0$, $x = 0$ is shown in figure.



$$y(0,\ 0) = \frac{0.8}{5} = 0.16\text{ m}$$

From the figure it is clear that $y_{max} = 0.16$ m

From the given equation, we see that at $t = 0$

$$y(x) = \frac{0.8}{16x^2 + 5} \text{ and } y(-x) = \frac{0.8}{16x^2 + 5}$$

Since $y(x) = y(-x)$, so pulse is symmetric.

Speed of pulse is

$$v = \frac{|\text{Coefficient of } t|}{|\text{Coefficient of } x|} = \frac{5}{4} = 1.25\text{ ms}^{-1}$$

So, it will travel a distance of 2.5 m in 2 s

Also, note that at $t = 1$ s, $x = -1.25$ m and at $t = 2$ s, $x = -2.5$ m, so we can say that the wave is travelling along negative x direction. **So, OPTIONS (B), (C) and (D) are correct.** Also, we can compare this with

$$y = \frac{a}{b + (x \pm vt)^2}, \text{ where } a = \frac{0.8}{16} \text{ and } b = \frac{5}{16}$$

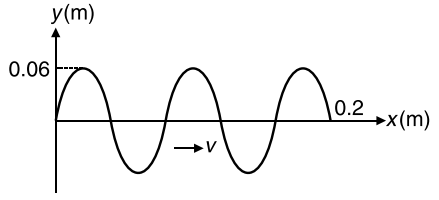
to get the desired results.

Test Your Concepts-I

Based on Wave Equation & Properties

(Solutions on page H.233)

1. For the wave shown in figure, write the wave equation if its position is shown at $t = 0$. Given, speed of wave is $v = 300\ \text{ms}^{-1}$.



2. A transverse harmonic wave of amplitude 0.02 m is generated at $x = 0$ at one end of a long horizontal string by a tuning fork of frequency 500 Hz. At a given instant of time, the displacement of the particle at $x = 0$ is zero, the displacement of a particle at $x = 0.1$ m is $-0.01\sqrt{3}$ m and that of a particle at $x = 0.8$ m is $0.01\sqrt{3}$. Find the velocity of the wave.

3. A pulse is propagating on a long stretched string along its length taken as positive x-axis. Shape of the string at $t = 0$ is given by

$$y = \begin{cases} \sqrt{a^2 - x^2}, & \text{when} \quad |x| \geq a \\ 0, & \text{when} \quad |x| \leq 0 \end{cases}$$

Study the propagation of this pulse, if it is travelling in positive x-direction with speed v.

4. Verify that the equation, $y = a\sin(\omega t - kx)$ satisfies the wave equation $\dfrac{\partial^2 y}{\partial t^2} = v^2 \dfrac{\partial^2 y}{\partial x^2}$. Find speed of wave and the direction in which it is travelling.

5. A progressive wave of frequency 500 Hz is travelling with a velocity of 360 ms^{-1}. How far apart are two points 60° out of phase?

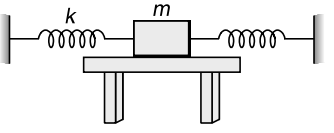
6. Out of the following functions, which one(s) represent(s) a wave?

(a) $\dfrac{1}{x + vt}$ (b) $\ln(x + vt)$

(c) $(x - vt)^2$ (d) $e^{-(x-vt)^2}$

7. A wave pulse moving to the right along the x-axis is represented by the wave function $y(x, t) = \dfrac{2}{(x - 3t)^2 + 1}$, where x and y are measured in cm and t is measured in seconds. Plot the wave function at $t = 0$, $t = 1$ s and $t = 2$ s.

8. One end of each of two identical springs, each of force-constant 0.5 Nm^{-1}, are attached on the opposite sides of a wooden block of mass 0.01 kg. The other ends of the springs are connected to separate rigid supports such that the springs are unstretched and are collinear in a horizontal plane. as shown in Figure.



To the wooden piece is fixed a pointer which touches a vertically moving plane paper. The wooden piece, kept on a smooth horizontal table is now displaced by 0.02 m along the line of springs and released. If the speed of paper is 0.1 ms^{-1}, find the equation of path traced by pointer on paper and the distance between two consecutive maxima on this path.

9. Equation of a transverse wave in a stretched string is given by $y = 2\sin\left[2\pi\left(\dfrac{x}{30} - \dfrac{t}{0.01}\right)\right]$, where y, x are in cm, t is in second. Calculate the amplitude, frequency, wavelength and wave velocity.

10. A travelling wave pulse is given by, $y = \dfrac{10}{5 + (x + 2t)^2}$, where x and y are in metre and t in second. In which direction and with what velocity is the pulse propagating? What is the amplitude of pulse?

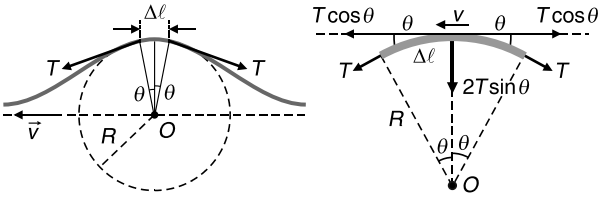
11. The wave function of a pulse is given by $y = \dfrac{3}{2 + (x - 4t)^2}$, where y is in metres and t is seconds. Determine the wave velocity of the pulse and indicate the direction of propagation of the wave.

12. Two pulses travelling on a same string are described by functions $y_1 = \dfrac{5}{(3x - 4t)^2 + 2}$ and $y_2 = \dfrac{-5}{(3x + 4t - 6)^2 + 2}$. In which direction does each pulse travel? At what time and point do the two waves cancel?

SPEED OF A TRANSVERSE WAVE IN A STRING

Let us consider a single symmetrical pulse travelling along a string with a speed v from left to right. If the amplitude of the pulse is small compared to the length of the string, the tension T will be approximately constant along the string.

To obtain the speed v of a wave on a stretched string, it is convenient to select a reference frame in which the pulse remains stationary i.e., we run along with the pulse keeping it constantly in our view. In this reference frame, it will appear as if the string is moving past us from right to left with speed v as shown in Figure.



Consider a small string element of length Δl, mass Δm within the pulse, such that it forms an arc of a circle or radius R and subtends an angle 2θ at the centre of that circle.

At the instant shown, this element is moving with speed v in a circular path, so it has a centripetal acceleration v^2/R. Forces acting on the segment are the tension T at each end. Horizontal components of tension T are equal and opposite, so they cancel. Vertical components of tension T point radially inward towards centre of the circular arc and provide the centripetal acceleration such that

$$2T\sin\theta = \frac{(\Delta m)v^2}{R} \qquad \ldots(1)$$

For small θ, $\sin\theta \approx \theta$ and if μ is the mass per unit length of the string, then mass of the element of length $\Delta l = 2R\Delta\theta$ is

$$\Delta m = \mu\Delta l = 2\mu R\theta$$

So, from equation (1), we get

$$2T\theta = (2\mu R\theta)\left(\frac{v^2}{R}\right)$$

$$\Rightarrow \quad v = \sqrt{\frac{T}{\mu}}$$

Conceptual Note(s)

If D is the diameter of the string, ℓ is its length and ρ is its density, then

$$\Rightarrow \quad m = \frac{\pi(D/2)^2\ell\rho}{\ell} = \frac{\pi D^2\rho}{4}$$

$$\Rightarrow \quad v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\pi D^2\rho/4}} = \frac{2}{D}\sqrt{\frac{T}{\pi\rho}} \qquad \ldots(1)$$

Since μ(the mass per unit length of string) is

$$\mu = \frac{m}{\ell} = \frac{A\ell\rho}{\ell} = A\rho$$

where A is area of cross-section of string, then from equation (1), we get

$$v = \sqrt{\frac{T}{\rho A}} = \sqrt{\frac{\text{Stress}}{\rho}} \qquad \left\{\because \frac{T}{A} = \text{Stress}\right\}$$

ILLUSTRATION 15

A harmonic wave with a wavelength of 20 cm, an amplitude of 3 cm and a velocity of 2 ms^{-1} travels on a string to the left along the x-axis. The mass per unit length of the string is 0.25 gm^{-1}.

(a) What are the frequency and period of the wave motion?
(b) How much is the tension in the string?
(c) What is the position function for the wave?

SOLUTION

(a) The frequency, $f = \frac{v}{\lambda} = \frac{2}{0.2} = 10$ Hz

The period $T = \frac{1}{f} = \frac{1}{10} = 0.1$ s

(b) We know that $v = \sqrt{\frac{T}{\mu}}$

$\Rightarrow \quad T = \mu v^2 = (0.25\times10^{-3})\times(2)^2 = 10^{-3}$ N

(c) In order to write the wave equation, we need to know and ω

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.2} = 10\pi \text{ radm}^{-1}$$

and $\omega = 2\pi f = 2\pi(10) = 20\pi$ rads^{-1}

Thus, the wave equation is $y = A\sin(kx + \omega t)$

$\Rightarrow \quad y = 0.03\sin[(10\pi)x + (20\pi)t]$

where x and y are in metre and t is in second.

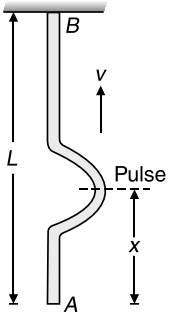
ILLUSTRATION 16

A uniform rope of mass 0.1 kg and length 2.45 m hangs from a ceiling. Calculate the speed of transverse wave in the rope at a point 0.5 m distant from the lower end. Also calculate the time taken by a transverse wave to travel the full length of the rope. If a particle is dropped from the ceiling at the instant wave starts from the free end of rope, then calculate the distance from the free end where the wave crosses the particle. Take $g = 9.8$ ms^{-2}.

SOLUTION

If M is the mass of the string of length L, the mass of length x of the string will be

$$M_1 = \mu x = \frac{M}{L}x$$



The tension in the string at a distance x from lower end A is $T = M_1 g = \left(\frac{M}{L}x\right)g$

$$\Rightarrow \quad v = \sqrt{\frac{T}{m}} = \sqrt{\frac{Mxg}{L(M/L)}} = \sqrt{xg} \qquad \ldots(1)$$

For $x = 0.5$ m, we get $v = \sqrt{0.5 \times 9.8} = 2.21\ \text{ms}^{-1}$

The tension and hence the velocity of the wave is different at different points of the string. So, if at point x the wave travels distance dx in time dt,

$$v = \frac{dx}{dt} = \sqrt{gx} \qquad \{\text{from (1)}\}$$

$$\Rightarrow \quad t = \int dt = \int_0^L \frac{dx}{\sqrt{gx}} = \frac{1}{\sqrt{g}}\int_0^L x^{-1/2}dx$$

$$\Rightarrow \quad t = 2\sqrt{\frac{L}{g}} = 2\sqrt{\frac{2.45}{9.8}} = 1\ \text{s}$$

Let y be the distance from the free end where the particle crosses the pulse. Then

for the particle, $t = \sqrt{\frac{2(L-y)}{g}} \qquad \left\{\because L - y = \frac{1}{2}gt^2\right\}$

for the pulse, $t = \sqrt{\frac{2y}{g}}$

At the instant of crossing t is same, so we get

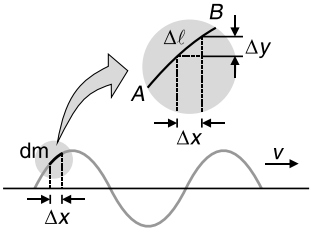
$$\sqrt{\frac{2(L-y)}{g}} = 2\sqrt{\frac{y}{g}}$$

$$\Rightarrow \quad 2L - 2y = 4y$$

$$\Rightarrow \quad y = \frac{2L}{6} = \frac{L}{3} = \frac{2.45}{3} \approx 0.82\ \text{m}$$

ENERGY DENSITY OF A TRANSVERSE WAVE

Consider a transverse wave to propagate in a string. When this wave moves along the string, it carries energy along the direction of travel. To calculate the energy density associated with the wave, let us consider a small segment of the string of lengthΔx as shown in Figure.



While oscillating, this segment has potential energy due to its stretching and kinetic energy due to its motion. The potential energy of a segment AB is the work done by the tension T in stretching the string, which is $T(\Delta l - \Delta x)$, where Δl is the length of the stretched segment and Δx is its original length. From the figure we see that

$$\Delta l = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

$$\Rightarrow \quad \Delta l = \Delta x\left[1 + \left(\frac{\Delta y}{\Delta x}\right)^2\right]^{\frac{1}{2}} \approx \Delta x\left[1 + \frac{1}{2}\left(\frac{\Delta y}{\Delta x}\right)^2\right]$$

$$\Rightarrow \quad \Delta l - \Delta x \approx \frac{1}{2}\left(\frac{\Delta y}{\Delta x}\right)^2 \Delta x$$

So, potential energy of this segment is

$$U = \frac{1}{2}T(\Delta l - \Delta x)^2 = \frac{1}{2}T\Delta x\left(\frac{\Delta y}{\Delta x}\right)^2$$

For very small increments i.e., when $\Delta x \to 0$, then $\frac{\Delta y}{\Delta x} \to \frac{\partial y}{\partial x}$

(since y is a simultaneous function of x and t, so this partial derivative indicates that we are interested in the variation of y with position at a fixed time). So, potential energy is

$$dU = \frac{1}{2}Tdx\left(\frac{\partial y}{\partial x}\right)^2$$

Potential energy per unit length is

$$\frac{dU}{dx} = \frac{1}{2}T\left(\frac{\partial y}{\partial x}\right)^2 \qquad \ldots(1)$$

The kinetic energy dK of the segment is

$$dK = \frac{1}{2}dm\left(\frac{\partial y}{\partial t}\right)^2$$

So, kinetic energy per unit length is

$$\frac{dK}{dx} = \frac{1}{2}\left(\frac{dm}{dx}\right)\left(\frac{\partial y}{\partial t}\right)^2 = \frac{1}{2}\mu\left(\frac{\partial y}{\partial t}\right)^2$$

Since, $\frac{\partial y}{\partial t} = -v\left(\frac{\partial y}{\partial x}\right)$

$$\Rightarrow \quad \frac{dK}{dx}=\frac{1}{2}\mu v^2\left(\frac{\partial y}{\partial x}\right)^2, \text{ where } v=\sqrt{\frac{T}{\mu}}$$

$$\Rightarrow \quad \frac{dK}{dx}=\frac{1}{2}\mu\left(\frac{T}{\mu}\right)\left(\frac{\partial y}{\partial x}\right)^2=\frac{1}{2}T\left(\frac{\partial y}{\partial x}\right)^2 \qquad \ldots(2)$$

The first interesting thing we observe is that the potential energy per unit length is equal to the kinetic energy per unit length of a transverse wave carried by the string.

So, total energy per unit length of the segment is

$$\frac{dE}{dx}=\frac{dU}{dx}+\frac{dK}{dx}=2\left[\frac{1}{2}T\left(\frac{\partial y}{\partial x}\right)^2\right]=T\left(\frac{\partial y}{\partial x}\right)^2$$

$$\Rightarrow \quad \frac{dE}{dx}=T\left(\frac{\partial y}{\partial x}\right)^2=\mu v^2\left(\frac{\partial y}{\partial x}\right)^2 \qquad \left\{\because v=\sqrt{\frac{T}{\mu}}\right\}$$

Assuming $y=A\sin(\omega t-kx)$, we get

$$\frac{\partial y}{\partial x}=-Ak\cos(\omega t-kx)$$

$$\Rightarrow \quad \frac{dE}{dx}=\mu v^2\left(\frac{\partial y}{\partial x}\right)^2=\mu v^2A^2k^2\cos^2(\omega t-kx)$$

$$\Rightarrow \quad \frac{dE}{dx}=\mu\omega^2A^2\cos^2(\omega t-kx) \qquad \left\{\because v=\frac{\omega}{k}\right\}$$

If S be the cross-sectional area of the string, then energy density associated with this segment is

$$\frac{dE}{Sdx}=\frac{\mu\omega^2A^2}{S}\cos^2(\omega t-kx)$$

Since, $\mu=\frac{dm}{dx}=\frac{\rho(Sdx)}{dx}=S\rho$

$$\frac{dE}{Sdx}=\rho\omega^2A^2\cos^2(\omega t-kx) \qquad \{\because \mu=S\rho\}$$

However, a wave can be very long, so the energy associated with each wavelength of the wave is

$$E=\int_0^{\lambda}dE=\mu\omega^2A^2\int_0^{\lambda}\cos^2(\omega t-kx)\,dx$$

If we take a snapshot of the wave at time $t=0$, then energy of a given element is

$$E=\int_0^{\lambda}dE=\mu\omega^2A^2\int_0^{\lambda}\cos^2(kx)\,dx$$

$$\Rightarrow \quad E=\int_0^{\lambda}dE=\frac{\mu\omega^2A^2}{2}\left[\int_0^{\lambda}dx+\int_0^{\lambda}\cos(2kx)\,dx\right]$$

$$\Rightarrow \quad E=\int_0^{\lambda}dE=\frac{\mu\omega^2A^2}{2}\left[\lambda+\frac{1}{2k}\sin(2kx)\Big|_0^{\lambda}\right]$$

$$\Rightarrow \quad E=\int_0^{\lambda}dE=\frac{\mu\omega^2A^2}{2}\left[\lambda+\frac{\sin(2k\lambda)}{2k}\right]$$

Since, $\sin(2k\lambda)=\sin(4\pi)=0 \qquad \{\because k=2\pi/\lambda\}$

$$\Rightarrow \quad E=\int_0^{\lambda}dE=\frac{1}{2}\mu\omega^2A^2\lambda$$

We can also say that the potential energy associated with each wavelength of the wave is equal to the kinetic energy and is given by

$$U=K=\frac{1}{2}\mu\omega^2A^2\int_0^{\lambda}\cos^2(\omega t-kx)\,dx=\frac{1}{4}\mu\omega^2A^2\lambda$$

POWER AND INTENSITY OF WAVE TRAVELLING THROUGH A STRETCHED STRING

If a wave is travelling in a stretched string of mass per unit length μ, then the power of the wave is given by

$$P_{\text{ins}}=\frac{dE}{dt}=\left(\frac{dE}{dx}\right)\left(\frac{dx}{dt}\right)=v\left(\frac{dE}{dx}\right)$$

Since, $\frac{dE}{dx}=\mu\omega^2A^2\cos^2(\omega t-kx)$

$$\Rightarrow \quad P_{\text{ins}}=\mu\omega^2A^2v\cos^2(\omega t-kx)$$

However, average power is given by

$$P_{\text{av}}=\langle P\rangle=\mu\omega^2A^2v\langle\cos^2(\omega t-kx)\rangle$$

Since, $\langle\cos^2(\omega t-kx)\rangle=\langle\sin^2(\omega t-kx)\rangle=1/2$

$$\Rightarrow \quad P_{\text{av}}=\frac{1}{2}\mu\omega^2A^2v$$

Note that $P\propto A^2$ and $P\propto\omega^2$

These results hold for all types of waves.

Intensity (I) is defined as the energy transmitted per second per unit area normal to the direction of propagation of the wave or the power transmitted across a unit area normal to the direction of propagation. Mathematically

$$I=\frac{1}{S}\left(\frac{dE}{dt}\right)=\frac{P}{S}=\left(\frac{dE}{Sdx}\right)\left(\frac{dx}{dt}\right)=uv$$

where, $u=\frac{dE}{Sdx}=\frac{dE}{dV}$ is energy density of the wave.

Since, we have discussed earlier, that energy density (u) of the wave is proportional to $A^2\omega^2$, so

$$I\propto A^2\omega^2$$

When a wavefront advances from a distance r_1 from a point source to a distance r_2, its surface area increases from $4\pi r_1^2$ to $4\pi r_2^2$. If there is no dissipation of energy, then $P=4\pi r_1^2I_1=4\pi r_2^2I_2$,

where I_1 and I_2 being the intensities at distances r_1 and r_2, respectively.

$$\Rightarrow \quad \frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$$

ABOUT AMPLITUDE

For a plane progressive wave, the amplitude remains constant i.e., it does not vary with distance.

For a spherical wave (i.e., wave starting from a point source), the amplitude varies inversely with distance from position of source i.e.,

$$A \propto \frac{1}{r} \qquad \left\{\because I \propto A^2 \propto \frac{1}{r^2}\right\}$$

For a cylindrical wave (i.e., wave starting from a linear source), the amplitude varies inversely as the square root of distance from the axis of source i.e.,

$$A \propto \frac{1}{\sqrt{r}} \qquad \left\{\because I \propto A^2 \propto \frac{1}{r}\right\}$$

ILLUSTRATION 17

A stretched string is forced to transmit transverse waves by means of an oscillator coupled to one end. The string has a diameter of 4 mm. The amplitude of the oscillation is 10^{-4} m and the frequency is 10 Hz. Tension in the string is 100 N and mass density of wire is 4.2×10^3 kgm^{-3}. Calculate the equation of the waves along the string, energy per unit volume of the wave, average energy flow per unit time across any section of the string and power required to drive the oscillator.

SOLUTION

Speed of transverse wave in a string of density ρ, area of cross section S is given by

$$v = \sqrt{\frac{T}{\rho S}} \qquad \{\because \ \mu = \rho S\}$$

$$\Rightarrow \quad v = \sqrt{\frac{100}{(4.2\times10^3)(\pi/4)(4\times10^{-3})^2}}$$

$$\Rightarrow \quad v = 43.53 \text{ ms}^{-1}$$

Since, $\omega = 2\pi f = 20\pi$ rads^{-1} $= 62.83$ rads^{-1}

$$\Rightarrow \quad k = \frac{\omega}{v} = 1.44 \text{ m}^{-1}$$

Equation of wave along the string is given by

$$y(x,\ t) = A\sin(\omega t - kx)$$

$$\Rightarrow \quad y(x,\ t) = 10^{-4}\sin(62.83t - 1.44x)$$

Energy per unit volume of the string is

$$u = \frac{1}{2}\rho\omega^2A^2 = \left(\frac{1}{2}\right)(4.2\times10^3)(62.83)^2(10^{-4})^2$$

$$\Rightarrow \quad u = 8.29\times10^{-2} \text{ Jm}^{-3}$$

Average energy flow per unit time is power, so

$$P = \left(\frac{1}{2}\rho\omega^2A^2\right)(sv) = (u)(sv)$$

$$\Rightarrow \quad P = (8.29\times10^{-2})\left(\frac{\pi}{4}\right)(4\times10^{-3})^2(43.53)$$

$$\Rightarrow \quad P = 4.53\times10^{-5} \text{ W}$$

So, the power required to drive the oscillator is obviously 4.53×10^{-5} W

REFLECTION OF STRING WAVE

If incident wave is $y = a\sin(\omega t - kx)$, then the equation of reflected wave takes the form

$$y = a'\sin(\omega t + kx)$$

where a' is new amplitude of reflected wave.

When string wave is reflected from a denser medium or rigid boundary, the wave suffers an additional phase change of π or a path change of $\lambda/2$. The equation of reflected wave in this case takes the form

$$y = a'\sin(\omega t + kx + \pi) = -a'\sin(\omega t + kx)$$

A mechanical wave is reflected and refracted at a boundary separating two media according to the usual laws of reflection and refraction.

Conceptual Note(s)

So, in reflection the following rules apply

(a) A wave coming from a medium where the wave velocity is larger, i.e., from a rarer to a denser medium, suffers a phase change of π or a path change of $\lambda/2$ (called **STOKE'S LAW**) on reflection.

(b) A wave coming from a denser to a rarer medium is reflected without any phase change.

(c) These rules apply in reflection of light also.

When a pulse travelling along a string reaches the end, it is reflected. If the end is fixed as shown in Figure 1, the pulse returns inverted.

This is because as the leading edge reaches the wall, the string pulls up the wall. According to Newton's Third Law, the wall will exert an equal and opposite force on the string at all instants. This force is therefore, directed first down and then up. It produces a pulse that is inverted but otherwise identical to the original.

The motion of free end can be studied by tying a ring at the end of string so that the ring can slide smoothly on the rod. The ring and rod maintain the tension but exert no transverse force.

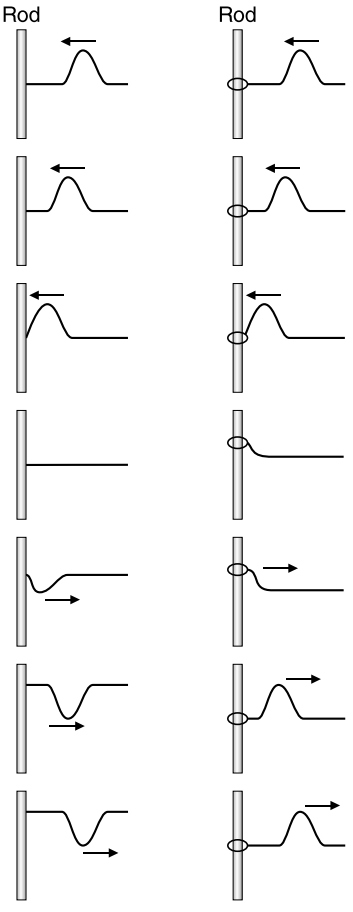


Figure 1 Figure 2

When a wave arrives at this free end, the ring slides along the rod. The ring reaches a maximum displacement. At this position the ring and the string come momentarily to rest as in the fourth drawing from the top in Figure 2. But the string is stretched in this position, giving increased tension, so the free end of the string is pulled back down and again a reflected pulse is produced, but now the direction of the displacement is the same as for the initial pulse.

WAVE PROPERTIES AFTER REFLECTION / REFRACTION (TRANSMISSION)

Any type of wave is associated with the following physical quantities:

(i) speed of wave (v)

(ii) frequency (f), time period (T) and angular frequency (ω)

(iii) wavelength (λ) and wave number (k)

(iv) amplitude (A) and intensity I and

(v) phase (ϕ)

Now, let us see what happens to these physical quantities when they are either reflected or transmitted.

S. No.	Wave Property	Reflection	Refraction (Transmission)
1.	v	does not change	changes
2.	f, T, w	do not change	do not change
3.	λ, k	do not change	change
4.	A, I	change	change
5.	ϕ	$\Delta\phi = 0$, from a rarer medium $\Delta\phi = \pi$, from a denser medium	does not change

PARTIAL REFLECTION AND TRANSMISSION

When a pulse encounters the boundary between a light string and a heavy string then partial reflection and partial transmission of the pulse takes place. Since the tensions are the same, the relative magnitudes of the wave velocities are determined by the mass densities. In Figure 1, the pulse approaches from the light string. The heavy string behaves somewhat like a wall but it can move, and so part of the original pulse is transmitted to the heavy string.

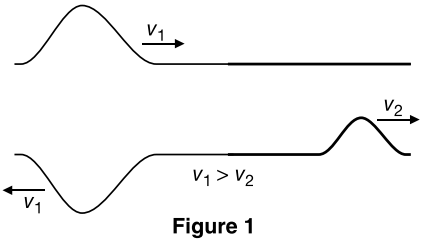


Figure 1

In Figure 2, the pulse approaches from a heavy string. The light string offers little resistance and now approximates a free end. Consequently, the reflected pulse is not inverted.

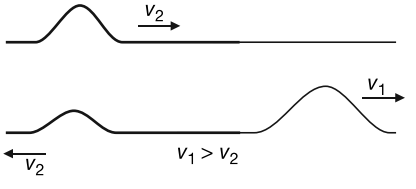


Figure 2

BOUNDARY CONDITIONS

As observed, when a travelling wave encounters the boundary of another medium, it is partially reflected and partially transmitted.

In this section let us investigate the division of the incident wave into reflected and transmitted components, as specified by the boundary conditions. For arriving at the results, we shall be considering two strings of linear mass density μ_1 and μ_2 joined at $x = L$ as shown in Figure 1. The rules

obtained with waves on a string can also be generalized to the reflection and transmission of other types of waves.

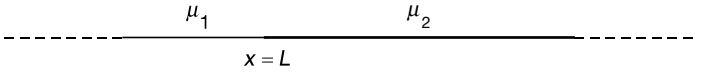


Figure 4.1

Our boundary is the junction between two strings of linear mass densities μ_1 and μ_2 as shown in figure. Both strings are under the same tension T.

Let the incident pulse at the boundary is represented by $y_i(x - v_1 t)$.

So, the reflected pulse is represented by $y_r(x + v_1 t)$ and the transmitted pulse by $y_t(x - v_2 t)$.

Both the incident and the reflected pulses travel on the left string at a speed $v_1 = \sqrt{\frac{T}{\mu_1}}$ while the transmitted pulse propagates to the right string at a speed $v_2 = \sqrt{\frac{T}{\mu_2}}$.

According to the Superposition Principle, the net vertical displacement of the left string is given by $y_i(x - v_1 t) + y_r(x + v_1 t)$.

Now, there are two independent conditions, the pulses must satisfy at the boundary.

CONDITION-1

The net vertical displacement on both sides of the boundary must be the same at all times as shown in Figure 2.

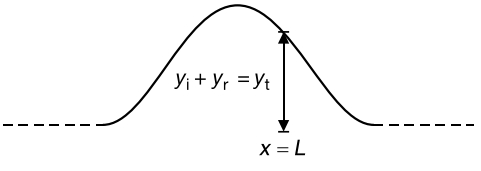


Figure 2

This is expressed mathematically as,

$$y_i(L - v_1 t) + y_r(L + v_1 t) = y_t(L - v_2 t) \quad \ldots(1)$$

where L, the position of the boundary has been substituted for x because $x = L$ at the boundary.

CONDITION-2

This condition is a consequence of the fact that the two strings must exert equal and opposite forces on each other at the junction. As Figure 3 illustrates $\vec{F}_1 = -\vec{F}_2$ only holds if the slopes of the two strings are the same at the junction. We therefore, have, $\left\{\frac{\partial}{\partial x}\left[y_i(x - v_1 t) + y_r(x + v_1 t)\right]\right\}_{x=L} =$

$$\left\{\frac{\partial}{\partial x}\left[y_t(x - v_2 t)\right]\right\}_{x=L} \quad (2)$$

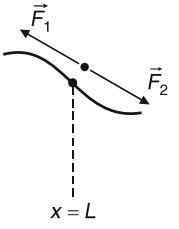


Figure 3

Solving Equations (1) and (2), we get

$$y_t(L - v_2 t) = \left(\frac{2v_2}{v_1 + v_2}\right) y_i(L - v_1 t)$$

and $$y_r(L + v_1 t) = \left(\frac{v_2 - v_1}{v_1 + v_2}\right) y_i(L - v_1 t)$$

The above result can also be explained as under. Suppose the equations of incident wave, reflected wave and transmitted wave are,

$$y_i = A_i \sin(kx - \omega t)$$

$$y_r = A_r \sin(kx + \omega t)$$

and $$y_t = A_t \sin(kx - \omega t)$$

where, $A_r = \left(\frac{v_2 - v_1}{v_1 + v_2}\right) A_i$ and $A_t = \left(\frac{2v_2}{v_1 + v_2}\right) A_i$

The above relations can also be derived from Energy Conservation Principle as shown below

By Law of Conservation of Energy, we have

$$\begin{pmatrix}\text{Average}\\ \text{Incident}\\ \text{Power}\end{pmatrix} = \begin{pmatrix}\text{Average}\\ \text{Reflected}\\ \text{Power}\end{pmatrix} + \begin{pmatrix}\text{Average}\\ \text{Transmitted}\\ \text{Power}\end{pmatrix}$$

$$\Rightarrow \quad P_i = P_r + P_t$$

$$\Rightarrow \quad \frac{1}{2}\mu_1 \omega^2 A_i^2 v_1 = \frac{1}{2}\mu_1 \omega^2 A_r^2 v_1 + \frac{1}{2}\mu_2 \omega^2 A_t^2 v_2$$

Since $v = \sqrt{\frac{T}{\mu}}$

$$\Rightarrow \quad \left(\frac{T}{v_1^2}\right)\omega^2 A_i^2 v_1 = \left(\frac{T}{v_1^2}\right)\omega^2 A_r^2 v_1 + \left(\frac{T}{v_2^2}\right)\omega^2 A_t^2 v_2$$

$$\Rightarrow \quad \frac{A_i^2}{v_1} = \frac{A_r^2}{v_1} + \frac{A_t^2}{v_2} \quad \ldots(1)$$

Further $A_i + A_r = A_t \quad \ldots(2)$

Solving these two equations for A_r and A_t, we get

$$A_r = \left(\frac{v_2 - v_1}{v_1 + v_2}\right) A_i \text{ and } A_t = \left(\frac{2v_2}{v_1 + v_2}\right) A_i$$

CASE 1:

When $v_1 < v_2$, i.e., medium 1 is denser and medium 2 is rarer, then both A_r and A_t are positive and also $A_t > A_i$ as shown in Figure 1.

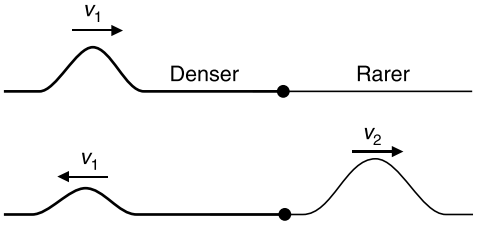


Figure 1 : For $v_1 < v_2$

CASE 2:

When $v_1 > v_2$ i.e., medium 1 is rarer and 2 is denser, then A_r = negative and $|A_r|$ and $|A_t|$ both are individually less than $|A_i|$. The negative value of A_r indicates that the reflected wave suffers a phase change of π as shown in Figure 2.

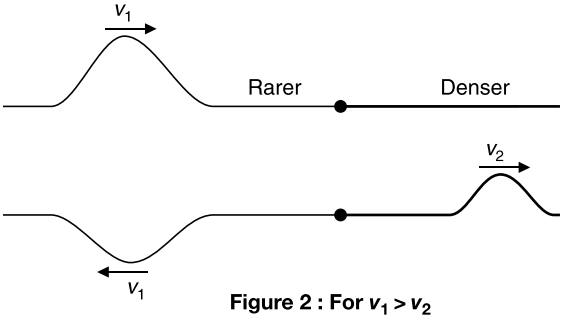


Figure 2 : For $v_1 > v_2$

CASE 3:

Free end of a string: In this case $\mu_2 \to 0$, so $v_2 \to \infty$ and so we get

$$A_t = 0 \text{ and } A_r = A_i$$

CASE 4:

Fixed end of a string: Since the fixed end is equivalent to a string of infinite linear mass density, $v_2 = \sqrt{\frac{T}{\mu_2}} \to 0$ and so we get

$$A_t = 0 \text{ and } A_r = -A_i$$

ILLUSTRATION 18

Two wires of different densities are soldered together end to end and are then stretched under tension T. The wave speed in the first wire is twice that in the second wire.

(a) If the amplitude of incident wave is A, what are amplitudes of reflected and transmitted waves?

(b) Assuming no energy loss in the wire, find the fraction of the incident power that is reflected at the junction and fraction of the same that is transmitted.

SOLUTION

(a) Since, $v_1 = 2v_2$ and $v = \sqrt{\frac{T}{\mu}}$

$$\Rightarrow \mu_1 = \frac{\mu_2}{4}$$

Since, $A_r = \left(\frac{v_2 - v_1}{v_1 + v_2}\right)A = \left(\frac{v_2 - 2v_2}{2v_2 + v_2}\right)A = -\frac{A}{3}$

and $A_t = \left(\frac{2v_2}{v_1 + v_2}\right)A = \left(\frac{2v_2}{2v_2 + v_2}\right)A = \frac{2}{3}A$

Here negative sign with A_r implies that the reflected wave suffers a phase change of π.

(b) Fraction of incident power reflected is

$$f_r = \frac{P_r}{P_i} = \frac{\frac{1}{2}\mu_1\omega^2 A_r^2 v_1}{\frac{1}{2}\mu_1\omega^2 A_i^2 v_1} = \left(\frac{A_r}{A_i}\right)^2 = \frac{1}{9}$$

Fraction of incident power transmitted is

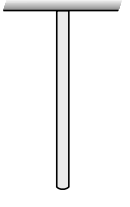
$$f_t = 1 - f_r = 1 - \frac{1}{9} = \frac{8}{9}$$

Test Your Concepts-II

Based on Transverse Wave in a String & Properties

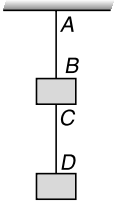
(Solutions on page H.234)

1. A wire of variable mass per unit length $\mu = \mu_0 x$, is hanging from the ceiling as shown in figure. The length of wire is ℓ_0. A small transverse disturbance is produced at its lower end. Find the time after which the disturbance will reach to the other ends.



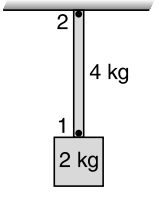
2. A wave pulse starts propagating in the positive x-direction along a non-uniform wire of length l with a mass per unit length given by $\mu = \mu_0 + \alpha x$ and under a tension 100 N. Find the time taken by the pulse to travel from the lighter end $(x = 0)$ to the heavier end where μ_0 and α are constant.

3. Two blocks each having a mass of 3.2 kg are connected by a wire CD and the system is suspended from the ceiling by another wire AB as shown in Figure. The linear mass density of the wire AB is 10 gm^{-1} and that of CD is 8 gm^{-1}. Calculate the speed of a transverse wave pulse produced in AB and in CD.



4. A transverse wave travelling in a string is given by the equation $y = A\sin(kx - \omega t)$. Calculate the potential energy per unit volume possessed by this wave.

5. A uniform rope of length 10 cm and mass 4 kg hangs vertically from a rigid support. A block of mass 2 kg is attached to the free end of the rope. A transverse pulse of wavelength 0.06 m is produced at the lower end of the rope, what is the wavelength of the pulse when it reaches the top of the rope?



6. The speed of a transverse wave travelling in a wire having a length 50 cm and mass 5 g is 80 ms^{-1}. The area of cross-section of the wire is 1 mm^2 and its Young's modulus is 16×10^{11} Nm^{-2}. Calculate the extension of the wire over its natural length.

7. One end of 12 m long rubber tube with a total mass of 900 g is fastened to a fixed support. A cord attached to the other and passes over a pulley and supports an object with a mass of 5 kg. The tube is struck a transverse blow at one end. Find the time required for the pulse to reach the other end. Take $g = 9.8$ ms^{-2}.

8. Energy is transmitted at a rate P_1 by a wave of frequency v_1, on a string with tension T_1. Assuming the amplitude to remain the same, calculate the new transmission rate in terms of P_1 if tension is increased to $4T_1$ and the wave frequency is decreased to $\frac{v_1}{4}$.

9. A steel wire of length 64 cm weighs 5 g. If it is stretched by a force of 8 N, calculate the speed of a transverse wave passing in it.

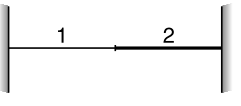
10. Two wires of different densities but same area of cross section are soldered together at one end and are stretched to a tension T. The velocity of a transverse wave in one wire is double of that in the second wire. Calculate the ratio of density of the first wire to that of the second wire.

11. A long wire PQR is made by joining two wires PQ and QR of equal radii. PQ has a length 4.8 m and mass 0.06 kg. QR has a length 2.56 m and mass 0.2 kg. The wire PQR is under a tension of 80 N. A sinusoidal wave pulse of amplitude 3.5 cm is sent along the wire PQ from the end P. No power is dissipated during the transmission of the wave pulse. Calculate

(a) The time taken by the wave pulse to reach the other end R and

(b) The amplitude of the reflected and transmitted wave pulse after the incident wave pulse crosses the joint Q .

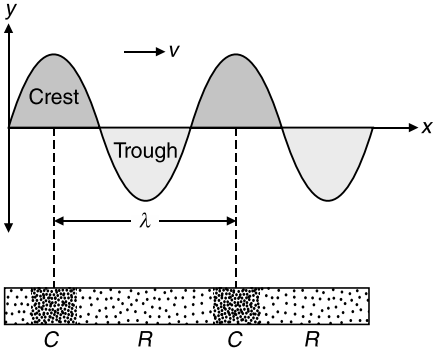
12. Two strings 1 and 2 are taut between two fixed supports (as shown in figure) such that the tension in both strings is same. Mass per unit length of 2 is more than that of 1. Explain which string is denser for a transverse travelling wave.



13. A string of length 40 cm and weighing 10 g is attached to a spring at one end and to a fixed wall at the other end. The spring has a spring constant of 160 Nm^{-1} and is stretched by 1 cm. If a wave pulse is produced on the string near the wall, calculate the time taken by the wave pulse to reach the spring.

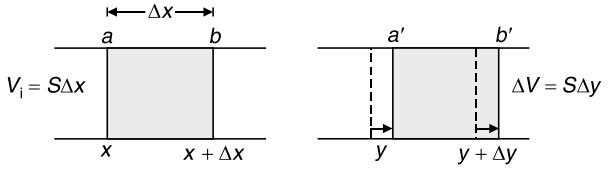
RELATION BETWEEN DISPLACEMENT WAVE AND PRESSURE WAVE FOR A LONGITUDINAL WAVE

A transverse wave propagates by means of crests and troughs and there is no change in pressure in a transverse wave.



A longitudinal wave propagates by means of compressions and rarefactions and there is always a pressure variation along longitudinal wave.

Consider a harmonic displacement wave moving through air contained in a long tube of cross sectional area S as shown in Figure.



The volume of gas that has a thickness Δx in the horizontal direction is $V_i = S\Delta x$. The change in volume ΔV is $S\Delta y$, where Δy is the difference between the value of y at $x+\Delta x$ and the value of y at x. From the definition of Bulk Modulus, the pressure variation in the gas is given by

$$\Delta P = -B\left(\frac{\Delta V}{V_i}\right) = -B\left(\frac{S\Delta y}{S\Delta x}\right) = -B\left(\frac{\Delta y}{\Delta x}\right)$$

When Δx approaches zero, the ratio $\frac{\Delta y}{\Delta x}$ becomes $\frac{\partial y}{\partial x}$ (This partial derivative indicates that we are interested in the variation of y with position at a fixed time). So, we get

$$\Delta P = -B\frac{\partial y}{\partial x} \qquad \ldots(1)$$

This is the equation which relates the displacement equation with the pressure equation.

If, $y = A\sin(\omega t - kx)$ …(2)

then, $\frac{\partial y}{\partial x} = kA\cos(\omega t - kx)$ …(3)

and from Equations (1) and (3), we get

$$\Delta P = BAk\cos(\omega t - kx) = (\Delta P)_m \cos(\omega t - kx) \qquad \ldots(4)$$

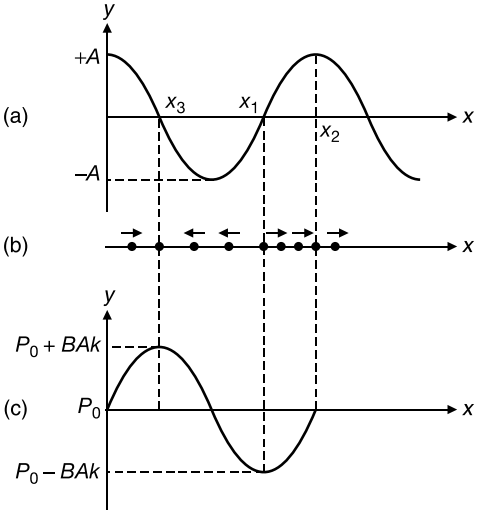
where, $(\Delta P)_m = BAk$ is the amplitude of pressure variation.

The pressure being maximum at compressions and minimum at rarefactions. In the loudest sound the change in pressure is $28\ \text{Nm}^{-2}$ and amplitude is nearly 10^{-5} m, while in the faintest sound the change in pressure is $0.002\ \text{Nm}^{-2}$ and amplitude is nearly 10^{-11} m.

Thus, a sound wave may be considered either as a displacement wave or as a pressure wave. We note that the pressure wave is 90° out of phase with the displacement wave.

From Equations (2) and (4), we observe that the pressure equation is 90° out of phase with displacement equation. When the displacement is zero, the pressure variation is either maximum or minimum and vice-versa.

Figure (a) shows displacement of air molecules from equilibrium in a harmonic sound wave versus position at some instants. Points x_1 and x_3 are points of zero displacement. Now from, Figure (b), we observe that, just to the left of x_1 , the displacement is negative, indicating that the gas molecules are displaced to left, away from point x_1 at this instant.



Just to the right of x_1, the displacement is positive, indicating that the molecules are displaced to the right, which is again away from point x_1. So, at point x_1 the pressure of the gas is minimum. So, if P_0 be the atmospheric pressure (normal pressure), then pressure at x_1 will be,

$$P(x_1) = P_0 - (\Delta P)_m = P_0 - BAk$$

At point x_3, the pressure (and hence the density also) is maximum because the molecules on both sides of that point are displaced toward point x_3. Hence,

$$P(x_3) = P_0 + (\Delta P)_m = P_0 + BAk$$

At point x_2 the pressure (and hence the density) does not change because the gas molecules on both sides of that point have equal displacements in the same direction and hence

$$P(x_2) = P_0$$

From Figure (a) and (c), we observe that the pressure change and displacement are 90° out of phase.

ILLUSTRATION 19

The pressure variation equation of a sound wave in air is given by

$$\Delta P = (0.02\ \text{Nm}^{-2})[\sin(500\ \text{s}^{-1})t - (3\ \text{m}^{-1})x]$$

Calculate the frequency, wavelength and the speed of sound wave in air. If the equilibrium pressure of air is $1.01\times10^5\ \text{Nm}^{-2}$, then calculate the maximum and minimum pressure at a point as the wave passes through that point.

SOLUTION

Since $\Delta P = 0.02\sin(500t - 3x)$ …(1)

$\Rightarrow \quad \Delta P = (\Delta P)_{\max}\sin(\omega t - kx)$ …(2)

Comparing equations (1) and (2), we get $k = 3\ \text{m}^{-1}$, $\Delta P_{\max} = 0.02\ \text{Nm}^{-2}$ and $\omega = 500\ \text{rads}^{-1}$

So, $f = \dfrac{\omega}{2\pi} = \dfrac{500}{2\pi} = \dfrac{250}{\pi}\ \text{Hz}$

Since $k = \dfrac{2\pi}{\lambda}$, so $\lambda = \dfrac{2\pi}{k} = \dfrac{2\pi}{3}\ \text{m}$

$\Rightarrow \quad v = f\lambda = \dfrac{250}{\pi}\times\dfrac{2\pi}{3} = \dfrac{500}{3}\ \text{ms}^{-1}$

If P_0 be the atmospheric pressure, then

$$P_{\max} = P_0 + (\Delta P)_{\max} = (1.01\times10^5 + 0.02)\ \text{Nm}^{-2}$$

$\Rightarrow \quad P_{\max} = 101000.02\ \text{Nm}^{-2}$

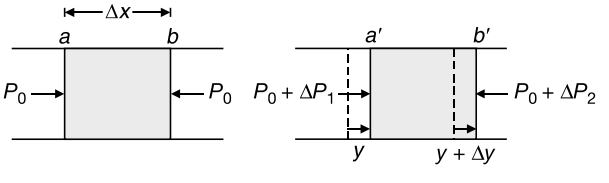
Also, $P_{\min} = P_0 - (\Delta P)_{\max} = (1.01\times10^5 - 0.02)\ \text{Nm}^{-2}$

$\Rightarrow \quad P_{\min} = 100999.98\ \text{Nm}^{-2}$

SPEED OF A LONGITUDINAL WAVES

Let us calculate the speed at which a longitudinal pulse propagates through a fluid (gas/liquid). To derive the expression for the wave equation, we shall be applying Newton's Second Law to the motion of an element of the fluid.

Consider a fluid element ab of thickness Δx confined to a tube of cross-sectional area S as shown in figure.



Assume that the equilibrium pressure of the fluid is P_0. Due to the disturbance, the section a of the element moves a distance y from its mean position and section b moves a distance $y+\Delta y$ to a new position b'. The pressure on the left side of the element becomes $P_0 + \Delta P_1$ and on the right side it becomes $P_0 + \Delta P_2$. If ρ is the equilibrium density, the mass of the element is $\rho S\Delta x$, because when the element moves its mass does not change, even though its volume and density do change.

The net force acting on the element is,

$$F = (\Delta P_1 - \Delta P_2)S$$

The net acceleration of the element is $a = \partial^2 y/\partial t^2$

Applying Newton's Second Law to the motion of the element, we get

$$F = (\Delta P_1 - \Delta P_2)S = \rho S\Delta x\frac{\partial^2 y}{\partial t^2} \quad \ldots(1)$$

Dividing both sides by Δx and observing that in the limit as $\Delta x\to 0$, we get

$$\frac{\Delta P_1 - \Delta P_2}{\Delta x} \to \frac{\partial P}{\partial x}$$

So, equation (1) becomes,

$$-\frac{\partial P}{\partial x} = \rho\frac{\partial^2 y}{\partial t^2} \quad \ldots(2)$$

The excess pressure ΔP may be written as

$$\Delta P = -B\frac{\partial y}{\partial x} \qquad \{\text{as discussed already}\}$$

When this is used in Equation (2), we get the wave equation as

$$\frac{\partial^2 y}{\partial x^2} = \frac{\rho}{B}\left(\frac{\partial^2 y}{\partial t^2}\right)$$

$$\Rightarrow \quad \frac{\partial^2 y}{\partial t^2} = \frac{B}{\rho}\frac{\partial^2 y}{\partial x^2}$$

Comparing this equation with the wave equation $\dfrac{\partial^2 y}{\partial t^2} = v^2\dfrac{\partial^2 y}{\partial x^2}$, we get the speed of longitudinal wave in a fluid (liquid or gas) to be

$$v = \sqrt{\frac{B}{\rho}}$$

When a longitudinal wave propagates in a solid rod or bar, the rod expands sideways slightly when it is compressed longitudinally and the speed of a longitudinal wave in a rod is given by

$$v = \sqrt{\frac{Y}{\rho}}$$

The speed of longitudinal waves in a medium of elasticity E and density ρ is given by

$$v = \sqrt{\frac{E}{\rho}}$$

For Liquids and Gases, E is the Bulk modulus of elasticity (B). For Solids, E is the Young's modulus (Y). So, we have

$$v_{\text{solid}} = \sqrt{\frac{Y}{\rho}} \text{ and } v_{\text{liquid/gas}} = \sqrt{\frac{B}{\rho}}$$

If the solid is extended, the speed will depend on the Bulk modulus B and the Shear modulus η and is given by

$$v = \sqrt{\frac{B + \frac{4\eta}{3}}{\rho}}$$

Conceptual Note(s)

(a) The density of a solid is much larger than that of a gas but the elasticity is larger by a greater factor. Hence longitudinal waves in a solid travel much faster than that in a gas.

(b) In a liquid the speed lies in between the two i.e.

$$v_{\text{solid}} > v_{\text{liquid}} > v_{\text{gas}}$$

(c) The speed at which a longitudinal pulse propagates through a fluid (gas/liquid) of Bulk's modulus B having density $\rho$is given by

$$v_{\text{liquid/gas}} = \sqrt{\frac{B}{\rho}}$$

(d) When a longitudinal wave propagates in a solid rod or bar, the rod expands sideways slightly when it is compressed longitudinally and the speed of a longitudinal wave in a rod is given by

$$v_{\text{solid}} = \sqrt{\frac{Y}{\rho}}$$

(e) The speed of longitudinal waves in a medium of elasticity E and density ρ is given by

$$v = \sqrt{\frac{E}{\rho}}$$

For Liquids and Gases, E is the Bulk modulus of elasticity (B) and for Solids, E is the Young's modulus (Y).

(f) If the solid is extended, the speed will depend on the Bulk modulus B and the Shear modulus η and is given by

$$v = \sqrt{\frac{B + \frac{4\eta}{3}}{\rho}}$$

ILLUSTRATION 20

The volumetric strain of water at a pressure of 10^5 Nm^{-2} is 5×10^{-5}. Calculate the speed of sound in water. Density of water is 10^3 kgm^{-3}.

SOLUTION

Bulk modulus of water is

$$B = \frac{\text{Normal stress}}{\text{Volume strain}} = \frac{10^5}{5\times10^{-5}} = 2\times10^9 \text{ Nm}^{-2}$$

Since, density of water is $\rho = 10^3$ kgm^{-3}

So, speed of sound in water is $v = \sqrt{\frac{B}{\rho}}$

$$\Rightarrow \quad v = \sqrt{\frac{2\times10^9}{10^3}} = 1.41\times10^3 \text{ ms}^{-1}$$

ILLUSTRATION 21

The Bulk modulus and modulus of rigidity for aluminium are 7.5×10^{10} Nm^{-2} and 2.1×10^{10} Nm^{-2} respectively. Determine the velocity of the waves in the medium. Density of aluminium is 2.7×10^3 kgm^{-3}.

SOLUTION

Given, Bulk modulus, $B = 7.5\times10^{10}$ Nm^{-2}

Modulus of rigidity, $\eta = 2.1\times10^{10}$ Nm^{-2}

Density, $\rho = 2.7\times10^3$ kgm^{-3}

So, velocity of longitudinal waves in aluminium is

$$v = \sqrt{\frac{B + \frac{4}{3}\eta}{\rho}} = \sqrt{\frac{7.5\times10^{10} + \frac{4}{3}\times2.1\times10^{10}}{2.7\times10^3}}$$

$$\Rightarrow \quad v = 6.18\times10^3 \text{ ms}^{-1}$$

RELATION BETWEEN PRESSURE WAVE AND DENSITY WAVE

Let us now find the relation between pressure wave and density wave from the definition of Bulk Modulus (B), we have

$$B = -\frac{dP}{dV/V} \qquad \ldots(1)$$

Since, volume $\text{Volume} = \frac{\text{Mass}}{\text{Density}} = \frac{M}{\rho}$

$$\Rightarrow \quad dV = -\left(\frac{m}{\rho^2}\right)d\rho = -\left(\frac{V}{\rho}\right)d\rho$$

$$\Rightarrow \quad \frac{dV}{V} = -\frac{d\rho}{\rho}$$

Substituting in equation (1), we get

$$d\rho = \frac{\rho(dP)}{B} = \frac{dP}{v^2} \qquad \left\{\because\ v = \sqrt{\frac{B}{\rho}}\right\}$$

This can be re-written as

$$\Delta\rho = \left(\frac{\rho}{B}\right)\Delta P = \frac{\Delta P}{v^2}$$

So, this relation relates the pressure equation with the density equation. For example, if we have

$$\Delta P = (\Delta P)_m \sin(kx - \omega t)$$

then, $\Delta\rho = (\Delta\rho)_m \sin(kx - \omega t)$

where, $(\Delta\rho)_m = \frac{\rho}{B}(\Delta P)_m = \frac{(\Delta P)_m}{v^2}$

So, the density equation is in phase with the pressure equation and is 90° out of phase with the displacement equation.

ILLUSTRATION 22

The pressure variation in a sound wave is given by

$$\Delta P = 8\cos\left(4.00x - 3000t + \frac{\pi}{4}\right)$$

Calculate the displacement amplitude. The density of the medium is 10^3 kgm^{-3}.

SOLUTION

Since, $\Delta P_0 = BAK = \rho v^2 Ak \qquad \{\because B = \rho v^2\}$

$$\Rightarrow \quad \Delta P_0 = \frac{\rho\omega^2}{k^2}Ak = \frac{\rho\omega^2 A}{k} \qquad \left\{\because v^2 = \frac{\omega^2}{k^2}\right\}$$

$$\Rightarrow \quad A = \frac{(\Delta P_0)k}{\rho\omega^2} = \frac{8\times 4.00}{10^3\times(3000)^2} = 3.55\times10^{-9}\ \text{m}$$

ILLUSTRATION 23

Find the displacement amplitude of a sound wave having a frequency of 100 Hz and a pressure amplitude of 10 Pa. If another sound wave of frequency 300 Hz has a displacement amplitude of 10^{-7} m, find the pressure amplitude of this wave. Speed of sound in air is 340 ms^{-1} and density of air is 1.29 kgm^{-3}.

SOLUTION

Since, $(\Delta P)_m = BAk$, where $k = \frac{\omega}{v} = \frac{2\pi f}{v}$

Since, $B = \rho v^2 \qquad \{\because\ v = \sqrt{B/\rho}\}$

$$\Rightarrow \quad (\Delta P)_m = (\rho v^2)(A)\left(\frac{2\pi f}{v}\right) = \frac{(\Delta P)_m}{2\pi v\rho f} \qquad \ldots(1)$$

$$\Rightarrow \quad A = \frac{10}{2\times3.14\times100\times1.29\times340} = 3.63\times10^{-5}\ \text{m}$$

From above equation (1), we get

$$(\Delta P)_m = 2\pi f\rho vA$$

$$\Rightarrow \quad (\Delta P)_m = 2\times3.14\times300\times1.29\times340\times10^{-7}$$

$$\Rightarrow \quad (\Delta P)_m = 8.26\times10^{-2}\ \text{Nm}^{-2}$$

ABOUT NATURE OF WAVES

For propagation of transverse mechanical waves, the medium must be rigid. Therefore, within solids the waves are transverse. A gas (air) has no rigidity, therefore the waves in a gas are always longitudinal. The waves on the surface of water are a combination of longitudinal and transverse waves, such types of waves are usually called **ripples**. In these waves the particles of medium vibrate up and down and back and forth simultaneously, describing ellipse in a vertical plane. The waves in a stretched string are always transverse. The wave on the surface of solids (e.g. rails) are longitudinal.

POWER AND INTENSITY OF WAVE TRAVELLING THROUGH A MEDIUM

When a particle executes harmonic oscillations, there exists oscillation energy which keeps on changing between kinetic and potential forms. In a progressive waves, this energy is transferred through the medium with a velocity v. The amplitude of the particle velocity is given as

$$v_0 = A\omega$$

If ρ is the density of the medium, the energy per unit volume is

$$U = \frac{1}{2}\rho v_0^2 = \frac{1}{2}\rho A^2\omega^2$$

If S is the area of cross-section of the medium, the energy associated with a volume $S\Delta x$, will be

$$\Delta E = U\,\Delta V = \frac{1}{2}\rho A^2\omega^2 S\,\Delta x$$

So, power (rate of transmission of energy) becomes

$$P = \frac{\Delta E}{\Delta t} = \frac{1}{2}\rho v\omega^2 A^2 S \qquad \left\{\text{as } \frac{\Delta x}{\Delta t} = v\right\}$$

The intensity is defined as average energy transmitted per unit normal area per second i.e., power per unit area. Hence, intensity

$$I = \frac{\Delta E}{S\Delta t} = \frac{P}{S} = \frac{1}{2}\rho v\omega^2 A^2$$

Further, as in case of sound wave, the displacement amplitude is related to the pressure amplitude through the relation $p_0 = \rho vA\omega$, so

$$I = \frac{1}{2}\rho v\omega^2\left(\frac{p_0}{\rho v\omega}\right)^2 = \frac{p_0^2}{2\rho v}$$

So, we observe that $I \propto A^2$ or $I \propto p_0^2$ and also $I \propto \omega^2$

The SI unit of intensity is Wm^{-2}.

ILLUSTRATION 24

An electric siren of 100 W sends out sound waves in air at a frequency of 1 kHz. Assuming that the sound energy travels equally in all directions in space, calculate. The intensity of sound at a point 100 m away from the source. If the velocity of sound v is 350 ms^{-1} and density of air $\rho = \mathbf{1.29}$ kgm^{-3}, determine the displacement amplitude of the air particles at that point.

SOLUTION

The intensity of sound at the point 100 m away from the source is

$$I = \frac{P}{\text{Area}} = \frac{P}{4\pi r^2} = \frac{100}{4 \times 3.141 \times (100)^2}$$

$$\Rightarrow \quad I = 7.96 \times 10^{-4} \text{ Wm}^{-2}$$

Now, we know that

$$I = \frac{1}{2}\rho v \omega^2 A^2$$

where, $I = 7.96 \times 10^{-4}$ Wm^{-2}, $\rho = 1.29$ kgm^{-3},

$v = 350$ ms^{-1} and $\omega = 2\pi \times 1 \times 10^3$ rads^{-1}

$$\Rightarrow \quad A^2 = \frac{2I}{\rho v \omega^2} = \frac{2 \times 7.96 \times 10^{-4}}{1.29 \times 350 \times (2\pi \times 10^3)^2}$$

$$\Rightarrow \quad A^2 = 8.94 \times 10^{-14}$$

$$\Rightarrow \quad A = 2.99 \times 10^{-7} \text{ m}$$

INTENSITY OF SOUND WAVE

Travelling sound waves, like all other travelling waves, transfer energy from one region of space to another. As discussed earlier, we define the intensity of a wave (denoted by I) to be the time average rate at which energy is transported by the wave, per unit area across a surface perpendicular to the direction of propagation. Since, we have already derived that

$$I = \frac{1}{2}\rho A^2 \omega^2 v \qquad \ldots(1)$$

For a sound wave, we have

$$(\Delta P)_m = BAk = BA\left(\frac{\omega}{v}\right)$$

$$\Rightarrow \quad \omega = \frac{(\Delta P)_m v}{BA}$$

Substituting this value in Equation (1), we get

$$I = \frac{1}{2}\rho A^2 \frac{(\Delta P)_m^2 v^2}{B^2 A^2} v \qquad \{\because \ \rho v^2 = B\}$$

$$\Rightarrow \quad I = \frac{v(\Delta P)_m^2}{2B} \qquad \ldots(2)$$

So, intensity of a sound wave can be calculated by using either of the Equations (1) or (2)

ILLUSTRATION 25

A point sound source is situated in a medium having Bulk modulus 1.6×10^5 Nm^{-2}. An observer standing at a distance 10 m from the source, writes down the equation for the wave as $y = A\sin(15\pi x - 6000\pi t)$; where y and x are in metre and t is in second. The maximum pressure, amplitude tolerable to the observer's ear is 24π Pa, then find the

(a) density of the medium
(b) displacement amplitude A of the waves received by the observer
(c) maximum power of the sound source.

SOLUTION

Since $k = 15\pi$ m^{-1} and $\omega = 6000\pi$ sec^{-1}

$$\Rightarrow \quad v = \omega/k = 400 \text{ ms}^{-1}$$

(a) Since, $v = \sqrt{B/\rho}$

$$\Rightarrow \quad \rho = \frac{B}{v^2} = \frac{1.6 \times 10^5}{(400)^2} = 1 \text{ kgm}^{-3}$$

(b) Pressure amplitude is $(\Delta P)_{\max} = BAk$

$$\Rightarrow \quad A = \frac{(\Delta P)_{\max}}{Bk} = \frac{24\pi}{1.6 \times 10^5 \times 15\pi} = 10^{-5} \text{ m}$$

$$\Rightarrow \quad A = 10 \ \mu\text{m}$$

(c) Intensity at distance r from a point source is given by

$I = \frac{W}{4\pi r^2}$. Also, $I = \frac{(\Delta P)_{\max}^2}{2\rho v}$

$$\Rightarrow \quad \frac{W}{4\pi(10)^2} = \frac{(24\pi)^2}{2 \times 1 \times 400}$$

$$\Rightarrow \quad W = 8.93 \times 10^3 \text{ watt}$$

ILLUSTRATION 26

A point like sound source emits sound of frequency 2000 Hz, uniformly in all directions. Its intensity at a distance of 6 m is 0.960 mW m^{-2}.

(a) Find the intensity at 30 m.
(b) At 6 m from the source, find the displacement and pressure amplitudes. (Atmospheric pressure $= 1 \times 10^5$ Pa and density of air $= 1.3$ kgm^{-3})

SOLUTION

(a) $I \propto \frac{1}{r^2}$

So, $0.960 \propto \frac{1}{6^2}$ $\qquad \ldots(1)$

and $I \propto \frac{1}{30^2}$ …(2)

Dividing (2) and (1), we get

$$\frac{I}{0.960} = \frac{6^2}{30^2}$$

$$\Rightarrow \quad I = 0.960\left(\frac{1}{5}\right)^2 = 0.038 \text{ mW m}^{-2}$$

(b) Now, $I = \frac{1}{2}\rho\omega^2 v A^2$

$$0.960\times10^{-3} = \frac{1}{2}\times1.3\times(2\pi\times2000)^2\times332\times A^2$$

$$\Rightarrow \quad A \approx 171 \text{ nm}$$

Also, $I = \frac{v(\Delta P_0)^2}{2B}$

$$\Rightarrow \quad \Delta P_0 = \sqrt{\frac{2BI}{v}} = \sqrt{\frac{2\times1.41\times10^5\times0.960\times10^{-3}}{332}}$$

where, $B = \gamma P$ and $\gamma_{\text{air}} \approx 1.41$

$$\Rightarrow \quad \Delta P_0 = 0.89 \text{ Pa}$$

SUPERSONIC AND SHOCK WAVES

When an object moves with a velocity greater than that of sound, it is termed as **Supersonic**. When such a supersonic body or plane travels in air, it produces energetic disturbance which moves in backward direction and diverges in the form of a cone. Such disturbances are called the **Shock Waves**.

The speed of supersonic is measured in **Mach number**. One Mach number is the speed of sound.

$$\text{Mach number} = \frac{\text{Speed of Supersonic}}{\text{Speed of Sound}}$$

INTENSITY LEVEL AND LOUDNESS OF SOUND

The term loudness describes the human perception of sound. A sound wave of higher intensity is perceived as louder than a wave of lower intensity. However, the relation is not linear. The sensation of loudness is roughly proportional to logarithm of intensity. The intensity level is defined by an arbitrary scale that corresponds roughly to sensation of loudness. Human ear responds to sound intensities over a wide range (from 10^{-12} Wm^{-2} to 1 Wm^{-2}). Therefore, instead of specifying intensity of sound in Wm^{-2}, we use a logarithmic scale of intensity called sound level, SL or L which is expressed in decibel (dB) and is given by

$$L(\text{in dB}) = L = 10\log_{10}\left(\frac{I}{I_0}\right)$$

where $I_0 = 10^{-12}$ Wm^{-2} is the threshold of hearing of human ear (the weakest sound audible to human ear). Unit of loudness level is decibel (dB) and

1 bel = 10 decibel

The decibel is a dimensionless unit.

A sound of intensity I_0 has

$$L = 10\log_{10}\left(\frac{I_0}{I_0}\right) = 0 \text{ dB}$$

The upper range of human hearing also called threshold of pain is 1 Wm^{-2} and it corresponds to

$$L = 10\log_{10}\left(\frac{1}{10^{-12}}\right) = 120 \text{ dB}$$

A sound 10 times more intense than I_0 has intensity level 10 dB, a sound 100 times more intense than I_0 has intensity level 20 dB and so on. *So, as the intensity of sound increases (or decreases) by a factor of 10, the intensity level increases (or decreases) by* 10 dB. The normal ear can distinguish between intensities that differ by about 1 dB.

The normal conversation is about 60 dB, city traffic noise is about 70 – 90 dB and a jet aircraft produces as much as 150 dB which can damage the ear.

NOISE LEVELS DUE TO DIFFERENT SOURCES

Source	Noise level in decibel	Intensity (Wm^{-2})
Threshold of hearing	0	10^{-12}
Rustle of leaves	10	10^{-11}
Average whispering	30	10^{-10}
Quiet radio in home	40	10^{-8}
Quiet automobile	50	10^{-7}
Ordinary conversation	60	3.2×10^{-6}
Busy Street traffic	70–90	10^{-5}
Noise factory	90	10^{-3}
Motorcycle rider and orchestra	100	10^{-2}
Threshold of pain	120	1
Machine Gun/Jack Hammer	130	10
Nearby Jet Airplane	150	10^{3}

If two intensity levels are I_1 and I_2, then number of decibels L_1 and L_2 for intensities I_1 and I_2 are

$$L_1 = 10\log_{10}\left(\frac{I_1}{I_0}\right) \text{ and } L_2 = 10\log_{10}\left(\frac{I_2}{I_0}\right)$$

$$\Rightarrow \quad L_2 - L_1 = 10\left[\log_{10}\left(\frac{I_2}{I_0}\right) - \log_{10}\left(\frac{I_1}{I_0}\right)\right]$$

$$\Rightarrow \quad L_2 - L_1 = 10\log_{10}\left(\frac{I_2}{I_1}\right)$$

So, if two sounds have intensity ratio 2, then they differ in sound level by

$$\Delta L = 10\log_{10}(2) = 10 \times 0.301 = 3 \text{ dB}$$

ILLUSTRATION 27

The loudness of a source of sound at a given location increases by 10 dB. By how many times does its intensity increase?

SOLUTION

Let the initial loudness be L_1 decibel, so we have

$$L_1 = 10\log_{10}\left(\frac{I}{I_0}\right)$$

The new loudness is now $L_1 + 10$. So, let us assume that the new intensity be nI i.e., becomes n times.

$$\Rightarrow \quad L_1 + 10 = 10\log_{10}\left(\frac{nI}{I_0}\right)$$

$$\Rightarrow \quad L_1 + 10 = 10\log_{10}\left(\frac{I}{I_0}\right) + 10\log_{10} n$$

$$\Rightarrow \quad L_1 + 10 = L_1 + 10\log n$$

$$\Rightarrow \quad \log n = 1$$

$$\Rightarrow \quad n = 10$$

Hence the loudness increases by 10 decibel when the intensity increases to 10 times the initial value. Similarly, the loudness decreases by 10 decibels when intensity drops to one tenth of the initial value.

ILLUSTRATION 28

What is the maximum possible sound level in dB of sound waves in air? Given that density of air is 1.3 kgm^{-3}, $v = 332$ ms^{-1} and atmospheric pressure $P = 1.01 \times 10^5$ Nm^{-2}.

SOLUTION

For maximum possible sound intensity, pressure amplitude of wave will be equal to atmospheric pressure. So, we have $p_0 = P = 1.01 \times 10^5$ Nm^{-2}

Since $I = \dfrac{p_0^2}{2\rho v}$, where $\rho = 1.3$ kgm^{-3}, $v = 232$ ms^{-1}

$$\Rightarrow \quad I = \frac{p_0^2}{2\rho v} = \frac{(1.01\times10^5)^2}{2\times1.3\times332} = 1.18\times10^7 \text{ Wm}^{-2}$$

Since, $L(\text{in dB}) = 10\log_{10}\left(\dfrac{I}{I_0}\right)$

$$\Rightarrow \quad L(\text{in dB}) \approx 10\log_{10}\left(\frac{10^7}{10^{-12}}\right) = 190 \text{ dB}$$

ILLUSTRATION 29

A fighter jet (subsonic) flies over head at an altitude of 100 m. The intensity of sound is observed to be 150 dB. At what altitude should this plane fly so that the intensity drops to the level of the threshold of pain (i.e., 120 dB)? Ignore the finite time required for the sound to reach the ground. Assume the jet to be a point source of sound.

SOLUTION

Since $L_2 - L_1 = 10\log_{10}\left(\dfrac{I_2}{I_1}\right)$

$$\Rightarrow \quad L_2 - L_1 = 120 - 150 = 10\log_{10}\left(\frac{I_2}{I_1}\right)$$

$$\Rightarrow \quad \log_{10}\left(\frac{I_2}{I_1}\right) = -3$$

$$\Rightarrow \quad \frac{I_2}{I_1} = \frac{r_1^2}{r_2^2} = 10^{-3} \qquad \left\{\because I \propto \frac{1}{r^2}\right\}$$

$$\Rightarrow \quad \frac{(100)^2}{r_2^2} = 10^{-3}$$

$$\Rightarrow \quad r_2^2 = 10^7$$

$$\Rightarrow \quad r_2 = 1000\sqrt{10} \text{ m} = 3162 \text{ m}$$

NEWTON'S FORMULA FOR SPEED OF SOUND IN A GAS

Newton assumed that the propagation of sound in a gas takes place under isothermal conditions, i.e., a compressed layer of air gradually loses heat to the surroundings and a rarefied layer gains heat from the surroundings, so that the temperature of air remains constant. Since for isothermal process

$$PV = \text{constant}$$

$$\Rightarrow \quad d(PV) = 0$$

$$\Rightarrow \quad PdV + VdP = 0$$

$$\Rightarrow \quad -\frac{dP}{dV/V} = P = B_{\text{iso}}$$

Since, $v = \sqrt{\dfrac{B}{\rho}}$ $\qquad \left\{\because B = -\dfrac{dP}{dV/V}\right\}$

$$\Rightarrow \quad v = \sqrt{\frac{B_{iso}}{\rho}} = \sqrt{\frac{P_{atm}}{\rho_{air}}} \qquad \{\text{Newton's Formula}\}$$

At STP, $P_{atm} = 1.013 \times 10^5$ Nm^{-2}, $\rho_{air} = 1.29$ kgm^{-3}

$$\Rightarrow \quad v = \sqrt{\frac{1.013 \times 10^5}{1.29}} \approx 280 \text{ ms}^{-1}$$

This value is about 16% less than the actual value, which is about 332 ms^{-1}. This large discrepancy shows that Newton made some error while calculating the speed of sound.

ILLUSTRATION 30

The speed of sound in air at *NTP* is 332 ms^{-1}. Calculate the percentage error in speed of sound as calculated from Newton's formula. Given that the density of air at 1 atm pressure is 1.293 kgm^{-3}.

SOLUTION

From Newton's formula, $v = \sqrt{\frac{P}{\rho}}$

where, $P = 1.01 \times 10^5$ Pa

$$\Rightarrow \quad v = \sqrt{\frac{1.01 \times 10^5}{1.293}} = 280 \text{ ms}^{-1}$$

Difference in velocity $= 332 - 280 = 52$ ms^{-1}

So, percentage error $= \frac{52}{332} \times 100 = 15.7\%$

LAPLACE'S CORRECTION FOR SPEED OF SOUND IN A GAS

Laplace pointed out that Newton's assumption of "**isothermal**" propagation is not correct. *The reason is that the compressions and rarefactions follow each other so rapidly that there is no time for the compressed layer (which is at a higher temperature) and the rarefied layer (which is at a lower temperature) to equalise their temperatures with the surroundings.*
As a result, the sound propagates under adiabatic and not under isothermal conditions. Since for an adiabatic process $PV^\gamma = \text{constant}$

$$\Rightarrow \quad d(PV^\gamma) = 0$$

$$\Rightarrow \quad P\gamma V^{\gamma-1}dV + V^\gamma dP = 0$$

$$\Rightarrow \quad B_{adi} = -\frac{dP}{dV/V} = \gamma P_{atm}, \text{ where } \gamma = \frac{C_P}{C_V}$$

$$\Rightarrow \quad v = \sqrt{\frac{\gamma P}{\rho}}$$

For air $\gamma = 1.4$, hence $v = 280\sqrt{1.4} \approx 330$ ms^{-1}, which agrees with the experimentally calculated value.

EFFECT OF EXTERNAL FACTORS ON SPEED OF SOUND

The speed of sound in a gas is given by

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$$

Effect of Change in Pressure

There is no effect of change of pressure (at constant temperature). This is because density also changes proportionately such that $P/\rho = \text{constant}$.

Effect of Change in Density

Since $v \propto \frac{1}{\sqrt{\rho}}$, so if ρ_1 and ρ_2 are the densities of two gases for which γ is the same, then the speeds of sound in them, v_1 and v_2 respectively, are related as

$$\frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}}$$

Effect of Change in Temperature

Since $v = \sqrt{\frac{\gamma RT}{M}}$ i.e., $v \propto \sqrt{T}$, so $\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$

If v_t is velocity of sound at t °C and v_0 is velocity of sound at 0 °C, then we have

$$v_t = v_0\sqrt{\frac{273+t}{273}} = v_0\left(1+\frac{t}{273}\right)^{\frac{1}{2}}$$

For $\frac{t}{273} \ll 1$, we have $\left(1+\frac{t}{273}\right)^{\frac{1}{2}} \approx 1+\frac{1}{2}\left(\frac{t}{273}\right)$

$$\Rightarrow \quad v_t \approx v_0\left(1+\frac{t}{546}\right)$$

Taking $v_0 = 332$ ms^{-1} we get $v_t \approx (332 + 0.61t)$ ms^{-1}

Thus for $\frac{t}{273} \ll 1$, the velocity increases by about 0.61 ms^{-1} for every degree rise of temperature.

Effect of Atomicity of Gas

Since $v = \sqrt{\frac{\gamma RT}{M}}$ i.e., $v \propto \sqrt{\frac{\gamma}{M}}$

So, $\frac{v_{He}}{v_{O_2}} = \sqrt{\left(\frac{\gamma_{He}}{\gamma_{O_2}}\right)\left(\frac{M_{O_2}}{M_{He}}\right)} = \sqrt{\left(\frac{5/3}{7/5}\right)\left(\frac{32}{4}\right)} = \sqrt{\frac{200}{21}}$

Effect of Change in Humidity

Density of water vapour is less than that of dry air (water vapours have a tendency to rise up). At S.T.P. it

is 0.8 kgm^{-3} whereas the density of dry air at S.T.P. is 1.29 kgm^{-3}. Hence, the speed of sound in air increases with increase of humidity.

Effect of Wind Velocity

If $\vec{v}_s$ be the velocity of sound in still air and wind blows at a speed $\vec{v}_w$ then velocity of sound $\vec{v}'$ in the medium is $\vec{v}' = \vec{v}_s + \vec{v}_w$

(a) If wind blows along the sound, then effective speed of sound becomes $v' = v_s + v_w$

(b) If wind blows opposite to sound, then effective speed of sound becomes $v' = v_s - v_w$

(c) If wind blows along the sound making an angle θ with it, then the effective speed of sound becomes $v' = v_s + v_w \cos\theta$

(d) If wind blows opposite to the sound making an angle θ with it, then the effective speed of sound becomes $v' = v_s - v_w \cos\theta$

Also keep in mind that the component of velocity of wind perpendicular to the velocity of sound does not affect the velocity of sound.

ILLUSTRATION 31

The velocity of sound in hydrogen at STP is 1400 ms^{-1}. Find the velocity of sound in a mixture with 3 parts by volume of oxygen and 2 parts by volume of hydrogen at 819 °C?

SOLUTION

$$v_{H_2} = \sqrt{\frac{\gamma RT}{M_{H_2}}} = \sqrt{\frac{\gamma R(273)}{M_{H_2}}} = 1400 \text{ ms}^{-1} \qquad \ldots(1)$$

Let the number of moles of hydrogen in the mixture be $2n$. The number of moles of oxygen will then be $3n$. The mean molar mass of a mixture is given by

$$M_{mix} = \frac{n_1M_1 + n_2M_2}{n_1 + n_2} = \frac{2nM_{H_2} + 3nM_{O_2}}{2n + 3n}$$

$$\Rightarrow \quad M_{mix} = \frac{2M_{H_2} + 3\left(16M_{H_2}\right)}{5} \qquad \left\{\because\ M_{O_2} = 16M_{H_2}\right\}$$

$$\Rightarrow \quad M_{mix} = \frac{50M_{H_2}}{5} = 10M_{H_2}$$

Since both these gases are diatomic, γ of the mixture remains unchanged i.e. $\gamma_{mix} = \gamma = 7/5$

At $T = 819\ °C = 1092\ K = 4(273\ K)$, speed of sound in mixture is given by

$$v_{mix} = \sqrt{\frac{\gamma_{mix}RT}{M_{mix}}} = \sqrt{\frac{\gamma R(1092)}{10M_{H_2}}} = \frac{2}{\sqrt{10}}\sqrt{\frac{\gamma R(273)}{M_{H_2}}}$$

From equation (1), we get

$$v_{mix} = \frac{2}{\sqrt{10}}(1400) = \frac{2800}{\sqrt{10}} = 280\sqrt{10} \text{ ms}^{-1}$$

ILLUSTRATION 32

Consider two points A and B such that the distance between them is d and temperature between them varies linearly from T_1 to T_2. What will be the time taken by sound waves to travel this distance if velocity of sound propagation in air varies as $v = \beta\sqrt{T}$.

SOLUTION

Since, $T(x) = T_1 + \left(\frac{T_2 - T_1}{d}\right)x$

Further $v = \beta\sqrt{T}$

$$\Rightarrow \quad v = \frac{dx}{dt} = \beta\sqrt{T_1 + \left(\frac{T_2 - T_1}{d}\right)x}$$

$$\Rightarrow \quad \int_0^t dt = \frac{1}{\beta}\int_0^d \frac{dx}{\sqrt{T_1 + \left(\frac{T_2 - T_1}{d}\right)x}}$$

$$\Rightarrow \quad t = \frac{2d}{\beta\left(\sqrt{T_1} + \sqrt{T_2}\right)}$$

ILLUSTRATION 33

At what temperature does the velocity of sound in air increases by 2% in comparison with velocity at 0 °C?

SOLUTION

If v_0 be the speed of sound at 0 °C and v_t be the speed at t °C, then

$$\frac{v_t}{v_0} = \sqrt{\frac{T}{T_0}} = \sqrt{\frac{t + 273}{273}}$$

According to problem, we have

$$v_t = v_0 + \frac{2}{100}v_0 = v_0(1 + 0.02) = (1.02)v_0$$

$$\Rightarrow \quad \frac{(1.02)v_0}{v_0} = \sqrt{\frac{t + 273}{273}}$$

$$\Rightarrow \quad t + 273 = (1.02)^2 \times 273 = 1.0404 \times 273$$

$$\Rightarrow \quad t = 284 - 273 = 11\ °C$$

Test Your Concepts-III

Based on Sound Waves & Properties

(Solutions on page H.235)

1. A point A is located at a distance $r = 1.5$ m from a point source of sound of frequency 600 Hz. The power of the source is 0.8 watt. Speed of sound in air is 340 ms^{-1} and density of air is 1.29 kgm^{-3}. Find at the point A,
(a) the pressure oscillation amplitude $(\Delta P)_m$
(b) the displacement oscillation amplitude A.

2. Calculate the ratio of speed of sound in hydrogen gas to the rms speed of hydrogen molecules.

3. A dog while barking delivers about 1 mW of power. Assuming this power to be uniformly distributed over a hemispherical area, calculate the sound level at a distance of 5 m. Also calculate the sound level if instead of one dog, five dogs start barking at the same time each delivering 1 mW of power.

4. Determine the speed of sound waves in water, and find the wavelength of a wave having a frequency of 242 Hz. Take $B_{water} = 2 \times 10^9$ Pa.

5. A sound wave propagating in air has a frequency of 5000 Hz. Calculate the percentage change in wavelength when the wave front, initially in a region where $T = 27$ °C, enters a region where the temperature decreases to 10 °C.

6. A window whose area is 2 m^2 opens on a street where the street noise results at the window an intensity level of 60 dB. How much acoustic power enters the window through sound waves? Now, if a sound absorber is fitted at the window, how much energy from the street will it collect in a day?

7. At what temperature will the speed of sound in hydrogen be the same as in oxygen at 100 °C? Molar masses of oxygen and hydrogen are in the ratio 16 : 1.

8. For aluminium, the bulk modulus of elasticity is 7.5×10^{10} Nm^{-2} and density is 2.70×10^3 kgm^{-3}. Calculate the velocity of longitudinal waves in aluminium.

9. Find the maximum increase in pressure when a sound wave produces an energy flow of 10^{-3} $Wattm^{-2}$. Velocity of sound = 340 ms^{-1}, density of air = 1.293 kgm^{-3}.

10. A plane longitudinal wave having angular frequency $\omega = 500$ sec^{-1} is travelling in positive x-direction in a medium of density $\rho = 1$ kgm^{-3} and bulk modulus 4×10^4 Nm^{-2}. The loudness at a point in the medium is observed to be 20 dB. Assuming at $x = 0$ initial phase of the medium particles to be zero, find the
(a) maximum pressure change in the medium
(b) equation of the wave.

11. A gas is a mixture of two parts by volume of hydrogen and one part by volume of nitrogen. If the velocity of sound in hydrogen at 0 °C is 1300 ms^{-1}, find the velocity of sound in the gaseous mixture at 27 °C.

12. For a person with normal hearing, the faintest sound that can be heard at a frequency of 400 Hz has a pressure amplitude of about 6×10^{-5} Pa. Calculate the corresponding intensity in Wm^{-2}. Take speed of sound in air as 344 ms^{-1} and density of air 1.2 kgm^{-3}.

13. The faintest sound that the human ear can detect at frequency 1 kHz corresponds to an intensity of about 10^{-12} Wm^{-2}. Determine the pressure amplitude and the maximum displacement associated with this sound assuming the density of the air = 1.3 kgm^{-3} and velocity of sound in air = 332 ms^{-1}.

REFLECTION OF SOUND WAVES

Reflection of sound waves from a rigid boundary (e.g. closed end of an organ pipe) is analogous to reflection of a string wave from rigid boundary in which reflection is accompanied by an inversion i.e. an abrupt phase change of π. This is consistent with the requirement of displacement amplitude to remain zero at the rigid end, because a medium particle at the rigid end cannot vibrate. Since the excess pressure and displacement corresponding to a same sound wave vary by $\pi/2$ in term of phase, so a displacement minima at the rigid end will be a point of pressure maxima. *This implies that the reflected pressure wave from the rigid boundary will have same phase as the incident wave, i.e., a compression pulse is reflected as a compression pulse and a rarefaction pulse is reflected as a rarefaction pulse.*

On the other hand, reflection of sound wave from a low pressure region (like open end of an organ pipe) is analogous to reflection of string wave from a free end. This point corresponds to a displacement maxima, so that incident and reflected displacement wave at this point must be in phase. This would imply that this point would be a minima for pressure wave (i.e. pressure at this point remains at its average value), and hence the reflected pressure wave would be out of phase by π with respect to the incident wave. i.e. a compression pulse is reflected as a rarefaction pulse and vice-versa.

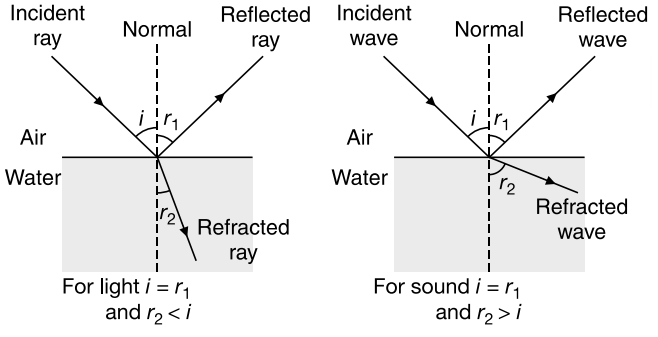
REFRACTION

A medium is said to be denser (relative to the other) if the speed of wave in this medium is less than the speed of the wave in the other medium. Rather we can say speed of a wave in a denser medium is less than its speed in the rarer medium. Thus, it is the speed of wave which decides whether the medium is denser or rarer for that particular wave and

$$v_{\text{denser}} < v_{\text{rarer}}$$

If a medium is denser for one type of wave then at the same time the same medium can be rarer for the other type of wave. For example, water is denser for electromagnetic (light) waves compared to air, because the speed of electromagnetic waves is less in water than in air. At the same time, for sound wave water is a rarer medium because speed of sound wave in water is more. The laws of refraction (or transmission from one medium to the other) and reflection remain the same i.e.,

The ray bends towards the normal if it travels from a rarer medium to a denser medium and vice-versa.



When sound wave passes from one homogeneous medium to another homogeneous medium, it deviates from its path. This phenomenon is called **refraction**. No phase change exists during refraction. If i and r are the angles of incidence and refraction, then **Snell's Law** states

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \text{constant}$$

where v_1 and v_2 are the velocities of sound in first and second medium respectively.

In reflection and refraction the frequency remains unchanged.

ECHO

The most common experience of sound reflection is the echo heard in large halls and in the neighbourhood of hills. An **echo** is simply the repetition of speaker's own voice caused by reflection at a distant surface e.g. a cliff, a row of buildings or any other extended surface.

If t is the time interval between production of sound from source and its echo at the site of source, then the distance between source and reflector(s) is given by

$$2s = vt$$

$$\Rightarrow \quad s = \frac{vt}{2}$$

where v is the velocity of sound.

As the persistence of hearing for human ear is 0.1 sec, therefore in order that an echo of short sound (e.g. shot or clapping) may be heard distinctly, the echo must come 0.1 sec later than the direct sound. In the case of articulate sound, it has been found that one can hear or pronounce distinctly not more than 5 syllables per second. Therefore, for monosyllabic sound the minimum time interval between sound and its echo is $\frac{1}{5}$ s, for disyllabic and trisyllabic sounds it is $\frac{2}{5}$ s and $\frac{3}{5}$ s respectively and so on.

If we hear the echo of monosyllabic sound that means sound travels from source to reflector and back from reflector to source in a collective time of $\frac{1}{5}$ s i.e. time to go from source to reflector is $\frac{1}{10}$ s.

Multiple echoes are produced when there are several reflecting surfaces. The stethoscope speaking tubes, whispering gallery and sounding boards are based upon the reflection of sound.

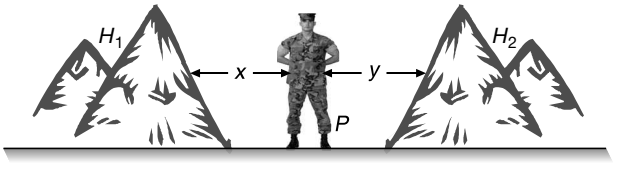
ILLUSTRATION 34

A person, standing between two parallel hills, fires a gun. He hears the first echo after 1.5 s and the second after 2.5 s. If the speed of sound is **332** ms^{-1}, calculate the distance between the hills. When will he hear the third echo?

SOLUTION

Let the person P be at a distance x from hill H_1 and at a distance y from H_2 as shown in figure. The time interval between the original sound and echoes from H_1 and H_2 will be respectively.

$$t_1 = \frac{2x}{v} \text{ and } t_2 = \frac{2y}{v}$$



Therefore, the distance between the hills is

$$x + y = \frac{v}{2}(t_1 + t_2) = \frac{332}{2}(1.5 + 2.5) = 664 \text{ m}$$

Now, as I echo will be from H_1 after time t_1, while II echo from H_2 after time t_2, III echo will be produced due to reflection of sound of I echo from H_2 or of II echo from H_1. Thus, the time after which III echo will be heard is

$$t_3 = t_1 + t_2 = 1.5 + 2.5 = 4 \text{ s}$$

The sound from both H_1 and H_2 will reach simultaneously. Note that it is an example of multiple or successive echoes which are not harmonic, as $t_1 = 1.5$ s, $t_2 = 2.5$ s and $t_3 = 4$ s.

ILLUSTRATION 35

The echo of a gunshot is heard 5 s after it is fired. Calculate the distance of the surface which reflects the sound. The velocity of sound is 340 ms^{-1}.

SOLUTION

Distance of the surface which reflects the sound is given by

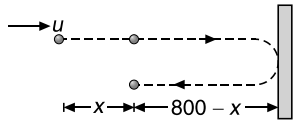
$$s = \frac{v \times t}{2} = \frac{340 \times 5}{2} = 170 \times 5 = 850 \text{ m}$$

ILLUSTRATION 36

An engine approaches a hill with a constant speed. When it is at a distance of 0.8 km it blows a whistle, whose echo is heard by the driver after 4 s. If speed of engine in air is 330 ms^{-1}. Calculate the speed of engine.

SOLUTION

Lets draw the situation according to the problem.



Distance travelled by sound when it again meets the person is

$$s = 800 + (800 - x)$$

$$\Rightarrow \quad s = 1600 - ut_0 = 1800 - u \times 4$$

$$\Rightarrow \quad \frac{1600 - 4u}{v} = 4$$

$$\Rightarrow \quad 1600 - 4u = 4v$$

$$\Rightarrow \quad 1600 - 4u = 4 \times 330$$

$$\Rightarrow \quad 4u = 1600 - 1320$$

$$\Rightarrow \quad 4u = 280$$

$$\Rightarrow \quad u = 70 \text{ ms}^{-1}$$

DIFFRACTION

When a wave is deflected by the corners of an obstacle (or is deviated from its straight line path), the phenomenon is called **diffraction**. For diffraction the wavelength of wave must be of the same order as the dimension of diffracting obstacle. For example sound has wavelength of the order of 1 m, so it is diffracted by doors and windows while light has wavelength of 4000 Å to 7800 Å, so it is diffracted by thin edge of blade and diffraction grating having nearly 15000 rulings per inch.

SUPERPOSITION OF WAVES: INTRODUCTION

When two or more wave trains travel in a region simultaneously they superpose each other, resulting in a variation of intensity in space and/or with time. This phenomenon is called **phenomenon of superposition of waves**.

SUPERPOSITION PRINCIPLE

If two or more waves arrive at a point simultaneously then the net displacement at that point is the algebraic sum of the displacement due to individual waves, i.e.

$$y = y_1 + y_2 + \ldots\ldots\ldots\ldots + y_n$$

where $y_1, y_2 \ldots\ldots\ldots\ldots, y_n$ are the displacements due to individual waves and y is the resultant displacement.
Suppose that two waves travel simultaneously along the same stretched string. Let $y_1(x,\ t)$ and $y_2(x,\ t)$ be the displacements that the string would experience if each wave travelled alone. The displacement of the string when the waves overlap is then the algebraic sum of the individual wave equations.

$$y'(x,\ t) = y_1(x,\ t) + y_2(x,\ t)$$

This summation of displacements along the string means that superimposing simply means that the waves algebraically add to produce a resultant wave (or net wave).
This is another example of the principle of superposition, which says that when several effects occur simultaneously, their net effect is the sum of the individual effects.

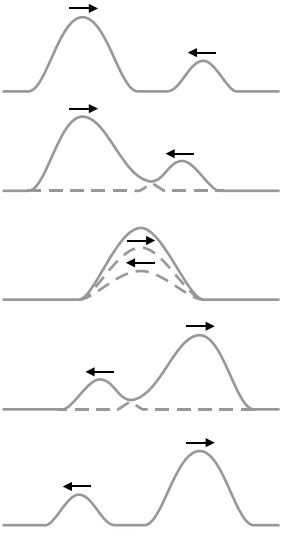


Figure shows a sequence of snapshots of two pulses travelling in opposite directions on the same stretched string. When the pulses overlap, the resultant pulse is their sum and we see the resultant wave, not the individual waves.

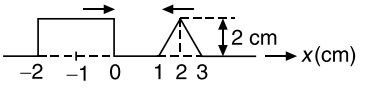
Also, note that the overlapping/superposition of the waves do not in any way alter the travel of each other and continue to move further individually after superposition as they were doing so before superposition.

Let us discuss three important cases.

	Interference	Beats	Stationary Waves
a)	Two waves of same amplitude or nearly same amplitude,	Two waves of same amplitude,	Two waves of same amplitude,
b)	having constant phase difference or no phase difference (such waves are called **coherent waves**),	having no phase difference,	having no phase difference or constant phase difference,
c)	moving in the same direction,	moving in same direction,	moving in opposite direction,
d)	having same velocity and frequency, on superposition will give rise to interference in which,	with same velocity, having slightly different frequency (Difference in frequency must not be greater than 10 Hz for human ear to observe), on superposition will give rise to beats in which,	with same velocity, having same frequency (or λ), on superposition will give rise to stationary waves in which,
e)	maxima and minima of intensity are obtained called constructive and destructive interference respectively. The position of Maxima and Minima remain fixed on screen and hence the phenomenon is called **Sustained interference**.	maxima and minima of intensity varying periodically with time are obtained.	maxima and minima of intensity varying periodically with path difference (x). The maxima of intensity are called Antinodes and minima of intensity are called Nodes.

ILLUSTRATION 37

In the arrangement shown in Figure a rectangular pulse and triangular pulse approaching each other. The pulse speed is 0.5 cms^{-1}. Sketch the resultant pulse at $t = 2$ s.



SOLUTION

In 2 s each pulse will travel a distance of 1 cm. The two pulse overlap between 0 and 1 cm. So, A_1 and A_2 can be added as shown in Figure.

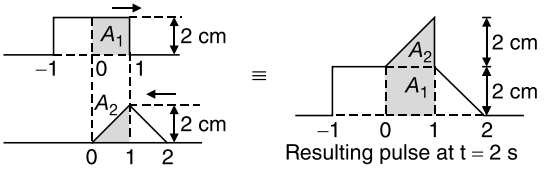


ILLUSTRATION 38

Sources separated by 20 m vibrate according to the equations $y_1 = 0.06\sin\pi t$ and $y_2 = 0.02\sin\pi t$. They send out waves along a rod with speed 3 ms^{-1}. Find the equation of motion of a particle 12 m from first source and 8 m from the second if y_1, y_2 are in metre.

SOLUTION

Since, $k = \dfrac{\omega}{v} = \dfrac{\pi}{3}$

$$\Rightarrow \quad y_1 = 0.06(\pi t - kx) = 0.06\sin\left(\pi t - \frac{\pi}{3}\times 12\right)$$

$$\Rightarrow \quad y_1 = 0.06\sin(\pi t - 4\pi)$$

Similarly, $y_2 = 0.02\sin(\pi t - kx') = 0.02\sin\left(\pi t - \dfrac{\pi}{3}\times 8\right)$

$$\Rightarrow \quad y_2 = 0.02\sin\left(\pi t - \frac{8\pi}{3}\right)$$

By Principle of Superposition $y = y_1 + y_2$

$$\Rightarrow \quad y = 0.06\sin(\pi t)\cos(4\pi) - 0.06\cos(\pi t)\sin(4\pi) + 0.02\sin(\pi t)\cos\left(\frac{8\pi}{3}\right) - 0.02\cos(\pi t)\sin\left(\frac{8\pi}{3}\right)$$

$$\Rightarrow \quad y = 0.05\sin(\pi t) - 0.0173\cos(\pi t)$$

INTERFERENCE

When two coherent waves of the same frequency moving in the same direction superpose, they give rise to variation of intensity in space – positions of maximum and minimum intensities appear alternately in the whole region of superposition. The positions of maxima and minima are fixed and this phenomenon is called **Sustained Interference**. The use of the term "**interference**" is generally restricted to this case only. Let the two waves be

$$y_1 = A_1\sin(\omega t - kx) \text{ and } y_2 = A_2\sin(\omega t - kx + \phi)$$

On superposition $y = y_1 + y_2$

$$\Rightarrow \quad y = A_1 \sin(\omega t - kx) + A_2 \sin(\omega t - kx + \phi)$$

$$\Rightarrow \quad y = A_1 \sin(\omega t - kx) + A_2 [\sin(\omega t - kx)\cos\phi + \cos(\omega t - kx)\sin\phi]$$

$$\Rightarrow \quad y = (A_1 + A_2\cos\phi)\sin(\omega t - kx) + (A_2 \sin\phi)\cos(\omega t - kx)$$

Let $A_1 + A_2\cos\phi = R\cos\theta$ and $A_2 \sin\phi = R\sin\theta$

$$\Rightarrow \quad y = R[\cos\theta \sin(\omega t - kx) + \sin\theta\cos(\omega t - kx)]$$

$$\Rightarrow \quad y = R\sin(\omega t - kx + \theta)$$

where, the resultant amplitude is

$$R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

and the phase angle $\theta = \tan^{-1}\left(\frac{A_2\sin\phi}{A_1 + A_2\cos\phi}\right)$

the resultant wave is also a harmonic wave of the same frequency.

CASE-I: CONSTRUCTIVE INTERFERENCE

Resultant Amplitude (R) is maximum when $\cos\phi = +1$,

$$\Rightarrow \quad \phi = 2n\pi \text{ where } n = 0, 1, 2, \ldots\ldots$$

$$\Rightarrow \quad x = (2n)\frac{\lambda}{2}, \text{ where } n = 0, 1, 2, 3, \ldots..$$

and $R_{max} = A_1 + A_2$

*The interference is said to be **constructive** when phase difference is an even multiple of π or path difference is an even multiple of $\lambda/2$.*

CASE-II: DESTRUCTIVE INTERFERENCE

The Resultant Amplitude is Minimum, when, $\cos\phi = -1$,

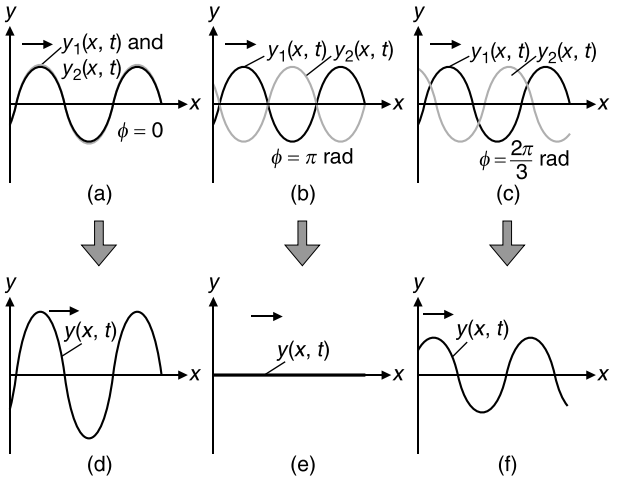
$$\Rightarrow \quad \phi = (2n+1)\pi \text{ where } n = 0,\ 1,\ 2.....$$

$$\Rightarrow \quad x = (2n+1)\frac{\lambda}{2}, \text{ where } n = 0, 1, 2, 3, ...$$

and $R_{min} = |A_1 - A_2|$

The interference is said to be destructive when phase difference is an odd multiple ofπ or path difference is an odd multiple of $\lambda/2$.

Two identical sinusoidal waves, $y_1(x,\ t)$ and $y_2(x, t)$, travel along a string in the positive direction of an x-axis. They interfere to give a resultant wave $y(x,\ t)$. The resultant wave is what is actually seen on the string. The phase difference (ϕ) between the two interfering waves is (a) 0 rad or 0°, (b) π rad or 180° and (c) $\frac{2\pi}{3}$ rad or 120°. The corresponding resultant waves are shown in (d), (e) and (f).



INTENSITY FACTOR

Since the intensity of a wave is proportional to the square of the amplitude, we have

$$I_R \propto R^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$$

This gives

$$I_{max} \propto (A_1 + A_2)^2 \text{ and } I_{min} \propto (A_1 - A_2)^2$$

$$\Rightarrow \quad \frac{I_{max}}{I_{min}} = \left(\frac{A_1 + A_2}{A_1 - A_2}\right)^2 = \left(\frac{r+1}{r-1}\right)^2$$

where $r = \frac{A_1}{A_2}$ is called the **amplitude ratio**. If I_1 and I_2 are intensities of two interfering waves, then

$$I_R = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi$$

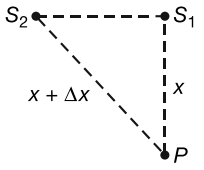
If $I_1 = I_2$, then $I_R = 4I\cos^2\left(\frac{\phi}{2}\right)$

In this case $I_{max} = 4I$ and $I_{min} = 0$

COHERENT SOURCES

Two sources which are either in phase or have a constant phase difference between them are called coherent sources. For the two sources to be coherent their frequencies must be same. But the reverse is not always true, i.e., the two different sources having the same frequency are not always coherent.

If the phase difference of the sources changes erratically with time, even if they have the same frequency the sources are said to be incoherent. So, light sources composed of a large number of same kind of atoms, which emit light of the same frequency are not coherent, because there are many atoms involved in each source and they do not oscillate in phase and hence are incoherent. Thus, two different light sources cannot be coherent.



Consider two coherent point sources of sound S_1 and S_2 which oscillate in phase with the same angular frequency ω. A point P is situated at a distance x from S_1 and $x+\Delta x$ from S_2, so that the path difference between the two waves reaching P from S_1 and S_2 is Δx. The displacement equations of two waves arriving at P are

$$y_1 = A_1 \sin(\omega t - kx)$$

and $y_2 = A_2 \sin[(\omega t - kx) + \phi]$

where, $\phi = \left(\frac{2\pi}{\lambda}\right)\Delta x$ is phase difference between two waves reaching P.

If the sources are incoherent, the phase difference between the sources keep on changing. At any point P, sometimes constructive and sometimes destructive interference takes place. If the intensity due to each source is I, the resultant intensity rapidly and randomly changes between $4I$ and zero, so that the average observable intensity is $2I$. If intensities due to individual sources is I_1 and I_2 the resultant intensity is

$$I = I_1 + I_2 \text{ (for incoherent sources)}$$

No interference effect is therefore observed. For observable interference, the sources must be coherent.

A good way to obtain a pair of coherent sources is to obtain two sound waves from the same source by dividing the original wave along two different paths and then combining them. The two waves then differ in phase only because of different paths travelled.

ILLUSTRATION 39

Two waves having intensity ratio of $100:1$, interfere with each other. Find the ratio of intensities at the maxima and minima

SOLUTION

The amplitude ratio,

$$\gamma = \frac{A_1}{A_2} = \frac{\sqrt{I_1}}{\sqrt{I_2}} = \sqrt{\frac{100}{1}} = 10$$

$$\Rightarrow \quad \frac{I_{\max}}{I_{\min}} = \frac{(\gamma+1)^2}{(\gamma-1)^2} = \frac{(10+1)^2}{(10-1)^2} = \frac{121}{81} = 1.494$$

ILLUSTRATION 40

Two coherent sound sources, each having a frequency of 400 Hz, are at distances $x_1 = 0.2$ m and $x_2 = 0.48$ m from a point. Calculate the intensity of the resultant wave at this point, if each wave has intensity of $60\ \text{Wm}^{-2}$ and the velocity of the waves in the medium is $448\ \text{ms}^{-1}$.

SOLUTION

Path difference,

$$\Delta x = x_2 - x_1 = 0.48 - 0.2 = 0.28 \text{ m}$$

The corresponding phase difference is given as

$$\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{(v/f)}\Delta x = \frac{2\pi f}{v}\Delta x$$

$$\Rightarrow \quad \phi = \frac{2\pi \times 400}{448} \times 0.28 = \frac{\pi}{2} \text{ rad}$$

Therefore, the intensity of the resultant wave is

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi = I_0 + I_0 + 2I_0 \cos\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \quad I = 2I_0 = 2 \times 60 = 120\ \text{Wm}^{-2}$$

ILLUSTRATION 41

Find the resultant amplitude and phase of a point at which N sinusoidal waves interfere. All the waves have same amplitude A and their phases increase in arithmetic progression of common difference ϕ.

SOLUTION

The diagram for their sum is shown in figure for $N = 6$. The resultant amplitude is A_R. The apex angle of every isosceles triangle is ϕ. So, the angle subtended by the resultant is $N\phi$. Since the heads of the vectors are all at the same distance r from the apex of the polygon, so

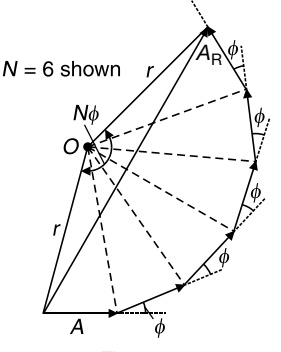


Figure 1

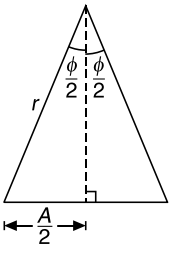


Figure 2

From Figure 1, we get

$$\frac{A_R}{2} = r\sin\left(\frac{N\phi}{2}\right) \qquad \ldots(1)$$

and from Figure 2, we get

$$\frac{A}{2} = r\sin\left(\frac{\phi}{2}\right) \qquad \ldots(2)$$

Dividing Equation (1) by (2), we get

$$\Rightarrow \quad A_R = A\frac{\sin(N\phi/2)}{\sin(\phi/2)}$$

Conceptual Note(s)

Consider $N = 2$ and $\phi = 90°$, then

$$A_R = \sqrt{2}A$$

Similarly, the above result can also be checked for other special cases.

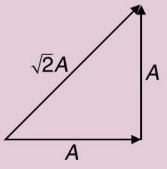
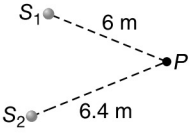


ILLUSTRATION 42

Two sources are placed from a person P as shown in figure. The speed of sound in air is 320 ms^{-1}. If sound signal is continuously varied from 500 Hz to 2500 Hz, for which frequency listener will hear minimum sound intensity.



SOLUTION

Path difference between waves arriving at point P is

$$\Delta x = S_2P - S_1P = 6.4 - 6 = 0.4 \text{ m}$$

For minimum sound intensity, we have

$$\Delta x = (2n+1)\frac{\lambda}{2} = \frac{(2n+1)}{2}\frac{v}{f}$$

$$\Rightarrow \quad f = \frac{(2n+1)v}{2\Delta x} = \frac{(2n+1)320}{2(0.4)} = 400(2n+1) \text{ Hz}$$

For $n = 0, 1, 2, \ldots\ldots$ we have

$f = 400$ Hz (for $n = 0$)

$f = 1200$ Hz (for $n = 1$)

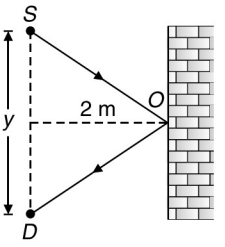
$f = 2000$ Hz (for $n = 2$)

$f = 2800$ Hz (for $n = 3$)

The frequencies are 1200 Hz and 2000 Hz

ILLUSTRATION 43

A source of sound emitting waves at 360 Hz is placed in front of a vertical wall, at a distance 2 m from it. A detector is also placed in front of the wall at the same distance from it. Find the minimum distance between the source and the detector for which the detector detects a maximum of sound. Take speed of sound in air = 360 ms^{-1}.



SOLUTION

Let the detector be at a distance of y metre from the source. Now the detector receives waves from two different paths.

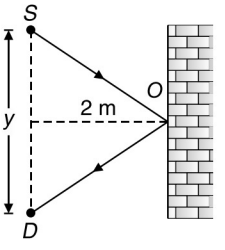
(i) Path SD

(ii) Path SOD

$$SD = y \text{ and } SOD = 2\sqrt{4 + \frac{y^2}{4}} = \sqrt{y^2 + 16}$$

So, path difference is given by

$$\Delta x = \sqrt{y^2 + 16} - y$$



For constructive interference at D i.e., for minimum value of y, we have

$$\Delta x = \lambda$$

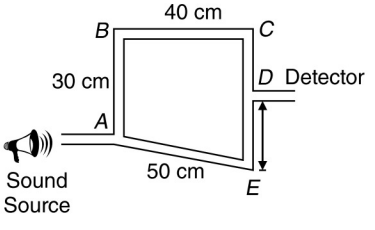
$$\Rightarrow \quad \sqrt{y^2 + 16} - y = \lambda = \frac{v}{f} = \frac{360}{360} = 1 \text{ m}$$

Solving this, we get

$$y = 7.5 \text{ m}$$

ILLUSTRATION 44

In the shown Figure $ABCDE$ is a tube which is open at A and D. A source of sound A is placed in front of A. If frequency of the source can be varied from 2000 Hz to 4000 Hz. Find frequencies at which a detector placed in front of D receives a maximum of intensity. (Given speed of sound = 340 ms^{-1})



SOLUTION

The path difference between the sounds reaching the detector from two parts of the tube is

$$\Delta x = ABCD - AED$$

$$\Rightarrow \quad \Delta x = [(30+40+30)-(30+50)] \text{ cm}$$

$$\Rightarrow \quad \Delta x = 20 \text{ cm} = 0.2 \text{ m}$$

If sound has frequency f and wavelength λ, then

$$\lambda = \frac{340}{f} \text{ m}$$

For maximum intensity at detector, we have

$$\Delta x = n\lambda, \text{ where } n = 1, 2, 3, \ldots\ldots$$

$$\Rightarrow \quad 0.2 \text{ m} = n\left(\frac{340}{f}\right)$$

$$\Rightarrow \quad f = \frac{n \times 3400}{2} \text{ Hz}$$

$$\Rightarrow \quad f = n \times 1700 \text{ Hz}$$

Now, for $n = 1$, $f = 1700$ Hz

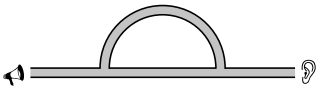
For $n = 2$, $f = 3400$ Hz

For $n = 3$, $f = 5100$ Hz and so, on

Since, frequency of the source is varied between 2000 Hz to 4000 Hz, so the frequency at which maxima is received by the detector is $f = 3400$ Hz.

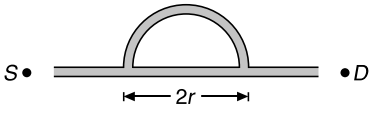
ILLUSTRATION 45

Figure shows a tube structure in which a sound signal is sent from one end and received at the other end. The semi-circular part has a radius of 20 cm. The frequency of the sound source can be varied electronically between 1000 and 4000 Hz. Find the frequencies at which maxima of intensity are detected. The speed of sound in air = 340 ms^{-1}.



SOLUTION

According to the question, the path difference between the waves arriving at D is given by



$$\Delta x = \pi r - 2r = (\pi - 2)r$$

For maximum intensity, we have path difference

$$\Delta x = (2n)\frac{\lambda}{2} = n\lambda$$

$$\Rightarrow \quad (\pi - 2)r = n\lambda = \frac{nv}{f}$$

$$\Rightarrow \quad f = \frac{nv}{(\pi-2)r} = \frac{n \times 340}{(3.14-2)\times 0.2} = 1491n \text{ Hz}$$

For $n = 1$, $f = 1491$ Hz

For $n = 2$, $f = 2982$ Hz

For $n = 3$, $f = 4473$ Hz

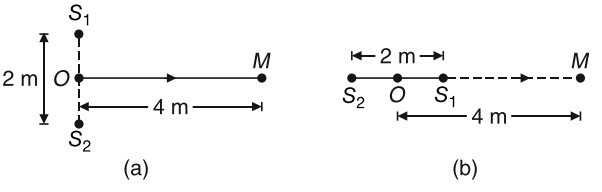
The frequencies in the given range are 1491 Hz and 2982 Hz

ILLUSTRATION 46

Two speakers connected to the same source of fixed frequency are placed 2 m apart in a box. A sensitive microphone placed at a distance of 4 m from their mid-point along the perpendicular bisector shows maximum response. The box is slowly rotated till the speakers are in line with the microphone. The distance between the mid-point of the speakers and microphone remains unchanged. Exactly 5 maximum responses are observed in the microphone in doing this. Calculate the wavelength of the sound wave.

SOLUTION

As shown in Figure (a), initially $S_1M = S_2M$, $\Delta x = 0$. Hence, there is principal maxima at M.



On rotation of speakers about O, when S_1 and S_2 are in line with microphone M as shown in Figure (b), 5 maximum response are observed. Therefore, we should have

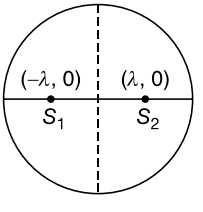
$$S_2M - S_1M = 5\lambda$$

$$\Rightarrow \quad S_2S_1 = 5\lambda$$

$$\Rightarrow \quad \lambda = \frac{S_2S_1}{5} = \frac{2}{5} = 0.4 \text{ m}$$

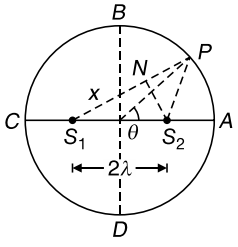
ILLUSTRATION 47

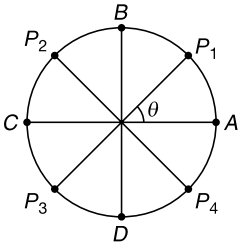
Two identical coherent sources S_1 and S_2 which emit sounds of wavelength λ in same phase are placed in opposite side of the centre of a circle of large radius (Considered to be origin) at $(-\lambda, 0)$ and $(\lambda, 0)$ as shown. Calculate angular position on the circle at which constructive interference occurs.



SOLUTION

From the shown Figure path difference between two sound waves coming from S_1 and S_2 at P.





$$\Delta x = S_1S_2\cos\theta = 2\lambda\cos\theta \qquad \{\because S_1S_2 = 2\lambda\}$$

For maxima i.e., constructive interference, we have

$$\Delta x = n\lambda$$

$$\Rightarrow \quad 2\lambda\cos\theta = n\lambda$$

$$\Rightarrow \quad \cos\theta = \frac{n}{2}$$

Since, $-1 \le \cos\theta \le 1$

$$\Rightarrow \quad -1 \le \frac{n}{2} \le 1$$

$$\Rightarrow \quad -2 \le n \le 2$$

n	–2	–1	0	1	2
$\cos\theta = n/2$	–1	$-1/2$	0	$1/2$	1
θ	180°	120° 240°	90° 270°	60° 300°	0°
Points	C	P_2, P_3	B, D	P_1, P_4	A

So, a total of 8 maxima are obtained on the circle.

Test Your Concepts-IV

Based on Interference

(Solutions on page H.237)

1. Two harmonic waves are represented in *SI* units by, $y_1(x, t) = 0.2\sin(x - 3t)$ and

$$y_2(x, t) = 0.2\sin(x - 3t + \phi)$$

Write the expression for the sum $y = y_1 + y_2$ for $\phi = \frac{\pi}{2}$ rad. If the phase difference ϕ between waves is unknown and amplitude of their sum is 0.32 m, calculate ϕ.

2. Two waves of equal frequencies have their amplitudes in the ratio of 3 : 5. They are superimposed on each other. Calculate the ratio of minimum and maximum intensities of the resultant wave.

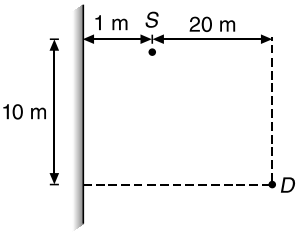
3. Three component sinusoidal waves progressing in the same direction along the same path have the same period but their amplitudes are A, $\frac{A}{2}$ and $\frac{A}{3}$. The phases of the variation at any position x on their path at time $t = 0$ and 0, $-\frac{\pi}{2}$ and $-\pi$ respectively. Find the amplitude and phase of the resultant wave.

4. Sound waves from a tuning fork A reach a point P by two separate paths ABP and ACP. When ACP is greater than ABP by 11.5 cm, there is silence at P. When the difference is 23 cm the sound becomes loudest at P and when 34.5 cm there is silence again and so on. Calculate the minimum frequency of the fork if the velocity of sound is taken to be 331.2 ms^{-1}.

5. A wave is represented by $y_1 = 10\cos(5x + 25t)$ where x is measured in meters and t in seconds. A second wave for which $y_2 = 20\cos\left(5x + 25t + \frac{\pi}{3}\right)$ interferes with the first wave. Calculate the amplitude and phase of the resultant wave.

6. Two loudspeakers S_1 and S_2 vibrating in the same phase, each emit sounds of frequency 220 Hz uniformly in all directions. S_1 has an acoustic output of 1.2×10^{-3} watt and S_2 has 1.8×10^{-3} watt. Consider a point P such that $S_1P = 0.75$ m and $S_2P = 3$ m. How are the phases arriving at P related? What is the intensity at P when both S_1 and S_2 are on? Speed of sound in air is 330 ms^{-1}.

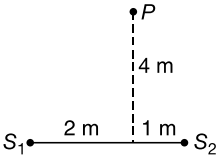
7. A sound source capable of producing sound of variable frequencies is located at a distance of 1 m from a reflecting wall as shown in Figure. A detector D is lying at a point shown in Figure. The various distances are also indicated. If the speed of sound in air is 340 ms^{-1}, find the frequencies of sound within the audible range, which will have maximum intensity at detector?



8. Two sound sources S_1 and S_2 emit pure sinusoidal waves in phase. If the speed of sound is 350 ms^{-1}, then for what frequencies,

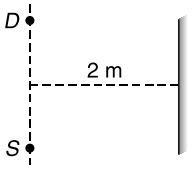
(a) constructive interference occurs at P?

(b) destructive interference occurs at P?



9. A sound wave of wavelength 40 cm enters the tube shown in Figure from the source. What must be the smallest radius r of the semi-circular part of the tube, so that minima would be received by the detector?

10. A source emitting sound of frequency 180 Hz is placed in front of an obstacle at a distance 2 m from it. A detector is also placed in front of obstacle at the same distance from it.

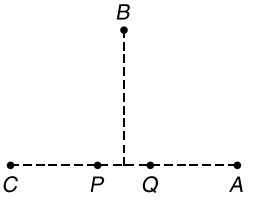


(a) Find the minimum distance between the source and detector for which the detector detects a maximum sound.

(b) How much further to the right must the obstacle be moved if the two waves are to be out of phase by 180°.

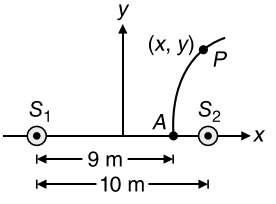
Speed of sound in air = 360 ms^{-1}.

11. Sources P and Q are two equally intense coherent sources emitting radiation of wavelength 20 m as shown in Figure.



The separation between P and Q is 5 m and the phase of P is ahead of that of Q by 90°. A, B and C are three distinct points of observation, each equidistant from the midpoint of PQ. Calculate the ratio of intensities of radiation arriving at A, B and C.

12. Two identical speakers 10 m apart are driven by the same oscillator with a frequency of $f = 21.5$ Hz Figure.



Explain why a receiver at point A records a minimum in sound intensity from the two speakers. If the receiver is moved in the plane of the speakers, what path should it take so that the intensity remains at a minimum? That is, determine the relationship between x and y (the coordinates of the receiver) that causes the receiver to record a minimum in sound intensity. Take the speed of sound to be 343 ms^{-1}.

STATIONARY WAVES

When two coherent waves of equal frequencies and equal amplitudes travelling through a region in opposite directions superpose, the resultant effect is a wave, which does not travel either way with time. These waves are called **stationary waves** or **standing waves** & their resultant amplitude varies periodically with distance. The name stationary for such type of wave is justified because there is no flow of energy along the stationary wave.

In practice, a stationary wave is formed when a wave train reflected at a boundary superimposes with the incident wave. The incident and the reflected waves then interfere to produce a stationary wave.

Stationary waves can be of two types

TRANSVERSE WAVES

Transverse stationary waves are formed in strings stretched between two points.

LONGITUDINAL WAVES

Longitudinal stationary waves are formed in air columns, e.g., in an organ pipe (open of closed).

STATIONARY WAVES PRODUCED ON REFLECTION FROM THE FREE END (RARER MEDIUM)

Let the incident wave be $y_i = A\sin(\omega t - kx)$

Then reflected wave is given by $y_r = A\sin(\omega t + kx)$

By Principle of Superposition, we have

$$y = y_i + y_r = A[\sin(\omega t - kx) + \sin(\omega t + kx)]$$

$$\Rightarrow \quad y = 2A\cos(kx)\sin(\omega t) = R\sin(\omega t)$$

This equation represents a stationary wave. We note that all the particles execute S.H.M. with a frequency equal to the frequency of the interfering waves. However,

unlike a progressive wave, the resultant amplitude is R given by

$$R = 2A\cos(kx)$$

$$\Rightarrow \quad I_R = R^2 = 4A^2\cos^2(kx)$$

Resultant intensity (I_R) is not the same for all the particles but varies with the location x of the particle. I_R **has maximum value** $4A^2$ when

$$\cos^2(kx) = 1$$

$$\Rightarrow \quad \cos(kx) = \pm 1$$

$$\Rightarrow \quad kx = n\pi, \text{ where } n = 0, 1, 2, 3,\ldots$$

$$\Rightarrow \quad x = \frac{n\lambda}{2}, \text{ where } n = 0, 1, 2, 3,\ldots \qquad \left\{\because k = \frac{2\pi}{\lambda}\right\}$$

The points where the amplitude is maximum are called **Antinodes.** Antinodes are obtained at x values given by

$$x = 0, \frac{\lambda}{2}, \frac{2\lambda}{2}, \frac{3\lambda}{2}, \frac{4\lambda}{2}, \ldots.$$

Distance between two successive antinodes is $\lambda/2$.
I_R **has the minimum value** i.e., zero when

$$\cos(kx) = 0$$

$$\Rightarrow \quad kx = (2n+1)\frac{\pi}{2}, \text{ where } n = 0, 1, 2, 3,\ldots$$

$$\Rightarrow \quad x = (2n+1)\frac{\lambda}{4}, \text{ where } n = 0, 1, 2, 3,\ldots$$

The points where the amplitude is zero are called **Nodes**. Nodes are obtained at x values given by

$$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \frac{7\lambda}{4}, \ldots.$$

Conceptual Note(s)

Here we also take a note that an antinode is always formed at the free end (i.e. at $x = 0$).

The formation of the reflected pulse is similar to the overlap of two pulses travelling in opposite directions. The net displacement at any point is given by the Principle of Superposition.

Figure shows two pulses with the same shape, travelling in opposite directions but not inverted relative to each other. Note that at one instant, the displacement of the free end is double the pulse height. Also note that an antinode is always formed at the free end.

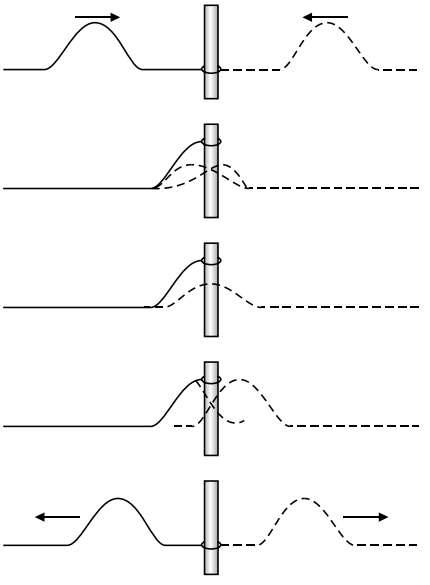


ILLUSTRATION 48

The standing wave $y = 2A\sin kx\cos\omega t$ in an elastic medium is the result of superposition of two travelling waves y_1 and y_2. If $y_1 = A\sin(\omega t - kx)$, determine the wave y_2.

SOLUTION

Since, $y = 2A\sin kx\cos\omega t$

$$\Rightarrow \quad y = A\sin(kx + \omega t) + A\sin(kx - \omega t)$$

Also, $y_1 = A\sin(\omega t - kx) = -A\sin(kx - \omega t)$
By the principle of superposition,

$$y = y_1 + y_2$$

$$\Rightarrow \quad y_2 = y - y_1 = [A\sin(kx + \omega t) + A\sin(kx - \omega t)] - [-A\sin(kx - \omega t)]$$

$$\Rightarrow \quad y_2 = A\sin(kx + \omega t) + 2A\sin(kx - \omega t)$$

$$\Rightarrow \quad y_2 = A\sin(\omega t + kx) - 2A\sin(\omega t - kx)$$

ILLUSTRATION 49

The vibration of a string of length 60 cm fixed at both ends are represented by the equation

$$y = 4\sin\left(\frac{\pi x}{15}\right)\cos(96\pi)$$

where x and y are in cm and t in second.

(a) What is the maximum displacement at $x = 5$ cm?
(b) Where are the nodes located along the string?
(c) What is the velocity of the particle at $x = 7.5$ cm and $t = 0.25$ s?
(d) Write down the equations of component waves whose superposition gives the above wave

SOLUTION

(a) For the point at $x = 5$ cm,

$$y = 4\sin\left(\frac{5\pi}{15}\right)\cos 96\pi t$$

Therefore, the maximum displacement is

$$A_s = 4\sin\left(\frac{5\pi}{15}\right) = 4\sin\left(\frac{\pi}{3}\right) = 2\sqrt{3} \text{ cm}$$

(b) At nodes, the amplitude of the stationary wave is zero. That is,

$$4\sin\left(\frac{\pi x}{15}\right) = 0$$

$$\Rightarrow \quad \frac{\pi x}{15} = 0,\ \pi,\ 2\pi,\ 3\pi,...$$

$$\Rightarrow \quad x = 0, 15, 30, 45, 60 \text{ cm}$$

The length of the string being 60 cm, its both ends are nodes.

(c) The velocity of a particle of the string is given as

$$v_p = \frac{dy}{dt} = -(96\pi)4\sin\left(\frac{\pi x}{15}\right)\sin(96\pi t)$$

Thus, the velocity of the particle at $x = 7.5$ cm and $t = 0.25$ s is given as

$$v_p = -(96\pi)4\sin\left(\frac{7.5\pi}{15}\right)\sin(96\pi \times 0.25)$$

$$\Rightarrow \quad v_p = -384\sin\left(\frac{\pi}{2}\right)\sin(24\pi) = -384 \times 1 \times 0 = 0$$

(d) $y = 4\sin\left(\frac{\pi x}{15}\right)\cos(96\pi t)$

$$\Rightarrow \quad y = 2\left[\sin\left(\frac{\pi x}{15} + 96\pi t\right) + \sin\left(\frac{\pi x}{15} - 96\pi t\right)\right]$$

$$\Rightarrow \quad y = 2\sin\left(\frac{\pi x}{15} + 96\pi t\right) + 2\sin\left(\frac{\pi x}{15} - 96\pi t\right)$$

So, $y_1 = 2\sin\left(\frac{\pi x}{15} + 96\pi t\right)$ and

$y_2 = 2\sin\left(\frac{\pi x}{15} - 96\pi t\right)$

STATIONARY WAVES PRODUCED ON REFLECTION FROM FIXED END (DENSER MEDIUM)

Let the incident wave be $y_i = A\sin(\omega t - kx)$

Then the reflected wave is given by Stroke's Law as

$$y_r = -A\sin(\omega t + kx)$$

On superposition $y = y_i + y_r$

$$\Rightarrow \quad y = A[\sin(\omega t - kx) - \sin(\omega t + kx)]$$

$$\Rightarrow \quad y = -2A\sin(kx)\cos(\omega t) = R\cos(\omega t)$$

where $R = -2A\sin(kx)$

$$\Rightarrow \quad I_R = R^2 = 4A^2\sin^2(kx)$$

I_R has the maximum value $4A^2$ when

$$\sin^2(kx) = 1$$

$$\Rightarrow \quad \sin(kx) = \pm 1$$

$$\Rightarrow \quad kx = (2n+1)\frac{\pi}{2}, \text{ where } n = 0,\ 1,\ 2,\ 3,...$$

$$\Rightarrow \quad x = (2n+1)\frac{\lambda}{4}, \text{ where } n = 0, 1, 2, 3,... \left\{\because\ k = \frac{2\pi}{\lambda}\right\}$$

The points where the amplitude is maximum are called **Antinodes.** Antinodes are obtained at x values given by

$$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \frac{7\lambda}{4}, ...$$

Distance between two successive antinodes is $\lambda/2$.

I_R has the minimum value i.e., zero when

$$\sin(kx) = 0$$

$$\Rightarrow \quad kx = n\pi, \text{ where } n = 0,\ 1,\ 2,...$$

$$\Rightarrow \quad x = n\frac{\lambda}{2} \qquad \left\{\because\ k = \frac{2\pi}{\lambda}\right\}$$

The points where the amplitude is zero are called **Nodes**. Nodes are obtained at x values given by

$$x = 0, \frac{\lambda}{2}, \frac{2\lambda}{2}, \frac{3\lambda}{2}, \frac{4\lambda}{2}, ...$$

Conceptual Note(s)

(a) Here we also take a note that a node is always formed at the fixed end (i.e. at $x = 0$).

(b) The distance between two successive nodes is $\frac{\lambda}{2}$. The distance between a node and the nearest consecutive antinode is $\frac{\lambda}{4}$. Nodes and Antinodes are formed alternately.

(c) In a longitudinal stationary wave, there is maximum **variation** of pressure, and hence density, at the **nodes**. There is **no variation** of pressure and density at the **antinodes**. Therefore, **Antinodes** may be called **Pressure Nodes** and **Nodes** may be called **Pressure Antinodes.**

The formation of the reflected pulse is similar to the overlap of two pulses travelling in opposite directions. The net displacement at any point is given by the Principle of Superposition.

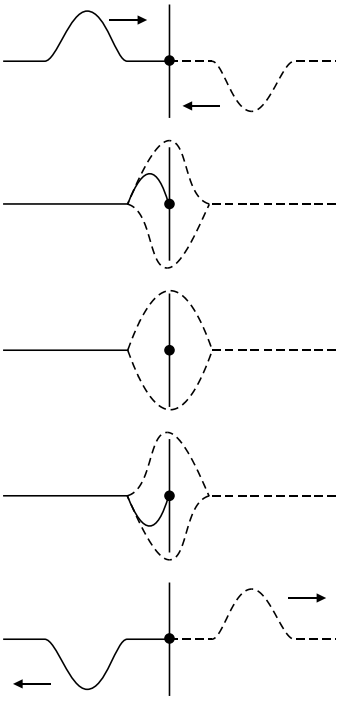


Figure shows two pulses with the same shape, one inverted with respect to the other, travelling in opposite directions. Because these two pulses have the same shape the net displacement of the point where the string is attached to the wall is zero at all times and hence a node is always obtained at the fixed end.

ILLUSTRATION 50

A standing wave is formed by two harmonic waves, $y_1 = A\sin(kx-\omega t)$ and $y_2 = A\sin(kx+\omega t)$ travelling on a string of density ρ, area of cross-section S, in opposite directions. Find the total mechanical energy between two adjacent nodes on the string.

SOLUTION

The distance between two adjacent nodes is $\frac{\lambda}{2}$ or $\frac{\pi}{k}$
Volume of string between two nodes is

$$V = \begin{pmatrix}\text{area of}\\ \text{cross-section}\end{pmatrix}\begin{pmatrix}\text{distance between}\\ \text{two nodes}\end{pmatrix}$$

$$\Rightarrow \quad V = (S)\left(\frac{\pi}{k}\right)$$

Energy density (energy per unit volume) of a travelling wave is given by

$$u = \frac{1}{2}\rho A^2\omega^2$$

A standing wave is formed by two identical waves travelling in opposite directions. So, energy stored between two nodes in a standing wave is twice the energy stored in distance π/k of a travelling wave.

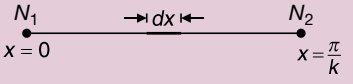
$$\Rightarrow \quad E = 2(\text{Energy Density})(\text{Volume})$$

$$\Rightarrow \quad E = 2\left(\frac{1}{2}\rho A^2\omega^2\right)\left(\frac{\pi S}{k}\right) = \frac{\rho A^2\omega^2\pi S}{k}$$

Alternate Method:
The equation of the standing wave

$$y = y_1 + y_2 = 2A\sin(kx)\cos(\omega t) = A(x)\cos(\omega t)$$

where, $A(x) = 2A\sin(kx)$



i.e., first node is at $x = 0$ and the next node is at $x = \frac{\pi}{k}$. Let us take an infinitesimal element of length dx at distance x from N_1. Mass of this element is $dm(=\rho S dx)$ and this element can be treated as point mass oscillating simple harmonically with angular frequency ω and amplitude $2A\sin(kx)$.
Hence, energy of this element is

$$dE = \frac{1}{2}(dm)(2A\sin kx)^2(\omega^2)$$

$$\Rightarrow \quad dE = \frac{1}{2}(\rho S dx)(2A\sin kx)^2\omega^2$$

Integrating this from $x = 0$ to $x = \frac{\pi}{k}$, we get

$$E = \frac{\rho A^2\omega^2\pi S}{k}$$

ILLUSTRATION 51

Consider a wave propagating in the negative x-direction whose frequency is 100 Hz. At $t = 5$ s, displacement associated with the wave is given by $y = 0.5\cos(0.1x)$ where x and y are measured in centimetre and t in second. Obtain the displacement (as a function of x) at $t = 10$ s. What is the wavelength and velocity associated with the wave?

SOLUTION

A wave travelling in negative x-direction can be represented as,

$$y(x, t) = A\cos(\omega t + kx + \phi)$$

At $t = 5$ s, $y(x,\ t = 5) = A\cos(5\omega + kx + \phi)$

Comparing this with the given equation, we get

$$A = 0.5 \text{ cm}, k = 0.1 \text{ cm}^{-1} \text{ and}$$

$$5\omega + \phi = 0 \qquad \ldots(1)$$

Since, $\lambda = \frac{2\pi}{k}$ i.e., $\lambda = \frac{2\pi}{0.1} = 20\pi$ cm

Also, $\omega = 2\pi f = 200\pi$ rads^{-1}

$$\Rightarrow \quad v = \frac{\omega}{k} = \frac{200\pi}{0.1} = 2000\pi \text{ cms}^{-1}$$

From Equation (1), we get $\phi = -5\omega$

At $t = 10$ s, we have

$$y = y(x, t = 10) = 0.5\cos(0.1x + 10\omega - 5\omega)$$

$$\Rightarrow \quad y = 0.5\cos(0.1x + 5\omega)$$

Substituting $\omega = 200\pi$, we get

$$y = y(x, t = 10) = 0.5\cos(0.1x + 1000\pi)$$

$$\Rightarrow \quad y = 0.5\cos(0.1x)$$

COMPARISON OF PROGRESSIVE AND STATIONARY WAVE

The following table compares a progressive wave with a stationary wave.

Sl. No.	Progressive	Stationary
1.	The wave advances with a constant speed.	The wave does not advance but remains confined in a particular region.
2.	The amplitude is the same for all the particles in the path of the wave.	The amplitude varies according to position, being zero at the nodes and maximum at the antinodes.
3.	All particles within one wavelength have different phases.	Phase of all particles between two adjacent nodes is the same. Particles in adjacent segments of length $\frac{\lambda}{2}$ have opposite phases.
4.	Energy is transmitted in the direction of propagation of the wave.	Energy is associated with the wave, but there is no transfer of energy across any section of the medium.

Conceptual Note(s)

(a) At nodes displacement of particles is always zero, so they are permanently at rest. But strain $\frac{\partial y}{\partial x}$ at nodes is maximum, so pressure and hence energy is maximum at nodes.

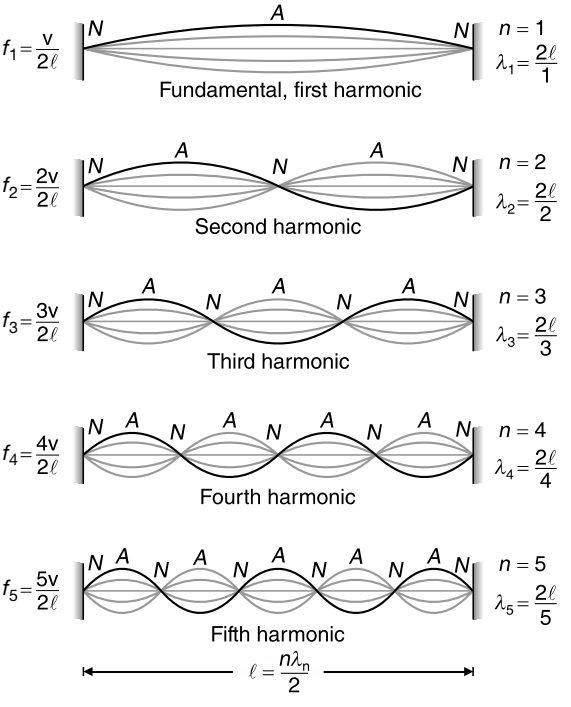
(b) At antinodes the displacement is maximum and strain $\frac{\partial y}{\partial x}$ is zero, so pressure and hence energy is minimum at antinodes.

(c) All particles between two consecutive nodes vibrate in same phase while the particles on opposite side of a node vibrate in opposite phase.

STATIONARY WAVES IN A STRING FIXED AT BOTH ENDS AND MODES OF VIBRATION OF A STRETCHED STRING

Consider a string of length l, stretched under a tension T between two fixed points. If the string is plucked and then released, a transverse wave travels along the string and is reflected at the ends.

A stationary wave is thus set up in the string. Since the end points are fixed, they are nodes. A string can vibrate in several modes. The first five modes and their frequencies are Shown in Figure.



Fundamental Mode or First Harmonic

String plucked at $\frac{l}{2}$ to get 1 loop

$$l = 2\left(\frac{\lambda_1}{4}\right) = \frac{v}{2f_1} \qquad \{\because v = f\lambda\}$$

$$\Rightarrow \quad f_1 = \frac{v}{2l} = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

Second Harmonic or First Overtone

String plucked at $\frac{l}{4}$ to get 2 loops

$$l = 4\left(\frac{\lambda_2}{4}\right) = \frac{v}{f_2} \qquad \{\because v = f\lambda\}$$

$$\Rightarrow \quad f_2 = \frac{v}{l} = 2\left(\frac{v}{2l}\right) = 2\left(\frac{1}{2l}\sqrt{\frac{T}{\mu}}\right) = 2f_1$$

Third Harmonic or Second Overtone

String plucked at $\frac{l}{6}$ to get 3 loops

$$l = 6\left(\frac{\lambda_3}{4}\right) = \frac{3v}{2f_3} \qquad \{\because v = f\lambda\}$$

$$\Rightarrow \quad f_3 = \frac{3v}{2l} = 3\left(\frac{v}{2l}\right) = 3\left(\frac{1}{2l}\sqrt{\frac{T}{\mu}}\right) = 3f_1$$

In general, if the string of length l is plucked at length $\frac{l}{2n}$, then it vibrates in n segments (loops) such that $l = \frac{n\lambda_n}{2}$ and hence we get the nth harmonic whose frequency is given by

$$f_n = n\left(\frac{1}{2l}\sqrt{\frac{T}{m}}\right) = nf_1.$$

If H represents Harmonic, O represents overtone and f_n represents the frequency of the n^{th} harmonic, then they are related to each other as shown in table.

H	**1**	**2**	**3**	**4**	**5**	**...**	**n**	**n + 1**
f_n	f_1	$2f_1$	$3f_1$	$4f_1$	$5f_1$	$\cdots$	nf_1	$(n+1)f_1$
O	–	1	2	3	4	...	$n-1$	n

Conceptual Note(s)

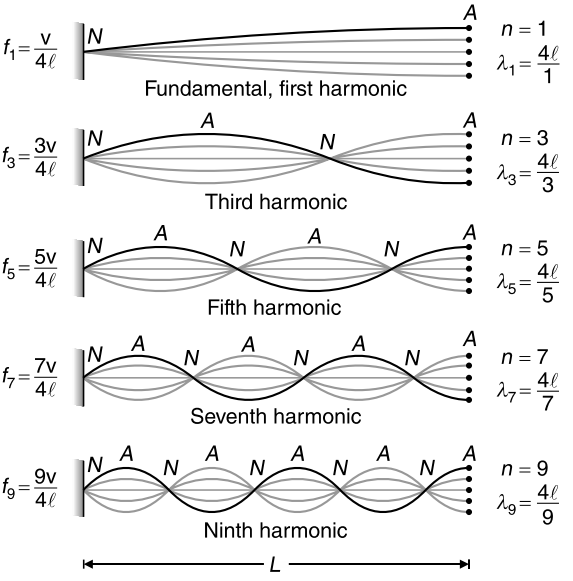
(a) Overtones are the frequencies which are counted leaving the first harmonic i.e. first overtone will be assigned to the frequency that happens the be the next immediate present frequency after first harmonic.
e.g., if third harmonic is present immediately after first harmonic then third harmonic is designated as first overtone.

(b) Whenever we are asked to compare the overtones we compare their respective harmonics.

(c) Harmonics may be either odd or even or both odd and even, but overtones are always both odd and even.

STATIONARY WAVES IN A STRING FIXED AT ONE END AND MODES OF VIBRATION OF A STRETCHED STRING

Similarly, stationary waves can be produced in string fixed at one end. *The first five modes and their frequencies are shown in Figure.*



In each mode of vibration shown, there is an odd number of quarter wave lengths in the length l such that

$$l = n\left(\frac{\lambda_n}{4}\right),\ n = 1,\ 3,\ 5,...$$

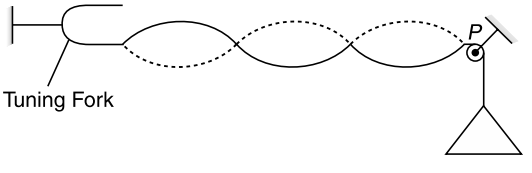
$$\Rightarrow \quad \lambda_n = \frac{v}{f_n} = \frac{4l}{n}$$

$$\Rightarrow \quad f_n = n\left(\frac{v}{4l}\right) = nf_1$$

where, $f_1 = \frac{v}{4l}$ is the fundamental frequency. The natural frequencies of this system are in the ratio $1:3:5:7:9:...$ which means that only odd harmonics are present and even harmonics are absent.

MELDE'S EXPERIMENT

One end of a horizontal string is attached to a prong of an electrically maintained tuning fork. The string passes over a pulley P and to the other end a desired weight can be attached. When the fork vibrates, a stationary wave is formed on the string. If the tension and/or the length of the string are suitably adjusted, the string can be made to vibrate in one of its modes with large amplitude. When this happens, string is in resonance with tuning fork and their frequencies are equal.



If n_1 and n_2 are the number of loops corresponding to tensions T_1 and T_2, then

$$n_1^2 T_1 = n_2^2 T_2$$

i.e. $n^2 T = \text{constant}$

Thus, if the number of loops is to be increased the tension should be decreased.

SONOMETER

It consists of a thin wire mounted on a large hollow wooden box. One end of the wire is fixed at A and the other end E carries a load. The wire passes over two sliding bridges B and C and a pulley D. Any disturbance created on the wire gets reflected by the bridges and hence a transverse stationary wave is formed between B and C. The length BC and tension are adjusted to produce resonance with a source of sound, e.g., tuning fork.

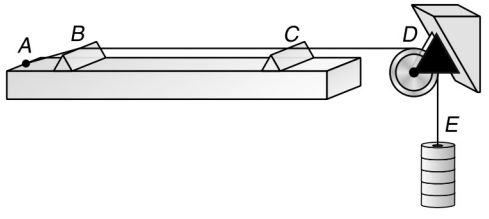


ILLUSTRATION 52

The fundamental frequency of a sonometer wire increases by 6 Hz if its tension is increased by 44% keeping the length constant. Find the change in the fundamental frequency of the sonometer wire, when the length of the wire is increased by 20% keeping the original tension in the wire.

SOLUTION

The fundamental frequency is given by

$$f = \frac{1}{2L}\sqrt{\frac{T}{m}}$$

Keeping the length of given wire constant, we have

$$\frac{f'}{f} = \left(\frac{T'}{T}\right)^{1/2}$$

Here, $f' = f + 6$ and $T' = T + 0.44T = 1.44T$

$$\Rightarrow \quad \frac{(f+6)}{f} = \sqrt{\frac{1.44T}{T}}$$

$$\Rightarrow \quad f = 30 \text{ Hz}$$

Now, keeping the original tension same the length of given wire is changed to

$$L'' = L + 0.020L = 1.20L$$

$$\Rightarrow \quad \frac{f''}{f} = \frac{L}{L''} = \frac{1}{1.20}$$

$$\Rightarrow \quad f'' = \frac{30}{1.2} = 25 \text{ Hz}$$

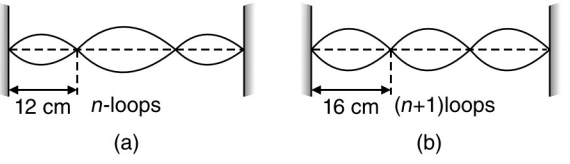
$$\Rightarrow \quad \Delta f'' - f = 25 - 30 = -5 \text{ Hz}$$

Thus, the fundamental frequency will decrease by 5 Hz.

ILLUSTRATION 53

A string fixed at both ends has consecutive standing wave modes for which the distances between adjacent nodes are 12 cm and 10 cm respectively. What the minimum possible length of the string?

SOLUTION



Let length of string be l then

From Figure (a) $12n = l$ ...(1)

From Figure (b) $10(n+1) = l$ ...(2)

From equation (1) and (2), we get

$$12n = 10(n+1)$$

$$\Rightarrow \quad 2n = 10$$

$$\Rightarrow \quad n = 5$$

From equation (1), we get $12 \times 5 = l$

$$\Rightarrow \quad l = 60 \text{ cm}$$

Therefore, the minimum possible length of the string can be 60 cm

ILLUSTRATION 54

The total length of a sonometer wire between fixed ends is 110 cm. Thus, bridges are placed to divide the length of the wire in the ratio 6 : 3 : 2. What is the minimum common frequency with which three parts can vibrate? Also calculate the ratio of number of loops formed in three parts. The tension in the wire is 400 N and the mass per unit length is 0.01 kgm^{-1}.

SOLUTION

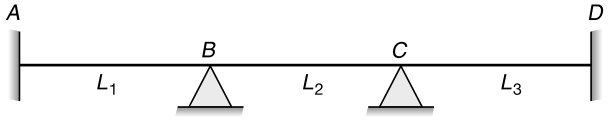
The length of each segment is given by

$$L_1 = \left(\frac{6}{6+3+2}\right)(110 \text{ cm}) = 0.6 \text{ m}$$

$$L_2 = \left(\frac{3}{6+3+2}\right)(110 \text{ cm}) = 0.3 \text{ m}$$

$$L_3 = \left(\frac{2}{6+3+2}\right)(110 \text{ cm}) = 0.2 \text{ m}$$

Let f_1, f_2 and f_3 be the fundamental frequencies of the segments AB, BC and CD respectively.



Since $f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$, so $fL = \frac{1}{2}\sqrt{\frac{T}{\mu}} = \text{constant}$

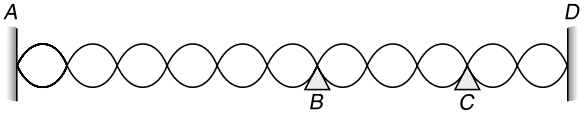
Also, $f_1L_1 = f_2L_2 = f_3L_3$ i.e., $0.6f_1 = 0.3f_2 = 0.2f_3$

Obviously, the sixth harmonic of AB, third harmonic of BC and second harmonic of CD coincide. Hence the lowest common frequency is given by

$$6f_1 = 3f_2 = 2f_3 = f_c$$

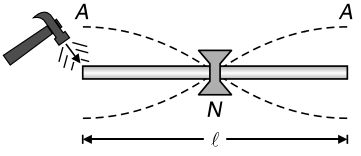
$$\Rightarrow \quad f_c = 6f_1 = 6\times\frac{1}{2L_1}\sqrt{\frac{T}{\mu}} = \frac{6}{2\times 0.6}\sqrt{\frac{400}{0.01}} = 1000 \text{ Hz}$$

Since nth *harmonic contains* n *loops, so AB vibrates in 6 loops, BC in 3 loops and CD in 2 loops*. Ratio of number of loop in three segments is 6 : 3 : 2.



VIBRATIONS OF A CLAMPED ROD

Let us discuss the oscillations of a rod clamped at its middle point on its length as shown in Figure.



Let the rod be gently hit at its one end due to which it begins to oscillate. In natural oscillations the rod vibrates at its lowest frequency and maximum wavelength, which we call fundamental mode of oscillations. With maximum wavelength, when transverse stationary waves setup in the rod, the free ends vibrate as antinodes and the clamped end as a node. If λ be wavelength of wave, then $l = \lambda_1/2$.

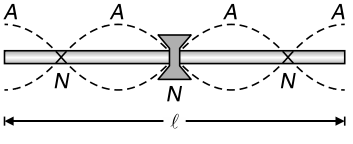
$$\Rightarrow \quad \lambda_1 = 2l \qquad \ldots(1)$$

The frequency of fundamental oscillations of a rod clamped at midpoint is

$$f_1 = \frac{v}{\lambda_1} = \frac{1}{2l}\sqrt{\frac{Y}{\rho}} \qquad \ldots(2)$$

where, Y is the Young's modulus and ρ is the density of the material of rod.

The next higher frequency at which rod vibrates is shown in Figure.



In this case if λ_3 be the wavelength of the waves in rod, then we have $l = 3\lambda_3/2$ and hence

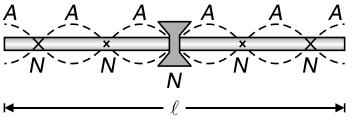
$$\lambda_3 = \frac{2l}{3} \qquad \ldots(3)$$

So, in this case, the oscillation frequency of rod is

$$f_3 = \frac{v}{\lambda_3} = 3\left(\frac{1}{2l}\sqrt{\frac{Y}{\rho}}\right) = 3f_1 \qquad \ldots(4)$$

This is called third harmonic or first overtone frequency of the clamped rod.

Similarly, the next higher frequency of oscillation is again shown in Figure



In this case if λ_5 be the wavelength of the waves in rod, we have $l = 5\lambda_5/2$ and hence

$$\lambda_5 = \frac{2l}{5} \qquad \ldots(5)$$

So, in this case, the oscillation frequency of rod is

$$f_5 = \frac{v}{\lambda_5} = 5\left(\frac{1}{2l}\sqrt{\frac{Y}{\rho}}\right) = 5f_1 \qquad \ldots(6)$$

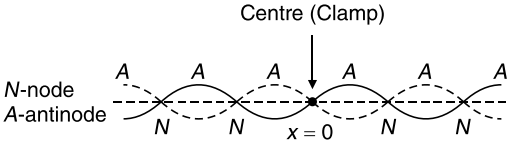
This is called fifth harmonic or second overtone frequency of the clamped rod.

ILLUSTRATION 55

A metallic rod of length 1 m is rigidly clamped at its mid-point. Longitudinal stationary waves are set up in the rod in such a way that there are two nodes on either side of the mid-point. The amplitude of an antinode is 2×10^{-6} m. Write the equation of motion at a point 2 cm from the mid-point and equations of the constituent waves in the rod. Take Young's modulus and density of the material of the rod to be 2×10^{11} Nm^{-2} and 8000 kgm^{-3})

SOLUTION

The wave pattern is shown in the Figure below.



From the diagram, we see that $1 \text{ m} = \frac{5\lambda}{2}$

$$\Rightarrow \quad \lambda = \frac{2}{5} = 0.4 \text{ m}.$$

The amplitude at the antinode is $2A = 2\times10^{-6}$

$$\Rightarrow \quad A = 1\times10^{-6}$$

The wave velocity v is given by

$$v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{2\times10^{11}}{8000}} = 5000 \text{ ms}^{-1}$$

Since $v = f\lambda$

$\Rightarrow \quad f(0.4) = 5000$

$\Rightarrow \quad f = 12500$ Hz

Also, $k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.4} = 5\pi$ and

$$\omega = 2\pi f = 2\pi(12,500) = 25000\pi$$

The equation of the stationary wave is therefore

$$y = 2A\sin kx\cos\omega t$$

$\Rightarrow \quad y = 2\times10^{-6}\sin(5\pi x)\cos(25000\pi t)$

At $x = 0.02$ m, we have

$$y = 2\times10^{-6}\sin\left(5\pi\times2\times10^{-2}\right)\cos(25000\pi t)$$

$\Rightarrow \quad y = 2\times10^{-6}\sin\left(\frac{\pi}{10}\right)\cos 25000\pi t$

The equations of the two waves are,

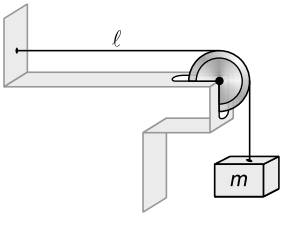
$$y_1 = A\sin(kx-\omega t) \text{ and } y_2 = A\sin(\omega t+kx)$$

So, $\quad y_1 = 10^{-6}\sin(5\pi x - 25000\pi t)$ and

$$y_2 = 10^{-6}\sin(25000\pi t + 5\pi x)$$

ILLUSTRATION 56

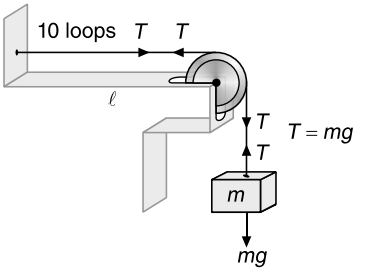
Figure shows a string stretched by a block going over a pulley. The string vibrates in its tenth harmonic in unison with a particular tuning fork. When a beaker containing water is brought under the block so that the block is completely dipped into the beaker, the string vibrates in its eleventh harmonic. Find the density of the material of the block.



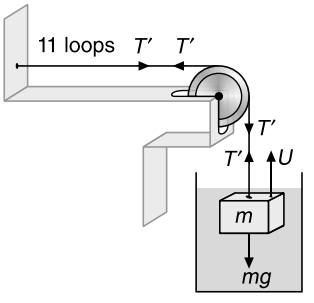
SOLUTION

According to the problem, we have $f = 10f_1$

$$\Rightarrow \quad f = \frac{10}{2l}\sqrt{\frac{T}{\mu}} = \frac{10}{2l}\sqrt{\frac{mg}{\mu}} \qquad \ldots(1)$$



When block is dipped in the beaker, then FBD of situation is shown



$$T' + U = mg$$

$$\Rightarrow \quad T' = mg - U = mg - \rho_\omega\left(\frac{m}{\rho_b}\right)g = mg\left(1-\frac{\rho_w}{\rho_b}\right)$$

where ρ_b is density of material of block and ρ_w is density of water.

$$\Rightarrow \quad f = \frac{11}{2l}\sqrt{\frac{T'}{\mu}} = \frac{11}{2l}\sqrt{\frac{mg}{\mu}\left(1-\frac{\rho_w}{\rho_b}\right)} \qquad \ldots(2)$$

Equating (1) and (2), we get

$$\frac{10}{2l}\sqrt{\frac{mg}{\mu}} = \frac{11}{2l}\sqrt{\frac{mg}{\mu}\left(1-\frac{\rho_w}{\rho_b}\right)}$$

$$\Rightarrow \quad 10 = 11\sqrt{1-\frac{\rho_w}{\rho_b}}$$

$$\Rightarrow \quad \frac{\rho_w}{\rho_b} = 1-\frac{100}{121} = \frac{21}{121}$$

$$\Rightarrow \quad \rho_b = \frac{121}{21}\rho_w = \frac{121}{21} \text{ gcc}^{-1} = 5.8 \text{ gcc}^{-1}$$

LONGITUDINAL STATIONARY WAVES IN AIR COLUMNS/ORGAN PIPES

An air filled pipe is called an organ pipe. There are two types of organ pipes

1. An open pipe which has both the ends open
2. A closed pipe which has one end closed and the other end open

If a tuning fork is placed near one end, which is open, a longitudinal wave travels in the air column. This is

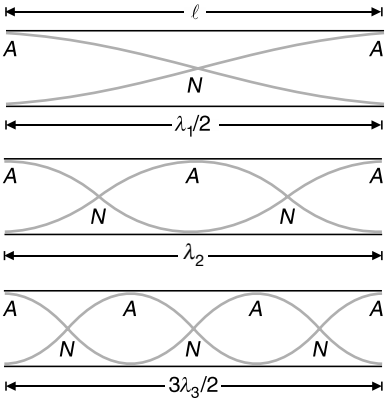
reflected back from the other end. The incident and the reflected waves superpose to produce a stationary wave. *It is important to note that a node is always formed at the closed end and an antinode at the open end.*

STATIONARY SOUND WAVES IN AN OPEN ORGAN PIPE

The concept of stationary waves can also be applied to sound waves in an air column (an air-filled pipe) called an organ pipe. Standing waves are the result of interference between longitudinal sound waves traveling in opposite directions.

Many other aspects of standing sound wave patterns are similar to those of string waves, like closed end of the pipe gets a displacement node (or pressure antinode) and the open end of the pipe gets a displacement antinode (or a pressure node).

The first three modes of vibration of an open pipe (open at both ends) are shown in Figure.



First Harmonic (Fundamental Mode)

$$l = 2\left(\frac{\lambda_1}{4}\right) = \frac{v}{2f_1} \qquad \{\because v = f\lambda\}$$

$$\Rightarrow \quad f_1 = \frac{v}{2l}$$

where v is the velocity of longitudinal wave.

Second Harmonic (First Overtone)

$$l = 4\left(\frac{\lambda_2}{4}\right) = \frac{v}{f_2} \qquad \{\because v = f\lambda\}$$

$$\Rightarrow \quad f_1 = \frac{v}{l} = 2\left(\frac{v}{2l}\right) = 2f_1$$

Third Harmonic (Second overtone)

$$l = 6\left(\frac{\lambda_3}{4}\right) = \frac{3v}{2f_3} \qquad \{\because v = f\lambda\}$$

$$\Rightarrow \quad f_3 = \frac{3v}{2l} = 3\left(\frac{v}{2l}\right) = 3f_1$$

Thus, in an open pipe all the harmonics (both odd and even) are present. For the nth harmonic

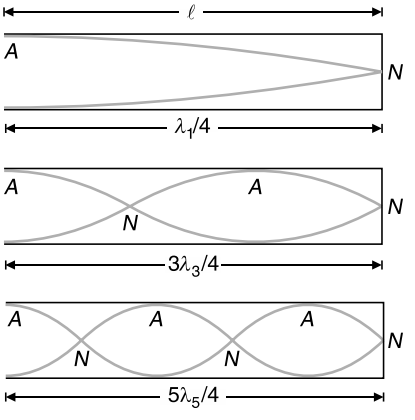
$$f_n = n\left(\frac{v}{2l}\right) = nf_1$$

If H represents Harmonic, O represents overtone and f_n represents the frequency of the n^{th} harmonic, then they are related to each other as shown in table.

H	**1**	**2**	**3**	**4**	**5**	**...**	**n**	**n + 1**
f_n	f_1	$2f_1$	$3f_1$	$4f_1$	$5f_1$	$\cdots$	nf_1	$(n+1)f_1$
O	–	1	2	3	4	...	$n-1$	n

CLOSED PIPE

The first three modes of vibration of a closed pipe (closed at one end and open at the other) are shown in Figure.



First Harmonic (Fundamental Mode)

$$l = 1\left(\frac{\lambda_1}{4}\right) = \frac{v}{4f_1} \qquad \{\because v = f\lambda\}$$

$$\Rightarrow \quad f_1 = \frac{v}{4l}$$

where v is the velocity of longitudinal wave .

Third Harmonic (First Overtone)

$$l = 3\left(\frac{\lambda_2}{4}\right) = \frac{3v}{4f_3} \qquad \{\because v = f\lambda\}$$

$$\Rightarrow \quad f_3 = \frac{3v}{4l} = 3\left(\frac{v}{4l}\right) = 3f_1$$

Please note here, that this frequency (the next immediate present frequency after fundamental harmonic) is thrice the fundamental and hence it has to be called as Third Harmonic or First Overtone.

Fifth Harmonic (Second overtone)

$$l = 5\left(\frac{\lambda_3}{4}\right) = \frac{5v}{4f_5} \qquad \{\because v = f\lambda\}$$

$$\Rightarrow \quad f_5 = \frac{5v}{4l} = 5\left(\frac{v}{4l}\right) = 5f_1$$

Again, please note here, that this frequency is five times the fundamental and hence it has to be called as Fifth Harmonic or Second Overtone.

So, in a closed organ pipe only odd harmonics are present (even harmonics are absent).

If H represents Harmonic, O represents overtone and f_n represents the frequency of the n^{th} harmonic, then they are related to each other as shown in table.

H	**1**	**3**	**5**	**7**	**9**	**...**	**n**	**n + 1**
f_n	f_1	$3f_1$	$5f_1$	$7f_1$	$9f_1$	$\cdots$	nf_1	$(2n+1)f_1$
O	–	1	2	3	4	...	$\frac{n-1}{2}$	n

Here the nth overtone is $(2n+1)$th harmonic, having frequency

$$f_{n\text{th overtone}} = (2n+1)\frac{v}{4l}$$

As an example, ratio of the seventh overtone to the second overtone in a closed organ pipe should be

$$\frac{f_{7\text{th overtone}}}{f_{2\text{nd overtone}}} = \frac{2(7)+1}{2(2)+1} = \frac{15}{5} = 3$$

END CORRECTION IN ORGAN PIPES

In the above discussion we have assumed that the antinode is formed exactly at the open end of a pipe. *However, the antinode is formed actually a little distance outside the open end. This is because the air outside is not a rigid boundary and therefore gets slightly compressed due to which it reflects back the incident wave.*

The correct effective length of a closed pipe is thus $l+e$, and that of an open pipe is $l+2e$, where e is called the end correction having approximate value $e = 0.3D$, where D is the internal diameter of the pipe. Hence, for a broad pipe the end correction is more compared to that of a narrow pipe.

Whenever we calculate the harmonic frequencies of oscillations of air column in organ pipe, then we must take into account the end corrections due to which the fundamental frequency of a closed organ pipe (having only one open end) of length l is

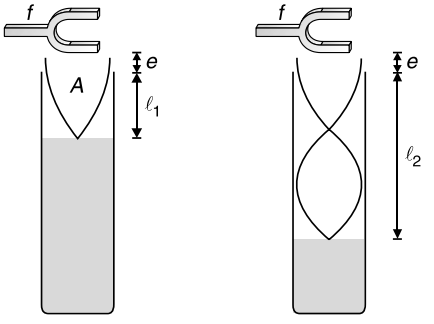
$$f_{\text{closed}} = \frac{v}{4(l+e)}$$

and fundamental frequency of an open organ pipe (having both ends open) of length l is

$$f_{\text{open}} = \frac{v}{2(l+2e)}$$

RESONANCE TUBE

It is used to determine velocity of sound in air by resonating the tuning fork of known frequency say f with the organ pipe. It also helps to calculate the end correction to be applied.



If l_1 and l_2 are lengths of first and second resonances also called as first resonance and second resonance lengths respectively, then we have

$$l_1 + e = \frac{\lambda}{4} \qquad \ldots(1)$$

$$l_2 + e = \frac{3\lambda}{4} \qquad \ldots(2)$$

Subtracting (1) from (2),

$$l_2 - l_1 = \frac{\lambda}{2} = \frac{v}{2f}$$

where v is the speed of sound in air at room temperature. So, we get

$$v = 2f(l_2 - l_1)$$

Dividing (2) by (1), we get $\frac{l_2+e}{l_1+e} = 3$ …(3)

$$\Rightarrow \quad e = \frac{l_2 - 3l_1}{2}$$

From equation (3), we get

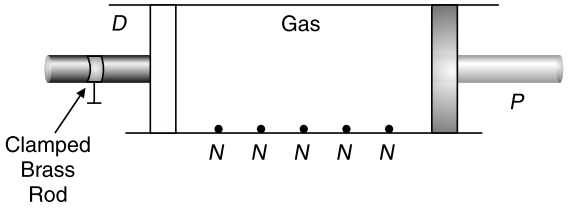
$$l_2 = 3l_1 + 2e$$

So, second resonance is obtained at length slightly more than thrice the length of first resonance.

KUNDT'S TUBE

Kundt's tube can be used to compare the speeds of sound in different gases. A horizontal glass tube fitted with a smoothly moving piston P at one end and a loosely fit disc D at the other is filled with a gas. A horizontal brass rod,

clamped at the middle is attached to D. A thin layer of lycopodium powder is spread inside the tube.



The outer half of the brass rod is rubbed along its length in one direction only by means of a wet or a rinsed cloth. This sets the rod and hence the air inside the tube into longitudinal vibrations. The piston is moved until resonance occurs. The lycopodium powder gets collected at the nodes in small heaps. The distance between two heaps gives $\frac{\lambda}{2}$.

Let λ_A and λ_B be the wavelengths corresponding to two gases. If v_A and v_B be the velocities of sound in them respectively, then, since frequency is the same in both the cases, we have

$$\frac{v_A}{v_B} = \frac{\lambda_A}{\lambda_B}$$

ILLUSTRATION 57

The air column in a pipe closed at one end is made to vibrate in its second overtone by a tuning fork of frequency 440 Hz. The speed of sound in air is 330 ms^{-1}. End correction may be neglected. Let p_0 denote the mean pressure at any point in the pipe, and Δp_0 the maximum amplitude of pressure variation.

(a) Find the length L of the air column?
(b) What is the amplitude of pressure variation at the middle of the column?
(c) What are the maximum and minimum pressure at the open end of the pipe?
(d) What are the maximum and minimum pressure at the closed-end of the pipe?

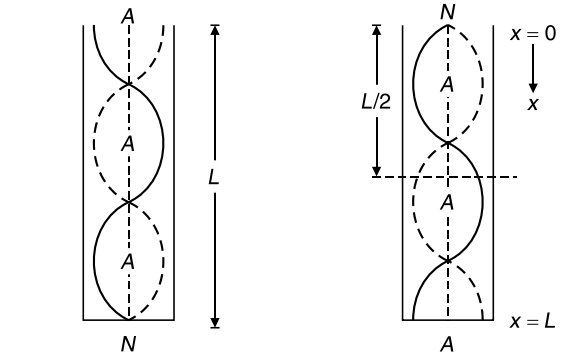
SOLUTION

(a) In case of a closed organ pipe, the fundamental frequency is $\left(\frac{v}{4L}\right)$ and only odd harmonics are present. Therefore, the second overtone will mean fifth harmonic i.e., $f = \frac{5v}{4L}$

$$\Rightarrow \quad L = \frac{5v}{4f} = \frac{5\times 330}{4\times 440} = \frac{15}{16} \text{ m}$$

(b) At the position of displacement antinode there is pressure node and vice-versa. The variation of the pressure amplitude of standing pressure waves along the length of the column $x = 0$ at its open end will be

$$p = \Delta p_0 \sin kx = \Delta p_0 \sin\left(\frac{2\pi}{\lambda}x\right) \qquad \left\{\text{as} \quad k = \frac{2\pi}{\lambda}\right\}$$



Now, for second overtone $L = \left(\frac{5}{4}\right)\lambda$, so at the middle, $x = \frac{L}{2} = \frac{5}{8}\lambda$

$$\Rightarrow \quad p = \Delta p_0 \sin\frac{2\pi}{\lambda}.\left(\frac{5}{8}\lambda\right) = \Delta p_0 \sin\left(\frac{5}{4}\pi\right)$$

$$\Rightarrow \quad |p| = \Delta p_0 \times \frac{1}{\sqrt{2}} = \frac{\Delta p_0}{\sqrt{2}}$$

At the open end, $x = 0$, $p = 0$.

$$\Rightarrow \quad p_{max} = p_{min} = p_0 \pm 0 = p_0$$

(b) The closed end is an antinode for the pressure wave. Therefore,

$$p_{max} = p_0 + \Delta p_0 \text{ and } p_{min} = p_0 - \Delta p_0$$

ILLUSTRATION 58

A tuning fork having frequency of 340 Hz is vibrated just above a cylindrical tube. The height of the tube is 120 cm. Water is slowly poured in. What is the minimum height of water required for resonance? $\left(v = 340 \ ms^{-1}\right)$

SOLUTION

As the tuning fork is in resonance with air column in the pipe closed at one end.

$$f = n\frac{v}{4L} \text{ with } n = 1, 3, 5, \ldots$$

Therefore, the length of air column in the pipe,

$$L = \frac{nv}{4f} = n\frac{340\times 100}{4\times 340} = 25n \text{ cm with } n = 1, 3, 5, \ldots$$

$$\Rightarrow \quad L = 25 \text{ cm}, 75 \text{ cm}, 125 \text{ cm}, \ldots.$$

Now, as the tube is 120 cm, the length of air column must be lesser than 120 cm. It can be only 25 cm or 75 cm. Further if h is the height of water filled in the tube.

$$L + h = 120 \text{ cm}$$

$$\Rightarrow \quad h = 120 - L$$

So h will be minimum when L is maximum, i.e., when $L = 75$ cm

$$\Rightarrow \quad h_{\min} = 120 - 75 = 45 \text{ cm}$$

ILLUSTRATION 59

A resonance air column resonates with a tuning fork of frequency 512 Hz at column lengths 16 cm and 49.2 cm. Find the end correction, and the velocity of sound wave in air.

SOLUTION

The resonance air column has one closed end, which is adjustable. Thus, if e is the end correction we have

$$L_1 + e = \frac{\lambda}{4} \text{ and } L_2 + e = \frac{3\lambda}{4}$$

$$\Rightarrow \quad \lambda = 4(L_1 + e) \text{ and } \lambda = \frac{4}{3}(L_2 + e)$$

$$\Rightarrow \quad 3(L_1 + e) = (L_2 + e)$$

$$\Rightarrow \quad e = \frac{L_2 - 3L_1}{2} = \frac{0.492 - 3 \times 0.16}{2} = 0.006 \text{ m}$$

Now, $v = f\lambda = f \times 4(L_1 + e)$

$$v = 512 \times 4 \times (0.16 + 0.006) = 340 \text{ ms}^{-1}$$

ILLUSTRATION 60

The water level in a vertical glass tube 1 m long can be adjusted to any position in the tube. A tuning fork vibrating at 660 Hz is held just over the open top end of the tube. At what positions of the water level will there be resonance. Assume the speed of sound to be 330 ms^{-1}.

SOLUTION

Resonance corresponds to a pressure antinode at closed end and pressure node at open end. Further, the distance between a pressure node and a pressure antinode is $\frac{\lambda}{4}$. For resonance, the length of air column l, must be

$$l = n\left(\frac{\lambda}{4}\right) = n\left(\frac{v}{4f}\right)$$

where, $n = 1, 3, 5, \ldots$.

$$\Rightarrow \quad l_1 = (1)\left(\frac{330}{4 \times 660}\right) = 0.125 \text{ m}$$

$$\Rightarrow \quad l_2 = 3l_1 = 0.375 \text{ m}$$

$$\Rightarrow \quad l_3 = 5l_1 = 0.625 \text{ m}$$

$$\Rightarrow \quad l_4 = 7l_1 = 0.875 \text{ m}$$

$$\Rightarrow \quad l_5 = 9l_1 = 1.125 \text{ m}$$

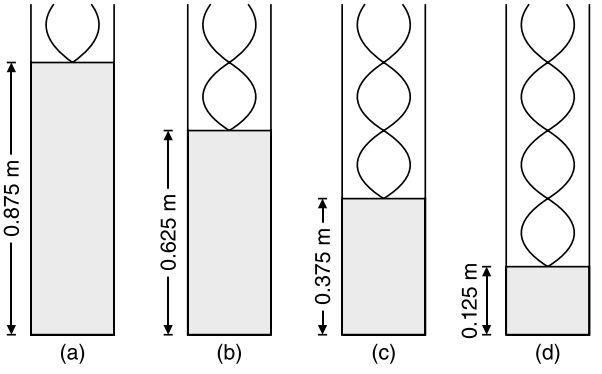
Since, $l_5 > 1$ m (the length of tube), the length of air columns can have the values from l_1 to l_4 only. Therefore, level of water at resonance will be

$$(1 - 0.125) \text{ m} = 0.875 \text{ m}$$

$$(1 - 0.375) \text{ m} = 0.625 \text{ m}$$

$$(1 - 0.625) \text{ m} = 0.375 \text{ m}$$

and $(1 - 0.875) \text{ m} = 0.125 \text{ m}$



In all the four cases shown in figure, the resonance frequency is 660 Hz but first one is the fundamental tone or first harmonic. Second is first overtone or third harmonic and so on.

ILLUSTRATION 61

The first overtone of an open organ pipe together with the first overtone of a closed organ pipe produces beats with a beat frequency 2.2 Hz. The fundamental frequency of the closed organ pipe is 110 Hz. Find the lengths of the pipes. Speed of sound in air, $v = 330$ ms^{-1}.

SOLUTION

Let the lengths of the closed and open organ pipes be l_1 and l_2. The fundamental frequency of the closed organ pipe is given by

$$f_1 = \frac{v}{4l_1}$$

$$\Rightarrow \quad 110 = \frac{330}{4l_1}$$

$$\Rightarrow \quad l_1 = \frac{330}{4 \times 110} = 0.75 \text{ m}$$

The frequency of the first overtone (third harmonic) of this pipe is thrice that of the fundamental mode. Hence it equals 330 Hz. This produces 2.2 beats per second with the first overtone (second harmonic) of an open organ pipe. Hence the frequency of second harmonic, f_2 of the open organ pipe must be

$$f_2 = (330 \pm 2.2) \text{ Hz}$$

$\Rightarrow \quad f_2 = 332.2$ Hz	$f_2 = 327.8$ Hz
$\Rightarrow \quad 2\left(\frac{v}{2l_2}\right) = 332.2$	$\frac{2v}{2l_2} = 327.8$
$\Rightarrow \quad \frac{330}{l_2} = 332.2$	$\frac{330}{l_2} = 327.8$
$\Rightarrow \quad l_2 = 0.99$ m	$l_2 = 1.0067$ m

ILLUSTRATION 62

An air column in a pipe closed at one end is made to vibrate in its second overtone by a tuning fork of frequency 440 Hz. The speed of sound in air is 330 ms^{-1}. The end correction may be neglected. Let P_0 denote the mean pressure at any point in the pipe and ΔP_0 the maximum amplitude of pressure variation.

(a) Find the length of the air column.
(b) What is the amplitude of pressure variation at the mid-point of the pipe?
(c) What are the maximum and minimum pressures at the open end of the pipe?
(d) What are the maximum and minimum pressure at the closed end of the pipe?

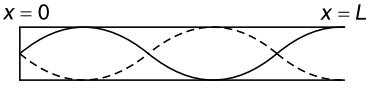
SOLUTION

(a) For the second overtone (fifth harmonic), the frequency is given by

$$f = \frac{5v}{4l}$$

$$\Rightarrow \quad 440 = \frac{5\times 330}{4l}$$

$$\Rightarrow \quad l = \frac{5\times 330}{4\times 440} = \frac{15}{16} \text{ m}$$



(b) Since, $\Delta P = \Delta P_0 \cos(kx)$

$$\Rightarrow \quad \frac{15}{16} = \frac{5\lambda}{4}$$

$$\Rightarrow \quad \lambda = \frac{3}{4} \text{ m}$$

$$\Rightarrow \quad k = \frac{2\pi}{\lambda} = \frac{2\pi}{3/4} = \frac{8\pi}{3}$$

At mid-point, $x = \frac{5\lambda}{8} = \frac{5}{8}\times\frac{3}{4} = \frac{15}{32}$

$$\Delta P = \pm\Delta P_0 \cos(kx)$$

$$\Rightarrow \quad \Delta P = \pm\Delta P_0 \cos\left(\frac{8\pi}{3}\times\frac{15}{32}\right)$$

$$\Rightarrow \quad \Delta P = \pm\Delta P_0 \cos\left(\frac{5\pi}{4}\right)$$

$$\Rightarrow \quad \Delta P = \pm\frac{\Delta P_0}{\sqrt{2}}$$

(c) The open end is a displacement antinode and therefore a pressure node. Therefore, the amplitude of pressure variation at the open end is zero. Hence,

$$P_{max} = P_{min} = P_0$$

(d) The closed end is a displacement node and therefore a pressure antinode, so

$$P_{max} = P_0 + \Delta P_0 \text{ and } P_{min} = P_0 - \Delta P_0$$

BEATS

If two coherent sources of slightly different frequencies send waves simultaneously in same region, then at each point there is a variation of intensity with time. In the case of sound waves, alternate loud and low sounds are heard. These variations in loudness are called **beats**. The time from each loud sound to the next loud sound is called **one beat period** and the number of such repetitions per second is called **beat frequency**.

Let the two waves be

$$y_1 = A\sin(2\pi f_1 t) \text{ and } y_2 = A\sin(2\pi f_2 t)$$

According to Principle of Superposition $y = y_1 + y_2$

$$\Rightarrow \quad y = A\left[\sin(2\pi f_1 t) + \sin(2\pi f_2 t)\right]$$

$$\Rightarrow \quad y = \left[2A\cos\left\{2\pi\left(\frac{f_1 - f_2}{2}\right)t\right\}\right]\sin\left\{2\pi\left(\frac{f_1 + f_2}{2}\right)t\right\}$$

$$\Rightarrow \quad y = R\sin(2\pi f t)$$

where, $R = 2A\cos\left\{2\pi\left(\frac{f_1 - f_2}{2}\right)t\right\}$ and $f = \frac{f_1 + f_2}{2}$

Intensity $\propto$ (Amplitude)^2

$$\Rightarrow \quad I_R \propto R^2$$

$$\Rightarrow \quad I_R = 4a^2\cos^2\left[2\pi\left(\frac{f_1 - f_2}{2}\right)t\right]$$

CONDITION FOR MAXIMA

I_R is MAXIMUM, when

$$\cos^2\left[2\pi\left(\frac{f_1 - f_2}{2}\right)t\right] = 1$$

$$\Rightarrow \quad \cos\left[2\pi\left(\frac{f_1 - f_2}{2}\right)t\right] = \pm 1$$

$$\Rightarrow \quad 2\pi\left(\frac{f_1 - f_2}{2}\right)t = n\pi;\ n = 0, 1, 2, 3,\ldots$$

$$\Rightarrow \quad t = \frac{n}{|f_1 - f_2|}; n = 0, 1, 2, 3,\ldots$$

So, we observe that MAXIMA are obtained at time values given by

$$t = 0, \frac{1}{|f_1 - f_2|}, \frac{2}{|f_1 - f_2|}, \frac{3}{|f_1 - f_2|},\ldots$$

CONDITION FOR MINIMA

I_R is MINIMUM, when

$$\cos^2\left[2\pi\left(\frac{f_1 - f_2}{2}\right)t\right] = 0$$

$$\Rightarrow \quad \cos\left[2\pi\left(\frac{f_1 - f_2}{2}\right)t\right] = 0$$

$$\Rightarrow \quad 2\pi\left(\frac{f_1 - f_2}{2}\right)t = (2n+1)\frac{\pi}{2}; n = 0, 1, 2, 3,\ldots$$

$$\Rightarrow \quad t = \frac{(2n+1)}{2|f_1 - f_2|}; n = 0, 1, 2, 3,\ldots$$

So, we observe that MINIMA are obtained at time values given by

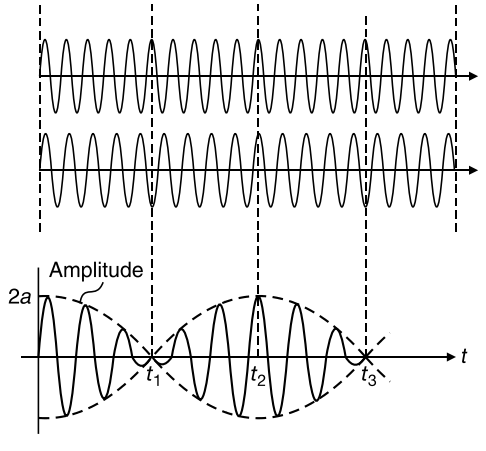
$$t = \frac{1/2}{|f_1 - f_2|}, \frac{3/2}{|f_1 - f_2|}, \frac{5/2}{|f_1 - f_2|},\ldots$$

Beat time (t_b) is the time interval between two consecutive maxima or minima. So,

$$t_b = \frac{1}{|f_1 - f_2|}$$

Further, beat frequency(f_b) is the reciprocal of beat time t_b. Hence,

$$f_b = \frac{1}{t_b} = |f_1 - f_2|$$



Conceptual Note(s)

We note that

(a) The resultant wave is also a harmonic wave with a frequency $f = \frac{1}{2}(f_1 + f_2)$

(b) The amplitude R is not constant but varies harmonically with a frequency $\frac{1}{2}|f_1 - f_2|$. The number of beats per second is twice this frequency because the ear perceives the magnitude of displacement and not its sign. Thus, the beat frequency f_b is $f_b = |f_1 - f_2|$

(c) If $|f_1 - f_2|$ is small, the beats can be heard. However, if $|f_1 - f_2|$ is large (more than 7), the beats produced are too rapid to be heard distinctly.

(d) Beats are used for tuning musical instruments and also to determine an unknown frequency.

TO FIND UNKNOWN FREQUENCY

Suppose we have two tuning forks A and B. The frequency of A is n Hz and that of B is **unknown**. When both are sounded together, they give x beats per second. Then possible frequency of B is $(n+x)$ or $(n-x)$ Hz.

If on loading B, the number of beats decreases, then frequency of B before loading was $(n+x)$ and if the number of beats increases, then frequency of B before loading will be $(n-x)$.

ILLUSTRATION 63

Two turning forks A and B produce 4 beats per second when sounded simultaneously. The frequency of A is known to be 256 Hz. When B is loaded with a little wax 4 beats per second are again produced. Find the frequency of B before and after loading.

SOLUTION

Since the beat frequency is 4 per second, the possible frequencies of fork B before loading are

$$(256+4) \text{ Hz and } (256-4) \text{ Hz}$$

or $\quad 260$ Hz and 252 Hz

After loading B with wax, again 4 beats/second are heard. Therefore, after loading, the possible frequencies are

$$(256+4) \text{ Hz and } (256-4) \text{ Hz}$$

or 260 Hz and 252 Hz

But, by loading a fork its frequency can only decrease. Hence, before loading fork B, its frequency can only be 260 Hz, so that after loading it decreases to 252 Hz.

ILLUSTRATION 64

Two tuning forks A and B when set vibrating, give 4 beats/s. If a prong of the fork A is filed, the beats are reduced to 2 s^{-1}. Determine the frequency of A, if that of B is 250 Hz.

SOLUTION

There are four beats between A and B therefore the possible frequencies of A are 246 Hz or 254 Hz (that is 250 ± 4) Hz.
Now when the prong of A is filed, its frequency becomes greater than the original frequency.
Assuming that the original frequency of A is 254, then on filing its frequency will become greater than 254.
The beats between A and B will be more than 4. But it is given that the beats are reduced to 2, therefore, 254 is not possible.
Therefore, the required frequency must be 246 Hz.
(This is true, because on filing the frequency may increase to 248, giving 2 beats with Q of frequency 250 Hz).

ILLUSTRATION 65

Wavelength of two notes in air is $\left(\frac{90}{175}\right)$ m and $\left(\frac{90}{173}\right)$ m respectively. Each of these notes produce 4 beats per second with a third note of a fixed frequency. Calculate the velocity of sound in air.

SOLUTION

Given: $\lambda_1 = \frac{90}{175}$ m and $\lambda_2 = \frac{90}{173}$

If f_1 and f_2 are the corresponding frequencies and v is the velocity of sound in air, we have

$$v = f_1\lambda_1 \quad \text{and} \quad v = f_2\lambda_2$$

$$\Rightarrow \quad f_1 = \frac{v}{\lambda_1} \quad \text{and} \quad f_2 = \frac{v}{\lambda_2}$$

Since $\lambda_1 < \lambda_2$, we must have $f_1 < f_2$. If f is the frequency of the third note, then

$$f_1 - f = 4 \text{ and } \quad f - f_2 = 4$$

$$\Rightarrow \quad f_1 - f_2 = 8$$

$$\Rightarrow \quad \frac{v}{\lambda_1} - \frac{v}{\lambda_2} = 8$$

$$\Rightarrow \quad v\left(\frac{175}{90} - \frac{173}{90}\right) = 8$$

$$\Rightarrow \quad v = 360 \text{ ms}^{-1}$$

ILLUSTRATION 66

A column of air and a tuning fork produces 4 beats/s. When sounding together, the tuning fork gives the lower note. The temperature of air is 15 °C. When the temperature falls to 10 °C, the two produce 3 beats/s. Find the frequency of the tuning fork.

SOLUTION

The frequency of the air column is given by $f = \frac{v}{\lambda}$, where v is the velocity of sound in air and λ is the wavelength. Since

$$v = \sqrt{\frac{\gamma RT}{M}}, \text{ so } f \text{ is dependent of temperature}$$

$$\Rightarrow \quad f \propto \sqrt{T}$$

Let the frequency of the tuning fork be f.
Then, the frequency of the air column at 15 °C is $f_1 = f + 4$
and the frequency of the air column at 10 °C is $f_2 = f + 3$
Since the frequency decreases with decrease temperature, so we have

$$\frac{f+4}{f+3} = \sqrt{\frac{288}{283}}$$

$$\Rightarrow \quad \frac{f+4}{f+3} = 1.00879$$

$$\Rightarrow \quad f + 4 = 1.00879f + 3.02638$$

$$\Rightarrow \quad 8.79 \times 10^{-3} f = 0.97362$$

$$\Rightarrow \quad f = 110.76 \text{ Hz}$$

ILLUSTRATION 67

Two wires are fixed on a sonometer. Their tensions are in the ratio 8 : 1, their lengths are in the ratio 36 : 35, the diameters are in the ratio 4 : 1 and densities are in the ratio 1 : 2. Find the frequencies of the beats produced if the note of the higher pitch has a frequency of 360 Hz.

SOLUTION

Given: $\frac{T_1}{T_2} = \frac{8}{1}, \frac{L_1}{L_2} = \frac{36}{35}, \frac{D_1}{D_2} = \frac{4}{1}, \frac{\rho_1}{\rho_2} = \frac{1}{2}$

If μ_1 and μ_2 are the linear densities, we have

$$\mu_1 = \pi\left(D_1^2/4\right)\rho_1 \text{ and } \mu_2 = \pi\left(D_2^2/4\right)\rho_2$$

Therefore, the ratio of linear densities is

$$\frac{\mu_1}{\mu_2} = \left(\frac{D_1}{D_2}\right)^2 \frac{\rho_1}{\rho_2} = \left(\frac{4}{1}\right)^2 \frac{1}{2} = \frac{8}{1}$$

The fundamental frequencies of the two wires are

$$f_1 = \frac{1}{2L_1}\sqrt{\frac{T_1}{\mu_1}} \text{ and } f_2 = \frac{1}{2L_2}\sqrt{\frac{T_2}{\mu_2}}$$

$$\Rightarrow \quad \frac{f_1}{f_2}=\frac{L_2}{L_1}\sqrt{\frac{T_1}{T_2}\times\frac{\mu_2}{\mu_1}}=\frac{35}{36}\sqrt{\frac{8}{1}\times\frac{1}{8}}=\frac{35}{36}$$

Since, $f_2 > f_1$, we have $f_2 = 360$ Hz

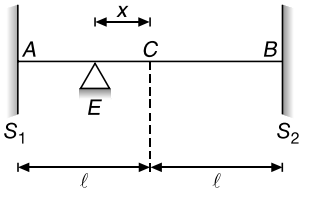
$$\Rightarrow \quad f_1=\left(\frac{35}{36}\right)\times 360 = 350 \text{ Hz}$$

Therefore, the beat frequency is

$$f_b = f_1 - f_2 = 350 \sim 360 = 10 \text{ Hz}$$

ILLUSTRATION 68

A string AB of length $2l$ is kept taut between two rigid supports S_1 and S_2. The tension in the string is T and its mass per unit length is μ. A knife edge E is placed at a distance $x(\ll l)$ from the midpoint C of AB. The two segments of the string are now made to vibrate in their fundamental modes. What beat frequency will be heard by the observer?



SOLUTION

Frequency of the section AE is $f_{AE}=\dfrac{1}{2(l-x)}\sqrt{\dfrac{T}{\mu}}$

Frequency of section EB is $f_{EB}=\dfrac{1}{2(l+x)}\sqrt{\dfrac{T}{\mu}}$

So, beat frequency is $f_b = f_{AE} - f_{EB}$

$$\Rightarrow \quad f_b=\frac{1}{2(l-x)}\sqrt{\frac{T}{\mu}}-\frac{1}{2(l+x)}\sqrt{\frac{T}{\mu}}$$

$$\Rightarrow \quad f_b=\frac{1}{2}\sqrt{\frac{T}{\mu}}\left[\frac{1}{l-x}-\frac{1}{l+x}\right]$$

$$\Rightarrow \quad f_b=\frac{x}{l^2-x^2}\sqrt{\frac{T}{\mu}}$$

$$\Rightarrow \quad f_b\approx\frac{x}{l^2}\sqrt{\frac{T}{\mu}} \qquad \{\because\ x\ll l,\ \text{so } x^2 \text{ is negligible}\}$$

PITCH (FREQUENCY)

The term pitch refers to the attribute of a sound sensation that distinguishes a "**high**" note from a "**low**" note. In physical description, it is identified with the frequency – the higher the frequency, the higher is the pitch. The pitch is the characteristic which distinguishes between a shrill (or sharp) sound and a grave (or flat) sound. A sound of **high pitch** is said to be **shrill** and of **low pitch** a **grave sound**. The pitch does not depend on intensity and loudness but depends on the frequency. The pitch of a sound changes due to **Doppler's effect**.

QUALITY (TIMBRE)

A musical instrument vibrates with many frequencies at the same time – a lowest frequency, called the **fundamental**, and its multiples, called **overtones**. The quality (or timbre) of any musical sound is determined by the number of overtones and their relative intensities. The sounds of different instruments are said to differ in quality. The quality of sound enables us to distinguish between two sounds having same loudness and pitch. The quality of sound depends on the presence of overtones. Due to quality of sound one can recognise the voice of his friend without seeing him.

OCTAVE AND INTERVAL

Higher (upper) octave of frequency f is $2f$

Lower octave of frequency f is $\dfrac{f}{2}$

Interval is the ratio of frequencies of successive notes.

MUSICAL SOUND AND MUSICAL SCALE

A sound produced by periodic vibrations is called a **musical sound**. A musical sound can be decomposed into harmonic components having simple frequency ratio like 1 : 2, 2 : 3, 3 : 5 etc.

A musical scale consists of a series of notes whose fundamental frequencies have specified ratios. There are eight notes which fix an octave – the frequency ratio of the first and the eighth is 1 : 2. The note of the lowest frequency is called the Keynote. There are two types of musical scales.

Major diatonic Scale: The frequency ratio of adjacent notes are either $\dfrac{9}{8}$ or $\dfrac{10}{9}$ or $\dfrac{16}{15}$ (simple ratio).

Whole tones: Interval between two tones whose frequencies bear the ratio $\dfrac{9}{8}$ or $\dfrac{10}{9}$.

Half tone: The interval between two notes of frequency ratio $\dfrac{16}{15}$ is called a **half tone**.

Symbol	Indian Name	Western Name	Frequency in the base 256 hz	Frequency Ratio of Intervals
C	*Sa*	*DO*	256	9/8 (Whole)
D	*Re*	*RE*	288	10/9 (Whole)
E	*Ga*	*MI*	320	16/15 (Half)
F	*Ma*	*FA*	341.3	9/8 (Whole)
G	*Pa*	*SOL*	384	10/9 (Whole)
A	*Dha*	*LA*	426.7	9/8 (Whole)
B	*Ni*	*SI*	480	16/15 (Half)
C_1	*Sa*	*DO*	512	

Equally Tempered Scale: It consists of 13 keys and 12 intervals. The intervals are equal and each being $2^{1/2}$.

Key	C	Cs	D	Ds	E	F	Fs	G	Gs	A	As	B	C′
	2^0	$2^{1/12}$	$2^{2/12}$	$2^{3/12}$	$2^{4/12}$	$2^{5/12}$	$2^{6/12}$	$2^{7/12}$	$2^{8/12}$	$2^{9/12}$	$2^{10/12}$	$2^{11/12}$	

i.e., that is the notes form a geometric progression.

AUDIBLE, INFRASONIC AND ULTRASONIC WAVES

Longitudinal mechanical waves possess large range of frequencies. The frequency range 20 Hz to 20,000 Hz (or 20 kHz)causes the sensation of hearing in human beings and is, therefore, called the **Audible Range**. Waves of frequencies below 20 Hz are called **Infrasonic Waves** and those above 20,000 Hz are called **Ultrasonic Waves**.
Audible waves are generated by vibrating strings, air columns, plates and membranes.
Infrasonic waves are usually generated by large sources. e.g., earthquake waves.

Ultrasonic waves are produced by piezoelectric effect, magnetostriction method and Galton's whistle. Some animals can hear ultrasound. Ultrasonics have various applications, e.g., determining depth of mines and sea, medical diagnosis and therapy, echo sounding, finding flaws in materials, stimulated plant growth destruction of living cells suspended in liquids kill bacteria and smaller animals like rats, frogs, fishes, removal of grease and dirt etc.

Test Your Concepts-V

Based on Stationary Waves & Beats

(Solutions on page H.239)

1. The displacement of a standing wave on a string is given by

$$y(x, t) = 0.4\sin(0.5x)\cos(30t)$$

where, x and y are in centimetre. Calculate the frequency, amplitude and wave speed of the component waves. Also calculate particle velocity at $x = 2.4$ cm, $t = 0.8$ s.

2. A string fixed at both ends has consecutive standing wave modes for which the distances between adjacent nodes are 18 cm and 16 cm respectively.
(a) What is the minimum possible length of the string in cm?
(b) If the tension is 10 N and the linear mass density is 4 gm^{-1}, what is the fundamental frequency, in Hz, to the nearest two digit integer.

3. The vibrations of a string fixed at both ends are described by the equation

$$y = (5\text{ mm})\sin\left[(1.57\text{ cm}^{-1})x\right]\sin\left[(314\text{ s}^{-1})t\right]$$

(a) What is the maximum displacement of the particle at $x = 5.66$ cm?
(b) What are the wavelengths and the wave speeds of the two transverse waves that combine to give the above vibration?
(c) What is the velocity of the particle at $x = 5.66$ cm at time $t = 2$ s?
(d) If the length of the string is 10 cm, locate the nodes and the antinodes. How many loops are formed in the vibration.

4. Consider a steel wire of length 1 m, mass 0.1 kg and cross-sectional area 10^{-6} m^2 in which longitudinal vibrations are setup.
(a) What is the fundamental frequency?
(b) The same wire is rigidly fixed at both ends and the temperature is lowered by 20 °C. If now transverse waves are set up by plucking it at the midpoint, calculate the fundamental frequency and compare this with the frequency in the first case.

Given: Coefficient of linear expansion of steel is $1.21\times10^{-5}\ (°C)^{-1}$ and Young's modulus of steel is $2\times10^{11}\ Nm^{-2}$.

5. A sonometer wire has a length of 114 cm between two fixed ends. Where should two bridges be placed so as to divide the wire into three segments whose fundamental frequencies are in the ratio 1:3:4?
6. A wire having a linear density of 0.05 gcm^{-1} is stretched between two rigid supports with a tension of 450 N. It is observed that the wire resonates at a frequency of 420 Hz. The next higher frequency at which the same wire resonates is 490 Hz. Find the length of the wire.
7. In a stationary wave pattern that forms as a result of reflection of waves from an obstacle the ratio of the amplitude at an antinode and a node is $\beta = 1.5$. What percentage of the energy passes across the obstacle?
8. A metallic rod of length 1 m is rigidly clamped at one of its end. The other end of the rod is free. Longitudinal stationary waves are set up in the rod in such a way that there are total six antinodes observed along the rod. If antinode amplitude is 4×10^{-6} m, Young's modulus is $6.4\times10^{10}\ Nm^{-2}$ and density of the rod is $5\times10^{3}\ kgm^{-3}$, calculate the
   (a) wavelength of the constituent waves.
   (b) frequency of the constituent waves.
   (c) equation of the motion at the mid-point of the rod.
9. A string is stretched with tension T between two rigid supports. A wave $y = A\sin(kx - \omega t)$ is produced in the string at left end. Stationary waves are formed due to reflection from the rigid supports. Calculate the
   (a) instantaneous kinetic energy contained in the string between two nodes.
   (b) maximum kinetic energy.
   (c) average kinetic energy over a cycle.
10. Calculate the velocity of sound in a gas in which two wavelength 204 cm and 208 cm produce 20 beats in 6 seconds.
11. An open organ pipe has a fundamental frequency of 300 Hz. The first overtone of a closed organ pipe has the same frequency as the first overtone of the open pipe. How long is each pipe? Given: Speed of sound in air is $v = 330\ ms^{-1}$.
12. A tube 1 m long is closed at one end. A stretched wire is placed near the open end. The wire is 0.3 m long and has a mass of 0.01 kg. It is held fixed at both ends and vibrates in its fundamental mode. It sets the air column in the tube into vibration at its fundamental frequency by resonance. Calculate the frequency of oscillation of the air column and tension in the wire if speed of sound in air is $v = 330\ ms^{-1}$.
13. The first overtone of an open organ pipe beats with the first overtone of a closed organ pipe with a beat frequency of 2.2 Hz. The fundamental frequency of the closed organ pipe is 110 Hz. Find the lengths of the pipes. Speed of sound in air is $v = 330\ ms^{-1}$
14. A 60 cm long flute can be considered to be a pipe open at both ends.
   (a) What is the fundamental frequency when all the holes are covered?
   (b) How far from the mouthpiece should a hole be uncovered for the fundamental frequency to be 330 Hz?
   Take speed of sound in air as $340\ ms^{-1}$.
15. A tuning fork produces 3 beats per second when sounded together with a fork of frequency 364 Hz. When the first fork is loaded with a little wax then the number of beats becomes two per second. What is the frequency of the first fork?
16. A string 25 cm long and having a mass of 2.5 g is under tension. A pipe closed at one end is 40 cm long. When the string is set vibrating in its first overtone and the air in the pipe in its fundamental frequency, 8 beats per second are heard. It is observed that decreasing the tension in the string decrease the beat frequency. If the speed of sound in air is $320\ ms^{-1}$, find the tension in the string.
17. Write the equation for the fundamental standing sound waves of antinode amplitude A, in a tube that is open at both ends. If the tube is 80 cm long and speed of the wave is $330\ ms^{-1}$.

DOPPLER EFFECT

You must have noticed the variation in the sound of a vehicle's horn as the vehicle moves past you. The frequency (or pitch) of the sound heard by you as the vehicle approaches you is higher than the frequency heard by you as it moves away from you. This experience is one example of the **Doppler effect.**

So, when a wave source and a listener are moving relative to each other, the received frequency is not the same as the frequency of the source. This effect is called Doppler effect and was first described by Austrian scientist Christian Andreas Doppler.

Doppler effect is also valid for electromagnetic waves. Doppler effect is observed when

(a) listener is moving but source is stationary
(b) source is moving but listener is stationary
(c) both the source and listener are moving

Case a) above involves the relative motion of the observer with respect to the medium while case b) does not. This fact makes the two cases different. *The basic effect of motion*

of the source is a change in wavelength whereas the basic effect of motion of the listener/observer/detector is a change in the number of wave crests received per second i.e., the frequency or pitch. Also, note that in the following discussion, all motions are relative to the medium.

Conceptual Note(s)

(a) Doppler effect is symmetric in electromagnetic waves as they do not require any medium to propagate but asymmetric in case of sound waves. Therefore, the observed frequency of sound when the observer is at rest and source is moving is different from that when the observer is moving and source is at rest. This can be easily understood from the first two cases discussed below.

(b) When the listener (detector) moves relative to air and the source is stationary relative to air, then the motion changes the frequency at which the listener intercepts wavefronts and thus changes the frequency of the sound wave heard by the listener.

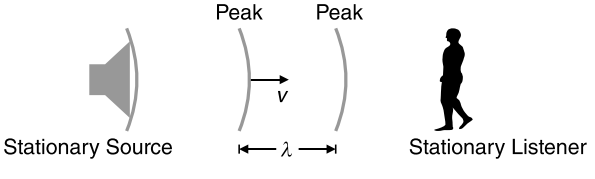
(c) When the source moves relative to the air and the listener (detector) is stationary relative to the air, then motion changes the wavelength of the sound wave and thus changes the frequency of the sound wave heard by the listener (since we know that frequency is inversely proportional to wavelength).

(d) The apparent frequency is more than the actual frequency if the source is moving towards the observer or the observer is moving towards the source. On the other hand, the apparent frequency is less than the actual frequency if the source is moving away from observer or observer is moving away from source.

(e) Doppler's effect of sound is asymmetric i.e., a source moving towards a stationary listener is not equivalent to a listener moving with same velocity towards a stationary source.

LISTENER AND SOURCE BOTH STATIONARY

For the Non-Doppler case, where the sound source and listener/receiver/detector/observer are both at rest with respect to the air (the medium in which the sound propagates) as shown in Figure.



If source produces waves of wavelength λ, frequency f that travel at wave speed v, then

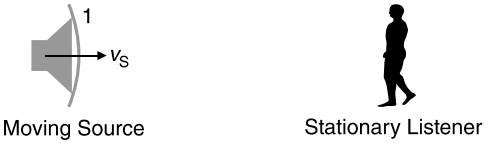
$$v = f\lambda$$

By stationary, we mean "stationary with respect to air/medium through which the sound wave propagates". Thus, when we say "a stationary observer", then no wind must be blowing.

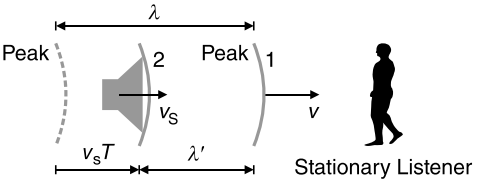
SOURCE APPROACHING A STATIONARY LISTENER

Let us find the equation for the shift in frequency, when the wave source is moving at speed v_S towards a stationary listener/receiver/detector/observer.

Let wave peak 1 be emitted by the source at a certain instant of time. Exactly one wave period T later, peak 1 has moved a distance $vT = \lambda$ (one wavelength) and then peak 2 is emitted as shown in Figure.



(a) Wave peak 1 is emitted



(b) Wave peak 2 is emitted exactly one wave period T later

Since, peak 2 is not emitted at the same position in space as peak 1 was emitted, because in time T the wave source also has moved distance $v_S T$ (assume that $v_S \ll v$). So, with each successive wave peak, the source is constantly advancing such that each wavelength gets diminished by the distance $v_S T$, due to which the listener perceives waves of wavelength $\lambda'$where

$$\lambda' = \lambda - v_S T \qquad \ldots(1)$$

Can we think about the other wave characteristics, such as wave speed and frequency?

Once the wave source releases a wave into the air, then the wave speed is determined by the properties of the medium (density, elastic response, temperature, etc.) and not on the manner the wave was produced. The wave forgets its history, i.e., whether its source was moving or not, hence the wave speed is still v.

Although the source is producing waves of frequency f it cannot be this frequency that the listener perceives, but a frequency f' which satisfies the equation $v = f'\lambda'$

Since $\lambda' = \lambda - v_S T$, so we get

$$v = f'(\lambda - v_S T) \qquad \ldots(2)$$

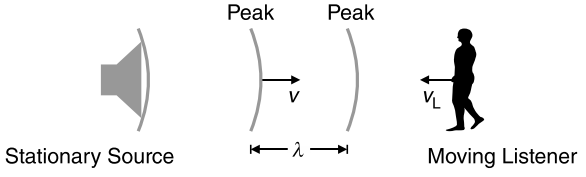
This frequency f' determines the pitch of sound heard by the listener. This frequency is also called the apparent frequency. Substituting $T = 1/f$ and $\lambda = v/f$ in equation (2), we get

$$v = f'\left(\frac{v}{f} - \frac{v_S}{f}\right)$$

$$\Rightarrow \quad f' = f\left(\frac{v}{v - v_S}\right)$$

LISTENER APPROACHING A STATIONARY SOURCE

Suppose the listener is approaching a stationary source with a speed v_L as shown in Figure.



The listener would measure the wavelength of the wave as λ just as in case both source and listener were at rest. However, since the listener is in motion relative to the air in which the waves have a speed v, so the listener perceives a higher speed of the waves relative to himself given by

$$v' = v_{\text{rel}} = v + v_L$$

Since $v' = f'\lambda$, so we get

$$f'\lambda = v + v_L \qquad \ldots(1)$$

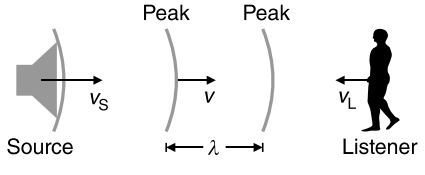
Also, $\lambda = \frac{v}{f}$, so equation (1) becomes

$$f'\left(\frac{v}{f}\right) = v + v_L$$

$$\Rightarrow \quad f' = f\left(\frac{v + v_L}{v}\right)$$

SOURCE AND LISTENER APPROACHING

When source and listener are approaching each other at shown in Figure, then by just combining the arguments given in the previous cases, we can write



$$f'(\lambda - v_S T) = v + v_L \qquad \ldots(1)$$

where, $T = \frac{1}{f}$, $\lambda = \frac{v}{f}$

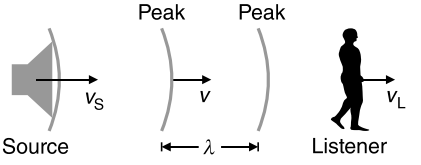
Substituting in equation (1), we get

$$f'\left(\frac{v}{f} - \frac{v_S}{f}\right) = v + v_L$$

$$\Rightarrow \quad f' = f\left(\frac{v + v_L}{v - v_S}\right)$$

SOURCE APPROACHING LISTENER AND BOTH MOVING IN SAME DIRECTION

When source is approaching listener and both are moving in the same direction as shown in Figure, then we can write



$$f'(\lambda - v_S T) = v - v_L \qquad \ldots(1)$$

where, $T = \frac{1}{f}$, $\lambda = \frac{v}{f}$

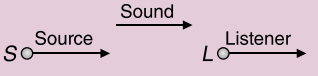
Substituting in equation (1), we get

$$f'\left(\frac{v}{f} - \frac{v_S}{f}\right) = v - v_L$$

$$\Rightarrow \quad f' = f\left(\frac{v - v_L}{v - v_S}\right)$$

Problem Solving Technique(s)

To learn about the general formula and apply it to various situations we proceed by executing the following set of instructions.



(a) Sound must travel from source (S) to listener (L).
(b) All velocities along the direction of sound are taken as positive.
(c) All velocities opposite to the direction of sound are taken as negative.
(d) The apparent frequency is taken as

$$f' = f\left(\frac{v - v_L}{v - v_S}\right)$$

(e) The above formula has to be suitably modified while applying to various situations.
(f) Above formula is valid only for velocities less than the velocity of sound.

The various possible cases and corresponding apparent frequencies f' are listed in the given table. Here, f is the actual frequency emitted by the source, f' is the apparent frequency heard by the listener, v_S is the speed of the source, v_L is the speed of the listener and v is the speed of sound.

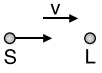
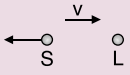
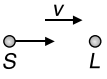
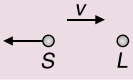
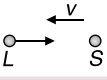
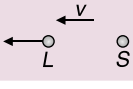
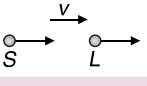
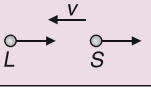
Sl. No.	Situation and Diagram	Apparent frequency
1.	Source moving towards a stationary listener 	$f' = \left(\frac{v}{v - v_S}\right) f \quad (f' > f)$
2.	Source moving away from a stationary listener 	$f' = \left(\frac{v}{v + v_S}\right) f \quad (f' < f)$
3.	Listener moving towards a stationary source 	$f' = \left(\frac{v + v_L}{v}\right) f \quad (f' > f)$
4.	Listener moving away from a stationary source 	$f' = \left(\frac{v - v_L}{v}\right) f \quad (f' < f)$
5.	Source moving towards a receding listener $(v_L > v_S)$ 	$f' = \left(\frac{v - v_L}{v - v_S}\right) f \quad (f' < f)$
6.	Listener moving towards a receding source $(v_S > v_L)$ 	$f' = \left(\frac{v + v_L}{v + v_S}\right) f \quad (f' < f)$
7.	Source and listener moving towards each other 	$f' = \left(\frac{v + v_L}{v - v_S}\right) f \quad (f' > f)$
8.	Source and listener moving away from each other 	$f' = \left(\frac{v - v_L}{v + v_s}\right) f \quad (f' < f)$

ILLUSTRATION 69

A van is moving with a speed of $3\ \text{ms}^{-1}$ towards a large wall. A person standing on the line of motion of the van observes the van to be moving away from him. The horn of the van is now blown. The frequency of the sound produced by the horn is $600\ \text{Hz}$. What is

(a) The frequency of sound heard by the person for the sound produced directly by the horn?
(b) The frequency of sound reflected by the wall?
(c) The beat frequency heard by the driver. The speed of sound in air is $330\ \text{ms}^{-1}$?

SOLUTION

(a) Since the van moves away from the person, the source is receding from the observer. Hence, the frequency v_a heard is given by,

$$f_a = f_0\left(\frac{v}{v + v_S}\right)$$

$$\Rightarrow \quad f_a = 600\left(\frac{330}{330 + 3}\right) = 594.6\ \text{Hz}$$

$$\Rightarrow \quad f_a \approx 595\ \text{Hz}$$

(b) The frequency of sound reflected by the wall is the same as the frequency of sound received by the wall. Now, the source is approaching the wall. Hence the frequency f_w of sound reflected by the wall is given by

$$f_w = f_0\left(\frac{v}{v - v_S}\right)$$

$$\Rightarrow \quad f_w = 600\left(\frac{330}{330 - 3}\right)$$

$$\Rightarrow \quad f_w = 605.5\ \text{Hz} = 606\ \text{Hz}$$

(c) The beat frequency heard is the difference between the two frequencies.

So, beat frequency is $f_b = 606 - 595$

$$\Rightarrow \quad f_b = 11\ \text{Hz}$$

Incidentally this beat frequency is not perceptible to the human ear because of the phenomenon of persistence of hearing. This persistence of hearing is about $\frac{1}{10}$th of a second. Therefore, the maximum beat frequency we can hear is about $10\ \text{Hz}$.

ILLUSTRATION 70

Two sources A and B are producing notes of frequency $680\ \text{Hz}$. A detector moves from A to B with a constant velocity $2\ \text{ms}^{-1}$. Find number of beats heard by the detector per second, if velocity of sound v is $340\ \text{ms}^{-1}$.

SOLUTION

The frequency of source A received by the detector

$$f_1 = \left(\frac{v - v_0}{v}\right) f \qquad \ldots(1)$$

The frequency of source B received by the observer

$$f_2 = \left(\frac{v + v_0}{v}\right) f \qquad \ldots(2)$$

Now number of beats heard by the detector per second i.e., beat frequency is $f_b = |f_1 - f_2|$

$$\Rightarrow \quad f_b = \left[\left(\frac{v + v_0}{v}\right) - \left(\frac{v - v_0}{v}\right)\right] = \frac{2v_0 f}{v}$$

$$\Rightarrow \quad f_b = \frac{2 \times 2 \times 680}{340} = 8 \text{ Hz}$$

Problem Solving Technique(s)

(a) Change in frequency measured by a stationary observer when a moving source crosses him

$$\Delta f = f_{\text{approach}} - f_{\text{recede}} = \left(\frac{2vv_s}{v^2 - v_s^2}\right) f$$

If $v_S \ll v$, then $v^2 - v_S^2 \approx v^2$

$$\Rightarrow \quad \Delta f \approx \left(\frac{2v_S}{v}\right) f$$

(b) Change in frequency measured by an observer as he passes by a stationary source

$$\Delta f = f_{\text{approach}} - f_{\text{recede}} = \left(\frac{2v_L}{v}\right) f$$

ILLUSTRATION 71

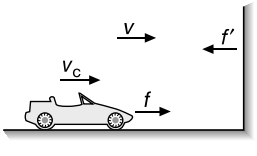
A car while blowing a horn of frequency f is approaching a wall with velocity v_C. If velocity of sound in air is v, then find number of beats heard by the driver per second.

SOLUTION

Frequency of the whistle heard by the driver directly from the source is

$$f_1 = f \qquad \ldots(1)$$

Frequency received by the wall is $f' = \left(\frac{v}{v - v_c}\right) f$



Since same frequency is reflected by the wall (as it is stationary), so let the driver receive the reflected frequency as $f_2 = f''$, then

$$f_2 = f'' = \left(\frac{v + v_C}{v}\right) f' = \left(\frac{v + v_C}{v}\right)\left(\frac{v}{v - v_C}\right) f$$

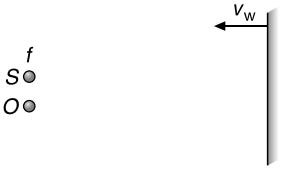
$$\Rightarrow \quad f_2 = \left(\frac{v + v_C}{v - v_C}\right) f \qquad \ldots(2)$$

Hence, number of beats heard by the driver per second i.e., beat frequency is $f_b = |f_1 - f_2|$

$$\Rightarrow \quad f_b = \left| f - \left(\frac{v + v_c}{v - v_c}\right) f \right| = \frac{2v_c f}{v - v_c}$$

ILLUSTRATION 72

A source S of frequency f and an observer are at rest and a wall is moving towards them with velocity v_w as shown.



If velocity of sound is v, calculate the number of beats heard by the observer per second.

SOLUTION

Frequency received by the observer directly from the source is

$$f_1 = f \qquad \ldots(1)$$

Frequency received by the wall is

$$f' = \left(\frac{v + v_w}{v}\right) f$$

Now as the wall is moving towards the observer, frequency heard by the observer after sound gets reflected from the wall is

$$f_2 = \left(\frac{v}{v - v_w}\right) f' = \left(\frac{v + v_w}{v - v_w}\right) f \qquad \ldots(2)$$

Number of beats heard by the observer per second i.e., beat frequency is $f_b = |f_1 - f_2|$

$$\Rightarrow \quad f_b = \left|1 - \frac{v + v_w}{v - v_w}\right| f = \left|\frac{v - v_w - v - v_w}{v - v_w}\right| f$$

$$\Rightarrow \quad f_b = \frac{2v_w f}{v - v_w}$$

ILLUSTRATION 73

A source of sound of frequency 256 Hz is moving rapidly towards a wall with a velocity of 5 ms^{-1}. Calculate the number of beats heard per second if sound travels at a speed of 330 ms^{-1}?

SOLUTION

CASE-I: When observer is between wall and the source

In this case, the observer is stationary and the source in motion towards observer. The apparent frequency for stationary observer is given by

$$f' = f\left(\frac{v}{v - v_s}\right)$$

where, $f = 256$ Hz, $v = 330\ \text{ms}^{-1}$, $v_s = 5\ \text{ms}^{-1}$

$$\Rightarrow \quad f' = 256\left(\frac{330}{330-5}\right) = \frac{256\times 330}{325}$$

$$\Rightarrow \quad f' = 259.93 \text{ Hz}$$

So, the apparent frequency received by the observer from source is 259.93 Hz.

Also, the observer receives sound reflected from the wall. Due to reflection, there is no change in the frequency of the sound and hence the frequency of reflected sound is also 259.93 Hz.

Hence, beat frequency is

$$f_b = |\Delta f| = 259.93 - 259.93 = 0 \text{ Hz}$$

CASE-II: When source is between wall and observer

For reflected sound, source is moving towards wall, so

$$f' = n\left(\frac{v}{v - v_s}\right) = \left(\frac{330}{330-5}\right)256 = 259.9 \text{ Hz}$$

For direct sound, source is moving away from the observer, so

$$f'' = f\left(\frac{v}{v + v_s}\right) = \left(\frac{330}{330+5}\right)256 = 252.2 \text{ Hz}$$

Hence, beat frequency is $f_b = |\Delta f| = f' - f''$

$$\Rightarrow \quad f_b = 259.9 - 252.2 = 7.7 \approx 8 \text{ Hz}$$

ILLUSTRATION 74

A source of sonic oscillations of frequency $f_0 = 1700$ Hz and a receiver are located at the same point. At the moment $t = 0$, the source starts receding from the receiver with constant acceleration $a = 10\ \text{ms}^{-2}$. Assuming the velocity of sound to be equal to $v = 340\ \text{ms}^{-1}$, find the oscillation frequency registered by the stationary receiver $t = 10$ seconds after the start of motion.

SOLUTION

Since, $f = \left(\frac{v}{v + v_s}\right) f_0$ …(1)

Let the sound emitted at time t_0 reaches the observer at time t. The time taken for the sound waves to reach the observer is $(t - t_0)$. So, the distance travelled by sound is $l = v(t - t_0)$

This is also equal to the distance of the source from the observer at time t_0. So,

$$v(t - t_0) = \frac{1}{2}at_0^2$$

$$\Rightarrow \quad at_0^2 + 2vt_0 - 2vt = 0$$

$$\Rightarrow \quad t_0 = \frac{-2v \pm \sqrt{4v^2 + 8avt}}{2a}$$

$$\Rightarrow \quad at_0 = v_s = \sqrt{v^2 + 2avt} - v$$

$$\Rightarrow \quad v + v_s = \sqrt{v^2 + 2avt}$$

$$\Rightarrow \quad f = \left(\frac{v}{\sqrt{v^2 + 2avt}}\right) f_0 \qquad \{\text{because of (1)}\}$$

$$\Rightarrow \quad f = \frac{f_0}{\sqrt{1 + \frac{2at}{v}}}$$

Substituting $f_0 = 1700$ Hz, $a = 10\ \text{ms}^{-2}$, $t = 10$ s and $v = 340\ \text{ms}^{-1}$, we get

$$f = 1349 \text{ Hz}$$

ILLUSTRATION 75

Two trains A and B simultaneously start moving along parallel tracks from a station along same direction. A starts with constant acceleration $2\ \text{ms}^{-2}$ from rest while B with the same acceleration but with initial velocity of $4\ \text{ms}^{-1}$. After 20 s passenger of A hears whistle of B. If frequency of whistle is 1194 Hz and velocity of sound in air is $322\ \text{ms}^{-1}$, calculate frequency observed by the passenger.

SOLUTION

Let t be the time when wave is emitted by B which is being received by A in 20 s. At 20 s train A will be at a distance

$$s_1 = \frac{1}{2}\times 2\times (20)^2 = 400 \text{ m}$$

from the starting point.

At time t, train B will be at a distance

$$s_2 = 40t + \frac{1}{2}\times 2\times t^2 = 40t + t^2$$

Now the distance $s_2 - s_1$ is travelled by the wave in time $(20 - t)$ sec with speed $322\ \text{ms}^{-1}$

$$\Rightarrow \quad 322(20 - t) = s_2 - s_1 = (40t + t^2) - 400$$

$$\Rightarrow \quad t^2 + 362t - 6840 = 0$$

$$\Rightarrow \quad t = \frac{-362 \pm \sqrt{(362)^2 + 4(6840)}}{2} = 18 \text{ s}$$

Velocity of source at $t = 18$ s is given by

$$v_s = u + at = (40) + 18(2) = 76\ \text{ms}^{-1}$$

{away from listener/observer}

and velocity of listener at $t = 20$ s is given by

$$v_L = at = (2)(20) = 40 \text{ ms}^{-1} \quad \{\text{towards the source}\}$$

So, apparent frequency $f_1 = \left(\frac{v + v_L}{v + v_s}\right) f$

$$\Rightarrow \quad f_1 = \left(\frac{322 + 40}{322 + 76}\right) \times 1194$$

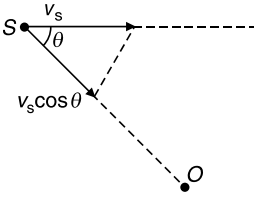
$$\Rightarrow \quad f_1 = 1086 \text{ Hz}$$

Conceptual Note(s)

In such type of problems, we first find the time t when the emitted wave was being received by the listener, in this case 20 sec. Now substitute v_L in the formula $f' = \left(\frac{v \pm v_L}{v \mp v_s}\right) f$ at time when wave is received and v_s when it is emitted (with proper signs). For example, in this equation the wave which is emitted at 18 s is being received by the listener at 20 s. So, we have put v_L at 20 s while v_s at 18 s.

SOURCE NOT MOVING TOWARDS OBSERVER

The previous cases were derived assuming that the motion is along the line joining the source and the observer. If the motion is along some other direction, the components of velocities along the line joining source and observer considered.



If at any instant the line joining the moving source and stationary observer makes an angle θ with the direction of motion of source, v_s is replaced by $v_s \cos\theta$.

$$f' = f\left(\frac{v}{v - v_s \cos\theta}\right)$$

Problem Solving Technique(s)

Note that **no Doppler's effect** will be observed in following cases:

(a) If the source and observer both move in the same direction with same speed, i.e.,

$$v_s = v_0 = u \text{ (say)}$$

Then, $f' = f\left(\frac{v + v_w - u}{v + v_w - u}\right) = f$

(b) If one is at the centre of a circle while the other is moving on it with uniform speed. In this situation, the component of u along the line of sight, i.e., radius, will be $u\cos 90° = 0$.

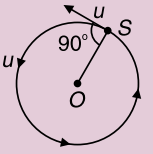
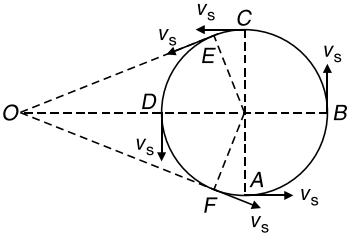


ILLUSTRATION 76

An observer is standing at O, far away from the circle A, B, C, D as shown and a source of frequency f is moving on the circle with constant speed v_S. Find maximum and minimum frequency ward by the observer.



SOLUTION

Observer will receive maximum frequency when the source is at E as in this case the source will be moving exactly towards the observer. Therefore

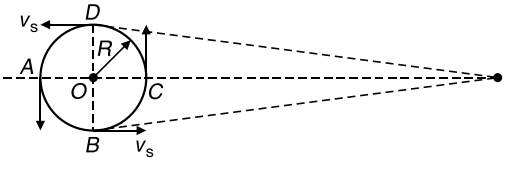
$$f_{\max} = \left(\frac{v}{v - v_S}\right) f$$

Observer will receive minimum frequency when the source is at F as in this case the source is moving exactly away from the observer. Therefore, minimum frequency received by the observer

$$f_{\min} = \left(\frac{v}{v + v_S}\right) f$$

ILLUSTRATION 77

A whistle of frequency 540 Hz rotates in a circle of radius 2 m at an angular speed of 15 rads^{-1}. What is the lowest and highest frequency heard by a listener, a long distance away at rest with respect to the centre of the circle? Can the apparent frequency be ever equal to actual? Given that the velocity of sound in air is 330 ms^{-1}.



SOLUTION

The linear speed of the source,

$$v_s = R\omega = 2\times 15 = 30 \text{ ms}^{-1}$$

Here, observer is at rest and the source is moving. Therefore, the apparent frequency is

$$f' = f\left(\frac{v}{v - v_s}\right)$$

The frequency f' will be lowest when the source is moving away from the observer, i.e., when the whistle is at point D i.e., $v_s = -30$ ms^{-1}

$$f'_{(\text{lowest})} = f\left(\frac{v}{v - v_s}\right) = 540\left(\frac{330}{330 + 30}\right) = 495 \text{ Hz}$$

The frequency f' will be highest when the source is moving towards the observer, i.e., when the whistle is at point B i.e., $v_s = +30$ ms^{-1}

$$f'_{(\text{highest})} = f\left(\frac{v}{v - v_s}\right) = 540\left(\frac{330}{330 - 30}\right) = 594 \text{ Hz}$$

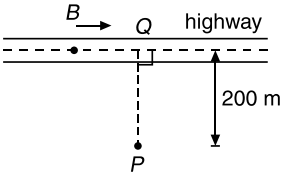
When the whistle is A or C, the velocity of source along the line of sight is $v_s \cos 90° = 0$, and hence

$$f' = f\left(\frac{v}{v - 0}\right) = f$$

Hence, at point A and C, the apparent frequency will be equal to the actual frequency.

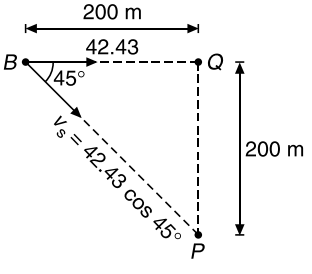
ILLUSTRATION 78

A person P is standing at a perpendicular distance of 200 m from point Q on a highway. A bus B is moving with a speed of 42.43 ms^{-1} towards point Q on the highway. The driver of the bus blows a horn of frequency 1300 Hz. What is the frequency of sound received by the person P, when the bus is distant 200 m from Q. Speed of sound in air is 330 ms^{-1}.



SOLUTION

The velocity component of the source along the line joining the source and the observer is



$$v_S = 42.43 \cos 45°,$$

$$\Rightarrow \quad v_S = \frac{42.43}{\sqrt{2}} \approx 30 \text{ ms}^{-1}$$

Since, $f_a = f_0\left(\frac{v}{v - v_S}\right)$

$$\Rightarrow \quad f_a = 1300\left(\frac{330}{330 - 30}\right) = 1430 \text{ Hz}$$

EFFECT OF MOTION OF MEDIUM

If v_m is the speed of the medium, then velocity of sound v is replaced by $v \pm v_m$, where "+" (positive) sign is taken if the medium moves in the direction of propagation of the wave, and "−" (negative) sign is taken if the medium moves opposite to the direction of propagation of the wave. There is no effect of motion of the medium alone.

WIND EFFECT

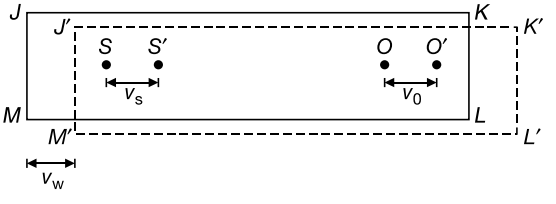
In case wind is blowing with speed v_w, the above formulae are modified by replacing v by $v + v_w$, if the wind blows in the same direction as v and $v - v_w$, if the wind blows in opposite direction to v.

Observer, Source and Medium in Motion

This is the most general case. All the formulae derived above, for both approaching and receiving source from observer, can be combined into a **single formula**,

$$f' = f\left(\frac{v + v_w - v_0}{v + v_w - v_s}\right)$$

While using the above formula, we should place the source S, to the left of the observer O, as shown in figure. Any velocity directed rightward is taken +ve and leftward −ve.



Conceptual Note(s)

If the wind blows with velocity w having direction making an angle θ with the direction of travel of sound towards observer, then velocity of sound v should be replaced by $v + w\cos\theta$.

If wind blows in the direction of sound, $\theta = 0$, so $\cos\theta = 1$ but if wind blows opposite to direction of sound, then $\theta = \pi$, so $\cos\theta = -1$.

ILLUSTRATION 79

A man standing near a railway track hears the whistle of an engine, which is approaching him with a velocity of 36 kmh^{-1}. What is the actual frequency of the whistle, if the apparent pitch of the whistle as heard by the man is 800 Hz? Velocity of sound is 350 ms^{-1}.

SOLUTION

Using the formula for most general case,

$$f' = f\left(\frac{v + v_w - v_0}{v + v_w - v_s}\right)$$

Keeping source to the left of observer, we get

$v = 350\ ms^{-1}$, $v_s = 36\ kmh^{-1} = 10\ ms^{-1}$, $v_0 = 0$, $v_w = 0$ and $f' = 800$ Hz

$$\Rightarrow \quad 800 = f\left(\frac{350}{350 - 10}\right)$$

$$\Rightarrow \quad f = \frac{800 \times (350 - 10)}{350} = 777.14 \text{ Hz}$$

ILLUSTRATION 80

A source and observer are approaching one another with a relative velocity of 30 ms^{-1}. The actual frequency of the sound emitted by the source is 1000 Hz. If the velocity of sound is 350 ms^{-1}, find the apparent pitch as heard by the observer in each of the following cases:

(a) Only the source moving, and the observer is stationary.
(b) Only the observer is moving and the source is stationary.
(c) The source is moving towards the observer with a velocity of 80 ms^{-1} and the observer is moving away from the source with a velocity of 50 ms^{-1}.

SOLUTION

Using the most general formula, and keeping the source on the left of the observer

$$f' = f\left(\frac{v + v_w - v_0}{v + v_w - v_s}\right)$$

(a) Here, $f = 1000$ Hz, $v_s = 30\ ms^{-1}$, $v = 350\ ms^{-1}$, $v_0 = 0$, $v_w = 0$

$$\Rightarrow \quad f' = 1000\left(\frac{350 - 0}{350 - 30}\right) = \frac{1000 \times 350}{320}$$

$$\Rightarrow \quad f' = 1093.75 \text{ Hz}$$

(b) Here, $f = 1000$ Hz, $v_s = 0$, $v_0 = -30\ ms^{-1}$, $v = 350\ ms^{-1}$, $v_w = 0$

$$\Rightarrow \quad f' = 1000\left(\frac{350 - (-30)}{350 - 0}\right) = \frac{1000 \times 380}{350}$$

$$\Rightarrow \quad f' = 1085.7 \text{ Hz}$$

(c) Here, $f = 1000$ Hz, $v_s = 80\ ms^{-1}$, $v_0 = 50\ ms^{-1}$, $v = 350\ ms^{-1}$, $v_w = 0$

$$\Rightarrow \quad f' = 1000\left(\frac{350 - 50}{350 - 80}\right) = \frac{1000 \times 300}{270}$$

$$\Rightarrow \quad f' = 1111.11 \text{ Hz}$$

Please note that, although the relative velocity between the source and observer is same in all three cases, still the apparent frequencies are different.

ILLUSTRATION 81

Two trains travelling at 108 kmh^{-1} are approaching one another. The first train gives a whistle of frequency 850 Hz. The wind is blowing at 36 kmh^{-1} in a direction from the first train towards the second. If the velocity of sound in still air is 340 ms^{-1}, find the apparent pitch of the whistle by a person in the second train

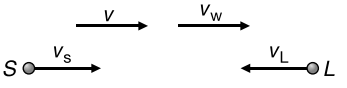
(a) before the trains meet, and
(b) after the trains have passed one another.

SOLUTION

We shall use the most general equation,

$$f' = f\left(\frac{v + v_w - v_L}{v + v_w - v_S}\right)$$

(a) Before the trains meet:



Here, $f = 850$ Hz, $v = 340\ ms^{-1}$,

$v_w = 36\ kmh^{-1} = 10\ ms^{-1}$,

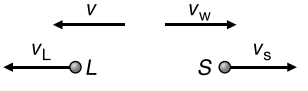
$v_S = 108\ kmh^{-1} = 30\ ms^{-1}$,

$v_L = -108\ kmh^{-1} = -30\ ms^{-1}$

So, $f' = 850\left(\frac{340 + 10 - (-30)}{340 + 10 - 30}\right) = \frac{850 \times 380}{320}$

$$\Rightarrow \quad f' = 1009.375 \text{ Hz}$$

(b) After the trains have passed one another:



Here, $f = 850$ Hz, $v = 340\ ms^{-1}$,

$v_w = -36\ kmh^{-1} = -10\ ms^{-1}$

$v_S = -108\ kmh^{-1} = -30\ ms^{-1}$

$v_L = 108\ kmh^{-1} = 30\ ms^{-1}$

$$\Rightarrow \quad f' = 850\left(\frac{340 - 10 - 30}{340 - 10 - (-30)}\right) = \frac{850 \times 300}{360}$$

$$\Rightarrow \quad f' = 708.33 \text{ Hz}$$

ILLUSTRATION 82

Two vehicles A and B are moving towards each other with same speed u. They blow horns of the same frequency f. Wind is blowing at speed w in the direction of motion of A. The driver of vehicle A hears the sound of horn blown by vehicle B and the sound of horn of his own vehicle after reflection from the vehicle B. Find the frequency and wavelength of both sounds received by A. Velocity of sound is v.

SOLUTION

From Doppler's Effect, the frequency of sound coming directly from B is given by

$$f_1 = \left[\frac{(v-w)+u}{(v-w)-u}\right] f$$

The wavelength is given by

$$\lambda_1 = \frac{\text{Velocity of sound w.r.t. observer}}{\text{Frequency of sound heard by observer}}$$

$$\Rightarrow \quad \lambda_1 = \frac{(v-w)+u}{f_1} = \frac{(v-w)+u}{\left[\frac{(v-w)+u}{(v-w)-u}\right] f} = \frac{(v-w)-u}{f}$$

The frequency of sound incident on B is given by

$$f' = \left[\frac{(v+w)+u}{(v+w)-u}\right] f$$

Now, B will act as a source of this frequency moving with velocity u towards the observer. The frequency of the sound reflected by B as heard by driver A is

$$f_2 = \left[\frac{(v-w)+u}{(v-w)-u}\right] f'$$

$$\Rightarrow \quad f_2 = \left[\frac{(v+w)+u}{(v+w)-u}\right]\left[\frac{(v-w)+u}{(v-w)-u}\right] f$$

$$\Rightarrow \quad \lambda_2 = \frac{(v-w)+u}{f_2} = \frac{(v-w)+u}{\left[\frac{(v+w)+u}{(v+w)-u}\right]\left[\frac{(v-w)+u}{(v-w)-u}\right] f}$$

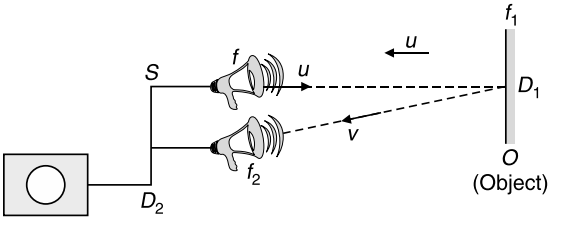
$$\Rightarrow \quad \lambda_2 = \frac{(v-w-u)(v+w-u)}{(v+w+u) f}$$

PRINCIPLE OF SONAR (OR RADAR)

The sound waves sent by the source S are reflected by the object O. The detector D_2 receives two notes one directly from the source S and other from the reflector O (which also acts as a virtual source). If these two frequencies are slightly different, their superposition produces beats. The beat frequency can be used to determine the speed by the object.

If the source S is at rest and a reflector is moving towards the source with speed u, the reflector (acting as detector D_1) will 'hear' the frequency

$$f_1 = f\left(\frac{v+u}{v}\right)$$



Now, this frequency is reflected by the object, which acts as a source moving with a speed u towards the detector D_2. Therefore, the frequency received back by D_2 at the site of the source is

$$f_2 = f_1\left(\frac{v}{v-u}\right) = f\left(\frac{v+u}{v}\right)\left(\frac{v}{v-u}\right) = f\left(\frac{v+u}{v-u}\right)$$

If $u \ll v$ using binomial theorem, we get

$$f_2 = f\left(1+\frac{u}{v}\right)\left(1-\frac{u}{v}\right)^{-1} = f\left(1+\frac{u}{v}\right)\left(1-(-1)\frac{u}{v}\right)$$

$$f_2 = f\left(1+\frac{u}{v}\right)^2 = f\left(1+\frac{2u}{v}\right)$$

So, beat frequency, $f_b = f_2 - f_1 = f\left(\frac{u}{v}\right)$

Thus, the speed of the moving object is given as

$$u = \frac{v}{2}\left(\frac{f_b}{f}\right)$$

ILLUSTRATION 83

A radar gun is a device which produces ultrasonic sound waves of a frequency f_0. These waves are then reflected by a moving object and received back by the device. The frequency of the waves thus received is f. Estimate the speed x of the moving object assuming it to be moving straight towards the radar gun. The speed of sound in air is $v (\gg x)$.

SOLUTION

Frequency, f' of sound waves received by the moving object is given by

$$f' = f_0\left(\frac{v+x}{v}\right)$$

The moving object reflects the sound waves of frequency f'. Since it is moving, so it acts as a moving source and hence frequency of sound waves f received from it by receiving unit of radar gun is given by

$$f = f'\left(\frac{v}{v-x}\right) = f_0\left(\frac{v+x}{v}\right)\left(\frac{v}{v-x}\right) = f_0\left(\frac{v+x}{v-x}\right)$$

$$\Rightarrow \quad f = f_0\left[\frac{1+(x/v)}{1-(x/v)}\right] = f_0\left(1+\frac{x}{v}\right)\left(1-\frac{x}{v}\right)^{-1}$$

Since $x/v \ll 1$, so from Binomial approximation, we get

$$\Rightarrow \quad f \approx f_0\left(1+\frac{x}{v}\right)\left(1+\frac{x}{v}\right) \approx f_0\left(1+\frac{2x}{v}\right)$$

$$\Rightarrow \quad \frac{f}{f_0} = 1+\frac{2x}{v}$$

$$\Rightarrow \quad \frac{f-f_0}{f} = \frac{2x}{v}$$

$$\Rightarrow \quad x = \frac{(f-f_0)v}{2}$$

DOPPLER'S EFFECT OF LIGHT

Till now we have studied that the Doppler effect observed in sound is asymmetric, i.e., the frequency change depends on whether the source is moving or the listener is moving, irrespective of the fact that they have same relative velocity. Doppler effect occurs in light also. However, the Doppler effect observed in light is symmetric, i.e., it depends only on the relative velocity of the source and the observer, irrespective of which of the two is moving.

This difference occurs because sound requires a material medium and v, v_S and v_L are measured relative to the medium. Light, on the other hand, does not require a medium for propagation and the speed of light is the same for any observer whether observer and/or the source is moving.

The formula for apparent frequency f' observed for a source emitting light of frequency f and moving with non-relativistic velocity $v_r\,(\ll c)$ with respect to the observer is

$$f' \approx f\left(1 \pm \frac{v_r}{c}\right)$$

Hence, we conclude that when a source approaches observer or vice versa, then frequency increases and wavelength decreases such that

$\frac{\Delta f}{f} = \pm\frac{v_r}{c}$	$\frac{\Delta\lambda}{\lambda} = \mp\frac{v_r}{c}$
$(+)$ Approach	$(-)$ Approach
$(-)$ Recede	$(+)$ Recede

RED SHIFT AND BLUE SHIFT

Since, from Doppler's Effect for Light we have

$$\frac{\Delta f}{f} = \pm\frac{v}{c} \text{ or } \frac{\Delta\lambda}{\lambda} = \mp\frac{v}{c}$$

Consider a source of light approaching an observer (or vice-versa), then the frequency must increase and hence wavelength should decrease. Since blue colour has minimum wavelength, so decrease in wavelength implies that a shift occurs towards the blue, hence this shift is termed as **Blue Shift**.

Further if a source of light recedes from an observer (or vice versa), then frequency must decrease and hence wavelength should increase. Since red colour has maximum wavelength, so increase in wavelength implies that the shift occurs towards the red colour and hence this shift is termed as **Red Shift**.

Our universe always exhibits red shift and hence it must be expanding in all the directions.

ILLUSTRATION 84

Assuming the earth to be moving towards a stationary star at a speed of $30\ \text{kms}^{-1}$, calculate the apparent wavelength of light emitted from the star if the real wavelength of light emitted by the star is $5875\ \text{Å}$.

SOLUTION

Here earth (observer) is approaching the star, so we gave

$$\Delta f = +\left(\frac{v}{c}\right)f \text{ or } \Delta\lambda = -\left(\frac{v}{c}\right)\lambda$$

where, v is the relative velocity of the source and c is the velocity of light.

So, the observer will notice an increase in frequency or decrease in wavelength.

$$\Rightarrow \quad |\Delta\lambda| = \left(\frac{30\times10^3}{3\times10^8}\right)5875\ \text{Å} = 5875\times10^{-4}\ \text{Å}$$

Hence, altered wavelength is

$$\lambda' = \lambda - \Delta\lambda$$

$$\Rightarrow \quad \lambda' = 5875\ \text{Å} - (5875\times10^{-4}\ \text{Å})$$

$$\Rightarrow \quad \lambda' = 0.9999\times5875\ \text{Å}$$

$$\Rightarrow \quad \lambda' = 5874.4125\ \text{Å}$$

ILLUSTRATION 85

How fast would you have to go through a red light to have it appear green if the respective wavelengths of red and green light are $620\ \text{nm}$ and $540\ \text{nm}$? Is it possible to achieve this speed on earth?

SOLUTION

Apparent wavelength is

$$\lambda_{\text{app}} = \lambda' = \lambda\left(1-\frac{v}{c}\right)$$

$$\Rightarrow \quad 540 = 620\left(1-\frac{v}{c}\right)$$

$$\Rightarrow \quad 1-\frac{v}{c} = \frac{27}{31}$$

$$\Rightarrow \quad \frac{v}{c} = \frac{4}{31}$$

$$\Rightarrow \quad v = \frac{4}{31}(3\times10^8) = 3.87\times10^7\ \text{ms}^{-1}$$

Since this speed in much larger than escape speed on earth, so this speed is not attainable.

ACOUSTICS OF BUILDINGS

Designing a building has to be done keeping in mind certain acoustical facts.

(a) There should not be **penetration** of sound between rooms, especially in hotels and studios of radio stations. For this the walls must be covered with sound absorbing materials and doors must have heavy curtains. Just the opposite is the requirement of a **'whispering gallery'**, in which the walls are made completely non-absorbing so that even whispers can be heard at diametrically opposite ends of a large gallery around a dome.

(b) **Echo:** In large auditoriums, echo creates lots of problems. If the reflected sound of any syllable comes after the next syllable directly reaching a listener, then there is confusion due to overlapping. Since the average interval between two syllables spoken by a person is about 0.2 s, those walls and ceilings should be made non-reflecting from where the sound comes back in more than 0.2 s.

(c) **Reverberation:** A sound once produced persists for some time due to repeated reflections in the hall. This phenomenon is called **Reverberation**. If the hall has many doors and windows and absorbing materials like curtains and furniture, the sound dies out quickly. Sound is also absorbed by the audience in the hall. The time taken by the sound intensity to fall by a factor of 10^{-6} is called the **Reverberation Time** of the hall. The reverberation time should neither be too large nor too small. For conference and lecture halls the reverberation time should be about 1 s. For a musical concert it should be a little larger.

Sabines reverberation formula for time is

$$T = \frac{CV}{AS}$$

where C = constant, V = Volume of hall,

A = Absorption coefficient,

S = Total surface area of enclosed space.

The reverberation time for a hall of 10^4 m^3 volume is between 1 s to 1.5 s.

(d) Lecture halls must possess **even distribution of sound.** For such halls, dome, curved surfaces and projections should be designed with great care. One method is to make the wall in front of the audience parabolic with the speaker at its focus. The reflected sound from this wall will then go equally everywhere. The other walls and ceilings should not be curved. Otherwise, they can produce improper focussing, increased intensity at some points and no sound at some other points (**Dead Centres**). Destructive interference can also take place between direct and reflected sound at some places.

Test Your Concepts-VI

Based on Doppler's Effect

(Solutions on page H.242)

1. Two tuning forks with natural frequencies 340 Hz each move relative to a stationary observer. One fork moves away from the observer while the other moves towards him at the same speed. The observer hears beats of frequency 3 Hz. Find the speed of the tuning fork. Given that the velocity of sound in air is 340 ms^{-1}.

2. A source of sound of frequency 1000 Hz moves to the right with a speed of 32 ms^{-1} relative to the ground. To its right is a reflecting surface moving to the left with a speed of 64 ms^{-1} relative to the ground. Given that the speed of sound in air is 332 ms^{-1}. Find the
(a) wavelength of the sound emitted in air by the source.
(b) number of waves arriving per second at the reflecting surface.
(c) speed of the reflected waves.
(d) wavelength of the reflected waves.

3. A whistle of frequency 540 Hz rotates in a circle of radius 2 m at a linear speed of 30 ms^{-1}. What is the lowest and highest frequency heard by an observer a long distance away at rest with respect to the centre of circle. Take speed of sound in air to be 330 ms^{-1}. Can the apparent frequency be ever equal to actual?

4. A siren emitting a sound of frequency 1000 Hz moves away from you toward a cliff at a speed of 10 ms^{-1}. Find the
(a) frequency of the sound, coming directly from the siren heard by you.
(b) frequency of the sound, reflected off the cliff heard by you.
(c) beat frequency heard by you.
Take the speed of sound in air to be 330 ms^{-1}.

5. A sonic source starts vibrating with a frequency of 1000 Hz and at the same time starts moving towards a receiver with an acceleration of 10 ms^{-2}. If the velocity of sound in air is 350 ms^{-1}
(a) find the wavelength of sound emitted exactly after source starts accelerating

(b) find the average frequency measured by the receiver over 1 sec after start.

6. A radar wave has a frequency of 7.8×10^9 Hz. The reflected wave from an airplane shows a frequency difference of 2.6×10^3 Hz on the higher side. Calculate velocity of the airplane in the line of sight.

7. Two cars A and B depart simultaneously from the same position and in same direction on a straight road. A starts with initial velocity 2 ms^{-1} and acceleration 2 ms^{-2} while B start with initial velocity 2 ms^{-1}. The driver of car A hears a sound of frequency 352 Hz emitted by car B after 10 sec, after the start, find the actual frequency of the sound as emitted by B.
Given: Velocity of the sound is $v = 330$ ms^{-1}.

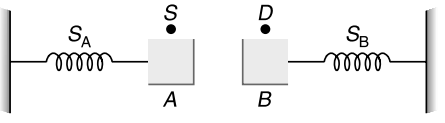
8. A stationary source emits sound of frequency $f_0 = 1000$ Hz. If a wind blows at the speed of $0.1v$, where v is the speed of sound, then find
(a) the percentage change in the wavelength towards the wind side of the source.
(b) the percentage change in the frequency for a stationary observer on the wind-side of the source.
(c) what happens when there is no wind, but the observer moves at speed $0.1v$ towards the source.

9. A sonometer wire under a tension of 64 N vibrating in its fundamental mode is in resonance with a vibrating tuning fork. The vibrating portion of the sonometer wire has a length of 10 cm and a mass of 1 g. The vibrating tuning fork is now moved away from the vibrating wire with a constant speed and an observer standing near the sonometer hears one beat per second. Calculate the speed with which the tuning fork is moved. The speed of sound in air is 300 ms^{-1}.

10. A source of sonic oscillations of frequency $f_0 = 1000$ Hz moves at right angles to the wall with a velocity $u = 0.17$ ms^{-1}. Two stationary receivers R_1 and R_2 are located on a straight line, coinciding with the trajectory of the source, in the following succession : R_1 source R_2 wall. Which receiver registers the beats and what is the beat frequency? The velocity of sound is equal to $v = 340$ ms^{-1}.

11. The wavelength of light coming from a distant galaxy is found to be 0.5% more than that coming from a source on earth. Calculate the velocity of the galaxy.

12. A source S emitting sound of 300 Hz is fixed on block A which is attached to the free end of a spring S_A as shown in the figure. The detector D fixed on block B attached to the free end of spring S_B detects this sound. The blocks A and B are simultaneously displaced towards each other through a distance of 1 m and then left to vibrate. Find the maximum and minimum frequencies of sound detected by D if the vibrational frequency of each block is 2 Hz. Take speed of sound = 340 ms^{-1}.



13. A railway engine is moving from east to west with a velocity of 72 kmh^{-1}. At the same time you are moving from west to east towards the engine in a car with a velocity of 54 kmh^{-1}. The wind is blowing from west to east with a velocity of 36 kmh^{-1}. If the velocity of sound in still air is 330 ms^{-1} and the actual frequency of the whistle of the engine is 1200 Hz, what will be the apparent frequency heard by you?

14. A fighter plane moving in a vertical loop with constant speed of radius R. The centre of the loop is at a height h directly overhead of an observer standing on the ground. The observer receives maximum frequency of the sound produced by the plane when it is nearest to him. Find the speed of the plane if velocity of sound in air is v.

SOLVED PROBLEMS

PROBLEM 1

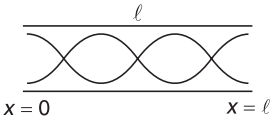
A 3 m long organ pipe open at both ends is driven to third harmonic standing wave. If the maximum amplitude of pressure oscillations is 1 percent of mean atmospheric pressure $\left(P_0 = 10^5 \text{ Nm}^{-2}\right)$. Find the maximum amplitude of particle displacement and density oscillations. Speed of sound $v = 332 \text{ ms}^{-1}$ and density of air $\rho = 1.03 \text{ kgm}^{-3}$.

SOLUTION

Given length of pipe is $l = 3$ m, so for third harmonic

$$3\left(\frac{\lambda}{2}\right) = l$$

$$\Rightarrow \quad \lambda = \frac{2l}{3} = \frac{2\times 3}{3} = 2 \text{ m}$$



The angular frequency is

$$\omega = 2\pi f = \frac{2\pi v}{\lambda} = \frac{(2\pi)(332)}{2} = 332\pi \text{ rads}^{-1}$$

The particle displacement $y(x,\ t)$ can be written as

$$y(x,\ t) = A\cos(kx)\sin(\omega t)$$

where, $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{2l/3} = \dfrac{3\pi}{l}$

and $\quad \omega = kv = \dfrac{3\pi v}{l} \qquad \left\{\because\ v = \dfrac{\omega}{k}\right\}$

$$\Rightarrow \quad y(x,\ t) = A\cos\left(\frac{3\pi x}{l}\right)\sin\left(\frac{3\pi vt}{l}\right)$$

The longitudinal oscillations of an air column can be viewed as oscillations of particle displacement or pressure wave or density wave. Pressure variation is related to particle displacement as

$$P(x) = -B\frac{\partial y}{\partial x} \qquad \{B = \text{Bulk modulus}\}$$

$$\Rightarrow \quad P(x) = \left(\frac{3BA\pi}{l}\right)\sin\left(\frac{3\pi x}{l}\right)\sin\left(\frac{3\pi vt}{l}\right)$$

The amplitude of pressure variation is

$$\Delta P_{\max} = \frac{3BA\pi}{l} = \frac{3\rho v^2 A\pi}{l} \qquad \left\{\because B = \rho v^2\right\}$$

$$\Rightarrow \quad A = \frac{\Delta P_{\max} l}{3\rho v^2 \pi}$$

Given that, $\Delta P_{\max} = P_0/100 = 10^3 \text{ Nm}^{-2}$

$$\Rightarrow \quad A = \frac{(10^3)(3)}{(3)(1.03)(332)^2(\pi)} = 0.0028 \text{ m} = 0.28 \text{ cm}$$

From definition of Bulk modulus (B), we have

$$B = -\frac{dP}{dV/V} \qquad \ldots(1)$$

Since, volume $V = \dfrac{m}{\rho}$

$$\Rightarrow \quad dV = -\frac{m}{\rho^2}d\rho = -\frac{Vd\rho}{\rho}$$

$$\Rightarrow \quad \frac{dV}{V} = -\frac{d\rho}{\rho}$$

Substituting in Equation (1), we get

$$d\rho = \frac{\rho(dP)}{B}$$

So, maximum amplitude of density oscillation is

$$\Delta\rho_{\max} = \frac{\rho}{B}\Delta P_{\max} = \frac{\Delta P_{\max}}{v^2} \qquad \left\{\because\ \frac{\rho}{B} = \frac{1}{v^2}\right\}$$

$$\Rightarrow \quad \Delta\rho_{\max} = \frac{10^3}{(332)^2} = 9\times 10^{-3} \text{ kgm}^{-3}$$

PROBLEM 2

A string 1 carries a harmonic wave. At a junction with string 2 it is partly reflected and partly transmitted. The linear mass density of the second string is four times that of the first string and that the boundary between the two strings is at $x = 0$. If the expression for the incident wave is, $y_i = A_i\cos(k_1 x - \omega_1 t)$

(a) What are the expressions for the transmitted and the reflected waves in terms of A_i, k_1 and ω_1?

(b) Show that the average power carried by the incident wave is equal to the sum of the average power carried by the transmitted and reflected waves.

SOLUTION

(a) Since, $v = \sqrt{\dfrac{T}{\mu}}$, $T_2 = T_1$ and $\mu_2 = 4\mu_1$

$$\Rightarrow \quad v_2 = \frac{v_1}{2} \qquad \ldots(1)$$

Also, the frequency does not change, so

$$\omega_1 = \omega_2 \qquad \ldots(2)$$

Since, $k = \dfrac{\omega}{v}$, so the wave number of harmonic waves in two strings are related as

$$k_2 = \frac{\omega_2}{v_2} = \frac{\omega_1}{v_1/2} = 2\left(\frac{\omega_1}{v_1}\right) = 2k_1 \qquad \ldots(3)$$

The amplitudes are given by

$$A_t = \left(\frac{2v_2}{v_1 + v_2}\right)A_i = \left[\frac{2(v_1/2)}{v_1 + (v_1/2)}\right] = \frac{2}{3}A_i \qquad \ldots(4)$$

and $A_r = \left(\frac{v_2 - v_1}{v_1 + v_2}\right) A_i = \left[\frac{(v_1/2) - v_1}{v_1 + (v_1/2)}\right] A_i$

$\Rightarrow \quad A_r = -\frac{A_i}{3}$ …(5)

So, from Equations (2), (3) and (4), the transmitted wave can be written as,

$$y_t = \frac{2}{3} A_i \cos(2k_1 x - \omega_1 t)$$

Similarly, the reflected wave is expressed as,

$$y_r = -\frac{A_i}{3} \cos(k_1 x + \omega_1 t)$$

$\Rightarrow \quad y_r = \frac{A_i}{3} \cos(k_1 x + \omega_1 t + \pi)$

The reflected ray suffers and additional phase change of π because the wave is reflected at the fixed end.

(b) The average power of a harmonic wave on a string of cross-sectional area S is given by

$$P = \frac{1}{2} \rho A^2 \omega^2 S v = \frac{1}{2} A^2 \omega^2 \mu v \qquad \{\because \rho S = \mu\}$$

So, $P_i = \frac{1}{2} \omega_1^2 A_i^2 \mu_1 v_1$ …(6)

$$P_t = \frac{1}{2} \omega_1^2 \left(\frac{2}{3} A_i\right)^2 (4\mu_1)\left(\frac{v_1}{2}\right) = \frac{4}{9} \omega_1^2 A_i^2 \mu_1 v_1 \quad \text{…(7)}$$

and $P_r = \frac{1}{2} \omega_2^2 \left(-\frac{A_i}{3}\right)^2 (\mu_1)(v_1) = \frac{1}{18} \omega_1^2 A_i^2 \mu_1 v_1$ …(8)

From Equations (6), (7) and (8), we observe that

$$P_i = P_t + P_r$$

PROBLEM 3

On a stormy day Ram asked Shyam to go to his friend's house which is towards north from his house. He asked him to go directly towards North. After sometime Ram observed Shyam going towards 37° West of North as wind was blowing at 36 kmhr^{-1} towards west. After 15 min Ram blew a whistle so as to give signal to Shyam to come back. Calculate the distance between Ram's house and Shyam when he heard the signal. If natural frequency of the whistle was 400 Hz, calculate the frequency as heard by Shyam. Assume that speed of sound in air is 330 ms^{-1} and Shyam is travelling at a speed of 16.67 ms^{-1}.

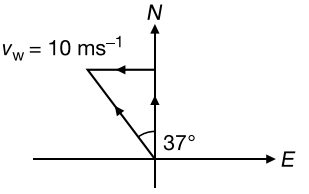
SOLUTION

Let Shyam hear the whistle after a time t, then total distance travelled by Shyam is

$$x = 16.67(15 \times 60) + 16.67t = 15000 + 16.67t$$

Distance travelled by sound in time t is

$$y = [330 + 10\cos(53°)]t = 336t$$



Since, $x = y$

$\Rightarrow \quad 15000 + 16.67t = 336t$

$\Rightarrow \quad t = \frac{45000}{958} = 47 \text{ s}$

So, distance is

$$x = y = 15000 + (16.67)(47) = 15783.3 \text{ m}$$

$\Rightarrow \quad x = y = 15.78 \text{ km}$

So, $f' = \left(\frac{330 + 10(3/5) - 16.67}{330 + 10(3/5)}\right)(400) = 380.15 \text{ Hz}$

PROBLEM 4

A standing wave is produced in a steel wire of mass 100 g tied to two fixed supports. The length of the string is 2 m and strain in it is 0.4%. The string vibrates in four loops. Assume one end of the string to be at $x = 0$, all particle to be at rest at $t = 0$ and maximum amplitude to be 3 mm, find

(a) wavelength and frequency of the wave
(b) equation of the standing wave
(c) equation of the travelling waves whose superposition is the given standing wave. Also find the velocity of these travelling waves.
(d) maximum kinetic energy of the wire.

(Given $\pi^2 = 10$, density of steel $= 4 \times 10^3$ kgm^{-3}, Young's modulus of steel $= 1.6 \times 10^{11}$ Nm^{-2})

SOLUTION

(a) Since the string vibrates in four loops, so

$$4\left(\frac{\lambda}{2}\right) = 2$$

$\Rightarrow \quad \lambda = 1 \text{ m}$

Since, $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\rho S}}$

Also, $Y = \frac{T/S}{\Delta l / l}$

$\Rightarrow \quad \frac{T}{S} = Y\left(\frac{\Delta l}{l}\right)$

Substituting the values, we get

$$v = 400 \text{ ms}^{-1}$$

$\Rightarrow \quad f = \frac{v}{\lambda} = \frac{400}{1} = 400 \text{ Hz}$

(b) $y = A\sin(kx)\cos(\omega t)$

Substituting $A = 3$ mm, $k = \dfrac{2\pi}{\lambda}$ and $\omega = 2\pi f$

$\Rightarrow \quad y = (3\text{ mm})\sin(2\pi x)\cos(800\pi t)$

(c) $y_1 = 1.5\sin(2\pi x - 800\pi t)$ and

$y_2 = 1.5\sin(2\pi x + 800\pi t)$

with $v = 400\text{ ms}^{-1}$

(d) Velocity of particle at

$$x = \frac{\partial y}{\partial t} = -\omega A\sin(kx)\sin(\omega t)$$

Kinetic energy of a small element of length dx at that position is

$$dK = \frac{1}{2}(\mu dx)v^2 = \frac{1}{2}\frac{m}{l}v^2 dx$$

So, total K.E. is $K = \dfrac{1}{2}\dfrac{m}{l}\omega^2 A^2\displaystyle\int_0^l \sin^2 kx dx$

$\Rightarrow \quad K = \dfrac{1}{4}m\omega^2 A^2$

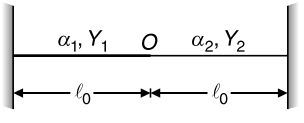
$\Rightarrow \quad K = 1.44$ J

PROBLEM 5

Two wires 1 and 2 of the same cross-sectional area $A = 10\text{ mm}^2$ and the same length but made of different materials are welded together and their ends are rigidly clamped between two walls, as shown in the figure. The respective Young's Moduli and the linear coefficients of thermal expansion are

$Y_1 = 10^9\text{ Nm}^{-2}$, $Y_2 = 2\times10^9\text{ Nm}^{-2}$

$\alpha_1 = 6\times10^{-4}\ ^\circ\text{C}$, $\alpha_2 = 3\times10^{-4}\ ^\circ\text{C}$



(a) If the temperature of the system is reduced by 20 °C, then

(i) find the tension in each wire

(ii) find the displacement of the joint O

(b) Find the first overtone frequency of the system if joint is a node and the mass per unit length of the wires are

$\mu_1 = 0.3\text{ kgm}^{-1}$ and $\mu_2 = 0.075\text{ kgm}^{-1}$; $l_0 = 1$ m

SOLUTION

(a) Since, we know that

$$\text{Stress} = \frac{T}{S} = Y\alpha\Delta\theta$$

$\Rightarrow \quad T = YS\alpha\Delta\theta$

where S is area of cross-section of the wire

$\Rightarrow \quad T_1 = (10^9)(10\times10^{-6})(6\times10^{-4})(20)$

$\Rightarrow \quad T_1 = 120$ N

Similarly, using the same formula, we get

$T_2 = 120$ N

So, displacement of the joint O is zero.

(b) Let n_1 loops be formed in wire 1 and n_2 loops in wire 2. Then $f_1 = f_2$, so we get

$$n_1\left(\frac{v_1}{2l_1}\right) = n_2\left(\frac{v_2}{2l_2}\right)$$

$$\Rightarrow \quad \frac{n_1}{\sqrt{\mu_1}} = \frac{n_2}{\sqrt{\mu_2}} \qquad \left\{\because\ v \propto \frac{1}{\sqrt{\mu}}\right\}$$

$$\Rightarrow \quad \frac{n_1}{n_2} = \sqrt{\frac{\mu_1}{\mu_2}} = \sqrt{\frac{0.3}{0.075}} = \frac{2}{1}$$

This ratio implies that in fundamental mode, 2 loops are formed on wire 1 and 1 loop is formed on wire 2. In first overtone frequency 4 loops are formed on wire 1 and 2 loops on wire 2.

Thus, the first overtone frequency is

$$f = 4\left(\frac{v_1}{2l_0}\right) = 4\left(\frac{1}{2l_0}\sqrt{\frac{T}{\mu_1}}\right) = 4\left(\frac{1}{2}\sqrt{\frac{120}{0.3}}\right)$$

$\Rightarrow \quad f = 40$ Hz

PROBLEM 6

A boat is travelling in a river with a speed 10 ms^{-1} along the stream flowing with a speed 2 ms^{-1}. From this boat, a sound transmitter is lowered into the river through a rigid support. The wavelength of the sound emitted from the transmitter inside the water is 14.45 mm. Assume that attenuation of sound in water and air is negligible.

(a) What will be the frequency detected by a receiver kept inside the river downstream?

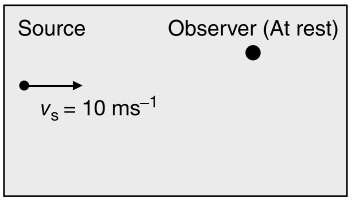
(b) The transmitter and the receiver are now pulled up into air. The air is blowing with a speed 5 ms^{-1} in the direction opposite the river stream. Determine the frequency of the sound detected by the receiver.

Given : Temperature of the air and water = 20 °C, Density of river water $= 10^3\text{ kgm}^{-3}$, Bulk modulus of the water is 2.088×10^9 Pa, Gas constant $R = 8.31\text{ Jmol}^{-1}\text{K}^{-1}$, Mean molecular mass of air $= 28.8\times10^{-3}\text{ kgmol}^{-1}$, $\dfrac{C_P}{C_V}$ for air is 1.4

SOLUTION

Velocity of sound in water is $v_w = \sqrt{\dfrac{B}{\rho}}$

$$\Rightarrow \quad v_w = \sqrt{\frac{2.088\times10^9}{10^3}} = 1445\text{ ms}^{-1}$$



Frequency of sound in water will be

$$f_0 = \frac{v_w}{\lambda_w} = \frac{1445}{14.45\times10^{-3}} \text{ Hz}$$

$$\Rightarrow \quad f_0 = 10^5 \text{ Hz}$$

(a) Frequency of sound detected by receiver (observer) at rest is given by

$$f_1 = f_0\left(\frac{v_w + v_r}{v_w + v_r - v_s}\right)$$

$$\Rightarrow \quad f_1 = (10^5)\left(\frac{1445+2}{1445+2-10}\right) \text{ Hz}$$

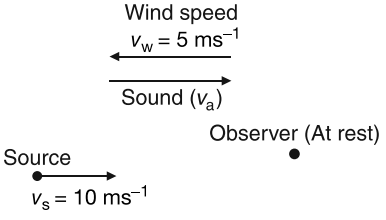
$$\Rightarrow \quad f_1 = 1.0069\times10^5 \text{ Hz}$$

(b) Velocity of sound in air is given by

$$v_a = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{(1.4)(8.31)(20+273)}{28.8\times10^{-3}}}$$

$$\Rightarrow \quad v_a = 344 \text{ ms}^{-1}$$

Since, frequency does not depend on the medium. Therefore, frequency in air is also $f_0 = 10^5$ Hz.



Frequency of sound detected by receiver (observer) in air is given by

$$f_2 = f_0\left(\frac{v_a - v_w}{v_a - v_w - v_s}\right) = 10^5\left(\frac{344-5}{344-5-10}\right) \text{ Hz}$$

$$\Rightarrow \quad f_2 = 1.0304\times10^5 \text{ Hz}$$

PROBLEM 7

The following equations represent transverse waves:

$z_1 = A\cos(kx - \omega t)$, $z_2 = A\cos(kx + \omega t)$ and

$z_3 = A\cos(ky - \omega t)$.

Identify the combination of the wave which will produce

(a) a standing wave

(b) a wave travelling in the direction making an angle of 45° with the positive x and positive y-axis.

In each case find the position at which the resultant intensity is always zero.

SOLUTION

(a) A standing wave is produced due to superposition of two waves of equal amplitude and frequency travelling in opposite direction with same speed. Here, the amplitude, the frequency and the velocity of all waves are equal; but z_1 is travelling along positive x-axis, z_2 along negative x-axis and z_3 along positive y-axis. So, superposition of z_1 and z_2 will produce stationary waves,

$$z = z_1 + z_2 = A\cos(\omega t - kx) + A\cos(\omega t + kx)$$

or $\quad z = 2A\cos kx\cos\omega t$

Now, as the amplitude of standing wave is $(2A\cos kx)$, the velocity is

$$I = A_s^2 = 4A^2\cos^2 kx$$

which will zero, if

$$\cos^2 kx = 0$$

$$\Rightarrow \quad kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$

$$\Rightarrow \quad x = \frac{\pi}{2k}, \frac{3\pi}{2k}, \frac{5\pi}{2k}, \ldots$$

$$\Rightarrow \quad x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots \qquad \left\{\because k = \frac{2\pi}{\lambda}\right\}$$

(b) The waves z_1 and z_3 are travelling along positive x-axis and positive y-axis respectively, with the same speed,

$$v = \frac{\omega}{k} = \frac{2\pi f}{2\pi/\lambda} = f\lambda$$

Therefore, the superposition of z_1 and z_3 will produce a wave travelling in a direction making an angle of 45° with positive x and y-axis. The resulting wave is given as

$$z = z_1 + z_3 = A\cos(\omega t - kx) + A\cos(\omega t - ky)$$

$$\Rightarrow \quad z = 2A\cos\left(\frac{k|x-y|}{2}\right)\cos\left[\omega t - k\left(\frac{x+y}{2}\right)\right]$$

The amplitude of this wave is

$$A_s = 2A\cos\left(\frac{k|x-y|}{2}\right)$$

$$\Rightarrow \quad I \propto \cos^2\left(\frac{k|x-y|}{2}\right)$$

Therefore, the intensity will be zero for

$$\left(\frac{k|x-y|}{2}\right) = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$

$$\Rightarrow \quad |x-y| = \frac{\pi}{k}, \frac{3\pi}{k}, \frac{5\pi}{k}, \ldots$$

$$\Rightarrow \quad |x-y| = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots \qquad \left\{\because k = \frac{2\pi}{\lambda}\right\}$$

PROBLEM 8

An aluminium wire of cross-sectional area 10^{-6} m^2 is joined to a steel wire of the same cross-sectional area. This compound wire is stretched on a sonometer pulled by a weight of 10 kg. The total length of the compound wire between the bridges is 1.5 m of which the aluminium wire is 0.6 m and the rest is steel wire. Transverse vibrations are set-up in the wire by using an external source of variable frequency. Find the lowest frequency of excitation for which the standing waves are formed such that the joint in the wire is a node. What is the total number of nodes at this frequency? Density of aluminium is 2.6×10^3 kgm^{-3} and that of steel is 1.04×10^4 kgm^{-3}. Take $g = 10$ ms^{-2}.

SOLUTION

Let n_a loops are formed in aluminium wire and n_s in steel. Then $f_a = f_s$

$$\Rightarrow \quad n_a\left(\frac{v_a}{2l_a}\right) = n_s\left(\frac{v_s}{2l_s}\right)$$

$$\Rightarrow \quad \frac{n_a}{n_s} = \left(\frac{v_s}{v_a}\right)\left(\frac{l_a}{l_s}\right)$$

Since, $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\rho S}} \propto \frac{1}{\sqrt{\rho}}$

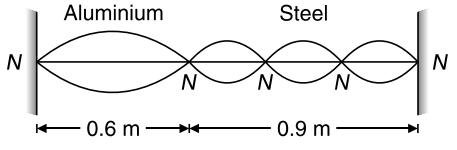
$$\Rightarrow \quad \frac{v_s}{v_a} = \sqrt{\frac{\rho_a}{\rho_s}}$$

$$\Rightarrow \quad \frac{n_a}{n_s} = \sqrt{\frac{\rho_a}{\rho_s}}\frac{l_a}{l_s}$$

Substituting the values, we get

$$\frac{n_a}{n_s} = \sqrt{\frac{2.6\times10^3}{1.04\times10^4}}\left(\frac{0.6}{0.9}\right) = \frac{1}{3}$$

i.e., at lowest frequency, one loop is formed in aluminium wire and three loops are formed in steel wire as shown in figure.



So, $f_{\min} = n_a\left(\frac{v_a}{2l_a}\right) = n_a\left(\frac{1}{2l_a}\sqrt{\frac{T}{\rho_a S}}\right)$

$$\Rightarrow \quad f_{\min} = (1)\left(\frac{1}{2\times0.6}\sqrt{\frac{100}{2.6\times10^3\times10^{-6}}}\right) = 163.4 \text{ Hz}$$

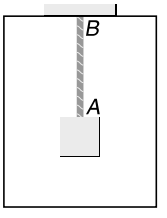
Total number of nodes are five as shown in figure.

Conceptual Note(s)

In such type of problems nature of junction will be known to us. Then we have to equate frequencies on the two sides. By equating the frequencies, we find $\frac{n_1}{n_2}$. Suppose this comes out to be 0.6. Write it, $\frac{n_1}{n_2} = \frac{3}{5}$. At lowest oscillation frequency 3 loops are formed on side 1 and 5 loops on side 2. At next higher frequency 4 loops will be formed on side 1 and 10 on side 2 and so on.

PROBLEM 9

A block of mass M is attached with a string of mass m and length l as shown in figure. The whole system is placed on a planet whose mass and radius is three times the mass and radius of earth. Find the



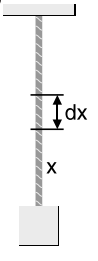
(a) time taken by a wave pulse to travel from one end A to B of the string.

(b) ratio of maximum and minimum velocity of wave pulse.

Assume the acceleration due to gravity on the earth to be g.

SOLUTION

(a) Since, $g' = \frac{3Gm}{(3R)^2} = \frac{g}{3}$



Consider a small element dx at a distance x from one end, then

$$v = \sqrt{\frac{T}{\mu}}$$

where, $\mu = \frac{m}{l}$ and $T = \left(M + \frac{m}{l}x\right)g'$

$$\Rightarrow \quad v = \frac{dx}{dt} = \sqrt{\frac{\left(M + \frac{m}{l}x\right)g}{3\left(\frac{m}{l}\right)}}$$

$$\Rightarrow \int_0^l \frac{dx}{\sqrt{Ml+mx}} = \sqrt{\frac{g}{3m}}\int_0^t dt$$

$$\Rightarrow t = 2\left(\frac{\sqrt{M+m}-\sqrt{M}}{\sqrt{m}}\right)\sqrt{\frac{3l}{g}}$$

(b) $v_{\min} = \sqrt{\frac{Mlg}{3m}}$ and $v_{\max} = \sqrt{\frac{(M+m)lg}{3m}}$

$$\Rightarrow \frac{v_{\max}}{v_{\min}} = \sqrt{1+\frac{m}{M}}$$

PROBLEM 10

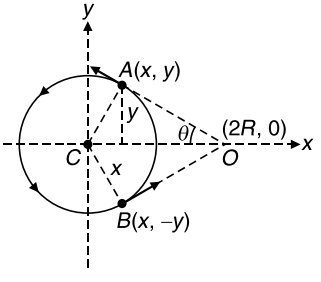
A source is moving along a circle $x^2+y^2=R^2$ with constant speed $v_s = \frac{330\pi}{6\sqrt{3}}$ ms^{-1} in clockwise direction while an observer is stationary at point $(2R,\ 0)$ with respect to the center of circle. Frequency emitted by the source is f.

(a) Find the coordinates of source when observer records the maximum and minimum frequency.

(b) Find the value of maximum and minimum frequency. Take speed of sound $v = 330$ ms^{-1}.

SOLUTION

(a) When the source is at B, the observed frequency is maximum and when it is at A, the observed frequency is minimum.



Let the co-ordinates of point A be $(x,\ y)$, then the equation of the circle is given by

$$x^2+y^2=R^2 \qquad \ldots(1)$$

$$d\left(x^2+y^2\right)=0$$

$$\Rightarrow 2xdx+2ydy=0$$

$$\Rightarrow \left(-\frac{dy}{dx}\right)=\frac{x}{y}=\tan\theta=\frac{y}{2R-x}$$

$$\Rightarrow 2Rx-x^2=y^2$$

$$\Rightarrow 2Rx=x^2+y^2=R^2$$

$$\Rightarrow x=\frac{R}{2}$$

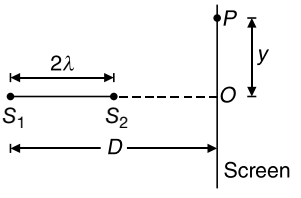
Substituting $x=\frac{R}{2}$ in (1), we get $y=\frac{\sqrt{3}}{2}R$

$$\Rightarrow A=\left(\frac{R}{2},\ \frac{\sqrt{3}}{2}R\right) \text{ and } B=\left(\frac{R}{2},\ -\frac{\sqrt{3}}{2}R\right)$$

(b) $f_{\max}=f_B=\left(\frac{v}{v-v_s}\right)f,\ f_{\min}=f_A=\left(\frac{v}{v+v_s}\right)f$

PROBLEM 11

Two coherent narrow slits emitting sound of wavelength λ in the same phase are placed parallel to each other at a small separation of 2λ. The sound is detected by moving a detector on the screen at a distance $D(\gg\lambda)$ from the slit S_1 as shown in figure. Find the distance y such that the intensity at P is equal to intensity at O.



SOLUTION

METHOD-I:

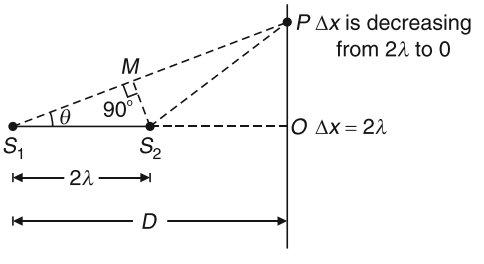
At point O on the screen the path difference between the sound waves reaching from S_1 and S_2 is 2λ (an even multiple of $\frac{\lambda}{2}$), so constructive interference is obtained at O. At a very large distance from point O on the screen the path difference is zero.

So, we can conclude that as we move away from point O on the screen path difference decreases from 2λ to zero. At O constructive interference is obtained (where $\Delta x=2\lambda$). So, next constructive interference will be obtained where $\Delta x=\lambda$. Hence,

$$S_1P-S_2P=\lambda$$

$$\Rightarrow \sqrt{D^2+y^2}-\sqrt{y^2+(D-2\lambda)^2}=\lambda$$

$$\Rightarrow \sqrt{D^2+y^2}-\lambda=\sqrt{y^2+(D-2\lambda)^2}$$



Squaring both sides, we get

$$D^2+y^2+\lambda^2-2\lambda\sqrt{D^2+y^2}=y^2+D^2+4\lambda^2-4\lambda D$$

$$\Rightarrow 2\sqrt{D^2+y^2}=4D-3\lambda$$

Since, $D \gg \lambda$, so we get

$$4D - 3\lambda \approx 4D$$

$$\Rightarrow \quad 2\sqrt{D^2 + y^2} = 4D$$

$$\Rightarrow \quad \sqrt{D^2 + y^2} = 2D$$

Again, squaring both sides, we get

$$D^2 + y^2 = 4D^2$$

$$\Rightarrow \quad y = \sqrt{3}D$$

METHOD-II:

Let $\Delta x = \lambda$ at angle θ as shown, so the path difference between the waves is

$$\Delta x = S_1M = 2\lambda\cos\theta$$

$$\Rightarrow \quad 2\lambda\cos\theta = \lambda \qquad \{\because\ \Delta x = \lambda\}$$

$$\Rightarrow \quad \theta = 60°$$

Now, $PO = S_1O\cot(30°)$

$$\Rightarrow \quad y = \sqrt{3}D$$

PROBLEM 12

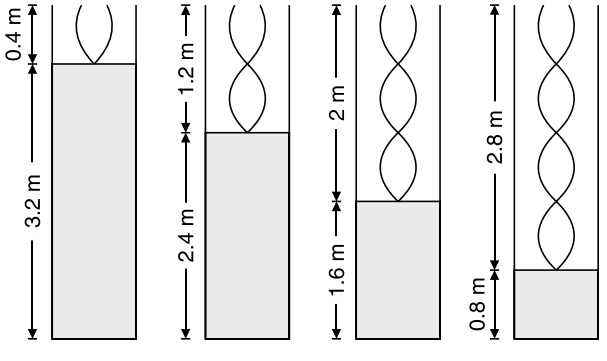
A 3.6 m long pipe resonates with a source of frequency 212.5 Hz when water level is at certain heights in the pipe. Find the heights of water level (from the bottom of the pipe) at which resonances occur. Neglect end correction. Now the pipe is filled to a height $H(= 3.6\ \text{m})$. A small hole is drilled very close to its bottom and water is allowed to leak. Obtain an expression for the rate of fall of water level in the pipe as a function of H. If the radii of the pipe and the hole are 2×10^{-2} m and 1×10^{-3} m respectively, calculate the time interval between the occurrence of first two resonances. Given that the speed of sound in air is 340 ms^{-1} and $g = 10\ \text{ms}^{-2}$.

SOLUTION

Speed of sound, $v = 340\ \text{ms}^{-1}$. Let l_0 be the length of air column corresponding to the fundamental frequency, then

$$\frac{v}{4l_0} = 212.5$$

$$\Rightarrow \quad l_0 = \frac{v}{4(212.5)} = \frac{340}{4(212.5)} = 0.4\ \text{m}$$



In closed pipe only odd harmonics are obtained. Now let l_1, l_2, l_3, l_4 etc., be the lengths corresponding to the 3rd harmonic, 5th harmonic, 7th harmonic, etc. Then

$$3\left(\frac{v}{4l_1}\right) = 212.5$$

$$\Rightarrow \quad l_1 = 1.2\ \text{m}$$

$$5\left(\frac{v}{4l_2}\right) = 212.5$$

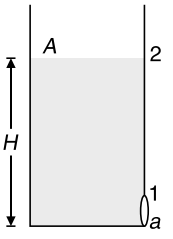
$$\Rightarrow \quad l_2 = 2\ \text{m}$$

$$7\left(\frac{v}{4l_3}\right) = 212.5$$

$$\Rightarrow \quad l_3 = 2.8\ \text{m and } 9\left(\frac{v}{4l_4}\right) = 212.5$$

$$\Rightarrow \quad l_4 = 3.6\ \text{m}$$

So, the heights of corresponding water levels are $(3.6-0.4)$ m, $(3.6-1.2)$ m, $(3.6-2)$ m and $(3.6-2.8)$ m Heights of corresponding water levels are 3.2 m, 2.4 m, 1.6 m and 0.8 m.



Let A and a be the area of cross-sections of the pipe and hole respectively. Then

$$A = \pi(2\times10^{-2})^2 = 1.26\times10^{-3}\ \text{m}^2$$

and $\quad a = \pi(10^{-3})^2 = 3.14\times10^{-6}\ \text{m}^2$

Since, velocity of efflux, $v = \sqrt{2gH}$

Applying Equation of Continuity at (1) and (2), we get

$$a\sqrt{2gH} = A\left(\frac{-dH}{dt}\right)$$

So, rate of fall of water level in the pipe is given by

$$\left(\frac{-dH}{dt}\right) = \frac{a}{A}\sqrt{2gH}$$

Substituting the values, we get

$$-\frac{dH}{dt} = \frac{3.14\times10^{-6}}{1.26\times10^{-3}}\sqrt{2\times10\times H}$$

$$\Rightarrow \quad -\frac{dH}{dt} = (1.11\times10^{-2})\sqrt{H}$$

$$\Rightarrow \quad \frac{dH}{\sqrt{H}} = -(1.11\times10^{-2})dt$$

Between first two resonances, the water level falls from 3.2 m to 2.4 m.

$$\Rightarrow \quad \int_{3.2}^{2.4}\frac{dH}{\sqrt{H}} = -(1.11\times10^{-2})\int_0^t dt$$

$\Rightarrow \quad 2\left(\sqrt{2.4}-\sqrt{3.2}\right)=-\left(1.11\times10^{-2}\right)t$

$\Rightarrow \quad t\approx 43$ second

PROBLEM 13

A train approaching a hill at a speed of 40 kmh^{-1} sounds a whistle of frequency 580 Hz when it is at a distance of 1 km from a hill. A wind with a speed of 40 kmh^{-1} is blowing in the direction of motion of the train. Find the

(a) frequency of the whistle as heard by an observer on the hill and
(b) distance from the hill at which the echo from the hill is heard by the driver and its frequency.

Given: Velocity of sound in air $v=1200$ kmh^{-1}.

SOLUTION

(a) Since, $f'=\left(\dfrac{v+w}{v+w-v_s}\right)f$

$\Rightarrow \quad f'=\left(\dfrac{1200+40}{1200+40-40}\right)(580)\approx 599$ Hz

(Also note that all velocities are in kmph, so do not bother to convert them to ms^{-1})

(b) For echo to be heard by the driver, the source is to be considered at the hill having frequency 599 Hz and the listener approaching with a speed of 40 kmhr^{-1} and the wind blowing at a speed of 40 kmhr^{-1} opposite to the direction of the waves. The frequency f'' is given by

$$f''=\left(\frac{1200-40+40}{1200-40}\right)(599)\approx 620 \text{ Hz}$$

Time taken by the wave to reach the hill,

$$t_1=\frac{1}{1200+40}=\frac{1}{1240} \text{ hr}$$

During this time the train moves towards the hill by a distance of

$$x=40\times\frac{1}{1240} \text{ km}=\frac{1}{31} \text{ km}$$

So, the distance between train and hill now is

$$\left(1-\frac{1}{31}\right)=\frac{30}{31} \text{ km}$$

Let the echo be heard after t hr after this instant, then

$$40t+(1200-40)t=\frac{30}{31}$$

$\Rightarrow \quad t=\dfrac{1}{1240}$ hr

The distance travelled by the train in this time is

$$x'=40\times\frac{1}{1240}=\frac{1}{31} \text{ km}$$

Therefore, the echo is heard by the driver when the train has moved a total distance

$$s=x+x'=\frac{1}{31}+\frac{1}{31}=\frac{2}{31} \text{ km}$$

So, distance from the hill is

$$l=1-s=1-\frac{2}{31}=0.935 \text{ km}$$

PROBLEM 14

Two sound sources driven in phase by the same amplifier are 2 m apart on the y-axis. At a point a very large distance from the y-axis, constructive interference is first heard at an angle $\theta_1=0.14$ rad with the x-axis and is next heard at $\theta_2=0.283$ rad. What is the

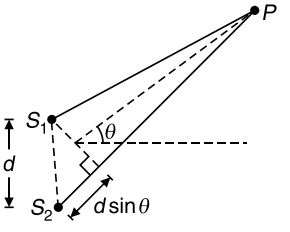
(a) wavelength of the sound waves from the sources?
(b) frequency of the sources?
(c) smallest angle for which the sound waves cancel?

Take speed of sound in air as 340 ms^{-1}.

SOLUTION

(a) At a point for away from the sources the path difference is

$$\Delta x=d\sin\theta$$



First constructive interference is obtained where

$$\Delta x=\lambda$$

$\Rightarrow \quad d\sin\theta_1=\lambda$ …(1)

Next constructive interference is obtained when, we get

$$\Delta x=2\lambda$$

$\Rightarrow \quad d\sin\theta_2=2\lambda$ …(2)

Subtracting Equation (1) from (2), we get

$$\lambda=d\left(\sin\theta_2-\sin\theta_1\right)$$

Substituting $d=2$ m,

$$\theta_1=0.14 \text{ rad}=8°$$

and $\theta_2=0.283$ rad $=16.21°$

we get, $\lambda=0.279$ m

(b) Frequency of the sources

$$f=\frac{v}{\lambda}=\frac{340}{0.279}=1219 \text{ Hz}$$

(c) The smallest angle where the sound waves cancel each other i.e., interference destructively is where, we have

$$\Delta x = d\sin\theta = \frac{\lambda}{2}$$

$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{\lambda}{2d}\right) = \sin^{-1}\left(\frac{0.279}{2\times 2}\right)$$

$$\Rightarrow \quad \theta = 4^\circ = 0.07 \text{ rad}$$

PROBLEM 15

A thin string is held at one end and oscillates vertically so that, $y(x=0,\, t) = 8\sin 4t\,(\text{cm})$. Neglect the gravitational force. The string's linear mass density is $0.2\ \text{kgm}^{-1}$ and its tension is 1 N. The string passes through a bath filled with 1 kg water. Due to friction heat is transferred to the bath. The heat transfer efficiency is 50%. Calculate how much time passes before the temperature of the bath rises one degree kelvin.

SOLUTION

Comparing the given equation with equation of a travelling wave,

$y = A\sin(kx \pm \omega t)$ at $x = 0$ we get

$A = 8\text{ cm} = 8\times 10^{-2}$ m

$\omega = 4\ \text{rads}^{-1}$

Speed of travelling wave,

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{1}{0.2}} = 2.236\ \text{ms}^{-1}$$

Since, $\mu = \dfrac{m}{l} = \rho S = 0.2\ \text{kgm}^{-1}$

where S is area of cross-section of string
The average power over a period is,

$$P = \frac{1}{2}(\rho S)\omega^2 A^2 v$$

Substituting the values, we get

$$P = \frac{1}{2}(0.2)(4)^2\left(8\times 10^{-2}\right)^2(2.236)$$

$$\Rightarrow \quad P = 2.29\times 10^{-2}\ \text{Js}^{-1}$$

The power transferred to the bath is,

$$P' = 0.5P = 1.145\times 10^{-2}\ \text{Js}^{-1}$$

To raise the temperature of 1 kg water by 1 degree kelvin, let the time taken be t, then

$$P't = mc\Delta T$$

where, c =specific heat of water $= 4.2\times 10^3\ \text{Jkg}^{-1}\text{K}^{-1}$

$$\Rightarrow \quad t = \frac{mc\Delta T}{P'} = \frac{(1)\left(4.2\times 10^{-3}\right)(1)}{1.145\times 10^{-2}}$$

$$\Rightarrow \quad t = 3.6\times 10^5\ \text{s} \approx 4.2\ \text{day}$$

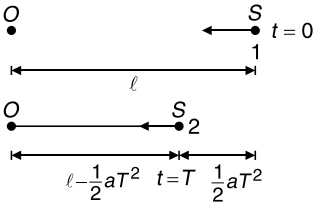
PROBLEM 16

A source emitting a sound of frequency f is placed at a large distance from an observer. The source starts moving towards the observer with a uniform acceleration a. Find the frequency heard by the observer corresponding to the wave emitted just after the source starts. The speed of sound in the medium is v.

SOLUTION

Suppose at $t = 0$, distance between source and observer is l. First wave pulse (say p_1) is emitted at this instant. This pulse will reach the observer after a time

$$t_1 = \frac{l}{v} \qquad \ldots(1)$$



Source will emit the next pulse (say p_2) after a time $T\left(=\dfrac{1}{f}\right)$. During this time the source will move a distance $\dfrac{1}{2}aT^2$ towards the observer. This pulse p_2 will reach the observer in a time

$$t_2 = T + \frac{\left(l - \frac{1}{2}aT^2\right)}{v} \qquad \ldots(2)$$

The changed time period as observed by the observer is

$$T' = t_2 - t_1 = T + \frac{l}{v} - \frac{1}{2}\frac{aT^2}{v} - \frac{l}{v}$$

Substituting $T' = \dfrac{1}{f'}$ and $T = \dfrac{1}{f}$ in the above equation, we get

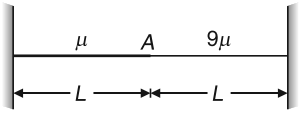
$$f' = \frac{2vf^2}{2vf - a}$$

Conceptual Note(s)

We have written $t_2 = T + \dfrac{\left(\ell - \frac{1}{2}aT^2\right)}{v}$, because the pulse p_2 is emitted after time T and then it takes a time $\dfrac{\left(\ell - \frac{1}{2}aT^2\right)}{v}$ to reach the observer. So, it reaches the observer in a time $T + \dfrac{\left(\ell - \frac{1}{2}aT^2\right)}{v}$.

PROBLEM 17

A light string is tied at one end to a fixed support and to a heavy string of equal length L at the other end as shown in figure. Mass per unit length of the strings are μ and 9μ and the tension is T. Find the possible values of frequencies such that point A is a node/antinode.



SOLUTION

When A is a Node

Let n_1 and n_2 be the number of complete loops formed on left and right side of point A, then

$$f_1 = f_2$$

$$\Rightarrow \quad n_1\left(\frac{v_1}{2L}\right) = n_2\left(\frac{v_2}{2L}\right), \text{ where } v \propto \frac{1}{\sqrt{\mu}}$$

$$\Rightarrow \quad \frac{n_1}{n_2} = \left(\frac{v_2}{v_1}\right) = \sqrt{\frac{\mu_1}{\mu_2}} = \frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \ldots, \text{etc.}$$

So, the possible frequencies are,

$$\frac{v_1}{2L}, 2\left(\frac{v_1}{2L}\right), \frac{3v_1}{2L}, \ldots, \text{etc.} \qquad \left\{\because v_1 = \sqrt{\frac{T}{\mu}}\right\}$$

OR $$\frac{1}{2L}\sqrt{\frac{T}{\mu}}, \frac{1}{L}\sqrt{\frac{T}{\mu}}, \frac{3}{2L}\sqrt{\frac{T}{2\mu}}, \ldots, \text{etc.}$$

When A is an Antinode

Let n_1 and n_2 be the number of complete loops on left and right side of point A, then

$$n_1\left(\frac{\lambda_1}{2}\right) + \frac{\lambda_1}{4} = L$$

$$\Rightarrow \quad f_1 = \frac{v_1}{L}\left(\frac{n_1}{2} + \frac{1}{4}\right) \qquad \{\because v = f\lambda\}$$

and $$n_2\left(\frac{\lambda_2}{2}\right) + \frac{\lambda_2}{4} = L$$

$$\Rightarrow \quad f_2 = \frac{v_2}{L}\left(\frac{n_2}{2} + \frac{1}{4}\right)$$

Substituting, $f_1 = f_2$ we get

$$\frac{2n_1 + 1}{2n_2 + 1} = \frac{1}{3}$$

This condition is met for

$$n_1 = 1, n_2 = 4$$

$$n_1 = 2, n_2 = 7$$

$$n_1 = 3, n_2 = 10 \text{ etc.}$$

$$\ldots \quad \ldots \quad \ldots$$

Therefore, the possible frequencies are

$$\frac{v_1}{L}\left(\frac{1}{2} + \frac{1}{4}\right), \frac{v_1}{L}\left(\frac{2}{2} + \frac{1}{4}\right), \frac{v_1}{L}\left(\frac{3}{2} + \frac{1}{4}\right), \ldots, \text{etc.}$$

OR $$\frac{3}{4L}\sqrt{\frac{T}{\mu}}, \frac{5}{4L}\sqrt{\frac{T}{\mu}}, \frac{7}{4L}\sqrt{\frac{T}{\mu}}, \ldots \text{ etc.}$$

PROBLEM 18

A string of linear mass density 5×10^{-3} kgm^{-1} is stretched under a tension of 65 N between two rigid supports 60 cm apart.

(a) If the string is vibrating in its second overtone so that the amplitude at one of its antinodes is 0.25 cm, what are the maximum transverse speed and acceleration of the string at antinodes?

(b) What are these quantities at a distance 5 cm from an node?

SOLUTION

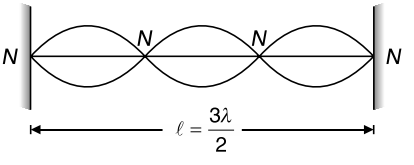
(a) In second overtone $l = \frac{3\lambda}{2}$

$$\Rightarrow \quad \lambda = \frac{2l}{3} = \frac{2\times60}{3} = 40 \text{ cm} = 0.4 \text{ m}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.4} = 5\pi \text{ m}^{-1}$$

Since, $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{65}{5\times10^{-3}}} = 114 \text{ ms}^{-1}$

$$\Rightarrow \quad \omega = kv = 570\pi \text{ rads}^{-1}$$



Maximum transverse speed at antinode is $v_{max} = A_0\omega$, where A_0 is amplitude of antinode

$$A_0 = 0.25 \text{ cm} = 2.5\times10^{-3} \text{ m}$$

So, maximum speed is

$$v_{max} = (2.5\times10^{-3})(570\pi) \text{ ms}^{-1} = 4.48 \text{ ms}^{-1}$$

Maximum acceleration is $a_{max} = \omega^2 A_0$

$$\Rightarrow \quad a_{max} = (570\pi)^2(2.5\times10^{-3}) \text{ ms}^{-2}$$

$$\Rightarrow \quad a_{max} = 8\times10^3 \text{ ms}^{-2}$$

(b) At a distance x from the node, the amplitude is

$$A = A_0 \sin(kx) = (2.5\times10^{-3})\sin(5\pi x) \text{ metre}$$

where x is in metre.

At $x = 5 \text{ cm} = 5\times10^{-2}$ m, we have

$$A = (2.5\times10^{-3})\sin(5\pi\times5\times10^{-2})$$

$\Rightarrow \quad A = 1.8\times10^{-3}$ m

So, maximum speed is $v_{max} = A\omega$

$\Rightarrow \quad v_{max} = (1.8\times10^{-3})(570\pi)$ ms^{-1} $= 3.22$ ms^{-1}

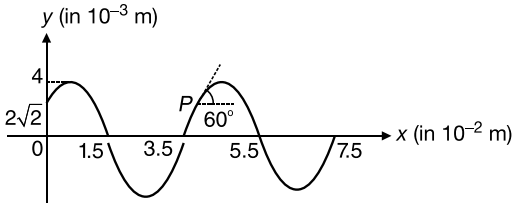
Also, maximum acceleration is $a_{max} = \omega^2 A$

$\Rightarrow \quad a_{max} = (570\pi)^2 (1.8\times10^{-3})$ ms^{-2}

$\Rightarrow \quad a_{max} = 5.8\times10^{3}$ ms^{-2}

PROBLEM 19

The figure shows a snapshot of a vibrating string at $t = 0$. The particle P is observed moving up with velocity $20\sqrt{3}$ cms^{-1}. The tangent at P makes an angle $60°$ with x-axis. Calculate the



(a) direction in which the wave is moving
(b) equation of the wave
(c) total energy carried by the wave per cycle of the string, assuming that μ, the mass per unit length of the string = 50 gm^{-1}.

SOLUTION

(a) Since, $v_P = -v\left(\frac{dy}{dx}\right)$. Also v_p and slope at P both are positive, v must be negative. Hence, wave is moving along negative x-axis.

(b) Since, $y = A\sin(\omega t - kx + \phi)$ …(1)

and $k = \frac{2\pi}{\lambda} = \frac{\pi}{2}$ cm^{-1}, $A = 4\times10^{-3}$ m $= 0.4$ cm

At $t = 0$, $x = 0$, $y = 2\sqrt{2}\times10^{-3}$ m $= 0.2\sqrt{2}$ cm

Substituting in equation (1), we get

$$\phi = \frac{\pi}{4} \text{ or } \frac{3\pi}{4}$$

Also, at $t = 0$, $x = 1.5$ cm, $y = 0$

Substituting in equation (1), we get

$$\phi = \frac{3\pi}{4}$$

Further, $20\sqrt{3} = -v\tan(60°)$

$\Rightarrow \quad v = -20$ cms^{-1}

$\Rightarrow \quad f = \frac{v}{\lambda} = 5$ Hz

$\Rightarrow \quad \omega = 2\pi f = 10\pi$

$\Rightarrow \quad y = 0.4\sin\left(10\pi t - \frac{\pi}{2}x + \frac{3\pi}{4}\right)$

(c) Since, $P = 2\pi^2 A^2 f^2 \mu v$

So, energy carried per cycle is

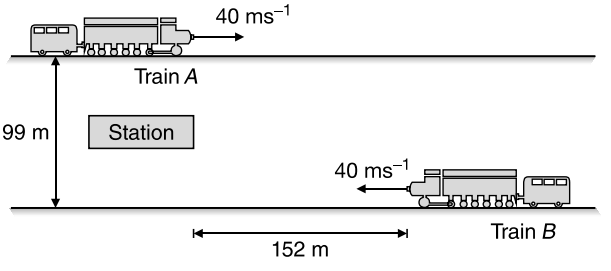
$$E = PT = \frac{P}{f} = 2\pi^2 A^2 f \mu v$$

Substituting the values, we get

$$E = 1.6\times10^{-5} \text{ J}$$

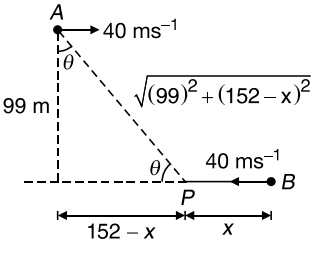
PROBLEM 20

Train A crosses a station with a speed of 40 ms^{-1} and whistles a short pulse of natural frequency 596 Hz. Another train B is approaching towards the same station with the same speed along a parallel track. Two tracks are 99 m apart. When train A whistles, the train B is 152 m away from the station as shown in Figure. If velocity of sound in air is 330 ms^{-1}, calculate the frequency of the pulse heard by driver of train B.



SOLUTION

From Figure if driver of train B receives the sound at point P as shown in Figure.



So, time taken by sound to go from A to P equals the time taken by the train to go from B to P i.e.,

$$\frac{\sqrt{(99)^2 + (152-x)^2}}{330} = \frac{x}{40}$$

$\Rightarrow \quad (4)^2\left[(99)^2 + (152)^2 + x^2 - 304x\right] = (33)^2 x^2$

$\Rightarrow \quad 1073x^2 + 4864x - 526480 = 0$

$$\Rightarrow \quad x = \frac{-4864 + \sqrt{(4864)^2 + 4(1073)(526480)}}{2(1073)}$$

$$\Rightarrow \quad x = \frac{-4864 + 47784}{2(1073)} = 20 \text{ m}$$

$$\Rightarrow \quad \cos\theta = \frac{132}{\sqrt{(99)^2+(132)^2}} = \frac{132}{165} = \frac{4}{5} = 0.8$$

Apparent frequency is given by

$$f_{app} = 596\left(\frac{330+40\cos\theta}{330-40\cos\theta}\right)$$

$$\Rightarrow \quad f_{app} = 596\left(\frac{330+32}{330-32}\right) = 724 \text{ Hz}$$

PROBLEM 21

A wire of length 2 m is fixed at both ends and is vibrating in its fundamental mode. The tension in the wire is 40 N and the mass of the wire is 0.1 kg. At the midpoint of the wire the amplitude is 2 cm.

(a) Find the maximum kinetic energy of the wire.

(b) At the instant the transverse displacement is given by $(0.02\text{ m})\sin\left(\frac{\pi x}{2}\right)$, what is the kinetic energy of the wire.

(c) At what position on the wire does the kinetic energy per unit length have its largest value?

(d) Where does the potential energy per unit length have its maximum value?

SOLUTION

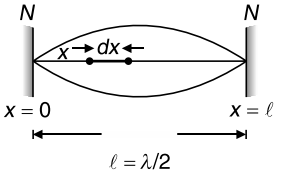
(a) In fundamental mode, $l = \frac{\lambda}{2}$

$\Rightarrow \quad \lambda = 2l = 4$ m

$\Rightarrow \quad k = \frac{2\pi}{\lambda} = \frac{\pi}{2}$ m^{-1}

Since, $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{40}{0.1/2}} = 28.3$ ms^{-1}

$\Rightarrow \quad \omega = vk = 44.5$ rads^{-1}



At the instant the transverse displacement is given by $R = (0.02\text{ m})\sin\left(\frac{\pi x}{2}\right)$, we have

$$R = A\sin(kx) = 2\times10^{-2}\sin\left(\frac{\pi x}{2}\right) \text{ m}$$

Maximum kinetic energy of an element dx is

$dK = \frac{1}{2}(dm)(R^2)(\omega^2)$, where $dm = \mu dx$

So, maximum kinetic energy of the wire is

$$K_{max} = \int dK = \int_0^1 \frac{1}{2}(\mu dx)R^2\omega^2$$

$$\Rightarrow \quad K_{max} = \frac{1}{2}A^2\omega^2\mu\int_0^1 \sin^2\left(\frac{\pi x}{2}\right)dx$$

$$\Rightarrow \quad K_{max} = \frac{1}{4}A^2\omega^2\mu\int_0^1 [1-\cos(\pi x)]dx$$

$$\Rightarrow \quad K_{max} = 9.9\times10^{-3}\int_0^2 [1-\cos(\pi x)]dx$$

$$\Rightarrow \quad K_{max} = 9.9\times10^{-3}\left[x-\frac{1}{\pi}\sin(\pi x)\right]_0^2$$

$$\Rightarrow \quad K_{max} \approx 0.02 \text{ J}$$

(b) At the given instant, we have

$$y = 0.02\sin\left(\frac{\pi x}{2}\right)$$

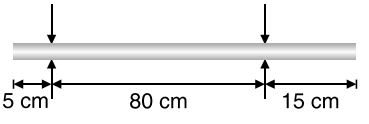
We observe that at, $y = 0$ at $x = 0$
and $y = 0.02$ m at $x = 1$ m
i.e., at the centre (or at the position of antinode) $y = 0.02$ m (the amplitude at that position). This implies that all the particles are at their extreme positions. Hence, the kinetic energy of the string is zero.

(c) Kinetic energy per unit length have its largest value at antinode or at $x = 1$ m.

(d) Potential energy per unit length have also its largest value at antinode or at $x = 1$ m

PROBLEM 22

A metal rod of length 1 m is clamped at two points as shown in the figure. Distance of the clamp from the two ends are 5 cm and 15 cm respectively. Find the minimum and next higher frequency of natural longitudinal oscillation of the rod. Given that Young's modulus of elasticity and density of aluminium are $Y = 1.6\times10^{11}$ Nm^{-2} and $\rho = 2500$ kgm^{-3} respectively.



SOLUTION

Speed of longitudinal waves in the rod

$$v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{1.6\times10^{11}}{2500}} = 8000 \text{ ms}^{-1}$$

At the clamped position, nodes will be formed. Between the clamps integral number of loops will be formed. Hence,

$$n_1\left(\frac{\lambda}{2}\right) = 80$$

$$\Rightarrow \quad n_1\lambda = 160 \qquad \ldots(1)$$

Between P and R, P is a fixed end and R is the free end. It means the number of loops between P and R will be an odd multiple of $\frac{\lambda}{4}$. So,

$$(2n_2 - 1)\frac{\lambda}{4} = 5$$

$$\Rightarrow \quad (2n_2 - 1)\lambda = 20 \qquad \ldots(2)$$

Also, between Q and S, we have

$$(2n_3 - 1)\frac{\lambda}{4} = 15$$

$$\Rightarrow \quad (2n_3 - 1)\lambda = 60 \qquad \ldots(3)$$

From Equations (1) and (2), we get

$$\frac{n_1}{2n_2 - 1} = \frac{160}{20} = 8 \qquad \ldots(4)$$

and from Equations (1) and (3), we get

$$\frac{n_1}{2n_3 - 1} = \frac{160}{60} = \frac{8}{3} \qquad \ldots(5)$$

For minimum frequency n_1, n_2 and n_3 should be least from Equations (4) and (5)

So, we get, $n_1 = 8$, $n_2 = 1$, $n_3 = 2$

$$\Rightarrow \quad \lambda = \frac{20}{2n_2 - 1} = 20 \text{ cm} \qquad \{\text{from Equation (2)}\}$$

$$\Rightarrow \quad \lambda = 0.2 \text{ m}$$

$$\Rightarrow \quad f_{\min} = \frac{v}{\lambda} = \frac{8000}{0.2} = 40 \text{ kHz}$$

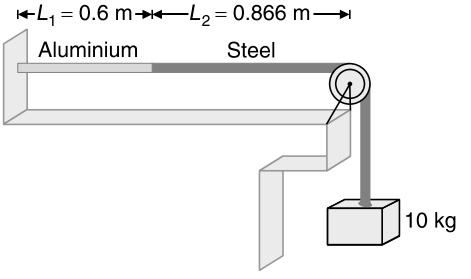
Next higher frequency corresponds to

$$n_1 = 24,\ n_2 = 2 \text{ and } n_3 = 5$$

$$f = 120 \text{ kHz}$$

PROBLEM 23

An aluminium wire of length $L = 60$ cm and of cross-sectional area $0.01\ \text{cm}^2$ is connected to a steel wire of the same cross-sectional area. The compound wire is loaded with a block of mass 10 kg, as shown in the figure, so that the distance L_2, from the joint to the supporting pulley is 86.6 cm Transverse waves are set up in the composite wire by using an external source of variable frequency. Density of aluminium is $2.6\ \text{gcm}^{-3}$ and density of steel is $7.8\ \text{gcm}^{-3}$.



(a) Find the lowest frequency of excitation for which standing waves are observed such that the joint in the wire is a node.

(b) What is total number of nodes observed at this frequency excluding the two at the ends of the wire?

SOLUTION

The distance between two consecutive nodes is equal to $\lambda/2$. The whole wire vibrates with the frequency of the tuning fork. The velocity of the wave in each part of the wire is $\sqrt{T/\mu}$. Since the mass per unit length in both the wires are different, so velocity of the wave in two parts of the wire is also different. Hence the wavelength is different in both parts of the wire (because frequency cannot be different)

Area of cross-section, $A = 0.01\ \text{cm}^2 = 10^{-6}\ \text{m}^2$

For aluminium wire, $\rho_1 = 2.6\ \text{gcm}^{-3} = 2.6 \times 10^3\ \text{kgm}^{-3}$

So, mass per unit length is

$$\mu_1 = \rho_1 A = (2.6 \times 10^3)(10^{-6}) = 2.6 \times 10^{-3}\ \text{kgm}^{-1}$$

For steel wire, $\rho_2 = 7.8\ \text{gcm}^{-3} = 7.8 \times 10^3\ \text{kgm}^{-3}$

So, mass per unit length is

$$\mu_2 = \rho_2 A = 7.8 \times 10^3 \times 10^{-6} = 7.8 \times 10^{-3}\ \text{kgm}^{-1}$$

Let the aluminium wire vibrate in n_1 harmonic, and the steel wire in n_2 harmonic. Then

For Aluminium Wire

$$n_1\left(\frac{\lambda_1}{2}\right) = L_1 \text{ i.e., } n_1\lambda_1 = 2L_1$$

$$\Rightarrow \quad f_1 = \frac{v_1}{\lambda_1} = \frac{n_1}{2L_1}\sqrt{\frac{T_1}{\mu_1}}$$

For Steel Wire

$$n_2\left(\frac{\lambda_2}{2}\right) = L_2 \text{ i.e., } n_2\lambda_2 = 2L_2$$

$$\Rightarrow \quad f_2 = \frac{v_2}{\lambda_2} = \frac{n_2}{2L_2}\sqrt{\frac{T_2}{\mu_2}}$$

Since tension $T_1 = T_2$ and whole wire vibrates with same frequency, so we have

$$f_1 = f_2$$

$$\Rightarrow \quad \frac{n_1}{2L_1}\sqrt{\frac{T}{\mu_1}} = \frac{n_2}{2L_2}\sqrt{\frac{T}{\mu_2}}$$

$$\Rightarrow \quad \frac{n_1}{2\times 0.6}\sqrt{\frac{T}{2.6\times 10^{-3}}} = \frac{n_2}{2\times 86.6}\sqrt{\frac{T}{7.8\times 10^{-3}}}$$

$$\Rightarrow \quad \frac{n_1}{n_2} = \frac{2}{5}$$

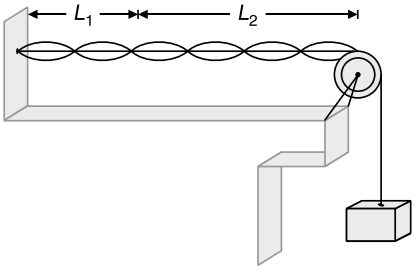
Therefore, the lowest frequency of excitation causes,

$n_1 = 2$ and $n_2 = 5$

(a) Now, the lowest frequency of excitation is

$$f = \frac{n_1}{2L_1}\sqrt{\frac{T}{\mu_1}} = \frac{2}{2\times 0.6}\sqrt{\frac{10\times 9.81}{2.6\times 10^{-3}}} = 323.74 \text{ Hz}$$

(b) The vibrating looks as shown in the figure



Therefore, total number of nodes observed is 6.

PROBLEM 24

The air column in a pipe closed at one end is made to vibrate in its second overtone by tuning fork of frequency 440 Hz. The speed of sound in air is 330 ms^{-1}. End corrections may be neglected. Let P_0 denote the mean pressure at any point in the pipe and ΔP_0 the maximum amplitude of pressure variation.

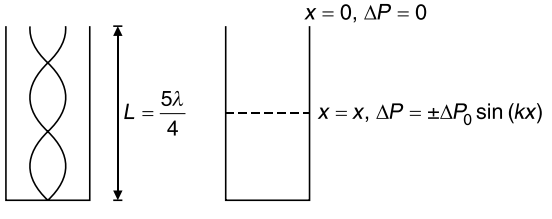
(a) Find the length L of the air column.
(b) What is the amplitude of pressure variation at the middle of the column?
(c) What are the maximum and minimum pressures at the open end of the pipe?
(d) What are the maximum and minimum pressures at the closed end of the pipe?

SOLUTION

(a) Frequency of second overtone of the closed pipe

$$f_5 = 5\left(\frac{v}{4L}\right) = 440 \text{ Hz} \qquad \{\text{Given}\}$$

$$\Rightarrow \quad L = \frac{5v}{4\times 440} \text{ m}$$



Substituting v = speed of sound in air = 330 ms^{-1}

$$\Rightarrow \quad L = \frac{5\times 330}{4\times 440} = \frac{15}{16} \text{ m}$$

$$\Rightarrow \quad \lambda = \frac{4L}{5} = \frac{4\left(\frac{15}{16}\right)}{5} = \frac{3}{4} \text{ m}$$

(b) Since, open end is a displacement antinode, therefore, it would be a pressure node. So, at $x = 0$; $\Delta P = 0$
Hence, pressure amplitude at $x = x$ can be written as

$$\Delta P = \pm \Delta P \sin(kx)$$

where $k = \frac{2\pi}{\lambda} = \frac{2\pi}{3/4} = \frac{8\pi}{3}$ m^{-1}

Therefore, pressure amplitude at

$x = \frac{L}{2} = \frac{\frac{15}{16}}{2}$ m $= \frac{15}{32}$ m will be given by

$$\Delta P = \pm \Delta P_0 \sin\left(\frac{8\pi}{3}\right)\left(\frac{15}{32}\right) = \pm \Delta P_0 \sin\left(\frac{5\pi}{4}\right)$$

$$\Rightarrow \quad \Delta P = \pm \frac{\Delta P_0}{\sqrt{2}}$$

(c) Open end is a pressure node, i.e., $\Delta P = 0$

Hence, $P_{\max} = P_{\min}$ = Mean pressure (P_0)

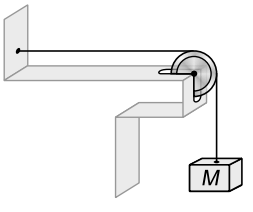
(d) Closed end is a displacement node or pressure antinode, so we have

$$P_{\max} = P_0 + \Delta P_0$$

$$P_{\min} = P_0 - \Delta P_0$$

PROBLEM 25

A string of length 1 m fixed at one end and on the other end a block of mass $M = 4$ kg is suspended. The string is set into vibrations and represented by equation $y = 6\sin\left(\frac{\pi x}{10}\right)\cdot\cos 100\pi t$ where x and y are in cm and t in seconds.



(a) Find the number of loops formed in the string.
(b) Find the maximum displacement of a point at $x = \frac{5}{3}$ cm.
(c) Calculate maximum kinetic energy of the string.
(d) Write down the equations of the component waves whose superposition gives the wave.

SOLUTION

(a) Comparing the given equation with $y = A\sin(kx)\cos(\omega t)$, we get

$$k = \frac{2\pi}{\lambda} = \frac{\pi}{10}\ \text{cm}^{-1} = 10\pi(\text{m}^{-1})$$

$$\lambda = 20\ \text{cm} = 0.2\ \text{m}$$

$$\Rightarrow\ \frac{\lambda}{2} = 10\ \text{cm}$$

Number of loops is $p = \dfrac{l}{\frac{\lambda}{2}} = 10$

(b) $A(x) = 6\sin\left(\dfrac{\pi x}{10}\right)$

Maximum displacement at $x = \dfrac{5}{3}$ cm is

$$A(x) = 6\sin\left[\left(\frac{\pi}{10}\right)\left(\frac{5}{3}\right)\right] = 3\ \text{cm}$$

(c) Considering an elemental length dx of string at a distance x from the left end. Maximum kinetic energy of this element is given by

$$dK = \frac{1}{2}(\mu dx)\left(\frac{A}{2}\right)(\omega^2)$$

$$\Rightarrow\ dK = \frac{1}{2}(\mu dx)(A\sin kx)^2\,\omega^2$$

$$\Rightarrow\ dK = \frac{1}{2}\left(\mu A^2\sin^2 kx\right)\left(\frac{T}{\mu}k^2\right)dx \qquad \left\{\because\ \omega^2 = \frac{T}{\mu}k^2\right\}$$

$$\Rightarrow\ dK = \frac{1}{4}TA^2k^2(1-\cos 2kx)\,dx$$

So, total kinetic energy of the string is

$$K = \int_0^l dK$$

$$\Rightarrow\ K = \frac{1}{4}TA^2k^2\int_0^l (1-\cos(2kx))\,dx$$

$$\Rightarrow\ K = \frac{1}{4}TA^2k^2\left(x - \frac{\sin 2kx}{2}\right)\Bigg|_0^l$$

$$\Rightarrow\ K = \frac{TA^2k^2l}{4}$$

Substituting the values, we get

$$K = \frac{(4\times 10)(6\times 10^{-2})^2(10\pi)^2(1)}{4} = 142.12\ \text{J}$$

(d) The equations of component waves are given by

$$y_1 = 3\sin\left(\frac{\pi x}{10} - 100\pi t\right)$$

and $y_2 = 3\sin\left(\dfrac{\pi x}{10} + 100\pi t\right)$

PROBLEM 26

The linear mass density of a non-uniform wire under constant tension decreases gradually along the wire so that an incident wave is transmitted without reflection. The wire is uniform for $-\infty < x \le 0$. In this region, a transverse wave has the form $y(x, t) = 0.003\cos(25x - 50t)$ where y and x are in metres and t is in seconds. From $x = 0$ to $x = 20$ m the linear mass density decreases gradually from μ_1 to $\mu_1/4$. For $20 < x \le \infty$, the linear mass density is $\mu = \mu_1/4$. Calculate

(a) the wave velocity for large value of x.
(b) amplitude of the wave for large value of x.
(c) $y(x, t)$ for $x > 20$ m.

SOLUTION

(a) Speed of wave for $x = 0$ is, $\dfrac{\omega}{k} = \dfrac{50}{25} = 2\ \text{ms}^{-1}$

Since, $v = \sqrt{\dfrac{T}{\mu}}$ i.e., $v \propto \dfrac{1}{\sqrt{\mu}}$

For $x > 20$ m, μ becomes $\dfrac{1}{4}$th its value at $x = 0$. Hence, speed of wave will become two times.
So, for $x > 20$ m, speed of wave will become 4 ms^{-1}.

(b) Equating the average power at $x = 0$ and for $x > 20$ m, we get

$$\frac{1}{2}\mu_1\omega^2A_1^2v_1 = \frac{1}{2}\left(\frac{\mu_1}{4}\right)\omega^2A_2^2v_2$$

$$\Rightarrow\ A_2 = 2A_1\sqrt{\frac{v_1}{v_2}} = 2\times 0.003\sqrt{\frac{2}{4}}$$

$$\Rightarrow\ A_2 = 0.0042\ \text{m}$$

(c) For $x > 20$ m, ω will remain same while k will become half so that speed becomes two times, because $v = \omega/k$. So, for $x > 20$ m, we have $A = 0.0042$ m, $k = \dfrac{25}{2} = 12.5\ \text{m}^{-1}$, $\omega = 50\ \text{rads}^{-1}$.

$$\Rightarrow\ y(x,\ t) = 0.0042\cos(12.5x - 50t)$$

PRACTICE EXERCISES

SINGLE CORRECT CHOICE TYPE QUESTIONS

This section contains Single Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. A transverse wave $y = 0.05\sin(20\pi x - 50\pi t)$ in metres, is propagating along +ve X-axis on a string. A light insect starts crawling on the string with the velocity of 5 cms^{-1} at $t = 0$ along the +ve X-axis from a point where $x = 5$ cm. After 5 s the difference in the phase of its position is equal to

(A) 150π (B) 250π
(C) -245π (D) -5π

2. A string under a tension of 100 N, emitting its fundamental note, gives 5 bps with a tuning fork. When the tension is increased to 121 N, again 5 beats per second are heard. The frequency of the fork is

(A) 105 N (B) 95 N
(C) 210 N (D) 190 N

3. Two tuning forks A and B give 5 bps. A resonates with a column of air 15 cm long, closed at one end, and B with a column 30.5 cm long, open at both ends. Neglecting end correction, the frequencies of A and B are respectively

(A) 300 Hz, 295 Hz (B) 295 Hz, 300 Hz
(C) 305 Hz, 300 Hz (D) 300 Hz, 305 Hz

4. A string fixed at both ends is vibrating in the lowest mode of vibration for which a point at quarter of its length from one end is a point of maximum displacement. The frequency of vibration in this mode is 100 Hz. What will be the frequency emitted when it vibrates in the next mode such that this point is again a point of maximum displacement?

(A) 400 Hz (B) 200 Hz
(C) 600 Hz (D) 300 Hz

5. The frequency of a sonometer wire is 100 Hz. When the weights producing the tensions are completely immersed in water the frequency becomes 80 Hz and on immersing the weights in a certain liquid the frequency becomes 60 Hz. The specific gravity of the liquid is

(A) 1.42 (B) 1.77
(C) 1.82 (D) 1.21

6. An organ pipe P_1, closed at one end and vibrating in its first overtone, and another pipe P_2 open at both ends and vibrating in its third overtone, are in resonance with a given tuning fork. The ratio of the length of P_1 to that of P_2 is

(A) $\frac{8}{3}$ (B) $\frac{3}{8}$
(C) $\frac{1}{2}$ (D) $\frac{1}{3}$

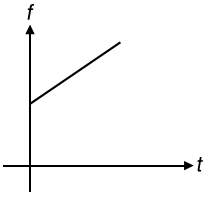
7. A tuning fork produces a wave of wavelength 110 cm in air at 0 °C. The wavelength at 25 °C would be

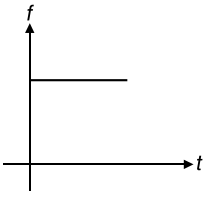
(A) 110 cm (B) 115 cm
(C) 120 cm (D) 130 cm

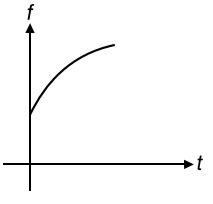
8. A 100 Hz sinusoidal wave is travelling in the positive x-direction along a string with a linear mass density of 3.5×10^{-3} kgm^{-1} and a tension of 35 N. At time $t = 0$, the point $x = 0$ has zero displacement and the slope of the string is $\frac{\pi}{20}$. Then select the wrong alternative

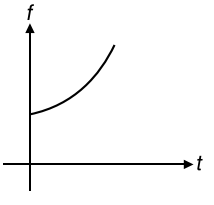
(A) Velocity of wave is 100 ms^{-1}
(B) Angular velocity is (200π) rads^{-1}
(C) Amplitude of wave is 0.025 m
(D) None of the above

9. An observer starts moving with uniform acceleration a towards a stationary sound source of frequency f_0. As the observer approaches the source, the apparent frequency f heard by the observer varies with time t as

(A) 

(B) 

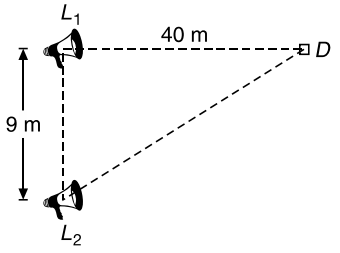
(C) 

(D) 

10. At $t = 0$, a transverse wave pulse in a wire is described by the function $y = \frac{6}{x^2 - 3}$ where x and y are in metre. The function $y(x, t)$ that describes this wave equation if it is travelling in the positive x direction with a speed of 4.5 ms^{-1} is

(A) $y = \frac{6}{(x + 4.5t)^2 - 3}$
(B) $y = \frac{6}{(x - 4.5t)^2 + 3}$
(C) $y = \frac{6}{(x + 4.5t)^2 + 3}$
(D) $y = \frac{6}{(x - 4.5t)^2 - 3}$

11. Two loudspeakers L_1 and L_2, driven by a common oscillator and amplifier, are arranged as shown. The frequency of the oscillator is gradually increased from zero and the detector at D records a series of maxima and minima. If the speed of sound is 330 ms^{-1} then the frequency at which the first maximum is observed is

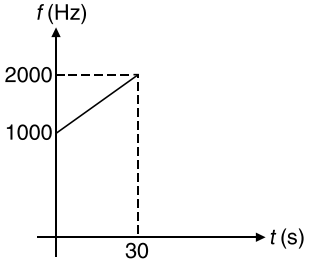


(A) 165 Hz (B) 330 Hz
(C) 495 Hz (D) 660 Hz

12. A glass tube 1 m long is filled with water. The water can be drained out slowly at the bottom of the tube. If a vibrating tuning fork of frequency 500 Hz is brought at the upper end of the tube and the velocity of sound is 300 ms^{-1}, then the total number of resonances obtained will be
(A) 4 (B) 3
(C) 2 (D) 1

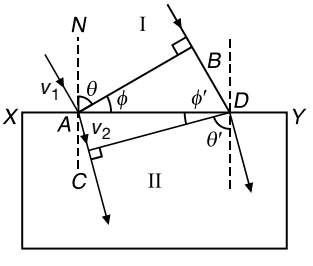
13. Two strings A and B, made of the same material, have same thickness. The length of A is half that of B while the tension on A is twice that on B. The ratio of the velocities of transverse waves in A and B is
(A) $\sqrt{2}:1$ (B) $2:1$
(C) $1:\sqrt{2}$ (D) $1:2$

14. A detector is released from rest over a source of sound of frequency $f_0 = 10^3$ Hz. The frequency observed by the detector at time t is plotted in the graph. The speed of sound in air is $\left(g = 10\ ms^{-2}\right)$



(A) 330 ms^{-1} (B) 350 ms^{-1}
(C) 300 ms^{-1} (D) 310 ms^{-1}

15. In the diagram a wavefront AB is incident from rarer Medium I to denser Medium II. Its position inside Medium II is CD. The ratio of velocity of sound in Medium I to Medium II i.e., $\frac{v_1}{v_2}$ is



(A) $\frac{\sin\phi}{\sin\theta}$ (B) $\frac{\sin\theta}{\sin\phi}$
(C) $\frac{BD}{AC}$ (D) $\frac{AB}{CD}$

16. A closed organ pipe and an open organ pipe of same length produce 2 beats when they are set into vibrations simultaneously in their fundamental mode. The length of open organ pipe is now halved and of closed organ pipe is doubled, the number of beats produced will be
(A) 8 (B) 7
(C) 4 (D) 2

17. A train is moving at 30 ms^{-1} in still air. The frequency of the locomotive whistle is 500 Hz and the speed of sound is 345 ms^{-1}. The apparent wavelengths of sound in front of and behind the locomotive are respectively
(A) 0.63 m, 0.80 m (B) 0.63 m, 0.75 m
(C) 0.60 m, 0.85 m (D) 0.60 m, 0.75 m

18. In PROBLEM 17, what would be the apparent wavelengths as heard by stationary listeners in front of and behind the locomotive if a wind of speed 10 ms^{-1} were blowing in the same direction as that in which the locomotive is travelling?
(A) 0.65 m, 0.73 m (B) 0.60 m, 0.73 m
(C) 0.65 m, 0.78 m (D) 0.60 m, 0.71 m

19. The amplitude of wave disturbance propagating in positive x-axis is given by $y = \frac{1}{1+x^2}$ at $t = 0$ and $y = \frac{1}{1+(x-1)^2}$ at $t = 2$ s, where x and y are in metres. The shape of the disturbance does not change during the propagation. The velocity of the wave is
(A) 1 ms^{-1} (B) 0.5 ms^{-1}
(C) 2 ms^{-1} (D) 4 ms^{-1}

20. Which of the following is not the standard form of a sine wave?
(A) $y = A\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$ (B) $y = A\sin(vt - kx)$
(C) $y = A\sin\omega\left(t - \frac{x}{v}\right)$ (D) $y = A\sin k(vt - x)$
where symbols have their usual meanings.

21. Two speakers connected to the same source of fixed frequency are placed 2.0 m apart in a box. A sensitive microphone placed at a distance of 4.0 m from their midpoint along the perpendicular bisector shows maximum response. The box is slowly rotated until the speakers are in line with the microphone. The distance between the midpoint of the speakers and the microphone remains unchanged. Exactly five maximum responses are observed in the microphone in doing this. The wavelength of the sound wave is
(A) 0.2 m (B) 0.4 m
(C) 0.6 m (D) 0.8 m

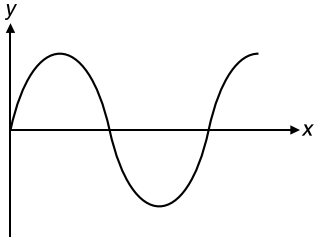
22. When beats are produced by two waves, viz., $y_1 = A\sin(1000\pi t)$ and $y_2 = A\sin(1008\pi t)$, the beat frequency will be
(A) 4 Hz (B) 8 Hz
(C) 4π Hz (D) 8π Hz

23. Two sound waves move in the same direction. If the average power transmitted across a cross section by them are equal while their wavelengths are in the ratio of 1 : 2. Their pressure amplitudes would be in the ratio of

(A) 1 (B) 2
(C) 4 (D) $\frac{1}{2}$

24. It is noticed that if the rate of clapping is 100 per minute, the original sound cannot be distinguished from the sound reflected from a wall. If the speed of sound is 330 ms^{-1}, the distance of wall from the man is
(A) 33 m (B) 99 m
(C) 132 m (D) 198 m

25. In a sine wave, position of different particles at time $t = 0$ is shown in figure. The equation for this wave if it is travelling along positive x-axis can be



(A) $y = A\sin(\omega t - kx)$ (B) $y = A\sin(kx - \omega t)$
(C) $y = A\cos(\omega t - kx)$ (D) $y = A\cos(kx - \omega t)$

26. A car, sounding a horn of frequency 1000 Hz, is moving directly towards a huge wall at a speed of15 ms^{-1}. If speed of sound is 340 ms^{-1}, then the frequency of the echo heard by the driver is
(A) 1046 Hz (B) 954 Hz
(C) 1092 Hz (D) 908 Hz

27. A source of frequency 10 kHz when vibrated over the mouth of a closed organ pipe is in unison at 300 K. The beats produced when temperature rises by 1 K is
(A) 30 Hz (B) 13.33 Hz
(C) 16.67 Hz (D) 40 Hz

28. A string of length $2l$, obeying Hooke's law, is stretched so that its extension is l. The speed of transverse waves in the string is v. If the string is further stretched so that the extension (including the previous extension) becomes $4l$, the speed of transverse waves in the string will be
(A) $\frac{v}{\sqrt{2}}$ (B) $\sqrt{2}v$
(C) $2v$ (D) $2\sqrt{2}v$

29. Two harmonic waves travelling in the same medium have frequency ratio 1 : 2 and intensity ratio 1 : 36. Their amplitude ratio is
(A) 1 : 3 (B) 1 : 6
(C) 1 : 8 (D) 1 : 72

30. A man standing in front of a mountain beats a drum at regular intervals. The rate of drumming is generally increased and he finds that the echo is not heard distinctly when the rate becomes 40 per minute. He then moves nearer to the mountain by 90 m and finds that echo is again not heard when the drumming rate becomes 60 per minute. The distance between the mountain and the initial position of the man is
(A) 205 m (B) 300 m
(C) 180 m (D) 270 m

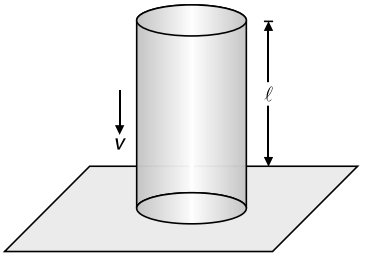
31. Two sounding bodies producing progressive waves given by $y_1 = 4\sin(400\pi t)$ and $y_2 = 3\sin(404\pi t)$, where t is in seconds, are situated near the ears of a person. The person will hear
(A) 2 beats per second with intensity ratio $\frac{4}{3}$ between maxima and minima.
(B) 2 beats per second with intensity ratio 49 between maxima and minima.
(C) 4 beats per second with intensity ratio 7 between maxima and minima.
(D) 4 beats per second with intensity ratio $\frac{4}{3}$ between maxima and minima.

32. The path difference between the two waves $y_1 = a_1\sin\left(\omega t - \frac{2\pi x}{\lambda}\right)$ and $y_2 = a_2\cos\left(\omega t - \frac{2\pi x}{\lambda} + \phi\right)$ is
(A) $\frac{\lambda}{2\pi}\phi$ (B) $\frac{\lambda}{2\pi}\left(\phi + \frac{\pi}{2}\right)$
(C) $\frac{2\pi}{\lambda}\left(\phi - \frac{\pi}{2}\right)$ (D) $\frac{2\pi}{\lambda}\phi$

33. The frequency of a radar is 780 MHz. When it is reflected from an approaching aeroplane, the apparent frequency is more than the actual frequency by 2.6 kHz. The speed of the aeroplane is
(A) 0.25 kms^{-1} (B) 0.5 kms^{-1}
(C) 1.0 kms^{-1} (D) 2.0 kms^{-1}

34. A string of mass 0.2 kgm^{-1} has length $l = 0.6$ m is fixed at both ends and stretched such that it has a tension of 80 N. The string vibrates in 3 segments with maximum amplitude of 0.5 cm. The maximum transverse velocity amplitude is
(A) 1.57 ms^{-1} (B) 6.28 ms^{-1}
(C) 3.14 ms^{-1} (D) 9.42 ms^{-1}

35. An open pipe of sufficient length is dipping in water with a speed v vertically. If at any instant l is length of tube above water then the rate at which fundamental frequency of pipe changes, is
(speed of sound = c)



(A) $\frac{cv}{2l^2}$ (B) $\frac{cv}{4l^2}$
(C) $\frac{c}{2v^2l^2}$ (D) $\frac{c}{4v^2l^2}$

36. In Melde's experiment, a string of length 0.8 m and mass 1.0 g vibrates in 4 segments when the tension in the string is 0.4 kgwt. The frequency of the fork is

(A) 70 Hz (B) 90 Hz
(C) 140 Hz (D) 180 Hz

37. In PROBLEM 36, the tension in kgwt required to make the string vibrate in 5 segments is

(A) $0.4\left(\frac{4}{5}\right)$ (B) $0.4\left(\frac{4}{5}\right)^2$
(C) $0.4\left(\frac{5}{4}\right)$ (D) $0.4\left(\frac{5}{4}\right)^2$

38. A string is under tension so that its length is increased by $\frac{1}{n}$ times its original length. The ratio of fundamental frequency of longitudinal vibrations and transverse vibrations will be

(A) $1:n$ (B) $n^2:1$
(C) $\sqrt{n}:1$ (D) $n:1$

39. Equations of a stationary and a travelling waves are as follows $y_1 = a\sin kx\cos\omega t$ and $y_2 = a\sin(\omega t - kx)$. The phase difference between two points $x_1 = \frac{\pi}{3k}$ and $x_2 = \frac{3\pi}{2k}$ are ϕ_1 and ϕ_2 respectively for the two waves. The ratio $\frac{\phi_1}{\phi_2}$ is

(A) 1 (B) $\frac{5}{6}$
(C) $\frac{3}{4}$ (D) $\frac{6}{7}$

40. The frequency of a sonometer wire is f. When the weights producing the tensions are completely immersed in water, then the frequency becomes $\frac{f}{2}$ and when the weights are immersed completely in a certain liquid frequency becomes $\frac{f}{3}$. The specific gravity of the liquid is

(A) $\frac{4}{3}$ (B) $\frac{16}{9}$
(C) $\frac{15}{12}$ (D) $\frac{32}{27}$

41. The equation of a travelling wave is given as $y = 5\sin[10\pi(t - 0.01x)]$ along the x-axis, where all quantities are in SI units. The phase difference between the points separated by a distance of 10 m along x-axis is

(A) $\frac{\pi}{2}$ (B) π
(C) 2π (D) $\frac{\pi}{4}$

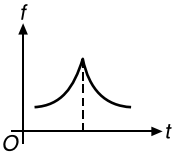
42. The ratio of the velocities of sound in hydrogen and carbon dioxide at S.T.P. is

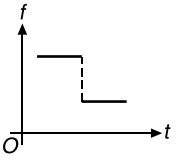
(A) $\sqrt{\frac{49}{45}\times 22}$ (B) $\sqrt{\frac{21\times 11}{10}}$
(C) $\sqrt{\frac{20}{11}\times 22}$ (D) $\sqrt{\frac{21}{25}\times 22}$

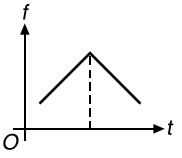
43. An air column, closed at one end and open at the other end, resonates with a tuning fork of frequency v when its length is 45 cm, 99 cm and at two other lengths in between these values. The wavelength of sound in the air column is

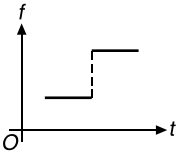
(A) 180 cm (B) 108 cm
(C) 54 cm (D) 36 cm

44. A source of sound emitting a note of constant frequency is moving towards a stationary listener, and then recedes from the listener with constant velocity maintained throughout the motion. The frequency heard by the listener (f) when plotted against time (t) will give the following curve (s).

(A) 

(B) 

(C) 

(D) 

45. A wave travelling along positive x-axis is given by $y = A\sin(\omega t - kx)$. If it is reflected from rigid boundary such that 80% amplitude is reflected, then equation of reflected wave is

(A) $y = A\sin(\omega t + kx)$ (B) $y = -0.8A\sin(\omega t + kx)$
(C) $y = 0.8A\sin(\omega t + kx)$ (D) $y = A\sin(\omega t + 0.8kx)$

46. In PROBLEM 45, if the reflecting boundary is free and yet 80% of amplitude gets reflected, the equation of reflected wave is

(A) $y = A\sin(\omega t + kx)$ (B) $y = -0.8A\sin(\omega t + kx)$
(C) $y = 0.8A\sin(\omega t + kx)$ (D) $y = -0.8A\sin(\omega t + 0.8kx)$

47. The speed of sound in air at 15 °C and 76 cm of mercury is 340 ms^{-1}. The speed of sound in air at 30 °C and 75 cm of mercury will be

(A) $340\sqrt{\frac{303}{288}}$ (B) $340\sqrt{\frac{288}{303}}$
(C) $340\sqrt{2}$ (D) $340\sqrt{\frac{2\times 75}{76}}$

48. If the intensity of sound is doubled, the intensity level will increase by nearly

(A) 1 dB (B) 2 dB
(C) 3 dB (D) 4 dB

49. In a plane progressive harmonic wave particle speed is always less than the wave speed if

(A) amplitude of wave is less than $\frac{\lambda}{2\pi}$
(B) amplitude of wave is greater than $\frac{\lambda}{2\pi}$
(C) amplitude of wave is less than λ
(D) amplitude of wave is greater than $\frac{\lambda}{\mu}$

50. The difference between the apparent frequencies of a source of sound as perceived by a stationary observer during its approach and recession is 2% of the actual frequency of the source. If the speed of sound is $300\ \text{ms}^{-1}$, the speed of the source is

(A) $12\ \text{ms}^{-1}$ (B) $6\ \text{ms}^{-1}$

(C) $1.5\ \text{ms}^{-1}$ (D) $3\ \text{ms}^{-1}$

51. A conveyor belt moves to the right with speed $v = 300\ \text{m min}^{-1}$. A pie man puts pies on the belt at a rate of 20 per minute while walking with speed $30\ \text{m min}^{-1}$ towards a receiver at the other end. The frequency with which they are received by the stationary receiver is

(A) 26.67 per minute (B) 30 per minute

(C) 22.22 per minute (D) 24 per minute

52. The fundamental frequency of a sonometer wire of length l is f_0. A bridge is now introduced at a distance of Δl from the centre of the wire ($\Delta l \ll l$). The number of beats heard if both sides of the bridges are set into vibration in their fundamental moves are

(A) $\dfrac{8 f_0 \Delta l}{l}$ (B) $\dfrac{f_0 \Delta l}{l}$

(C) $\dfrac{2 f_0 \Delta l}{l}$ (D) $\dfrac{4 f_0 \Delta l}{l}$

53. Wavelengths of two notes in air are $\dfrac{80}{195}$ m and $\dfrac{80}{193}$ m. Each note produces 5 bps with a third note of a fixed frequency. The speed of sound in air is

(A) $300\ \text{ms}^{-1}$ (B) $340\ \text{ms}^{-1}$

(C) $375\ \text{ms}^{-1}$ (D) $400\ \text{ms}^{-1}$

54. Velocity of sound in an open organ pipe is $330\ \text{ms}^{-1}$. The frequency of wave is 1.1 kHz and the length of tube is 30 cm. To which harmonic does this frequency corresponds

(A) 2^{nd} (B) 3^{rd}

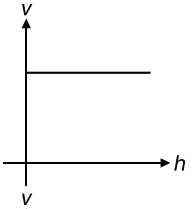
(C) 4^{th} (D) 5^{th}

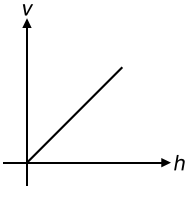
55. Two stretched wires are in unison. If the tension in one of the wires is increased by 1%, 3 beats are produced in2 s. The initial frequency of each wire is

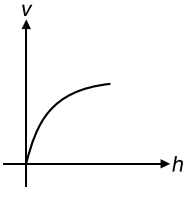
(A) 150 Hz (B) 200 Hz

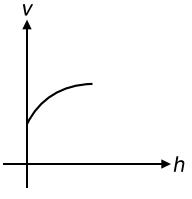
(C) 300 Hz (D) 450 Hz

56. A uniform rope having mass m hangs vertically from a rigid support. A transverse wave pulse is produced at the lower end. The speed (v) of wave pulse varies with height (h) from the lower end as

(A) 

(B) 

(C) 

(D) 

57. Two identical sonometer wires have a fundamental frequency of 500 Hz when kept under the same tension. The percentage increase in the tension of one wire that would cause an occurrence of 5 bps, when both wires vibrate together, is

(A) 1% (B) 1.5%

(C) 2% (D) 4%

58. A string stretched by a weight of 4 kg is vibrating in its fundamental mode. The additional weight required to produce an octave of the first is

(A) 4 kg (B) 8 kg

(C) 12 kg (D) 16 kg

59. The minimum distance of a reflector to hear the echo of monosyllabic sound is (speed of sound is $330\ \text{ms}^{-1}$)

(A) 16.5 m (B) 33 m

(C) 165 m (D) 330 m

60. Two wires of radii r and $2r$ are welded together end to end. The combination is used as a sonometer wire and is kept under a tension T. The welded point is mid-way between the bridges. The ratio of the number of loops formed in the wires, such that the joint is a node when stationary vibrations are set up in the wires, is

(A) $\dfrac{1}{4}$ (B) $\dfrac{1}{3}$

(C) $\dfrac{1}{2}$ (D) $\dfrac{2}{3}$

61. A star emits light of wavelength λ and it is receding from the earth with a speed v_s. The shift in the wavelength of the spectral line observed on the earth is

(A) $\lambda \dfrac{v_s^2}{c^2}$ (B) $-\lambda \dfrac{v_s^2}{c^2}$

(C) $-\lambda \dfrac{v_s}{c}$ (D) $\lambda \dfrac{v_s}{c}$

62. The frequency of the whistle of an engine is 600 Hz. It is moving with a speed of $30\ \text{ms}^{-1}$ towards a stationary observer. The apparent frequency is (speed of sound is $330\ \text{ms}^{-1}$)

(A) 630 Hz

(B) 660 Hz

(C) 570 Hz

(D) 540 Hz

63. Two waves of the same frequency and amplitude superpose to produce a resultant disturbance of the same amplitude. The phase difference between the waves is

(A) ZERO (B) $\dfrac{\pi}{3}$

(C) $\dfrac{\pi}{2}$ (D) $\dfrac{2\pi}{3}$

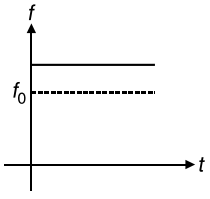
64. A tuning fork of frequency 340 Hz is vibrated just above a cylindrical tube of length 120 cm. Water is slowly poured in the tube. If the speed of sound in air is $340\ \text{ms}^{-1}$, then the minimum height of water required for resonance is

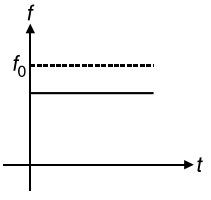
(A) 25 cm (B) 45 cm

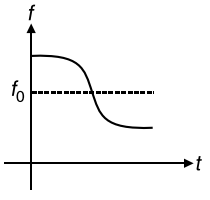
(C) 75 cm (D) 95 cm

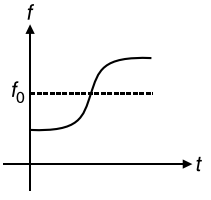
65. Source and observer both start moving simultaneously from origin one along x-axis and the other along y-axis with speed of source is 2 (speed of observer). The graph between the apparent frequency observed by observer (f) and time (t) would be

(Here f_0 =natural frequency of source)

(A) 

(B) 

(C) 

(D) 

66. The velocities of sound in an ideal gas at temperature T_1 and T_2 are found to be V_1 and V_2 respectively. If the r.m.s. velocities of the molecules of the same gas at the same temperatures T_1 and T_2 are v_1 and v_2, respectively, then

(A) $v_2 = v_1\left(\frac{V_1}{V_2}\right)$ (B) $v_2 = v_1\left(\frac{V_2}{V_1}\right)$

(C) $v_2 = v_1\left(\sqrt{\frac{V_2}{V_1}}\right)$ (D) $v_2 = v_1\left(\sqrt{\frac{V_1}{V_2}}\right)$

67. The maximum pressure variation that the human ear can tolerate in loud sound is about 30 Nm^{-2}. The corresponding maximum displacement for a sound wave in air having a frequency of 10^3 Hz is (take velocity of sound in air as 300 ms^{-1} and density of air 1.5 kgm^{-3})

(A) $\frac{2\pi}{3}\times 10^{-2}$ m (B) $\frac{2\times 10^{-4}}{\pi}$ m

(C) $\frac{\pi}{3}\times 10^{-2}$ m (D) $\frac{10^{-4}}{3\pi}$ m

68. The equation of a wave is $y = 4\sin\left\{\frac{\pi}{2}\left(2t + \frac{x}{8}\right)\right\}$ where y, x are in cm and time in second. The amplitude, wavelength, velocity and frequency of the wave are, respectively,

(A) 4 cm, 32 cm, 16 cms^{-1}, 0.5 Hz

(B) 8 cm, 16 cm, 32 cms^{-1}, 1.0 Hz

(C) 4 cm, 32 cm, 32 cms^{-1}, 0.5 Hz

(D) 8 cm, 16 cm, 16 cms^{-1}, 1.0 Hz

69. In PROBLEM 68, the phase difference between two positions of the same particle which are occupied at time interval of 0.4 s is

(A) 0.2π (B) 0.4π

(C) 0.6π (D) 0.8π

70. In PROBLEM 68, the phase difference at any instant between two particles 12 cm apart is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$

(C) $\frac{3\pi}{4}$ (D) π

71. When a tuning fork A of frequency 100 Hz is sounded with a tuning fork B, the number of beats per second is 2. On putting some wax on the prongs of B, the number of beats per second becomes 1. The frequency of the fork B is

(A) 98 Hz (B) 99 Hz

(C) 101 Hz (D) 102 Hz

72. In a hall, a person receives direct sound waves from a source 120 m away. He also receives waves from the same source which reach him after being reflected from the 25 m high ceiling at a point half-way between them. The two waves interfere constructively for wavelengths (in metres) of

(A) $20, \frac{20}{3}, 4, \ldots..$ (B) $10, 5, \frac{10}{3}, \ldots..$

(C) 30, 20, 10, …. (D) 35, 25, 15, …..

73. When a source of sound approaches a stationary observer the intensity of the sound wave

(A) is increased

(B) is decreased

(C) remains constant

(D) may increase or decrease depending on the speed of source

74. A source of sound and a listener are both moving in the same direction, the source following the listener. If the respective velocities of sound, source and listener are v, v_s and v_l, then the ratio of the actual frequency of the source and the apparent frequency as received by the listener is

(A) $\frac{v - v_l}{v - v_s}$ (B) $\frac{v - v_s}{v - v_l}$

(C) $\frac{v + v_l}{v + v_s}$ (D) $\frac{v + v_s}{v + v_l}$

75. An open and a closed pipe have same length. The ratio of frequencies of their n^{th} overtone is

(A) $\frac{n+1}{2n+1}$ (B) $\frac{2(n+1)}{2n+1}$

(C) $\frac{n}{2n+1}$ (D) $\frac{n+1}{2n}$

76. String 1 has twice the length, twice the radius, twice the tension and twice the density of another string 2. The relation between the fundamental frequencies of 1 and 2 is

(A) $f_1 = 2f_2$ (B) $f_1 = 4f_2$

(C) $f_2 = 4f_1$ (D) $f_1 = f_2$

77. Identical wires A and B of different materials are hung from the ceiling of a room. The density of wire A is greater than the density of wire B. Identical wave pulses are produced at the bottom of respective wires. The time taken by the pulse to reach the top is

(A) greater for wire A
(B) greater for wire B
(C) same for both the wires
(D) cannot be determined

78. An open organ pipe of length l is sounded together with another open organ pipe of length $l+x$ in their fundamental tones. Speed of sound in air is v. The beat frequency heard will be $(x \ll l)$

(A) $\frac{vx}{4l^2}$ (B) $\frac{vl^2}{2x}$
(C) $\frac{vx}{2l^2}$ (D) $\frac{vx^2}{2l}$

79. A sample of oxygen at N.T.P. has volume V and a sample of hydrogen at N.T.P. has volume$4V$. Both the gases are mixed and the mixture is maintained at N.T.P. If the speed of sound in hydrogen at N.T.P. is $1270\ \text{ms}^{-1}$, that in the mixture will be

(A) $317\ \text{ms}^{-1}$ (B) $635\ \text{ms}^{-1}$
(C) $830\ \text{ms}^{-1}$ (D) $950\ \text{ms}^{-1}$

80. A wave equation is given by $y = A\cos(\omega t - kx)$, where symbols have their usual meanings. If v_p is the maximum particle velocity and v is the wave velocity of the wave then
(A) v_p can never be equal to v
(B) $v_p = v$ for $\lambda = 2\pi A$
(C) $v_p = v$ for $\lambda = \frac{A}{2\pi}$
(D) $v_p = v$ for $\lambda = \frac{A}{\pi}$

81. A police car with a siren of frequency 8 kHz is moving with uniform velocity $36\ \text{kmhr}^{-1}$ towards a tall building which reflects the sound waves. The speed of sound in air is $320\ \text{ms}^{-1}$. The frequency of the siren heard by the car driver is
(A) 8.50 kHz (B) 8.25 kHz
(C) 7.75 kHz (D) 7.50 kHz

82. The first overtone of an open pipe has frequency n. The first overtone of a closed pipe of the same length will have frequency

(A) $\frac{n}{2}$ (B) $2n$
(C) $\frac{3n}{4}$ (D) $\frac{4n}{3}$

83. In a Kundt's tube, stationary waves of frequency 1000 Hz are produced. If the distance between 6 successive nodes is 82.5 cm, the speed of sound in the gas filled in the tube is

(A) $300\ \text{ms}^{-1}$ (B) $330\ \text{ms}^{-1}$
(C) $360\ \text{ms}^{-1}$ (D) $390\ \text{ms}^{-1}$

84. If the amplitude of a wave at a distance r from a point source is A, the amplitude at a distance $2r$ will be

(A) $2A$ (B) A
(C) $\frac{A}{2}$ (D) $\frac{A}{4}$

85. The fundamental frequency of a sonometer wire is n. If its length, diameter and tension are doubled, the material of the wire remaining the same, the new fundamental frequency will be

(A) n (B) $\frac{n}{\sqrt{2}}$
(C) $\frac{n}{2}$ (D) $\frac{n}{2\sqrt{2}}$

86. A wire of density $9\times10^3\ \text{kgm}^{-3}$ is stretched between two clamps 1 m apart and is subjected to an extension of 4.9×10^{-4} m. If Young's modulus of the wire is $9\times10^{10}\ \text{Nm}^{-2}$, the lowest frequency of the transverse vibrations in the wire is
(A) 35 Hz (B) 70 Hz
(C) 105 Hz (D) 140 Hz

87. A sonometer wire vibrates with a frequency f. It is replaced by another wire of three times the diameter if tension and other parameters remain the same, the frequency of vibration of the wire will be

(A) $\frac{f}{9}$ (B) $\frac{f}{3}$
(C) $3f$ (D) $9f$

88. The lengths of two open organ pipes are l and $l+\Delta l$, $(\Delta l \ll l)$. If v is the speed of sound then the frequency of beats between them will be approximately (neglect end correction)

(A) $\frac{v}{2l}$ (B) $\frac{v}{4l}$
(C) $\frac{v\Delta l}{l^2}$ (D) $\frac{v\Delta l}{2l^2}$

89. A train is moving with a constant speed along a circular track. The engine of the train emits a sound of frequency f. The frequency heard by the guard at rear end of the train
(A) is less than f
(B) is greater than f
(C) is equal to f
(D) may be greater than, less than or equal to f depending on the factors like speed of train, length of train and radius of circular track

90. Which of the following equations does not represent a progressive wave

(A) $y = A\sin\left\{\omega\left(t - \frac{x}{c}\right)\right\}$
(B) $y = A\sin\left\{\frac{2\pi}{\lambda}(ct - x)\right\}$
(C) $y = A\sin\left\{2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)\right\}$
(D) $y = A\sin\left\{2\pi\left(\frac{t}{T} - \frac{x}{c}\right)\right\}$

91. Speed of transverse wave in a string of density $100\ \text{kgm}^{-3}$ and area of cross-section $10\ \text{mm}^2$ under a tension of 10^3 N is

(A) $100\ \text{ms}^{-1}$ (B) $1000\ \text{ms}^{-1}$
(C) $200\ \text{ms}^{-1}$ (D) $2000\ \text{ms}^{-1}$

92. A person speaking normally produces a sound intensity of 40 dB at a distance of 1 m. If the threshold intensity for reasonable audibility is 20 dB, the maximum distance at which he can be heard clearly is
(A) 4 m (B) 5 m
(C) 10 m (D) 20 m

93. If two coherent waves of the same frequency but different amplitudes superpose at a point
(A) there will be no interference.
(B) the intensity of sound at any point will vary with time.
(C) there will be interference but the minimum intensity will not be zero.
(D) there will be interference with minimum intensity equal to zero.

94. Two interfering waves of the same frequency have amplitudes in the ratio $1:3$. If the intensity of the first wave is I, the intensity at the maxima of interference is
(A) $16I$ (B) $8I$
(C) $4I$ (D) $64I$

95. In PROBLEM 94, the intensity at the minima is
(A) ZERO (B) $2I$
(C) $4I$ (D) $8I$

96. A source of sound is moving towards a stationary observer with a speed of $50\ \text{ms}^{-1}$. The observer measures the frequency of the source as 1000 Hz. The speed of sound is $350\ \text{ms}^{-1}$. The apparent frequency measured by the observer when the source is moving away after crossing the observer is
(A) 750 Hz (B) 850 Hz
(C) 1150 Hz (D) 1250 Hz

97. A 10 watt source of sound of frequency 1000 Hz sends out waves in air. The displacement amplitude at a distance of 10 m from the source is (speed of sound in air $= 340\ \text{ms}^{-1}$ and density of air $= 1.29\ \text{kgm}^{-3}$)
(A) 0.62 μm (B) 4.2 μm
(C) 1.6 μm (D) 0.96 μm

98. Two tuning forks A and B vibrating simultaneously produce 5 bps. Frequency of B is 512 Hz. If one arm of A is filed, the number of beats per second increases. Frequency of A is
(A) 502 Hz (B) 507 Hz
(C) 517 Hz (D) 522 Hz

99. If λ_1, λ_2 and λ_3 are the wavelengths of the waves giving resonance with the fundamental, first and second overtones respectively of a closed organ pipe. Then the ratio of wavelengths $\lambda_1 : \lambda_2 : \lambda_3$ is
(A) $1:2:3$ (B) $1:\frac{1}{3}:\frac{1}{5}$
(C) $1:3:5$ (D) $5:3:1$

100. In order to increase the frequency of transverse oscillations of a stretched wire by 50%, its tension must be increased by
(A) 50% (B) 100%
(C) 125% (D) 150%

101. Speed of sound wave in a gas is v_1 and r.m.s. speed of molecules of the gas at the same temperature is v_2. Then
(A) $v_1 = v_2$ (B) $v_1 < v_2$
(C) $v_1 > v_2$ (D) $v_1 \le v_2$

102. A vibrating stretched string resonates with a tuning fork of frequency 512 Hz when the length of the string is 0.5 m. The length of the string required to vibrate resonantly with a tuning fork of frequency 256 Hz would be
(A) 0.25 m (B) 0.75 m
(C) 1.0 m (D) 2.0 m

103. Consider two pipes each having a length of 2 m. One is closed at one end and the other is open at both ends. The speed of sound in air is $340\ \text{ms}^{-1}$. The frequency at which both can resonate is
(A) 340 Hz (B) 510 Hz
(C) 42.5 Hz (D) None of these

104. If l_1 and l_2 are the lengths of air column for the first and second resonance when a turning fork of frequency n is sounded on a resonance tube. Taking into account the end correction, the distance of the antinode from the top end of the resonance tube is
(A) $2(l_2 - l_1)$ (B) $\frac{1}{2}(2l_1 - l_2)$
(C) $\frac{l_2 - 3l_1}{2}$ (D) $\frac{l_2 - l_1}{2}$

105. The speed of sound wave in a gas, in which two waves of wavelengths 1 m and 1.02 m produce 6 beats per second is
(A) $350\ \text{ms}^{-1}$ (B) $306\ \text{ms}^{-1}$
(C) $380\ \text{ms}^{-1}$ (D) $410\ \text{ms}^{-1}$

106. Two interfering waves of the same frequency have an intensity ratio $16:1$. The ratio of intensities at the maxima and the minima is
(A) $\frac{25}{16}$ (B) 9
(C) 4 (D) $\frac{25}{9}$

107. A string of length L is stretched by $\frac{L}{20}$ and speed of transverse wave along it is v. The speed of wave when it is stretched by $\frac{L}{10}$ will be (assume that Hooke's Law is applicable)
(A) $2v$ (B) $\frac{1}{\sqrt{2}}v$
(C) $\sqrt{2}v$ (D) $4v$

108. Two identical sounds s_1 and s_2 reach at a point P in phase. The resultant loudness at point P is n decibel higher than the loudness of s_1. The value of n is
(A) 2 (B) 4
(C) 5 (D) 6
Given $\log_{10}(2) = 0.3$

109. The frequency of the horn of a car as perceived by a stationary observer towards whom the car is moving differs from the actual frequency by 2.5%. If the speed of the sound in air is $320\ \text{ms}^{-1}$, the speed of the car is

(A) 6 ms^{-1} (B) 7.2 ms^{-1}

(C) 7.8 ms^{-1} (D) 9 ms^{-1}

110. If the tension in a string stretched between two fixed points is made four times, the frequency of the second harmonic will become

(A) two times (B) three times

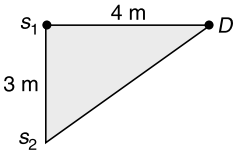
(C) four times (D) six times

111. Two sound waves, each of amplitude A and frequency ω, superpose at a point with a phase difference of $\frac{\pi}{2}$. The amplitude and the frequency of the resultant wave are, respectively

(A) $\frac{A}{\sqrt{2}}, \frac{\omega}{2}$ (B) $\frac{A}{\sqrt{2}}, \omega$

(C) $\sqrt{2}A, \frac{\omega}{2}$ (D) $\sqrt{2}A, \omega$

112. In the figure the intensity of waves arriving at D from two coherent sources s_1 and s_2 is I_0. The wavelength of the wave is $\lambda = 4$ m. Resultant intensity at D will be



(A) $4I_0$ (B) I_0

(C) $2I_0$ (D) ZERO

113. A stretched string of length 1 m has mass per unit length 0.5 g. The tension in the string is 20 N. If it is plucked at a distance of 25 cm from one end, the frequency of vibration will be

(A) 100 Hz (B) 200 Hz

(C) 300 Hz (D) 400 Hz

114. If v_0 and v denote the sound velocity and the r.m.s. velocity of the molecules in a gas, then

(A) $v_o > v$

(B) $v_0 = v$

(C) $v_0 = v\sqrt{\frac{\gamma}{3}}$

(D) v_0 and v are not related

115. A transverse sine wave of amplitude 10 cm and wavelength 200 cm travels from left to right along a long horizontal stretched string with a speed of 100 cms^{-1}. Take the origin at left end of the string. At time $t = 0$ the left end of the string is at the origin and is moving downward. Then the equation of the wave will be (in C.G.S. system)

(A) $y = 10\sin(0.01\pi x - \pi t)$

(B) $y = 10\sin(\pi t - 0.01\pi x)$

(C) $y = 10\sin(0.02\pi x - 0.01\pi t)$

(D) $y = 10\sin(\pi t - 0.02\pi x)$

116. The speed of sound in air at S.T.P. is 300 ms^{-1}. If the air pressure becomes double, the temperature remaining the same, the speed of sound would become

(A) 1200 ms^{-1} (B) 600 ms^{-1}

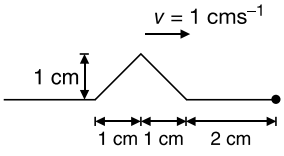
(C) $300\sqrt{2}$ ms^{-1} (D) 300 ms^{-1}

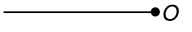
117. When a sound wave of frequency 300 Hz passes through a medium, the maximum displacement of a particle of the medium is 0.1 cm. The maximum velocity of particle is equal to

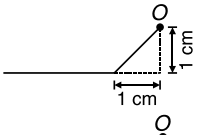
(A) 60π cms^{-1} (B) 30π cms^{-1}

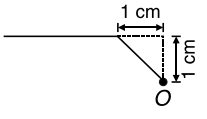
(C) 30 cms^{-1} (D) 60 cms^{-1}

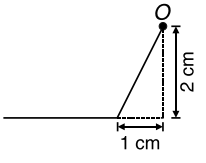
118. A wave pulse on a string has the dimension shown in figure. The wave speed is $v = 1$ cms^{-1}. If point O is a free end. The shape of wave at time $t = 3$ s is



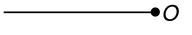
(A) 

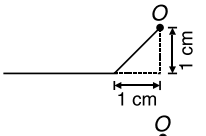
(B) 

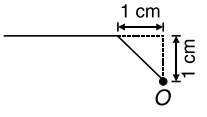
(C) 

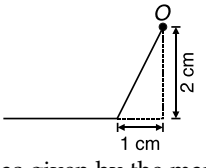
(D) 

119. In PROBLEM 118, shape of the wave at time $t = 3$ s if O is a fixed end will be

(A) 

(B) 

(C) 

(D) 

120. A tuning fork, whose frequency as given by the manufacturer is 512 Hz, is being tested using an accurate oscillator. It is found that they produce 2 beats per second when the oscillator reads 514 Hz, and 6 beats second when it reads of 510 Hz. The actual frequency of the fork is

(A) 508 Hz (B) 512 Hz

(C) 516 Hz (D) 518 Hz

121. A travelling wave is partly reflected and partly transmitted from a rigid boundary. Let a_i, a_r and a_t be the amplitudes of incident wave, reflected wave and transmitted wave and I_i, I_r and I_t be the corresponding intensities. Then choose the correct alternative

(A) $\frac{I_i}{I_r} = \left(\frac{a_i}{a_r}\right)^2$ (B) $\frac{I_i}{I_t} = \left(\frac{a_i}{a_t}\right)^2$

(C) $\frac{I_r}{I_t} = \left(\frac{a_r}{a_t}\right)^2$ (D) $\frac{I_i}{I_r} = \left(\frac{a_r}{a_i}\right)^2$

122. A uniform circular hoop is rotating clockwise in the absence of gravity with tangential speed v_0. If v be the velocity of transverse waves travelling on this string then,
(A) $v \neq v_0$ (B) $v < v_0$
(C) $v > v_0$ (D) $v = v_0$

123. Two sound waves have intensities of 10 and 500 μWcm^{-2}. How many decibels is the sound louder than the other?
(A) 7 dB (B) 1.7 dB
(C) 2.7 dB (D) 3.7 dB

124. In a resonance tube, the first resonance is obtained when the level of water in the tube is at 16 cm from the open end. Neglecting end correction, the next resonance will be obtained when the level of water from the open end is
(A) 24 cm (B) 32 cm
(C) 48 cm (D) 64 cm

125. Waves from three sources of the same intensity and frequencies $(n-1)$, n and $(n+1)$ Hz superpose. The number of beats per second is
(A) 0 (B) 1
(C) 2 (D) 3

126. The apparent wavelength of the light from a star, moving away from the earth is 0.01% more than its real wavelength. The speed of the star with respect to the earth is
(A) 10 kms^{-1} (B) 15 kms^{-1}
(C) 30 kms^{-1} (D) 60 kms^{-1}

127. The temperature at which the speed of sound in air becomes double its value at 0 °Cis
(A) 1092 °C (B) 819 K
(C) 819 °C (D) 546 °C

128. The equation for the vibration of a string fixed at both ends vibrating in its third harmonic is given by $y = 2\text{ cm}\sin\left[(0.6\text{ cm}^{-1})x\right]\cos\left[(500\pi s^{-1})t\right]$. The length of the string is
(A) 24.6 cm (B) 12.5 cm
(C) 20.6 cm (D) 15.7 cm

129. A stretched string of length 2 m vibrates in 4 segments. The distance between consecutive nodes is
(A) 0.5 m (B) 0.25 m
(C) 1.0 m (D) 0.75 m

130. An echo repeats two syllables. If the speed of sound is 330 ms^{-1}, then the minimum distance of reflecting surface is
(A) 16.5 m (B) 33.0 m
(C) 66.0 m (D) 330 m

131. A motion is described by $y = 3e^{x}e^{-3t}$ where y, x are in metre and t is in second.
(A) This represents equation of progressive wave propagating along $-x$ direction with 3 ms^{-1}.
(B) This represents equation of progressive wave propagating along $+x$ direction with 3 ms^{-1}.
(C) This does not represent a progressive wave equation.
(D) Data is insufficient to arrive at any conclusion of this sort.

132. A wave of frequency 500 Hz has a velocity 360 ms^{-1}. The distance between two nearest points which are 60° out of phase, is
(A) 0.7 cm (B) 12 cm
(C) 70 cm (D) 120 cm

133. The frequency of the first harmonic of a string stretched between two points is 100 Hz. The frequency of the third overtone is
(A) 200 Hz (B) 300 Hz
(C) 400 Hz (D) 600 Hz

134. The three lowest frequencies (in Hz) with which a 20 m long pipe, closed at one end, can vibrate are (speed of sound is 340 ms^{-1})
(A) 425, 850, 1275
(B) 425, 1275, 2125
(C) 900, 1800, 2700
(D) 900, 2700, 4500

135. 65 tuning forks are arranged in order of increasing frequency. Any two successive forks produce 4 bps when sounded together. If the last fork gives an octave of the first, the frequency of the first fork is
(A) 252 Hz (B) 256 Hz
(C) 260 Hz (D) 264 Hz

136. The fundamental not produced by an open organ pipe has frequency n. The fundamental note produced by a closed organ pipe of the same length will have frequency
(A) $\frac{n}{2}$ (B) $2n$
(C) $\frac{n}{4}$ (D) $4n$

137. The ratio of the velocity of sound in a monatomic gas to that in a triatomic gas having same molar mass, under similar conditions of temperature and pressure, is
(A) 1.12 (B) 1.25
(C) 1.50 (D) 1.62

138. The equation of a wave disturbance is given as $y = 0.02\cos\left(\frac{\pi}{2} + 50\pi t\right)\cot(10\pi x)$, where x and y are in metres and t in seconds. Select the incorrect statement.
(A) Antinode occurs at $x = 0.3$ m
(B) The wavelength is 0.2 m
(C) The speed of the constituent waves is 4 ms^{-1}
(D) Node occurs at $x = 0.15$ m

139. The minimum length of a tube, open at both ends, that resonates with a tuning fork of frequency 350 Hz is (velocity of sound in air is 350 ms^{-1})
(A) 0.25 m (B) 0.5 m
(C) 1 m (D) 2 m

140. A tuning fork and a sonometer give 5 bps both when the length of the wire is 1 m and 1.05 m. The frequency of the fork is
(A) 420 Hz (B) 410 Hz
(C) 210 Hz (D) 205 Hz

141. A source and a listener are both moving towards each other with speed $\frac{v}{10}$, where v is the speed of sound. If the frequency of the note emitted by the source is f, the frequency heard by the listener would be nearly

(A) $1.11f$ (B) $1.22f$

(C) $1.27f$ (D) f

142. A sonometer wire is vibrating in the second overtone. In the wire there are

(A) two nodes and two antinodes.
(B) one node and two antinodes.
(C) four nodes and three antinodes.
(D) three nodes and three antinodes.

143. Waves from two sources of intensities I and $4I$ interfere at a point. The resultant intensity at a point where the phase difference is $\frac{\pi}{2}$ is

(A) $9I$ (B) $5I$

(C) $3I$ (D) I

144. A sufficiently long closed organ pipe has a small hole at its bottom. Initially the pipe is empty. Water is poured into the pipe at a constant rate. The fundamental frequency of the air column in the pipe

(A) continuously increases
(B) first increases and then becomes constant
(C) continuously decreases
(D) first decreases and then becomes constant

145. A sound has an intensity of 2×10^{-8} Wm^{-2}. Its intensity level in decibels is $(\log_{10}2=0.3)$

(A) 23 (B) 3

(C) 43 (D) 4.3

146. The fractional change in tension in a sonometer wire of fixed length to produce a note one octave lower than before is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$

(C) $\frac{2}{3}$ (D) $\frac{3}{4}$

147. Speed of sound wave is v. If a reflector moves towards a stationary source emitting waves of frequency f with speed u, the wavelength of reflected wave will be

(A) $\frac{v-u}{v+u}f$ (B) $\frac{v+u}{v}f$

(C) $\frac{v+u}{v-u}f$ (D) $\frac{v-u}{v}f$

148. A siren placed at a railway platform is emitting sound of frequency 5 kHz. A passenger sitting in a moving train A records a frequency of 5.5 kHz while the train approaches the siren. During his return journey in a different train B he records a frequency of 6.0 kHz while approaching the same siren. The ratio of velocity of train B to that of train A is

(A) $\frac{242}{252}$ (B) 2

(C) $\frac{5}{6}$ (D) $\frac{11}{6}$

149. A progressive wave of frequency 500 Hz is travelling with a speed of 350 ms^{-1}. A compressional maximum appears at a place at a given instant. The minimum time interval after which a rarefaction maximum occurs at the same point is

(A) $\frac{1}{250}$ s (B) $\frac{1}{500}$ s

(C) $\frac{1}{1000}$ s (D) $\frac{1}{350}$ s

150. In PROBLEM 149, the minimum distance between a centre of compression and a centre of rarefaction at any instant is

(A) 1.4 m (B) 0.70 m
(C) 0.35 m (D) 0.175 m

151. If the amplitude of sound is doubled and the frequency reduced to one-fourth, the intensity will

(A) increase by a factor of 2
(B) decrease by a factor of 2
(C) decrease by factor of 4
(D) remain unchanged

152. A standing wave is maintained in a homogeneous string of cross-sectional area a and density ρ. It is formed by the superposition of two waves travelling in opposite directions given by the equation

$$y_1 = a\sin(\omega t - kx) \text{ and } y_2 = 2a\sin(\omega t + kx)$$

The total mechanical energy confined between the sections corresponding to the adjacent antinodes is

(A) $\frac{3\pi s\rho\omega^2a^2}{2k}$ (B) $\frac{\pi s\rho\omega^2a^2}{2k}$

(C) $\frac{5\pi s\rho\omega^2a^2}{2k}$ (D) $\frac{2\pi s\rho\omega^2a^2}{2k}$

153. If the intensity of sound increases by a factor of10^5, the increase in the intensity level is

(A) 5 dB (B) 10 dB
(C) 25 dB (D) 50 dB

154. The ratio of intensities between two coherent sound sources is 4 : 1. The difference of loudness decibels (dB) between maximum and minimum intensities, when they interfere in space is

(A) $10\log(2)$ (B) $20\log(3)$

(C) $20\log_e(3)$ (D) $20\log(2)$

155. The intensity in Wm^{-2} of a 70 dB noise is

(A) 10^{-5} (B) 10^{-7}
(C) 10^5 (D) 10^7

156. Two sound sources are moving in opposite directions with velocities v_1 and v_2 $(v_1 > v_2)$. Both are moving away from a stationary observer. The frequency of both the sources is 900 Hz. For what value of $v_1 - v_2$ the beat frequency

observed by the observer is 6 Hz. Given, speed of sound $v = 300\ \text{ms}^{-1}$ and that v_1 and v_2 both $\ll v$.

(A) $1\ \text{ms}^{-1}$ (B) $2\ \text{ms}^{-1}$

(C) $3\ \text{ms}^{-1}$ (D) $4\ \text{ms}^{-1}$

157. A source of sound is moving with a constant speed of $20\ \text{ms}^{-1}$ emitting a note of a fixed frequency. The ratio of the frequencies observed by a stationary observer when the source is approaching him and after it has crossed him is

(A) 9 : 8 (B) 8 : 9

(C) 10 : 9 (D) 9 : 10

158. A stretched rope having linear mass density $5\times10^{-2}\ \text{kgm}^{-1}$ is under a tension of 80 N. The power that has to be supplied to the rope to generate harmonic waves at a frequency of 60 Hz and an amplitude of 6 cm is

(A) 215 W (B) 251 W

(C) 512 W (D) 521 W

159. The frequency changes by 10% as the source approaches a stationary observer with constant speed v_s. What would be the percentage change in frequency as the source recedes the observer with the same speed. Given that $v_s \ll v$ (v = speed of sound in air)

(A) 14.3% (B) 20%

(C) 16.7% (D) 10%

160. A column of air at 51 °C and a tuning fork produce 4 bps when sounded together. As the temperature of the air column is decreased, the number of bps tends to decrease and when the temperature is 16 °C, the two produce 1 bps. The frequency of the fork is

(A) 50 Hz (B) 75 Hz

(C) 100 Hz (D) 150 Hz

161. A train moves towards a stationary observer with $34\ \text{ms}^{-1}$. The train sounds a whistle and its frequency registered by the observer is f_1. If the train's speed is reduced to $17\ \text{ms}^{-1}$, the frequency registered is f_2. If the speed of sound is $340\ \text{ms}^{-1}$, then the ratio $\frac{f_1}{f_2}$ is

(A) $\frac{18}{19}$ (B) $\frac{1}{2}$

(C) 2 (D) $\frac{19}{18}$

162. An engine is moving on a circular track with a constant speed. It is blowing a whistle of frequency 500 Hz. The frequency received by an observer standing stationary at the centre of the track is

(A) 500 Hz.

(B) more than 500 Hz.

(C) less than 500 Hz.

(D) more or less than 500 Hz depending on the actual speed of the engine.

163. Two tuning forks of frequencies 256 Hz and 258 Hz are sounded together. The time interval, between two consecutive maxima heard by an observer is

(A) 0.5 s (B) 2 s

(C) 250 s (D) 252 s

164. How many times more intense is a 90 dB sound than a 40 dB sound?

(A) 2.5 (B) 5

(C) 50 (D) 10^5

165. Two steel wires of the same length are stretched by the same tension. The frequency of the fundamental note emitted by one is four times that of the other. The ratio of their diameters is

(A) 1 : 2 (B) 1 : 4

(C) 2 : 1 (D) 4 : 1

166. First overtone frequency of a closed organ pipe is equal to the first overtone frequency of an open organ pipe. Further nth harmonic of closed organic pipe is also equal to the mth harmonic of open pipe, where n and m are

(A) 5, 4 (B) 7, 5

(C) 9, 6 (D) 7, 3

167. A source of sound is travelling towards a stationary observer. The frequency of sound heard by the observer is 25% more than the actual frequency. If the speed of sound is v, that of the source is

(A) $\frac{v}{5}$ (B) $\frac{v}{4}$

(C) $\frac{v}{3}$ (D) $\frac{v}{2}$

168. A cylindrical tube, open at both ends, has a fundamental frequency f in air. The tube is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column is now

(A) $\frac{f}{2}$ (B) f

(C) $\frac{3f}{4}$ (D) $2f$

169. A closed organ pipe and an open organ pipe of same length produce four beats in their fundamental mode when sounded together. If length of the open organ pipe is increased, then the number of beats will

(A) increase

(B) decrease

(C) remain constant

(D) may increase or decrease

170. In an experiment it was found that string vibrates in n loops when a mass M is placed on the pan. What mass should be placed on the pan to make it vibrate in $2n$ loops with same frequency. (Neglect the mass of pan)

(A) $2M$ (B) $\frac{1}{4}M$

(C) $4M$ (D) $\frac{1}{2}M$

171. For a certain organ pipe, three successive resonance frequencies are observed at 425, 595 and 765 Hz. The speed of sound in air is $340\ \text{ms}^{-1}$. The pipe is a

(A) closed pipe of length 1 m.

(B) closed pipe of length 2 m.

(C) open pipe of length 1 m.

(D) open pipe of length 2 m.

172. In PROBLEM 171, the fundamental frequency of the pipe is
(A) 425 Hz (B) 340 Hz
(C) 170 Hz (D) 85 Hz

173. The ratio of the velocities of sound in hydrogen and oxygen at S.T.P. is
(A) 16 : 1 (B) 8 : 1
(C) 4 : 1 (D) 2 : 1

174. A heavy rope is suspended from a rigid support. A wave pulse is set up at the lower end, then
(A) the pulse will travel with uniform speed
(B) the pulse will travel with increasing speed
(C) the pulse will travel with decreasing speed
(D) the pulse cannot travel through the rope

175. A boat of anchor is rocked by waves of velocity 25 ms^{-1}, having crests 100 m apart. They reach the boat once every
(A) 4.0 s (B) 8.0 s
(C) 2.0 s (D) 0.25 s

176. For the stationary wave

$$y(\text{in cm}) = 4\sin\left(\frac{\pi x}{15}\right)\cos(96\pi t)$$

the distance between a node and the nearest antinode is
(A) 7.5 cm (B) 15 cm
(C) 22.5 cm (D) 30 cm

177. For a certain organ pipe three successive resonance frequencies are observed at 425 Hz, 595 Hz and 765 Hz respectively. If the speed of sound in air is 340 ms^{-1}, then the length of the pipe is
(A) 2 m (B) 0.4 m
(C) 1 m (D) 0.2 m

178. In a stationary wave that forms as a result of reflection of waves from an obstacle, the ratio of the amplitude at an antinode to the amplitude at node is n. The fraction of energy reflected is
(A) $\left(\frac{n-1}{n}\right)^2$ (B) $\left(\frac{n-1}{n+1}\right)^2$
(C) $\left(\frac{1}{n}\right)^2$ (D) $\left(\frac{n}{n+1}\right)^2$

179. Consider 10 identical sources of sound all giving the same frequency but having phase angles which are random. If the average intensity of each source is I_0, the average of resultant intensity due to all these ten sources will be
(A) I_0 (B) $\sqrt{10}\, I_0$
(C) $10I_0$ (D) $100I_0$

180. The fundamental frequency of a closed organ pipe is 50 Hz. The frequency of the second overtone is
(A) 100 Hz (B) 150 Hz
(C) 200 Hz (D) 250 Hz

181. A tuning fork of frequency 90 Hz is sounded and moved towards a stationary observer with a speed equal to one-tenth the speed of sound. The note heard by the observer will have a frequency
(A) 100 (B) 110
(C) 80 (D) 70

182. The speed of sound in a gas in which two waves of wavelengths 50 cm and 50.4 cm produce 6 beats per second is
(A) 338 ms^{-1} (B) 350 ms^{-1}
(C) 378 ms^{-1} (D) 400 ms^{-1}

183. When a source of sound of frequency f crosses a stationary observer with a speed v_s ($\ll$ speed of sound v), the apparent change in frequency Δf is given by
(A) $\frac{2fv_s}{v}$ (B) $2fvv_s$
(C) $\frac{2fv}{v_s}$ (D) $\frac{fv_s}{v}$

MULTIPLE CORRECT CHOICE TYPE QUESTIONS

This section contains Multiple Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

1. A wire of 9.8×10^{-3} kgm^{-1} passes over a frictionless light pulley fixed on the top of a frictionless inclined plane which makes an angle of 30° with the horizontal. Masses m and M are tied at the two ends of wire such that m rests on the plane and M hangs freely vertically downwards. The entire system is in equilibrium and a transverse wave propagates along the wire with a velocity of 100 ms^{-1}.
(A) $m = 20$ kg (B) $M = 5$ kg
(C) $\frac{m}{M} = \frac{1}{2}$ (D) $\frac{m}{M} = 2$

2. A wave equation which gives the displacement along the y-direction is given by

$$y = 10^{-4}\sin(60t + 2x)$$

where x and y are in metres and t is time in seconds. This represents a wave
(A) travelling with a velocity of 30 ms^{-1} in the negative x-direction
(B) of wavelength πm
(C) of frequency $\frac{30}{\pi}$ hertz
(D) of amplitude 10^{-4} m travelling along the negative x-direction

3. The displacement of a particle in a medium due to a wave travelling in the x-direction through the medium is given by $y = a\sin(\alpha t - \beta x)$ where t is time in second, α and β are constants.
(A) The frequency of the wave is α.
(B) The time period of the wave is $\frac{2\pi}{\alpha}$.

(C) The wavelength of the wave is $\frac{2\pi}{\beta}$.

(D) The velocity of the wave is $\frac{\alpha}{\beta}$.

4. Any progressive wave equation in differential form is

(A) $\frac{1}{\omega^2}\frac{\partial^2 y}{\partial t^2} = \frac{1}{k^2}\frac{\partial^2 y}{\partial x^2}$ (B) $\frac{1}{\omega}\frac{\partial y}{\partial t} = -\frac{1}{k}\frac{\partial y}{\partial x}$

(C) $\frac{1}{\omega^2}\frac{\partial^2 y}{\partial t^2} = -\frac{1}{k^2}\frac{\partial^2 y}{\partial x^2}$ (D) $\frac{1}{\omega}\frac{\partial y}{\partial t} = \frac{1}{k}\frac{\partial y}{\partial x}$

5. A plane progressive wave of frequency 25 Hz amplitude 2.5×10^{-5} m and initial phase zero propagates along negative x-direction with a velocity of 300 ms^{-1}. At any instant, the phase difference between the oscillations at two points 6 m apart along the line is ϕ and the corresponding amplitude difference is A.

(A) $A = 0$ (B) $\phi = 0$

(C) $A = 2.5\times10^{-5}$ m (D) $\phi = \pi$

6. A transverse sinusoidal wave of amplitude a, wavelength λ and frequency f is travelling on a stretched string. The maximum speed of any point on the string is $\frac{v}{10}$, where v is the speed of propagation of the wave. If $a = 10^{-3}$ m and $v = 10$ ms^{-1}, then λ and f are given by

(A) $\lambda = 2\pi\times10^{-2}$ m (B) $\lambda = 10^{-3}$ m

(C) $f = \frac{10^3}{2\pi}$ Hz (D) $f = 10^4$ Hz

7. The tension in a stretched string fixed at both ends is changed by 2%, the fundamental frequency is found to get changed by 15 Hz. Select the correct statement(s).

(A) Wavelength of the string of fundamental frequency does not change

(B) Velocity of propagation of wave changes by 2%

(C) Velocity of propagation of wave changes by 1%

(D) Original frequency is 1500 Hz

8. Two identical straight wires are stretched so as to produce 6 beats per second when vibrating simultaneously. On changing the tension slightly in one of them, the beat frequency remains unchanged. Denoting by T_1, T_2 the higher and the lower initial tension in the strings, then it could be said that while making the above changes in tension

(A) T_2 was decreased (B) T_2 was increased

(C) T_1 was decreased (D) T_1 was increased

9. To raise the pitch of a stringed musical instrument the player can

(A) loosen the string. (B) tighten the string.

(C) shorten the string. (D) lengthen the string.

10. Two vehicles, each moving with speed u on the same horizontal straight road, are approaching each other. Wind blows along the road with velocity w. One of these vehicles blows a whistle of frequency f_1. An observer in the other vehicle hears the frequency of the whistle to be f_2. The speed of sound in still air is V. The correct statement(s) is (are)

(A) If the wind blows from the observer to the source, $f_2 > f_1$

(B) If the wind blows from the source to the observer, $f_2 > f_1$

(C) If the wind blows from observer to the source, $f_2 < f_1$

(D) If the wind blows from the source to the observer, $f_2 < f_1$

11. $Y(x,t) = \frac{0.8}{\left[(4x+5t)^2+5\right]}$ represents a moving pulse where x and y are in metres and in t second. Then

(A) pulse is moving in positive x-direction

(B) in 2 s it will travel a distance of 2.5 m

(C) its maximum displacement is 0.16 m

(D) it is a symmetric pulse

12. An air column in a pipe, which is closed at one end, will be in resonance with a vibrating tuning fork of frequency 264 Hz, if the length of the column in cm is

(A) 31.25 (B) 62.50

(C) 93.75 (D) 125

13. A metallic wire of length ℓ is held between two rigid supports. If the wire is cooled through a temperature t, Y is the Young's modulus of elasticity, ρ is the density and α is the thermal coefficient of linear expansion of the wire, then the frequency of oscillation is proportional to

(A) $\frac{1}{\ell}$ (B) $\sqrt{Y}$

(C) $\sqrt{\frac{\alpha}{\rho}}$ (D) $\sqrt{t}$

14. A wave is represented by the equation

$$y = A\sin\left(10\pi x + 15\pi t + \frac{\pi}{3}\right)$$

where x is in meters and t is in seconds. The expression represents

(A) a wave travelling in the positive x-direction with a velocity 1.5 ms^{-1}

(B) a wave travelling in the negative x-direction with a velocity 1.5 ms^{-1}

(C) a wave travelling in the negative x-direction with a wavelength 0.2 m

(D) a wave travelling in the positive x-direction with a wavelength 0.2 m

15. In a plane progressive harmonic wave

(A) phase difference between displacement and acceleration of particle is zero

(B) phase difference between displacement and acceleration of particle is π

(C) phase difference between displacement and velocity of particle is $\frac{\pi}{2}$

(D) phase difference between velocity and acceleration of particle is $\frac{\pi}{2}$

16. As a wave propagates
(A) the wave intensity remains constant for a plane wave
(B) the wave intensity decreases as the inverse of the distance from the source for a spherical wave
(C) the wave intensity decreases as the inverse square of the distance from the source for a spherical wave
(D) total intensity of the spherical wave over the spherical surface centered at the source remains constant at all times

17. The velocity of sound in air is affected by change in the
(A) atmospheric pressure. (B) moisture content of air.
(C) temperature of air. (D) composition of air.

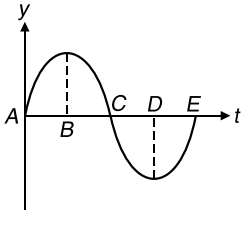
18. A driver in a stationary car blows a horn which produces monochromatic sound waves of frequency 1000 Hz normally towards a reflecting wall. The wall approaches the car with a speed of 3.3 ms^{-1}.
(A) The frequency of sound reflected from wall and heard by the driver is 1020 Hz.
(B) The frequency of sound reflected from wall and heard by the driver is 980 Hz.
(C) The percentage increase in frequency of sound after reflection from wall is 2%.
(D) The percentage decrease in frequency of sound after reflection from wall is 2%.

19. A horizontal stretched string, fixed at two ends, is vibrating in its fifth harmonic according to the equation,

$$y(x, t) = (0.01\text{ m})\sin[(62.8\text{ m}^{-1})x]\cos[(628\text{ s}^{-1})t].$$

Assuming $\pi = 3.14$, the correct statement(s) is (are)
(A) The number of nodes is 5
(B) The length of the string is 0.25 m
(C) The maximum displacement of the midpoint of the string, from its equilibrium position is 0.01 m
(D) The fundamental frequency is 100 Hz

20. Sound wave is travelling along positive x-direction. Displacement (y) of particles from their mean positions at any time t is shown in figure. Choose the correct alternative(s)

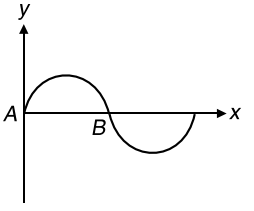


(A) particle located at E has its velocity in negative x-direction
(B) particle located at D has zero velocity
(C) particles located near C are under compression
(D) change in pressure at D is zero

21. Standing waves can be produced
(A) on a string clamped at both ends
(B) on a string clamped at one end and free at the other
(C) when incident wave gets reflected from a wall
(D) when two identical waves with a phase difference of π are moving in the same direction

22. Which of the following functions of x and t represents a progressive wave
(A) $y = \sin(4t - 3x)$ (B) $y = \dfrac{1}{4 + (4t - 3x)^2}$
(C) $y = \dfrac{1}{4t + 3k}$ (D) All of the above

23. The figure shows an instantaneous profile of a rope carrying a progressive wave moving from left to right, then

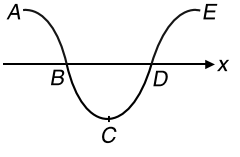


(A) the phase at A is greater than the phase at B
(B) the phase at B is greater than the phase at A
(C) A is moving upwards
(D) B is moving upwards

24. The equation of a wave travelling on a string is given by $y = 8\sin[(5\text{ m}^{-1})x - (4\text{ s}^{-1})t]$. Then
(A) velocity of wave is 0.8 ms^{-1}
(B) the displacement of a particle of the string at $t = 0$ and $x = \dfrac{\pi}{30}$ m from the mean position is 4 m
(C) the displacement of a particle from the mean position at $t = 0$, $x = \dfrac{\pi}{30}$ m is 8 m
(D) velocity of the wave is 8 ms^{-1}

25. The equation of a wave disturbance is given as $y = 0.02\sin\left(\dfrac{\pi}{2} + 50\pi t\right)\cos(10\pi x)$ where x and y are in metres and t is in second. Select the correct statement(s).
(A) The wavelength of wave is 0.2 m.
(B) The displacement node occurs at $x = 0.15$ m.
(C) The displacement antinode occurs at $x = 0.3$ m.
(D) The speed of constituent waves is 0.2 ms^{-1}.

26. The figure represents a longitudinal wave travelling in positive x-direction. Then



(A) part ABC represents compression
(B) part ABC represents rarefaction
(C) part CDE represents compression
(D) part CDE represents rarefaction

27. A sound wave of frequency f travels horizontally to the right. It is reflected from a large vertical plane surface moving to left with a speed v. The speed of sound in medium is C

(A) the number of wave striking the surface per second is $f\dfrac{(c+v)}{c}$

(B) the wavelength of the reflected wave is $\dfrac{c(c-v)}{f(c+v)}$

(C) the frequency of the reflected wave is $f\dfrac{(c+v)}{(c-v)}$

(D) the number of beats heard by a stationary listener to the left of the reflecting surface is $\dfrac{vf}{c-v}$

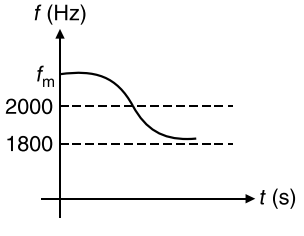
28. When a wave goes from one medium to another, there is a change in the
(A) velocity (B) amplitude
(C) frequency (D) wavelength

29. For a certain stretched string, three consecutive resonance frequencies are observed as 105, 175, 245 Hz respectively. Then select the correct alternative(s)
(A) The string is fixed at both ends
(B) The string is fixed at one end only
(C) The fundamental frequency is 35 Hz
(D) The fundamental frequency is 52.5 Hz

30. The equation of a stationary wave in a string is $y=(4\text{ mm})\sin\left[(3.14\text{ m}^{-1})x\right]\cos\omega t$. Select the correct alternative(s).
(A) The amplitude of component waves is 2 mm
(B) The amplitude of component waves is 4 mm
(C) The smallest possible length of string is 0.5 m
(D) The smallest possible length of string is 1.0 m

31. A uniform rope of mass M length L hangs vertically from the ceiling, with its lower end free. A disturbance on the rope travelling upwards starting from the lower end has a velocity v at a point P at distance x from the lower end.
(A) Tension at point P is Mg
(B) $v=\sqrt{xg}$
(C) $v=\sqrt{2xg}$
(D) Tension at point P is $\left(\dfrac{M}{L}\right)xg$

32. A stationary observer receives a sound of frequency $f_0=2200$ Hz. The apparent frequency f varies with time as shown in figure. Speed of sound $=300\text{ ms}^{-1}$. Choose the correct alternative(s)



(A) speed of source is 66.7 ms^{-1}
(B) f_m shown in figure cannot be greater than 2500 Hz
(C) speed of source is 33.33 ms^{-1}
(D) f_m shown in figure cannot be greater than 2250 Hz

33. Velocity of sound in air is 320 ms^{-1}. A pipe closed at one end has a length of 1 m. Neglecting end corrections, the air column in the pipe can resonate for sound of frequency
(A) 80 Hz
(B) 240 Hz
(C) 320 Hz
(D) 400 Hz

34. Mechanical waves
(A) are longitudinal only.
(B) are transverse only.
(C) can be both longitudinal and transverse.
(D) require a medium for propagation.

35. The $(x,\ y)$ co-ordinates of the corners of a square plate are $(0,0)$, $(L,0)$, (L,L) and $(0,L)$. The edges of the plate are clamped and transverse standing waves are set up in it. If $u(x,y)$ denotes the displacement of the plate at the point (x,y) at some instant of time, the possible expression(s) for u is (are) ($a=$positive constant)

(A) $a\cos\left(\dfrac{\pi x}{2L}\right)\cos\left(\dfrac{\pi y}{2L}\right)$

(B) $a\sin\left(\dfrac{\pi x}{L}\right)\sin\left(\dfrac{\pi y}{L}\right)$

(C) $a\sin\left(\dfrac{\pi x}{L}\right)\sin\left(\dfrac{2\pi y}{L}\right)$

(D) $a\cos\left(\dfrac{2\pi x}{L}\right)\sin\left(\dfrac{\pi y}{L}\right)$

36. A closed organ pipe of length 1.2 m vibrates in its first overtone mode. The pressure variation is maximum at
(A) 0.8 m from the open end
(B) 0.4 m from the open end
(C) closed end
(D) 1.0 m from the open end

37. In a wave motion $y=a\sin(kx-\omega t)$, y can represent
(A) electric field
(B) magnetic field
(C) displacement
(D) pressure

38. The equation $y=4+2\sin(6t-3x)$ represents a wave motion with
(A) amplitude 6 units
(B) amplitude 2 units
(C) wave speed 2 units
(D) wave speed $\dfrac{1}{2}$ unit

REASONING BASED QUESTIONS

This section contains Reasoning type questions, each having four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Each question contains STATEMENT 1 and STATEMENT 2. You have to mark your answer as

Bubble (A) If both statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1.
Bubble (B) If both statements are TRUE but STATEMENT 2 is not the correct explanation of STATEMENT 1.
Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE.
Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 is TRUE.

1. **Statement-1:** If oil of density higher than of water is used in place of water in a resonance tube, the frequency decreases.
Statement-2: Frequency does not depend on change of medium in resonance tube.

2. **Statement-1:** In Doppler's effect, the red shift means, shifting towards the red end of the spectrum.
Statement-2: In red shift, the apparent wavelength increases.

3. **Statement-1:** Solids can support both longitudinal and transverse waves, but only longitudinal waves can propagate in gases.
Statement-2: Solids possess two types of elasticity.

4. **Statement-1:** A tuning fork is in resonance with a closed pipe. But the same tuning fork cannot be in resonance with an open pipe of the same length.
Statement-2: The same tuning fork will not be in resonance with open pipe of same length due to end correction of pipe.

5. **Statement-1:** Coefficient of adiabatic elasticity of air is greater than the coefficient of isothermal elasticity.
Statement-2: Heat is exchanged freely in an isothermal change, but not in an adiabatic change.

6. **Statement-1:** Sound waves cannot be polarised.
Statement-2: Only transverse waves can be polarised.

7. **Statement-1:** After Laplace's correction for Newton's formula for finding the speed of sound in gases, we got $v = \sqrt{\frac{\gamma P}{\rho}}$.
Statement-2: According to Laplace, the wave propagates so much fast that it does not find any time to interact with its surroundings.

8. **Statement-1:** The fundamental frequency of an open organ pipe increases as the temperature is increased.
Statement-2: As the temperature increases the velocity of sound increases more rapidly than length of pipe.

9. **Statement-1:** Intensity of sound waves does not change when the listener moves towards or away from stationary source.
Statement-2: The motion of listener causes the apparent change in wavelength.

10. **Statement-1:** Velocity of particles while crossing mean position (in stationary waves) varies from maximum at antinodes to zero at nodes.
Statement-2: Amplitude of vibration at antinodes is maximum and at nodes, the amplitude is zero and all particles between two successive nodes cross the mean position together.

11. **Statement-1:** The bells are made of metals and not of wood.
Statement-2: Wood offers high damping on the sound waves.

12. **Statement-1:** The sound of a train coming from some distance can be easily detected by placing out ears near the rails.
Statement-2: Sound travels faster in air than solids.

13. **Statement-1:** We can recognise our friend by listening their voices.
Statement-2: The quality of sound produced by different persons are different.

14. **Statement-1:** The change in air pressure at constant temperature effects the speed of sound.
Statement-2: The speed of sound in gases is proportional to the square root of absolute temperature.

15. **Statement-1:** In a stationary wave, there is not transfer of energy.
Statement-2: There is no outward motion of disturbance from one particle to adjoining particle in a stationary wave.

16. **Statement-1:** Whistle of the approaching railway engine is shriller than the receding engine.
Statement-2: Apparent frequency of railway engine is both cases is same.

17. **Statement-1:** In everyday life the Doppler's effect is observed readily for sound waves than light waves.
Statement-2: Velocity of light is greater than the sound.

18. **Statement-1:** In a sound wave, a displacement node is a pressure antinode and vice-versa.
Statement-2: Displacement node is a point of minimum displacement.

19. **Statement-1:** If two waves of same amplitude, produce a resultant wave of same amplitude, then the phase difference between them will be $120°$.
Statement-2: The resultant amplitude of two waves is equal to sum of amplitude of two waves.

20. **Statement-1:** The error in Newton's formula of velocity of sound in air was 16%.
Statement-2: The experimental value of velocity of sound in air was not accurate.

LINKED COMPREHENSION TYPE QUESTIONS

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct. (For the sake of competitiveness there may be a few questions that may have more than one correct options)

Comprehension 1

The vibrations of a string of length 60 cm fixed at both ends are represented by the equation

$$y = 4\sin\left(\frac{\pi x}{15}\right)\cos(96\pi t)$$

where x and y are in cm and t in seconds.
Based on the above facts, answer the following questions.

1. Maximum displacement of a point at $x = 5$ cm is
(A) 4 cm (B) 2 cm
(C) $2\sqrt{3}$ cm (D) None of these

2. The nodes are located along the string at x values given by
(A) 0, 7.5 cm, 15 cm, ...
(B) 0, 15 cm, 30 cm, ...
(C) 0, 30 cm, 60 cm, ...
(D) None of these

3. The velocity of the particle at $x = 7.5$ cm at $t = 0.25$ s is
(A) 0 (B) 384 cms^{-1}
(C) $192\sqrt{2}$ cms^{-1} (D) None of these

4. The stationary wave $y = 4\sin\left(\frac{\pi x}{15}\right)\cos(96\pi t)$ is obtained by the superposition of two waves y_1 and y_2. Then
(A) $y_1 = 4\sin\left(\frac{\pi x}{15} - 96\pi t\right)$, $y_2 = 4\sin\left(\frac{\pi x}{15} + 96\pi t\right)$
(B) $y_1 = 2\sin\left(\frac{\pi x}{15} - 96\pi t\right)$, $y_2 = 2\sin\left(\frac{\pi x}{15} - 96\pi t\right)$
(C) $y_1 = 2\sin\left(\frac{\pi x}{15} + 96\pi t\right)$, $y_2 = 2\sin\left(\frac{\pi x}{15} + 96\pi t\right)$
(D) $y_1 = 2\sin\left(\frac{\pi x}{15} - 96\pi t\right)$, $y_2 = 2\sin\left(\frac{\pi x}{15} + 96\pi t\right)$

Comprehension 2

An observer standing on a railway crossing receives frequencies of 2.2 kHz and 1.8 kHz when the train approaches and recedes from the stationary observer. The speed of sound in air is 300 ms^{-1}. Based on the above facts, answer the following questions.

5. The actual frequency is
(A) 2.2 kHz (B) 1.98 kHz
(C) 1.8 kHz (D) 2.4 kHz

6. The velocity of the train is
(A) 5 ms^{-1} (B) 10 ms^{-1}
(C) 20 ms^{-1} (D) 30 ms^{-1}

Comprehension 3

The air column in a pipe closed at one end is made to vibrate in its second overtone by tuning fork of frequency 440 Hz. The speed of sound in air is 330 ms^{-1}. End corrections may be neglected. Let P_0 denote the mean pressure at any point in the pipe and ΔP_0 the maximum amplitude of pressure variation. Based on the above facts, answer the following questions.

7. The length L of the air column is
(A) $\frac{16}{15}$ m (B) $\frac{15}{16}$ m
(C) $\frac{5}{3}$ m (D) $\frac{3}{5}$ m

8. The corresponding wavelength to vibrate in second overtone is
(A) $\frac{2}{3}$ m (B) $\frac{3}{2}$ m
(C) $\frac{4}{3}$ m (D) $\frac{3}{4}$ m

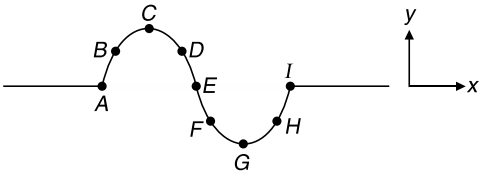
9. The amplitude of the pressure variation (ΔP) at the middle of the column is given by
(A) $\Delta P = \pm\Delta P_0$ (B) $\Delta P = \pm\frac{\Delta P_0}{2}$
(C) $\Delta P = \pm\frac{\Delta P_0}{\sqrt{2}}$ (D) $\Delta P = \pm\frac{\sqrt{3}\Delta P_0}{2}$

10. If $P_{\max}$ and $P_{\min}$ be the maximum and the minimum pressures at the open end of the pipe, then
(A) $P_{\max} = P_{\min} < P_0$ (B) $P_{\max} = P_{\min} > P_0$
(C) $P_{\max} = P_{\min} = P_0$ (D) $P_{\max} = P_{\min} = \frac{P_0}{2}$

11. If $P'_{\max}$ and $P'_{\min}$ be the maximum and the minimum pressures at the closed end of the pipe, then
(A) $P'_{\max} = P_0 + \Delta P_0$; $P'_{\min} = P_0$
(B) $P'_{\max} = P_0 + \Delta P_0$; $P'_{\min} = P_0 - \Delta P_0$
(C) $P'_{\max} = P_0 - \Delta P_0$; $P_{\min} = P_0 - 2\Delta P_0$
(D) $P'_{\max} = P_0$; $P'_{\min} = P_0 - \Delta P_0$

Comprehension 4

A progressive wave pulse is generated on a string is travelling in –ve X direction as shown in figure. Based on above information, answer the following questions.



12. Kinetic energy is maximum for the particle
(A) D (B) E
(C) F (D) G

13. Potential energy is maximum for the particle
(A) D (B) E
(C) F (D) G

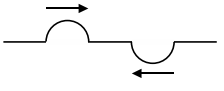
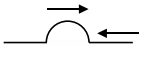
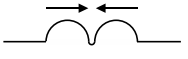
14. Particle B and D are moving respectively
(A) $\uparrow\uparrow$ (B) $\downarrow\downarrow$
(C) $\uparrow\downarrow$ (D) $\downarrow\uparrow$

Comprehension 5

When two pulses are allowed to pass through a string in opposite direction with same speed, the shape of string varies. Each pulse passes the overlap region as if the other pulse were not present there. The superposition principle can be applied to deal with the two waves in opposite direction. If two wave pulses are identical in shape, except that one is inverted with respect to another, at some instant displacement will be zero, but not their velocities. When reflection takes place from denser medium's surface or a rigid body, there will be phase difference of π. The corresponding path difference and time difference can be analysed. Based on above information, answer the following questions.

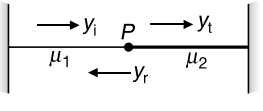
15. If the wave travelling in the string is given by $y = A\cos(\omega t + kx + \phi)$, then the wave reflected from denser medium's surface is
(A) $A\sin(\omega t + kx + \phi)$ (B) $A\sin(\omega t + kx)$
(C) $-A\cos(\omega t + kx + \phi)$ (D) $-A\cos(\omega t - kx + \phi)$

16. When displacement is zero as two wave pulses are moving in opposite direction, then which of the following will remain non-zero?
(A) Amplitude of each wave.
(B) Velocity of each wave.
(C) Both (A) and (B).
(D) Net amplitude and net velocity.

17. As the two waves are moving in opposite direction in a string, which of the following is not possible
(A) (B) 
(C)  (D) 

Comprehension 6

Consider two stretched strings 1 and 2, joined at a point (P) as shown in the figure. The linear mass densities of the two strings are μ_1 and μ_2 as shown in the figure.



The tension in both the strings is same. Let a transverse wave represented by $y_i = A_i\sin(k_i x - \omega t)$ be incident at the joint from the left string, as shown in the figure. Then, some part of the wave is transmitted to the right string and some part is reflected to the left string. The transmitted and the reflected waves can be represented by equations

$$y_t = A_t\sin(k_t x - \omega t) \text{ (transmitted wave)}$$

$$y_r = A_r\sin(k_i x + \omega t) \text{ (reflected wave)}$$

where, all the terms in above equations have their usual meanings. In the above two equations, ω is same in both transmitted and reflected waves. This is due to the fact that frequency of the wave remains unchanged when a wave travels in different media. Assuming origin to be at the point P (shown in figure), then at the point P, we must have the same value for y as well as the slope. So, mathematically, at the point P, we must have

$$y_i + y_r = y_t \text{ (at point } P) \quad \ldots(1)$$

$$\frac{\partial y_i}{\partial x} + \frac{\partial y_r}{\partial x} = \frac{\partial y_t}{\partial x} \text{ (at point } P) \quad \ldots(2)$$

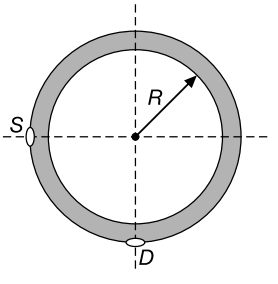
Based on the above facts, answer the following questions.

18. Amplitude of reflected wave is
(A) $A_r = \dfrac{2\left(\sqrt{\mu_1}\right)}{\left(\sqrt{\mu_1}+\sqrt{\mu_2}\right)}A_i$ (B) $A_r = \dfrac{\left(\sqrt{\mu_1}-\sqrt{\mu_2}\right)}{\left(\sqrt{\mu_1}+\sqrt{\mu_2}\right)}A_i$
(C) $A_r = \dfrac{\left(\sqrt{\mu_1}-\sqrt{\mu_2}\right)}{\left(\sqrt{\mu_1}+\sqrt{\mu_2}\right)}A_i$ (D) None of these

19. Amplitude of transmitted wave is
(A) $A_t = \dfrac{2\left(\sqrt{\mu_1}\right)}{\left(\sqrt{\mu_1}+\sqrt{\mu_2}\right)}A_i$ (B) $A_t = \dfrac{\left(\sqrt{\mu_1}-\sqrt{\mu_2}\right)}{\left(\sqrt{\mu_1}+\sqrt{\mu_2}\right)}A_i$
(C) $A_t = \dfrac{\left(\sqrt{\mu_1}\right)}{\left(\sqrt{\mu_1}+\sqrt{\mu_2}\right)}A_i$ (D) None of these

Comprehension 7

A narrow tube is bent in the form of circle of radius R as shown. Two small holes S and D are made in the tube at the positions right angles to each other. A source placed at S generates a wave of intensity I_0 which is equally divided into two parts. One part travels along the longer path, while the other travels along the shorter path. Both the part waves meet at point D where a detector is placed. Based on above information, answer the following questions.



20. Maximum intensity produced at D is given by
(A) $4I_0$ (B) $3I_0$
(C) $2I_0$ (D) I_0

21. The maximum value of wavelength λ to produce a maximum at D is given by
(A) πR (B) $2\pi R$
(C) $\dfrac{\pi R}{2}$ (D) $\dfrac{3\pi R}{2}$

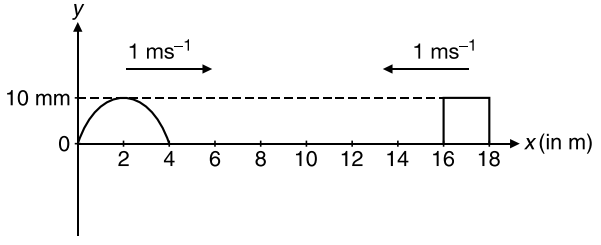
22. The maximum value of wavelength λ to produce a minimum at D is given by

(A) πR (B) $2\pi R$

(C) $\frac{\pi R}{2}$ (D) $\frac{3\pi R}{2}$

Comprehension 8

A sinusoidal pulse is generated at $x = 0$ and $t = 0$ on a string lying along x-axis. The pulse is travelling with speed $1\ \text{ms}^{-1}$. A rectangular pulse is generated at $t = 0$ s and is travelling in opposite direction with speed $1\ \text{ms}^{-1}$ as shown in figure. Based on the above facts, answer the following questions.



23. The velocity of particle at $x = 4$ m and $t = 0$ s is

(A) ZERO (B) $\frac{\pi}{4}\ \text{cms}^{-1}$

(C) $\frac{\pi}{2}\ \text{cms}^{-1}$ (D) $\frac{\pi}{6}\ \text{cms}^{-1}$

24. The displacement of particle at $x = 9$ m and $t = 8$ s is

(A) $\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right)$ cm (B) $(\sqrt{2}+1)$ cm

(C) $\left(\frac{3}{2}\right)$ cm (D) $(\sqrt{2})$ cm

25. The velocity of particle at $x = 9$ m and $t = 8$ s is

(A) $\frac{\pi}{4}\ \text{cms}^{-1}$ in negative y-direction

(B) $\frac{\pi}{4}\ \text{cms}^{-1}$ in positive y-direction

(C) $\frac{\pi}{4\sqrt{2}}\ \text{cms}^{-1}$ in negative y-direction

(D) $\frac{\pi}{2\sqrt{2}}\ \text{cms}^{-1}$ in negative y-direction

Comprehension 9

When a wire is fixed between two rigid supports, and we pluck the mid-point of string, it starts to vibrate between two extreme positions. During this oscillation, some points of string vibrate with maximum amplitude while other points vibrate with minimum amplitude. These are known as antinodes and nodes respectively. Suppose we have a 2 m wire which is fixed at both ends and which is vibrating in its fundamental mode, the tension in the wire is 40 N and the mass of the wire is 0.1 kg . At the mid-point of wire, the amplitude is 2 cm. At any instant the amplitude at x is calculated from the expression $y = 0.02\ \text{m}\sin\left(\frac{\pi x}{2}\right)$, then answer the following questions.

26. The distance between any two consecutive nodal points is

(A) $\frac{\lambda}{2}$ (B) λ

(C) 2λ (D) 3λ

27. The angular frequency of the wire is (approximately)

(A) 44 rad sec^{-1} (B) 88 rad sec^{-1}

(C) 22 rad sec^{-1} (D) 176 rad sec^{-1}

28. The kinetic energy of a particle of the wire is zero

(A) at the nodal points at all times

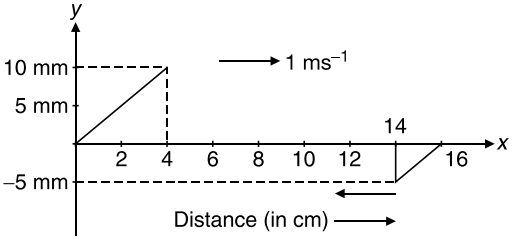
(B) at centre or position of antinodes at all times

(C) at $x = 1.5$ m at all times

(D) at $x = 0.5$ m at all times

Comprehension 10

Two wave pulses are travelling in opposite direction with speed $1\ \text{ms}^{-1}$. Figure shows the shape of pulse at $t = 0$. Based on the above facts, answer the following questions.



29. The speed of particle at $x = 2$ cm and $t = 0$ is

(A) $1\ \text{ms}^{-1}$ (B) $0.75\ \text{ms}^{-1}$

(C) $0.5\ \text{ms}^{-1}$ (D) $0.25\ \text{ms}^{-1}$

30. The displacement of particle at $x = 8$ cm and $t = 6$ sec is

(A) 10 mm (B) 5 mm

(C) –5 mm (D) ZERO

31. The speed of particle at $x = 8$ cm and $t = 6$ sec is

(A) ZERO (B) $0.125\ \text{ms}^{-1}$

(C) $0.25\ \text{ms}^{-1}$ (D) None of these

Comprehension 11

The displacement of the medium in a sound wave is given by the equation $y_i = A\cos(ax + bt)$ where A, a and b are positive constants. The wave is reflected by an obstacle situated a $x = 0$. The intensity of the reflected wave is 0.64 times that of the incident wave. Based on the above facts, answer the following questions.

32. The wavelength of the incident wave is

(A) $\lambda = \frac{\pi}{a}$ (B) $\lambda = \frac{\pi}{2a}$

(C) $\lambda = \frac{2\pi}{a}$ (D) $\lambda = \frac{4\pi}{a}$

33. The frequency of the incident wave is

(A) $f = \frac{b}{\pi}$ (B) $f = \frac{b}{2\pi}$

(C) $f = \frac{b}{4\pi}$ (D) $f = \frac{b}{3\pi}$

34. The equation for the reflected wave is

(A) $y_r = 0.8A\cos(ax - bt)$

(B) $y_r = -0.8A\cos(ax + bt)$

(C) $y_r = -0.8A\cos(ax - bt)$

(D) None of these

35. For the resultant wave formed after reflection, if v_{max} and v_{min} be the maximum and the minimum values of the particle speeds in the medium, then

(A) $v_{max} = 0.2$ Ab, $v_{min} = 0$

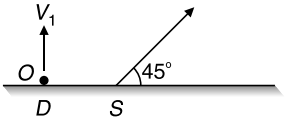
(B) $v_{max} = 0.8$ Ab, $v_{min} = 0.2$ Ab

(C) $v_{max} = -1.8$ Ab, $v_{min} = 0.2$ Ab

(D) $v_{max} = 1.8$ Ab, $v_{min} = 0$

Comprehension 12

Consider the situation in which a source emitting sound of frequency $f_0 = 960$ Hz is projected with velocity $10\sqrt{2}$ ms^{-1} at angle $\theta = 45°$ making with horizontal. Simultaneously a detector is projected at speed $v_1 = 10$ ms^{-1} vertically upwards as shown in the figure. (Take velocity of sound as 310 ms^{-1} and air to be still). After the source and detector fall to the ground, they come to rest.



Taking acceleration due to gravity as 10 ms^{-2}. Based on above information, answer the following questions.

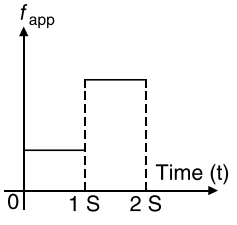
36. The maximum apparent frequency received by the detector is equal to

(A) 992 Hz (B) 930 Hz

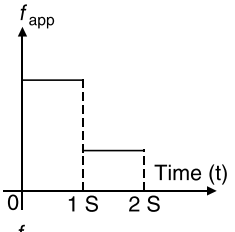
(C) 960 Hz (D) 682 Hz

37. The graph of f_{app} vs time is best represented as (for time $0 \le t \le 2$ s)

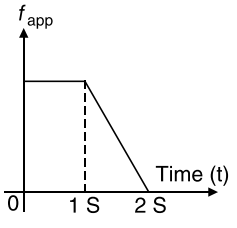
(A)



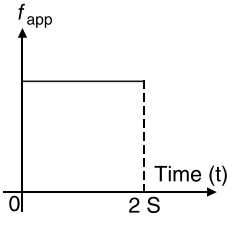
(B)



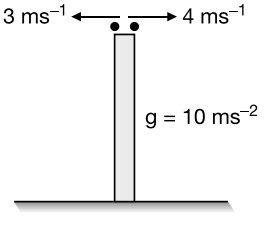
(C)



(D)



38. Suppose the source and detector are projected horizontally in opposite directions from a high tower as shown, then



(A) The apparent frequency received by detector will be constant till the source and detector fall on ground.

(B) The apparent frequency received by detector will continuously decrease till the source and detector fall on ground.

(C) The apparent frequency must be same as frequency emitted by source at least once during the complete journey.

(D) The apparent frequency received by detector will continuously increase till the source and detector fall on ground.

Comprehension 13

A number of waveforms can be described in terms of the combination of travelling waves which can be analysed by using the Principle of Superposition. Consider two wave pulses y_1 and y_2 described by $y_1 = \dfrac{5}{(3x-4t)^2+2}$ and $y_2 = \dfrac{-5}{(3x+4t-6)^2+2}$ travelling on same string. Based on the above facts, answer the following questions.

39. The direction in which each pulse is travelling is

(A) y_1 is in positive x-axis, y_2 is in positive x-axis

(B) y_1 is in negative x-axis, y_2 is in negative x-axis

(C) y_1 is in positive x-axis, y_2 is in negative x-axis

(D) y_1 is in negative x-axis, y_2 is in positive x-axis

40. The time when the two waves cancel everywhere is

(A) 1 s (B) 0.5 s

(C) 0.25 s (D) 0.75 s

41. The point where two waves always cancel is

(A) 0.25 m (B) 0.5 m

(C) 0.75 m (D) 1 m

Comprehension 14

A point sound source is situated in a medium of bulk modulus 1.6×10^5 Nm^{-2}. An observer standing at a distance 10 m from the source, writes down the equation for the wave as $y = A\sin(15\pi x - 6000\pi t)$, where y and x are in metre and t is in second. The maximum pressure amplitude received by the observer's ear is 24π Pa. Based on the above facts, answer the following questions.

42. The density of the medium is

(A) 1 kgm^{-3} (B) 2 kgm^{-3}

(C) 3 kgm^{-3} (D) $\frac{1}{2}$ kgm^{-3}

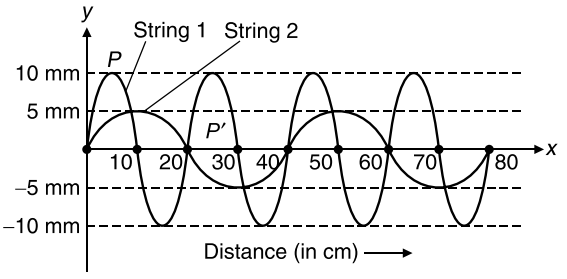
43. If another sound wave of the same form, having same phase is emitted from some other point, then the intensity of sound at a point which is equidistant from both the sources is

(A) $0.72\pi^2$ (B) $1.44\pi^2$

(C) $2.16\pi^2$ (D) $2.88\pi^2$

Comprehension 15

Two identical strings 1 and 2 are stretched by the same force and waves are generated in the two strings travelling in +ve x-direction. Shapes of string 1 and 2 at some instant is shown in figure. Based on the above facts, answer the following questions.



44. Ratio of intensity of wave in the string 1 and 2 is

(A) 1 : 1 (B) 1 : 4

(C) 16 : 1 (D) 4 : 1

45. Phase difference between point P (on first string) and P' (on second string) at the instant shown is

(A) $\frac{\pi}{2}$ (B) $\frac{2\pi}{3}$

(C) π (D) ZERO

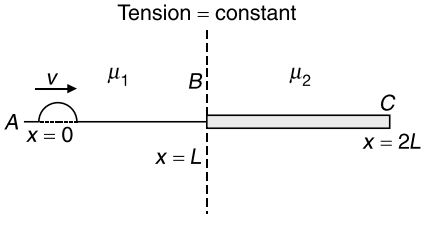
46. Phase difference between particles of string 1 and 2 at $x = 20$ cm at the instant shown is

(A) ZERO (B) $\frac{\pi}{2}$

(C) $\frac{2\pi}{3}$ (D) π

Comprehension 16

The speed of a transverse wave depends on the linear mass density of the string and the tension in the string. Consider two strings of equal lengths to be joined at B. The mass of the string BC is four times the mass of string AB. If a wave pulse is generated in string AB, which travels towards boundary at B with speed v and the equation of incident pulse is given by $y_i = A_i \sin(\omega t - kx)$. Based on above information, answer the following questions.



47. Amplitude of wave reflected back after incident on boundary at point B

(A) $-\frac{A_i}{3}$ (B) $-\frac{2A_i}{3}$

(C) $\frac{A_i}{3}$ (D) $\frac{2A_i}{3}$

48. Speed of transmitted wave on string BC is

(A) v (B) $\frac{v}{2}$

(C) $2v$ (D) None of these

49. Equations of reflected and transmitted waves respectively are

(A) $y_r = -\frac{A_i}{3}\sin(\omega t + kx),\ y_t = -\frac{2A_i}{3}\sin(\omega t - 2kx)$

(B) $y_r = \frac{A_i}{3}\sin(\omega t + kx),\ y_t = \frac{2A_i}{3}\sin(\omega t + 2kx)$

(C) $y_r = -\frac{A_i}{3}\sin(\omega t + kx),\ y_t = \frac{2A_i}{3}\sin(\omega t - 2kx)$

(D) $y_r = -\frac{A_i}{3}\sin(\omega t - kx),\ y_t = \frac{2A_i}{3}\sin\left(\omega t + \frac{k}{2}x\right)$

Comprehension 17

A thin string is held at one end and oscillated vertically, so that $y(x = 0,\ t) = 8\sin(4t)$ cm. The string's density is $0.2\ \text{kgm}^{-1}$ and its tension is 1 N. The string passes through a bath filled with 1 kg water. Due to friction, heat is transferred to the bath. The heat transfer efficiency is 50%. Specific heat of water is $1\ \text{kcalkg}^{-1}\text{K}^{-1}$. Based on the above facts, answer the following questions.

50. The wave function representing the wave on string is

(A) $y(x, t) = 8\sin\left(t - \frac{4}{\sqrt{5}}x\right)$

(B) $y(x, t) = 8\sin\left(4t - \frac{4}{\sqrt{5}}x\right)$

(C) $y(x, t) = 8\sin\left(4t - \frac{1}{\sqrt{5}}x\right)$

(D) $y(x, t) = 8\sin(t - x)$

51. The average power transported by wave over one period is

(A) 0.02 W (B) 0.2 W

(C) 2 W (D) 20 W

52. The time taken for the temperature of bath to rise by 1 K is

(A) 4×10^5 s (B) 40 s

(C) 4 s (D) 4×10^2 s

Comprehension 18

A train approaching a hill at a speed of $40\ \text{kmhr}^{-1}$ sounds a whistle of frequency 580 Hz when it is at a distance of 1 km from a hill. A wind with a speed of $40\ \text{kmhr}^{-1}$ is blowing in the direction of motion of the train. Taking velocity of sound in air = $1200\ \text{kmhr}^{-1}$. Based on the above facts, answer the following questions.

53. The frequency of the whistle as heard by an observer on the hill is approximately

(A) 499 Hz (B) 599 Hz

(C) 699 Hz (D) 799 Hz

54. The distance from the hill at which the echo from the hill is heard by the driver is

(A) 935 m (B) 900 m

(C) 1000 m (D) 1035 m

55. The echo frequency heard by the driver is approximately

(A) 321 Hz (B) 421 Hz

(C) 521 Hz (D) 621 Hz

MATRIX MATCH/COLUMN MATCH TYPE QUESTIONS

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

If the correct matches are A → p, s and t; B → q and r; C → p and q; and D → s and t; then the correct darkening of bubbles will look like the following:

	p	q	r	s	t
A	Ⓟ	Ⓠ	ⓡ	Ⓢ	Ⓣ
B	Ⓟ	Ⓠ	Ⓡ	Ⓢ	Ⓣ
C	Ⓟ	Ⓠ	ⓡ	Ⓢ	Ⓣ
D	Ⓟ	Ⓠ	ⓡ	Ⓢ	Ⓣ

1. Match the standing waves formed in **COLUMN-II** due to plane progressive waves and also the conditions in **COLUMN-I**.

COLUMN-I	COLUMN-II
(A) Incident wave is $y_i = A\sin(kx - \omega t)$	(p) $y = 2A\cos(kx)\sin(\omega t)$
(B) Incident wave is $y_i = A\cos(kx - \omega t)$	(q) $y = 2A\sin(kx)\sin(\omega t)$
(C) $x = 0$ is rigid support	(r) $y = 2A\sin(kx)\cos(\omega t)$
(D) $x = 0$ is flexible support	(s) $y = 2A\cos(kx)\cos(\omega t)$

2. The equation of a travelling wave is given by (all quantities are in SI units) $y = (0.02)\sin 2\pi(10t - 5x)$. Match the quantities in **COLUMN-I** with the SI values in **COLUMN-II**.

COLUMN-I	COLUMN-II (in SI units)
(A) Speed of wave	(p) 10
(B) Angular frequency of wave	(q) 0.4π
(C) Wavelength of wave	(r) 2
(D) Maximum particle speed	(s) 0.2

3. For a closed organ pipe, match the quantities in **COLUMN-I** with respective values in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Third overtone frequency is x times the fundamental frequency x, equals	(p) 3
(B) Number of nodes in second overtone	(q) 4
(C) Number of antinodes in third overtone	(r) 5
(D) Harmonic corresponding to second overtone	(s) 7

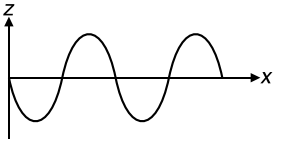
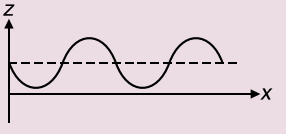
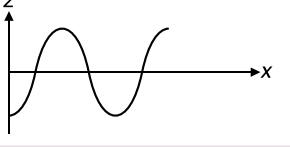
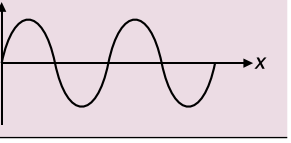
4. A wave is transmitted from a denser to rarer medium. Then match the following

COLUMN-I	COLUMN-II
(A) Frequency of wave	(p) will increase
(B) Speed of wave	(q) will decrease
(C) Wavelength of wave	(r) will remain unchanged
(D) Amplitude of wave	(s) may increase or decrease

5. The displacement equation of a standing wave in air is given by

$$y = A\cos(kx)\cos(\omega t)$$

Match the physical quantities in the **COLUMN-I**, to the correct plots in the **COLUMN-II**. Note that the physical quantities in **COLUMN-I** have been denoted by symbol Z in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Displacement y of the particles at $t = \frac{T}{2}$	(p) 
(B) Particle velocity at $t = \frac{T}{4}$	(q) 
(C) Change in pressure of the medium at $t = 0$	(r) 
(D) Density of the medium at $t = \frac{T}{2}$	(s) 

6. A source emits sound of frequency f. The source and listener both have same speed. For the apparent frequency heard by listener match the following

COLUMN-I	COLUMN-II
(A) Listener is approaching the source but source is receding from the listener	(p) more than f
(B) Listener and source both approaching towards each other	(q) less than f
(C) Listener and source both receding from each other	(r) equal to f
(D) Source is approaching but listener is receding	

7. From a single source, two wave trains are sent in two different strings. String 2 is 4 times heavy than string 1. The two wave equations are: (area of cross-sectional and tension of both strings is same).

$$y_1 = A\sin(\omega_1 t - k_1 x) \text{ and } y_2 = 2A\sin(\omega_2 t - k_2 x)$$

Suppose $u =$ energy density, $P =$ power transmitted, $I =$ intensity of the wave and v be velocity of the wave, then match the following

COLUMN-I	COLUMN-II
(A) $\frac{u_1}{u_2} =$	(p) $\frac{1}{8}$
(B) $\frac{P_1}{P_2} =$	(q) $\frac{1}{16}$
(C) $\frac{I_1}{I_2} =$	(r) 2
(D) $\frac{v_1}{v_2} =$	(s) $\frac{1}{4}$

8. In the equation, $y = A\sin 2\pi\left(ax + bt + \frac{\pi}{4}\right)$, match the quantities in **COLUMN-I** to the values in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Frequency of wave	(p) a
(B) Wavelength of wave	(q) b
(C) Phase difference between two points $\frac{1}{4a}$ distance apart	(r) π
(D) Phase difference of a point after a time interval of $\frac{1}{8b}$	(s) $\frac{\pi}{2}$
	(t) None of these

9. Match the quantities in **COLUMN-I** to respective properties in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) In refraction	(p) Speed of wave does not change
(B) In reflection	(q) Wavelength is decreased
(C) In refraction from rarer to denser medium	(r) Frequency does not change
(D) In reflection from a denser medium	(s) Phase change of π takes place

10. Regarding speed of sound in gas match the following

COLUMN-I	COLUMN-II
(A) Temperature of gas is made 4 times and pressure 2 times	(p) Speed becomes $2\sqrt{2}$ times
(B) Only pressure is made 4 time without change in temperature	(q) Speed becomes 2 times
(C) Only temperature is changed to 4 times	(r) Speed remains unchanged
(D) Molecular mass of the gas is made 4 times	(s) Speed remains half

11. The equation of a stationary wave (all quantities are in SI units) is given by $y = (0.06)\sin(2\pi x)\cos(5\pi t)$. Match the quantities in **COLUMN-I** to the values (in SI units) in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Amplitude of constituent wave is	(p) 0.06
(B) Position of node is at x_1, given by	(q) 0.5
(C) Position of anti node is at x_2, given by	(r) 0.25
(D) Amplitude at $x = \frac{3}{4}$ m is	(s) 0.03
	(t) None of these

12. Fundamental frequency of closed pipe is 100 Hz and that of an open pipe is 200 Hz. Match the following $\left(v_s = 330\ \text{ms}^{-1}\right)$

COLUMN-I	COLUMN-II
(A) Length of closed pipe	(p) 0.825 m
(B) Length of open pipe	(q) 1.65 m
(C) Lowest harmonic of closed pipe which is equal to any of the harmonic of open	(r) 2
(D) Ratio of second overtone of open pipe to the first overtone of closed pipe is	(s) 5
	(t) None of these

INTEGER/NUMERICAL ANSWER TYPE QUESTIONS

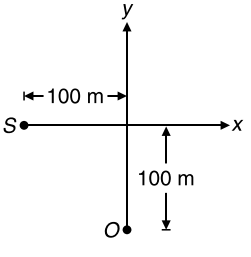
In this section, the answer to each question is a numerical value obtained after doing series of calculations based on the data given in the question(s).

1. A road passes at some distance from a standing man. A truck is coming on the road with some acceleration. The truck driver blows a whistle of frequency 500 Hz when the line joining the truck and the man makes an angle of θ with the road. The man hears a note having a frequency of 600 Hz when the truck is closest to him. Also, the speed of truck has got doubled during this time. Find the value of θ, in degree.

2. An organ pipe open at both ends sounds in unison with a tuning fork at 20 °C. When the fork and the pipe are sounded together at 30 °C, 5 beats per second are heard. Determine the frequency of the fork, in Hz, assuming that it is not affected by the temperature change.

3. A stationary source is emitting sound at a fixed frequency f_0, which is reflected by two cars approaching the source. The difference between the frequencies of sound reflected from the cars is 1.2% of f_0. What is the difference in the speeds of the cars (in km per hour) to the nearest integer? The cars are moving at constant speeds much smaller than the speed of sound which is 330 ms^{-1}.

4. At $t = 0$, a source of sonic oscillations S and an observer O start moving along x and y axes with velocity 5 ms^{-1} and 10 ms^{-1} respectively. The figure gives their position at $t = 0$. The frequency of sonic oscillations of source is 1000 Hz. Obtain the frequency of signals, in Hz, received by the observer after 5 seconds. Speed of sound is 330 ms^{-1}.



5. A simple harmonic wave of amplitude 8 units travels along positive x-axis. At any given instant of time, for a particle at a distance of 10 cm from the origin, the displacement is +6 units and for a particle at a distance of 25 cm from the origin, the displacement is +4 units. Calculate the wavelength, in cm.

6. In a stationary wave pattern that forms as a result of reflection of waves from an obstacle the ratio of the amplitude at an antinode and a node is $\beta = 1.5$. What percentage of the energy passes across the obstacle?

7. An astronaut approaching moon is sending radio signal of 5×10^9 Hz and finds out that the frequency shift of the echo received is 10^3 Hz. Calculate his speed of approach in ms^{-1}.

8. A guitar string is 90 cm long and has a fundamental frequency of 124 Hz. Where should it be pressed, in cm, from one end to produce a fundamental frequency of 186 Hz.

9. Two radio stations broadcast their programmes at the same amplitude A_0 and intensity I_0. Their frequency difference $f_1 - f_2$ is 10^3 Hz. A detector receives the signals from the two stations simultaneously. It can detect signals of intensity $\geq 2I_0$. Find the time, in μs, for which the detector remains idle in each cycle of the intensity of the signal.

10. A point source of sound emits a constant power with intensity inversely proportional to the square of the distance from the source. By how many decibels does the sound intensity level drop when you move from point P_1 to P_2. Distance of P_2 from the source is two times the distance of source from P_1.

11. Standing waves are set up in a string of length 240 cm clamped horizontally at both ends. The separation between any two consecutive points where the displacement amplitude is $3\sqrt{2}$ mm is 20 cm.
    (a) Find the maximum displacement amplitude, in mm.
    (b) Determine the overtone in which the string is vibrating.

12. A wave moves with speed 300 ms^{-1} on a wire which is under a tension of 50 N. By how much the tension must be changed, in newton, to increase the speed to 312 ms^{-1}?

13. A source of sound of frequency 900 Hz moves uniformly along a straight line with velocity 0.8 times velocity of sound. An observer is located at a distance $l = 250$ m from the line. Find the
    (a) frequency of the sound, in Hz, at the instant when the source is closest to the observer.
    (b) distance of the source, in metre, when he observes no change in the frequency.

14. At a distance 20 m from a point source of sound the loudness level is 30 dB. Neglecting the damping, find
(a) the loudness at 10 m from the source, in decibel.
(b) the distance from the source (in metre) at which sound is not heard.

15. A string has a linear mass density μ of 625 gm^{-1} and is stretched with a tension T of 49 N. A wave of frequency 125 Hz and amplitude 9 mm, is travelling along the string. At what average rate (in watt) is the wave transporting energy through the string?

16. A uniform rope of length 12 m and mass 6 kg hangs vertically from a rigid support. A block of mass 2 kg is attached to the free end of the rope. A transverse pulse of wavelength 0.06 m is produced at the lower end of the rope. What is the wavelength of the pulse, in cm, when it reaches the top of the rope.

17. A long glass tube is held vertically, dipping into water, while a tuning fork of frequency 500 Hz is repeatedly struck and held over the open end. Strong resonance is obtained, when the length of the tube above the surface of water is 60 cm and again 85 cm, but not at any intermediate point. Find the speed of sound in air, in ms^{-1}.

18. Calculate the velocity of transverse wave, in ms^{-1}, in a string of mass 3×10^{-2} kg length 2 mto which a load of 15 kg is attached. Take $g = 10\ ms^{-2}$

19. A sonometer wire has a total length of 1.1 m between the fixed ends. Where should the two bridges be placed from the ends, in cm, below the wire so that the three segments of the wire have their fundamental frequencies in the ratio 1 : 2 : 3?

20. A string of mass m is fixed at both ends. The fundamental tone oscillation are excited in the string with angular frequency ω and maximum displacement amplitude A. The total energy contained in the string is $\frac{m\omega^2A^2}{x}$, where x is an integer from zero to 9. Find x.

21. A taut string for which $\mu = 5\times10^{-2}\ kgm^{-1}$ is under a tension of 80 N. How much power, in watt, must be supplied to the string to generate sinusoidal waves at a frequency of 60 Hz and an amplitude of 6 cm?

22. The vibrations from an 800 Hz tuning fork set up standing waves in a string clamped at both ends. The wave speed in the string is known to be 400 ms^{-1} for the tension used. The standing wave is observed to have four antinodes. Calculate the length of the string in metre.

23. A wire made of material of Young's modulus $20\times10^{10}\ Nm^{-2}$ is stretched by 0.1%. If its length is one metre and density of the material of the wire is 8000 kgm^{-3}. Calculate the fundamental frequency of wire, in Hz.

24. Third overtone of a closed organ pipe of length 7 cm is in unison with fourth harmonic of an open organ pipe. Find the length of open organ pipe, in cm.

25. Calculate the speed of longitudinal waves, in ms^{-1}, in the following gases at 0 °C and 1 atm $(=10^5\ Pa)$.
(a) Oxygen for which the Bulk's modulus is 1.41×10^5 Pa and density is 1.43 kgm^{-3}.
(b) Helium for which the Bulk's modulus is 1.7×10^5 Pa and the density is 0.18 kgm^{-3}.

26. A string fixed at both ends is vibrating in the lowest possible mode of vibration for which a point at quarter of its length from one end is a point of maximum displacement. The frequency of vibration in this mode is 100 Hz. What will be the frequency emitted, in Hz, when it vibrates in the next mode such that this point is again a point of maximum displacement?

ARCHIVE: JEE MAIN

1. **[Online September 2020]**
Two identical strings X and Z made of same material have tension T_X and T_Z in them. It their fundamental frequencies are 450 Hz and 300 Hz, respectively, then the ratio $\frac{T_X}{T_Z}$ is

(A) 0.44 (B) 1.5
(C) 2.25 (D) 1.25

2. **[Online September 2020]**
A wire of density $9\times10^{-3}\ kgcm^{-3}$ is stretched between two clamps 1 m apart. The resulting strain in the wire is 4.9×10^{-4}. The lowest frequency of the transverse vibrations in the wire is (Young's modulus of wire $Y = 9\times10^{10}\ Nm^{-2}$) (to the nearest integer)_____.

3. **[Online September 2020]**
A uniform thin rope of length 12 m and mass 6 kg hangs vertically from a rigid support and a block of mass 2 kg is attached to its free end. A transverse short-wave train of wavelength 6 cm is produced at the lower end of the rope. What is the wavelength of the wave train (in cm) when it reaches the top of the rope?

(A) 9 (B) 12
(C) 6 (D) 3

4. **[Online September 2020]**
For a transverse wave travelling along a straight line, the distance between two peaks (crests) is 5 m, while the distance between one crest and one trough is 1.5 m. The possible wavelengths (in m) of the waves are

(A) 1, 2, 3, …

(B) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \ldots$

(C) 1, 3, 5, ….

(D) $\frac{1}{1}, \frac{1}{3}, \frac{1}{5}, \ldots$

5. [Online September 2020]

In a resonance tube experiment when the tube is filled with water up to height of 17.0 cm from bottom, it resonates with a given tuning fork. When the water level is raised the next resonance with the same tuning fork occurs at a height of 24.5 cm. If the velocity of sound in air is 330 ms^{-1}, the tuning fork frequency is

(A) 1100 Hz (B) 3300 Hz
(C) 2200 Hz (D) 550 Hz

6. [Online September 2020]

Assume that the displacement s of air is proportional to the pressure difference (Δp) created by a sound wave. Displacement s further depends on the speed of sound (v), density of air (ρ) and the frequency (f). If $\Delta p \sim 10$ Pa, $v \sim 300$ ms^{-1}, $p \sim 1$ kgm^{-3} and $f \sim 1000$ Hz, then s will be the order of (take multiplicative constant to be 1)

(A) 10 mm
(B) $\frac{3}{100}$ mm
(C) 1 mm
(D) $\frac{1}{10}$ mm

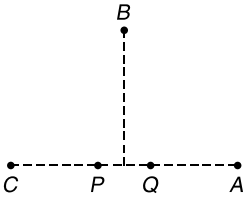
7. [Online September 2020]

A driver in a car, approaching a vertical wall notices that the frequency of his car horn, has changed from 440 Hz to 480 Hz, when it gets reflected from the wall. If the speed of sound in air is 345 ms^{-1}, then the speed of the car is

(A) 36 $kghr^{-1}$ (B) 24 $kmhr^{-1}$
(C) 18 $kmhr^{-1}$ (D) 54 $kmhr^{-1}$

8. [Online September 2020]

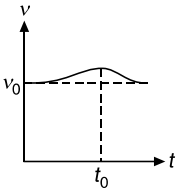
In the figure below, P and Q are two equally intense coherent sources emitting radiation of wavelength 20 m. The separation between P and Q is 5 m and the phase of P is ahead of that of Q by 90°. A, B and C are three distinct points of observation, each equidistant from the midpoint of PQ. The intensities of radiation at A, B, C will be in the ratio

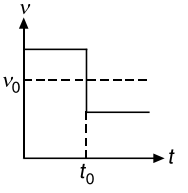


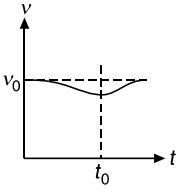
(A) 0 : 1 : 2
(B) 4 : 1 : 0
(C) 0 : 1 : 4
(D) 2 : 1 : 0

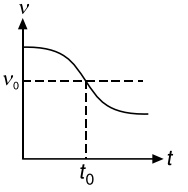
9. [Online September 2020]

A sound source S is moving along a straight track with speed v, and is emitting sound of frequency ν_0 (see figure). An observer is standing at a finite distance, at the point O, from the track. The time variation of frequency heard by the observer is best represented by: (t_0 represents the instant when the distance between the source and observer is minimum)

(A)


(B)


(C)


(D)


10. [Online January 2020]

Speed of a transverse wave on a straight wire (mass 6.0g, length 60 cm and area of cross-section 1.0 mm^2) is 90 ms^{-1}. If the Young's modulus of wire is 16×10^{11} Nm^{-2}, the extension of wire over its natural length is

(A) 0.02 mm (B) 0.04 mm
(C) 0.03 mm (D) 0.01 mm

11. [Online January 2020]

A stationary observer receives sound from two identical tuning forks, one of which approaches and the other one recedes with the same speed (much less than the speed of sound). The observer hears 2 bps. The oscillation frequency of each tuning fork is $\nu_0 = 1400$ Hz and the velocity of sound in air is 350 ms^{-1}. The speed of each tuning fork is close to

(A) $\frac{1}{8}$ ms^{-1} (B) $\frac{1}{2}$ ms^{-1}
(C) 1 ms^{-1} (D) $\frac{1}{4}$ ms^{-1}

12. [Online January 2020]

A one meter long (both ends open) organ pipe is kept in a gas that has double the density of air at STP. Assuming the speed of sound in air at STP is 300 ms^{-1}, the frequency difference between the fundamental and second harmonic of this pipe is_____ Hz.

13. [Online January 2020]

A transverse wave travels on a taut steel wire with a velocity of v when tension in it is 2.06×10^4 N. When the tension is changed to T, the velocity changed to $\frac{v}{2}$. The value of T is close to

(A) 10.2×10^2 N
(B) 5.15×10^3 N
(C) 2.50×10^4 N
(D) 30.5×10^4 N

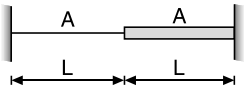
14. [Online January 2020]

Three harmonic waves having equal frequency v and same intensity I_0, have phase angles 0, $\frac{\pi}{4}$ and $-\frac{\pi}{4}$ respectively. When they are superimposed the intensity of the resultant wave is close to

(A) $5.8I_0$ (B) $0.2I_0$
(C) I_0 (D) $3I_0$

15. [Online January 2020]
A wire of length L and mass per unit length 6.0×10^{-3} kgm^{-1} is put under tension of 540 N. Two consecutive frequencies that it resonates at are 420 Hz and 490 Hz. Then L in meters is
(A) 8.1 m (B) 5.1 m
(C) 1.1 m (D) 2.1 m

16. [Online April 2019]
A wire of length $2L$, is made by joining two wires A and B of same length but different radii r and $2r$ and made of the same material. It is vibrating at a frequency such that the joint of the two wires forms a node. If the number of anti-nodes in wire A is p and that in B is q then the ratio $p:q$ is



(A) 4 : 9 (B) 1 : 2
(C) 3 : 5 (D) 1 : 4

17. [Online April 2019]
The pressure wave, $P=0.01\sin[1000t-3x]$ Nm^{-2}, corresponds to the sound produced by a vibrating blade on a day when atmospheric temperature is 0 °C. On some other day when temperature is T, the speed of sound produced by the same blade and at the same frequency is found to be 336 ms^{-1}. Approximate value of T is
(A) 4 °C (B) 12 °C
(C) 15 °C (D) 11 °C

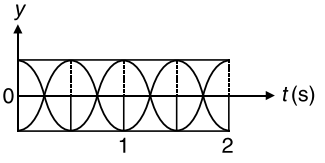
18. [Online April 2019]
A string is clamped at both the ends and it is vibrating in its 4th harmonic. The equation of the stationary wave is $y=0.3\sin(0.157x)\cos(200\pi t)$. The length of the string is (All quantities are in SI units)
(A) 60 m (B) 20 m
(C) 40 m (D) 80 m

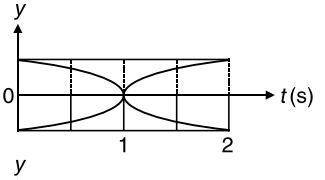
19. [Online April 2019]
Two cars A and B are moving away from each other in opposite directions. Both the cars are moving with a speed of 20 ms^{-1} with respect to the ground. If an observer in car A detects a frequency 2000 Hz of the sound coming from car B, what is the natural frequency of the sound source in car B? (speed of sound in air = 340 ms^{-1})
(A) 2150 Hz (B) 2300 Hz
(C) 2060 Hz (D) 2250 Hz

20. [Online April 2019]
A string 2.0 m long and fixed at its ends is driven by a 240 Hz vibrator. The string vibrates in its third harmonic mode. The speed of the wave and its fundamental frequency is
(A) 320 ms^{-1}, 120 Hz (B) 320 ms^{-1}, 80 Hz
(C) 180 ms^{-1}, 80 Hz (D) 180 ms^{-1}, 120 Hz

21. [Online April 2019]
A stationary source emits sound waves of frequency 500 Hz. Two observers moving along a line passing through the source detect sound to be of frequencies 480 Hz and 530 Hz. Their respective speeds are …… in ms^{-1}. (Given speed of sound = 300 ms^{-1})
(A) 12, 16 (B) 16, 14
(C) 8, 18 (D) 12, 18

22. [Online April 2019]
The correct figure that shows, schematically, the wave pattern produced by superposition of two waves of frequencies 9 Hz and 11 Hz, is

(A)


(B)


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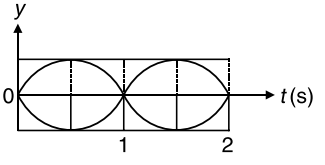

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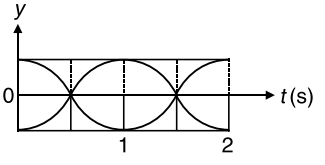


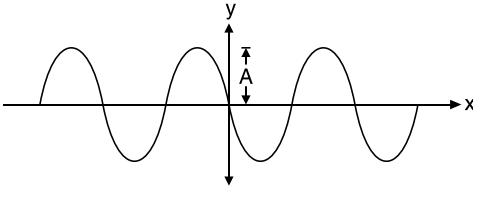
23. [Online April 2019]
A source of sound S is moving with a velocity of 50 ms^{-1} towards a stationary observer. The observer measures the frequency of the source as 1000 Hz. What will be the apparent frequency of the source when it is moving away from the observer after crossing him? (Take velocity of sound in air is 350 ms^{-1})
(A) 857 Hz (B) 1143 Hz
(C) 807 Hz (D) 750 Hz

24. [Online April 2019]
A submarine (A) travelling at 18 kmh^{-1} is being chased along the line of its velocity by another submarine (B) travelling at 27 kmh^{-1}. B sends a sonar signal of 500 Hz to detect A and receives a reflected sound of frequency ν. The value of ν is close to (Speed of sound in water = 1500 ms^{-1})
(A) 499 Hz (B) 504 Hz
(C) 507 Hz (D) 502 Hz

25. [Online April 2019]
A progressive wave travelling along the positive x-direction is represented by

$$y(x,\ t)=A\sin(kx-\omega t+\phi)$$

Its snapshot at $t=0$ is given in the Figure.



For this wave, the phase ϕ is

(A) π (B) $-\frac{\pi}{2}$

(C) $\frac{\pi}{2}$ (D) 0

26. [Online April 2019]

A small speaker delivers 2 W of audio output. At what distance from the speaker will one detect 120 dB intensity sound? [Given reference intensity of sound as 10^{-12} Wm^{-2}]

(A) 30 cm (B) 40 cm

(C) 10 cm (D) 20 cm

27. [Online April 2019]

Two sources of sound S_1 and S_2 produce sound waves of same frequency 660 Hz. A listener is moving from source S_1 towards S_2 with a constant speed u ms^{-1} and he hears 10 beats/s. The velocity of sound is 330 ms^{-1}. Then, u equals

(A) 10.0 ms^{-1} (B) 5.5 ms^{-1}

(C) 2.5 ms^{-1} (D) 15.0 ms^{-1}

28. [Online April 2019]

A tuning fork of frequency 480 Hz is used in an experiment for measuring speed of sound (v) in air by resonance tube method. Resonance is observed to occur at two successive lengths of the air column, $l_1 = 30$ cm and $l_2 = 70$ cm. Then, v is equal to

(A) 332 ms^{-1} (B) 384 ms^{-1}

(C) 379 ms^{-1} (D) 338 ms^{-1}

29. [Online January 2019]

A heavy ball of mass M is suspended from the ceiling of a car by a light string of mass $m (m \ll M)$. When the car is at rest, the speed of transverse waves in the string is 60 ms^{-1}. When the car has acceleration a, the wave-speed increases to 60.5 ms^{-1}. The value of a, in terms of gravitational acceleration g, is closest to

(A) $\frac{g}{30}$ (B) $\frac{g}{5}$

(C) $\frac{g}{20}$ (D) $\frac{g}{10}$

30. [Online January 2019]

A musician using an open flute of length 50 cm produces second harmonic sound waves. A person runs towards the musician from another end of a hall at a speed of 10 kmh^{-1}. If the wave speed is 330 ms^{-1}, the frequency heard by the running person shall be close to

(A) 500 Hz (B) 753 Hz

(C) 333 Hz (D) 666 Hz

31. [Online January 2019]

A train moves towards a stationary observer with speed 34 ms^{-1}. The train sounds a whistle and its frequency registered by the observer is f_1. If the speed of the train is reduced to 17 ms^{-1}, the frequency registered is f_2. If speed of sound is 340 ms^{-1}, then the ratio $\frac{f_1}{f_2}$ is

(A) $\frac{21}{20}$ (B) $\frac{20}{19}$

(C) $\frac{18}{17}$ (D) $\frac{19}{18}$

32. [Online January 2019]

A string of length 1 m and mass 5 g is fixed at both ends. The tension in the string is 8.0 N. The string is set into vibration using an external vibrator of frequency 100 Hz. The separation between successive nodes on the string is close to

(A) 33.3 cm (B) 10.0 cm

(C) 16.6 cm (D) 20.0 cm

33. [Online January 2019]

Equation of travelling wave on a stretched string of linear density 5 gm^{-1} is $y = 0.03\sin(450t - 9x)$ where distance and time are measured in SI units. The tension in the string is

(A) 10 N (B) 7.5 N

(C) 5 N (D) 12.5 N

34. [Online January 2019]

A person standing on an open ground hears the sound of a jet aeroplane, coming from north at an angle 60° with ground level. But he finds the aeroplane right vertically above his position. If v is the speed of sound, speed of the plane is

(A) $\frac{2v}{\sqrt{3}}$ (B) $\frac{\sqrt{3}}{2}v$

(C) $\frac{v}{2}$ (D) v

35. [Online January 2019]

A travelling harmonic wave is represented by the equation $y(x, t) = 10^{-3}\sin(50t + 2x)$, where x and y are in meter and t is in seconds. Which of the following is a correct statement about the wave?

(A) The wave is propagating along the negative x-axis with speed 25 ms^{-1}

(B) The wave is propagating along the positive x-axis with speed 100 ms^{-1}

(C) The wave is propagating along the negative x-axis with speed 100 ms^{-1}

(D) The wave is propagating along the positive x-axis with speed 25 ms^{-1}

36. [Online January 2019]

A resonance tube is old and has jagged end. It is still used in the laboratory to determine velocity of sound in air. A tuning fork of frequency 512 Hz produces first resonance when the tube is filled with water to a mark 11 cm below a reference mark, near the open end of the tube. The experiment is repeated with another fork of frequency 256 Hz which produces first resonance when water reaches a mark 27 cm below the reference mark. The velocity of sound in air, obtained in the experiment, is close to

(A) 322 ms^{-1}

(B) 341 ms^{-1}

(C) 328 ms^{-1}

(D) 335 ms^{-1}

37. [2018]
A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is 2.7×10^{3} kgm^{-3} and its Young's modulus is 9.27×10^{10} Pa. What will be the fundamental frequency of the longitudinal vibrations?
(A) 5 kHz (B) 2.5 kHz
(C) 10 kHz (D) 7.5 kHz

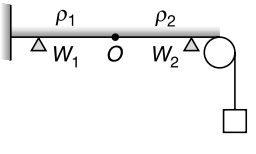
38. [Online 2018]
A tuning fork vibrates with frequency 256 Hz and gives one beat per second with the third normal mode of vibration of an open pipe. What is the length of the pipe? (Speed of sound in air is 340 ms^{-1})
(A) 190 cm (B) 180 cm
(C) 200 cm (D) 220 cm

39. [Online 2018]
5 beats/second are heard when a tuning fork is sounded with a sonometer wire under tension, when the length of the sonometer wire is either 0.95 m or 1 m. The frequency of the fork will be
(A) 251 Hz (B) 300 Hz
(C) 195 Hz (D) 150 Hz

40. [Online 2018]
Two sitar strings, A and B, playing the note Dha are slightly out of tune and produce beats of frequency 5 Hz. The tension of the string B is slightly increased and the beat frequency is found to decrease by 3 Hz. If the frequency of A is 425 Hz, the original frequency of B is
(A) 428 Hz (B) 430 Hz
(C) 420 Hz (D) 422 Hz

41. [Online 2018]
The end correction of a resonance column is 1 cm. If the shortest length resonating with the tuning fork is 10 cm, the next resonating length should be
(A) 36 cm (B) 40 cm
(C) 28 cm (D) 32 cm

42. [Online 2017]
Two wires W_1 and W_2 have the same radius r and respective densities ρ_1 and ρ_2 such that $\rho_2 = 4\rho_1$. They are joined together at the point O, as shown in the figure. The combination is used as a sonometer wire and kept under tension T. The point O is midway between the two bridges. When a stationary wave is set up in the composite wire, the joint is found to be a node. The ratio of the number of antinodes formed in W_1 to W_2 is



(A) 1:1 (B) 1:2
(C) 1:3 (D) 4:1

43. [Online 2017]
A standing wave is formed by the superposition of two waves travelling in opposite directions. The transverse displacement is given by

$$y(x,\,t)=0.5\sin\left(\frac{5\pi}{4}x\right)\cos(200\pi t)$$

What is the speed of the travelling wave moving in the positive x direction? (x and t are in meter and second, respectively.)
(A) 180 ms^{-1} (B) 160 ms^{-1}
(C) 120 ms^{-1} (D) 90 ms^{-1}

44. [2016]
A uniform string of length 20 m is suspended from a rigid support. A short-wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the support is (take $g = 10$ ms^{-2})
(A) $\sqrt{2}$ s (B) $2\pi\sqrt{2}$ s
(C) 2 s (D) $2\sqrt{2}$ s

45. [2016, 2012]
A pipe open at both ends has a fundamental frequency f in air. The pipe is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column is now
(A) f (B) $\frac{f}{2}$
(C) $\frac{3f}{4}$ (D) $2f$

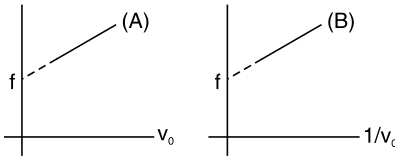
46. [Online 2016]
Two engines pass each other moving in opposite directions with uniform speed of 30 ms^{-1}. One of them is blowing a whistle of frequency 540 Hz. Calculate the frequency heard by driver of second engine before they pass each other. Speed of sound is 330 ms^{-1}.
(A) 450 Hz
(B) 540 Hz
(C) 270 Hz
(D) 648 Hz

47. [Online 2016]
A toy car blowing its horn is moving with a steady speed of 5 ms^{-1} away from a wall. An observer, towards whom the toy car is moving, is able to hear 5 bps. If the velocity of sound in air is 340 ms^{-1}, the frequency of the horn of the toy car is close to
(A) 680 Hz (B) 510 Hz
(C) 340 Hz (D) 170 Hz

48. [2015]
A train is moving on a straight track with speed 20 ms^{-1}. It is blowing its whistle at the frequency of 1000 Hz. The percentage change in the frequency heard by a person standing near the track as the train passes him is (speed of sound = 320 ms^{-1}) close to
(A) 6% (B) 12%
(C) 18% (D) 24%

49. [Online 2015]
A bat moving at $10\ \text{ms}^{-1}$ towards a wall sends a sound signal of $8000\ \text{Hz}$ towards it. On reflection it hears a sound of frequency f. The value of f in Hz is close to (speed of sound $= 320\ \text{ms}^{-1}$)
(A) 8258 (B) 8516
(C) 8000 (D) 8424

50. [Online 2015]
A source of sound emits sound waves at frequency f_0. It is moving towards an observer with fixed speed $v_s (v_s < v$, where v is the speed of sound in air). If the observer were to move towards the source with speed v_0,one of the following two graphs (A and B) will give the correct variation of the frequency f heard by the observer as v_0 is changed. The variation of f with v_0 is given correctly by



(A) graph A with slope $= \dfrac{f_0}{(v - v_s)}$
(B) graph A with slope $= \dfrac{f_0}{(v + v_s)}$
(C) graph B with slope $= \dfrac{f_0}{(v - v_s)}$
(D) graph B with slope $= \dfrac{f_0}{(v + v_s)}$

51. [2014]
A pipe of length $85\ \text{cm}$ is closed from one end. Find the number of possible natural oscillations of air column in the pipe whose frequencies lie below $1250\ \text{Hz}$. The velocity of sound in air is $340\ \text{ms}^{-1}$.
(A) 4 (B) 12
(C) 8 (D) 6

52. [2013]
A sonometer wire of length $1.5\ \text{m}$ is made of steel. The tension in it produces an elastic strain of 1%. What is the fundamental frequency of steel if density and elasticity of steel are $7.7 \times 10^3\ \text{kgm}^{-3}$ and $2.2 \times 10^{11}\ \text{Nm}^{-2}$ respectively?
(A) 188.5 Hz (B) 178.2 Hz
(C) 200.5 Hz (D) 770 Hz

53. [2011]
The transverse displacement $y(x, t)$ of a wave on a string is given by $y(x, t) = e^{-(ax^2 + bt^2 + 2\sqrt{ab}xt)}$. This represents a
(A) wave moving in $+x$ direction with speed $\sqrt{\dfrac{a}{b}}$
(B) wave moving in $-x$ direction with speed $\sqrt{\dfrac{b}{a}}$
(C) standing wave of frequency $\sqrt{b}$
(D) standing wave of frequency $\dfrac{1}{\sqrt{b}}$

54. [2010]
The equation of a wave on a string of linear mass density $0.04\ \text{kgm}^{-1}$ is given by

$$y = 0.02(\text{m})\sin\left[2\pi\left(\frac{t}{0.04(\text{s})} - \frac{x}{0.50(\text{m})}\right)\right]$$

The tension in the string is
(A) 6.25 N (B) 4.0 N
(C) 12.5 N (D) 0.5 N

55. [2009]
Three sound waves of equal amplitudes have frequencies $(\nu - 1), \nu, (\nu + 1)$. They superpose to give beats. The number of beats produced per second will be
(A) 4 (B) 3
(C) 2 (D) 1

56. [2009]
A motor cycle starts from rest and accelerates along a straight path at $2\ \text{ms}^{-2}$. At the starting point of the motor cycle there is a stationary electric siren. How far has the motor cycle gone when the driver hears the frequency of the siren at 94% of its value when the motor cycle was at rest? (Speed of sound $= 330\ \text{ms}^{-1}$).
(A) 49 m (B) 98 m
(C) 147 m (D) 196 m

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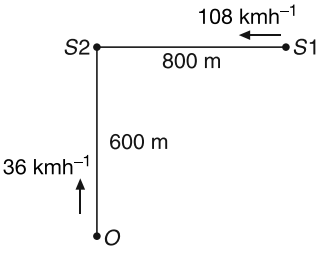
Single Correct Choice Type Problems

(In this section each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct)

1. [IIT-JEE 2012]
A student is performing the experiment of resonance column. The diameter of the column tube is $4\ \text{cm}$. The frequency of the tuning fork is $512\ \text{Hz}$. The air temperature is $38\ ^\circ\text{C}$ in which the speed of sound is $336\ \text{ms}^{-1}$. The zero of the meter scale coincides with the top end of the resonance column tube. When the first resonance occurs, the reading of the water level in the column is
(A) 14 cm (B) 15.2 cm
(C) 16.4 cm (D) 17.6 cm

2. [IIT-JEE 2010]
A hollow pipe of length $0.8\ \text{m}$ is closed at one end. At its open end a $0.5\ \text{m}$ long uniform string is vibrating in its second harmonic and it resonates with the fundamental frequency of the pipe. If the tension in the wire is $50\ \text{N}$ and the speed of sound is $320\ \text{ms}^{-1}$, the mass of the string is
(A) 5 gram (B) 10 gram
(C) 20 gram (D) 40 gram

3. [IIT-JEE 2008]
A transverse sinusoidal wave moves along a string in the positive x-direction at a speed of $10\ \text{cms}^{-1}$. The wavelength of the wave is $0.5\ \text{m}$ and its amplitude is $10\ \text{cm}$. At a particular time t, the snap-shot of the wave is shown in figure. The velocity of point P when its displacement is $5\ \text{cm}$ is



(A) $\frac{\sqrt{3}\pi}{50}\hat{j}$ ms^{-1} (B) $-\frac{\sqrt{3}\pi}{50}\hat{j}$ ms^{-1}

(C) $\frac{\sqrt{3}\pi}{50}\hat{i}$ ms^{-1} (D) $-\frac{\sqrt{3}\pi}{50}\hat{i}$ ms^{-1}

4. [IIT-JEE 2008]

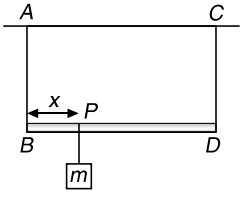
A vibrating string of certain length l under a tension T resonates with a mode corresponding to the first overtone (third harmonic) of an air column of length 75 cm inside a tube closed at one end. The string also generates 4 bps when excited along with a tuning fork of frequency n. Now when the tension of the string is slightly increased the number of beats reduces to 2 bps. Assuming the velocity of sound in air to be 340 ms^{-1}, the frequency n of the tuning fork in Hz is

(A) 344 (B) 336

(C) 117.3 (D) 109.3

5. [IIT-JEE 2006]

A massless rod BD is suspended by two identical massless strings AB and CD of equal lengths. A block of mass m is suspended from point P such that BP is equal to x. If the fundamental frequency of the left wire is twice the fundamental frequency of right wire, then the value of x is



(A) $\frac{l}{5}$ (B) $\frac{l}{4}$

(C) $\frac{4l}{5}$ (D) $\frac{3l}{4}$

6. [IIT-JEE 2005]

A tuning fork of 512 Hz is used to produce resonance in a resonance tube experiment. The level of water at first resonance is 30.7 cm and at second resonance is 63.2 cm. Assuming that the speed of sound is 330 ms^{-1}, the error in calculating speed of sound is

(A) 204.1 cms^{-1} (B) 110 cms^{-1}

(C) 58 cms^{-1} (D) 280 cms^{-1}

7. [IIT-JEE 2005]

An open pipe is in resonance in 2nd harmonic with frequency f_1. Now one end of the tube is closed and frequency is increased to f_2 such that the resonance again occurs in nth harmonic. Choose the correct option

(A) $n=3,\ f_2=\frac{3}{4}f_1$ (B) $n=3,\ f_2=\frac{5}{4}f_1$

(C) $n=5,\ f_2=\frac{5}{4}f_1$ (D) $n=5,\ f_2=\frac{3}{4}f_1$

8. [IIT-JEE 2004]

A source of sound of frequency 600 Hz is placed inside water. The speed of sound in water is 1500 ms^{-1} and in air it is 300 ms^{-1}. The frequency of sound recorded by an observer who is standing in air is

(A) 200 Hz (B) 3000 Hz

(C) 120 Hz (D) 600 Hz

9. [IIT-JEE 2004]

A closed organ pipe of length L and an open organ pipe contain gases of densities ρ_1 and ρ_2 respectively. The compressibility of gases are equal in both the pipes. Both the pipes are vibrating in their first overtone with same frequency. The length of the open organ pipe is

(A) $\frac{L}{3}$ (B) $\frac{4L}{3}$

(C) $\frac{4L}{3}\sqrt{\frac{\rho_1}{\rho_2}}$ (D) $\frac{4L}{3}\sqrt{\frac{\rho_2}{\rho_1}}$

10. [IIT-JEE 2003]

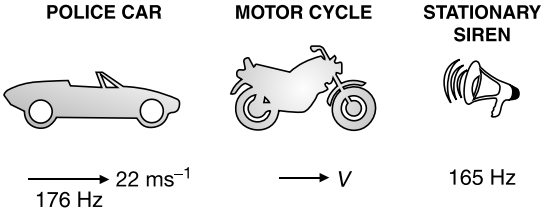
In the experiment for the determination of the speed of sound in air using the resonance column method, the length of the air column that resonates in the fundamental mode, with a tuning fork is 0.1 m. When this length is changed to 0.35 m, the same tuning fork resonates with the first overtone. Calculate the end correction

(A) 0.012 m (B) 0.025 m

(C) 0.05 m (D) 0.024 mg

11. [IIT-JEE 2003]

A police car moving at 22 ms^{-1} chases a motorcyclist. The police man sounds his horn at 176 Hz, while both of them move towards a stationary siren of frequency 165 Hz. Calculate the speed of the motorcycle. If its is given that the motorcyclist does not observe any beats



(A) 33 ms^{-1} (B) 22 ms^{-1}

(C) ZERO (D) 11 ms^{-1}

12. [IIT-JEE 2002]

A sonometer wire resonates with a given tuning fork forming standing waves with five antinodes between the two bridges when a mass of 9 kg is suspended from the wire. When this mass is replaced by M, the wire resonates with the same tuning fork forming three antinodes for the same position of bridges. The value of M is

(A) 25 kg (B) 5 kg

(C) 12.5 kg (D) $\frac{1}{25}$ kg

13. [IIT-JEE 2002]

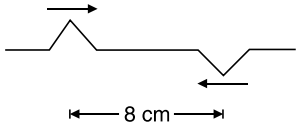
A siren placed at a railway platform is emitting sound of frequency 5 kHz. A passenger sitting in a moving train A records a frequency of 5.5 kHz, while the train approaches the siren. During his return journey in a different train B he records a frequency of 6 kHz while approaching the same siren. The ratio of the velocity of train B to that train A is

(A) $\frac{242}{252}$ (B) 2

(C) $\frac{5}{6}$ (D) $\frac{11}{6}$

14. [IIT-JEE 2001]

Two pulses in a stretched string, whose centres are initially 8 cm apart, are moving towards each other as shown in the figure. The speed of each pulse is 2 cms^{-1}. After 2 seconds the total energy of the pulses will be



(A) zero
(B) purely kinetic
(C) purely potential
(D) partly kinetic and partly potential

15. [IIT-JEE 2001]

The ends of a stretched wire of length L are fixed at $x = 0$ and $x = L$. In one experiment the displacement of the wire is $y_1 = A\sin\left(\frac{\pi x}{L}\right)\sin\omega t$ and energy is E_1 and in other experiment its displacement is $y_2 = A\sin\left(\frac{2\pi x}{L}\right)\sin 2\omega t$ and energy is E_2. Then

(A) $E_2 = E_1$ (B) $E_2 = 2E_1$

(C) $E_2 = 4E_1$ (D) $E_2 = 16E_1$

16. [IIT-JEE 2000]

Two vibrating strings of the same material but of lengths L and $2L$ have radii $2r$ and r respectively. They are stretched under the same tension. Both the strings vibrate in their fundamental modes, the one of length L with frequency v_1 and the other with frequency v_2. The ratio $\frac{v_1}{v_2}$ is given by

(A) 2 (B) 4

(C) 8 (D) 1

17. [IIT-JEE 2000]

A train moves towards a stationary observer with speed 34 ms^{-1}. The train sounds a whistle and its frequency registered by the observer is f_1. If the train's speed is reduced to 17 ms^{-1}, the frequency registered is f_2. If the speed of sound is 340 ms^{-1} then the ratio $\frac{f_1}{f_2}$ is

(A) $\frac{18}{19}$ (B) $\frac{1}{2}$

(C) 2 (D) $\frac{19}{18}$

18. [IIT-JEE 2000]

Two monoatomic ideal gases 1 and 2 of molecular masses m_1 and m_2 respectively are enclosed in separate containers kept at the same temperature. The ratio of the speed of sound in gas 1 to that in gas 2 is given by

(A) $\sqrt{\frac{m_1}{m_2}}$ (B) $\sqrt{\frac{m_2}{m_1}}$

(C) $\frac{m_1}{m_2}$ (D) $\frac{m_2}{m_1}$

19. [IIT-JEE 1999]

The ratio of the speed of sound in nitrogen gas to that in helium gas, at 300 K is

(A) $\sqrt{\left(\frac{2}{7}\right)}$ (B) $\sqrt{\left(\frac{1}{7}\right)}$

(C) $\frac{\left(\sqrt{3}\right)}{5}$ (D) $\frac{\left(\sqrt{6}\right)}{5}$

20. [IIT-JEE 1998]

A string of length 0.4 m and mass 10^{-2} kg is tightly clamped at its ends. The tension in the string is 1.6 N. Identical wave pulses are produced at one end at equal intervals of time Δt. The value of Δt which allows constructive interference between successive pulses is

(A) 0.05 s (B) 0.10 s

(C) 0.20 s (D) 0.40 s

21. [IIT-JEE 1997]

A travelling wave in a stretched string is described by the equation, $y = A\sin(kx - \omega t)$. The maximum particle velocity is

(A) $A\omega$ (B) $\frac{\omega}{k}$

(C) $\frac{d\omega}{dk}$ (D) $\frac{x}{\omega}$

22. [IIT-JEE 1997]

A whistle giving out 450 Hz approaches a stationary observer at a speed of 33 ms^{-1}. The frequency heard by the observer (in Hz) is

(A) 409 (B) 429

(C) 517 (D) 500

23. [IIT-JEE 1996]

The extension in a string, obeying Hooke's Law, is x. The speed of sound in the stretched string is v. If the extension in the string is increased to $1.5x$, the speed of sound will be

(A) $1.22v$ (B) $0.61v$

(C) $1.50v$ (D) $0.75v$

24. [IIT-JEE 1996]

An open pipe is suddenly closed at one end with the result that the frequency of third harmonic of the closed pipe is found to be higher by 100 Hz than the fundamental frequency of the open pipe. The fundamental frequency of the open pipe is

(A) 200 Hz (B) 300 Hz

(C) 240 Hz (D) 480 Hz

25. [IIT-JEE 1995]
An object of specific gravity ρ is hung from a thin steel wire. The fundamental frequency for transverse standing waves in the wire is 300 Hz. The object is immersed in water, so that one half of its volume is submerged. The new fundamental frequency (in Hz) is

(A) $300\left(\frac{2\rho-1}{2\rho}\right)^{1/2}$ (B) $300\left(\frac{2\rho}{2\rho-1}\right)^{1/2}$

(C) $300\left(\frac{2\rho}{2\rho-1}\right)$ (D) $300\left(\frac{2\rho-1}{2\rho}\right)$

26. [IIT-JEE 1992]
The displacement y of a particle executing periodic motion is given by

$$y = 4\cos^2\left(\frac{1}{2}t\right)\sin(1000t)$$

This expression may be considered to be a result of the superposition of

(A) two (B) three
(C) four (D) five

independent harmonic motions.

27. [IIT-JEE 1988]
An organ pipe P_1 closed at one end vibrating in its first harmonic and another pipe P_2 open at ends vibrating in its third harmonic are in resonance with a given tuning fork. The ratio of the length of P_1 and P_2 is

(A) $\frac{8}{3}$ (B) $\frac{3}{8}$

(C) $\frac{1}{6}$ (D) $\frac{1}{3}$

28. [IIT-JEE 1988]
A wave represented by the equation $y = a\cos(kx-\omega t)$ is superposed with another wave to form a stationary wave such that the point $x=0$ is a node. The equation for the other wave is

(A) $y = a\sin(kx\times\omega t)$ (B) $y = -a\cos(kx+\omega t)$
(C) $y = -a\cos(kx-\omega t)$ (D) $y = -a\sin(kx-\omega t)$

29. [IIT-JEE 1987]
The displacement of particles in a string stretched in the x-direction is represented by y. Among the following expressions for y, those describing wave motion are

(A) $\cos kx\sin\omega t$ (B) $k^2x^2-\omega^2t^2$
(C) $\cos^2(kx+\omega t)$ (D) $\cos(k^2x^2-\omega^2t^2)$

30. [IIT-JEE 1986]
A tube, closed at one end and containing air, produces, when excited, the fundamental note of frequency 512 Hz. If the tube is opened at both ends the fundamental frequency that can be excited is (in Hz)

(A) 1024 (B) 512
(C) 256 (D) 128

31. [IIT-JEE 1984]
A transverse wave is described by the equation $y = y_0\sin 2\pi\left(ft-\frac{x}{\lambda}\right)$. The maximum particle velocity is equal to four times the wave velocity if

(A) $\lambda = \frac{\pi y_0}{4}$ (B) $\lambda = \frac{\pi y_0}{2}$
(C) $\lambda = \pi y_0$ (D) $\lambda = 2\pi\, y_0$

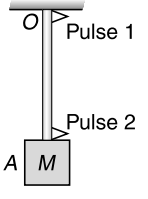
32. [IIT-JEE 1981]
A cylindrical tube, open at both ends, has a fundamental frequency f in air. The tube is dipped vertically in water so that half of its in water. The fundamental frequency of the air column in now

(A) $\frac{f}{2}$ (B) $\frac{3f}{4}$
(C) f (D) $2f$

Multiple Correct Choice Type Problems

(In this section each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct)

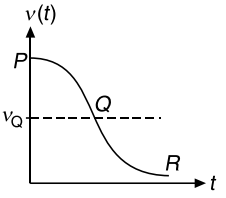
1. [JEE (Advanced) 2017]
A block M hangs vertically at the bottom end of a uniform rope of constant mass per unit length. The top end of the rope is attached to a fixed rigid support at O. A transverse wave pulse (Pulse 1) of wavelength λ_0 is produced at point O on the rope. The pulse takes time T_{OA} to reach point A (Pulse 2) without disturbing the position of M it takes time T_{AO} to reach point O. Which of the following options is/are correct?



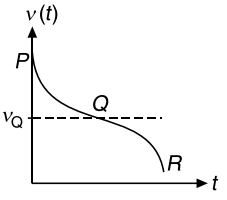
(A) The time $T_{AO} = T_{OA}$
(B) The wavelength of Pulse 1 becomes longer when it reaches point A
(C) The velocity of any pulse along the rope is independent of its frequency and wavelength
(D) The velocities of the two pulses (Pulse 1 and Pulse 2) are the same at the mid-point of rope

2. [JEE (Advanced) 2016]
Two loudspeakers M and N are located 20 m apart and emit sound at frequencies 118 Hz and 121 Hz, respectively. A car is initially at a point P, 1800 m away from the mid-point Q of the line MN and moves towards Q constantly at 60 kmh^{-1} along the perpendicular bisector of MN. It crosses Q and eventually reaches a point R, 1800 m away from Q. Let $v(t)$ represent the beat frequency measured by a person sitting in the car at time t. Let v_P, v_Q and v_R be the beat frequencies measured at locations P, Q and R respectively. The speed of sound in air is 300 ms^{-1}. Which of the following statement(s) is (are) true regarding the sound heard by the person?

(A) $v_P + v_R = 2v_Q$
(B) The rate of change in beat frequency is maximum when the car passes through Q
(C) The plot below represents schematically the variation of beat frequency with time



(D) The plot below rep represents schematically the variations of beat frequency with time



3. [JEE (Advanced) 2014]

A student is performing an experiment using a resonance column and a tuning fork of frequency $244\ s^{-1}$. He is told that the air in the tube has been replaced by another gas (assume that the column remains filled with the gas). If the minimum height at which resonance occurs is (0.350 ± 0.005) m, the gas in the tube is

(Useful information: $\sqrt{167RT} = 640\ J^{\frac{1}{2}}\ mole^{-\frac{1}{2}}$; $\sqrt{140RT} = 590\ J^{\frac{1}{2}}\ mole^{-\frac{1}{2}}$. The molar masses M in grams are given in the options. Take the value of $\sqrt{\frac{10}{M}}$ for each gas as given there.**)**

(A) Neon $\left(M = 20,\ \sqrt{\frac{10}{20}} = \frac{7}{10}\right)$

(B) Nitrogen $\left(M = 28,\ \sqrt{\frac{10}{28}} = \frac{3}{5}\right)$

(C) Oxygen $\left(M = 32,\ \sqrt{\frac{10}{32}} = \frac{9}{16}\right)$

(D) Argon $\left(M = 36,\ \sqrt{\frac{10}{36}} = \frac{17}{32}\right)$

4. [JEE (Advanced) 2014]

One end of a taut string of length 3 m along X-axis is fixed at $x = 0$. The speed of the waves in the string is $100\ ms^{-1}$. The other end of the string is vibrating in the y-direction so that stationary waves are set up in the string. The possible waveform(s) of these stationary wave is (are)

(A) $y(t) = A\sin\frac{\pi x}{6}\cos\frac{50\pi t}{3}$

(B) $y(t) = A\sin\frac{\pi x}{3}\cos\frac{100\pi t}{3}$

(C) $y(t) = A\sin\frac{5\pi x}{6}\cos\frac{250\pi t}{3}$

(D) $y(t) = A\sin\frac{5\pi x}{2}\cos 250\pi t$

5. [JEE (Advanced) 2013]

Two vehicles, each moving with speed u on the same horizontal straight road, are approaching each other. Wind blows along the road with velocity w. One of these vehicles blows a whistle of frequency f_1. An observer in the other vehicle hears the frequency of the whistle to be f_2. The speed of sound in still air is V. The correct statement(s) is (are)

(A) If the wind blows from the observer to the source, $f_2 > f_1$

(B) If the wind blows from the source to the observer, $f_2 > f_1$

(C) If the wind blows from observer to the source, $f_2 < f_1$

(D) If the wind blows from the source to the observer, $f_2 < f_1$

6. [IIT-JEE 2012]

A person blows into open-end of a long pipe. As a result, a high-pressure pulse of air travels down the pipe. When this pulse reaches the other end of the pipe,

(A) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is open

(B) a low-pressure pulse starts travelling up the pipe, if the other end of the pipe is open

(C) a low-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed

(D) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed

7. [IIT-JEE 2012]

A horizontal stretched string, fixed at two ends, is vibrating in its fifth harmonic according to the equation,

$$y(x,\ t) = (0.01\ m)\sin\left[(62.8\ m^{-1})x\right]\cos\left[(628\ s^{-1})t\right]$$

Assuming $\pi = 3.14$, the correct statement(s) is (are)

(A) The number of nodes is 5

(B) The length of the string is 0.25 m

(C) The maximum displacement of the midpoint of the string, from its equilibrium position is 0.01 m

(D) The fundamental frequency is 100 Hz

8. [IIT-JEE 2009]

A student performed the experiment to measure the speed of sound in air using resonance air-column method. Two resonances in the air-column were obtained by lowering the water level. The resonance with the shorter air-column is the first resonance and that with the longer air column is the second resonance. Then,

(A) the intensity of the sound heard at the first resonance was more than that at the second resonance

(B) the prongs of the tuning fork were kept in a horizontal plane above the resonance tube

(C) the amplitude of vibration of the ends of the prongs is typically around 1 cm

(D) the length of the air-column at the first resonance was somewhat shorter than $\frac{1}{4}$th of the wavelength of the sound in air

9. [IIT-JEE 1999]

Standing waves can be produced

(A) on a string clamped at both ends

(B) on a string clamped at one end and free at the other

(C) when incident wave gets reflected from a wall

(D) when two identical waves with a phase difference of π are moving in the same direction

10. [IIT-JEE 1999]

In a wave motion $y = a\sin(kx - \omega t)$, y can represent

(A) electric field (B) magnetic field

(C) displacement (D) pressure

11. [IIT-JEE 1999]

$Y(x, t) = \dfrac{0.8}{\left[(4x + 5t)^2 + 5\right]}$ represents a moving pulse where x and y are in metres and in t second. Then

(A) pulse is moving in positive x-direction

(B) in 2 s it will travel a distance of 2.5 m

(C) its maximum displacement is 0.16 m

(D) it is a symmetric pulse

12. [IIT-JEE 1999]

As a wave propagates

(A) the wave intensity remains constant for a plane wave

(B) the wave intensity decreases as the inverse of the distance from the source for a spherical wave

(C) the wave intensity decreases as the inverse square of the distance from the source for a spherical wave

(D) total intensity of the spherical wave over the spherical surface centered at the source remains constant at all times

13. [IIT-JEE 1998]

A transverse sinusoidal wave of amplitude a, wavelength λ and frequency f is travelling on a stretched string. The maximum speed of any point on the string is $\dfrac{v}{10}$, where v is the speed of propagation of the wave. If $a = 10^{-3}$ m and $v = 10$ ms^{-1}, then λ and f are given by

(A) $\lambda = 2\pi \times 10^{-2}$ m (B) $\lambda = 10^{-3}$ m

(C) $f = \dfrac{10^3}{2\pi}$ Hz (D) $f = 10^4$ Hz

14. [IIT-JEE 1998]

The (x, y) co-ordinates of the corners of a square plate are $(0, 0)$, $(L, 0)$, (L, L) and $(0, L)$. The edges of the plate are clamped and transverse standing waves are set up in it. If $u(x, y)$ denotes the displacement of the plate at the point (x, y) at some instant of time, the possible expression(s) for u is (are) (a = positive constant)

(A) $a\cos\left(\dfrac{\pi x}{2L}\right)\cos\left(\dfrac{\pi y}{2L}\right)$ (B) $a\sin\left(\dfrac{\pi x}{L}\right)\sin\left(\dfrac{\pi y}{L}\right)$

(C) $a\sin\left(\dfrac{\pi x}{L}\right)\sin\left(\dfrac{2\pi y}{L}\right)$ (D) $a\cos\left(\dfrac{2\pi x}{L}\right)\sin\left(\dfrac{\pi y}{L}\right)$

15. [IIT-JEE 1995]

A wave disturbance in a medium is described by $y(x, t) = 0.02\cos\left(50\pi t + \dfrac{\pi}{2}\right)\cos(10\pi x)$, where x and y are in metres and t in seconds

(A) A displacement node occurs at $x = 0.15$ m

(B) An antinode occurs at $x = 0.3$ m

(C) The wavelength of the wave is 0.2 m

(D) The speed of the wave is 5 ms^{-1}

16. [IIT-JEE 1995]

A sound wave of frequency f travels horizontally to the right. It is reflected from a large vertical plane surface moving to left with a speed v. The speed of sound in medium is C

(A) the number of wave striking the surface per second is $f\dfrac{(c + v)}{c}$

(B) the wavelength of the reflected wave is $\dfrac{c(c - v)}{f(c + v)}$

(C) the frequency of the reflected wave is $f\dfrac{(c + v)}{(c - v)}$

(D) the number of beats heard by a stationary listener to the left of the reflecting surface is $\dfrac{vf}{c - v}$

17. [IIT-JEE 1991]

Two identical straight wires are stretched so as to produce 6 beats per second when vibrating simultaneously. On changing the tension slightly in one of them, the beat frequency remains unchanged. Denoting by T_1, T_2 the higher and the lower initial tension in the strings, then it could be said that while making the above changes in tension

(A) T_2 was decreased

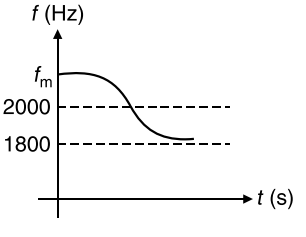
(B) T_2 was increased

(C) T_1 was decreased

(D) T_1 was increased

18. [IIT-JEE 1990]

A stationary observer receives a sound of frequency $f_0 = 2200$ Hz. The apparent frequency f varies with time as shown in figure. Speed of sound = 300 ms^{-1}. Choose the correct alternative(s)



(A) speed of source is 66.7 ms^{-1}

(B) f_m shown in figure cannot be greater than 2500 Hz

(C) speed of source is 33.33 ms^{-1}

(D) f_m shown in figure cannot be greater than 2250 Hz

19. [IIT-JEE 1989]
Velocity of sound in air is 320 ms^{-1}. A pipe closed at one end has a length of 1 m. Neglecting end corrections, the air column in the pipe can resonate for sound of frequency
(A) 80 Hz (B) 240 Hz
(C) 320 Hz (D) 400 Hz

20. [IIT-JEE 1985]
An air column in a pipe, which is closed at one end, will be in resonance with a vibrating tuning fork of frequency 264 Hz, if the length of the column in cm is
(A) 31.25 (B) 62.50
(C) 93.75 (D) 125

21. [IIT-JEE 1981]
A wave equation which gives the displacement along the y-direction is given by

$$y = 10^{-4}\sin(60t + 2x)$$

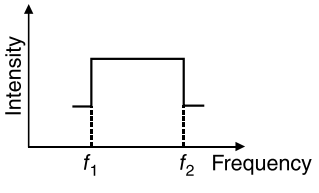
where x and y are in metres and t is time in seconds. This represents a wave
(A) travelling with a velocity of 30 ms^{-1} in the negative x-direction
(B) of wavelength πm
(C) of frequency $\frac{30}{\pi}$ hertz
(D) of amplitude 10^{-4} m travelling along the negative x-direction

Linked Comprehension Type Questions

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct. (For the sake of competitiveness there may be a few questions that may have more than one correct options)

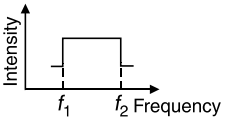
Comprehension 1

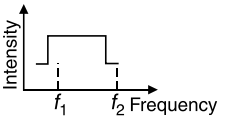
Two trains A and B are moving with speeds 20 ms^{-1} and 30 ms^{-1} respectively in the same direction on the same straight track, with B ahead of A. The engines are at the front ends. The engine of train A blows a long whistle.

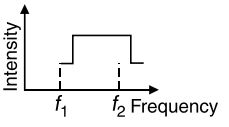


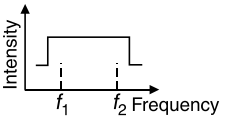
Assume that the sound of the whistle is composed of components varying in frequency from $f_1 = 800$ Hz to $f_2 = 1120$ Hz, as shown in the figure. The spread in the frequency (highest frequency – lowest frequency) is thus 320 Hz. The speed of sound in still air is 340 ms^{-1}. Based on the above facts, answer the following questions.

1. [IIT-JEE 2007]
The speed of sound of the whistle is
(A) 340 ms^{-1} for passengers in A and 310 ms^{-1} for passengers in B
(B) 360 ms^{-1} for passengers in A and 310 ms^{-1} for passengers in B
(C) 310 ms^{-1} for passengers in A and 360 ms^{-1} for passengers in B
(D) 340 ms^{-1} for passengers in both the trains

2. [IIT-JEE 2007]
The distribution of the sound intensity of the whistle as observed by the passengers in train A is best represented by
(A) 

(B) 

(C) 

(D) 

3. [IIT-JEE 2007]
The spread of frequency as observed by the passengers in train B is
(A) 310 Hz (B) 330 Hz
(C) 350 Hz (D) 290 Hz

Comprehension 2

Two plane harmonic sound waves are expressed by the equations.

$$y_1(x, t) = A\cos(0.5\pi x - 100\pi t)$$

$$y_2(x, t) = A\cos(0.46\pi x - 92\pi t)$$

(All parameters are in SI)
Based on the above facts, answer the following questions.

4. [IIT-JEE 2006]
How many times does an observer hear maximum intensity in one second?
(A) 4 (B) 10
(C) 6 (D) 8

5. [IIT-JEE 2006]
The speed of the sound is
(A) 200 ms^{-1}
(B) 180 ms^{-1}
(C) 192 ms^{-1}
(D) 96 ms^{-1}

6. [IIT-JEE 2006]
At $x = 0$, how many times the amplitude of $y_1 + y_2$ becomes zero in one second?
(A) 192 (B) 48
(C) 100 (D) 96

Matrix Match/Column Match Type Questions

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

If the correct matches are A → p, s and t; B → q and r; C → p and q; and D → s and t; then the correct darkening of bubbles will look like the following:

	p	q	r	s	t
A	Ⓟ	Ⓠ	ⓡ	Ⓢ	Ⓣ
B	Ⓟ	Ⓠ	Ⓡ	Ⓢ	ⓣ
C	Ⓟ	Ⓠ	ⓡ	Ⓢ	ⓣ
D	Ⓟ	Ⓠ	ⓡ	Ⓢ	Ⓣ

Answer Q. 1 and Q. 2 by appropriately matching the lists based on the information given in the paragraph.

A musical instrument is made using four different metal strings. 1, 2, 3 and 4 with mass per unit length μ, 2μ, 3μ and 4μ respectively. The instrument is played by vibrating the strings by varying the free length in between the range L_0 and $2L_0$. It is found that in string-1 (μ) at free length L_0 and tension T_0 the fundamental mode frequency is f_0.

COLUMN-I gives the above four strings while **COLUMN-II** lists the magnitude of some quantity.

COLUMN-I	COLUMN-II
(A) String-1 (μ)	(p) 1
(B) String-2 (2μ)	(q) $\frac{1}{2}$
(C) String-3 (3μ)	(r) $\frac{1}{\sqrt{2}}$
(D) String-4 (4μ)	(s) $\frac{1}{\sqrt{3}}$
	(t) $\frac{3}{16}$
	(u) $\frac{1}{16}$

1. [JEE (Advanced) 2019]

If the tension in each string is T_0, the correct match for the highest fundamental frequency in f_0 units will be

(A) A-p, B-q, C-t, D-s
(B) A-p, B-r, C-s, D-q
(C) A-q, B-s, C-r, D-p
(D) A-q, B-p, C-r, D-t

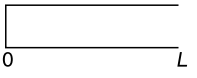
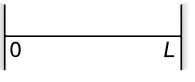
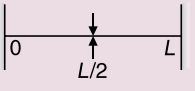
2. [JEE (Advanced) 2019]

The length of the strings 1, 2, 3 and 4 are kept fixed at L_0, $\frac{3L_0}{2}$, $\frac{5L_0}{4}$ and $\frac{7L_0}{4}$ respectively. Strings 1, 2, 3 and 4 are vibrated at their 1^{st}, 3^{rd}, 5^{th}, and 14^{th} harmonics, respectively such that all the strings have same frequency. The correct match for the tension in the four strings in the units of T_0 will be

(A) A-p, B-r, C-t, D-u
(B) A-p, B-q, C-t, D-u
(C) A-p, B-q, C-r, D-t
(D) A-t, B-q, C-r, D-u

3. [IIT-JEE 2011]

COLUMN-I shows four systems, each of the same length L, for producing standing waves. The lowest possible natural frequency of a system is called its fundamental frequency, whose wavelength is denoted as λ_f, Match each system with statements given in **COLUMN-II** describing the nature and wavelength of the standing waves.

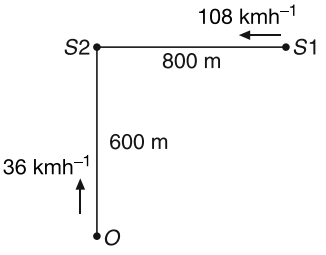
COLUMN-I	COLUMN-II
(A) Pipe closed at one end 	(p) Longitudinal waves
(B) Pipe open at both ends	(q) Transverse waves
(C) Stretched wire clamped at both ends 	(r) $\lambda_f = L$
(D) Stretched wire clamped at both ends and at mid-point 	(s) $\lambda_f = 2L$
	(t) $\lambda_f = 4L$

Integer/Numerical Answer Type Questions

In this section, the answer to each question is a numerical value obtained after series of calculations based on the data provided in the question(s).

1. [JEE (Advanced) 2019]

A train $S1$, moving with a uniform velocity of 108 kmh^{-1}, approaches another train $S2$ standing on a platform. An observer O moves with a uniform velocity of 36 kmh^{-1} towards $S2$, as shown in figure. Both the trains are blowing whistles of same frequency 120 Hz. When O is 600 m away from $S2$ and distance between $S1$ and $S2$ is 800 m, the number of beats heard by O is ..... [Speed of the sound = 330 ms^{-1}]



2. [JEE (Advanced) 2017]

A stationary source emits sound of frequency $f_0 = 492$ Hz. The sound is reflected by a large car approaching the source with a speed of 2 ms^{-1}. The reflected signal is received by the source and superposed with the original. What will be the beat frequency of the resulting signal in Hz? (Given that the speed of sound in air is 330 ms^{-1} and the car reflects the sound at the frequency it has received).

3. [JEE (Advanced) 2016]

Four harmonic waves of equal frequencies and equal intensities I_0 have phase angles 0, $\frac{\pi}{3}$, $\frac{2\pi}{3}$ and π. When they are superposed, the intensity of the resulting wave is nI_0. The value of n is

4. [IIT-JEE 2010]

When two progressive waves $y_1 = 4\sin(2x - 6t)$ and $y_2 = 3\sin\left(2x - 6t - \frac{\pi}{2}\right)$ are superimposed, the amplitude of the resultant wave is x unit. Find x.

5. [IIT-JEE 2009]

A 20 cm long string, having a mass of 1 g, is fixed at both the ends. The tension in the string is 0.5 N. The string is set into vibrations using an external vibrator of frequency 100 Hz. Find the separation (in cm) between the successive nodes on the string.

ANSWER KEYS–TEST YOUR CONCEPTS AND PRACTICE EXERCISES

Test Your Concepts-I (Based on Wave Equation & Properties)

1. $(0.06\text{ m})\sin(23562t-78.5x)$
2. 150 ms^{-1}
3. NO ANSWER
4. NO ANSWER
5. [0.12 m]
6. $e^{-(x-vt)^2}$
7. NO ANSWER
8. $y=0.02\cos(10t-100x)$ metre, 0.0628 m
9. 30 ms^{-1}
10. $v=2$ ms^{-1} along $-x$ direction, 2 units
11. 4 ms^{-1}, along $+x$ direction
12. y_1 along $+x$, y_2 along $-x$, 0.75 s, 1 m

Test Your Concepts-II (Based on Transverse Wave in a String & Properties)

1. $\sqrt{\dfrac{8\ell_0}{g}}$
2. $\dfrac{1}{15\alpha}\left[(\mu_0+\alpha l)^{\frac{3}{2}}-\mu_0^{\frac{3}{2}}\right]$
3. 80 ms^{-1}, 63.24 ms^{-1}
4. $\dfrac{1}{2}\rho A^2\omega^2\cos^2(kx-\omega t)$
5. $0.06\sqrt{3}$ m
6. 0.02 mm
7. 0.47 s
8. $\dfrac{P_1}{8}$
9. [32 ms^{-1}]
10. [0.25]
11. (a) 0.14 s, (b) –1.5 cm, 2 cm
12. String 2
13. [0.05 s]

Test Your Concepts-III (Based on Sound Waves & Properties)

1. (a) 4.98 Nm^{-2} (b) 3×10^{-6} m
2. $\sqrt{\dfrac{7}{15}}$
3. [68 dB, 75 dB]
4. 5.84 m
5. 3%
6. 0.173 J
7. –249.7 °C
8. 5.27×10^{3} ms^{-1}
9. 0.94 Nm^{-2}
10. (a) 2×10^{-4} Nm^{-2} (b) $y=2\times10^{-9}\sin\left(500t-\dfrac{5}{2}x\right)$
11. 591 ms^{-1}
12. 4.4×10^{-12} Wm^{-2}
13. 1.1×10^{-11} m

Test Your Concepts-IV (Based on Interference)

1. $0.28\sin\left(x-3t+\dfrac{\pi}{4}\right)$, ±1.29 rad
2. $\dfrac{1}{16}$
3. $\dfrac{5}{6}A$, $-\tan^{-1}\left(\dfrac{3}{4}\right)$
4. 1440 Hz
5. 26.46 m, $(5x+25t+0.714)$ rad
6. $\phi_1=\pi$, $\phi_2=4\pi$, 0.6×10^{-3} W
7. [$(188n)$ Hz]
8. (a) 1000 Hz, 2000 Hz, 3000 Hz,….
   (b) 500 Hz, 1500 Hz, 2500 Hz,….
9. 17.54 cm
10. (a) 3 m, (b) 0.6 m
11. [2:1:0]
12. $\sqrt{(x+5)^2+y^2}-\sqrt{(x-5)^2+y^2}=(16n-8)$ $n=1, 2, 3,\ldots..$

Test Your Concepts-V (Based on Stationary Waves & Beats)

1. (a) $\dfrac{15}{\pi}$ Hz, 0.2 cm, 60 cms^{-1} (b) 10.12 cms^{-1}
2. (a) 144 cm (b) 17 Hz
3. (a) 2.56 mm (b) 4 cm, 2 ms^{-1}
   (c) 78.5 cms^{-1}
   (d) Nodes at $x=0$, 2 cm, 4 cm, ….., Number of loops = 5
4. (a) $500\sqrt{2}$ Hz (b) 11 Hz
5. 18 cm, 24 cm, 72 cm
6. 2.142 m
7. $0.04I_i$
8. (a) 0.36 m (b)9939 Hz,
   (c) $y=2.57\times10^{-6}\sin(62400t+\phi)$
9. (a) $\pi kA^2T\sin^2(\omega t)$ (b) πkA^2T
   (c) $\dfrac{\pi kA^2T}{2}$

10. $353.6\ \text{ms}^{-1}$
11. 41.25 cm
12. 82.5 Hz, 81.675 N
13. Closed organ pipe is $\ell_1 = 0.75$ m, Open pipe is either $\ell_2 = 0.99$ m or 1.0067 m
14. (a) 283.33 Hz (b) 51.5 cm
15. 367 Hz
16. 27.04 N
17. $A\cos(3.93x)\sin(1297t)$

Test Your Concepts-VI (Based on Doppler's Effect)

1. $1.5\ \text{ms}^{-1}$
2. (a) 0.364 m (b)1320 Hz
   (c) $332\ \text{ms}^{-1}$ (d) 0.2 m
3. $f_{\min} = 495$ Hz, $f_{\max} = 594$ Hz
4. (a) 970.6 Hz (b) 1031.3 Hz
   (c) 60.7 Hz
5. (a) 0.35 m (b) 1014.5 Hz
6. $180\ \text{kmh}^{-1}$
7. 374.8 Hz
8. (a) 10% (b) No change
   (c) Percentage increase in frequency is 10%
9. $0.752\ \text{ms}^{-1}$
10. R_1
11. $1.5\times10^{6}\ \text{ms}^{-1}$
12. $f_{\min} = 278.62$ Hz, $f_{\max} = 323$ Hz
13. 1340 Hz
14. $\dfrac{Rv\cos^{-1}(R/h)}{\sqrt{h^2-R^2}}$

Single Correct Choice Type Questions

1. B	2. A	3. C	4. D	5. B	6. B	7. B	8. D	9. A	10. D
11. B	12. D	13. B	14. C	15. C	16. B	17. B	18. A	19. B	20. B
21. B	22. A	23. A	24. B	25. B	26. C	27. C	28. D	29. A	30. D
31. B	32. B	33. B	34. A	35. B	36. C	37. B	38. C	39. D	40. D
41. B	42. A	43. D	44. B	45. B	46. C	47. A	48. C	49. A	50. D
51. C	52. A	53. D	54. A	55. C	56. C	57. C	58. C	59. B	60. C
61. D	62. B	63. D	64. B	65. B	66. B	67. D	68. A	69. B	70. C
71. D	72. B	73. A	74. B	75. B	76. C	77. C	78. C	79. B	80. B
81. A	82. C	83. B	84. C	85. D	86. A	87. B	88. D	89. C	90. D
91. B	92. C	93. C	94. A	95. C	96. A	97. D	98. C	99. B	100. C
101. B	102. C	103. D	104. C	105. B	106. D	107. C	108. D	109. C	110. A
111. D	112. C	113. B	114. C	115. A	116. D	117. A	118. D	119. A	120. C
121. A	122. D	123. B	124. C	125. C	126. C	127. C	128. D	129. A	130. C
131. B	132. B	133. C	134. B	135. B	136. A	137. A	138. C	139. B	140. D
141. B	142. C	143. B	144. B	145. C	146. D	147. C	148. B	149. C	150. C
151. C	152. C	153. D	154. B	155. A	156. B	157. A	158. C	159. D	160. A
161. D	162. A	163. A	164. D	165. B	166. C	167. A	168. B	169. D	170. B
171. A	172. D	173. C	174. B	175. A	176. A	177. C	178. B	179. C	180. D
181. A	182. C	183. A							

Multiple Correct Choice Type Questions

1. A, D	2. A, B, C, D	3. B, C, D	4. A, B	5. A, D
6. A, C	7. B, C, D	8. B, C	9. B, C	10. A, B
11. B, C, D	12. A, C	13. A, B, C, D	14. B, C	15. B, C, D
16. A, C, D	17. B, C, D	18. A, C	19. B, C	20. A, B, C, D
21. A, B, C	22. A, B	23. A, D	24. A, B	25. A, B, C
26. A, D	27. A, B, C	28. A, B, D	29. B, C	30. A, D
31. B, D	32. C, D	33. A, B, D	34. C, D	35. B, C
36. B, C	37. A, B, C, D	38. B, C		

Reasoning Based Questions

1. D	**2.** A	**3.** B	**4.** C	**5.** B	**6.** A	**7.** A	**8.** A	**9.** B	**10.** A
11. A	**12.** C	**13.** A	**14.** D	**15.** B	**16.** C	**17.** B	**18.** B	**19.** C	**20.** C

Linked Comprehension Type Questions

1. C	**2.** B	**3.** A	**4.** D	**5.** B	**6.** D	**7.** B	**8.** D	**9.** C	**10.** C
11. B	**12.** B	**13.** B	**14.** C	**15.** D	**16.** C	**17.** D	**18.** B	**19.** A	**20.** C
21. A	**22.** B	**23.** B	**24.** A	**25.** C	**26.** A	**27.** A	**28.** A	**29.** D	**30.** D
31. B	**32.** C	**33.** B	**34.** C	**35.** D	**36.** B	**37.** D	**38.** A	**39.** C	**40.** D
41. D	**42.** A	**43.** B	**44.** C	**45.** A	**46.** D	**47.** A	**48.** B	**49.** C	**50.** B
51. A	**52.** A	**53.** B	**54.** A	**55.** D					

Matrix Match/Column Match Type Questions

1.	A → (p, r)	B → (q, s)	C → (q, r)	D → (p, s)
2.	A → (r)	B → (p)	C → (q)	D → (s)
3.	A → (s)	B → (p)	C → (q)	D → (r)
4.	A → (r)	B → (p)	C → (p)	D → (p)
5.	A → (r)	B → (r)	C → (s)	D → (q)
6.	A → (r)	B → (p)	C → (q)	D → (r)
7.	A → (q)	B → (p)	C → (p)	D → (r)
8.	A → (q)	B → (t)	C → (s)	D → (t)
9.	A → (r)	B → (p, r)	C → (q, r)	D → (p, r, s)
10.	A → (q)	B → (r)	C → (q)	D → (s)
11.	A → (s)	B → (q)	C → (r)	D → (p)
12.	A → (p)	B → (p)	C → (t)	D → (r)

Integer/Numerical Answer Type Questions

1. 60	**2.** 296	**3.** 7	**4.** 1030	**5.** 250
6. 96	**7.** 30	**8.** 60	**9.** 500	**10.** 6
11. (a) 6, (b) 5	**12.** 4	**13.** (a) 2500, (b) 320	**14.** (a) 36, (b) 632	**15.** 43, 7
16. 12	**17.** 350	**18.** 100	**19.** 60, 20	**20.** 4
21. 512	**22.** 1	**23.** 79	**24.** 8	**25.** (a) 314, (b) 972
26. 300				

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1. C	**2.** 35	**3.** B	**4.** D	**5.** C	**6.** B	**7.** D	**8.** D	**9.** D	**10.** C
11. D	**12.** 106	**13.** B	**14.** A	**15.** D	**16.** B	**17.** A	**18.** D	**19.** D	**20.** B
21. D	**22.** A	**23.** D	**24.** D	**25.** A	**26.** B	**27.** C	**28.** D	**29.** B	**30.** D
31. D	**32.** D	**33.** D	**34.** C	**35.** A	**36.** C	**37.** A	**38.** C	**39.** C	**40.** C
41. D	**42.** B	**43.** B	**44.** D	**45.** A	**46.** D	**47.** D	**48.** B	**49.** B	**50.** A
51. D	**52.** B	**53.** B	**54.** A	**55.** C	**56.** B				

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Single Correct Choice Type Problems

1. B	**2.** B	**3.** C	**4.** A	**5.** A	**6.** D	**7.** C	**8.** D	**9.** C	**10.** B
11. B	**12.** A	**13.** B	**14.** B	**15.** C	**16.** D	**17.** D	**18.** B	**19.** C	**20.** B
21. A	**22.** D	**23.** A	**24.** A	**25.** A	**26.** B	**27.** C	**28.** B	**29.** A	**30.** A
31. B	**32.** C								

Multiple Correct Choice Type Problems

1. A, C, D	**2.** A, B, C	**3.** D	**4.** A, C, D	**5.** A, B
6. A, B, D	**7.** B, C	**8.** A, D	**9.** A, B, C	**10.** A, B, C, D
11. B, C, D	**12.** A, C, D	**13.** A, C	**14.** B, C	**15.** A, B, C, D
16. A, B, C	**17.** B, C	**18.** C, D	**19.** A, B, D	**20.** A, C
21. A, B, C, D				

Linked Comprehension Type Questions

1. B	**2.** A	**3.** A	**4.** A	**5.** A	**6.** C

Matrix Match/Column Match Type Questions

1. A → (p, r, s, q)	B → (q, p, r, t)	C → (q, s, r, p)	D → (p, q, t, s)
2. A → (p, q, r, t)	B → (p, q, t, u)	C → (p, r, t, u)	D → (t, q, r, u)
3. A → (p, t)	B → (p, s)	C → (q, s)	D → (q, r)

Integer/Numerical Answer Type Questions

1. 8.13	**2.** 6	**3.** 3	**4.** 5	**5.** 5

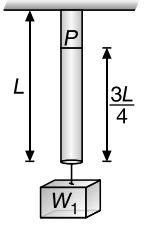
HINTS AND EXPLANATIONS

Test Your Concepts-I
(Based on Young's Modulus, Longitudinal Stress and Strain)

1. Since the wire is uniform so the weight of wire below point P is $\frac{3W}{4}$

Total force at point P is $F = W_1 + \frac{3W}{4}$

At point P, Stress $= \frac{\text{Force}}{\text{Area}} = \frac{W_1 + \frac{3W}{4}}{S}$



2. $2T_C + T_S = mg$ …(1)

$\Delta l_C = \Delta l_S$

$\Rightarrow \quad \frac{T_C l}{AY_C} = \frac{T_S l}{AY_S}$

Since, $Y_S = 2Y_C$

$\Rightarrow \quad \frac{T_C}{T_S} = \frac{1}{2}$ …(2)

Solving these two equations, we get

$T_C = \frac{mg}{4}$ and $T_S = \frac{mg}{2}$

3. Increment in the wire of length AB is

$$\Delta L_{AB} = \frac{MgL_{AB}}{AY} = \frac{10 \times 10 \times 0.1}{10^{-7} \times 2.5 \times 10^{10}}$$

$\Rightarrow \quad \Delta L_{AB} = 4 \times 10^{-3}$ m $= 4$ mm

So, displacement of point B is

$\Delta L_B = \Delta L_{AB} = 4 \times 10^{-3}$ m $= 4$ mm

Increment in the wire of length BC is

$$\Delta L_{BC} = \frac{MgL_{BC}}{AY} = \frac{10 \times 10 \times 0.2}{10^{-7} \times 4 \times 10^{10}}$$

$\Rightarrow \quad \Delta L_{BC} = 5 \times 10^{-3}$ m $= 5$ mm

So, displacement of the point C is

$\Delta L_C = \Delta L_B + \Delta L_{BC} = 4$ mm $+ 5$ mm $= 9$ mm

Increment in the wire of length CD is

$$\Delta L_{CD} = \frac{MgL_{CD}}{AY} = \frac{10 \times 10 \times 0.15}{10^{-7} \times 4 \times 10^{10}}$$

$\Rightarrow \quad \Delta L_{CD} = 15 \times 10^{-3}$ m $= 15$ mm

So, displacement of the point D is

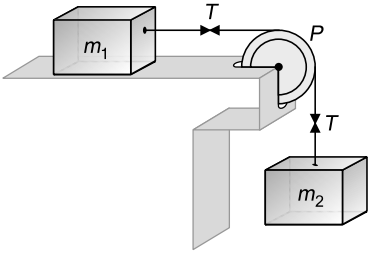
$\Delta L_D = \Delta L_C + \Delta L_{CD} = 9$ mm $+ 15$ mm

$\Rightarrow \quad \Delta L_D = 24$ mm

4. $\Delta l = \frac{Fl}{AY}$

$\Rightarrow \quad \Delta l \propto \frac{1}{Y}$

5. If the system moves with acceleration a and T is the tension in the string W_2 then by comparing this condition from standard case $T = \frac{m_1 m_2}{m_1 + m_2} g$



In the given example

$m_1 = (m + m) = 2m$ and $m_2 = m$

$\Rightarrow \quad \text{Tension} = \frac{(m)(2m)g}{m + 2m} = \frac{2}{3}mg$

$\Rightarrow \quad \text{Stress} = \frac{T}{a} = \frac{2}{3a}mg$

and $\text{Strain} = \frac{\text{Stress}}{\text{Young's modulus}} = \frac{2mg}{3aY}$

6. At first sight, it looks that rod B is stronger as it supports a larger load. But we cannot compare strengths without having a common basis of comparison. Since the rods have different cross-sectional area, therefore, the strengths can be compared by determining the load capacity per unit area.

For rod *A*

$$\sigma_1 = \frac{W_1}{A_1} = \frac{600 \text{ N}}{10 \text{ mm}^2} = \frac{600}{10 \times 10^{-6}}$$

$\sigma_1 = 60 \times 10^6$ Nm^{-2}

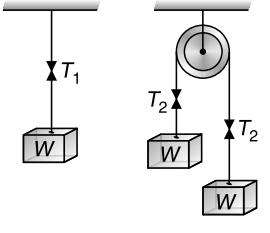
For rod *B*

$$\sigma_2 = \frac{W_2}{A_2} = \frac{6000 \text{ N}}{1000 \text{ mm}^2} = \frac{6000}{1000 \times 10^{-6}}$$

$\sigma_2 = 6 \times 10^6$ Nm^{-2}

Since $\sigma_1 > \sigma_2$, therefore, the material of rod A is stronger than that of rod B.

7. Since elongation in the wire is directly proportional to the tension in the wire



In first case $T_1 = W$ and in second case

$T_2 = \text{Thrust} = \frac{2W \times W}{W + W} = W$

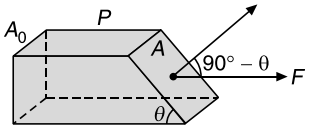
Since, $\frac{T_1}{T_2}=1$

$\Rightarrow \quad \frac{l_1}{l_2}=1$

$\Rightarrow \quad l_2=l_1=1\text{ mm}$

8. Normal force $F_n=F\sin\theta$

Area $A=\frac{A_0}{\sin\theta}$



$\Rightarrow \quad \sigma_n=\frac{F_n}{A}=\frac{F\sin\theta}{A_0/\sin\theta}=\frac{F}{A_0}\sin^2\theta$

$\Rightarrow \quad \frac{F_{\max}\sin^2\theta}{A_0}=\sigma_0$

$\Rightarrow \quad F_{\max}=\frac{\sigma_0 A_0}{\sin^2\theta}$

9. Since, $\Delta l=\frac{FL}{AY}$, so for a given stretching force, we have

$$\Delta l\propto\frac{1}{AY}$$

Let three wires have young's moduli $2Y$, $2Y$, Y and cross sectional areas A, $2A$ and $3A$ respectively, so we have

$$\Delta l_1:\Delta l_2:\Delta l_3=\frac{1}{A_1Y_1}:\frac{1}{A_2Y_2}:\frac{1}{A_3Y_3}$$

$$\Rightarrow \quad \Delta l_1:\Delta l_2:\Delta l_3=\frac{1}{A(2Y)}:\frac{1}{(2A)(2Y)}:\frac{1}{(3A)Y}$$

$$\Rightarrow \quad \Delta l_1:\Delta l_2:\Delta l_3=\frac{1}{2}:\frac{1}{4}:\frac{1}{3}$$

$$\Rightarrow \quad \Delta l_1:\Delta l_2:\Delta l_3=6:3:4$$

10. The maximum load that can be hung from a copper wire of diameter 0.42 mm is given by

$$F_{\max}=(\sigma_{\max})A$$

$$\Rightarrow \quad F_{\max}=(3\times10^8)\left(\frac{\pi}{4}\right)(0.42\times10^{-3})^2$$

$$\Rightarrow \quad F_{\max}=41.6\text{ N}$$

The percentage of length of wire that will stretch is given by

$$\frac{\Delta l}{l}\times100=\frac{F}{2YA}\times100$$

$$\Rightarrow \quad \frac{\Delta l}{l}\times100=\frac{41.6\times100}{2(1.1\times10^{11})\left(\frac{\pi}{4}\right)(0.42\times10^{-3})^2}$$

$$\Rightarrow \quad \frac{\Delta l}{l}\times100\%=0.136\%$$

11. The cross-sectional area of column is

$$A=\pi r^2=\pi(0.045\text{ m})^2$$

$$\Rightarrow \quad A=6.36\times10^{-3}\text{ m}^2$$

Since, $Y=\left(\frac{F}{A}\right)\left(\frac{\Delta L}{L}\right)$

$$\Rightarrow \quad \Delta L=\frac{FL}{AY}=\frac{mgL}{AY}$$

$$\Rightarrow \quad \Delta L=\frac{(8\times10^4)(9.8\text{ N})(4\text{ m})}{(6.36\times10^{-3}\text{ m}^2)(1.9\times10^{11}\text{ Nm}^{-2})}$$

$$\Rightarrow \quad \Delta L=2.6\times10^{-3}\text{ m}=2.6\text{ mm}$$

12. $Y=\frac{Fl}{A\Delta l}=\frac{(5000)(4)}{(0.5\times10^{-4})(0.2\times10^{-2})}$

$$\Rightarrow \quad Y=2.0\times10^{11}\text{ Nm}^{-2}$$

13. Stress is defined as

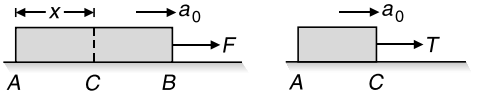
$$\sigma=\frac{F}{A}$$

Since both F and A are same, hence the ratio of stresses is 1. Strain is given by

$$\text{Strain}=\frac{\text{Stress}}{Y}\propto\frac{1}{Y}$$

$$\Rightarrow \quad \frac{(\text{Strain})_{\text{steel}}}{(\text{Strain})_{\text{copper}}}=\frac{Y_{\text{copper}}}{Y_{\text{steel}}}=\frac{13}{20}$$

14. Free body diagram of AB is shown in Figure.



Tension T at C is given by

$$T=(m_{AC})a_0=(xS\rho)a_0$$

$$\Rightarrow \quad \text{Stress}=\frac{T}{S}=\rho x a_0$$

At $x=\frac{L}{2}$, Stress $=\frac{\rho L a_0}{2}$

Total elongation is

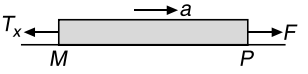
$$\Delta l=\int_0^L\frac{(\text{Stress})\,dx}{Y}$$

$$\Rightarrow \quad \Delta l=\int_0^L\frac{(\rho x a_0)\,dx}{Y}=\frac{\rho a_0 L^2}{2Y}$$

15. **(a)** Acceleration of the rod, $a=\frac{F}{m}$

$$F-T_x=(m_{PM})a=\left(\frac{m}{l}x\right)\left(\frac{F}{m}\right)$$

$$\Rightarrow \quad T_x=F\left(1-\frac{x}{l}\right)$$



(b) Stress $\sigma=\frac{F}{A}=\frac{T_x}{A}=\frac{F}{A}\left(1-\frac{x}{l}\right)$

(c) Change in length $\Delta l = \int_0^{\ell} \frac{T_x dx}{AY}$

$$\Rightarrow \quad \Delta l = \int_0^{l} \frac{F\left(1-\frac{x}{l}\right)dx}{AY} = \frac{Fl}{2AY}$$

(d) Strain, $\frac{\Delta l}{l} = \frac{F}{2AY}$

16. **(a)** $\sigma = \frac{F}{A} = \frac{8000 \times 9.8}{\frac{\pi}{4} \times (15 \times 10^{-2})^2} = 4.4 \times 10^6 \text{ Nm}^{-2}$

(b) $\varepsilon = \frac{\sigma}{Y} = \frac{4.4 \times 10^6}{2.0 \times 10^{11}} = 2.2 \times 10^{-5}$

(c) $\Delta l = L\varepsilon = 3.0 \times 2.2 \times 10^{-5} = 6.6 \times 10^{-5} \text{ m}$

17. Increase in circumference of the ring is

$$\Delta l = 2\pi(R - r)$$

$$\Rightarrow \quad \text{Strain} = \frac{2\pi(R-r)}{2\pi r} = \frac{R-r}{r}$$

Since, we know that

$$\text{Stress} = Y(\text{Strain}) = Y\left(\frac{R-r}{r}\right)$$

$$\Rightarrow \quad \text{Tension} = (\text{Stress})A = Ybd\left(\frac{R-r}{r}\right)$$

Test Your Concepts-II (Based on Elastic Energy, Energy Density and Poisson's Ratio)

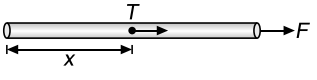
1. The elastic energy is given by

$$U = \frac{1}{2} \times Y \times (\text{strain})^2 \times \text{volume}$$

$$\Rightarrow \quad U = \frac{1}{2} \times 2 \times 10^{11} \times (10^{-3})^2 \times \frac{3.1}{7800}$$

$$\Rightarrow \quad U = 39.74 \text{ J}$$

2. The tension T in the string at a distance x from its free end is given as



$$T = F\left(\frac{x}{l}\right)$$

Hence the stress is

$$\sigma = \frac{T}{A} = \frac{F}{Al}x$$

Since we know that

$$dU = \frac{(\text{Stress})^2}{2Y}dV$$

where $dV = Adx$

$$\Rightarrow \quad U = \frac{1}{2Y}\int \sigma^2 dV = \frac{1}{2Y}\int\left(\frac{F}{Al}x\right)^2 Adx$$

$$\Rightarrow \quad U = \frac{1}{2Y}\int_0^{l} \frac{F^2}{A^2l^2}x^2 Adx$$

$$\Rightarrow \quad U = \frac{F^2 l}{6AY}$$

3. Energy stored

$$U = \frac{1}{2}(\text{stress})(\text{strain})(\text{volume})$$

$$\Rightarrow \quad U = \frac{1}{2}\left(\frac{F}{A}\right)\left(\frac{\Delta l}{l}\right)(Al)$$

$$\Rightarrow \quad U = \frac{1}{2}F\Delta l$$

$$\Rightarrow \quad U = \frac{1}{2}(100)(0.3 \times 10^{-3})$$

$$\Rightarrow \quad U = 0.015 \text{ J}$$

4. The stress in steel wire,

$$\sigma = \frac{F}{A} = \frac{36}{1.2 \times 10^{-7}} = 3 \times 10^8 \text{ Nm}^{-2}$$

As Young's modulus of elasticity, $Y = \frac{\text{Stress}}{\text{Strain}}$

$$\text{Strain} = \frac{\text{Stress}}{Y} = \frac{3 \times 10^8}{1.8 \times 10^{11}} = \frac{5}{3} \times 10^{-3}$$

Increase of length is

$$\Delta l = l(\text{strain}) = 2(1.67 \times 10^{-3})$$

$$\Rightarrow \quad \Delta l = 3.34 \times 10^{-3} \text{ m} = 3.34 \text{ mm}$$

Work done is given by

$$W = \frac{1}{2}(\text{stress})(\text{strain})(\text{volume})$$

$$\Rightarrow \quad W = \frac{1}{2}(3 \times 10^8)\left(\frac{5}{3} \times 10^{-3}\right)(2 \times 1.2 \times 10^{-7})$$

$$\Rightarrow \quad W = 6 \times 10^{-2} \text{ J} = 60 \text{ mJ}$$

5. Work done = Potential energy stored

$$\Rightarrow \quad W = \frac{1}{2}k(\Delta l)^2$$

Since, $k = \frac{YA}{l}$

$$\Rightarrow \quad W = \frac{1}{2}\left(\frac{YA}{l}\right)(\Delta l)^2$$

Substituting the values, we get

$$W = \frac{1}{2}\frac{(2 \times 10^{11})(10^{-6})}{(2)}(0.1 \times 10^{-3})^2$$

$$\Rightarrow \quad W = 5 \times 10^{-4} \text{ J}$$

6. By Law of Conservation of Energy, we have

$$\begin{pmatrix}\text{Loss in}\\ \text{Gravitational}\\ \text{Potential Energy}\end{pmatrix} = \begin{pmatrix}\text{Gain in Elastic}\\ \text{Potential Energy}\\ \text{in Both Leg Bones}\end{pmatrix}$$

$$\Rightarrow \quad mgh = 2 \times \left(\frac{1}{2}\text{stress} \times \text{strain} \times \text{volume}\right)$$

Since we have, $m = 45 \text{ kg}$, $h = 2 \text{ m}$, $L = 0.50 \text{ m}$ and $A = 5 \times 10^{-4} \text{ m}^2$, so we get

$$mgh = (45)(10)(2) = 900 \text{ J}$$

$$\Rightarrow \quad 900 = 2\left[\frac{1}{2}(0.9\times10^{8})\varepsilon(2.5\times10^{-4})\right]$$

$$\Rightarrow \quad \text{Strain} = \varepsilon = 0.04$$

$$\Rightarrow \quad Y = \frac{\text{Stress}}{\text{Strain}} = \frac{0.9\times10^{8}}{0.04} = 2.25\times10^{9} \text{ Nm}^{-2}$$

7. A stretched catapult has elastic potential energy stored in it. So, elastic potential energy stored in both the rubber strings is

$$U = \left(\frac{1}{2}\frac{YAl^2}{L}\right)\times 2$$

This energy U, when imparted to the stone, will make it rise through a vertical height h so that the energy possessed by the stone is

$$U = mgh$$

By Law of Conservation of Energy, we get

$$\Rightarrow \quad \frac{YAl^2}{L} = mgh$$

$$\Rightarrow \quad Y = \frac{mghL}{Al^2} = \frac{100\times10^{-3}\times10\times25\times10^{-1}}{10\times10^{-6}\times(5\times10^{-2})^2}$$

$$\Rightarrow \quad Y = 1\times10^{8} \text{ Nm}^{-2}$$

8. Given that, $l = 4.0$ m, $\Delta l = 2\times10^{-3}$ m, $A = 2.0\times10^{-6}$ m^2, $Y = 2.0\times10^{11}$ Nm^{-2}

The energy density of stretched wire

$$u_e = \frac{1}{2}\times\text{stress}\times\text{strain}$$

$$\Rightarrow \quad u_e = \frac{1}{2}\times Y\times(\text{strain})^2$$

$$\Rightarrow \quad u_e = \frac{1}{2}\times2.0\times10^{11}\times\left(\frac{(2\times10^{-3})}{4}\right)^2$$

$$\Rightarrow \quad u_e = 0.25\times10^{5} = 2.5\times10^{4} \text{ Jm}^{-3}$$

$$\text{Now,} \quad \begin{pmatrix}\text{Elastic}\\ \text{potential}\\ \text{energy}\end{pmatrix} = \begin{pmatrix}\text{energy}\\ \text{density}\end{pmatrix}\times(\text{volume})$$

$$\Rightarrow \quad U = 2.5\times10^{4}\times(2.0\times10^{-6})\times4.0 \text{ J}$$

$$\Rightarrow \quad U = 20\times10^{-2} = 0.20 \text{ J}$$

Test Your Concepts-III (Based on Shear Modulus, Tangential Stress, Shear Strain)

1. $$\text{Shear Stress} = \frac{\text{Tangential Force}}{\text{Area of Face}}$$

$$\Rightarrow \quad \text{Shear Stress} = \frac{0.50 \text{ N}}{15\times10^{-4} \text{ m}^2} = 333 \text{ Pa}$$

$$\begin{pmatrix}\text{Shear}\\ \text{Strain}\end{pmatrix} = \frac{\begin{pmatrix}\text{Displacement of Upper Face}\\ \text{wrt Lower Fixed Face}\end{pmatrix}}{(\text{Separation between Faces})}$$

$$\Rightarrow \quad \text{Shear Strain} = \frac{0.4 \text{ cm}}{3 \text{ cm}} = 0.133$$

Shear modulus is given by

$$\eta = \frac{\text{Tangential Stress}}{\text{Shear strain}}$$

$$\Rightarrow \quad \eta = \frac{333 \text{ Pa}}{0.133} = 2.5 \text{ kPa}$$

2. If the top face moves a distance Δx relative to the bottom face, then magnitude of the force required to produce this change in shape is given by

$$F = \eta A\frac{\Delta x}{L_0}$$

$$\Rightarrow \quad \eta = \frac{FL_0}{A\Delta x}$$

where, $A = (0.07)(0.07)$ m^2 is the area of the top surface, and $L_0 = 0.03$ m is the separation between the top surface and bottom surface.

$$\Rightarrow \quad \eta = \frac{FL_0}{A\Delta x} = \frac{(0.49)(0.03)}{(0.07)(0.07)(6\times10^{-3})}$$

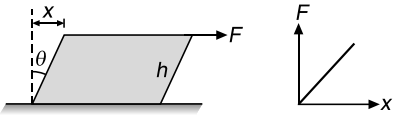
$$\Rightarrow \quad \eta = 500 \text{ Nm}^{-2}$$

This is a small value for modulus of rigidity and so the butter block can be deformed easily.

3. Since, $\text{Stress} = \eta(\text{Strain})$

$$\Rightarrow \quad \sigma = \frac{F}{A} = \eta\left(\frac{x}{h}\right)$$

$$\Rightarrow \quad x = \frac{\sigma h}{\eta}$$



Since force changes linearly with x, we can write its work done as

$$W = \frac{1}{2}Fx = \frac{1}{2}(\sigma h^2)\left(\frac{\sigma h}{\eta}\right)$$

$$\Rightarrow \quad W = \frac{\sigma^2 h^3}{2\eta}$$

4. The shear stress,

$$\frac{F}{A} = \frac{mg}{A} = \frac{(160)(10)}{3.2\times10^{-4}}$$

$$\Rightarrow \quad \frac{F}{A} = 5\times10^{6} \text{ Nm}^{-2}$$

The vertical deflection Δl of the right end of the bar is

$$\Delta l = \left(\frac{F}{A}\right)\frac{L_0}{\eta}$$

$$\Rightarrow \quad \Delta l = \frac{(5\times10^{6})(0.10)}{8\times10^{10}} = 6.25\times10^{-6} \text{ m}$$

Test Your Concepts-IV (Based on Bulk's Modulus, Normal Stress and Volumetric Strain)

1. Volume of the spherical body $V = \frac{4}{3}\pi R^3$

$$\Rightarrow \quad \frac{\Delta V}{V} = 3\frac{\Delta R}{R}$$

$$\Rightarrow \quad \frac{\Delta R}{R} = \frac{1}{3}\frac{\Delta V}{V} \qquad \ldots(1)$$

Now Bulk modulus is defined as

$$B = -V\frac{\Delta P}{\Delta V}$$

Since $\Delta P = \frac{mg}{A}$

$$\Rightarrow \left|\frac{\Delta V}{V}\right| = \frac{\Delta P}{B} = \frac{mg}{AB} \quad \ldots(2)$$

Substituting the value of $\frac{\Delta V}{V}$ from equation (2) in equation (1) we get

$$\frac{\Delta R}{R} = \frac{mg}{3AB}$$

2. Since, $B = \frac{-dP}{dV/V} = \frac{1.8\times10^6}{\left(\frac{0.3}{800}\right)} = 4.8\times10^9 \text{ Nm}^{-2}$

Compressibility k is given by

$$k = \frac{1}{B} = \frac{1}{4.8\times10^9} = 2.1\times10^{-10} \text{ m}^2\text{N}^{-1}$$

3. $B = \frac{\Delta p}{\frac{\Delta V}{V}} = \frac{\rho gh}{\left(\frac{\Delta V}{V}\right)}$

$$\Rightarrow B = \frac{(10^3)(9.8)(180)}{\left(\frac{0.1}{100}\right)}$$

$$\Rightarrow B = 1.76\times10^9 \text{ Nm}^{-2}$$

4. $B = \frac{-\Delta P}{\frac{\Delta V}{V}}$

$$\Rightarrow \left|\frac{\Delta V}{V}\times100\right| = \frac{\Delta P}{B}\times100 = \frac{1.01\times10^5}{4\times10^{10}}\times10^2$$

$$\Rightarrow \left|\frac{\Delta V}{V}\times100\right| = 2.525\times10^{-4}\%$$

5. Pressure increases with depth of a liquid. At a depth h below the water surface increase in pressure is given by

$$\Delta P = \rho gh$$

Using the equation, $\Delta\rho = \rho\frac{\Delta P}{B}$

we get $\Delta\rho = \frac{\rho(\rho gh)}{B} = \frac{\rho^2 gh}{B}$

Substituting the values, we have

$$\Delta\rho = \frac{(10^3)^2(10)(10^3)}{2.3\times10^9} = 4.33 \text{ kgm}^{-3}$$

6. The changed density is given by

$$\rho' = \frac{\rho}{1-\frac{dp}{B}}$$

Substituting the value, we have

$$\rho' = \frac{11.4}{1-\frac{2\times10^8}{8\times10^9}}$$

$$\Rightarrow \rho' = 11.69 \text{ gcm}^{-3}$$

7. Here, $B = 2.3\times10^9 \text{ Nm}^{-2}$

$$\Rightarrow B = \frac{2.3\times10^9}{1.01\times10^5}$$

$$\Rightarrow B = 2.27\times10^4 \text{ atm}$$

(a) Compressibility k is

$$k = \frac{1}{B} = \frac{1}{2.27\times10^4} = 4.4\times10^{-5} \text{ atm}^{-1}$$

(b) Here, $\frac{\Delta V}{V} = -0.1\% = -0.001$

Required increase in pressure,

$$\Delta P = B\times\left(-\frac{\Delta V}{V}\right)$$

$$\Rightarrow \Delta P = 2.27\times10^4\times0.001$$

$$\Rightarrow \Delta P = 22.7 \text{ atm}$$

8. Since, $U = \frac{(\text{Stress})^2}{2B}$

$$\Rightarrow U = \frac{(\rho gh)^2}{2B}$$

Here, $\frac{1}{B}$ = compressibility $= 4.9\times10^{-10} \text{ m}^2\text{N}^{-1}$

$$\Rightarrow U = \frac{(10^3\times9.8\times10^3)^2}{2}\times4.9\times10^{-10}$$

$$\Rightarrow U = 235\times10^2 \text{ Jm}^{-3} = 23.5 \text{ kJm}^{-3}$$

9. Since Bulk's modulus is

$$B = \frac{-\Delta P}{\left(\frac{\Delta V}{V}\right)}$$

Substituting the values, we get

$$B = \frac{(1.165-1.01)\times10^5}{\left(\frac{10}{100}\right)} = 1.55\times10^5 \text{ Nm}^{-2}$$

Test Your Concepts-V (Based on Fluid Properties, Pressure and Pascal's Law)

1. Density of oil in CGS units is numerically equal to the relative density. So, we have

$$\rho_{cgs} = (\text{RD}) \text{ gcm}^{-3} = 0.8 \text{ gcm}^{-3}$$

$$\Rightarrow \rho_{SI} = 800 \text{ kgm}^{-3}$$

2. Volume of fat in $1000\ \text{cm}^3$ of milk is

$$V = 4\% \times 1000\ \text{cm}^3 = 40\ \text{cm}^3$$

Mass of $40\ \text{cm}^3$ fat is

$$m = \rho V$$

$$\Rightarrow \quad m = \left(865\ \text{kgm}^{-3}\right)\left(40 \times 10^{-6}\ \text{m}^3\right)$$

$$\Rightarrow \quad m = 0.0346\ \text{kg}$$

Density of skimmed milk is

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{(1.032 - 0.0346)\text{kg}}{(1000 - 40) \times 10^{-6}\ \text{m}^3}$$

$$\Rightarrow \quad \rho = 1039\ \text{kgm}^{-3}$$

3. Since, $P = \dfrac{F}{A}$

$$\Rightarrow \quad F = PA = \left(1.01 \times 10^5\ \text{Nm}^{-2}\right)\left(2 \times 10^{-4}\ \text{m}^2\right)$$

$$\Rightarrow \quad F \approx 20\ \text{N}$$

4. Pressure (and hence force) on the two equal base areas are identical. Since force is exerted by water on the sides of the vessels also, which has a non-zero vertical component when the sides of the vessel are not perfectly normal to the base. This net vertical component of force by water on the sides of the vessel is greater for the first vessel than the second. Hence, the vessels weigh different whereas force on the base is the same in the two cases.

5. The pressure at the surface is equal to the atmospheric pressure $P_0 = 10^5\ \text{Nm}^{-2}$.
The pressure at the bottom of the beaker is
$P = P_0 + h\rho g$, where

$$h\rho g = (0.1)(13600)(10) = 13600\ \text{Nm}^{-2}$$

$$\Rightarrow \quad P = 10^5\ \text{Nm}^{-2} + 13600\ \text{Nm}^{-2}$$

$$\Rightarrow \quad P \approx 1.14 \times 10^5\ \text{Nm}^{-2}$$

So, force exerted by the mercury at the bottom of the beaker is

$$F = PA$$

where, $A = \pi\left(\dfrac{4}{100}\right)^2 \approx 5 \times 10^{-3}\ \text{m}^2$

$$\Rightarrow \quad F = \left(1.14 \times 10^5\right)\left(5 \times 10^{-3}\right) = 570\ \text{N}$$

6. Absolute pressure $P = P_a + \rho g h$
where $P_a = 1.01 \times 10^5\ \text{Pa}$

$$\rho = 1.03 \times 10^3\ \text{kgm}^{-3}$$

$$\Rightarrow \quad P = 1.01 \times 10^5\ \text{Pa} + 1.03 \times 10^3 \times 10 \times 500\ \text{Pa}$$

$$\Rightarrow \quad P = 1.01 \times 10^5\ \text{Pa} + 51.5 \times 10^5\ \text{Pa}$$

$$\Rightarrow \quad P = 52.5 \times 10^5\ \text{Pa}$$

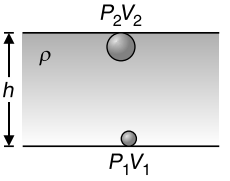
$$\Rightarrow \quad P \approx 52\ \text{atm}$$

7. Since the bell is brought gradually to the surface of the lake, so the pressure volume relation for air inside the bell is isothermal. So according to Boyle's law, we have

$$P_1V_1 = P_2V_2$$

$$\Rightarrow \quad (P_0 + h\rho_w g)V_1 = P_0V_2$$

$$\Rightarrow \quad V_2 = \left(1 + \frac{h\rho_w g}{P_0}\right)V_1$$



Since $P_2 = P_0 = 70$ cm of Hg, so we have

$$P_2 = 70 \times 13.6 \times 1000\ \text{Nm}^{-2}$$

$$\Rightarrow \quad V_2 = \left(1 + \frac{47.6 \times 10^2 \times 1 \times 1000}{70 \times 13.6 \times 1000}\right)V_1$$

$$\Rightarrow \quad V_2 = (1 + 5)50 = 300\ \text{cm}^3$$

8. Force at the bottom of vessel equals the weight of liquid. So, we have

$$F_{\text{bottom}} = (\pi R^2)h\rho g$$

Force on side walls of vessel

$$F_{\text{curved}} = \left(\frac{h}{2}\rho g\right)(2\pi Rh)$$

According to the problem, we have

$$\pi R^2 \rho g = \left(\frac{h}{2}\rho g\right)(2\pi Rh)$$

$$\Rightarrow \quad h = R$$

9. Difference of pressure between sea level and the top of hill is

$$\Delta P = (h_1 - h_2) \times \rho_{Hg} \times g$$

$$\Rightarrow \quad \Delta P = (75 - 50) \times 10^{-2} \times \rho_{Hg} \times g \qquad \ldots(1)$$

and pressure difference due to h meter of air is

$$\Delta P = h \times \rho_{\text{air}} \times g \qquad \ldots(2)$$

Equating (1) and (2), we get

$$h \times \rho_{\text{air}} \times g = (75 - 50) \times 10^{-2} \times \rho_{Hg} \times g$$

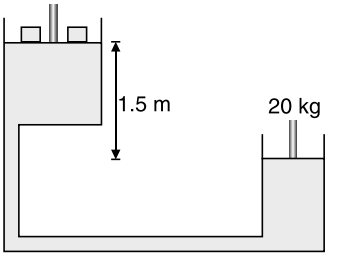
$$\Rightarrow \quad h = 25 \times 10^{-2}\left(\frac{\rho_{Hg}}{\rho_{\text{air}}}\right) = 25 \times 10^{-2} \times 10^4$$

$$\Rightarrow \quad h = 2500\ \text{m} = 2.5\ \text{km}$$

10. Since pressure at same level in the liquid should be the same, so we must have

$$P_{\text{larger piston}} + h\rho g = P_{\text{smaller piston}} \qquad \ldots(1)$$

where $h = 1.5$ m and $\rho = 750\ \text{kgm}^{-3}$



If P_{atm} be the atmospheric pressure, then pressure on the smaller piston is

$$P_{\substack{\text{smaller}\\\text{piston}}} = P_{\text{atm}} + \frac{20 \times 9.8}{\pi \times \left(5 \times 10^{-2}\right)^2}\ \text{Nm}^{-2}$$

and pressure on the larger piston is

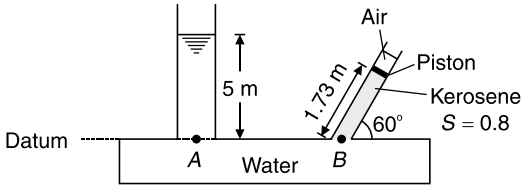
$$P_{\substack{\text{larger}\\\text{piston}}} = P_{\text{atm}} + \frac{F}{\pi\times(17.5\times10^{-2})^2}\ \text{Nm}^{-2}$$

Substituting these values in equation (1), we get

$$\frac{20\times9.8}{\pi(5\times10^{-2})^2} - \frac{F}{\pi(17.5\times10^{-2})^2} = (1.5)(750)(9.8)$$

$$\Rightarrow\quad F = 1.3\times10^3\ \text{N} = 1.3\ \text{kN}$$

11. Let p_a be the air pressure above the piston and p_0 be the atmospheric pressure. Then by applying Pascal's Law at points A and B, we get

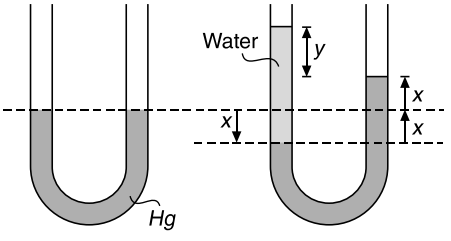


$$p_0 + \rho_w g(5) = p_a + \rho_k g(1.73)\sin60°$$

$$p_a = (10^3)(10)(5) + 10^5 - (800)(10)(1.73)\frac{\sqrt{3}}{2}$$

$$\Rightarrow\quad p_a = 138\ \text{kPa}$$

12. The situation is shown in Figure.



If water depresses the mercury by x, the mercury in the other limb will rise by x above its initial level (as fluids are incompressible), so we have

$$(27.2)\rho_w g = (2x)\rho_{\text{Hg}} g$$

$$\Rightarrow\quad 27.2 = 2(13.6)x$$

$$\Rightarrow\quad x = 1\ \text{cm}$$

Also, from the diagram, we see that

$$y + 2x = 27.2\ \text{cm}$$

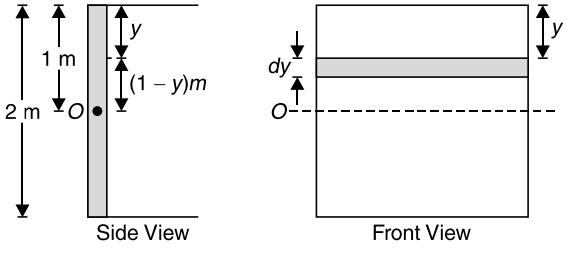
$$\Rightarrow\quad y = 25.2\ \text{cm}$$

13. The pressure at depth $y = \rho g y$

Force on small element step $= 3\rho g y\,dy$

Torque about O due to this force

$$d\tau = 3\rho g y\,dy(1-y)$$



Total torque $\tau = \int d\tau = 3\rho g\int_0^2 (y - y^2)dy$

$$\Rightarrow\quad \tau = 3\rho g\left(\frac{y^2}{2} - \frac{y^3}{3}\right)\Big|_0^2$$

$$\Rightarrow\quad \tau = 3\rho g\left(2 - \frac{8}{3}\right) = -2\rho g = -2\times10^4\ \text{Nm}^{-1}$$

Negative sign indicates that gate tends to rotate in clockwise direction.

14. In equilibrium

$$\frac{600\times10}{800\times10^{-4}} = \frac{F}{25\times10^{-4}} + h\rho g$$

$$\Rightarrow\quad \frac{F}{25\times10^{-4}} = \frac{60}{8}\times10^4 - 8\times(0.75\times10^3)\times10$$

$$\Rightarrow\quad \frac{F}{25\times10^{-4}} = 1.5\times10^4$$

$$\Rightarrow\quad F = 37.5\ \text{N}$$

15. Let P be the pressure in C. Then

$$P + h_1\rho_w g = P + h_2\rho g$$

$$\Rightarrow\quad \rho = \left(\frac{h_1}{h_2}\right)\rho_w = \left(\frac{10}{12}\right)(1.0) = 0.83\ \text{gcm}^{-3}$$

16. Pressure at A is

$$P_A = P_{\text{atm}} - \rho_1 g\ell\sin\theta \qquad \ldots(1)$$

Pressure at B is

$$P_B = P_{\text{atm}} + \rho_2 gh \qquad \ldots(2)$$

Also, we can say that pressure at B is

$$P_B = P_A + \rho_3 g\ell\sin\theta \qquad \ldots(3)$$

$$\Rightarrow\quad P_{\text{atm}} + \rho_2 gh = P_A + \rho_3 g\ell\sin\theta$$

$$\Rightarrow\quad P_{\text{atm}} + \rho_2 gh = P_{\text{atm}} - \rho_1 g\ell\sin\theta + \rho_3 g\ell\sin\theta$$

$$\Rightarrow\quad \sin\theta = \frac{\rho_2 h}{(\rho_3 - \rho_1)\ell}$$

17. Let the pressure of the liquid just below the piston be P. The forces acting on the piston are

(a) its weight, mg (downward)

(b) force due to the air above it, P_0A (downward)

(c) force due to the liquid below it, PA (upward).

For the piston to be in equilibrium, we have

$$PA = P_0A + mg$$

$$\Rightarrow\quad P = P_0 + \frac{mg}{A}$$

18. The force exerted by liquid on the gate is equal to the product of the average pressure acting on the gate with the vertical projected area of the gate.

$$F_H = P_{\text{av}}\left(\begin{matrix}\text{Vertical Projected}\\\text{Area of the Gate}\end{matrix}\right)$$

$$\Rightarrow\quad F_H = \left(\frac{P_A + P_B}{2}\right)(2R\ell)$$

$$\Rightarrow\quad F_H = \left(\frac{R\rho g + 3R\rho g}{2}\right)(2R\ell) = 4\rho g\ell R^2$$

Test Your Concepts-VI (Based on Pressure in Accelerating Fluids)

1. For liquid not to spill out of the tube, we have

$$h\rho g = l\rho a$$

$$\Rightarrow \quad a = \frac{hg}{l}$$

2. Let us assume that the water does not spill when the cylinder rotates with an angular speed of 10 rad s^{-1}, so the rise (h) in the level of water given by

$$h = \frac{r^2\omega^2}{2g}$$

$$\Rightarrow \quad h = \frac{\omega^2 r^2}{2g} = \frac{(20)^2(10)^2}{2\times 1000} = 10 \text{ cm}$$

Therefore, rise in the liquid level at the periphery is 10 cm. For water to just spill over the sides, the maximum available height $(h_{\max})$ is

$$h_{\max} = (50-30) \text{ cm} = 20 \text{ cm}.$$

Since, $h_{\max} = \frac{(\omega')^2 R^2}{2g}$

$$\Rightarrow \quad 20\times 10^{-2} = \frac{(\omega')^2 (20\times 10^{-2})^2}{2\times 10}$$

$$\Rightarrow \quad (\omega')^2 = 100$$

$$\Rightarrow \quad \omega' = 10 \text{ rads}^{-1}$$

Hence, frequency of rotation is

$$f' = \frac{\omega'}{2\pi} = \frac{5}{\pi} \text{ s}^{-1}$$

3. Pressure difference due to horizontal length of liquid is

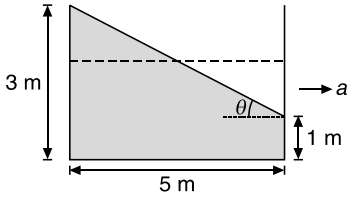
$$\Delta P = \int_0^L \rho\omega^2 x dx$$

Since the liquid is at rest w.r.t. the frame attached to the tube, so we have

$$\Delta P = \frac{\rho\omega^2 L^2}{2} = h\rho g$$

$$\Rightarrow \quad h = \frac{\omega^2 L^2}{2g}$$

4. The level of liquid should rise by 1 m on the left side and hence fall by 1 m on the right side.



$$\Rightarrow \quad \frac{a}{g} = \tan\theta = \frac{3-1}{5} = \frac{2}{5}$$

$$\Rightarrow \quad a = 0.4g$$

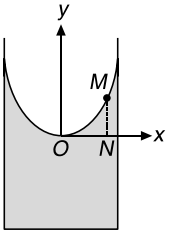
$$\Rightarrow \quad a = 4 \text{ ms}^{-2}$$

5. Since the tank is closed, so we do not need to take the atmospheric pressure into account inside the tank. Applying Pascal's equation, we get

$$P_A + h\rho g + l\rho a = P_B$$

$$\Rightarrow \quad P_B - P_A = (hg + la)\rho$$

6. In the figure shown, suppose the coordinates of point M are (x, y) with respect to the coordinate axes. Then, Points M and O are open to atmosphere.



$$\Rightarrow \quad p_M = p_O = p_{\text{atm}}$$

Now, $p_M - p_N = -\rho g y$

$$\Rightarrow \quad p_O - p_N = -\frac{\rho\omega^2 x^2}{2}$$

$$\Rightarrow \quad p_{\text{atm}} - p_N = -\frac{\rho\omega^2 x^2}{2} \qquad \ldots(1)$$

Also, due to rotation of the fluid we have

$$p_N - p_O = \frac{\rho\omega^2 x^2}{2}$$

$$\Rightarrow \quad p_N - p_{\text{atm}} = \frac{\rho\omega^2 x^2}{2} \qquad \ldots(2)$$

On adding equations (1) and (2), we get

$$0 = -\rho g y + \frac{\rho\omega^2 x^2}{2}$$

$$\Rightarrow \quad y = \frac{\omega^2 x^2}{2g}$$

This is the required equation of free surface of the liquid and we can see that this is an equation of a parabola.

Test Your Concepts-VII (Based on Archimedes' Principle and Buoyancy)

1. Let V be the total volume and V_i the volume of ice piece immersed in water. For equilibrium of ice piece, we have

Weight = Upthrust

$$\Rightarrow \quad V\rho_{\text{ice}} g = V_{\text{imm}}\rho_w g$$

Substituting in above equation, we get

$$\frac{V_{\text{imm}}}{V} = \frac{900}{1000} = 0.9$$

i.e., the fraction of volume outside the water,

$$f = 1 - 0.9 = 0.1$$

2. If V be the volume of the block, then

$$W = V\rho g \qquad \ldots(1)$$

When completely immersed in water, it weighs W', so we have

$$W' = W - U \qquad \ldots(2)$$

where, $U = V\rho_w g$

$$\Rightarrow \quad \frac{W'}{W} = \frac{W-U}{W} = \frac{V\rho g - V\rho_w g}{V\rho g} = 1 - \frac{\rho_w}{\rho}$$

$$\Rightarrow \quad \frac{\rho_w}{\rho} = 1 - \frac{W'}{W}$$

$$\Rightarrow \quad \frac{\rho_w}{\rho} = \frac{W - W'}{W}$$

$$\Rightarrow \quad \rho = \left(\frac{W}{W - W'}\right)\rho_w$$

3. Let the mass of the metal of specific gravity 11.4 be m, then the mass of the second metal of specific gravity 7.4 should be $(96 - m)$.

Volume of first metal is $V_1 = \frac{m}{11.4}$ cm^3

Volume of second metal is $V_2 = \frac{96 - m}{7.4}$ cm^3

Total volume of the alloy is $V = \frac{m}{11.4} + \frac{96 - m}{7.4}$

Buoyancy force on the alloy piece when immersed in water is

$$U = V_{\text{imm}}\rho_w g = \left(\frac{m}{11.4} + \frac{96 - m}{7.4}\right) \text{ gwt}$$

Apparent weight in water is

$$W_{\text{app}} = W - U$$

$$\Rightarrow \quad W_{\text{app}} = 96 - \left(\frac{m}{11.4} + \frac{96 - m}{7.4}\right)$$

According to the given problem, $W_{\text{app}} = 86$ gwt

$$\Rightarrow \quad 96 - \left(\frac{m}{11.4} + \frac{96 - m}{7.4}\right) = 86$$

$$\Rightarrow \quad \frac{m}{11.4} + \frac{96 - m}{7.4} = 10$$

Solving we get

$$m = 62.7 \text{ g}$$

So, mass of second metal is

$$m' = 96 - m = 96 - 62.7 = 33.3 \text{ g}$$

4. The density of the sphere is

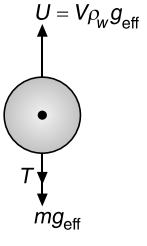
$$\rho = 0.5\rho_w = 500 \text{ kgm}^{-3}$$

If m be the mass of the sphere, V be the volume of the sphere immersed in water is

$$V = \frac{m}{(0.5)\rho_w}$$

The upthrust U acting on the sphere is

$$U = V\rho_w g_{\text{eff}} = \left[\frac{m}{(0.5)\rho_w}\right]\rho_w g_{\text{eff}} = 2mg_{\text{eff}}$$



where, $g_{\text{eff}} = g + a$ and $a = 2$ ms^{-2}.

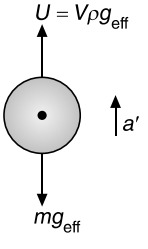
$$\Rightarrow \quad U = 2m(g + a) = (2)(2)(10 + 2) = 48 \text{ N}$$

Since the arrangement is in equilibrium with respect to the tank, so we have

$$U = T + mg_{\text{eff}}$$

$$\Rightarrow \quad T = U - mg_{\text{eff}} = 48 - 2(10 + 2) = 24 \text{ N}$$

When the thread snaps, tension T disappears and let the sphere now starts accelerating upwards with an acceleration a' with respect to the tank as in the free body diagram shown in Figure.



$$\Rightarrow \quad U - mg_{\text{eff}} = ma'$$

$$\Rightarrow \quad 48 - 2(10 + 2) = 2a'$$

$$\Rightarrow \quad a' = 12 \text{ ms}^{-2}$$

5. Density of ball is $\rho_{\text{ball}} = \frac{10}{15} = \frac{2}{3}$ gcm^{-3}

At a depth 10 m, acceleration of ball is

$$a = \frac{U - mg}{m} = \frac{V\rho_{\text{water}}g - V\rho_{\text{ball}}g}{V\rho_{\text{ball}}}$$

$$\Rightarrow \quad a = \left(\frac{\rho_{\text{water}} - \rho_{\text{ball}}}{\rho_{\text{ball}}}\right)g = \left(\frac{\rho_{\text{water}}}{\rho_{\text{ball}}} - 1\right)g$$

$$\Rightarrow \quad a = 10\left(\frac{3}{2} - 1\right) = 5 \text{ ms}^{-2}$$

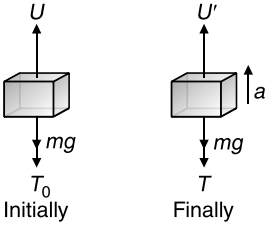
Time taken to reach the surface is

$$t = \sqrt{\frac{2h}{a}} = \sqrt{\frac{2 \times 10}{5}} = 2 \text{ s}$$

6. Let m be the mass of block. Initially for the equilibrium of block, we have

$$U = T_0 + mg \qquad \ldots(1)$$

Let U be the upthrust acting on the block. The free body diagrams for the initial and final situation of the block are shown in Figure.



When the lift is accelerated upwards, then we have

$$U' - T - mg = ma \qquad \ldots(2)$$

where $U' = V\rho_w(g + a) = U\left(1 + \frac{a}{g}\right)$ $\qquad \ldots(3)$

Solving equations (1), (2) and (3), we get

$$T = T_0\left(1 + \frac{a}{g}\right)$$

7. Average pressure at bottom of vessel is

$$P = P_{\text{atm}} + h\rho g + \frac{W_{\text{eff}}}{A}$$

where, $A = 16 \times 10^{-4}$ m^2 and

$$W_{\text{eff}} = mg - U$$

$$\Rightarrow \quad W_{\text{eff}} = (0.1)(10) - \left(\frac{4}{100}\right)^3(1000)(10)$$

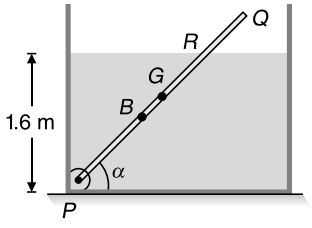
$$\Rightarrow \quad W_{\text{eff}} = 1 - 0.64 = 0.36 \text{ N}$$

$$\Rightarrow \quad P = 10^5 + (0.04)(1000)(10) + \frac{0.36}{16 \times 10^{-4}}$$

$$\Rightarrow \quad P = 10^5 + 400 + 225$$

$$\Rightarrow \quad P = 1.00625 \times 10^5 \text{ Pa}$$

8. Let G be the centre of gravity of the rod PQ, then G is the mid-point of PQ and B be the mid-point of the immersed portion PR of the rod. This point B is called the Centre of Buoyancy as shown in Figure.



Since the hinge is 1.6 m below the free surface of water, so we have

$$PR = 1.6 \text{ cosec}\alpha$$

Volume of PR is

$$V_{PR} = (1.6 \times 9.5 \times 10^{-4})\text{cosec}\alpha$$

Weight of water displaced by PR is

$$W_{PR} = (1.6)(9.5 \times 10^{-4})(10^3)(9.8)\text{cosec}\alpha$$

$$W_{PR} = 14.896\,\text{cosec}\alpha$$

Hence, the buoyant force is $14.896\,\text{cosec}\alpha$ acting vertically upwards at B.

The weight of the rod is 25 N acting vertically downwards at G. For equilibrium of the rod, net torque due to forces acting on the rod about the point A is zero. Hence, we have

$$(14.896\,\text{cosec}\alpha)(PB\cancel{\cos\alpha}) = (25)(PG\cancel{\cos\alpha})$$

Since, $PB = \dfrac{PR}{2}$, so we get

$$(14.896\,\text{cosec}\alpha)(0.8\,\text{cosec}\alpha) = (25)(1.5)$$

$$\Rightarrow \sin^2\alpha = 0.32$$

$$\Rightarrow \sin\alpha = 0.56$$

$$\Rightarrow \alpha = 34.3°$$

Further, let N_y be the reaction at hinge in vertically downward direction (because no horizontal component of any force exists). Then considering the translatory equilibrium of rod in vertical direction, we get

$$N_y + W = U$$

$$\Rightarrow N_y = U - W$$

$$\Rightarrow N_y = 14.896\,\text{cosec}(34.3°) - 25$$

$$\Rightarrow N_y = 26.6 - 25$$

$$\Rightarrow N_y = 1.6 \text{ N (downwards)}$$

9. If cavity volume is V, then the loss in weight of the piece of copper must be equal to the upthrust experienced by it. So, we have

$$\left(\frac{264}{8.8} + V\right)\rho_w \cancel{g} = (264 - 221)\cancel{g} = 43\cancel{g}$$

$$\Rightarrow V = 43 - \frac{264}{8.8} = 13 \text{ cm}^3$$

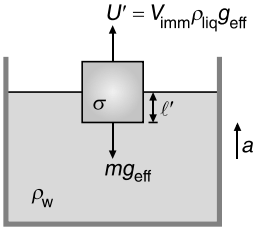
10. Assuming the density of water to be ρ_w and that of block to be σ. Initially, when beaker is at rest and when the block is floating with its length l inside water, then

$$U = mg$$

$$\Rightarrow (L^2 l)\rho_w g = (L^3\sigma)g$$

$$\Rightarrow l = L\left(\frac{\sigma}{\rho_w}\right) \quad \ldots(1)$$

Now, when beaker is accelerating upwards with an acceleration a, then let the length of the cube inside water is l' as shown in Figure.



Consider the frame attached to the beaker, then in this frame the block is in equilibrium, so we have

$$U' = mg_{\text{eff}}$$

$$\Rightarrow (L^2 l')\rho_w \cancel{(g+a)} = (L^3)\sigma\cancel{(g+a)}$$

$$\Rightarrow l' = L\left(\frac{\sigma}{\rho_w}\right)$$

$$\Rightarrow \Delta l = l' - l = 0$$

11. If container is at rest, let tension in string be T. Then, we have

$$T + mg = U$$

$$\Rightarrow T = U - mg = V(\rho_l - \rho_s)g$$

$$\Rightarrow T = (1.2 - 0.8) \times 1000 \times 980$$

$$\Rightarrow T = 3.92 \times 10^5 \text{ dyne} = 3.92 \text{ N}$$

When the container accelerates upwards, let the tension in the thread be T' and the upthrust acting on the block be U'. In the frame attached to the beaker, the block is in equilibrium, so we have

$$T' + mg_{\text{eff}} = U'$$

where, $U' = V\rho_l g_{\text{eff}}$ and $g_{\text{eff}} = g + a$

$$\Rightarrow T' = V\rho_l(g+a) - m(g+a)$$

$$\Rightarrow T' = V\rho_l(g+a) - V\rho_s(g+a)$$

$$\Rightarrow T' = V(\rho_l - \rho_s)(g+a)$$

$$\Rightarrow T = (1000)(1.2 - 0.8)(980 + 520)$$

$$\Rightarrow T = 6 \times 10^5 \text{ dyne} = 6 \text{ N}$$

Test Your Concepts-VIII (Based on Viscosity and Terminal Speed)

1. The velocity gradient in vertical direction is

$$\frac{dv}{dx} = \frac{18}{5} = 1 \text{ s}^{-1}$$

The magnitude of the force of viscosity is

$$F = \eta A \frac{dv}{dx}$$

The shearing stress is

$$\frac{F}{A} = \eta \frac{dv}{dx} = \left(10^{-2} \text{ poise}\right)\left(1 \text{ s}^{-1}\right) = 10^{-3} \text{ Nm}^{-2}$$

2. If r be the radius of small rain drop, then the terminal speed of the small drop is

$$v_T \propto r^2 \qquad \ldots(1)$$

If R be the radius of large drop, then equating the initial and the final volumes, we get

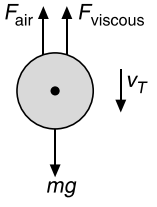
$$\frac{4}{3}\pi R^3 = 27\left(\frac{4}{3}\pi r^3\right)$$

$$\Rightarrow \quad R = 3r$$

$$\Rightarrow \quad \frac{v_T'}{v_T} = \left(\frac{R}{r}\right)^2 = 9$$

$$\Rightarrow \quad v_T' = 9 \text{ ms}^{-1}$$

3. The free body diagram of the ball is shown in Figure.
At $v = v_T$, the net force acting on the drop is zero.



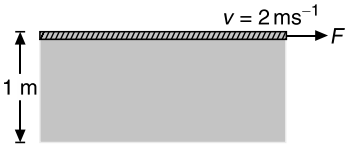
$$\Rightarrow \quad mg = F_{air} + F_{viscous}$$

$$\Rightarrow \quad \frac{4}{3}\pi R^3 dg = KRv_T + 6\pi\eta R v_T$$

$$\Rightarrow \quad v_t = \frac{4\pi R^2 gd}{3(6\pi\eta + K)}$$

4. The speed gradient is given by

$$\frac{\Delta v}{\Delta x} = \frac{2-0}{1-0} = 2 \text{ ms}^{-1}\text{m}^{-1}$$



From, Newton's law of viscous force,

$$|F| = \eta A \frac{\Delta v}{\Delta x}$$

$$\Rightarrow \quad |F| = \left(0.01 \times 10^{-1}\right)(2)(2)$$

$$\Rightarrow \quad |F| = 4 \times 10^{-3} \text{ N}$$

So, to keep the plate moving, a force of 4×10^{-3} N must be applied.

5. Terminal velocity of the largest particle which is just about to settle at the bottom of the vessel is

$$v_T = \frac{10 \times 10^{-2}}{3600} = \frac{1}{36000} \text{ ms}^{-1}$$

If r be the radius of that particle, then

$$v_T = \frac{2}{9}\frac{r^2}{\eta}(\rho - \sigma)g \qquad \ldots(1)$$

Substituting $\rho = 4 \times 10^3 \text{ kgm}^{-3}$, $\eta = \frac{0.01}{10} \text{ Nsm}^{-3}$ and $v_T = \frac{1}{36000} \text{ ms}^{-1}$ in equation (1), we get

$$r = 2 \times 10^{-6} \text{ m}$$

6. Velocity of ball just before striking the water surface is

$$v = \sqrt{2gh} \qquad \ldots(1)$$

Terminal velocity of ball inside water is

$$v = \frac{2}{9}r^2 g\frac{(\rho - 1)}{\eta} \qquad \ldots(2)$$

Equating (1) and (2), we get

$$\sqrt{2gh} = \frac{2}{9}\frac{r^2 g}{\eta}(\rho - 1)$$

$$\Rightarrow \quad h = \frac{2}{81}r^4\left(\frac{\rho - 1}{\eta}\right)^2 g$$

7. Area of each square metal plate is

$$A = (10 \text{ cm}) \times (10 \text{ cm}) = 100 \text{ cm}^2 = 0.01 \text{ m}^2$$

The viscous force is

$$F = 150 \text{ dyne} = 150 \times 10^{-5} \text{ N}$$

Relative speed between the plates is

$$\Delta v = 5 \text{ cms}^{-1} = 0.05 \text{ ms}^{-1}$$

Let the separation between the plates be Δx, then magnitude if the viscous force is

$$F = \eta A \frac{\Delta v}{\Delta x}, \text{ where } \eta = 0.001 \text{ Pl}$$

$$\Rightarrow \quad 150 \times 10^{-5} \text{ N} = \frac{0.001 \times 0.01 \times 0.05}{\Delta x}$$

$$\Rightarrow \quad \Delta x = \frac{0.05}{150} \text{ m} = \frac{1}{3000} \text{ m} = 0.033 \text{ cm}$$

8. Since, $R = \frac{\rho v d}{\eta}$

where, $v = 30 \text{ cms}^{-1} = 0.3 \text{ ms}^{-1}$, $\rho = 1060 \text{ kgm}^{-3}$

$$d = 2 \times 1.0 \text{ cm} = 0.02 \text{ m}$$

$$\eta = 4 \text{ mPas} = 4 \times 10^{-3} \text{ Pas} = 4 \times 10^{-3} \text{ Nsm}^{-2}$$

$$\Rightarrow \quad R = \frac{(1060)(0.3)(0.02)}{\left(4 \times 10^{-3}\right)} = 1590$$

Since $R < 2000$, so this flow will be laminar.

9. Since the rate of flow of liquid through the tubes is the same, so we have

$$Q = \frac{\pi r^4 P}{8\eta l} = \text{constant}$$

$$\Rightarrow \quad \frac{\pi(r/2)^4 P_2}{8\eta(2l)} = \frac{\pi(r/3)^4 P_3}{8\eta(l/2)}$$

$$\Rightarrow \quad \frac{P_2}{P_3} = \frac{64}{81}$$

10. According to Poiseuille equation, we have

$$Q = \frac{\pi R^4 \Delta P}{8\eta l} = \text{constant}$$

Also, according to the problem, the pressure gradient maintained is also constant, so we have

$$\frac{\Delta P}{l} = \text{constant}$$

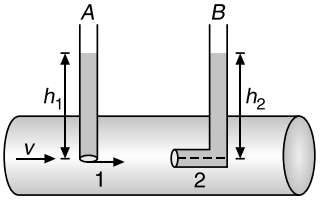
$$\Rightarrow \quad R^4 \propto \eta$$

$$\Rightarrow \quad \frac{R_2^4}{R_1^4} = \frac{\eta_2}{\eta_1} = \frac{12.96}{0.01} = 1296$$

$$\Rightarrow \quad \frac{R_2}{R_1} = (1296)^{1/4} = 6$$

Test Your Concepts-IX (Based on Equation of Continuity, Bernoulli's Theorem and Applications)

1. The liquid will not flow through the tubes. There is no speed of the liquid inside tubes. Let us take two points 1 (just outside the tube A) and 2 (just inside the tube B) as shown in Figure.



The speed of liquid at a point 2 is zero.
Using Bernoulli's equation between point 1 and 2 give

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + 0$$

$$\Rightarrow \quad P_2 - P_1 = \frac{1}{2}\rho v^2 \qquad \{\because v_1 = v \text{ and } v_2 = 0\}$$

$$\Rightarrow \quad \rho g h_2 - \rho g h_1 = \frac{1}{2}\rho v^2$$

$$\Rightarrow \quad h_2 - h_1 = \frac{v^2}{2g}$$

2. The maximum height to which the water rises will correspond to the height of the liquid when the inflow of water per second in the bucket equals the outflow of the water per second from the orifice.

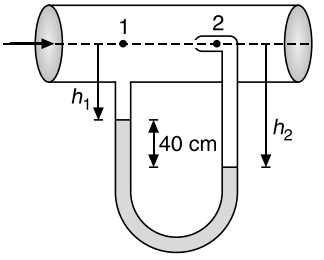
$$1.3 \times 10^{-4} = av = a\sqrt{2gh_m}$$

$$\Rightarrow \quad h_m = \frac{(1.3 \times 10^{-4})^2}{2a^2 g} = \frac{(1.3 \times 10^{-4})^2}{2(10^{-4})^2 (9.8)}$$

$$\Rightarrow \quad h_m = 0.086 \text{ m}$$

$$\Rightarrow \quad h_m = 8.6 \text{ cm}$$

3. Let P_1 be pressure of air at 1 and P_2 be the pressure of air at 2 as shown in Figure.



Since relative density of alcohol is 0.8, so density of alcohol is $0.8 \text{ gcm}^{-3} = 800 \text{ kgm}^{-3}$. Also, we are given that $P_1 - P_2 = 40$ cm of alcohol, so we have

$$P_2 - P_1 = (0.4 \text{ m})\rho_{\text{alcohol}} g$$

$$\Rightarrow \quad P_2 - P_1 = (0.4)(800)(10) = 3200 \text{ Nm}^{-2}$$

If v_1 is velocity of air at the nozzle and v_2, at the other end of the tube, then according to Bernoulli's Theorem, we have

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

Just when the air enters the tube at 2, its velocity becomes zero, so $v_2 = 0$ and $v_1 = v$.

$$\Rightarrow \quad P_2 - P_1 = \frac{1}{2}\rho v^2$$

$$\Rightarrow \quad \frac{1}{2}\rho v^2 = 3200$$

$$\Rightarrow \quad v^2 = 6400$$

$$\Rightarrow \quad v = 80 \text{ ms}^{-1}$$

4. **CASE-1: For $h_1 = 4H$**

If h be the height of the free surface above the ground, then

$$h = h_1 + 6H = 4H + 6H = 10H$$

Maximum range will be obtained when

$$h_2 = \frac{h}{2} = 5H$$

and this maximum range will be,

$$R_{\max} = h = 10H$$

CASE-2: For $h_1 = 8H$

Here, $h = h_1 + 6H = 8H + 6H = 14H$

Since, $\frac{h}{2}$ i.e. $7H$ lies inside the platform. So, maximum range will be obtained from the bottommost point lying in the liquid container.

$$\Rightarrow \quad h_2 = 6H$$

and this maximum range will be

$$R_{\max} = 2\sqrt{h_2(h - h_2)}$$

$$\Rightarrow \quad R_{\max} = 2\sqrt{6H \times 8H} = 8\sqrt{3}H$$

5. Given that, $h_2 - h_1 = 0.51$ m
Since, we know that the thrust due to liquid of density ρ emerging with a speed v from an orifice of area A is

$$F = v\left(\frac{dm}{dt}\right) = v\left(\frac{A(dx)\rho}{dt}\right) = Av^2\rho$$

This thrust is opposite to the velocity of the liquid emerging from the orifice.

So, $F_1 = Av_1^2\rho = A(2gh_1)\rho$, opposite to v_1

and $F_2 = Av_2^2\rho = A(2gh_2)\rho$, opposite to v_2

$\Rightarrow \quad F_{net} = F_2 - F_1 = 2g\rho A(h_2 - h_1)$

$\Rightarrow \quad F_{net} = 2\times 9.8\times 10^3 \times 0.5\times 10^{-4}(0.51)$

$\Rightarrow \quad F_{net} = 0.5 \text{ N}$

6. The force of thrust acting on the slider is

$$F_t = v_r\left(\frac{dm}{dt}\right) = (v-V)^2(A\rho)$$

$\Rightarrow \quad F_t = (20-10)^2(0.005)(1000) = 500 \text{ N}$

So, acceleration of the slider is

$$a = \frac{F_t - \mu_k mg}{m}$$

$\Rightarrow \quad a = \dfrac{500-(0.25)(40)(10)}{40} = 10 \text{ ms}^{-2}$

At terminal velocity (v_T) of slider, we have

$a_{net} = 0$

$\Rightarrow \quad F_t = f_k = \mu_k mg$

$\Rightarrow \quad (v-v_T)^2(A\rho) = \mu_k mg$

$\Rightarrow \quad v_T = v - \sqrt{\dfrac{\mu_k mg}{A\rho}} = 20 - \sqrt{\dfrac{(0.25)(40)(10)}{(0.005)(1000)}}$

$\Rightarrow \quad v_T = 20 - 4.5 = 15.5 \text{ ms}^{-1}$

7. Applying Bernoulli's Theorem, we get

$$\frac{1}{2}\rho v^2 = \rho gh + \frac{mg}{A} \qquad \ldots(1)$$

where, $h = 1.0 - 0.5 = 0.5$ m and area of piston is $A = 0.5 \text{ m}^2$.

From equation (1), we get

$$v = \sqrt{2gh + \frac{2mg}{\rho A}}$$

$\Rightarrow \quad v = \sqrt{2\times 10\times 0.5 + \dfrac{2\times 20\times 10}{10^3\times 0.5}}$.

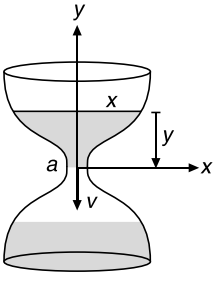
$\Rightarrow \quad v = 3.3 \text{ ms}^{-1}$

So, the speed with which it hits the surface is

$$v' = \sqrt{v^2 + 2gh'} = \sqrt{(3.25)^2 + (2\times 9.8\times 0.5)}$$

$\Rightarrow \quad v' = 4.51 \text{ ms}^{-1}$

8. At any instant t, let the level of liquid in the clock jug be y as shown in Figure. After time $t+dt$, the level of water in the jug be $y-dy$. Let the water exit the drain hole with a speed v, then from Torricelli's Theorem, we have



$$v = \sqrt{2gy}$$

Also, the rate of flow of liquid through the drain hole is

$$Q = \frac{dV}{dt} = av = a\sqrt{2gy}$$

where, $\dfrac{dV}{dt} = (\pi x^2)\left(-\dfrac{dy}{dt}\right)$

$\Rightarrow \quad a\sqrt{2gy} = \pi x^2\left(-\dfrac{dy}{dt}\right) \qquad \ldots(1)$

According to the problem, we have

$$-\frac{dy}{dt} = \frac{4\times 10^{-2}}{3600} = 1.11\times 10^{-5} \text{ ms}^{-1}$$

$$a = \pi r^2 = \pi(2\times 10^{-3})^2$$

$\Rightarrow \quad a = 1.26\times 10^{-8} \text{ m}^2$

Substituting these values in equation (1), we get

$$(1.26\times 10^{-5})\sqrt{2\times 9.8\times y} = \pi(1.11\times 10^{-5})\cdot x^2$$

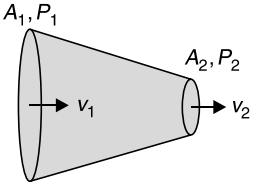
$\Rightarrow \quad y = 0.4x^4$

9. From continuity equation,

$$A_1v_1 = A_2v_2$$

$\Rightarrow \quad \dfrac{v_1}{v_2} = \dfrac{A_2}{A_1} = \dfrac{\pi r_2^2}{\pi r_1^2} = \left(\dfrac{r_2}{r_1}\right)^2 = \left(\dfrac{0.04}{0.1}\right)^2 = \dfrac{4}{25} \qquad \ldots(1)$

From Bernoulli's equation,



$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$\Rightarrow \quad v_2^2 - v_1^2 = \dfrac{2(P_1 - P_2)}{\rho}$

$\Rightarrow \quad v_2^2 - v_1^2 = \dfrac{2\times 10}{1.25\times 10^3} = 1.6\times 10^{-2} \text{ m}^2\text{s}^{-2} \qquad \ldots(2)$

Solving equations (1) and (2), we get

$v_2 \approx 0.128 \text{ ms}^{-1}$

Rate of volume flow through the tube

$$Q = A_2v_2 = (\pi r_2^2)v_2$$

$\Rightarrow \quad Q = \pi(0.04)^2(0.128)$

$\Rightarrow \quad Q = 6.43\times 10^{-4} \text{ m}^3\text{s}^{-1}$

10. Since, $Q = av = -A\left(\dfrac{dy}{dt}\right)$

$\Rightarrow \quad a\sqrt{2gy} = -A\left(\dfrac{dy}{dt}\right)$

$\Rightarrow \quad dt = -\left(\dfrac{A}{a}\right)\dfrac{dy}{\sqrt{2gy}}$

$\Rightarrow \quad \displaystyle\int_0^t dt = -\left(\frac{A}{a}\right)\frac{dy}{\sqrt{2gy}}$

$\Rightarrow \quad t = \dfrac{400\times 2}{\sqrt{2\times 9.8}} \approx 180 \text{ s} \approx 3 \text{ minute}$

When the water level in tank is maintained always at a level of 1 m above the orifice, then

$$t = \frac{\text{Total Volume}}{av} = \frac{Ah}{a\sqrt{2gh}}$$

$\Rightarrow \quad t = \dfrac{A}{a}\sqrt{\dfrac{h}{2g}} = 400\sqrt{\dfrac{1.0}{2\times 9.8}}$

$\Rightarrow \quad t \approx 90 \text{ s} = 1.5 \text{ minute}$

11. Applying Bernoulli's equation between points (1) and (2), we get

$$P_1+\frac{1}{2}\rho v_1^2+\rho g h_1=P_2+\frac{1}{2}\rho v_2^2+\rho g h_2$$

Since, area of reservoir $\gg$ area of pipe, so we have

$$v_1\approx 0$$

Also $P_1=P_2=P_{\text{atm}}$

$$\Rightarrow\quad v_2=\sqrt{2g(h_1-h_2)}=\sqrt{2\times 10\times 5}=10\ \text{ms}^{-1}$$

The minimum pressure in the bend will be at A. Therefore, applying Bernoulli's equation between 1 and A, we get

$$P_1+\frac{1}{2}\rho v_1^2+\rho g h_1=P_A+\frac{1}{2}\rho v_A^2+\rho g h_A$$

Since, $v_1\approx 0$ and from equation of continuity, we have $v_A=v_2$.

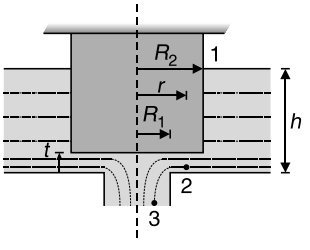
$$\Rightarrow\quad P_A=P_1+\rho g(h_1-h_A)-\frac{1}{2}\rho v_2^2$$

Substituting the values, we get

$$P_A=(10^5)+(1000)(10)(-1)-\frac{1}{2}(1000)(10)^2$$

$$\Rightarrow\quad P_A=4\times 10^4\ \text{Nm}^{-2}$$

12. Let v_2 be the speed of liquid at section 2 and v_3 at section 3 as shown in Figure.



Using Equation of Continuity at 2 and 3, we get

$$A_2v_2=A_3v_3$$

$$\Rightarrow\quad (2\pi rt)v_2=(2\pi R_1t)v_3$$

$$\Rightarrow\quad \frac{v_2}{v_3}=\frac{R_1}{r}\qquad\ldots(1)$$

If t is the thickness of clearance, then on applying the Bernoulli's equation at 1, 2 and 3, we get

$$P_0+0+\rho g h=P+\frac{1}{2}\rho v_2^2+0=P_0+\frac{1}{2}\rho v_3^2+0\qquad\ldots(2)$$

From equation (1) and (2), we get

$$P=P_0+\rho g h\left(1-\frac{R_1^2}{r^2}\right)$$

13. Time taken for the level to fall from h to h' is

$$t=\frac{A}{A_0}\sqrt{\frac{2}{g}}\left[\sqrt{h}-\sqrt{h'}\right]$$

So, the time taken for the level to fall from H to $\frac{H}{2}$ is

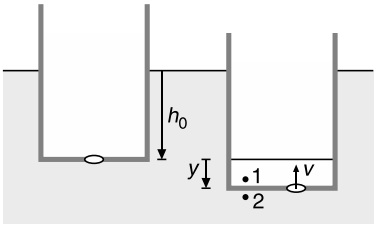
$$t_1=\frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{H}-\sqrt{\frac{H}{2}}\right]$$

and the time taken for the level to fall from $\frac{H}{2}$ to zero is

$$t_2=\frac{A}{a}\sqrt{\frac{2}{g}}\left(\sqrt{\frac{H}{2}}-0\right)$$

$$\Rightarrow\quad \frac{t_1}{t_2}=\frac{1-\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}-0}=\sqrt{2}-1$$

14. In this problem, we note that the level of water in the can will rise as much as the can sinks in water. At any instant, let the can sink in water through y as shown in Figure.



Just above the bottom of can, at the point 1, pressure is

$$P_1=P_0+\rho g y$$

Just below the bottom of can, at the point 2, pressure is

$$P_2=P_0+\rho g(h_0+y)$$

$$\Rightarrow\quad \Delta P=P_2-P_1=\rho g h_0$$

Due to this difference in pressure, water will start flowing inside the can, such that

$$\Delta P=\frac{1}{2}\rho v^2$$

$$\Rightarrow\quad v=\sqrt{\frac{2\Delta P}{\rho}}=\sqrt{2gh_0}$$

So, the time taken to sink completely is

$$t=\frac{A(h-h_0)}{av}=\frac{A(h-h_0)}{a\sqrt{2gh_0}}$$

15. $$\frac{1}{2}\rho_a v^2=\rho_w g h$$

$$\Rightarrow\quad v=\sqrt{\frac{2gh\rho_w}{\rho_a}}=\sqrt{\frac{2\times 9.8\times 10^{-2}\times 10^3}{1.3}}$$

$$\Rightarrow\quad v=12.3\ \text{ms}^{-1}$$

16. Considering ZPEL at the ground, then on applying Bernoulli's Theorem, we get

$$\rho_2 g h+\rho_1 g(h-y)=\frac{1}{2}\rho_1 v^2$$

$$\Rightarrow\quad v=\sqrt{2g\left(\frac{\rho_2}{\rho_1}h+h-y\right)}$$

$$\Rightarrow\quad v=\sqrt{20\left[\left(\frac{600}{900}\right)(0.6)+0.6-0.2\right]}$$

$$\Rightarrow\quad v=4\ \text{ms}^{-1}$$

Due to this velocity, the thrust force acting on the tank is

$$F=av^2\rho_1$$

$$\Rightarrow\quad F=(900)(5\times 10^{-4})(4)^2=7.2\ \text{N}$$

The normal reaction force is given by

$$N = mg = Ah(\rho_1 + \rho_2)g$$

$$\Rightarrow \quad N = (0.5)(0.6)(900 + 600)(10)$$

$$\Rightarrow \quad N = 4500 \text{ N}$$

So, the limiting friction is

$$f_L = \mu N = (0.01)(4500) = 45 \text{ N}$$

Since, $f_L > F$, so the minimum force required to keep the tank in equilibrium is

$$F_{\min} = 0$$

and the maximum force required to keep the tank in equilibrium is

$$F_{\max} = 45 + 7.2 = 52.2 \text{ N}$$

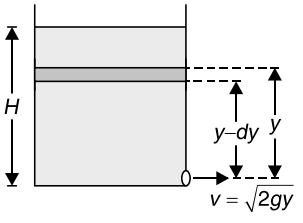
17. Now let at some time t the level of liquid inside the container be y. The speed of the liquid coming out of the hole will be $\sqrt{2gy}$

Applying equation of continuity

$$\Rightarrow \quad \left(\sqrt{2gy}\right)a = -A\frac{dy}{dt}$$

$$\Rightarrow \quad -\frac{a}{A}\int_0^t dt = \int_H^0 \frac{dy}{\sqrt{2gy}}$$

$$\Rightarrow \quad t = \frac{A}{a}\sqrt{\frac{2H}{g}}$$



18. At depth y, the velocity of efflux is $v = \sqrt{2gy}$

Force acting on small cross-sectional area $dA = bdy$ is

$$dF = (dA)v^2\rho = (bdy)(2gy)\rho$$

$$\Rightarrow \quad dF = \rho b(2gy)dy = 2bg\rho y dy$$

$$\Rightarrow \quad F = \int_{0.25}^{0.75} 2\rho g b y dy = 2\rho g b\left(\frac{y^2}{2}\right)\Big|_{0.25}^{0.75}$$

$$\Rightarrow \quad F = (10^3)(9.8)(10^{-3})\left[(0.75)^2 - (0.25)^2\right]$$

$$\Rightarrow \quad F = 4.9 \text{ N}$$

19. The volume flow rate in the line is

$$Q = \frac{dV}{dt} = Av = \pi(6.5\times10^{-3})^2(1.2)$$

$$\Rightarrow \quad Q = 1.6\times10^{-4} \text{ m}^3\text{s}^{-1}$$

When water flows out through the shower head with a speed v', then Q remains the same.

$$\Rightarrow \quad Q = \frac{dV}{dt} = nav'$$

$$\Rightarrow \quad v' = \frac{dV/dt}{na} = \frac{1.6\times10^{-4}}{12\times\pi\times(4.6\times10^{-4})^2}$$

$$\Rightarrow \quad v' = 20 \text{ ms}^{-1}$$

20. Applying Bernoulli's equation at points 1 and 2, we get

$$p_1 + \frac{1}{2}(2\rho)v_1^2 + (2\rho)gh_1 = p_2 + \frac{1}{2}(2\rho)v_2^2 + (2\rho)gh_2$$

where, $v_1 \approx 0$, $v_2 = v$, $h_1 = h_2$, $p_2 = p_0$

and $p_1 = p_0 + \rho gH + 2\rho gH = p_0 + 3\rho gH$

Substituting these values in the Bernoulli's equation, we get

$$p_0 + 3\rho gH = p_0 + \rho v^2$$

$$\Rightarrow \quad v = \sqrt{3gH}$$

Since this velocity is horizontal, so time taken by the liquid to fall to the ground is

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2H}{g}} \qquad \{\because h = H\}$$

So, the required range of the liquid is

$$R = vt = \left(\sqrt{3gH}\right)\left(\sqrt{\frac{2H}{g}}\right)$$

$$\Rightarrow \quad R = \sqrt{6}H$$

21. The speed v with which liquid comes out of the orifice is

$$v = \sqrt{2g\left(\begin{array}{c}\text{depth of orifice below}\\ \text{free surface of liquid}\end{array}\right)}$$

$$\Rightarrow \quad v = \sqrt{2gh_1}\text{, along the horizontal}$$

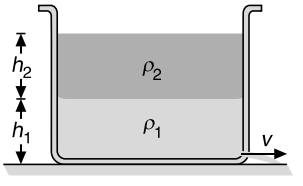
Let t be the time taken by the liquid to hit the ground, then since v is horizontal, so we have

$$t = \sqrt{\frac{2h_2}{g}}$$

The range is given by

$$R = vt = \left(\sqrt{2gh_1}\right)\left(\sqrt{\frac{2h_2}{g}}\right) = 2\sqrt{h_1h_2}$$

22. The arrangement discussed in problem is shown in Figure.



Assuming the atmospheric pressure to be P_a, then on applying the Bernoulli's Theorem, we get

$$P_a + \frac{1}{2}\rho_1 v^2 = P_a + \rho_1 gh_1 + \rho_2 gh_2$$

$$\Rightarrow \quad v = \sqrt{2g\left(h_1 + h_2\frac{\rho_2}{\rho_1}\right)}$$

$$\Rightarrow \quad v = \sqrt{2\times9.8\left(0.3 + \frac{0.2\times600}{1000}\right)}$$

$$\Rightarrow \quad v = 2.87 \text{ ms}^{-1}$$

23. Time taken to empty the complete tank is

$$t = \frac{A}{a}\sqrt{\frac{2H}{g}}$$

Since, $t_{H\to0} = t_{H\to\frac{H}{3}} + t_{\frac{H}{3}\to0}$

$$\Rightarrow \quad \frac{A}{a}\sqrt{\frac{2H}{g}} = t_0 + \frac{A}{a}\sqrt{\frac{2\left(\frac{H}{3}\right)}{g}}$$

$$\Rightarrow \quad \frac{A}{a}\sqrt{\frac{2\left(\frac{H}{3}\right)}{g}} = \frac{A}{a}\sqrt{\frac{2H}{g}} - t_0$$

$$\Rightarrow \quad t_{\frac{H}{3}\to 0} = \frac{A}{a}\sqrt{\frac{2H}{g}} - t_0$$

Test Your Concepts-X (Based on Surface Tension, Surface Energy, Excess Pressure and Capillarity)

1. Since, $h = \frac{2T\cos\theta}{r\rho g}$, so the surface tension of the liquid is

$$T = \frac{rh\rho g}{2} \qquad \{\because\ \theta = 0^\circ\}$$

$$\Rightarrow \quad T = \frac{(0.025)(3)(1.5)(980)}{2}$$

$$\Rightarrow \quad T = 55 \text{ dynecm}^{-1}$$

Hence excess pressure inside a spherical bubble is

$$\Delta p = \frac{4T}{R} = \frac{(4)(55)}{(0.5)} = 440 \text{ dynecm}^{-2}$$

2. When the ring is about to leave the water surface, surface tension force on it is

$$F_{ST} = 2\pi RT + 2\pi rT = 2\pi(R+r)T$$

Spring force $F_s = kx$

$$\Rightarrow \quad kx = 2\pi(R+r)T + mg$$

$$\Rightarrow \quad T = \frac{kx - mg}{2\pi(R+r)}$$

3. $h = \frac{2T\cos\theta}{r\rho g}$

Substituting the proper values, we have

$$h = \frac{(2)(7\times10^{-2})\cos 0^\circ}{(0.5\times10^{-3})(10^3)(9.8)} = 2.86\times10^{-2} \text{ m}$$

$$\Rightarrow \quad h = 2.86 \text{ cm}$$

4. In general, $2\pi T\cos\theta = (\pi r^2 h'\rho)g$

$$\Rightarrow \quad 2T\cos\theta = h'r\rho g$$

Now $r = R\cos\theta$

$$R = \frac{2T}{h'\rho g} = \frac{2\times0.07}{1\times10^{-2}\times1000\times9.8}$$

$$\Rightarrow \quad R = 1.4\times10^{-3} \text{ m} = 1.4 \text{ mm}$$

5. Excess pressure inside first bubble is $\Delta P_1 = \frac{4T}{r}$

Excess pressure inside second bubble is

$$\Delta P_2 = \frac{4T}{R}$$

So, excess of pressure on the two sides of the separating film is

$$\Delta P = \Delta P_1 - \Delta P_2 = 4T\left(\frac{1}{r} - \frac{1}{R}\right) \qquad \ldots(1)$$

If R' be the radius of the interface film, then we have

$$\Delta P = \frac{4T}{R'} \qquad \ldots(2)$$

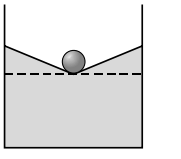
From equations (1) and (2), we get

$$\frac{4T}{R'} = 4T\left(\frac{1}{r} - \frac{1}{R}\right)$$

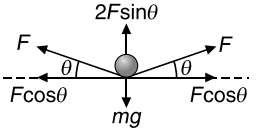
$$\Rightarrow \quad \frac{1}{R'} = \frac{1}{r} - \frac{1}{R}$$

$$\Rightarrow \quad R' = \frac{Rr}{R-r}$$

6. Let the mass of the needle be m. Since the liquid surface gets distorted, so the force due to surface tension acting on both sides of the needle makes an angle θ with the horizontal. The forces acting on the needle are F, F and mg as shown in Figure.



Cross-sectional View



On resolving the forces, the horizontal components cancel. However, for vertical equilibrium, we have

$$2F\sin\theta = mg \text{, where } F = Tl$$

$$\Rightarrow \quad m = \frac{2F\sin\theta}{g} = \frac{2Tl\sin\theta}{g}$$

For m to be maximum, $\sin\theta = \text{MAX} = 1$

$$\Rightarrow \quad m_{\max} = \frac{2Tl}{g}$$

7. $h = \frac{2T\cos\theta}{r\rho g}$

$$\Rightarrow \quad hr = \frac{2T\cos\theta}{\rho g} = \text{constant}$$

$$\Rightarrow \quad h_1 r_1 = h_2 r_2$$

$$\Rightarrow \quad h_2 = \frac{h_1 r_1}{r_2}$$

Substituting $\frac{r_2}{r_1} = \frac{1}{3}$, we get

$$h_2 = (2)(3) = 6 \text{ cm}$$

8. Let R be the radius of the big drop formed by the combination of 8 small droplets. Since volume of the big drop equals the total volume of 8 small droplets, so we have

$$\frac{4}{3}\pi R^3 = 8\left(\frac{4\pi}{3}r^3\right)$$

$$\Rightarrow \quad R^3 = 8r^3$$

$$\Rightarrow \quad R = 2r = 2(5\times10^{-4} \text{ m}) = 10^{-3} \text{ m}$$

Final surface area of the big drop is

$$A_f = 4\pi R^2 = 4\pi(10^{-3}\text{ m})^2$$

$$\Rightarrow \quad A_f = 4\pi \times 10^{-6}\text{ m}^2$$

Total initial surface area of 8 small droplets is

$$A_i = 8(4\pi r^2) = 8 \times 4\pi(5 \times 10^{-4}\text{ m})^2$$

$$\Rightarrow \quad A_i = 8\pi \times 10^{-6}\text{ m}^2$$

Decrease in surface area is

$$A_i - A_f = 8\pi \times 10^{-6}\text{ m}^2 - 4\pi \times 10^{-6}\text{ m}^2$$

$$\Rightarrow \quad A_i - A_f = |\Delta A| = 4\pi \times 10^{-6}\text{ m}^2$$

Energy evolved is

$$W = T|\Delta A| = (72 \times 10^{-3})(4\pi \times 10^{-6})\text{ J}$$

$$\Rightarrow \quad W = 9 \times 10^{-7}\text{ J}$$

9. Since pressure on concave side is greater than that on convex side, so pressure at A and B is

$$P_A = P_0 - \frac{2T}{r_1} \text{ and } P_B = P_0 - \frac{2T}{r_2}$$

So, the pressure difference between A and B is

$$\Delta P = 2T\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

If this pressure difference corresponds to a height equal to h units of the liquid, then we have

$$2T\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = h\rho g$$

$$\Rightarrow \quad h = \frac{2T}{\rho g}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\Rightarrow \quad h = \frac{2 \times 0.07}{1000 \times 9.8}\left(\frac{1}{1 \times 10^{-3}} - \frac{1}{1.5 \times 10^{-3}}\right)$$

$$\Rightarrow \quad h = 4.76\text{ mm}$$

10. Since a soap bubble has two free surfaces, so the initial surface area of the soap bubble is

$$A_{\text{initial}} = 2(4\pi r_1^2) = 2(4\pi)(0.1)^2\text{ m}^2$$

Final surface area of the soap bubble is

$$A_{\text{final}} = 2(4\pi r_2^2) = 2(4\pi)(0.2)^2\text{ m}^2$$

So, change in surface area of the soap bubble is

$$\Delta A = 8\pi\left[(0.2)^2 - (0.1)^2\right] = 0.24\pi\text{ m}^2$$

Hence the work done W is given by

$$W = T\Delta A = (25 \times 10^{-3})(0.24\pi)$$

$$\Rightarrow \quad W = 6\pi \times 10^{-3}\text{ J}$$

$$\Rightarrow \quad W \approx 19\text{ mJ}$$

11. Since a film has two free surfaces, so initial surface area of the film is

$$A_i = 2(l \times b) = 2(10 \times 0.5)\text{ cm}^2 = 10^{-3}\text{ m}^2$$

Final surface area of the film is

$$A_f = 2[10 \times (0.5 + 0.1)]\text{ cm}^2$$

$$\Rightarrow \quad A_f = 1.2 \times 10^{-3}\text{ m}^2$$

So, the required work done is

$$W = T\Delta A = (72 \times 10^{-3})(1.2 - 1) \times 10^{-3}\text{ J}$$

$$\Rightarrow \quad W = 72 \times 10^{-3} \times 2 \times 10^{-4} = 144 \times 10^{-7}\text{ J}$$

12. Let r_1 and r_2 be the radii of two given soap bubbles and r be that of the coalesced bubble.

Since a soap bubble has two free surfaces, so if W_1 and W_2 are the potential energies of the two given bubbles, then we have

$$W_1 = 2(4\pi r_1^2)T \text{ and } W_2 = 2(4\pi r_2^2)T$$

Potential energy of the bubble formed due to coalescing of the two bubbles is

$$W = 2(4\pi r^2)T$$

Since total energy of system is conserved, so we have

$$8\pi r^2 T = 8\pi r_1^2 T + 8\pi r_2^2 T$$

$$\Rightarrow \quad r^2 = r_1^2 + r_2^2 = (6)^2 + (8)^2 = 100$$

$$\Rightarrow \quad r = 10\text{ cm}$$

13. The depth of the water column, when the water just enters the hollow sphere through the hole is $h = 40$ cm. Let R be the radius of the hole in the hollow sphere, then on immersing the hollow sphere (having hole) in water, an air bubble is formed at the hole. The radius of this air bubble is approximately equal to the radius of the hole.

The pressure corresponding to water depth of 40 cm will try to push water into the hollow sphere, whereas the excess pressure inside the air bubble opposes the entry of water. So, water will enter the hollow sphere just when the excess of pressure inside the air bubble becomes equal to the pressure exerted by 40 cm column of water.

$$\Rightarrow \quad P_i - P_0 = \Delta P = \frac{2T}{R} \qquad \ldots(1)$$

Also, $P_i - P_0 = h\rho g = (0.40)(10^3)(10)$

$$\Rightarrow \quad P_i - P_0 = \Delta P = 4 \times 10^3\text{ Nm}^{-2} \qquad \ldots(2)$$

Substituting in equation (2), we get

$$R = \frac{2T}{\Delta P} = \frac{2(75 \times 10^{-3})}{4 \times 10^3} = 3.75 \times 10^{-6}\text{ m}$$

14. Total initial surface energy of 1000 droplets is

$$U_i = (1000)\left[4\pi(0.5 \times 10^{-3})^2\right](0.475)$$

$$\Rightarrow \quad U_i \approx 1.5 \times 10^{-3}\text{ J}$$

When the drops combine, then volume remains the same. So, we have

$$\Rightarrow \quad 1000\left[\frac{4\pi}{3}(0.5)^3\right] = \frac{4\pi}{3}R^3$$

$$\Rightarrow \quad R = 10(0.5\text{ mm}) = 5\text{ mm}$$

Final surface energy of the bigger drop is

$$U_f = 4\pi(5 \times 10^{-3})^2(0.475) = 0.15 \times 10^{-3}\text{ J}$$

Loss in surface energy is

$$U_i - U_f = 1.5 \times 10^{-3} - 0.15 \times 10^{-3}$$

$$\Rightarrow \quad U_i - U_f = 1.35 \times 10^{-3}\text{ J}$$

Single Correct Choice Type Questions

1. Equating the rate of flow of liquid

$$Q = \frac{\pi r^4 P}{8\eta\ell}$$

$$\Rightarrow \quad Q_1 = Q_2$$

$$\Rightarrow \quad \frac{\pi(r)^4 P_1}{8\eta\ell} = \frac{\pi(2r)^4 P_2}{8\eta(2\ell)}$$

$$\Rightarrow \quad \frac{P_1}{P_2} = 8$$

Hence, the correct answer is (C).

2. Increase in tension of wire is

$$F = YA\alpha\Delta\theta$$

$$\Rightarrow \quad F = 8\times10^{-6}\times2.2\times10^{11}\times10^{-2}\times10^{-4}\times5 = 8.8\text{ N}$$

Hence, the correct answer is (D).

3. Let R_1 and R_2 are the radii of soap bubbles before and after collapsing. Then

$$V = 2\left(\frac{4}{3}\pi R_1^3\right) - \frac{4}{3}\pi R_2^3 \qquad \ldots(1)$$

$$S = 2\left(8\pi R_1^2\right) - 8\pi R_2^2 \qquad \ldots(2)$$

Since, $P_1V_1 = P_2V_2$

$$\Rightarrow \quad 2\left(\frac{4}{3}\pi R_1^3\right)\left(P + \frac{4T}{R_1}\right) = \left(P + \frac{4T}{R_2}\right)\left(\frac{4}{3}\pi R_2^3\right)$$

$$\Rightarrow \quad 2R_1^3\left(P + \frac{4T}{R_1}\right) = R_2^3\left(P + \frac{4T}{R_2}\right)$$

$$\Rightarrow \quad P\left(2R_1^3 - R_2^3\right) = 4T\left(R_2^2 - 2R_1^2\right) \qquad \ldots(3)$$

From equation (1), $4\pi\left(2R_1^3 - R_2^3\right) = 3V$

From equation (2), $8\pi\left(R_2^2 - 2R_1^2\right) = -S$

Substituting in equation (3), we get

$$3PV + 4ST = 0$$

Hence, the correct answer is (A).

4. Since $Y = \frac{F/A}{x/L}$

$$\Rightarrow \quad Y = \frac{FL}{Ax}$$

For wires of same material Y is the same.

$$\Rightarrow \quad \frac{FL}{Ax} = \text{constant}$$

$$\Rightarrow \quad x \propto \frac{L}{A}$$

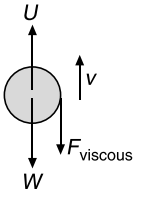
$$\Rightarrow \quad \frac{x_1}{x_2} = \frac{L_1A_2}{L_2A_1}$$

$$\Rightarrow \quad \frac{x_1}{x_2} = \frac{1}{2}\left(\frac{1}{4}\right)$$

$$\Rightarrow \quad \frac{x_1}{x_2} = \frac{1}{8}$$

Hence, the correct answer is (B).

5. Forces acting on the ball are



Upthrust, $U = 2V\rho g$

Viscous drag, $F_{\text{viscous}} = 6\pi\eta rv$

and weight of ball $W = V\rho g$

Net force acting on ball in the upward direction is

$$F_{\text{net}} = U - W - F_{\text{viscous}}$$

Since U and W are constants whereas F_{viscous} increases with increase in v. Hence, acceleration of the ball will go on decreasing.

Hence, the correct answer is (B).

6. For constant temperature,

$$P_iV_i = P_fV_f$$

$$\Rightarrow \quad \left(P_1 + \frac{4T}{r}\right)\left(\frac{4}{3}\pi r^3\right) = \left(P_2 + \frac{4T}{r/2}\right)\left[\frac{4}{3}\pi\left(\frac{r}{2}\right)^3\right]$$

$$\Rightarrow \quad P_2 = 8P_1 + \frac{24T}{r}$$

Hence, the correct answer is (A).

7. Since, $W = \frac{1}{2}(\text{Tension})(\text{Extension})$

Further

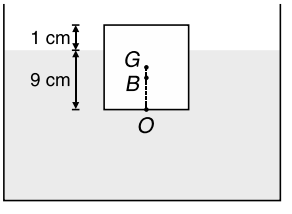
$$T = YA\alpha t \text{ and}$$

$$\Delta\ell = \ell\alpha t$$

$$\Rightarrow \quad W = \frac{1}{2}(YA\alpha t)(\ell\alpha t)$$

Hence, the correct answer is (B).

8. Relative density of ice is 0.9 i.e., 90% volume of ice is immersed in water. When ice melts completely level of water does not change.



OB = 4.5 cm, OG = 5 cm

Since, $GB = h = 0.5$ cm

$$\Rightarrow \quad \Delta U = -mgh$$

$$\Rightarrow \quad \Delta U = -(0.1)^3(900)(10)\left(0.5\times10^{-2}\right)\text{ J}$$

$$\Rightarrow \quad \Delta U = -0.045\text{ J}$$

Hence, the correct answer is (D).

9. The pressure at the interface must be same, calculated via either tube. Since, both tubes are open to the atmosphere, we must have

$$h_1\rho_1 g = h_2\rho_2 g$$

$$\Rightarrow \quad h_1\rho_1 = h_2\rho_2$$

$$\Rightarrow \quad h\rho = \text{constant}$$

$$\Rightarrow \quad h \propto \frac{1}{\rho}$$

Hence, the correct answer is (B).

10. $W = 2\pi(r_1 + r_2)T$

$\Rightarrow \quad T = \dfrac{W}{2\pi(r_1 + r_2)} = \dfrac{7.48 \times 10 \times 10^{-3}}{2\pi \times 17 \times 10^{-2}}$

$\Rightarrow \quad T = 70 \times 10^{-3}\ \text{Nm}^{-1}$

Hence, the correct answer is (D).

11. $\dfrac{1}{2}\left(\dfrac{YA}{L}\right)(\Delta\ell)^2 = \dfrac{1}{2}mv^2$

$\Rightarrow \quad Y = \dfrac{mv^2L}{A(\Delta\ell)^2} = \dfrac{0.02 \times 400 \times 0.42 \times 4}{\pi \times 36 \times 10^{-6} \times 0.04}$

$\Rightarrow \quad Y = 2.3 \times 10^6\ \text{Nm}^{-2}$

So, order is 10^6.

Hence, the correct answer is (D).

12. $2T\ell\cos\theta = mg = \pi r^2\ell\rho g$

$\Rightarrow \quad r = \sqrt{\dfrac{2T}{\pi\rho g}}$

Hence, the correct answer is (A).

13. $\dfrac{4T}{R} = \dfrac{4T}{r_1} - \dfrac{4T}{r_2} = \dfrac{4T}{r} - \dfrac{4T}{2r} = \dfrac{4T}{2r}$

$\Rightarrow \quad R = 2r$

Hence, the correct answer is (C).

14. $\Delta L = \dfrac{FL}{YA}$

$\Rightarrow \quad \dfrac{L_A}{Y_A r_A^2} = \dfrac{L_B}{Y_B r_B^2}$

$\Rightarrow \quad r_A = r_B\sqrt{\dfrac{L_A}{L_B}\dfrac{Y_B}{Y_A}}$

$\Rightarrow \quad r_A = (2\ \text{mm})\sqrt{\dfrac{2 \times 2 \times 4}{3 \times 7}}$

$\Rightarrow \quad r_A = \dfrac{4}{4.58} \times 2 = 1.7\ \text{mm}$

Hence, the correct answer is (D).

15. Since $-\dfrac{Adh}{dt} = \dfrac{\pi(h\rho g)r^4}{8\eta\ell}$

$\Rightarrow \quad \displaystyle\int_0^t dt = \int_H^{H/2} -\frac{8\eta\eta\ell A}{\pi\rho g r^4}$

$\Rightarrow \quad t = \dfrac{8\eta\ell A}{\pi\rho g r^4}\ln(2)$

Hence, the correct answer is (C).

16. $U = \dfrac{1}{2} \times \dfrac{(\text{stress})^2}{Y} \times \text{volume} = \dfrac{1}{2} \times \dfrac{F^2 \times A \times L}{A^2 \times Y}$

$\Rightarrow \quad U = \dfrac{1}{2} \times \dfrac{F^2L}{AY} = \dfrac{1}{2} \times \dfrac{(50)^2 \times 0.2}{1 \times 10^{-4} \times 1 \times 10^{11}}$

$\Rightarrow \quad U = 2.5 \times 10^{-5}\ \text{J}$

Hence, the correct answer is (B).

17. Given that, $h\rho g = 10^5\ \text{Pa}$

$\Rightarrow \quad P_x = \left(h - \dfrac{h}{5}\right)\rho g = 0.8h\rho g$

$\Rightarrow \quad P_x = 0.8 \times 10^5\ \text{Pa}$

Hence, the correct answer is (B).

18. When ice melts into water its volume decreases. Hence, over all level should decrease. Now suppose m is the mass of ice, V_1 is volume immersed in water and V_2 the volume immersed in oil. For floating condition, we have

Weight = Upthrust

$\Rightarrow \quad mg = V_1\rho_w g + V_2\rho_0 g$

$\Rightarrow \quad V_1 = \dfrac{m - V_2\rho_0}{\rho_w}$

$\Rightarrow \quad V_1 = \dfrac{m}{\rho_w} - \dfrac{V_2\rho_0}{\rho_w}$ ...(1)

When ice melts, m mass of ice converts into m mass of water. Volume of water so formed is

$V_3 = \dfrac{m}{\rho_w}$ ...(2)

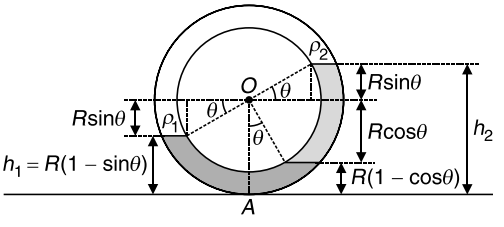
From equations (1) and (2), $V_3 > V_1$

So, the interface level will rise.

Hence, the correct answer is (A).

19. Since pressure at A due to the left column and the right column of the liquid should be the same, so we have

$(P_A)_{\text{due to left column}} = (P_A)_{\text{due to right column}}$ ...(1)



Due to left column

Pressure at A is given by

$(P_A)_{\text{left}} = R(1 - \sin\theta)\rho_1 g$

Due to right column

Pressure at A is given by

$(P_A)_{\text{right}} = R(\sin\theta + \cos\theta)\rho_2 g + (1 - \cos\theta)\rho_1 g$

Substitution these both in equation (1), we get

$\rho_1(1 - \cos\theta) + \rho_2(\sin\theta + \cos\theta) = \rho_1(1 - \sin\theta)$

$\Rightarrow \quad \dfrac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \dfrac{\rho_1}{\rho_2}$

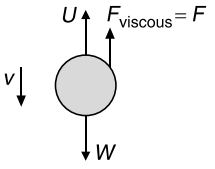
$\Rightarrow \quad \tan\theta = \dfrac{\rho_1 - \rho_2}{\rho_1 + \rho_2}$

$\Rightarrow \quad \tan 30° = \dfrac{\rho_1 - \rho_2}{\rho_1 + \rho_2}$

$\Rightarrow \quad \dfrac{\rho_1}{\rho_2} = \dfrac{\sqrt{3} + 1}{\sqrt{3} - 1}$

Hence, the correct answer is (D).

20. The free body diagram for the first case is shown in Figure.

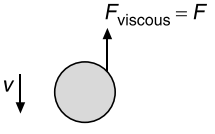


If U is the upthrust, F is the viscous force and W is the weight of the body, then at terminal velocity, we have

$$U + F = W$$

$$\Rightarrow \quad F = V(\rho - \sigma)g = V\sigma g \qquad \{\because \rho = 2\sigma\}$$

The free body diagram for the second case (when gravity is absent) is shown in Figure.



In this case, only the viscous drag F acts on the body. So, initial retardation of the ball is

$$a = \frac{F}{m} = \frac{V\sigma g}{2\sigma V} = \frac{g}{2}$$

This retardation with gradually decrease as the speed of the ball decreases and finally the ball will stop.

Hence, the correct answer is (D).

21. $E(\text{dissipated}) = \frac{1}{2}k\Delta x^2$

$$\Rightarrow \quad \frac{1}{2}\times 800 \times \left(\frac{2\times 2}{100\times 100}\right) = \frac{16}{100} J$$

$$\Rightarrow \quad \frac{16}{100} = \frac{1}{2}\times 400 \times \Delta T + 1 \times 4184 \times \Delta T$$

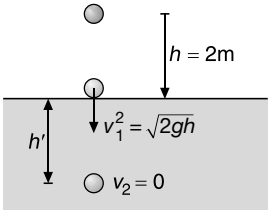
$$\Rightarrow \quad \frac{16}{100} = (200 + 4184)\Delta T = 4384\Delta T$$

$$\Rightarrow \quad \Delta T = \frac{16}{4384\times 100} = 3.6\times 10^{-5} \text{ K}$$

Hence, the correct answer is (A).

22. Retardation of the ball inside the water is

$$a = \frac{U - W}{m} = \frac{V(1)g - V(0.8)g}{V(0.8)} = \frac{g}{4}$$



Since, $v_2^2 = v_1^2 - 2ah'$

where, $v_2^2 = 0$ and $v_1^2 = 2gh$, so

$$\Rightarrow \quad h' = 8 \text{ m}$$

Hence, the correct answer is (A).

23. According to Pascal's Law, for an incompressible liquid confined to a closed rigid container, the excess pressure is transmitted equally in all the directions.

So, the increment in pressure at each point is $\Delta P = \frac{F}{A}$

Hence, the correct answer is (A).

24. $V_1 = \frac{\pi \text{Pr}^4}{8\eta\ell_1}$, $V_2 = \frac{\pi \text{Pr}^4}{8\eta\ell_2}$, $V_3 = \frac{\pi \text{Pr}^4}{8\eta\ell_3}$

and $V = \frac{\pi \text{Pr}^4}{8\eta\ell}$

Since, $V = V_1 + V_2 + V_3$

Substituting the values, we get

$$\ell = \frac{\ell_1\ell_2\ell_3}{\ell_1\ell_2 + \ell_1\ell_3 + \ell_2\ell_3}$$

$$\Rightarrow \quad \ell = \frac{(1)(2)(3)}{(1)(2)+(1)(3)+(2)(3)} = \frac{6}{11} \text{ m}$$

Hence, the correct answer is (A).

25. $\frac{4}{3}\pi R^3 = 10^6\left(\frac{4}{3}\pi r^3\right)$

$$\Rightarrow \quad R = (10^2 r)$$

Now, $U_i = 10^6(4\pi r^2)T$ and $U_f = (4\pi R^2)T$

$$\Rightarrow \quad \frac{U_f}{U_i} = \frac{1}{10^2}$$

Hence, the correct answer is (B).

26. Acceleration of container is given to be

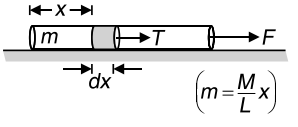
$$a = a_0(\hat{i} - \hat{j} + \hat{k})$$

Since there is no gravity, so the pressure difference will be only due to the acceleration of container. All points other than point H, are acted upon by a pseudo force. Hence, at point H pressure developed is zero.

Hence, the correct answer is (D).

27. $F = Ma$

$$\Rightarrow \quad a = \frac{F}{M}$$



$$\Rightarrow \quad T = ma = \left(\frac{M}{L}\right)x\frac{F}{M} = \frac{F}{L}x$$

Since, $Y = \frac{T/A}{d\ell/dx} = \frac{\left(\frac{F}{L}x\right)dx}{Ad\ell}$

$$\Rightarrow \quad \int_0^{\Delta\ell} d\ell = \frac{F}{ALY}\int_0^L x\,dx = \frac{FL^2}{2ALY} = \frac{FL}{2AY}$$

$$\Rightarrow \quad \frac{\Delta L}{L} = \frac{F}{2AY}$$

Hence, the correct answer is (C).

28. Since stress is shown on x-axis and strain on y-axis, so we have

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{1}{\tan\theta} = \frac{1}{\text{slope}}$$

So, elasticity of wire P is minimum and of wire R is maximum

Hence, the correct answer is (D).

29. Upthrust acting on cylinder is

$$U = \left(\frac{m}{\rho}\right)\sigma g$$

According to Newton's Third Law, force exerted by the cylinder on the liquid is also $\left(\frac{m}{\rho}\right)\sigma g$

So, increase in pressure is $\Delta P = \frac{m\sigma g}{\rho s}$

Hence, the correct answer is (C).

30. $\text{Stress} = \frac{400}{\pi r^2} \le 379\times10^6 \text{ Nm}^{-2}$

$\Rightarrow \quad r^2 \ge \frac{400}{379\times10^6\pi}$

$\Rightarrow \quad 2r \ge 1.15 \text{ mm}$

Hence, the correct answer is (B).

31. In steady flow the viscous force is balanced by the force due to the pressure difference at the back and front ends. So, we have

$$P_1\pi r^2 - P_2\pi r^2 = 4\pi\eta L v_m$$

$\Rightarrow \quad v_m = \frac{(\Delta P)r^2}{4\eta L} \qquad \{\because P_1 - P_2 = \Delta P\}$

Hence, the correct answer is (B).

32. $U = \frac{(\text{Normal Stress})^2}{2K}$

$\Rightarrow \quad U = \frac{1}{2}\beta(hdg)^2$

Hence, the correct answer is (A).

33. Mass of liquid in capillary is

$m = \pi r^2 h\rho$, where $h = \frac{2T}{r\rho g}$

$\Rightarrow$ Mass $\propto r$

When r is doubled, then mass of water in capillary will also become two times.

Hence, the correct answer is (C).

34. $\frac{\Delta V}{V} = (1-2\sigma)\frac{\Delta L}{L}$

$\Rightarrow \quad \frac{\Delta V}{V} = (1-0.4)2\times10^{-3}$

$\Rightarrow \quad \frac{\Delta V}{V} = 1.2\times10^{-3}$

So, percentage value is 0.12%

Hence, the correct answer is (A).

35. $F = YA\alpha\Delta t = (2\times10^{11})(3\times10^{-6})(10^{-5})(20-10)$

$\Rightarrow \quad F = 60 \text{ N}$

Hence, the correct answer is (C).

36. Maximum stress on the wire will be at highest point (at point of suspension).

$\Rightarrow \quad \text{Stress} = \sigma = \frac{W}{A} = \frac{A\ell\rho g}{A}$

Hence, the correct answer is (D).

37. Since $Y = \frac{FL}{A|\Delta L|}$

$\Rightarrow \quad \Delta L = \frac{FL}{AY}$

Since $\Delta L = L\alpha T$

$\Rightarrow \quad \alpha = \frac{F}{AYT}$

$\Rightarrow \quad \gamma = 3\alpha = \frac{3F}{\pi r^2 YT}$

Hence, the correct answer is (B).

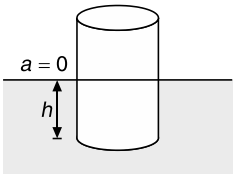
38. For a freely falling vessel, we have $g_{\text{eff}} = 0$

$\Rightarrow$ Upthrust $= 0$

Therefore, the volume of block immersed in the liquid can be any arbitrary value.

Hence, the correct answer is (C).

39. Let h height of float submerged in water as shown in Figure.



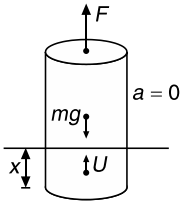
For equilibrium, we have

$$mg = U$$

$\Rightarrow \quad (AHd)g = (Ah\rho)g$

$\Rightarrow \quad h = \frac{Hd}{\rho}$

Now when force F is applied, then for minimum work, the float is slowly pulled out. When submerged length of float is x, then



$$(F + \rho Axg) - mg = 0$$

$\Rightarrow \quad F = mg - \rho Axg$

$\Rightarrow \quad W = \int F dx = \int (mg - \rho Axg)dx$

$\Rightarrow \quad W = mg\int dx - \rho Ag\int_0^h x dx$

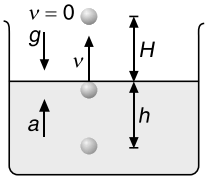
$\Rightarrow \quad W = mgh - \frac{\rho Agh^2}{2}$

$\Rightarrow \quad W = (\rho Ah)gh - \frac{\rho Agh^2}{2} = \frac{\rho Agh^2}{2}$

$\Rightarrow \quad W = \frac{\rho Ag}{2}\left(\frac{Hd}{\rho}\right)^2 = \frac{Agd^2H^2}{2\rho}$

Hence, the correct answer is (C).

40. Volume of ball is $V = \frac{m}{\rho}$



Acceleration of ball inside the liquid is

$$a = \frac{F_{\text{net}}}{m} = \frac{\text{upthrust} - \text{weight}}{m}$$

$\Rightarrow \quad a = \frac{\left(\frac{m}{\rho}\right)(3\rho)(g) - mg}{m} = 2g$, upwards

Velocity of ball while reaching at surface is

$$v = \sqrt{2ah} = \sqrt{4gh}$$

The ball will jump to a height, so we have

$$H = \frac{v^2}{2g} = \frac{4gh}{2g} = 2h$$

Hence, the correct answer is (C).

41. Let area of ice-cuboid excluding hole be A, then for floating, we have

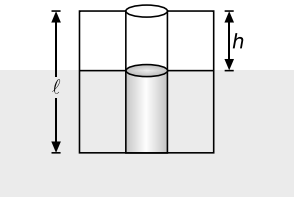
$$\begin{pmatrix}\text{Weight of Ice}\\ \text{Block}\end{pmatrix} = \begin{pmatrix}\text{Weight of Liquid}\\ \text{Displaced}\end{pmatrix}$$

$$\Rightarrow \quad W_{\text{ice block}} = U$$

$$\Rightarrow \quad A\rho_{\text{ice}} lg = A\rho_\omega (l-h)g$$

$$\Rightarrow \quad \frac{9l}{10} = l - h$$

$$\Rightarrow \quad h = l - \frac{9l}{10} = \frac{l}{10}$$



Hence, the correct answer is (D).

42. $$\frac{\Delta P_1}{\Delta P_2} = \frac{0.01}{0.02} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{4T/r_1}{4T/r_2} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{r_1}{r_2} = 2$$

$$\Rightarrow \quad \frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3 = 8$$

Hence, the correct answer is (A).

43. Let y be the height of liquid at some instant. Then

$$-\frac{dy}{dt} = \text{constant} \qquad \{\text{given}\}$$

From Equation of Continuity, we get

$$(\pi x^2)\left(-\frac{dy}{dt}\right) = a\sqrt{2gy} \qquad \ldots(1)$$

where, a is area of hole

Since, π, $\left(-\frac{dy}{dt}\right)$, a and g are constants. Hence, squaring equation (1), we get

$$y = kx^4$$

Hence, the correct answer is (A).

44. Let h be the height of water inside the capillary.
Total upward force tending to pull the water up supports the weight of the water.

$$\Rightarrow \quad T(2\pi r_1 + 2\pi r_2) = h(\pi r_2^2 - \pi r_1^2)\rho g$$

$$\Rightarrow \quad h = \frac{2T}{(r_2 - r_1)\rho g}$$

$$\Rightarrow \quad h = \frac{2\times 7\times 10^{-2}}{(1\times 10^{-3})(10^3)(10)}$$

$$\Rightarrow \quad h = 14\times 10^{-3}\ \text{m} = 1.4\ \text{cm}$$

Hence, the correct answer is (A).

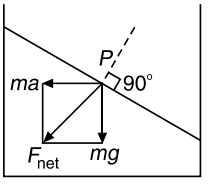
45. $$(\Sigma Q)_{\text{inflow}} = (\Sigma Q)_{\text{outflow}}$$

$$\Rightarrow \quad (4\times 10^{-6}) + Q + (4\times 10^{-6}) = (8\times 10^{-6}) + (2\times 10^{-6}) + (5\times 10^{-6}) + (6\times 10^{-4})$$

$$\Rightarrow \quad Q = 13\times 10^{-6}\ \text{m}^3\text{s}^{-1}$$

Hence, the correct answer is (C).

46. Net force on the free surface of the liquid in equilibrium (from accelerated frame) should be perpendicular to it. Forces on a water particle P on the free surfaces have been shown in Figure. In the figure ma is the pseudo force.



Hence, the correct answer is (C).

47. Area of wire is $A = \pi r^2$

Since $Y = \dfrac{\frac{F}{A}}{\frac{\Delta\ell}{\ell_0}}$

$$\Rightarrow \quad \frac{Mg}{\pi r^2} = \frac{\Delta\ell}{\ell_0} Y$$

$$\Rightarrow \quad \frac{Mg}{\pi r^2} = \left(\frac{4\times 10^{-3}}{2}\right) Y \qquad \ldots(1)$$

Mass of load of volume V is given by

$$M = V(8\rho_w)$$

Now when load is immersed in liquid, then

$$\frac{8V\rho_w g - 2V\rho_w g}{\pi r^2} = \frac{\Delta\ell'}{\ell_0} Y \qquad \ldots(2)$$

$$\Rightarrow \quad \frac{6V\rho_w g}{\pi r^2} = \frac{\Delta\ell'}{\ell_0} Y$$

$$\Rightarrow \quad \frac{\Delta\ell'}{4\times 10^{-3}} = \frac{6V\rho_w g}{8V\rho_w g}$$

$$\Rightarrow \quad \Delta\ell' = \frac{6}{8}\times 4\times 10^{-3}\ \text{m}$$

$$\Rightarrow \quad \Delta\ell' = 3\times 10^{-3}\ \text{m} = 3\ \text{mm}$$

Hence, the correct answer is (D).

48. Let m_1 be the mass of concrete and m_2 be the mass of wood
From Law of Floatation, we have

$$W = U$$

$$\Rightarrow \quad (m_1 + m_2)g = \left(\frac{m_1}{2.5} + \frac{m_2}{0.5}\right)(1)(g)$$

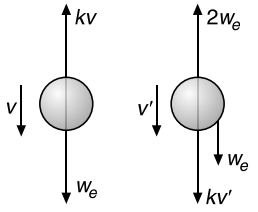
$$\Rightarrow \quad \frac{m_1}{m_2} + 1 = \frac{2}{5}\left(\frac{m_1}{m_2}\right) + 2$$

$$\Rightarrow \quad \frac{3}{5}\left(\frac{m_1}{m_2}\right) = 1$$

$$\Rightarrow \quad \frac{m_1}{m_2} = \frac{5}{3}$$

Hence, the correct answer is (A).

49. If W_e be the effective weight, then we have



$$kv = W_e \quad \ldots(1)$$

In equilibrium, we have

$$2W_e - W_e = kv' \quad \ldots(2)$$

From equations (1) and (2), we get

$$v' = v = 1 \text{ ms}^{-1}$$

Hence, the correct answer is (A).

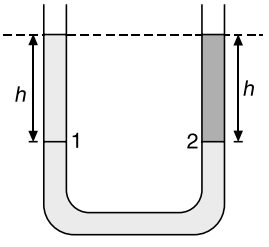
50. Since, $\Delta P = P_2 - P_1 = \rho g \Delta H$

$\Rightarrow \quad 3.03 \times 10^6 = 10^3 \times 10 \times \Delta H$

$\Rightarrow \quad \Delta H \simeq 300 \text{ m}$

Hence, the correct answer is (C).

51. Since $P_1 = P_2$



$\Rightarrow \quad P_0 + \rho_w g h = P_0 + \rho g h$

$\Rightarrow \quad \rho = \rho_w$

Hence, the correct answer is (A).

52. Energy of satellite at radius r is

$$E = -\frac{GMm}{2r}$$

Due to friction losses (due to air) energy of the satellite goes on decreasing i.e. r goes on increasing. Since, orbital speed

$$v_0 \propto \frac{1}{\sqrt{r}}$$

or it goes on increasing till it finally falls back on the earth.

Hence, the correct answer is (A).

53. $\text{Reading} = \begin{pmatrix}\text{weight of}\\ \text{bucket of}\\ \text{water}\end{pmatrix} + \begin{pmatrix}\text{magnitude of}\\ \text{upthrust of}\\ \text{block}\end{pmatrix}$

$$\text{Reading} = (10g) + \frac{1}{2}\left(\frac{7.2}{7.2\rho_w}\right)\rho_w g$$

$$\text{Reading} = 10.5g = 10.5 \text{ kg}$$

Hence, the correct answer is (B).

54. Volume flow rate is $Q = av = \dfrac{V}{t}$

$\Rightarrow \quad v_1 = \dfrac{V}{A_1 t} = \dfrac{120 \times 10^{-3}}{5 \times 10^{-4} \times 2 \times 60}$

Since, $A_1 v_1 = A_2 v_2$

and $A_2 = \dfrac{A_1}{5}$

$\Rightarrow \quad v_2 = 5v_1 = 5(2) = 10 \text{ ms}^{-1}$

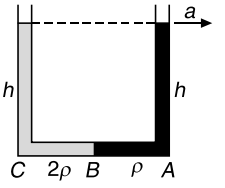
Now, $t_0 = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2 \times 1}{10}}$

$\Rightarrow \quad t_0 = 0.447 \text{ s}$

$\Rightarrow \quad R = v_2 t_0 = 4.47 \text{ m}$

Hence, the correct answer is (C).

55. For the given situation, liquid of density 2ρ should be behind that of ρ as both liquids are at same level as shown in Figure. Pressure in right limb at point A is given by



$$P_A = P_{\text{atm}} + \rho g h$$

Since, $P_B = P_A + \rho a \dfrac{l}{2} = P_{\text{atm}} + \rho g h + \rho a \dfrac{l}{2}$

Pressure in left limb at point C is given by

$$P_C = P_{\text{atm}} + (2\rho) g h \quad \ldots(1)$$

Also, $P_C = P_B + (2\rho) a \dfrac{l}{2}$

$\Rightarrow \quad P_C = P_{\text{atm}} + \rho g h + \dfrac{3}{2}\rho a l \quad \ldots(2)$

From (1) and (2), we get

$$P_{\text{atm}} + \rho g h + \frac{3}{2}\rho a l = P_{\text{atm}} + 2\rho g h$$

$\Rightarrow \quad h = \left(\dfrac{3a}{2g}\right) l$

Hence, the correct answer is (B).

56. The mercury will not come out from this hole as the pressure outside the tube is equal to atmospheric pressure which is greater than the pressure inside the tube which is $P_0 - \rho g H$.

Hence, the correct answer is (D).

57. Applying continuity equation, we get

$$v_A a_A = v_B a_B$$

Since $a_A = a_B$ so $v_A = v_B$

Applying Bernoulli's equation, we get

$$P_A + \rho g h_A + \frac{1}{2}\rho v_A^2 = P_B + \rho g h_B + \frac{1}{2}\rho v_B^2$$

Since $v_A = v_B$ and $h_A = h_B$, so we get

$$P_A = P_B$$

Hence, the correct answer is (C).

58. To measure the atmospheric pressure, same length of tubes containing mercury are required, no matter how many tubes are used as

$$P_{\text{atm}} = \rho_{Hg} g h$$

Hence, the correct answer is (D).

59. $W = 2(T)(\Delta A) = 2T(4\pi r^2)$

$\Rightarrow \quad W = 8 \times 3 \times 10^{-2} \times 3.14 \times (10^{-2})^2$

$\Rightarrow \quad W = 75.36 \times 10^{-6} \text{ J} = 75.36 \text{ μJ}$

Hence, the correct answer is (C).

60. The required pressure should exceed the atmospheric pressure by an amount that can balance the hydrostatic pressure of the water column and the capillary pressure in the air bubble with a radius r.

$$\Rightarrow \quad P = \rho gh + \frac{2T}{r}$$

$$\Rightarrow \quad P = (10^3)(10)(2\times10^{-2}) + \frac{2\times7\times10^{-2}}{(00.5)(10^2)}$$

$$\Rightarrow \quad P = 200 + 280 = 480 \text{ Nm}^{-2}$$

Hence, the correct answer is (A).

61. Let L_0 be the unstretched length and L_3 be the length under a tension of 9 N.

$$\Rightarrow \quad Y = \frac{4L_0}{A(L_1 - L_0)} = \frac{5L_0}{A(L_2 - L_0)} = \frac{9L_0}{A(L_3 - L_0)} \qquad \ldots(1)$$

$$\Rightarrow \quad \frac{4L_0}{A(L_1 - L_0)} = \frac{5L_0}{A(L_2 - L_0)}$$

$$\Rightarrow \quad L_0 = 5L_1 - 4L_2 \qquad \ldots(2)$$

Also, from (1), we get

$$\frac{4}{(L_1 - L_0)} = \frac{9}{(L_3 - L_0)}$$

$$\Rightarrow \quad 4L_3 - 4L_0 = 9L_1 - 9L_0$$

$$\Rightarrow \quad 4L_3 = 9L_1 - 5L_0$$

$$\Rightarrow \quad 4L_3 = 9L_1 - 25L_1 + 20L_2$$

$$\Rightarrow \quad L_3 = 5L_2 - 4L_1$$

Hence, the correct answer is (B).

62. Since, Strain $= 10^{-3}$

and Work done $= \frac{1}{2}$(Tension)(Extension)

$$\Rightarrow \quad W = \frac{1}{2}Fx$$

$$\Rightarrow \quad W = \frac{1}{2}\left(\frac{YAx}{L}\right)x$$

$$\Rightarrow \quad W = \frac{1}{2}\frac{(2\times10^{11})(10^{-6})(10^{-6})}{(1)}$$

$$\Rightarrow \quad W = 0.1 \text{ J}$$

Hence, the correct answer is (A).

63. $\rho = \frac{M}{V}$

$$\Rightarrow \quad \frac{\Delta\rho}{\rho} = -\frac{\Delta V}{V}$$

$$\Rightarrow \quad \frac{\Delta V}{V} = -0.1\%$$

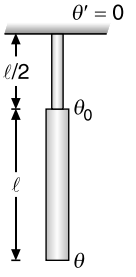
Since, $B = -\frac{\Delta P}{\left(\frac{\Delta V}{V}\right)}$

$$\Rightarrow \quad \Delta P = (2\times10^9)\left(\frac{0.1}{100}\right)$$

$$\Rightarrow \quad \Delta P = 2\times10^6 \text{ Nm}^{-2}$$

Hence, the correct answer is (C).

64. $\tau = C(\theta) = \frac{\pi\eta r^4\theta}{2L} = \text{Constant}$



$$\Rightarrow \quad \frac{\pi\eta r^4(\theta - \theta_0)}{2\ell} = \frac{\pi\eta\left(\frac{r}{2}\right)^4(\theta_0 - \theta')}{2\left(\frac{\ell}{2}\right)}$$

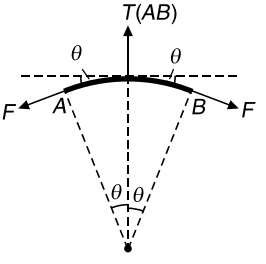
$$\Rightarrow \quad \frac{(\theta - \theta_0)}{2} = \frac{\theta_0}{16}$$

$$\Rightarrow \quad \theta_0 = \frac{8}{9}\theta$$

Hence, the correct answer is (D).

65. Let F be tension in the thread, then

$$2F\sin\theta = T(AB)$$



$$\Rightarrow \quad 2F\theta = T(2R\theta)$$

$$\Rightarrow \quad F = RT$$

Hence, the correct answer is (C).

66. Velocity of body just before touching the lake surface is,

$$v = \sqrt{2gh}$$

Retardation in the lake,

$$a = \frac{\text{upthrust} - \text{weight}}{\text{mass}}$$

$$\Rightarrow \quad a = \frac{V\rho g - V\rho g}{V\rho} = \left(\frac{\sigma - \rho}{\rho}\right)g$$

So, maximum depth is

$$d_{\max} = \frac{v^2}{2a} = \frac{h\rho}{\sigma - \rho}$$

Hence, the correct answer is (A).

67. From the ideal gas equation

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

For a gas the more the Bulk's modulus, the more is elasticity. Since

$$B \propto P$$

$$\Rightarrow \quad \frac{B_2}{B_1} = \frac{E_2}{E_1} = \frac{P_2}{P_1} = \frac{V_1}{V_2}\times\frac{T_2}{T_1} = \left(\frac{1}{4}\right)\times\left(\frac{400}{300}\right) = \frac{1}{3}$$

$$\Rightarrow \quad B_2 = \frac{B_1}{3}$$

So, elasticity will become $\frac{1}{3}$ times.

Hence, the correct answer is (D).

68. Inside the satellite, we have $g_{\text{eff}} = 0$

So, there will be no pressure difference inside the mercury. Hence, the mercury wil rise to full length of the tube, i.e., 90 cm because air pressure outside the tube is 76 cm of mercury, while pressure above the tube is zero.

Hence, the correct answer is (B).

69. When value of Poisson's ratio is 0.5, then $\Delta V = 0$ (always). Otherwise, this can be shown as follows.

$$\frac{\Delta V}{V} = \frac{\Delta(\pi r^2 L)}{\pi r^2 L}$$

$$\Rightarrow \quad \frac{\Delta V}{\mathrm{V}} = \frac{\Delta L}{L} + \frac{2\Delta r}{r}$$

Since, $\sigma = -\dfrac{\Delta r/r}{\Delta L/L}$

$$\Rightarrow \quad \frac{\Delta r}{r} = -\sigma\left(\frac{\Delta L}{L}\right)$$

$$\Rightarrow \quad \frac{\Delta V}{V} = \frac{\Delta L}{L}(1-2\sigma)$$

For $\sigma = 0.5$

$$\frac{\Delta V}{V} = 0$$

Hence, the correct answer is (D).

70. $U = \dfrac{1}{2}\mathrm{Y}(\text{Strain})^2$

$$\Rightarrow \quad U = \frac{1}{2}(2\times 10^{11})\left(\frac{0.5}{100}\right)^2$$

$$\Rightarrow \quad U = 2.5\times 10^{6}\ \mathrm{Jm}^{-3}$$

Hence, the correct answer is (A).

71. Force constant is $k = \dfrac{YA}{L}$

$$\Rightarrow \quad k \propto Y$$

$$\Rightarrow \quad \frac{k_A}{k_B} = \frac{Y_A}{Y_B} = 2$$

Hence, the correct answer is (B).

72. $\rho g h = \rho a L$

$$\Rightarrow \quad h = \frac{aL}{g}$$

Hence, the correct answer is (B).

73. Flow rate of water $(Q) = 100\ \mathrm{ltmin}^{-1}$

$$\Rightarrow \quad Q = \frac{100\times 10^{-3}}{60} = \frac{5}{3}\times 10^{-3}\ \mathrm{m}^3$$

Since $Q = Av$

So, velocity of flow is $v = \dfrac{Q}{A} = \dfrac{5\times 10^{-3}}{3\times\pi\times(5\times 10^{-2})^2}$

$$\Rightarrow \quad v = \frac{10}{15\pi} = \frac{2}{3\pi}\ \mathrm{ms}^{-1}$$

$$\Rightarrow \quad v = 0.2\ \mathrm{ms}^{-1}$$

Reynold's number $n_R = \dfrac{Dv\rho}{\eta}$

$$\Rightarrow \quad n_R = \frac{(10\times 10^{-2})\times\dfrac{2}{3\pi}\times 1000}{1} \simeq 2\times 10^4$$

Order of n_R is 10^4

Hence, the correct answer is (B).

74. Liquid cannot be stretched as a wire. Therefore, young's modulus is not defined for a liquid. Similarly, the free surface of a liquid cannot sustain the shear stress. Hence, shear modulus is also not defined for a liquid. Only bulk modulus is defined for a liquid.

Hence, the correct answer is (A).

75. Volume Flow Rate $= \dfrac{0.74}{60}\ \mathrm{m}^3\mathrm{s}^{-1}$

Speed of efflux $= \dfrac{0.74\times 10^4}{60\times\pi\times 4}\ \mathrm{ms}^{-1} = \sqrt{2gh}$

$$\Rightarrow \quad 9.82 = \sqrt{2\times 10\times h}$$

$$\Rightarrow \quad h = 4.8\ \mathrm{m}$$

Hence, the correct answer is (C).

76. $\Delta\ell = \dfrac{FL}{AY}$

$$\Rightarrow \quad \frac{\Delta\ell_S}{\Delta\ell_B} = \left(\frac{F_S}{F_B}\right)\left(\frac{L_S}{L_B}\right)\left(\frac{A_B}{A_S}\right)\left(\frac{Y_B}{Y_S}\right)$$

$$\Rightarrow \quad \frac{\Delta\ell_S}{\Delta\ell_B} = \left(\frac{3M}{2M}\right)(a)\left(\frac{1}{b^2}\right)\left(\frac{1}{c}\right) = \frac{3a}{2b^2c}$$

Hence, the correct answer is (B).

77. According to Law of Floatation, $W = U$

$$\Rightarrow \quad V\sigma g = \left(\frac{4V}{5}\right)\rho_\omega g \qquad \ldots(1)$$

Also, $V\sigma g = \dfrac{V}{2}\rho_\omega g + \dfrac{V}{2}\rho_{\text{oil}} g$ $\qquad \ldots(2)$

$$\Rightarrow \quad \left(\frac{\rho_\omega}{2} + \frac{\rho_{\text{oil}}}{2}\right) = \frac{4}{5}\rho_\omega$$

$$\Rightarrow \quad \frac{\rho_{\text{oil}}}{2} = \rho_\omega\left(\frac{4}{5} - \frac{1}{2}\right) = \frac{3}{10}\rho_\omega$$

$$\Rightarrow \quad \rho_{\text{oil}} = \frac{3}{5}\rho_\omega = 0.6\rho_\omega$$

Hence, the correct answer is (D).

78. Since, decrease in weight equals the decrease in upthrust.

$$\Rightarrow \quad mg = V\rho_w g$$

$$\Rightarrow \quad m = V\rho_w$$

If A is area of the base, then

$$(0.2) = (2\times 10^{-2})(A)(10^3)$$

$$\Rightarrow \quad A = 10^{-2}\ \mathrm{m}^2 = 100\ \mathrm{cm}^2 = \ell^2$$

$$\Rightarrow \quad \ell = 10\ \mathrm{cm}$$

Hence, the correct answer is (D).

79. $\dfrac{4}{3}\pi R^3 = 27\left(\dfrac{4}{3}\pi r^3\right)$

$$\Rightarrow \quad R^3 = 27r^3$$

$$\Rightarrow \quad R = 3r$$

Since $v \propto r^2$

$$\Rightarrow \quad \frac{v_1}{v_2}=\left(\frac{R}{r}\right)^2=9$$

Hence, the correct answer is (B).

80. According to Law of Floatation, $W=U$

$$\Rightarrow \quad (d_1+d_2)LAg=\frac{3}{2}LAdg$$

$$\Rightarrow \quad (d_1+d_2)=\frac{3}{2}d$$

Since, $d_1<d_2$

$$\Rightarrow \quad d_1<\frac{3}{4}d$$

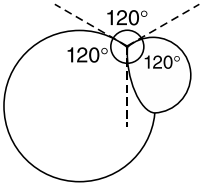
Hence, the correct answer is (A).

81. Upthrust is $U=V_{\text{imm}}\rho_{\text{liq}}g_{\text{eff}}$

Since value of g_{eff} will decrease, so upthrust will also decrease.

Hence, the correct answer is (C).

82. At any point of contact the forces of surface tension balance each other. Therefore, $\theta=120°$.



Hence, the correct answer is (D).

83. Weight of rod is

$$W=(0.012)(1)(2\times10^3)(10)=240\text{ N}$$

Buoyancy force on rod is

$$U=(0.012)(1)(10^3)(10)=120\text{ N}$$

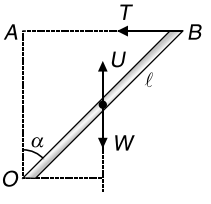
For equilibrium, $\Sigma\tau_O=0$

The free body diagram of rod is shown in Figure.

$$\Rightarrow \quad \tau=(240-120)\left(\frac{\sin\alpha}{2}\right)=45(\cos\alpha)$$

$$\Rightarrow \quad \tan\alpha=\frac{90}{120}=\frac{3}{4}$$

$$\Rightarrow \quad \alpha=37°$$

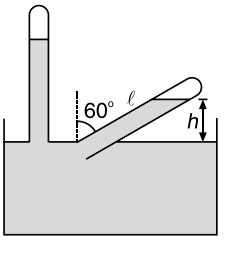


Hence, the correct answer is (B).

84. Since, $\ell\cos60°=h=76$ cm

$$\Rightarrow \quad \frac{\ell}{2}=76\text{ cm}$$

$$\Rightarrow \quad \ell=152\text{ cm}$$



Hence, the correct answer is (A).

85. For floating, $W=U$

$$\Rightarrow \quad W_{\text{sphere}}+W_{\text{chain}}=U$$

$$W=mg+(\lambda h)g$$

Now $U=V_{\text{sphere}}\rho_{\text{water}}g+V_{\text{chain}}\rho_{\text{water}}g$

$$\Rightarrow \quad U=V_{\text{sphere}}(3\rho_{\text{sphere}})g+V_{\text{chain}}\left(\frac{\rho_{\text{chain}}}{7}\right)g$$

$$\Rightarrow \quad U=3mg+\frac{1}{7}(m_{\text{chain}}g)$$

$$\Rightarrow \quad U=3mg+\frac{\lambda hg}{7}$$

Since $W=U$

$$\Rightarrow \quad mg+\lambda hg=3mg+\frac{\lambda hg}{7}$$

$$\Rightarrow \quad 2mg=\frac{6\lambda hg}{7}$$

$$\Rightarrow \quad h=\frac{7m}{3\lambda}$$

Hence, the correct answer is (B).

86. The centre of gravity of liquid in the first column is $\frac{h_1}{2}$ and that in the second column is $\frac{h_2}{2}$.

Let Initial Potential Energy be U_i

$$\Rightarrow \quad U_i=(Ah_1\rho)g\left(\frac{h_1}{2}\right)+(Ah_2\rho)g\left(\frac{h_2}{2}\right)$$

$$\Rightarrow \quad U_i=\frac{1}{2}A\rho g\left(h_1^2+h_2^2\right)$$

When the water column in the two equavalises then equivalent height is $\frac{h_1+h_2}{2}$ and the centre of gravity in both the columns is $\frac{h_1+h_2}{4}$. If U_f be the final potential energy of liquid in both the columns then

$$U_f=2\left[A\left(\frac{h_1+h_2}{2}\right)\rho\left(\frac{h_1+h_2}{4}\right)g\right]$$

$$\Rightarrow \quad U_f=\frac{1}{4}A\rho g(h_1+h_2)^2$$

Since work done is equal to the decrease in potential energy, so

$$W=U_i-U_f$$

$$\Rightarrow \quad W=A\rho g\left(\frac{h_1-h_2}{2}\right)^2$$

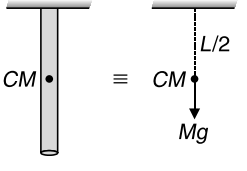
Hence, the correct answer is (B).

87. $Y=\dfrac{\text{Stress}}{\text{Strain}}$

$$\text{Stress}=\frac{ALDg}{A}=LDg$$

$$\text{Strain}=\frac{\Delta L}{L_{eff}}=\frac{\ell}{(L/2)}$$

$$\Rightarrow \quad Y=\frac{LDg}{(2\ell/L)}$$

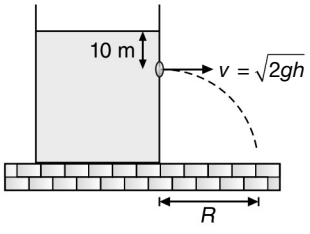


$\Rightarrow \quad \ell = \dfrac{L^2 D g}{2Y}$

Hence, the correct answer is (B).

88. Area enclosed by hysterisis loop gives the energy loss in the process of stretching and unstretching of rubber band and this loss will appear in the form of heating.
Hence, the correct answer is (A).

89. Range becomes twice if speed of efflux becomes twice.
Since, $v = \sqrt{2gh}$
Therefore, h should become 4 times or 40 m
So, an extra pressure equivalent to 30 m of water should be applied.



Since, $1 \text{ atm} = 0.76 \times 13.6 \text{ m}$ of water
$\Rightarrow \quad 1 \text{ atm} = 10.336 \text{ m}$ of water
$\Rightarrow \quad 30 \text{ m}$ of water ≈ 3 atm
Hence, the correct answer is (D).

90. $B = \dfrac{\Delta P}{\Delta V/V} = \dfrac{h\rho g}{\Delta V/V} = \dfrac{200 \times 10^3 \times 10}{0.1/100} = 2 \times 10^9$
Hence, the correct answer is (D).

91. $F = 2000 \text{ N}$, $L = 6 \text{ m}$, $\ell = 0.5 \text{ cm}$, $A = 10^{-6} \text{ m}^2$
$\Rightarrow \quad Y = \dfrac{FL}{A\ell} = \dfrac{2000 \times 6}{10^{-6} \times 0.5 \times 10^{-2}}$
$\Rightarrow \quad Y = 2.35 \times 10^{12} \text{ Nm}^{-2}$
Hence, the correct answer is (A).

92. $U = \dfrac{1}{2} \times \dfrac{YA\ell^2}{L}$
$\Rightarrow \quad U = \dfrac{1}{2} \times \dfrac{2 \times 10^{11} \times 3 \times 10^{-6} \times (1 \times 10^{-3})^2}{4}$
$\Rightarrow \quad U = 0.075 \text{ J}$
Hence, the correct answer is (C).

93. Change in pressure $\Delta P = \rho g h$
By definition, Bulk modulus is

$$B = -\frac{\Delta P}{\Delta V/V}$$

But for a fixed mass, we have $\Delta m = 0$
$\Rightarrow \quad 0 = \Delta m = \rho \Delta V + V \Delta \rho$
$\Rightarrow \quad \dfrac{\Delta V}{V} = -\dfrac{\Delta \rho}{\rho}$
$\Rightarrow \quad \Delta \rho = \dfrac{\rho \Delta P}{B} = \dfrac{\rho(\rho g h)}{B} = \dfrac{\rho^2 g h}{B}$
Hence, the correct answer is (C).

94. Let area of hose pipe be A, then $F = Av^2\rho$
The thrust force will be due to the liquid whose momentum changes.
$\Rightarrow \quad F = \left(\dfrac{\rho A}{4}\right)v^2 + \left(\dfrac{\rho A}{4}\right)(2v^2)$
So, pressure P is given by

$$P = \frac{3\rho A v^2}{4A} = \frac{3\rho}{4} v^2$$

Hence, the correct answer is (A).

95. $\Delta P = \rho_{\text{oil}} g h_{\text{oil}} + \rho_w g h_w$
$\Rightarrow \quad \Delta P = (600)(10)(10 \times 10^{-2}) + (1000)(10)(2 \times 10^{-2})$
$\Rightarrow \quad \Delta P = 800 \text{ Nm}^{-2} = 800 \text{ Pa}$
Hence, the correct answer is (D).

96. Given $(50)^3 \times \dfrac{30}{100} \times (1) \times g = M_{\text{cube}} g$ …(1)
Let m be the mass placed on block for just submerging it, then
$(50)^3 \times (1) \times g = (M_{\text{cube}} + m) g$ …(2)
Subtracting (1) from (2), we get
$mg = (50)^3 \times g(1 - 0.3) = 125 \times 0.7 \times 10^3 \text{ g}$
$\Rightarrow \quad m = 87.5 \text{ kg}$
Hence, the correct answer is (C).

97. Inside the water, weight is equal to upthrust.
So, apparent weight is zero.
Hence, the correct answer is (D).

98. Fraction of volume immersed is

$$f = \frac{\rho_s}{\rho_l}$$

$\Rightarrow \quad f \propto \dfrac{1}{\rho_l}$
$\Rightarrow \quad \dfrac{f_1}{f_2} = \dfrac{\rho_{l_2}}{\rho_{l_1}}$
$\Rightarrow \quad \rho_{l_2} = \left(\dfrac{f_1}{f_2}\right)\rho_{l_1} = \dfrac{\left(V - \dfrac{V}{3}\right)}{\left(\dfrac{V}{3}\right)}\rho_{l_1}$
$\Rightarrow \quad \rho_{l_2} = 2\rho_{l_1}$
Hence, the correct answer is (B).

99. $F = \eta A \dfrac{dv}{dy} = \dfrac{(2)(20 \times 10^{-4})(10^{-2})}{10^{-3}}$
$\Rightarrow \quad F = 4 \times 10^{-2} \text{ N}$
Hence, the correct answer is (D).

100. Since, $g_{\text{eff}} = 0$, so pressure inside and pressure outside the hole will be same.
Hence, the correct answer is (D).

101. Since $\Sigma F_{\text{upwards}} = \Sigma F_{\text{downwards}}$
$\Rightarrow \quad N + kx = mg$
$\Rightarrow \quad N = 20 - 1 = 19 \text{ N}$
Hence, the correct answer is (B).

CHAPTER 1

102. Since, $\Delta P = \frac{1}{2}\rho v^2$

$\Rightarrow \quad v = \sqrt{\frac{2\Delta P}{\rho}} = \sqrt{\frac{2\times 5\times 10^5}{1000}} = 31.5 \text{ ms}^{-1}$

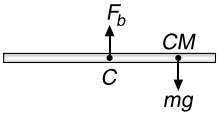
Hence, the correct answer is (B).

103. Torque about C of rod is

$(mg)\frac{l}{4} = I\alpha$

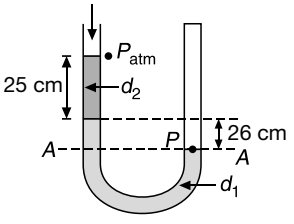
$\Rightarrow \quad I\alpha = (\pi r^2)(l)(\rho)(g)\left(\frac{l}{4}\right)$

$\Rightarrow \quad \alpha = \frac{\pi r^2 l^2 g\rho}{4I}$



Hence, the correct answer is (B).

104. Pressure in a liquid at same level is same, so using Pascal's equation from open end, we get



$P_{atm} + yd_2g + xd_1g = P$

In C.G.S. units, we have

$P_{atm} + (13.6\times 2)(25)g + (13.6)(26)g = P$

$\Rightarrow \quad P_{atm} + 13.6g(50+26) = P$

Since, $P_{atm} = 76\times 13.6\times g$

$\Rightarrow \quad 2P_{atm} = P$

$\Rightarrow \quad P_{atm} = \frac{P}{2}$

Hence, the correct answer is (C).

105. Weight of wire is $A\ell\rho g$

$\Rightarrow \quad W = A\ell\rho g$

Breaking Stress $= \frac{W}{A}$

$\Rightarrow$ Breaking Stress $= \ell\rho g$

$\Rightarrow \quad \ell = \frac{10^6}{3\times 10^3(10)}$

$\Rightarrow \quad \ell = \frac{1}{3}\times 100$

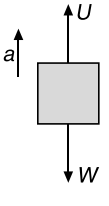
$\Rightarrow \quad \ell \simeq 34 \text{ m}$

Hence, the correct answer is (B).

106. Let m be the mass of block and a its acceleration in upward direction. Then

$a = \frac{\text{upthrust} - \text{weight}}{m} = \frac{U-W}{m}$

$\Rightarrow \quad a = \frac{\left(\frac{m}{0.5\times 10^3}\right)(10^3)(g+a_0) - mg}{m}$



$\Rightarrow \quad a = 2\left(g + \frac{g}{2}\right) - g = 2g$

Acceleration of block relative to water is

$a_r = a - a_0 = 2g - \frac{g}{2} = \frac{3g}{2}$

Since, $h = \frac{1}{2}a_r t^2$

$\Rightarrow \quad t = \sqrt{\frac{2h}{a_r}} = \sqrt{\frac{2h}{\frac{3}{2}g}} = 2\sqrt{\frac{h}{3g}}$

$\Rightarrow \quad t = 2\sqrt{\frac{1.2}{3\times 10}} = 0.4 \text{ s}$

Hence, the correct answer is (B).

107. In a freely falling system, $g_{eff} = 0$ and since

$U = V_{imm}\rho_{liquid}g_{eff}$

So, upthrust $U = 0$

Hence, the correct answer is (A).

108. $\ell = \frac{FL}{\pi r^2 Y}$

$\Rightarrow \quad r^2 \propto \frac{1}{Y}$ (F, L and ℓ are constant)

$\Rightarrow \quad \frac{r_2}{r_1} = \left(\frac{Y_1}{Y_2}\right)^{1/2} = \left(\frac{7\times 10^{10}}{12\times 10^{10}}\right)^{1/2}$

$\Rightarrow \quad r_2 = 1.5\times\left(\frac{7}{12}\right)^{1/2} = 1.145 \text{ mm}$

$\Rightarrow$ Diameter $= 2r_2 = 2.29$ mm

Hence, the correct answer is (C).

109. $B = \frac{\Delta P}{\Delta V / V}$

$\Rightarrow \quad B = \frac{(1.165 - 1.01)\times 10^5}{10/100} = \frac{0.155\times 10^5}{1/10}$

$\Rightarrow \quad B = 1.55\times 10^5$ Pa

Hence, the correct answer is (D).

110. Shearing strain

$\phi = \frac{x}{L} = \frac{0.02 \text{ cm}}{10 \text{ cm}}$

$\Rightarrow \quad \phi = 0.002$

Hence, the correct answer is (D).

111. According to Newton's Second Law, we have

$F = \frac{\Delta p}{\Delta t} = \left(\frac{\Delta m}{\Delta t}\right)(\Delta v)$

$\Rightarrow \quad F = \rho\left(\frac{\Delta V}{\Delta t}\right)(2v)$

$\Rightarrow \quad F = \rho(Av)(2v) = 2\rho Av^2$

Hence, the correct answer is (C).

112. Since, $F = YA\alpha\Delta T$

$\Rightarrow \quad \frac{F_1}{F_2} = \frac{\alpha_1}{\alpha_2} = \frac{3}{2}$

Hence, the correct answer is (A).

113. $v_T = \frac{2r^2(\rho - \sigma)g}{9\eta}$

$\Rightarrow \quad v_T \propto r^2$

Let R be the radius of bigger drop, then by equating the volumes, we get

$$\frac{4}{3}\pi R^3 = 2\left(\frac{4}{3}\pi r^3\right)$$

$\Rightarrow \quad R = (2)^{1/3} r$

Hence, the terminal velocity of big drop

$$v' = (2)^{2/3} v = (4)^{1/3} v$$

Hence, the correct answer is (D).

114. For $\theta = 0°$, height h to which liquid rises in the capillary is

$$h = \frac{2T}{r\rho g}$$

$\Rightarrow \quad \Delta U = mg\frac{h}{2} = \left(\pi r^2 h\rho\right)g\frac{h}{2}$

$\Rightarrow \quad \Delta U = \frac{\pi r^2 \rho g}{2}\left(\frac{4T^2}{r^2\rho^2 g^2}\right) = \frac{2\pi T^2}{\rho g}$

Hence, the correct answer is (B).

115. Speed of efflux at a depth h is given by

$$v = \sqrt{2gh}$$

Since volume of water flowing out per second from both the holes are equal, so According to Equation of Continuity, we get

$$a_1 v_1 = a_2 v_2$$

$\Rightarrow \quad (L^2)\sqrt{2g(y)} = \pi R^2 \sqrt{2g(4y)}$

$\Rightarrow \quad R = \frac{L}{\sqrt{2\pi}}$

Hence, the correct answer is (A).

116. $\Delta P_A = \rho g h = \rho a \ell$

Hence, the correct answer is (B).

117. Length of rod inside the water is

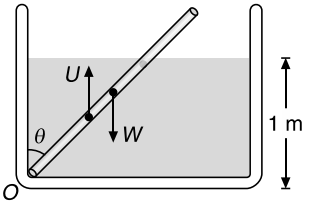
$$\ell = 1\sec\theta = \sec\theta$$

Upthrust $U = \left(\frac{2}{2}\right)(\sec\theta)\left(\frac{1}{500}\right)(1000)(10)$

$\Rightarrow \quad U = 20\sec\theta$

Weight of rod is $W = mg = 2 \times 10 = 20$ N

For rotational equilibrium of rod net torque about O should be zero, i.e. $\Sigma\tau_O = 0$



$\Rightarrow \quad F\left(\frac{\sec\theta}{2}\right)(\sin\theta) = W(1\sin\theta)$

$\Rightarrow \quad \frac{20}{2}\sec^2\theta = 20$

$\Rightarrow \quad \theta = 45°$

$\Rightarrow \quad F = 20\sec 45° = 20\sqrt{2}$ N

For vertical equilibrium of rod, force exerted by the hinge on the rod will be $(20\sqrt{2} - 20)$ N downwards or 8.28 N $\approx$ 8.3 N downwards.

Hence, the correct answer is (C).

118. For a capillary tube, $h \propto \frac{1}{r}$

Also, $M \propto \pi r^2 h$

$\Rightarrow \quad M \propto r$

Hence, the correct answer is (A).

119. Let ρ_s be the density of sugar solution, so

$$\rho_s > \rho_w$$

If m be the mass of ice, then for floating condition we have,

$$W = U$$

$\Rightarrow \quad mg = V_{\text{initial}}\rho_s g$

$\Rightarrow \quad V_{\text{initial}} = \frac{m}{\rho_{\text{sugar}}} = \frac{m}{\rho_s}$ …(1)

When ice melts, m mass of ice becomes m mass of water, so volume of this water formed is

$$V_{\text{final}} = \frac{m}{\rho_w}$$ …(2)

Since, $\rho_w < \rho_s$

$\Rightarrow \quad V_{\text{final}} > V_{\text{initial}}$. Hence, level will increase.

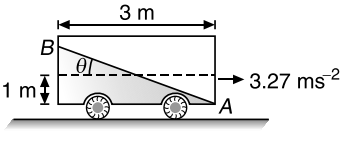
Hence, the correct answer is (A).

120. Density of liquid decrease with increase in temperature and level of liquid and hence, $h \propto \frac{1}{\rho}$

$\Rightarrow \quad h_2 > h_1 \qquad \{\because \rho_{100\,°C} < \rho_{50\,°C}\}$

Hence, the correct answer is (C).

121. The liquid is in equilibrium with respect to container, if θ be the angle made by liquid surface with horizontal as shown in Figure, then given as



$$\tan\theta = \frac{a}{g} = \frac{1}{3}$$

Depth of water at point A is

$$h_A = 1 - 1.5\tan\theta = 0.5 \text{ m}$$

Depth of water at point B is

$$h_B = 1 + 1.5\tan\theta = 1.5 \text{ m}$$

Hence, the correct answer is (C).

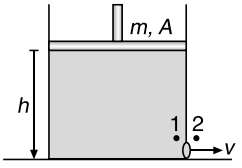
122. $F = YA\alpha\Delta T$

$\Rightarrow \quad F = (10^{11})(10^{-4})(10^{-5})(100)$

$\Rightarrow \quad F = 10^4$ N

Hence, the correct answer is (B).

123. Applying Bernoulli's theorem at 1 and 2, we get



$$\Rightarrow \quad \rho gh + \frac{mg}{A} = \frac{1}{2}\rho v^2$$

$$\Rightarrow \quad v = \sqrt{2gh + \frac{2mg}{\rho A}} = \sqrt{2\left(gh + \frac{mg}{\rho A}\right)}$$

Hence, the correct answer is (B).

124. If coefficient of volume expansion is α and rise in temperature is ΔT then $\Delta V = V\alpha\Delta T$

$$\Rightarrow \quad \frac{\Delta V}{V} = \alpha\Delta T$$

Volume elasticity i.e., Bulk's Modulus is defined as

$$\beta = \frac{P}{\Delta V/V}$$

$$\Rightarrow \quad \beta = \frac{P}{\alpha\Delta T}$$

$$\Rightarrow \quad \Delta T = \frac{P}{\alpha\beta}$$

Hence, the correct answer is (A).

125. $x \propto \dfrac{\ell}{d^2}$

$$\Rightarrow \quad x \propto \frac{\ell}{r^2}$$

$$\Rightarrow \quad \frac{x_2}{x_1} = \frac{r_1^2}{r_2^2}$$

$$\Rightarrow \quad x_2 = (3)\frac{1}{\left(\frac{1}{2}\right)^2}$$

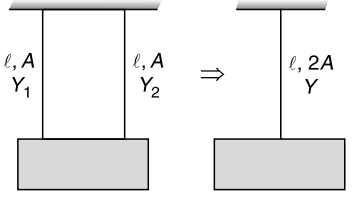
$$\Rightarrow \quad x_2 = 3 \times 4 = 12 \text{ mm}$$

Hence, the correct answer is (D).

126. Equivalent spring constant of a wire is given by

$$k = \frac{YA}{\ell}$$

Since, $k_{eq} = k_1 + k_2$



$$\Rightarrow \quad \frac{Y(2A)}{\ell} = \frac{Y_1 A}{\ell} + \frac{Y_2 A}{\ell}$$

$$\Rightarrow \quad Y = \frac{Y_1 + Y_2}{2}$$

Hence, the correct answer is (B).

127. According to Law of Floatation, we have

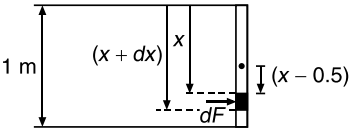
$$W = U$$

$$\Rightarrow \quad Mg = \left(\frac{3}{4}a^3\right)\rho g$$

$$\Rightarrow \quad a = \left(\frac{4M}{3\rho}\right)^{\frac{1}{3}}$$

Hence, the correct answer is (D).

128. For equilibrium, the net torque (τ) about hinge is zero. Let us calculate torque on the gate due to liquid. Consider a section of gate at distance x from water surface, having a cross-sectional width dx as shown in Figure. If dF be the infinitesimal force acting on this element, then



$$dF = PdA = (\rho gx)(1dx)$$

Torque on this element is

$$d\tau = (dF)r_\perp = (\rho gx)(x - 0.5)dx$$

So, net anticlockwise torque is

$$\tau = \int_0^1 d\tau = \left(\frac{\rho g}{12}\right)$$

Net clockwise torque due to external applied force is

$$\tau = F(0.5) = \frac{F}{2}$$

Equating the two torques, we get

$$F = \frac{\rho g}{6}$$

Hence, the correct answer is (C).

129. Work done against gravity

$$W_1 = mgh = (V\rho)gh$$

Work done against pressure difference is

$$W_2 = (\Delta P)V = (h\rho g)V$$

$$\Rightarrow \quad \text{Total work done } W = W_1 + W_2 = 2h\rho gV$$

$$\Rightarrow \quad P = \frac{W}{t} = \frac{2h\rho gV}{t}$$

$$\Rightarrow \quad P = \frac{(2)(6)(10^3)(10)(300 \times 10^{-3})}{60} = 600 \text{ W}$$

Hence, the correct answer is (A).

130. $Y = 10^4 \text{ Nm}^{-2}$, $A = 2 \times 10^{-4} \text{ m}^2$,

$F = 2 \times 10^5 \text{ dyne} = 2 \text{ N}$

Since, $\ell = \dfrac{FL}{AY} = \dfrac{2 \times L}{2 \times 10^{-4} \times 10^4} = L$

So, final length is the sum of initial length and increment i.e.

$$L_{final} = 2L$$

Hence, the correct answer is (C).

131. Let ρ be the density of liquid. Then

$$F_1 = (\Delta P)a = h\rho ga \qquad \ldots(1)$$

In the second case, for the liquid striking the disc elastically, we have

$$F_2 = \frac{\Delta p}{\Delta t} = \frac{2(\Delta m)v}{\Delta t} = \frac{2(\rho a \Delta x)v}{\Delta t}$$

$$\Rightarrow \quad F_2 = 2av\rho\left(\frac{\Delta x}{\Delta t}\right) = 2av^2\rho$$

where, $v = \sqrt{2gh}$ {due to Torricelli's Theorem}

$$\Rightarrow \quad F_2 = 2av^2\rho = 2a\left(\sqrt{2gh}\right)^2\rho = 4h\rho ga \qquad \ldots(2)$$

From equations (1) and (2), we get

$$\frac{F_1}{F_2} = \frac{1}{4}$$

Hence, the correct answer is (D).

132. By definition, Bulk's modulus is given by

$$K = \frac{dP}{\left(\frac{dV}{V}\right)}$$

$$\Rightarrow \quad K = \frac{mg/A}{\left(\frac{dV}{V}\right)}$$

$$\Rightarrow \quad \frac{dV}{V} = \frac{mg}{AK}$$

For a sphere

$$V = \frac{4\pi}{3}R^3$$

$$\Rightarrow \quad dV = \frac{4\pi}{3}d(R^3)$$

$$\Rightarrow \quad dV = \frac{4\pi}{3}(3R^2dR)$$

$$\Rightarrow \quad \frac{dV}{V} = 3\left(\frac{dR}{R}\right)$$

$$\Rightarrow \quad 3\left(\frac{dR}{R}\right) = \frac{mg}{AK}$$

$$\Rightarrow \quad \frac{dR}{R} = \frac{mg}{3AK}$$

Hence, the correct answer is (B).

133. If length of the wire is doubled then strain $= 1$

$$\Rightarrow \quad Y = \text{Stress} = \frac{\text{Force}}{\text{Area}} = \frac{2\times10^5}{2} = 10^5 \text{ dynecm}^{-2}$$

Hence, the correct answer is (B).

134. $$F = \frac{YA\ell}{L} = \frac{9\times10^{10}\times\pi\times4\times10^{-6}\times0.1}{100} = 360\pi \text{ N}$$

Hence, the correct answer is (A).

135. Since, $\Delta\ell = \left(\frac{\ell}{YA}\right)W$

i.e. $\Delta\ell$ vs W graph is a straight line passing through origin (as shown in questions also). The slope of $\Delta\ell$ vs W graph is $\frac{\ell}{YA}$.

$$\Rightarrow \quad \text{Slope} = \frac{\ell}{YA}$$

$$\Rightarrow \quad Y = \left(\frac{\ell}{A}\right)\left(\frac{1}{\text{slope}}\right)$$

$$\Rightarrow \quad Y = \left(\frac{1}{10^{-6}}\right)\frac{(80-20)}{(4-1)\times10^{-4}} = 2\times10^{11} \text{ Nm}^{-2}$$

Hence, the correct answer is (A).

136. Increase in length due to own weight is

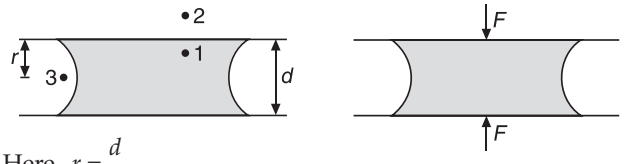
$$\Delta\ell = \int_0^L \frac{(L-x)mg\,dx}{LAY} = \frac{mgL}{2AY}$$

Since, $m = \lambda L$

$$\Rightarrow \quad Y = \frac{mgL}{2A\ell} = \frac{\lambda gL^2}{2A\ell}$$

Hence, the correct answer is (B).

137. For cylindrical surface $\Delta P = \frac{T}{r}$ $\left(\text{not } \frac{2T}{r}\right)$



Here, $r = \frac{d}{2}$

$$\Rightarrow \quad \Delta P = \frac{2T}{d}$$

$$\Rightarrow \quad P_2 - P_1 = \frac{2T}{d} = \Delta P$$

$$\Rightarrow \quad F = (\Delta P)A = (\Delta P)\left(\frac{V}{d}\right) = \frac{(\Delta P)m}{\rho d} \qquad \left\{\because A = \frac{V}{d}\right\}$$

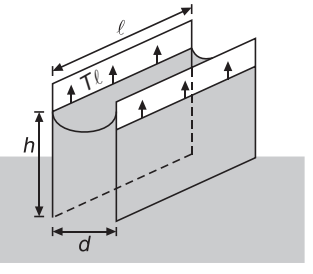
$$\Rightarrow \quad F = \frac{2Tm}{\rho d^2}$$

Hence, the correct answer is (C).

138. When two parallel plates with the spacing 'd' are placed in water reservoir, then liquid will rise till force due to surface tension balances the weight of liquid, so we have

$$2Tl\cos(0°) = (\rho lhd)g$$

$$\Rightarrow \quad h = \frac{2T}{\rho gd}$$



Hence, the correct answer is (B).

139. According to Newton's Second Law, we have

$$F = \frac{\Delta p}{\Delta t} = \left(\frac{\Delta m}{\Delta t}\right)(\Delta v)$$

$$\Rightarrow \quad F = \rho\left(\frac{\Delta V}{\Delta t}\right)(v_1 + v_2) = \rho V(v_1 + v_2)$$

Hence, the correct answer is (D).

CHAPTER 1

140. $-\frac{dh}{dt}=\sqrt{2gh}$

$\Rightarrow \quad a\sqrt{2gh}=Q$

$\Rightarrow \quad 10^{-4}\sqrt{2gh}=10^{-4}$

$\Rightarrow \quad h=\frac{1}{2g}$

$\Rightarrow \quad h=5.1\text{ cm}$

Hence, the correct answer is (A).

141. Since, $\frac{T_{Hg}}{T_{\text{water}}}=7.5$, $\frac{\rho_{Hg}}{\rho_w}=13.6$ and

$$\frac{\cos\theta_{Hg}}{\cos\theta_W}=\frac{\cos 135^\circ}{\cos 0^\circ}=\frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \frac{R_{Hg}}{R_{\text{water}}}=\left(\frac{S_{Hg}}{S_W}\right)\left(\frac{\rho_W}{\rho_{Hg}}\right)\left(\frac{\cos\theta_{Hg}}{\cos\theta_W}\right)$$

$$\Rightarrow \quad \frac{R_{Hg}}{R_{\text{water}}}=7.5\times\frac{1}{13.6}\times\frac{1}{\sqrt{2}}=0.4=\frac{2}{5}$$

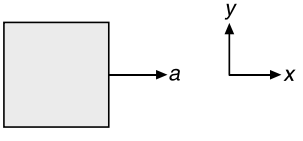
Hence, the correct answer is (D).

142. When a fluid (gas or liquid) is accelerated in positive x direction, then pressure decreases in positive x direction. Change in pressure has differential relation given by

$$\frac{dP}{dx}=-\rho a$$

when ρ is the density of fluid.

Therefore, pressure is lower in front side.



Hence, the correct answer is (B).

143. According to Bernoulli's Theorem

$$\Delta P=\frac{1}{2}\rho v^2$$

$$\Rightarrow \quad v=\sqrt{\frac{2\Delta P}{\rho}}=\sqrt{\frac{2\times 0.5\times 10^5}{10^3}}=10\text{ ms}^{-1}$$

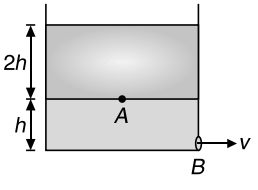
Hence, the correct answer is (D).

144. Pressure at point A is

$$P_A=P_{\text{atm}}+\rho g(2h)$$

Applying Bernoulli's theorem between points A and B, we get

$$P_{\text{atm}}+2\rho gh+\rho g(2h)=P_{\text{atm}}+\frac{1}{2}(2\rho)v^2$$



$\Rightarrow \quad v=2\sqrt{gh}$

Hence, the correct answer is (B).

145. $P=P_0+h\rho g$

$\Rightarrow \quad h=\frac{P-P_0}{\rho g}$

i.e., h depends on P and ρ. So, we cannot conclude anything by seeing h only.

Hence, the correct answer is (D).

146. Since $P=\frac{W}{t}$, we have

$$\Rightarrow \quad P=\frac{\Delta K}{\Delta t}=\frac{\frac{1}{2}(\Delta m)v^2}{\Delta t}$$

$$\Rightarrow \quad P=\frac{1}{2}\left[A\left(\frac{\Delta x}{\Delta t}\right)\rho\right]v^2$$

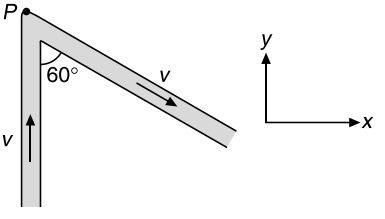
Since, $\frac{\Delta x}{\Delta t}=v$

$\Rightarrow \quad P\propto v^3$

Hence, the correct answer is (C).

147. Along x and y directions along in momentum of water is

$$|\Delta\vec{p}_x|=mv\sin 60^\circ=\frac{\sqrt{3}}{2}mv$$



$$|\Delta\vec{p}_y|=\frac{mv}{2}+mv=\frac{3}{2}mv$$

$$\Rightarrow \quad |\Delta\vec{p}_{\text{net}}|=\sqrt{\Delta p_x^2+\Delta p_y^2}=mv\sqrt{\frac{9}{4}+\frac{3}{4}}$$

$\Rightarrow \quad |\Delta\vec{p}_{\text{net}}|=\sqrt{3}mv$

Force on bend is rate of change of momentum, so we get

$$|\vec{F}_{\text{net}}|=\sqrt{3}\left(\frac{dm}{dt}\right)v=\sqrt{3}\rho Av^2$$

Hence, the correct answer is (A).

148. Since, $v_T=\frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$

$$\Rightarrow \quad v_T=\frac{2\times(0.003)^2(1260\times 2-1260)\times 10}{9\times 1.26}$$

$\Rightarrow \quad v_T=0.02\text{ ms}^{-1}$

$\Rightarrow \quad t=\frac{d}{v_T}=\frac{10\times 10^{-2}}{0.02}=5\text{ s}$

Hence, the correct answer is (D).

149. $\frac{dV}{V}=(1-2\sigma)\frac{dL}{L}$

$\Rightarrow \quad \frac{dV}{V}=2\times 2\times 10^{-3}=4\times 10^{-3}$ $\quad \left\{\because \sigma=0.5=\frac{1}{2}\right\}$

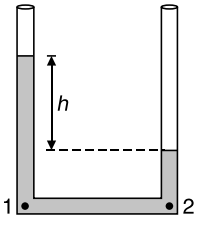
So, percentage change in volume is $4\times 10^{-1}=0.4\%$

Hence, the correct answer is (B).

150. Let P_1 and P_2 be the pressures at the bottom of the left end and right end of the tube respectively. Then

$$F = (P_1 - P_2)A = \rho g h A$$

when A is the cross section of tube



The mass of the liquid in the horizontal portion is

$$m = \rho L A$$

Since, $F = ma$

$$\Rightarrow \quad \rho g h A = \rho L A a$$

$$\Rightarrow \quad h = \frac{aL}{g}$$

Hence, the correct answer is (D).

151. Bulk's Modulus B is

$$B = \frac{(100)(10^3)(10)}{\frac{0.1}{100}}$$

$$\Rightarrow \quad B = 10^9 \text{ Nm}^{-2}$$

Hence, the correct answer is (B).

152. When the vessel is stationary, then

Weight = Upthrust

$$\Rightarrow \quad V\rho_w g = V_i \rho_\ell g$$

(ρ_w = density of wood and ρ_L = density of liquid)

$$\Rightarrow \quad \frac{V_i}{V} = \frac{\rho_w}{\rho_L} \qquad \ldots(1)$$

When the vessel moves upwards, then

$$(\text{upthrust}) - (\text{weight}) = (\text{mass})(\text{acceleration})$$

$$\Rightarrow \quad U' - W = ma$$

$$\Rightarrow \quad V_i' \rho_L \left(g + \frac{g}{2}\right) - V\rho_w g = \frac{V\rho_w g}{2}$$

$$\Rightarrow \quad \frac{V_i'}{V} = \frac{\rho_w}{\rho_L} \qquad \ldots(2)$$

From equation (1) and (2), we get

$$\frac{V_i}{V} = \frac{V_i'}{V}$$

i.e., fraction (or percentage) of volume immersed in liquid remains unchanged and is independent of value of g.

Hence, the correct answer is (C).

153. $F = \dfrac{YA\ell}{L}$

$$\Rightarrow \quad F \propto r^2 \qquad \{\because\ Y,\ \ell \text{ and } L \text{ are constants}\}$$

If diameter is made four times, then force required will become 16 times. i.e., 16×10^3 N

Hence, the correct answer is (B).

154. Let h be the height of the hill, then pressure difference equals the pressure due to h metre of air.

$$\Rightarrow \quad (75 - 50)(10^{-2})\rho_{\mu g} g = h\rho_{\text{air}} g$$

$$\Rightarrow \quad h = (25 \times 10^{-2}) \times \frac{\rho_{Hg}}{\rho_{\text{air}}}$$

$$\Rightarrow \quad h = (25 \times 10^{-2})(10^4) \text{ m} = 2.5 \text{ km}$$

Hence, the correct answer is (B).

155. Thermal Stress $= Y\alpha\Delta T$

$$\Rightarrow \quad Y_1\alpha_1\Delta T = Y_2\alpha_2\Delta T$$

$$\Rightarrow \quad \frac{Y_1}{Y_2} = \frac{\alpha_2}{\alpha_1} = \frac{3}{2}$$

Hence, the correct answer is (C).

156. The tension on heavier sphere is upwards and on lighter sphere is downwards. So, we have

$$(250 \times 10^{-6})(800)g + T = (250 \times 10^{-6})\rho_w g \qquad \ldots(1)$$

and $(250 \times 10^{-6})(1200)g + T = (250 \times 10^{-6})\rho_w g \qquad \ldots(2)$

From equations (1) and (2), we get

$$T = 0.5 \text{ N}$$

Hence, the correct answer is (B).

157. Torque due to hydrostatic force about centre of sphere is zero because hydrostatic force passes through the centre. So, horizontal force on the cylinder due to two liquids must cancel out.

$$\Rightarrow \quad F_{\text{LHS}} = F_{\text{RHS}}$$

If L be the length of cylinder, then

$$(P_{\text{avg}})_{\text{left}}\, hL = (P_{\text{avg}})_{\text{right}}\, RL$$

$$\Rightarrow \quad \frac{1}{2}(2\rho g h)(hL) = \frac{1}{2}(3\rho g R)(RL)$$

$$\Rightarrow \quad h = \sqrt{\frac{3}{2}}R$$

Hence, the correct answer is (B).

158. By Law of Conservation of Energy, we get

$$v_2^2 = v_1^2 + 2gh \qquad \ldots(1)$$

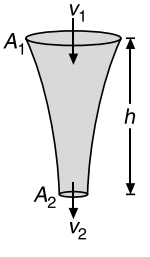
This can also be found by applying Bernoulli's theorem between 1 and 2 as shown in Figure.

From continuity equation

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \left(\frac{A_1}{A_2}\right) v_1 \qquad \ldots(2)$$

Substituting value of v_2 from equation (2) in equation (1), we get



$$\left(\frac{A_1^2}{A_2^2}\right) v_1^2 = v_1^2 + 2gh$$

$$\Rightarrow \quad A_2^2 = \frac{A_1^2 v_1^2}{v_1^2 + 2gh}$$

$$\Rightarrow \quad A_2 = \frac{A_1 v_1}{\sqrt{v_1^2 + 2gh}}$$

Substituting the given values, we get

$$A_2 = \frac{(10^{-4})(1)}{\sqrt{(1)^2 + 2(10)(0.15)}} = 5 \times 10^{-5} \text{ m}^2$$

Hence, the correct answer is (C).

159. Since $y = \dfrac{x^2\omega^2}{2g}$

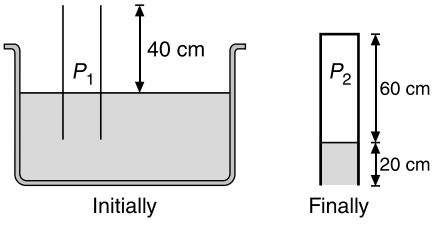
At $x = r$, we get $y = \dfrac{r^2\omega^2}{2g}$

Since $\omega = 4\pi$ rads^{-1}, $r = \dfrac{5}{100}$ m

$\Rightarrow \quad y = \dfrac{16\pi^2 \times 25 \times 10^{-4}}{2 \times 10} = 1.9 \text{ cm} \approx 2.0 \text{ cm}$

Hence, the correct answer is (D).

160. Initially $P_1 = P_0 = P_{atm}$



Since temperature is constant, so

$P_1V_1 = P_2V_2$

$\Rightarrow \quad P_2 = P_1\left(\dfrac{V_1}{V_2}\right) = P_0\left(\dfrac{40}{60}\right) = \dfrac{2}{3}P_0$

Finally, when top of tube is closed, then

$P_2 + (20 \text{ cm of Hg}) = P_0$

$\Rightarrow \quad \dfrac{2}{3}P_0 + (20 \text{ cm of Hg}) = P_0$

$\Rightarrow \quad \dfrac{P_0}{3} = (20 \text{ cm of Hg})$

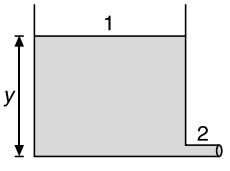
$\Rightarrow \quad P_0 = 60 \text{ cm of Hg}$

Hence, the correct answer is (C).

161. The velocity of fluid at the hole when the level of liquid in the container be $y (< h)$ at any instant is

$$v_2 = \sqrt{\frac{2gy}{1 + \left(\dfrac{a^2}{A^2}\right)}} \qquad \ldots(1)$$

Using continuity equation at the two cross-sections 1 and 2, we get



$\Rightarrow \quad Av_1 = av_2$

$\Rightarrow \quad v_1 = \left(\dfrac{a}{A}\right)v_2$

So, acceleration a_1 of top surface is

$\dfrac{dv_1}{dt} = \dfrac{dv_1}{dt} \times \dfrac{dy}{dy}$

$\Rightarrow \quad a_1 = \dfrac{dv_1}{dt} = v_1\dfrac{dv_1}{dy}$

$\Rightarrow \quad a_1 = \dfrac{a}{A}(v_2)\dfrac{d}{dy}\left(\dfrac{a}{A}v_2\right)$

$\Rightarrow \quad a_1 = \dfrac{a^2}{A^2}v_2\dfrac{d}{dy}(v_2)$

$\Rightarrow \quad a_1 = -\dfrac{a^2}{A^2}\sqrt{2gy}\sqrt{2g}\dfrac{1}{2\sqrt{y}}$

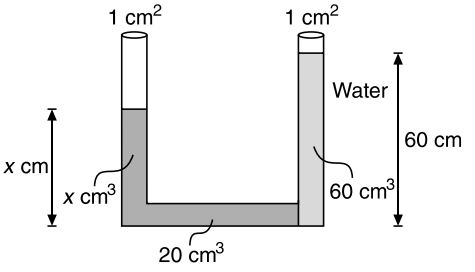
$\Rightarrow \quad a_1 = -\dfrac{ga^2}{A^2}$

Hence, the correct answer is (D).

162. Since area of cross-section is 1 cm^2, so height to which water should rise is 60 cm .

Hence, $h_1\rho_1 g = h_2\rho_2 g$

$\Rightarrow \quad x(4) = (60)(1)$



$\Rightarrow \quad x = 15$ cm

So, total volume of liquid required is

$V = (20 + 15) \text{ cm}^3 = 35 \text{ cm}^3$

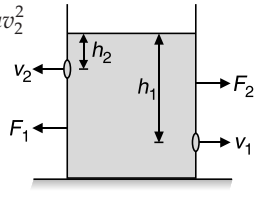
Hence, the correct answer is (D).

163. Thrust force $F = F_1 - F_2 = \rho a v_1^2 - \rho a v_2^2$

$\Rightarrow \quad F = \rho a(2gh_1) - \rho a(2gh_2)$

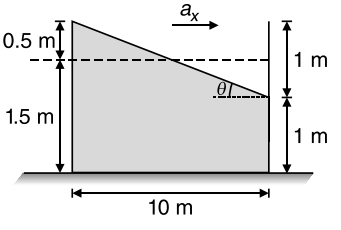
$\Rightarrow \quad F = 2\rho ag(h_1 - h_2)$

$\Rightarrow \quad F = 2\rho agh$



Hence, the correct answer is (C).

164. Figure shows the limiting state of oil surface in the tank at the instant when just spilling begins to start from rear end



Since, $\tan\theta = \dfrac{a_x}{g} = \dfrac{v}{gt}$

Also, $\tan\theta = \dfrac{1}{10}$

$\Rightarrow \quad t = \dfrac{10 \times 20}{10} = 20 \text{ s}$

Hence, the correct answer is (A).

165. Viscous force $F = mg\sin\theta$

$\Rightarrow \quad \eta(a^2)\dfrac{v}{t} = mg\sin(37°) = \dfrac{3}{5}mg$

$\Rightarrow \quad \eta a^2\left(\dfrac{v}{t}\right) = \dfrac{3}{5}(a^3\rho)g$

$\Rightarrow \quad \eta = \dfrac{3\rho agt}{5v}$

Hence, the correct answer is (A).

166. Total force at height $\dfrac{3L}{4}$ from its lower end is

F = Weight suspended + Weight of $\dfrac{3}{4}$ of the chain

$\Rightarrow \quad F = W_1 + \left(\dfrac{3W}{4}\right)$

Hence, Stress $= \frac{F}{A} = \frac{W_1 + \left(\frac{3W}{4}\right)}{S}$

Hence, the correct answer is (C).

167. $Y = 3B(1-2\sigma)$

$\Rightarrow \quad \sigma = \frac{3B - Y}{6K}$

$\Rightarrow \quad \sigma = \frac{3\times 11\times 10^{10} - 7.25\times 10^{10}}{6\times 11\times 10^{10}}$

$\Rightarrow \quad \sigma = 0.39$

Hence, the correct answer is (C).

168. Since $\eta = \frac{\frac{F}{A}}{\frac{x}{L}}$

$$F = -\eta\frac{Ax}{L}$$

(Negative sign indicates that restoring force is directed towards the mean position)

$\Rightarrow \quad M\ddot{x} = -\eta\frac{Ax}{L}$

$\Rightarrow \quad \ddot{x} + \frac{\eta A}{ML}x = 0$

Comparing with

$$\ddot{x} + \omega^2 x = 0$$

we get $\omega = \sqrt{\frac{\eta A}{ML}}$

$\Rightarrow \quad T = 2\pi\sqrt{\frac{ML}{\eta A}}$

Since $A = L^2$

$\Rightarrow \quad T = 2\pi\sqrt{\frac{M}{\eta L}}$

Hence, the correct answer is (D).

169. For points to lie on same stream line

$$P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$$

Sum of all three terms are different at three points A, B and C.

Hence, the correct answer is (D).

170. $W = \frac{1}{2}\frac{YA\ell^2}{L}$

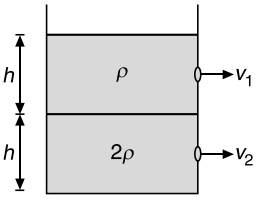
$\Rightarrow \quad 0.4 = \frac{1}{2}\times\frac{Y\times 10^{-6}\times(0.2\times 10^{-2})^2}{1}$

$\Rightarrow \quad Y = 2\times 10^{11}\ \text{Nm}^{-2}$

Hence, the correct answer is (C).

171. Since, $v_1 = \sqrt{2g\left(\frac{h}{2}\right)} = \sqrt{gh}$ …(1)

According to Bernoulli's theorem, we have



$$\rho gh + 2\rho g\left(\frac{h}{2}\right) = \frac{1}{2}(2\rho)v_2^2$$

$\Rightarrow \quad v_2 = \sqrt{2gh}$ …(2)

$\Rightarrow \quad \frac{v_1}{v_2} = \frac{1}{\sqrt{2}}$

Hence, the correct answer is (D).

172. $T = 2\pi\sqrt{\frac{M}{K_{\text{eq}}}} = 2\pi\sqrt{\frac{M(YA+KL)}{YAK}}$

Hence, the correct answer is (B).

173. As the air is pumped out buoyancy due to air will become zero. Hence, $V_2 > V_1$

Hence, the correct answer is (C).

174. $P - P_0 = \frac{4T}{R}$

$\Rightarrow \quad P = \frac{4T}{R} + P_0$

$$V = \frac{4}{3}\pi R^3 = kt$$

$\Rightarrow \quad R = \left(\frac{3kt}{4\pi}\right)^{1/3}$

$\Rightarrow \quad P = P_0 + \frac{4T}{R} = P_0 + 4T\left(\frac{4\pi}{3kT}\right)^{1/3}$

$\Rightarrow \quad P = m\left(\frac{1}{t^{1/3}}\right) + c$

Hence, the correct answer is (D).

175. F_1 will decrease and F_2 will increase. So f_1 may or may not be greater than f_2.

Total weight of system in both conditions will remain same. Hence,

$$f_1 + f_2 = F_1 + F_2$$

Hence, the correct answer is (A).

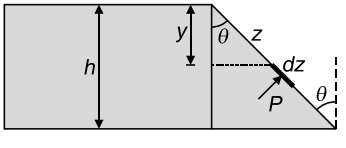
176. Since $F = PA = (\rho gh)A$

$\Rightarrow \quad F = (10^3)(10)(99+1)\times 10^{-2}\times 10^{-2}$

$\Rightarrow \quad F = 100\ \text{N}$

Hence, the correct answer is (D).

177. Since, $z = y\sec\theta$



$\Rightarrow \quad dz = dy\sec\theta$

Also, $P = \rho g y$ and

$$dA = bdz = bdy\sec\theta$$

$$\Rightarrow \quad dF = PdA = (\rho g b \sec\theta) y dy$$

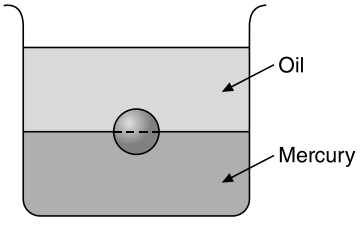
$$\Rightarrow \quad F = \int_0^h dF = \frac{1}{2}\rho b h^2 g \sec\theta$$

Hence, the correct answer is (C).

178. For floating of sphere, we have

$$W = U$$

$$\Rightarrow \quad V\rho_m g = \frac{V}{2}\rho_{Hg} g + \frac{V}{2}\rho_{\text{oil}}$$



$$\Rightarrow \quad \rho_m = \frac{\rho_{Hg} + \rho_{\text{oil}}}{2} = \frac{13.6 + 0.8}{2}$$

$$\Rightarrow \quad \rho_m = \frac{14.4}{2} = 7.2$$

Hence, the correct answer is (C).

179. Let $m_A = 2m$, $m_B = 3m$, density of liquid be ρ and density of block B to be 2ρ

Acceleration of system, when block B is inside the liquid is

$$a_1 = \frac{m_A g - (m_B g - \text{upthrust on } B)}{m_A + m_B}$$

$$\Rightarrow \quad a_1 = \frac{2mg - \left(3mg - \frac{3m}{2\rho}\cdot\frac{1}{2}\rho g\right)}{5\text{ m}}$$

$$\Rightarrow \quad a_1 = \frac{g}{10} = \text{constant}$$

However, when the block is outside the liquid, then the acceleration of system (in opposite direction) is

$$a_2 = \frac{m_B g - m_A g}{m_A + m_B} = \frac{3mg - 2mg}{5m} = \frac{g}{5}$$

Since, a_1 and a_2 are constants, motion is periodic but not simple harmonic.

Hence, the correct answer is (A).

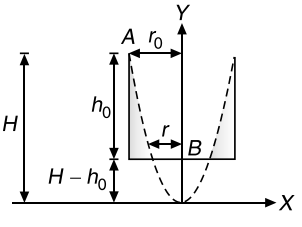
180. Area of bottom is $A_0 = \pi r_0^2$

If r is radius of the exposed bottom, then

$$A = \frac{A_0}{2}$$

$$\Rightarrow \quad \pi r^2 = \frac{1}{2}\pi r_0^2$$

$$\Rightarrow \quad r = \frac{r_0}{\sqrt{2}}$$



Due to rotation of cylinder, the liquid surface has an equation given by

$$y = \frac{\omega^2 x^2}{2g}$$

At $y = H$, $x = r_0$

and at $y = H - h_0$, $x = \frac{r_0}{\sqrt{2}}$

$$\Rightarrow \quad h_0 = \frac{\omega^2}{2g}\left(\frac{r_0^2}{2}\right)$$

$$\Rightarrow \quad \omega = 2\sqrt{\frac{gh_0}{r_0}}$$

$$\Rightarrow \quad \omega = \frac{2\sqrt{10\times 0.1}}{0.02}$$

$$\Rightarrow \quad \omega = 100 \text{ rads}^{-1}$$

Hence, the correct answer is (C).

181. Rate of flow $Q = \dfrac{\Delta P}{\left(\frac{8\eta L}{\pi R^4}\right)}$

$$\Rightarrow \quad Q \propto (\Delta P)(R^4)$$

Since, ΔP is increased two times, whereas radius is reduced to half, so rate of flow of liquid will become $\frac{1}{8}$ times.

Hence, the correct answer is (D).

182. For wires of same material, we have

$$\frac{F\ell}{Ax} = \text{constant}$$

For same tension applied on all, we have

$$x \propto \frac{\ell}{A}$$

$$\Rightarrow \quad x \propto \frac{\ell}{d^2}$$

Extension produced x is maximum for dimensions given in OPTION **(A)**.

Hence, the correct answer is (C).

183. Acceleration a of body is

$$a = \frac{\text{upthrust} - \text{weight}}{\text{mass}}\text{, upwards}$$

$$\Rightarrow \quad a = \frac{V\sigma g - V\rho g}{V\rho} = \left(\frac{\sigma}{\rho} - 1\right)g\text{, upwards}$$

Hence, the correct answer is (A).

184. $$B = \frac{\Delta P}{\Delta V/V} = \frac{h\rho g}{0.1/100} = \frac{200\times 10^3 \times 9.8}{1/1000}$$

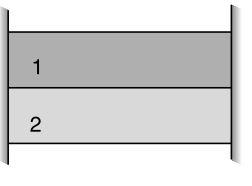
$$\Rightarrow \quad B = 19.6\times 10^8 \text{ Nm}^{-2}$$

Hence, the correct answer is (A).

185. Thermal stress $\sigma = \dfrac{F}{A} Y\alpha\Delta T$

Given, $\sigma_1 = \sigma_2$

$$\Rightarrow \quad Y_1\alpha_1\Delta T = Y_2\alpha_2\Delta T$$



$$\Rightarrow \quad \frac{Y_1}{Y_2}=\frac{\alpha_2}{\alpha_1}=\frac{3}{2}$$

Hence, the correct answer is (C).

186. Since $P=P_0+\rho gh+\dfrac{2T}{r}$

$$\Rightarrow \quad P=(10^5)+(10^3)(10)(10)+\frac{2\times7\times10^{-2}}{10^{-3}}$$

$$\Rightarrow \quad P=2.0014\times10^5\ \text{Nm}^{-2}$$

Hence, the correct answer is (D).

187. According to Archimedes Principle, we have

Loss in Weight = Upthrust.

$$\Rightarrow \quad (38.2-36.2)g=\left(V_{\text{gold}}+V_{\text{cavity}}\right)\rho_w g$$

$$\Rightarrow \quad 2=\left(\frac{38.2}{19.3}\right)+V_{\text{cavity}} \qquad \{\because\ \rho_w=1\ \text{gcc}^{-1}\}$$

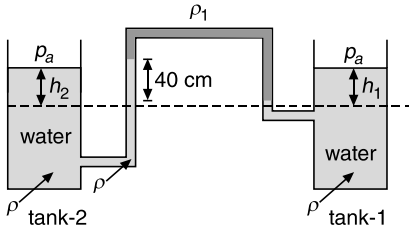
$$\Rightarrow \quad V_{\text{cavity}}=0.02\ \text{cm}^3$$

Hence, the correct answer is (C).

188. Using Pascal's equation from first to second tank surface, we get

$$P_a+h_1\rho g-40\rho_1 g+40\rho g=P_a+h_2\rho g$$

$$\Rightarrow \quad h_2\rho g-h_1\rho g=40\rho g-40\rho_1 g$$



as $\rho_1=0.9\rho$

$$(h_2-h_1)\rho g=40\rho g-36\rho g$$

$$\Rightarrow \quad h_2-h_1=4\ \text{cm}$$

Hence, the correct answer is (B).

189. ℓ will decrease because the block moves up, h will decrease because the coin will displace the volume of water (V_1) equal to its own volume, when it is in the water whereas when it is on the block it will displace the volume of water (V_2), whose weight is equal to weight of coin and since, density of coin is greater than the density of water $V_1<V_2$.

Hence, the correct answer is (D).

190. $$\frac{Y_A}{Y_B}=\frac{\tan\theta_A}{\tan\theta_B}=\frac{\tan60}{\tan30}=\frac{\sqrt{3}}{\frac{1}{\sqrt{3}}}=3$$

$$\Rightarrow \quad Y_A=3Y_B$$

Hence, the correct answer is (D).

191. $$\eta=\frac{F}{A\theta}$$

$$\Rightarrow \quad \eta=\frac{5\times10^5}{\left(\frac{10}{100}\right)^2 0.001}$$

$$\Rightarrow \quad \eta=5\times10^{10}\ \text{Nm}^{-2}$$

Hence, the correct answer is (C).

192. For maximum range, orifice should be at the middle of ground and free surface of liquid i.e.,

$$x=\frac{2H+H}{2}=1.5H$$

Since, this point lies in the tank. So, hole should be made at this point.

Hence, the correct answer is (C).

193. $$\ell=\frac{FL}{AY}$$

$$\Rightarrow \quad \ell\propto\frac{L}{r^2}\ (F \text{ and } Y \text{ are constant})$$

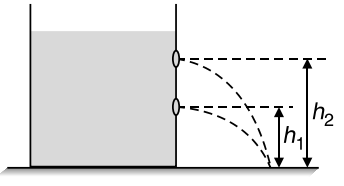
$$\Rightarrow \quad \frac{\ell_2}{\ell_1}=\frac{L_2}{L_1}\times\left(\frac{r_1}{r_2}\right)^2=2\times\left(\frac{1}{2}\right)^2=\frac{1}{2}$$

$$\Rightarrow \quad \ell_2=\frac{\ell_1}{2}$$

i.e., the change in the length of other wire is $\dfrac{\ell}{2}$

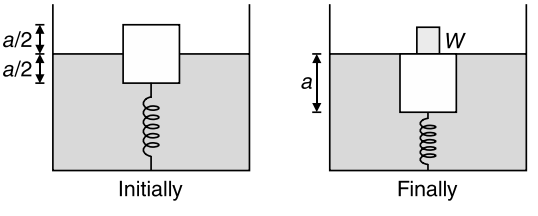
Hence, the correct answer is (C).

194. If H is the height of the liquid surface, then for same range, we have $h_2=H-h_1$ and for maximum range, we have $h=\dfrac{H}{2}=\dfrac{h_1+h_2}{2}$



Hence, the correct answer is (D).

195. Since, $\rho_{\text{block}}=\dfrac{1}{2}\rho_{\text{water}}$, so 50% of its volume is immersed in water



When weight is put over the block, then half of the volume of block is further immersed in water. Therefore,

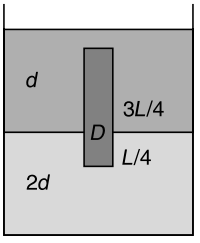
$$W=(\text{Extra Upthrust})+(\text{Spring Force})$$

$$\Rightarrow \quad W=(a)(a)\left(\frac{a}{2}\right)(2\rho)(g)+k\left(\frac{a}{2}\right)$$

$$\Rightarrow \quad W=a\left(a^2\rho g+\frac{k}{2}\right)$$

Hence, the correct answer is (D).

196. Considering vertical equilibrium of cylinder, we get



$$\begin{pmatrix}\text{Weight}\\ \text{of}\\ \text{cylinder}\end{pmatrix}=\begin{pmatrix}\text{Upthrust due}\\ \text{to upper}\\ \text{liquid}\end{pmatrix}+\begin{pmatrix}\text{Upthrust due}\\ \text{to lower}\\ \text{liquid}\end{pmatrix}$$

$$\Rightarrow \left(\frac{A}{5}\right)(L)Dg=\left(\frac{A}{5}\right)\left(\frac{3L}{4}\right)(d)g+\left(\frac{A}{5}\right)\left(\frac{L}{4}\right)(2d)(g)$$

$$\Rightarrow D=\left(\frac{3}{4}\right)d+\left(\frac{1}{4}\right)(2d)$$

$$\Rightarrow D=\frac{5}{4}d$$

Hence, the correct answer is (A).

197. Since, $W=U$

$$\Rightarrow (A\times 0.5\times 900)g+(100)g=A\times 0.5\times 1000\times g$$

$$\Rightarrow A=2\ \text{m}^2$$

Hence, the correct answer is (D).

198. When the vessel is at rest, then for equilibrium of block, we have

$$U+kx_1=mg \qquad \ldots(1)$$

where, x_1 is the compression in the spring. Also, if l_1 is the length of the spring in the first case, then

$$x_1=l_0-l_1 \qquad \ldots(2)$$

When the vessel accelerates vertically downwards with an acceleration a, then from the reference frame attached to the vessel, the block is in equilibrium, so

$$U+kx_2=m(g-a) \qquad \ldots(3)$$

where, x_2 is the new extension in the spring. Also, if l_2 is the length of the spring in the second case, then

$$x_2=l_0-l_2 \qquad \ldots(4)$$

From equations (1) and (3), we conclude that

$$x_2<x_1$$

So, from equations (2) and (4), we get

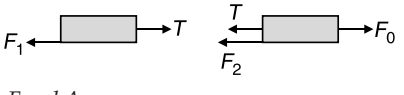
$$l_2>l_1$$

Hence the spring length will increase.

Hence, the correct answer is (D).

199. Since, viscous force F is proportional to area A, so let $F=kA$

$v=$ constant (i.e. $a=0$)



So, $T=F_1=kA_1 \qquad \ldots(1)$

$$F_0=T+F_2=kA_1+kA_2$$

$$\Rightarrow F_0=k(A_1+A_2) \qquad \ldots(2)$$

Dividing equation (2) by equation (1), we get

$$\frac{T}{F_0}=\frac{A_1}{A_1+A_2}$$

$$\Rightarrow T=\frac{F_0A_1}{A_1+A_2}$$

Hence, the correct answer is (C).

200. Buoyant force is

$$U=V_{\text{imm}}\rho_{\text{liq}}g$$

Since, $V_{\text{imm}}\rho_{\text{liq}}$ and g all are same w.r.t. O_1 and O_2, so U is same.

Hence, the correct answer is (A).

201. $$\rho=\frac{M}{V} \qquad \ldots(1)$$

$$\rho+\Delta\rho=\frac{M}{V-\Delta V} \qquad \ldots(2)$$

$$\Rightarrow \Delta\rho=\frac{M}{V-\Delta V}-\frac{M}{V}$$

$$\Rightarrow \Delta\rho=\frac{M}{V}\left[\frac{1}{1-\frac{\Delta V}{V}}-1\right]$$

$$\Rightarrow \Delta\rho=\rho\left[\frac{1}{\left(1-\frac{P}{K}\right)}-1\right] \qquad \left\{\because K=\frac{P}{\left(\frac{\Delta V}{V}\right)}\right\}$$

$$\Rightarrow \Delta\rho=\frac{\rho P}{K-P}$$

Hence, the correct answer is (C).

202. Since mercury meniscus is convex, so the pressure just inside the hole will be less than the outside pressure by $\frac{2T}{r}$

$$\Rightarrow h\rho g=\frac{2T}{r}$$

$$\Rightarrow h=\frac{2T}{r\rho g}$$

Hence, the correct answer is (C).

203. Since, $W=\frac{1}{2}(\text{Tension})(\text{Extension})$

$$\Rightarrow W=\frac{1}{2}Fx$$

$$\Rightarrow \frac{W_1}{W_2}=\frac{F_1}{F_2}$$

$$\Rightarrow \frac{W_1}{W_2}=\frac{YA_1x/L_1}{YA_2x/L_2}$$

$$\Rightarrow \frac{W_1}{W_2}=\frac{A_1L_2}{A_2L_2}$$

$$\Rightarrow W_2=(4)(2)(2)$$

$$\Rightarrow W_2=16\ \text{J}$$

Hence, the correct answer is (A).

204. For an isothermal process

$$PV = \text{constant}$$

$$\Rightarrow \quad PdV + VdP = 0$$

$$\Rightarrow \quad \frac{dP}{dV} = -\left(\frac{P}{V}\right)$$

Since bulk modulus is

$$B = -\left(-\frac{dP}{dV/V}\right) = -\left(\frac{dP}{dV}\right)V$$

$$\Rightarrow \quad B = -\left[\left(-\frac{P}{V}\right)V\right] = P$$

$$\Rightarrow \quad B = P$$

Similarly, adiabatic bulk modulus is given by $B = \gamma P$

Hence, the correct answer is (B).

205. Let x be the fraction of its volume which is hollow. Then weight of shell is

$$W = (V - xV)(5 \times 1000)g$$

Weight of water displaced is equal to upthrust i.e.

$$V \times 1000 \times g$$

Since, $\begin{pmatrix}\text{Weight of water}\\ \text{displaced}\end{pmatrix} = \begin{pmatrix}\text{Loss in}\\ \text{weight}\end{pmatrix}$

$$\Rightarrow \quad V(1000)g = \frac{1}{2}(V - xV)(5 \times 1000)g$$

$$\Rightarrow \quad x = \frac{3}{5}$$

Hence, the correct answer is (A).

206. $\Delta P = h\rho g = 2T\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = 2T\left(\frac{r_2 - r_1}{r_1 r_2}\right)$

$$\Rightarrow \quad T = \frac{\rho g h r_1 r_2}{2(r_2 - r_1)}$$

Hence, the correct answer is (A).

207. Let the spring constant be k. When the piece is hanging in air, then for equilibrium, we have

$$kx = mg$$

$$\Rightarrow \quad k(0.01) = (0.01)(10)$$

$$\Rightarrow \quad k = 10 \text{ Nm}^{-1}$$

The volume of the copper piece is

$$V_{Cu} = \frac{0.01 \text{ kg}}{9000 \text{ kgm}^{-3}} = \frac{1}{9} \times 10^{-5} \text{ m}^3$$

This is also the volume of water displaced when the piece is immersed in water. Since the force of buoyancy is equal to the weight of the liquid displaced. So,

$$U = \left(\frac{1}{9} \times 10^{-5}\right)(1000)(10) = 0.011 \text{ N}$$

If the elongation of the spring is x' when the piece is immersed in water, then for equilibrium condition of the piece, we have

$$kx' = 0.1 \text{ N} - 0.011 \text{ N} = 0.089 \text{ N}$$

Since $k = 10 \text{ Nm}^{-1}$

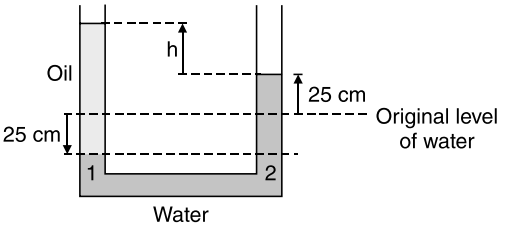
$$\Rightarrow \quad x = \frac{0.089}{10} \text{ m} = 0.0089 \text{ m} = 0.89 \text{ cm}$$

Hence, the correct answer is (B).

208. $P_1 = P_2$

$$\Rightarrow \quad P_0 + \rho_0 g h_0 = P_0 + \rho_w g h_w$$

$$\Rightarrow \quad \rho_0 h_0 = \rho_w h_w$$



$$\Rightarrow \quad (0.8)(h + 50) = (1)(50)$$

$$\Rightarrow \quad h = 12.5 \text{ cm}$$

Hence, the correct answer is (B).

209. Given that compressibility k is 5×10^{-5} per unit atmospheric pressure

$$\Rightarrow \quad k = \frac{5 \times 10^{-5}}{10^5} = 5 \times 10^{-10} \text{ N}^{-1}\text{m}^2$$

$$\Rightarrow \quad B = \frac{1}{k} = \frac{1}{5} \times 10^{10} \text{ Nm}^{-2}$$

Since, $\Delta V = \frac{V \Delta P}{B}$

$$\Rightarrow \quad \Delta V = \frac{(100)(100 \times 10^5)}{\frac{1}{5} \times 10^{10}} = 0.5 \text{ cm}^3$$

Hence, the correct answer is (A).

210. $\Delta P = h\rho g = \frac{2T}{r}$

$$\Rightarrow \quad h = \frac{2T}{r\rho g}$$

Hence, the correct answer is (C).

211. $\ell = \frac{L^2 dg}{2Y} = \frac{(10)^2 \times 1500 \times 10}{2 \times 5 \times 10^8} = 15 \times 10^{-4} \text{ m}$

Hence, the correct answer is (A).

212. Let a be the length of each side of the cube, then

$$200 \times g = (2) \times (a^2) \times 1 \times g$$

$$\Rightarrow \quad a = 10 \text{ cm}$$

Hence, the correct answer is (C).

213. At terminal speed, net force on the ball is zero, so $F = 0$.

$$\Rightarrow \quad (\text{Weight}) = (\text{Upthrust}) + (\text{Viscous force})$$

$$\Rightarrow \quad W = U + F$$

$$\Rightarrow \quad \frac{4}{3}\pi r^3 \rho_1 g = \frac{4}{3}\pi r^3 \rho_2 g + k r v_T$$

$$\Rightarrow \quad v_T = \frac{4\pi g r^2}{3k}(\rho_1 - \rho_2)$$

Hence, the correct answer is (A).

214. Let density of material of sphere (in gcm^{-3}) be ρ

Applying the condition of floatation, we get

$$\text{Weight} = \text{Upthrust}$$

$$\Rightarrow \quad V\rho g = \frac{V}{2}\rho_{\text{oil}} g + \frac{V}{2}\rho_{Hg} g$$

$$\Rightarrow \quad \rho = \frac{\rho_{oil}}{2} + \frac{\rho_{Hg}}{2} = \frac{0.8}{2} + \frac{13.6}{2} = 7.2 \text{ gcm}^{-3}$$

Hence, the correct answer is (C).

215. $\text{Stress} = \dfrac{F}{A} = \dfrac{4 \times 3.1\pi}{\pi \times (2 \times 10^{-3})^2}$

$$\Rightarrow \quad \text{Stress} = 3.1 \times 10^6 \text{ Nm}^{-2}$$

Hence, the correct answer is (B).

216. Since, $h = \dfrac{2T\cos\theta}{r\rho g}$

$$\Rightarrow \quad 10 = \frac{2T_1 \cos 0°}{r\rho_1 g} \qquad \ldots(1)$$

$$\Rightarrow \quad -3.42 = \frac{2T_2 \cos 135°}{r\rho_2 g} \qquad \ldots(2)$$

From these two equations, we get

$$\frac{T_1}{T_2} = \frac{10(\cos 135°)}{(-3.42)(\cos 0°)\rho_2} = \frac{(10)\left(-\frac{1}{\sqrt{2}}\right)(1)}{(-3.42)(1)(13.6)}$$

$$\Rightarrow \quad \frac{T_1}{T_2} = 1 : 6.5$$

Hence, the correct answer is (D).

217. For equilibrium, we have

Surface Tension Force + Upthrust = Weight

$$\Rightarrow \quad (2\pi rT) + \left(\frac{2}{3}\pi r^3\right)\rho_w g = \left(\frac{4}{3}\pi r^3\right)\rho_s g$$

Substituting the values, we get

$r \approx 1.2$ mm

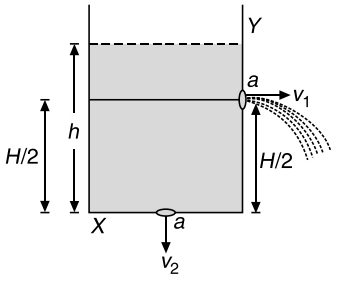
Hence, the correct answer is (C).

218. Efflux speed at holes X and Y respectively is

$$v_1 = \sqrt{2g\left(\frac{h-H}{2}\right)} \text{ and } v_2 = \sqrt{2gh}$$

Applying equation of continuity at top surface and holes, we get

$$A\left(-\frac{dh}{dt}\right) = a(v_1 + v_2)$$



$$\Rightarrow \quad A\left(-\frac{dh}{dt}\right) = a\left[\sqrt{2g\left(\frac{h-H}{2}\right)} + \sqrt{2gh}\right]$$

$$\Rightarrow \quad -\frac{A}{a\sqrt{2g}} \int_H^{\frac{H}{2}} \frac{dh}{\sqrt{h} + \sqrt{h - \frac{H}{2}}} = \int_0^t dt$$

$$\Rightarrow \quad t = \frac{2A}{3a}(\sqrt{2} - 1)\sqrt{\frac{H}{g}}$$

Hence, the correct answer is (B).

219. When the water reaches the lower end of the tube a convex meniscus will be formed such that

$$\Delta P = 2 \times \frac{2T}{r} = \frac{4T}{r}$$

$$\Rightarrow \quad \frac{2T}{r} = \rho g h$$

Also, $\Delta P = \rho g h'$

$$\Rightarrow \quad h' = 2h$$

Hence, the correct answer is (D).

220. Since, $F = YA\left(\dfrac{\Delta L}{L}\right) = YA\alpha\Delta T \qquad \left\{\because \dfrac{\Delta L}{L} = \alpha\Delta T\right\}$

$$\Rightarrow \quad F = (2 \times 10^{11})(10^{-6})(1.1 \times 10^{-5})(20)$$

$$\Rightarrow \quad F = 44 \text{ N}$$

Hence, the correct answer is (D).

221. Let tension in the thread be F

Surface tension force is acting radially outwards.

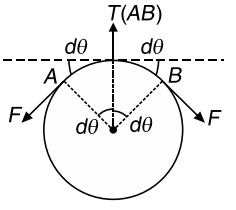
On section AB, we have

$$2F\sin(d\theta) = T(AB) = T(2Rd\theta)$$

Since, $\sin d\theta \approx d\theta$

$$\Rightarrow \quad 2Fd\theta = 2TRd\theta$$

$$\Rightarrow \quad F = 2TR$$

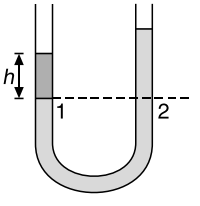


Hence, the correct answer is (D).

222. Surface area decreases and hence, the surface energy also decreases.

Hence, the correct answer is (B).

223. Since, $P_1 = P_2$

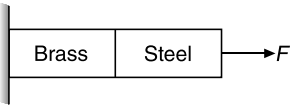


$$\Rightarrow \quad P_0 + \rho_1 g h = P_0 + \rho_2 g h$$

$$\Rightarrow \quad \rho_1 = \rho_2$$

Hence, the correct answer is (B).

224.



Corresponding to the stress (σ), total elongation is

$$\Delta\ell_{net} = \frac{\sigma L_1}{Y_1} + \frac{\sigma L_2}{Y_2}$$

$$\Rightarrow \quad \sigma = \Delta\ell\left(\frac{Y_1 Y_2}{Y_1 + Y_2}\right)$$

$$\Rightarrow \quad \sigma = 0.2 \times 10^{-3} \times \left(\frac{120 \times 60}{180}\right) \times 10^9$$

$\Rightarrow \quad \sigma = 8 \times 10^6 \text{ Nm}^{-2}$

Hence, the correct answer is (A).

225. We know that, $h = \dfrac{2T\cos\theta}{R\rho g}$

$\Rightarrow \quad h \propto \dfrac{1}{R}$

Since, $M = (\pi R^2 h)\rho$

$\Rightarrow \quad M \propto R^2 h$

$\Rightarrow \quad M \propto R$

Radius is doubled, so mass in the capillary tube will also become two times.

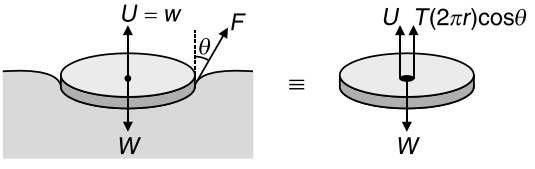
Hence, the correct answer is (A).

226. Since, $h = \dfrac{2T\cos\theta}{r\rho g}$

For a freely falling lift $g_{\text{eff}} = 0$, so $h \to \infty$ or it will fill the entire length of the tube.

Hence, the correct answer is (B).

227. Surface tension force on liquid is downwards, but on the disc it is upwards as shown in Figure.



For equilibrium of disc, we have

$W = U + F = w + 2\pi Tr\cos\theta$

Hence, the correct answer is (C).

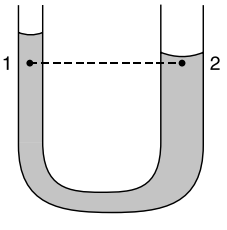
228. The average velocity in the first half of the distance $< v$, while in the second half the average velocity is v. Therefore, $t_1 > t_2$. The work done against gravity in both halves is $\dfrac{mg\ell}{2}$.

Hence, the correct answer is (D).

229. Since $P_1 = P_2$

$\Rightarrow \quad \left(P_0 - \dfrac{2T}{r_1}\right) + \rho gh = \left(P_0 - \dfrac{2T}{r_2}\right)$

$\Rightarrow \quad T = \dfrac{\rho g h r_1 r_2}{2(r_2 - r_1)}$



Hence, the correct answer is (A).

230. $\ell = \dfrac{FL}{AY} = \dfrac{FL^2}{(AL)Y} = \dfrac{FL^2}{VY}$

If volume is fixed then $\ell \propto L^2$

Hence, the correct answer is (C).

231. Since loss in weight equals the upthrust, so we get

$(w - w_1) = V\rho_w g \quad \ldots(1)$

$(w - w_2) = V\rho_L g \quad \ldots(2)$

Dividing equation (2) by (1), we get

$\dfrac{\rho_L}{\rho_w} = \dfrac{w - w_2}{w - w_1} = \text{Relative Density of liquid}$

Hence, the correct answer is (B).

232. Since, $W_{\text{app}} = W_{\text{actual}} - U = W - V_S\rho_L g$

At higher temperature, we have

$U' = V_S'\rho_L' g$

$\Rightarrow \quad \dfrac{U'}{U} = \left(\dfrac{V_S'}{V_S}\right)\left(\dfrac{\rho_L'}{\rho_L}\right)$

$\Rightarrow \quad \dfrac{U'}{U} = \dfrac{(1 + \gamma_S \Delta T)}{(1 + \gamma_L \Delta T)}$

Since, $\gamma_S < \gamma_L$

$\Rightarrow \quad U' < U$

$\Rightarrow \quad W_{\text{app}}' > W_{\text{app}}$

Hence, the correct answer is (C).

233. $\dfrac{Mg\ell}{A\Delta\ell} = Y$

$\Rightarrow \quad \Delta\ell_{\text{Mechanical}} = \dfrac{Mg\ell}{AY}$

Since $\Delta\ell_{\text{Thermal}} = \ell\alpha\Delta T = \ell\alpha \times 20$

$\Rightarrow \quad \dfrac{Mg\ell}{AY} = 20\alpha\ell$

$\Rightarrow \quad M = \dfrac{20 \times 10^{-5} \times \pi \times 1 \times 10^{-6} \times 10^{11}}{10}$

$\Rightarrow \quad M = 6.28 \text{ kg}$

Hence, the correct answer is (C).

234. Since $\Delta P = \dfrac{4T}{r}$ and $r_A > r_B$

$\Rightarrow \quad P_A < P_B$

So, the air rushes from B to A

Hence, the correct answer is (A).

235. Speed of efflux is given by

$v = \sqrt{2gh}$

$\Rightarrow \quad \dfrac{dV}{dt} = av = a\sqrt{2gh}$

This is independent of ρ_{liquid}.

Hence, the correct answer is (C).

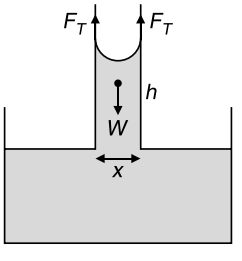
236. Let L be the width of plates (perpendicular to paper inwards), then surface tension force in upward direction equals weight of liquid that rises to a height h

$2F_T = W$

$\Rightarrow \quad 2(TL\cos\theta) = V\rho g$

$\Rightarrow \quad 2TL\cos\theta = (Lxh)\rho g$

$\Rightarrow \quad h = \dfrac{2T\cos\theta}{x\rho g}$



Hence, the correct answer is (B).

237. Let the velocity at A be v_A and that at B be v_B, then by Equation of Continuity, we get

$$\frac{v_B}{v_A}=\frac{30}{15}=2$$

Applying Bernoulli's equation, we get

$$P_A+\frac{1}{2}\rho v_A^2=P_B+\frac{1}{2}\rho v_B^2$$

$$\Rightarrow\quad P_A-P_B=\frac{1}{2}\rho(2v_A)^2-\frac{1}{2}\rho v_A^2=\frac{3}{2}\rho v_A^2$$

$$\Rightarrow\quad 600=\frac{3}{2}(1000)v_A^2$$

$$\Rightarrow\quad v_A=\sqrt{0.4}=0.63\ \text{ms}^{-1}$$

The rate of flow is

$$Q=(30)(0.63)=1800\ \text{cm}^3\text{s}^{-1}$$

Hence, the correct answer is (C).

Multiple Correct Choice Type Questions

1. Speed gradient is $\frac{\Delta v}{\Delta x}=\frac{2\ \text{ms}^{-1}}{1\ \text{m}}=2\ \text{s}^{-1}$

and $F=\eta A\frac{\Delta v}{\Delta x}=(10^{-3})(10)(2)=0.02\ \text{N}$

$\Rightarrow\quad F=0.02\ \text{N}$

Hence, (A) and (C) are correct.

2. $(RD)_{\text{metal}}=\frac{W_{\text{air}}}{W_{\text{air}}-W_{\text{water}}}=\frac{210}{210-180}=7$

Since, loss in weight of the body immersed in liquid is equal to the upthrust, so we get

$$(210-120)=\left(\frac{210}{7}\right)(RD)_{\text{liquid}}$$

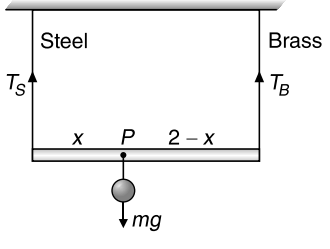
$\Rightarrow\quad (RD)_{\text{liquid}}=3$

Hence, (B) and (C) are correct.

3. For a tube with uniform cross-sectional area, speed will remain same and hence pressure will be the same if both the points lie at the same horizontal level.
However, if the cross-sectional area is different, then too pressure may be same at points lying on different heights (i.e. points not on the same level).
Hence, (B) and (D) are correct.

4. When stress in both wires are equal, then $\frac{T_S}{A_S}=\frac{T_B}{A_B}$

$$\Rightarrow\quad \frac{T_S}{T_B}=\frac{A_S}{A_B}=\frac{10^{-3}}{2\times10^{-3}}=\frac{1}{2}\qquad\ldots(1)$$



For, system to be in equilibrium, conserving moments about P, we get

$$T_S x=T_B(2-x)$$

$$\Rightarrow\quad \frac{T_S}{T_B}=\frac{2-x}{x}\qquad\ldots(2)$$

From Equations (1) and (2), we get

$$x=1.33\ \text{m}$$

So $x=1.33$ m, if stress is equal in both wires.

Since, $\text{Strain}=\frac{\text{Stress}}{Y}$

If strain in both wires is equal, then $\frac{T_S/A_S}{Y_S}=\frac{T_B/A_B}{Y_B}$

$$\Rightarrow\quad \frac{T_S}{T_B}=\frac{A_SY_S}{A_BY_B}=\frac{(10^{-3})(2\times10^{11})}{(2\times10^{-3})(10^{11})}=1\qquad\ldots(3)$$

From Equations (2) and (3), we get

$$x=1\ \text{m}$$

So $x=1$ m, if strain is equal in both wires.
Hence, (A) and (C) are correct.

5. When the second ball having $\rho<\rho_L$ falls from the same height and hits the liquid surface, then we see that the buoyant force acting on the ball is more than its weight due to which the ball experiences a constant retardation up to a certain depth and then starts rising. So, it will return to its original position, i.e. at the same height, in a time $t_2>t_1$.
When $\rho_L=\rho$, then net force on the ball inside the liquid is zero and hence it continues to move with the constant velocity independent of its depth.
The acceleration of the ball inside and outside the liquid is constant, so the motion of the ball cannot be simple harmonic.
Hence, (B), (C) and (D) are correct.

6. According to continuity equation, $Av=\text{constant}$

Since $A_2<A_1$, so $\quad v_2>v_1$

According to Bernoulli's equation, we have

$$P+\frac{1}{2}\rho v^2=\text{constant}\qquad\{\because\ h=\text{constant}\}$$

Since $v_2>v_2$, so $P_2<P_1$

The volume of liquid flowing per second or the mass of liquid flowing per second through both the sections of tube is constant.
Hence, (A), (C) and (D) are correct.

7. The more is the modulus of elasticity, the more is the resistance offered to external deforming forces.
Hence, (B) and (C) are correct.

8. Area of cross-section is $A=\frac{\pi d^2}{4}$

Since, $\Delta L=\frac{FL}{AY}=\frac{FL}{(\pi d^2/4)Y}$

$$\Rightarrow\quad \Delta L\propto\frac{1}{d^2}$$

$$\Rightarrow\quad \frac{\Delta L_B}{\Delta L_A}=\frac{(2d)^2}{(d)^2}$$

$$\Rightarrow\quad \Delta L_B=4\Delta L_A$$

Also, $\text{Strain}=\frac{\Delta L}{L}=\frac{4F}{\pi d^2Y}$

$\Rightarrow \quad \text{Strain} \propto \dfrac{1}{d^2}$

$\Rightarrow \quad (\text{Strain})_B = 4(\text{Strain})_A$

Hence, (B) and (D) are correct.

9. For the siphon to work, the pressure inside the lower face of the tube should be more than the atmospheric pressure for which the condition $h_2 > h_1$ must be obeyed. The height h_1 should be less than the height of corresponding liquid barometer, because otherwise the liquid will not rise to that level.
Hence, (A) and (D) are correct.

10. According to Equation of Continuity, the rate of flow of liquid or volume of liquid flowing per second through both the sections of the tube is the same, i.e.

$$A_1 v_1 = A_2 v_2$$

According to Bernoulli's Theorem, we have

$$\frac{1}{2}\rho\left(v_2^2 - v_1^2\right) = \rho g h$$

$\Rightarrow \quad v_2^2 - v_1^2 = 2gh$

Also, as a consequence of Bernoulli's Theorem, the energy per unit mass of the liquid is the same in both sections of the tube.
Hence, (A), (B) and (D) are correct.

11. The speed of efflux of the liquid coming out of the orifice is

$$v = \sqrt{2gy}$$

The range x of the liquid is

$$R = x = 2\sqrt{y(H-y)}$$

So, as y is increased, x increases, becomes maximum and then decreases.

x is maximum for $y = \dfrac{H}{2}$ and $x_{\max} = H$

Hence, (B), (C) and (D) are correct.

12. Since the elastic potential energy is $U = \dfrac{1}{2}kx^2$.

So U-x graph is a parabola symmetric about U axis.
From the graph, we see that at $x = 0.2$ mm, $U = 0.2$ J

$\Rightarrow \quad 0.2 = \dfrac{1}{2}k\left(2\times10^{-4}\right)^2$

$\Rightarrow \quad k = 10^7 \text{ Nm}^{-1}$

Also, the spring constant of a wire is $k = \dfrac{YA}{L}$

$\Rightarrow \quad \dfrac{A}{L} = \dfrac{k}{Y} = \dfrac{10^7}{2\times10^{11}} = 5\times10^{-5} \qquad \ldots(1)$

Given that the volume of the wire is

$$AL = 200\times10^{-6} \text{ m}^3 \qquad \ldots(2)$$

Solving Equations (1) and (2), we get

$A = 10^{-4}$ m^2 and $L = 2$ m

Hence, (B) and (C) are correct.

13. At the level of interface between ρ_2 and ρ_3, pressures will be equal from both sides. Hence,

$$\rho_1 g h + \rho_3 g\left(\frac{h}{2}\right) = \rho_2 g h$$

$\Rightarrow \quad \rho_3 = 2(\rho_2 - \rho_1)$

From this expression, we conclude that $\rho_2 > \rho_1$.
Hence, (B) and (D) are correct.

14. The fraction of volume of body immersed inside liquid is given by

$$f = \frac{\rho_s}{\rho_l}$$

Since ρ_s and ρ_l are same, so

$$f_1 = f_2 = f_3$$

Also, we note that base area in the third case is uniform, so h_3 is minimum.
Hence, (B), (C) and (D) are correct.

15. The work done in slowly stretching the wire through l is $W = mgl$.
So, loss in gravitational potential energy of the wire is mgl.

Since, $l = \dfrac{mgl}{AY}$ …(1)

and $U_{\text{elastic}} = \dfrac{1}{2}Kl^2 = \dfrac{1}{2}\left(\dfrac{YA}{L}\right)l^2$ …(2)

From Equations (1) and (2), we get

$$U_{\text{elastic}} = \frac{1}{2}mgl$$

$\Rightarrow \quad \Delta H = W - U_{\text{elastic}} = \dfrac{mgl}{2}$

So, half of work done is stored as elastic energy and the remaining is lost as heat.
Hence, (B), (C) and (D) are correct.

16. The reaction force exerted by the liquid on the tube is

$$F = Av^2\rho$$

Hence, (A), (B) and (C) are correct.

17. As we move up from the base of the vessels, the vessel C is narrowing and hence the height of liquid in vessel C is maximum. So, pressure at base of vessel C is maximum. Therefore, force on the base of vessel C is maximum and is given by

$$F_3 = (P_0 + h_3\rho g)A.$$

In equilibrium, net force on all the three vessels equals the weight of liquid, which is same for all the three vessels.
Hence, (B) and (C) are correct.

18. Viscosity of a liquid decreases with increase in temperature, whereas the viscosity of a gas increases with increase in temperature. The surface tension of the liquid decreases with an increase in temperature and for a contact angle $\theta = 90°$ the liquid neither rises nor falls.
Hence, (B) and (C) are correct.

19. Elastic forces are not always conservative. Elastic forces are conservative as long as the loading and deloading curves are coincident even if the curves are not linear.
Hence, elastic forces may be conservative even when Hooke's law is not obeyed if loading and deloading curves are coincident.
Hence, (B) and (D) are correct.

20. Area of steel rod, $A_S = 16\ \text{cm}^2$

Collective area of two brass rods is

$$A_B = 2 \times 10 = 20\ \text{cm}^2$$

Also, $F = 5000$ kgf

If σ_S be the stress in steel and σ_B be the stress in brass, then

$$\begin{pmatrix}\text{Decrease in length}\\ \text{of steel rod}\end{pmatrix} = \begin{pmatrix}\text{Decrease in length}\\ \text{of brass rod}\end{pmatrix}$$

$$\Rightarrow \quad \left(\frac{\sigma_S}{Y_S}\right)L_S = \left(\frac{\sigma_B}{Y_B}\right)L_B$$

$$\Rightarrow \quad \sigma_S = \left(\frac{Y_S}{Y_B}\right)\left(\frac{L_B}{L_S}\right)\sigma_B = \left(\frac{2\times 10^6}{10^6}\right)\left(\frac{20}{30}\right)\sigma_B$$

$$\Rightarrow \quad \sigma_S = \frac{4}{3}\sigma_B \qquad \ldots(1)$$

Since, $F = \sigma_S A_S + \sigma_B A_B$

$$\Rightarrow \quad 5000 = 16\sigma_S + 20\sigma_B \qquad \ldots(2)$$

From Equations (1) and (2), we get

$$\sigma_B = 120.9\ \text{kgfcm}^{-2} \text{ and } \sigma_S = 161.2\ \text{kgfcm}^{-2}$$

Hence, (C) and (D) are correct.

21. Since, $\Delta L = \dfrac{WL}{AY}$

So, ΔL can be increased by making W two times or by making L two times or by making A half.

Hence, (A), (B) and (C) are correct.

22. The pressure at the base of the vessel is $P_{\text{base}} = (2h)\rho g$ and so the force exerted by the liquid at the base of container is $F_{\text{base}} = P_{\text{base}}A_2 = 2h\rho g A_2$.

The weight is in the beaker is

$$W = (A_1 + A_2)h\rho g < 2h\rho g A_2$$

The force on the vertical walls of the vessel cancel. So, for equilibrium, we have

$$F_X + W = F_{\text{base}}$$

$$\Rightarrow \quad F_X = F_{\text{base}} - W = 2hA_2\rho g - (A_1 + A_2)h\rho g$$

$$\Rightarrow \quad F_X = (A_2 - A_1)h\rho g \text{, downwards}$$

Hence the force on the wall at X is downwards.

Hence, (A), (B), (C) and (D) are correct.

23. Since extension $\Delta x = \dfrac{Fl}{AY}$

$$\Rightarrow \quad F = \left(\frac{AY}{L}\right)\Delta x$$

i.e. F versus Δx graph is a straight line of slope $\dfrac{YA}{L}$ passing through the origin.

Since $(\text{Slope})_B > (\text{Slope})_A$

$$\Rightarrow \quad \left(\frac{YA}{L}\right)_B > \left(\frac{YA}{L}\right)_A$$

$$\Rightarrow \quad (A)_B > (A)_A$$

Since the wires are of same material, so

$$Y_A = Y_B$$

Hence, the correct answer is (C).

24. Since $F = 6\pi\eta r v$

So, $F \propto v$ and $F \propto r$ or $F \propto A^{1/2}$ or $F \propto V^{1/3}$

Hence, (B), (C) and (D) are correct.

25. Since the liquid in which the block is immersed applies an upthrust on the block, so an equal force will be exerted (from Newton's Third Law) on the liquid. Hence, A will read less than 2 kg and B more than 5 kg.

Hence, (B) and (C) are correct.

26. Since, loss in weight equals the upthrust, so we have

$$(27 - 18)g = V_{\text{imm}}\rho_{\text{liq}}g = V\rho_w g$$

$$\Rightarrow \quad V = \frac{9}{1000}\ \text{m}^3 = 9000\ \text{cm}^3$$

Let V_C be the volume of the cavity inside the iron casting, then

$$(27000)g = (V - V_C)\rho_{\text{iron}}g$$

$$\Rightarrow \quad V_C = 9000 - \frac{27000}{7.8} = 5538\ \text{cm}^3$$

Hence, (A) and (C) are correct.

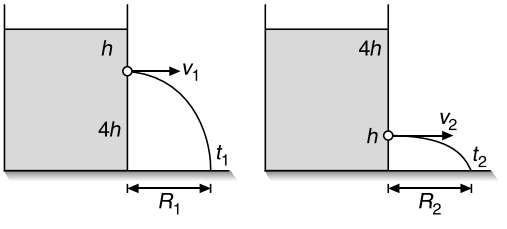
27. In an accelerated container, weight remains unchanged (apparent weight changes) while pressure and upthrust increases.

If m is mass of block and a be its acceleration, then the equation motion of block in new situation is

$$U' - W = ma$$

Hence, (B) and (C) are correct.

28. The situation discussed in the problem is shown in Figure.



Since $v = \sqrt{2gh}$, where h is the depth of the orifice below the free surface of the liquid. So, we have

$$v_1 = \sqrt{2gh} \text{ and } v_2 = \sqrt{2g(4h)} = 2\sqrt{2gh}$$

$$\Rightarrow \quad \frac{v_1}{v_2} = \frac{1}{2}$$

Also, $t = \sqrt{\dfrac{2h'}{g}}$, where h' is the height of the orifice.

$$\Rightarrow \quad \frac{t_1}{t_2} = \sqrt{\frac{h_1'}{h_2'}} = \sqrt{\frac{4h}{h}} = 2$$

The range of the liquid is

$$R = vt = 2\sqrt{h \times h'}$$

$$\Rightarrow \quad \frac{R_1}{R_2} = \sqrt{\frac{(h)(4h)}{(4h)(h)}} = 1$$

Hence, (A), (C) and (D) are correct.

29. Since the liquid will be in equilibrium, so the side walls of the conical flask will exert a downward force on the liquid due to which net force on the base of the conical flask will increase and hence $W_1 > W_2$.
If F be the force exerted by the liquid on the walls of the flask, then for the flask we have
$F + W_2 = W_1$
$\Rightarrow \quad F = W_1 - W_2$
Hence, (B) and (D) are correct.

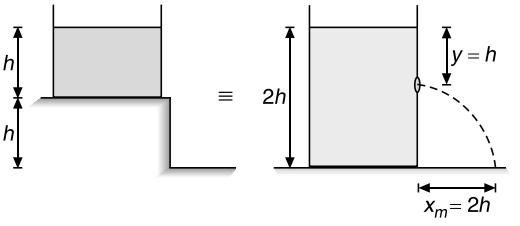
30. When $F = \frac{mg}{3}$, then tension in the wire Q and P respectively are
$$T_Q = \frac{mg}{3} \text{ and } T_P = mg + \frac{mg}{3} = \frac{4mg}{3}$$
Since stress is $\sigma = \frac{T}{\pi r^2}$
$$\Rightarrow \quad \frac{\sigma_P}{\sigma_Q} = \left(\frac{T_P}{T_Q}\right)\left(\frac{r_Q}{r_P}\right)^2$$
When $r_P = r_Q$, then $\sigma_P = 4\sigma_Q$ and hence P breaks.
When $r_P < 2r_Q$, then $\sigma_P > \sigma_Q$ and hence P breaks.
When $r_P = 2r_Q$, then $\sigma_A = \sigma_B$ and hence either P or Q may break.
Hence, (A), (B) and (C) are correct.

31. The level of water in beaker will fall if initially the impurity pieces plus ice system was floating and on the melting of ice the impurities sink.
The level of water in beaker will remain unchanged if initially the impurities plus ice system was floating and on melting the impurities also float.
Hence, (C) and (D) are correct.

32. In air, $a_1 = g$, downwards
In liquid, $a_2 = \frac{\text{upthrust} - \text{weight}}{\text{mass}} = \frac{U - W}{m}$
$$\Rightarrow \quad a_2 = \frac{(V)(2\rho)(g) - (V)(\rho)(g)}{(V\rho)}$$
$\Rightarrow \quad a_2 = g$, upwards
So, $a_1 \neq a_2$ because the balls have accelerations of same value but different directions.
On entering the non-viscous liquid, the ball will be retarded to zero velocity and then will be accelerated upwards towards the surface. It comes out of the surface with the same acceleration and will rise to the same height from which it had started falling and in this process the motion becomes periodic but not harmonic (because acceleration is constant).
Hence, (A), (C) and (D) are correct.

33. Work done against the elastic force is
$$W = \frac{1}{2}k(\Delta l)^2 = \frac{1}{2}kl^2 = \frac{1}{2}\left(\frac{YA}{L}\right)l^2 = \frac{YAl^2}{2L}$$
This work done against the elastic force equals the elastic potential energy stored in the wire, so
$$U_{\text{elastic}} = \frac{YAl^2}{2L}$$
Hence, (A) and (C) are correct.

34. This is similar to the case as if a tank is filled with a liquid up to a height of $2h$.



In that case, range becomes maximum when hole is punched at the middle of the tank i.e. at $y = h$ and the maximum range is equal to the level of liquid in the tank.
Hence, (A) and (C) are correct.

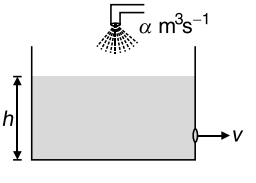
35. Initial rate of flow of volume through holes 1 and 2 respectively is
$$Q_1 = \frac{dV_1}{dt} = a_1 v_1 = \pi(2R)^2\sqrt{2gh} \quad \ldots(1)$$
$$Q_2 = \frac{dV_2}{dt} = a_2 v_2 = \pi R^2\sqrt{2g(16h)} \quad \ldots(2)$$
From Equations (1) and (2), we can see that
$Q_1 = Q_2$
After some time, v_1 and v_2 both will decrease, but decrease in the value of v_1 will be more rapid compared to v_2. So,
$Q_1 < Q_2$
Hence, (B) and (D) are correct.

36. Since $v = \sqrt{2gh}$, where h is the depth of the orifice from the free surface. Since $h_2 = \frac{h_1}{4}$, so $v_2 = \frac{v_1}{2}$
$\Rightarrow \quad v_1 = 2v_2$
Further $Q = Av$
Given that $A_1 = 2A_2$ and $v_1 = 2v_2$
$\Rightarrow \quad A_1 v_1 = A_2 v_2$
$\Rightarrow \quad Q_1 = Q_2$
Hence, (B) and (C) are correct.

37. If ice floating on water melts, then the level of liquid (if the liquid is water) will remain the same. However, if the liquid is denser than water then the level of liquid will rise on melting of ice and vice versa.
Hence, (A), (C) and (D) are correct.

38. Since, pressure increases with depth in vertical direction and in horizontal direction it increases in the direction opposite to acceleration. So, pressure is maximum at point D and minimum at B.
Hence, (B) and (D) are correct.

39. Level of water in the vessel will be maximum when the rate of inflow of water in the vessel equals the rate of outflow of water from the vessel
$\Rightarrow \quad \alpha = av = a\sqrt{2gh}$
$$\Rightarrow \quad h = \frac{\alpha^2}{2ga^2}$$
So, $h \propto \alpha^2$ and $h \propto a^{-2}$
Hence, (B) and (C) are correct.



40. The liquid is incompressible and the area of cross-section of the tube is same at both the points, so

$$v_A = v_B$$

However, the pressure at the point B will be more than the pressure at the point A.

Hence, (B) and (C) are correct.

41. The siphon will work when the atmospheric pressure helps the liquid to flow out of the vessel.

Hence, (B) and (C) are correct.

42. Since viscous force equals the weight, so

$$6\pi\eta r v = \frac{4}{3}\pi r^3 \rho g \quad \ldots(1)$$

Substituting the values, we can find r. Also

$$v \propto r^2$$

Hence, (B) and (C) are correct.

43. Since the terminal velocity is

$$v_T = \frac{2}{9}\left(\frac{r^2}{\eta}\right)(\rho_{\text{oil}} - \rho_{\text{air}})g \approx \frac{2}{9}\left(\frac{r^2}{\eta}\right)\rho_{\text{oil}}$$

$$\Rightarrow \quad 5\times10^{-4} = \frac{2}{9}\left(\frac{r^2}{1.8\times10^{-5}}\right)(900)$$

$$\Rightarrow \quad r = 6.7\times10^{-6} \text{ m}$$

When r is halved, terminal velocity will become one fourth the initial value.

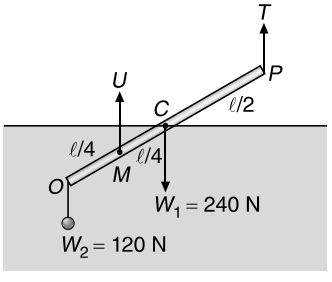
Hence, (A) and (C) are correct.

44. For translational equilibrium of the rod, we have

$$T + U = W_1 + W_2 = 360 \text{ N} \quad \ldots(1)$$

Since, $U = \left(\frac{V}{2}\right)\rho_w g = \left(\frac{V}{2}\right)(10^3)(10)$

$$\Rightarrow \quad U = 0.5\times10^4 V \quad \ldots(2)$$



For rotational equilibrium of the rod, we have

$$\Sigma\tau_{\text{about } M} = 0$$

$$\Rightarrow \quad 120\left(\frac{l}{4}\right) + T\left(\frac{3l}{4}\right) = 240\left(\frac{l}{4}\right)$$

$$\Rightarrow \quad T = \frac{240-120}{3} = 40 \text{ N}$$

So, from equations (1) and (2), we get

$$V = 6.4\times10^{-2} \text{ m}^3$$

The point of application of the buoyant force is passing through the centre of the immersed part of the rod.

Hence, (A), (B) and (C) are correct.

45. $U = V_{\text{immersed}}\rho_{\text{liquid}}g$

Since, g is different on moon and on the earth.

Hence only (C) is a correct statement and all others incorrect.

Hence, (A), (B) and (D) are correct.

46. Radius of meniscus is $r = \frac{R}{\cos\theta}$

Due to spherical surface $\Delta P = \frac{2\sigma}{r}$

$$\Rightarrow \quad \Delta P = \frac{2\sigma\cos\theta}{R}$$

$$\Rightarrow \quad P_0 - P_A = \Delta P = \frac{2\sigma\cos\theta}{R}$$

$$\Rightarrow \quad P_A = P_0 - \frac{2\sigma\cos\theta}{R}$$

Since $P_B = P_0$, so $P_A = P_B - \frac{2\sigma\cos\theta}{R}$

Also, $P_A = P_B - h\rho g = P_0 - h\rho g$

Hence, (A), (B) and (C) are correct.

Reasoning Based Questions

1. When a rain drop falls in air (viscous medium) then after falling through same height, the viscous drag balances the weight of the drop and then through rest height its velocity is constant or it attains a terminal velocity.

Hence, the correct answer is (A).

2. From the formula, excess pressure = $\frac{2T}{r}$ Here, T is surface tension, r is the radius of liquid drop. Hence, excess pressure is inversely proportional to radius and hence, the surface area. Therefore, the excess pressure inside a smaller drop is large as compared to the larger drop due to which smaller drop of liquid resists deforming forces better than a larger drop.

Hence, the correct answer is (C).

3. The barometer has to measure the atmospheric pressure which has a constant value, so when it accelerates upwards, then $g_{\text{eff}}(=g+a)$ increases and hence h decreases. Also, due to the increase in value of g, upthrust U also increases. So, both are true but Statement-2 is not the reason to Statement-1.

Hence, the correct answer is (B).

4. Usually the air will not strike the wings of the aeroplane with large velocity, so to get the lift, the aeroplane runs for some distances on the runway before taking off. Due to the special shape of wings, the velocity of the layers of air above the wings increases and hence pressure decreases. Due to this aeroplane gets an uplift.

Hence, the correct answer is (A).

5. As we go deeper in the lake, the density of water increases due to pressure.

Hence, the correct answer is (A).

6. Since, the particle of dust is like spheres of very small radii and when it acquires the terminal velocity they begin to settle down. As from the definition the terminal velocity of dust particles is directly proportional to the square of its radii. Thus, the terminal velocity of dust particle is very small and so, they settle down in a closed room after sometime.

Hence, the correct answer is (C).

7. Area of cross-section is different. So, heights are different. In pressure height is more important.
Hence, the correct answer is (D).

8. A body much displace as much amount of liquid greater than the actual weight of the body to float in it. In case of floating, no net downward force acts on the body.
Hence, the correct answer is (B).

9. When the ball enters the liquid it may accelerate or retard depending on the density of ball and the density of liquid, because if $\rho_{ball} > \rho_{liquid}$, then $W > U$ and the ball will accelerate and if $\rho_{ball} < \rho_{liquid}$, then $W < U$ and the ball will decelerate.
Hence, the correct answer is (A).

10. Bulk modulus of elasticity measures how good the body is to regain its original volume on being compressed. Therefore, it represents incompressibility of the material.

Bulk modulus $B = -\dfrac{\Delta P}{\Delta V / V} = -\dfrac{(\Delta P)V}{\Delta V}$

Where ΔP = change in pressure, ΔV = change in volume
Here, negative sign implies that when the pressure increases, volume decreases and vice-versa.
Hence, the correct answer is (B).

11. Speed will also depend on h.
Hence, the correct answer is (D).

12. With increase in depth the pressure increases from the formula $P \propto h$. Therefore the force perpendicular to the walls of dam increases. Hence, the dam must have greater strength at base than at top. Due to this dam are made thicker at the base than at the top.
Hence, the correct answer is (A).

13. If length is doubled, Δl will also become double but Y will remain the same as it is the property of the material, i.e. it changes only when the material changes.
Hence, the correct answer is (D).

14. A hydraulic lift is an arrangement used to multiply the force. When a force is applied hydraulic pressure is transmitted in all directions. This is the working principle of Pascal's laws.
Hence, the correct answer is (B).

15. When a needle is placed carefully on the surface of water, if floats on the surface of water due to surface tension of water which does not allow the needle to sink. On the other hand, if the ball is placed on the surface of water, the surface tension of water is not sufficient to keep it floating so it sinks down.
Hence, the correct answer is (C).

16. Excess pressure is inversely proportional to radius of the bubble.
Hence, the correct answer is (A).

17. In a small drop, the force due to surface tension is very large as compared to its weight and hence it is spherical shape. A big drop becomes oval in shape due to its large weight. The surface tension of liquid decreases with increase of temperature.
Hence, the correct answer is (B).

18. Bulk modulus is related to volume change and volume change is possible in all three states, however Young's modulus is related to length change, which is possible only in solids.
Hence, the correct answer is (B).

19. From definition, elasticity is the measure of tendency of the body to regain its original configuration. Since, steel is deformed less than rubber hence, steel is more elastic.
Hence, the correct answer is (A).

20. Force of buoyancy,

$$U = V\rho_{air}g$$

Since, ρ_{air} is negligible. Hence, U is negligible
Hence, the correct answer is (C).

21. Stress is the internal force per unit area of body. If the same force is applied to the rubber and steel, then strain in rubber is more. It means the steel is more elastic than rubber.
Hence, the correct answer is (C).

22. $p_1 = p_0 + \rho g h$

$p_2 = p_0 + \rho g(2h) \neq 2p_1$

Hence, the correct answer is (D).

23. Breaking load depends on the area of cross-section and is independent of length of rod. Hence,

$$\text{Breaking load} = \begin{pmatrix}\text{breaking}\\ \text{stress}\end{pmatrix} \times \begin{pmatrix}\text{cross-sectional}\\ \text{area}\end{pmatrix}$$

Hence, the correct answer is (C).

24. Since there is no atmosphere on moon, so atmospheric pressure is zero. Hence barometer height is zero.
Hence, the correct answer is (D).

25. On applying load on lead and rubber wires of same cross-sectional area, the strain produced in lead is much less than rubber wires. Now from the relation, $Y =$ Young's modulus

$$\Rightarrow \quad Y = \frac{\text{Stress}}{\text{Strain}}$$

The Young's modulus of elasticity is greater for lead wires. Hence, lead is elastic than rubber.
Hence, the correct answer is (A).

27. According to the ascent formula $h = \dfrac{2T\cos\theta}{r\rho g}$

$$\Rightarrow \quad h \propto \frac{1}{r}$$

From the relation if radius is less then, height h to which the liquid will rise is greater.
Hence, the correct answer is (A).

28. Pressure at P is less. As liquid is flowing at P while liquid is at rest at Q.
Hence, the correct answer is (D).

29. As the temperature decreases in winter. So, on decreasing temperature, the coefficient of viscosity of engine oil and the lubricants increases due to which the machine parts get jammed in winter.
Hence, the correct answer is (A).

Linked Comprehension Type Questions

1. Mass of water = (Volume)(density)

$\Rightarrow \quad m_o = (AH)\rho$

$\Rightarrow \quad H = \dfrac{m_o}{A\rho}$ …(1)

Velocity of efflux

$$V = \sqrt{2gH} = \sqrt{2g\frac{m_o}{A\rho}} = \sqrt{\frac{2m_o g}{A\rho}}$$

Hence, the correct answer is (D).

2. Thrust force on the container due to draining out of liquid from the bottom is given by,

$$F = \begin{pmatrix}\text{density of}\\ \text{liquid}\end{pmatrix}\begin{pmatrix}\text{area of}\\ \text{hole}\end{pmatrix}\begin{pmatrix}\text{velocity}\\ \text{of efflux}\end{pmatrix}^2$$

$\Rightarrow \quad F = \rho a V^2$

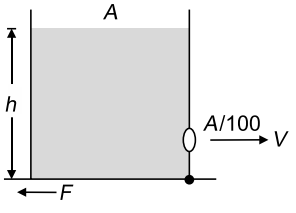
$\Rightarrow \quad F = \rho\left(\dfrac{A}{100}\right)V^2 = \rho\left(\dfrac{A}{100}\right)\left(\dfrac{2m_o g}{A\rho}\right)$

$\Rightarrow \quad F = \dfrac{m_o g}{50}$

So, acceleration of the container is

$$a = \frac{F}{m_o} = \frac{g}{50}$$

$\Rightarrow \quad a = \dfrac{g}{50}$

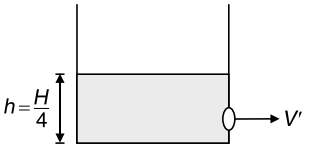


Hence, the correct answer is (B).

3. Velocity of efflux when 75% liquid has been drained out i.e., height of liquid,

$$h = \frac{H}{4} = \frac{m_o}{4A\rho}$$

$\Rightarrow \quad V' = \sqrt{2gh} = \sqrt{2g\left(\dfrac{m_o}{4A\rho}\right)}$



$\Rightarrow \quad V' = \sqrt{\dfrac{m_o g}{2A\rho}}$

Hence, the correct answer is (B).

4. Unless the upthrust becomes equal to weight of the cube, it will be in contact with vessel and normal reaction between the two will be

$N = mg - U$

As liquid starts collecting in the vessel at a constant rate, U increases linearly with time and hence N decreases linearly with time. Once, the upthrust equals the weight of the cube, then $N = 0$.

Hence, the correct answer is (A).

5. When contact between the cube and the vessel breaks, then

W = Upthrust

$\Rightarrow \quad (10\times10\times10)\times0.5\times g = 10\times10\times h\times1\times g$

$\Rightarrow \quad h = 5$ cm

Now, $Q\times t = (20\times20 - 10\times10)\times h$

$\Rightarrow \quad t = \dfrac{300\times5}{50} = 30$ s

Hence, the correct answer is (B).

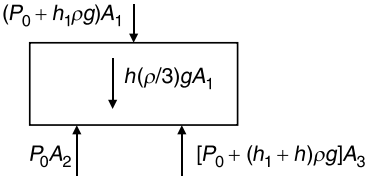
6. Let A_1 = Area of cross-section of cylinder $= 4\pi r^2$

A_2 = Area of base of cylinder in air $= \pi r^2$

and A_3 = Area of base of cylinder in water

$\Rightarrow \quad A_3 = A_1 - A_2 = 3\pi r^2$

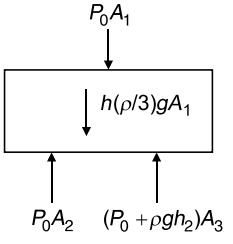
Drawing free body diagram of cylinder



Equating the net downward forces and net upward forces, we get $h_1 = \dfrac{5}{3}h$

Hence, the correct answer is (C).

7. Again, equating the forces, we get



$h_2 = \dfrac{4h}{9}$

Hence, the correct answer is (A).

8. For $h_2 < \dfrac{4h}{9}$, buoyant force will further decrease.

Hence the cylinder remains at its original position.

Hence, the correct answer is (A).

9. For the orifice (i.e. hole) at a depth h, we have

$$\Delta P_1 = \frac{1}{2}\rho v_1^2$$

$\Rightarrow \quad h\rho g = \dfrac{1}{2}\rho v_1^2$

$\Rightarrow \quad v_1 = \sqrt{2gh}$

Similarly, for the orifice at a depth $2h$, we have

$$\Delta P_2 = \frac{1}{2}(2\rho)v_2^2$$

$\Rightarrow \quad (\rho gh + 2\rho gh) = \rho v_2^2$

$\Rightarrow \quad v_2 = \sqrt{3gh}$

$\Rightarrow \quad \dfrac{v_1}{v_2} = \sqrt{\dfrac{2}{3}}$

Hence, the correct answer is (C).

10. For the orifice at a depth h, the speed v_1 will not change, because speed of efflux does not depend on the density of liquid below the orifice. When the density of lower liquid is increased and made $2n\rho$, where $n > 1$, then for orifice at a depth $2h$, we have

$$\Delta P_2 = \frac{1}{2}(2n\rho)v_2^2$$

$\Rightarrow \quad \rho gh + 2(n\rho)gh = n\rho v_2^2$

$\Rightarrow \quad v_2 = \sqrt{\dfrac{gh + 2ngh}{n}} = \sqrt{2gh + \dfrac{gh}{n}}$

Since, $n > 1$

$\Rightarrow \quad v_2 < \sqrt{3gh}$

Hence, the correct answer is (B).

11. Considering vertical equilibrium of cylinder:

$$\begin{pmatrix}\text{Weight}\\ \text{of}\\ \text{cylinder}\end{pmatrix} = \begin{pmatrix}\text{upthrust due}\\ \text{to upper}\\ \text{liquid}\end{pmatrix} + \begin{pmatrix}\text{upthrust due}\\ \text{to lower}\\ \text{liquid}\end{pmatrix}$$

Note that h_1 and $h_2 \neq \dfrac{H}{2}$

$\Rightarrow \quad \left(\dfrac{A}{5}\right)(L)D.g = \left(\dfrac{A}{5}\right)\left(\dfrac{3L}{4}\right)(d)g + \left(\dfrac{A}{5}\right)\left(\dfrac{L}{4}\right)(2d)(g)$

$\Rightarrow \quad D = \left(\dfrac{3}{4}\right)d + \left(\dfrac{1}{4}\right)(2d)$

$\Rightarrow \quad D = \dfrac{5}{4}d$

Hence, the correct answer is (C).

12. Considering vertical equilibrium of two liquids and the cylinder.

$$(P - P_o)A = \begin{pmatrix}\text{weight of}\\ \text{two liquids}\end{pmatrix} + \begin{pmatrix}\text{weight of}\\ \text{cylinder}\end{pmatrix}$$

$$\Rightarrow \quad P = P_o + \frac{\begin{pmatrix}\text{weight of}\\ \text{two liquids}\end{pmatrix} + \begin{pmatrix}\text{weight of}\\ \text{cylinder}\end{pmatrix}}{A} \qquad \ldots(1)$$

Now, weight of cylinder

$$W = \left(\frac{A}{5}\right)(L)(D)(g) = \left(\frac{A}{5}Lg\right)\left(\frac{5}{4}d\right) = \frac{ALdg}{4}$$

Weight of upper liquid $= \left(\dfrac{H}{2}Adg\right)$ and

Weight of lower liquid $= \dfrac{H}{2}A(2d)g = HAgd$

Total weight of two liquids $= \dfrac{3}{2}HAgd$

From equation (1) pressure at the bottom of the container will be

$$P = P_o + \frac{\left(\dfrac{3}{2}\right)HAgd + \dfrac{ALgd}{4}}{A}$$

$\Rightarrow \quad P = P_o + \dfrac{gd}{4}(L + 6H)$

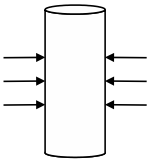
Hence, the correct answer is (B).

13. Young's modulus is property of material of the wire, so the ratio of the Young's moduli is 1
Hence, the correct answer is (C).

14. On crossing the yield region, the material will experience the breaking stress and further elongation causes reduction in stress and breaking of the wire.
Hence, the correct answer is (C).

15. Since the stress-strain curve flattens on crossing the elastic region, so $x > y$
Hence, the correct answer is (D).

16. Liquid A is applying the hydrostatic force on cylinder from all the sides. So, net force is zero.

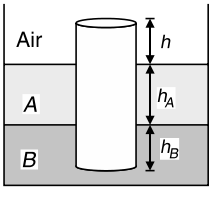


Hence, the correct answer is (A).

17. In equilibrium:
Weight of cylinder = Net upthrust on the cylinder.
Let s be the area of cross-section of the cylinder, then

$$weight = (s)(h + h_A + h_B)\rho_{\text{cylinder}}g$$

and upthrust on the cylinder = upthrust due to liquid A + upthrust due to liquid $B = sh_A\rho_A g + sh_B\rho_B g$



Equating these two

$$s(h + h_A + h_B)\rho_{\text{cylinder}}g = sh_{APA}g = h_A\rho_A + h_B\rho_B$$

$\Rightarrow \quad (h + h_A + h_B)\rho_{\text{cylinder}} = h_A\rho_A + h_B\rho_B$

Substituting,

$h_A = 1.02$ cm , $h_B = 0.8$ cm and $\rho_A = 0.7$ gcm^{-3}

$\rho_B = 1.02$ gcm^{-3} and $\rho_{\text{cylinder}} = 0.8$ gcm^{-3}

In the above equation, we get

$h = 0.25$ cm

Hence, the correct answer is (C).

18. Net upward force = Extra Upthrust $= (sh)\rho_B g$

$\Rightarrow$ Net acceleration $a = \dfrac{\text{Force}}{\text{mass of cylinder}}$

$\Rightarrow \quad a = \dfrac{sh\rho_B g}{s(h + h_A + h_B)\rho_{\text{cylinder}}}$

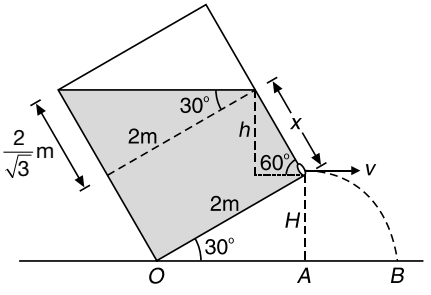
$\Rightarrow \quad a = \dfrac{h\rho_B g}{(h + h_A + h_B)\rho_{\text{cylinder}}}$

Substituting the values of h, h_A, h_B, ρ_B and ρ_{cylinder}, we get $a = \dfrac{g}{6}$ (upwards)

Hence, the correct answer is (B).

19. When the vessel is tilted such that no liquid spills, then the volume of liquid should remain unchanged in both the configurations of the vessel. Hence, we have

$V_1 = V_2$



From the figures, we observe that

$$(2)(2)(2)=(2)\left[\frac{1}{2}\left(x+\left(x+\frac{2}{\sqrt{3}}\right)\right)(2)\right]$$

$\Rightarrow \quad x \approx 1.42$ m

Since, $h = x\sin(60°) = 1.23$ m

$\Rightarrow \quad v=\sqrt{2gh}=\sqrt{2\times10\times1.23}=4.96\approx 5\ \text{ms}^{-1}$

Hence, the correct answer is (C).

20. Since, $H = 2\sin 30° = 1$ m

$\Rightarrow \quad t=\sqrt{\frac{2H}{g}}=\frac{1}{\sqrt{5}}$ s

Hence, the correct answer is (C).

21. Since $OA = 2\cos 30° = \sqrt{3}$ m

and $AB = vt = (5)\left(\frac{1}{\sqrt{5}}\right)=\sqrt{5}$ m

$\Rightarrow \quad OB = OA + AB = \left(\sqrt{3}+\sqrt{5}\right)$ m

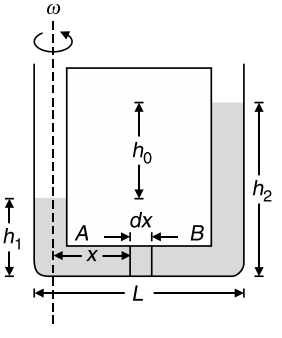
Hence, the correct answer is (C).

22. Force on small element of length dx

$$dF = dmx\omega^2$$

where, dm is the mass of the small element given by

$$dm = (Adx\rho)x\omega^2$$



Total force, $F=\int_0^L A\rho\omega^2 x\,dx = A\rho\omega^2\int_0^L x\,dx$

$$F = A\rho\omega^2\left|\frac{x^2}{2}\right|_0^2=\frac{A\rho\omega^2L^2}{2}$$

Hence, the correct answer is (C).

23. From the left side, point B receives atmospheric pressure, pressure from column of height h_1 and pressure from centrifugal force.

Hence, the correct answer is (D).

24. $P_0+h_2\rho g = P_0+h_1\rho g+\frac{\rho\omega^2L^2}{2}$

$\Rightarrow \quad (h_2-h_1)\rho g=\frac{\rho\omega^2L^2}{2}$

$\Rightarrow \quad h_0=\frac{\omega^2L^2}{2g}$

Hence, the correct answer is (B).

25. $v=\sqrt{2gh}=\sqrt{2\times10\times(4+1)}=10\ \text{ms}^{-1}$

Hence, the correct answer is (C).

26. Applying Bernoulli's theorem at A and B, we get

$$P_0+0+0=P_B+\frac{1}{2}\rho v^2+\rho g(1.5)$$

$\Rightarrow \quad P_B=1.01\times10^5-\frac{1}{2}(900)(10)^2-(900)(10)(1.5)$

$\Rightarrow \quad P_B=4.25\times10^4\ \text{Nm}^{-2}$

Hence, the correct answer is (C).

27. Applying Bernoulli's theorem at B and C

$$P_B+\frac{1}{2}\rho g(2.5)=P_C+\frac{1}{2}\rho v^2$$

$\Rightarrow \quad P_C=4.25\times10^4+(900)(10)(2.5)$

$\Rightarrow \quad P_C=6.5\times10^4\ \text{Nm}^{-2}$

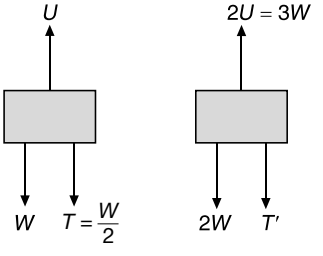
Hence, the correct answer is (B).

28. When the string is cut, tension becomes zero i.e., net upward force on the block becomes $\frac{W}{2}$, i.e. net upward acceleration experienced by the block $\frac{g}{2}$ or $5\ \text{ms}^{-2}$.

$\Rightarrow \quad t=\sqrt{\frac{2s}{a}}=\sqrt{\frac{2\times2}{5}}=\frac{2}{\sqrt{5}}$ s

Hence, the correct answer is (D).

29. If weight is doubled, then upthrust will also be doubled because weight can be increased only by doubling the volume of the block.



Before, $U = W+\frac{W}{2}=\frac{3W}{2}$

After, $2U = 3W = 2W + T'$

$\Rightarrow \quad T' = W = 2T$

When string is cut in the second case, then net upward acceleration will be

$$a'=\frac{F_{\text{net}}}{m'}=\frac{3W-2W}{(2W/g)}=\frac{g}{2}=a$$

So, time taken by block to reach the liquid surface will be same in both the cases.

Hence, $x = 2$ and $y = 1$

$\Rightarrow \quad \frac{x}{y} = 2$

Hence, the correct answer is (B).

30. Given: $A_1 = 4\times10^{-3}\ \text{m}^2$, $A_2 = 8\times10^{-3}\ \text{m}^2$,

$h_1 = 2\ \text{m}$, $h_2 = 5\ \text{m}$

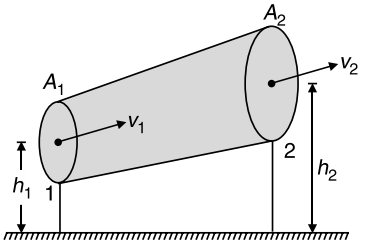
$v_1 = 1\ \text{ms}^{-1}$ and $\rho = 10^3\ \text{kgm}^{-3}$

From continuity equation, we have

$A_1v_1 = A_2v_2$

$\Rightarrow \quad v_2 = \left(\frac{A_1}{A_2}\right)v_1$

$\Rightarrow \quad v_2 = \left(\frac{4\times10^{-3}}{8\times10^{-3}}\right)\left(1\ \text{ms}^{-1}\right)$



$\Rightarrow \quad v_2 = \frac{1}{2}\ \text{ms}^{-1} = 0.5\ \text{ms}^{-1}$

Applying Bernoulli's equation at section 1 and 2

Hence, the correct answer is (A).

31. $P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$ ...(1)

$\Rightarrow \quad P_1 - P_2 = \rho g\left(h_2 - h_1\right) + \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$ ...(2)

Work done per unit volume by the pressure as the fluid flows from P to Q

$W_1 = P_1 - P_2 = \rho g\left(h_2 - h_1\right) + \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$

{from equation (1)}

$\Rightarrow \quad W_1 = \left\{\left(10^3\right)(9.8)(5-2) + \frac{1}{2}\left(10^3\right)\left(\frac{1}{4} - 1\right)\right\}\text{Jm}^{-3}$

$\Rightarrow \quad W_1 = (29400 - 375)\ \text{Jm}^{-3} = 29025\ \text{Jm}^{-3}$

Hence, the correct answer is (C).

32. Work done per unit volume by the gravity as the fluid flows from P to Q

$W_2 = \rho g\left(h_2 - h_1\right) = \left[\left(10^3\right)(9.8)(5-2)\right]\text{Jm}^{-3}$

$\Rightarrow \quad W_2 = 29400\ \text{Jm}^{-3}$

Hence, the correct answer is (B).

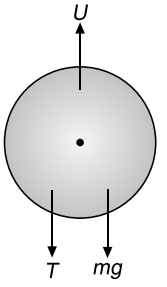
33. Volume of liquid displaced equals the volume of the ball, i.e. V, so mass of liquid that overflows is ρV.

Hence, the correct answer is (A).

34. When the weight mg is added, the weight $V\rho g$ of liquid is overflowed.

Hence, the correct answer is (C).

35. The free body diagram of the forces acting on the sphere is shown in Figure.



For, equilibrium, we have

$U = T + mg$

$\Rightarrow \quad V\rho g = T + mg$

$\Rightarrow \quad T = (V\rho - m)g$

Hence, the correct answer is (A).

36. In this case too, when the weight mg is added, the weight $V\rho g$ of liquid is overflowed.

Hence, the correct answer is (C).

37. Applying Bernoulli's theorem at 1 and 2

$$P_o + dg\left(\frac{H}{2}\right) + 2dg\left(\frac{H}{2} - h\right) = P_o + \frac{1}{2}(2d)v^2$$

Here, v is velocity of efflux at 2.

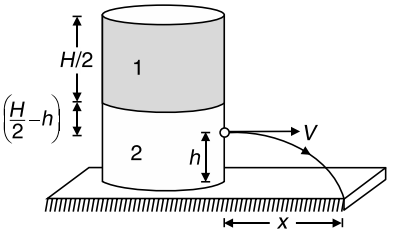
Solving this, we get

$$v = \sqrt{(3H - 4h)\frac{g}{2}} = \sqrt{\left(\frac{3H}{2} - 2h\right)g}$$

Hence, the correct answer is (A).

38. Time taken to reach the liquid to the bottom will be

$t = \sqrt{\frac{2h}{g}}$



Horizontal distance x travelled by the liquid is

$\Rightarrow \quad x = vt = \sqrt{(3H - 4h)\frac{g}{2}}\left(\sqrt{\frac{2h}{g}}\right)$

$\Rightarrow \quad x = \sqrt{h(3H - 4h)}$

Hence, the correct answer is (B).

39. For x to be maximum

$\frac{dx}{dh} = 0$

$\Rightarrow \quad \frac{1}{2\sqrt{h(3H - 4h)}}(3H - 8h) = 0$

$\Rightarrow \quad h = \frac{3H}{8}$

Therefore, x will be maximum at $h = \frac{3H}{8}$.

Hence, the correct answer is (B).

40. The maximum value of x will be

$$x_m = \sqrt{\left(\frac{3H}{8}\right)\left[3H - 4\left(\frac{3H}{8}\right)\right]}$$

$$x_m = \frac{3}{4}H$$

Hence, the correct answer is (C).

41. Initially, $\Delta L = \frac{FL}{AY}$

$$\Rightarrow \quad \Delta L = \frac{10 \times 1}{10^{-3} \times 2 \times 10^5}$$

$$\Rightarrow \quad \Delta L = 0.05 \text{ m} = 5 \text{ cm}$$

Hence, the correct answer is (B).

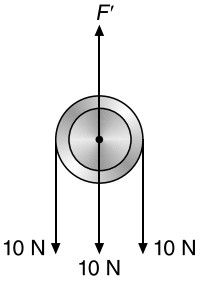
42. Force constant of string is

$$k = \frac{\text{Force}}{\text{Elongation}} = \frac{F}{\frac{FL}{AY}}$$

$$\Rightarrow \quad k = \frac{YA}{L} = \frac{1 \times 10^5 \times 10^{-3}}{1} = 200 \text{ Nm}^{-1}$$

Initial elastic potential energy of string

$$U_i = \frac{1}{2}k(0.05)^2 = \frac{1}{2}(200)\left(25 \times 10^{-4}\right) = 25 \times 10^{-2} \text{ J}$$



Let after force $F = 10$ N is applied extra elongation is x, then

$$0.25 + 30x = \frac{1}{2}(200)\left(x^2 + 25 \times 10^{-4} + 0.1x\right)$$

$$0.25 + 30x = 100x^2 + 0.25 + 10x$$

$$\Rightarrow \quad 100x^2 = 20x$$

$$\Rightarrow \quad x = \frac{20}{100} = \frac{1}{5} \text{ m} = 0.2 \text{ m}$$

$$\Rightarrow \quad x = 20 \text{ cm}$$

$$\Rightarrow \quad x_{\max} = 20 + 5 = 25 \text{ cm}$$

Hence, the correct answer is (C).

43. When the displacement of the pulley is 25 cm, the string gets loosened on both the sides. So, point A moves down by 50 cm.

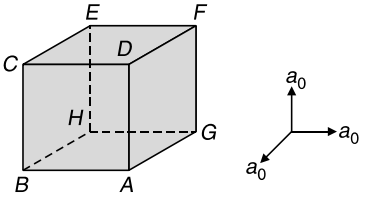
Hence, the correct answer is (D).

Matrix Match/Column Match Type Questions

1. A → (p, t)
B → (q)
C → (r)
D → (s)

Since $\vec{a} = a_0\hat{i} + a_0\hat{j} + a_0\hat{k}$, so

$$a_x = a_0, \; a_y = a_0, \; a_z = a_0$$



Assuming the cube to be of side length ℓ, we observe that in the gravity free space, we get

$$P_A - P_D = \ell\rho a_y = \ell\rho a_0$$

$$P_G - P_A = \ell\rho a_z = \ell\rho a_0$$

Similarly,

$$P_B - P_A = \ell\rho a_0, \; P_C - P_D = \ell\rho a_0$$

$$P_F - P_D = \ell\rho a_0, \; P_H - P_E = \ell\rho a_0$$

$$P_E - P_C = \ell\rho a_0, \; P_E - P_F = \ell\rho a_0$$

$$P_H - P_B = \ell\rho a_0, \; P_H - P_G = \ell\rho a_0$$

$$P_G - P_F = \ell\rho a_0$$

From these equations, we can also, see that

$$P_A = P_F, \; P_E = P_B$$

2. A → (t, r)
B → (p, r)
C → (q, r)
D → (s)

From, $Q = A_1v_1 = A_2v_2$ or $v \propto \frac{1}{A}$

At a point where area of cross-section is less, volume of liquid flowing per second, i.e. the rate of flow of liquid is same but speed is more and vice-versa. Also, when area is less, then v is more and consequently due to Bernoulli's Theorem pressure is less and hence $P_{\text{narrow}} < P_{\text{broad}}$, so $\Delta P < 0$. The rate of flow of liquid and the speed are also positive.

3. A → (q)
B → (r)
C → (s)
D → (p)

Stoke's law gives viscous force acting on a spherical body falling through a liquid column of infinite length.

Equation of continuity i.e. $a_1v_1 = a_2v_2$ is based on Law of Conservation of Mass

Bernoulli's Theorem is based on Law of Conservation of Energy

Surface tension is the phenomenon in which the free surface of the liquid behaves like a stretched elastic membrane.

4. A → (p, q, r)
B → (q, s)
C → (q)
D → (q, s)

In case of solid, all three elastic constants are found. In case of liquid and gas only bulk modulus is defined. The concept of surface tension is also applicable for liquids.

5. A → (p, r, s, t)
B → (p, t)
C → (p, q, r, s)
D → (p, r, s, t)

Since $k = \frac{YA}{L}$ and $T = 2\pi\sqrt{\frac{m}{k}}$

Also, work done $W = \frac{1}{2}kx^2$

$\Rightarrow \quad W = \frac{1}{2}\left(\frac{YA}{L}\right)x^2$

Thermal stress $= Y\alpha\Delta T$
i.e. thermal stress depends upon coefficient of linear expansion, Young's modulus of wire and rise in temperature ΔT.
Stress $= Y(\text{strain}) = Y\alpha\Delta T$

6. A → (s)
B → (p)
C → (p)
D → (p)
Contact angle between silver and water is $90°$, so $h = 0$

Capillary rise in glass tube is $h = \frac{2T}{r\rho g}$

Capillary rise in parallel glass plate with separation Δt is

$$h = \frac{2T\ell}{(\ell\rho g)\Delta t}$$

$\Rightarrow \quad h = \frac{2T}{r\rho g}$

The capillary rise in co-axial cylinder is

$$h = \frac{2T}{\rho g(r_2 - r_1)} = \frac{2T}{r\rho g}$$

7. A → (r)
B → (q)
C → (p)
D → (s)
Conceptual

8. A → (p, q, s)
B → (p, r, s)
C → (p, q, s)
D → (p)
For a liquid drop and air bubble, the excess pressure is

$$\Delta P = \frac{2T}{R}$$

For a soap bubble, the excess pressure is

$$\Delta P = \frac{4T}{R}$$

Also, surface tension decreases with increase in temperature

9. A → (r)
B → (t)
C → (p)
D → (p)
E → (q)

The pressure per unit speed gradient equals the viscosity η, so we have

$$[\eta] = ML^{-1}T^{-1}$$

$$[\text{Surface Tension}] = MT^{-2}$$

$$[E] = ML^{-1}T^{-2}$$

$$[u] = ML^{-1}T^{-2}$$

$$[n_R] = M^0L^0T^0$$

10. A → (q)
B → (p, q, r, s)
C → (q)
D → (p, q, r, s)
Conceptual

11. A → (p)
B → (q, r)
C → (s)
D → (s)

$F_2 - F_1 = U = W$

For liquid-1, the horizontal pair of forces cancel out.
So, net force is zero.

12. A → (r)
B → (s)
C → (q)
D → (p)
Conceptual

13. A → (p)
B → (q, r, s)
C → (p, s)
D → (q, r)
When cohesive forces are greater than adhesive forces, then shape of meniscus is concave from the liquid side and pressure is greater on concave side due to surface tension.

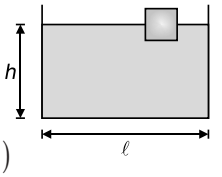
14. A → (r)
B → (p)
C → (r)
D → (p)
According to Torricelli's Theorem, we have

$v = \sqrt{2gh_1}$ and $R = 2\sqrt{h_1h_2}$

where, h_1 is the depth of the orifice from the free surface and h_2 is the height of the orifice from the ground.

15. A → (r)
B → (s)
C → (p)
D → (q)
Conceptual

16. A → (p)
B → (p, s)
C → (q, s)
D → (p, r)



Since, $U = V_{\text{imm}}\rho_{\text{liq}}g_{\text{eff}} = V_{\text{imm}}\rho_{\text{liq}}(g + a_y)$

$$\Delta P_y = h\rho(g + a_y) \qquad \ldots(1)$$

$$\Delta P_x = \ell \rho a_x \qquad \ldots(2)$$

In both equations (1) and (2), pressure increases opposite to the acceleration of the container.

17. A → (q)
B → (p)
C → (r)
D → (t)

If the orifice be at a depth h below the free surface of the liquid filled to a height H, then we have

$$v = \sqrt{2gh}$$

$$t = \sqrt{\frac{2h}{g}} \text{ and}$$

$$R = 2\sqrt{h(H-h)}$$

At the mountain, value of g will be less than its value at the surface of the earth, so v will decrease, t will increase and R will remain the same.

Range will be maximum when the orifice is at the middle of the liquid level that fills the beaker and not at the middle of the beaker.

18. A → (q)
B → (p)
C → (r)
D → (q)

Let the cube be of length L, density ρ_s be floating in a liquid of density ρ_ℓ

Since the cube is floating, so weight of cube is balanced by upthrust. Hence

$$\rho_s ALg = \rho_\ell Axg$$

$$\Rightarrow \quad x = \left(\frac{\rho_s}{\rho_\ell}\right)L$$

So, if ρ_ℓ is increased, x decreases and vice-versa.

When base area and density of cube remain same, then on increasing the length of cube, x also increases

Since x is independent of g, so when the whole system is accelerated upwards, then value of x will remain the same.

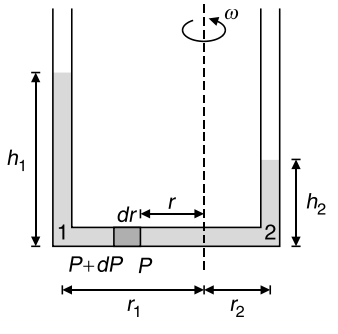
Also, when cube is replaced by another cube of same size but lesser density, then x decreases.

Integer/Numerical Answer Type Questions

1. Consider a small element of length dr at a distance r from the axis of rotation. Considering the equilibrium of this element.

$$(P+dP) - p = \rho\omega^2 r dr$$

$$\Rightarrow \quad dP = \rho\omega^2 r dr$$



On integrating between 1 and 2, we get

$$P_1 - P_2 = \rho\omega^2 \int_{-r_2}^{r_1} r dr$$

$$\Rightarrow \quad P_1 - P_2 = \frac{\rho\omega^2}{2}\left(r_1^2 - r_2^2\right)$$

Also, we note that $P_1 - P_2 = (h_1 - h_2)\rho g$

$$\Rightarrow \quad h_1 - h_2 = \frac{\omega^2}{2g}\left(r_1^2 - r_2^2\right)$$

$$\Rightarrow \quad h_1 - h_2 = \frac{(2\pi)^2}{2(10)}\left[(0.5)^2 - (0.25)^2\right]$$

$$\Rightarrow \quad h_1 - h_2 = 0.37 \text{ m} = 37 \text{ cm}$$

2. We know change in pressure,

$$\Delta P = -B\frac{\Delta V}{V_0}$$

$$\Rightarrow \quad \Delta P = -\left(2.6\times10^{10} \text{ Nm}^{-2}\right)\frac{-1.0\times10^{-10} \text{ m}^3}{1.0\times10^{-6} \text{ m}^3}$$

$$\Rightarrow \quad \Delta P = 2.6\times10^{6} \text{ Nm}^{-2}$$

where we have expressed the volume V_0 of the cube at the ocean's surface as

$$V_0 = \left(1.0\times10^{-2} \text{ m}\right)^3 = 1.0\times10^{-6} \text{ m}^3.$$

Since the pressure increases by 1.0×10^4 Nm^{-2} per meter of depth, the depth is

$$h = \frac{\Delta P}{\text{Pressure Gradient}} = \frac{2.6\times10^6 \text{ Nm}^{-2}}{1.0\times10^4 \frac{\text{Nm}^{-2}}{\text{m}}}$$

$$\Rightarrow \quad h = 260 \text{ m}$$

3. Since, $\Delta l = \frac{mgl}{2AY} = \frac{\rho(\pi r^2 l)gl}{2(\pi r^2)Y}$

$$\Rightarrow \quad \Delta l = \frac{gl^2\rho}{2Y} = \frac{(9.8)(5)^2(8000)}{2\times2\times10^{11}}$$

$$\Rightarrow \quad \Delta l = 4.9\times10^{-6} \text{ m} = 4.9 \ \mu\text{m}$$

$$\Rightarrow \quad U = \frac{1}{2}k(\Delta l)^2 = \frac{1}{2}\frac{YA}{l}(\Delta l)^2$$

$$\Rightarrow \quad U = \frac{1}{2}\times\frac{Y\times\pi r^2}{l}(\Delta l)^2$$

$$\Rightarrow \quad U = \frac{(2\times10^{11})(3.14)(6\times10^{-3})^2(4.9\times10^{-6})^2}{2\times5}$$

$$\Rightarrow \quad U = 5.43\times10^{-5} \text{ J} = 54.3 \ \mu\text{J}$$

4. Work done in stretching the wire from its initial length through $0.61 \text{ mm} (= 0.61\times10^{-3} \text{ m})$ when the stretching force is $3 \text{ kgwt} (= 3\times9.8 \text{ N})$, i.e.,

$$W_1 = \frac{1}{2}\times\text{Stretching force}\times\text{extension}$$

$$\Rightarrow \quad W_1 = \frac{1}{2}\times(3\times9.8)\times\left(0.61\times10^{-3}\right) = 0.9\times10^{-2} \text{ J}$$

Work done in stretching the wire from its initial length through $1.02\ \text{mm}(=1.02\times10^{-3}\ \text{m})$ when the stretching force is $5\ \text{kgwt}(=5\times9.8\ \text{N})$, i.e.,

$$W_2=\frac{1}{2}\times(5\times9.8)\times(1.02\times10^{-3})=2.5\times10^{-2}\ \text{J}$$

Work done in stretching the wire from $0.61\ \text{mm}$ to $1.02\ \text{mm}$ is

$$W=(W_2-W_1)=2.5\times10^{-2}\ \text{J}-0.9\times10^{-2}\ \text{J}$$

$$\Rightarrow\quad W=1.6\times10^{-2}\ \text{J}=16\ \text{mJ}$$

5. Velocity of flow is

$$v=\sqrt{2gh}=\sqrt{2\times10\times3.6}=6\sqrt{2}=8.5\ \text{ms}^{-1}$$

Rate of flow is

$$Q=Av=\pi\left(\frac{4\times10^{-2}}{\sqrt{\pi}}\right)^2(8.5)$$

$$\Rightarrow\quad Q=(16\times10^{-4})(8.5)=13.6\times10^{-3}\ \text{m}^3\text{s}^{-1}$$

$$\Rightarrow\quad Q=13.6\times10^{3}\ \text{cm}^3\text{s}^{-1}=13600\ \text{cm}^3\text{s}^{-1}$$

Applying Bernoulli's theorem between surface and A, we get

$$P_{\text{atm}}=P+\frac{1}{2}\rho v^2+\rho gh$$

$$\Rightarrow\quad P=P_{\text{atm}}-\frac{1}{2}\rho v^2-\rho gh$$

$$\Rightarrow\quad P=10^5-\frac{1}{2}(10^3)(6\sqrt{2})^2-(10^3)(10)(1.8)$$

$$\Rightarrow\quad P=4.6\times10^4\ \text{Nm}^{-2}=46\ \text{kNm}^{-2}$$

6. The excess of pressure inside the air bubble blown at the lower end of the tube is

$$P_i-P_0=\frac{2T}{R}$$

Given that, $T_{Hg}=0.465\ \text{Nm}^{-1}$, $\rho_{Hg}=13600\ \text{kgm}^{-3}$ and the radius of the tube is

$$R=\frac{1}{2}\ \text{mm}=0.5\ \text{mm}=5\times10^{-4}\ \text{m}$$

$$\Rightarrow\quad P_i-P_0=\frac{2T}{R}=\frac{2\times0.465}{5\times10^{-4}}=1860\ \text{Nm}^{-2}$$

$$\Rightarrow\quad P_i-P_0=\frac{1860}{13600\times9.8}=0.014\ \text{m of Hg}$$

$$\Rightarrow\quad P_i-P_0=14\ \text{mm of Hg}$$

7. Writing equation of motion for the block

$$T-mg\sin30^\circ=ma \qquad \ldots(1)$$

For the sphere

$$\text{Weight}-\text{Buoyant force}-T=ma \qquad \ldots(2)$$

$$\Rightarrow\quad mg-\frac{mg}{2}-T=ma$$

Solving, we get

$$a=0$$

8. **For steel wire:**

Total force on steel wire $F_1=(4+6)\times10=100\ \text{N}$

Radius of steel wire $r_1=\left(\frac{0.25}{2}\right)\text{cm}=0.125\times10^{-2}\ \text{m}$

Young's modulus of steel $Y_1=2.0\times10^{11}\ \text{Pa}$

Since, $Y_1=\frac{F_1\times l_1}{A_1\times\Delta l_1}=\frac{F_1\times l_1}{\pi r_1^2\times\Delta l_1}$

Elongation of the steel wire is $\Delta l_1=\frac{F_1\times l_1}{\pi r_1^2\times Y_1}$

$$\Rightarrow\quad \Delta\ell_1=\frac{100\times1.5\times7}{22\times(0.125\times10^{-2})^2\times2\times10^{11}}$$

$$\Rightarrow\quad \Delta\ell_1=1.49\times10^{-4}\ \text{m}\approx0.15\ \text{mm}$$

For brass wire:

Total force on brass wire $F_2=6\times10=60\ \text{N}$

Radius of brass wire $r_2=\left(\frac{0.25}{2}\right)\text{cm}=0.125\times10^{-2}\ \text{m}$

$Y_2=9\times10^{11}\ \text{Pa}$, $l_2=1.0\ \text{m}$, $\Delta l_2=?$

Elongation of the brass wire is $\Delta l_2=\frac{F_2\times l_2}{\pi r_2^2\times Y_2}$

$$\Rightarrow\quad \Delta\ell_2=\frac{60\times1.0\times7}{22\times(0.125\times10^{-2})^2\times(0.9\times10^{11})}$$

$$\Rightarrow\quad \Delta\ell_2=1.3\times10^{-4}\ \text{m}=0.13\ \text{mm}$$

9. According to Bernoulli's Theorem, we have

$$P_1+\frac{1}{2}\rho v_1^2=P_2+\frac{1}{2}\rho v_2^2$$

$$\Rightarrow\quad P_1-P_2=\frac{1}{2}\rho(v_2^2-v_1^2)$$

$$\Rightarrow\quad P_1-P_2=\frac{1}{2}(1.3)(120^2-90^2)$$

$$\Rightarrow\quad P_1-P_2=4095\ \text{Nm}^{-2}\approx4.1\ \text{kPa}$$

Lift force is given by

$$F=A(P_1-P_2)$$

$$\Rightarrow\quad F=(4.1\times10^3)(10\times2)=8.2\times10^4\ \text{N}$$

$$\Rightarrow\quad F=82\ \text{kN}$$

10. The distance Δx that the top surface of the disc moves relative to the bottom surface is given by

$$\Delta x=\frac{FL_0}{\eta A}$$

where F is the magnitude of the shearing force, L_0 is the thickness of the cartilage, A is the cross-sectional area and η is the shear modulus. Since the cross-section is circular, so $A=\pi r^2$, where r is the radius.

$$\Delta x=\frac{(11\ \text{N})(7.0\times10^{-3}\ \text{m})}{(1.2\times10^7\ \text{Nm}^{-2})\pi(3.0\times10^{-2}\ \text{m})^2}$$

$$\Rightarrow\quad \Delta x=2.3\times10^{-6}\ \text{m}=2.3\ \mu\text{m}$$

11. Since, $\Delta l=\frac{Fl}{AY}=\frac{Fl}{(\pi d^2/4)Y}$

$$\Rightarrow\quad d=\sqrt{\frac{4Fl}{\pi(\Delta l)Y}}=\sqrt{\frac{4\times400\times3}{3.14\times0.2\times10^{-2}\times2.1\times10^{11}}}$$

$$\Rightarrow\quad d=1.91\times10^{-3}\ \text{m}=1.91\ \text{mm}$$

12. Change in pressure, $\Delta P=501\ \text{atm}-1\ \text{atm}$

$$\Rightarrow\quad \Delta P=500\ \text{atm}=500\times1.01\times10^5\ \text{Pa}$$

Let ρ' be the density of water at the given depth, then

$$\rho' = \frac{\rho}{1-(\Delta V/V)} = \frac{\rho}{1-(\Delta P/B)}$$

$$\Rightarrow \quad \rho' = \frac{1.0\times10^3}{1-2.525\times10^{-2}} = 1025 \text{ kgm}^{-3}$$

13. The horizontal reaction force on vessel due to ejection of liquid is

$F = Av^2\rho$, where $v^2 = 2gh$

For the vessel to slide, we must have

$F \geq f_{\text{limiting}}$

$$\Rightarrow \quad Av^2\rho \geq \mu mg$$

$$\Rightarrow \quad A \geq \frac{\mu mg}{\rho(2gh)}$$

$$\Rightarrow \quad A \geq \frac{(0.4)(100)}{2(2)(1000)} = 0.01 \text{ m}^2$$

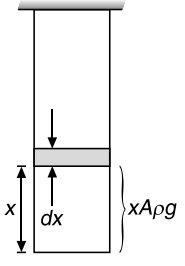
If d is the diameter of the hole, then

$$A = \frac{\pi d^2}{4}$$

$$\Rightarrow \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(0.01)}{3.14}} = 0.1128 \text{ m}$$

$$\Rightarrow \quad d = 11.3 \text{ cm}$$

14. Consider an element as shown in the figure.



Stress in the element $= \frac{\text{Force}}{\text{Area}} = \frac{xA\rho g}{A} = x\rho g$

Now, elastic potential energy stored in the wire is

$$dU = \frac{1}{2}(\text{Stress})(\text{Strain})(\text{Volume})$$

$$\Rightarrow \quad dU = \frac{1}{2}\frac{(\text{Stress})^2}{Y}(\text{Volume})$$

$$\Rightarrow \quad dU = \frac{1}{2}\frac{(x\rho g)^2}{Y}Adx = \frac{1}{2}\frac{\rho^2 g^2 A}{Y}x^2dx$$

Total elastic potential energy is $U = \frac{1}{2}\frac{\rho^2 g^2 A}{Y}\int_0^L x^2dx$

$$U = \frac{\rho^2 g^2 AL^3}{6Y}$$

$$\Rightarrow \quad p = 6$$

15. $\Delta l = \frac{Fl}{AY} = \frac{1000\times100}{4\times2\times10^6}$

$$\Rightarrow \quad \Delta\ell = 0.0125 \text{ cm} = 0.125 \text{ mm}$$

16. Since, the terminal velocity is given by

$$v_T = \frac{2r^2}{9}\frac{(\rho-\sigma)g}{\eta}$$

Also, we know $v = \frac{s}{t}$

$$\Rightarrow \quad \frac{s}{t} = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$$

$$\Rightarrow \quad r^2 = \frac{9s}{2t}\frac{\eta}{(\rho-\sigma)g} \qquad \ldots(1)$$

Given that, $s = 2\times10^{-2}$ m, $t = 1$ h $= 3600$ s

$$\eta = 1\times10^{-2} \text{ poise} = 1\times10^{-3} \text{ kgm}^{-1}\text{s}^{-1}$$

Substituting these given values, we get

$$r^2 = \frac{9}{2}\left(\frac{2\times10^{-2}}{3600}\right)\frac{1\times10^{-3}}{(1.8\times10^3-1\times10^3)\times10}$$

$$\Rightarrow \quad r^2 = \frac{9}{36}\times\frac{1}{8}\times10^{-10} = \frac{1}{32}\times10^{-10}$$

$$\Rightarrow \quad r = \sqrt{\frac{100}{32}}\times10^{-6} \text{ m} = 1.77 \ \mu\text{m}$$

So, the diameter is

$D = 2r = 2\times1.77 \ \mu\text{m} = 3.54 \ \mu\text{m}$

17. Maximum stress on the upper string is

$$\sigma_{\max} = \frac{(m_1+m_2+m)g}{0.006\times10^{-4}}$$

$$\Rightarrow \quad 8\times10^8 = \frac{(10+20+m)\times10}{0.006\times10^{-4}}$$

$$\Rightarrow \quad m = 18 \text{ kg}$$

Maximum stress on the upper string is

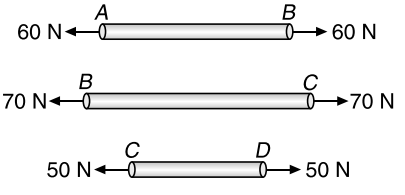
$$\sigma_{\max} = \frac{(m_1+m')g}{0.003\times10^{-4}}$$

$$\Rightarrow \quad 8\times10^8 = \frac{(10+m')\times10}{0.003\times10^{-4}}$$

$$\Rightarrow \quad m' = 14 \text{ kg}$$

So, answer is 14 kg and lower string will break earlier.

18. The action of forces on each part of rod is shown in Figure.



We know that the extension due to external force F is given by,

$$\Delta L = \frac{FL}{AY}$$

$$\Delta L_{AB} = \frac{(60\times10^3)\times1.5}{1\times2\times10^{11}} = 4.5\times10^{-7} \text{ m}$$

$$\Delta L_{BC} = \frac{(70\times10^3)X_1}{1\times2\times10^{11}} = 3.5\times10^{-7} \text{ m and}$$

$$\Delta L_{CD} = \frac{(50\times10^3)X_2}{1\times2\times10^{11}} = 5.0\times10^{-7} \text{ m}$$

The total extension is given by

$$\Delta L = \Delta L_{AB} + \Delta L_{BC} + \Delta L_{CD}$$

$$\Rightarrow \quad \Delta L = 4.5\times10^{-7} + 3.5\times10^{-7} + 5.0\times10^{-7}$$

$$\Rightarrow \quad \Delta L = 13\times10^{-7}\ \text{m} = 1.3\ \mu\text{m}$$

19. $$\frac{F}{A_{\min}} = \sigma_m$$

$$\Rightarrow \quad \frac{F}{\left(\frac{\pi d_{\min}^2}{4}\right)} = \sigma_m$$

$$\Rightarrow \quad d_{\min} = \sqrt{\frac{4F}{\pi\sigma_m}}$$

$$\Rightarrow \quad d_{\min} = \sqrt{\frac{4\times10\times9.8}{3.14\times1.5\times10^8}} = 9.1\times10^{-4}\ \text{m} = 0.91\ \text{mm}$$

20. Using equation of continuity, $A_1v_1 = A_2v_2$

$$v_2 = \left(\frac{A_1}{A_2}\right)v_1 \text{ or } v_2 = \left(\frac{\pi r_1^2}{\pi r_2^2}\right)v_1 = \left(\frac{r_1}{r_2}\right)^2 v_1$$

Substituting the values, we get

$$v_2 = \left(\frac{1.0\times10^{-2}}{2.0\times10^{-2}}\right)^2 (2)$$

$$\Rightarrow \quad v_2 = 0.5\ \text{ms}^{-1} = 50\ \text{cms}^{-1}$$

21. Taking the subscript G for ground floor and F for first floor, then according to Bernoulli's theorem, we have

$$P_G + \rho g h_G + \frac{1}{2}\rho v_G^2 = P_F + \rho g h + \frac{1}{2}\rho v_F^2$$

Assuming zero potential energy level (ZPEL) at the ground floor, then $h_G = 0$.

$$\Rightarrow \quad P_G + \frac{1}{2}\rho v_G^2 = P_F + \rho g h + \frac{1}{2}\rho v_F^2 \qquad \ldots(1)$$

According to Equation of Continuity, we have

$$A_F v_F = A_G v_G$$

$$\Rightarrow \quad v_F = \frac{A_G v_G}{A_F} = \frac{\pi(2.5)^2\times3}{\pi(1.25)^2} = 12\ \text{ms}^{-1}$$

From equation (1), we get

$$P_F = P_G - \frac{1}{2}\rho\left(v_F^2 - v_G^2\right) - \rho g h$$

$$\Rightarrow \quad P_F = 317600 - \left(10^3\right)\left(\frac{144-9}{2} + (10)(25)\right)$$

$$\Rightarrow \quad P_F = 317600 - 317500 = 100\ \text{Nm}^{-2}$$

22. The stress should not exceed the elastic limit otherwise the wire will suffer permanent deformation. So,

$$\frac{F}{A} = 2.48\times10^8\ \text{Pa}$$

Area of cross section of the wire is

$$A = \pi r^2 = 3.14\times\left(1\times10^{-3}\right)^2 = 3.14\times10^{-6}\ \text{m}^2$$

$$\Rightarrow \quad F = 779\ \text{N}$$

Also, $\text{Strain} = \dfrac{\text{Stress}}{Y} = \dfrac{2.48\times10^8}{2\times10^{11}} = 1.24\times10^{-3}$

$$\Rightarrow \quad \frac{\Delta L}{L} = 1.24\times10^{-3}$$

$$\Rightarrow \quad \Delta L = 1.24\times10^{-3}\times10\ \text{m} = 1.24\ \text{cm}$$

The stress should not exceed the ultimate strength.

$$\frac{F}{A} = 4.89\times10^8\ \text{Pa} \qquad \{\text{Take } b \ll r\}$$

$$\Rightarrow \quad F = 4.89\times10^8\times3.14\times10^{-6} = 1535\ \text{N} \approx 1.5\ \text{kN}$$

23. $$\rho' = \frac{\rho}{1-\frac{dP}{B}} \approx \rho\left(1+\frac{dP}{B}\right)$$

$$\Rightarrow \quad \Delta\rho = \rho' - \rho = \frac{\rho(dP)}{B}$$

$$\Rightarrow \quad \Delta\rho = \frac{\rho(\rho g h)}{B}$$

$$\Rightarrow \quad \Delta\rho = \frac{(1030)^2\times9.8\times400}{2\times10^9}$$

$$\Rightarrow \quad \Delta\rho = 2.0\ \text{kgm}^{-3}$$

24. Let the atmospheric pressure be P_0. Then, pressure at A is

$$P_A = P_0 + h(1000)g \qquad \ldots(1)$$

Similarly, pressure at B is

$$P_B = P_0 + (0.02\ \text{m})(13600)g \qquad \ldots(2)$$

Since these pressures are equal as A and B because these points are at the same horizontal level, so equating (1) and (2), we get

$$h = (0.02\ \text{m})(13.6) \approx 27\ \text{cm}$$

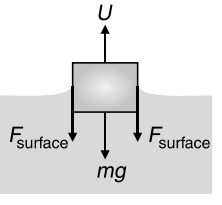
25. If surface tension is ignored, then according to the Laws of Floatation, the upthrust acting on the cube must balance its weight. So, if x be the length of the cube immersed inside the water, then

$$W = U$$

$$\Rightarrow \quad \left(800\times10^{-3}\right)g = \left[(0.1)^2 x\right]\rho_w g$$

$$\Rightarrow \quad x = 0.08\ \text{m} \qquad \ldots(1)$$

Now, we know that since water wets the cube and the angle of contact is zero, so the force due to surface tension acts vertically downwards as shown in Figure and hence the cube is buoyed down due to surface tension.



Let x' be the new immersed length at which the cube is in equilibrium, then we have

$$mg + F = U' \qquad \ldots(2)$$

where, F is the force due to surface tension acting on the four edges of the cube. So

$$F = 4TL = 4(0.07)(0.1)\ \text{N} = 0.028\ \text{N}$$

So, from equation (2) we get

$$\left(800\times10^{-3}\right)g + 0.028 = \left[(0.1)^2 x'\right]\rho_w g$$

$$\Rightarrow \quad x' = \frac{8 + 0.028}{100} = 0.08028 \text{ m}$$

Therefore, the additional distance is

$$x' - x = 0.00028 \text{ m} = 280\ \mu\text{m}$$

26. Force constant of the wire is

$$k = \frac{F}{\Delta L} = \frac{F}{\frac{FL}{AY}} = \frac{YA}{L}$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{YA}{Lm}}$$

$$\Rightarrow \quad \sqrt{\frac{YA}{Lm}} = 140$$

$$\Rightarrow \quad \frac{Y \times 4.9 \times 10^{-7}}{1 \times 0.1} = 140 \times 140$$

$$\Rightarrow \quad Y = \frac{140 \times 140}{49 \times 10^{-7}} = 4 \times 10^9$$

$$\Rightarrow \quad x = 4$$

27. The stress in either cable is

$$\text{Stress} = \frac{F}{A} = \frac{F}{\pi r^2} \qquad \ldots(1)$$

Now, for equal stresses in the cables, we have

$$\frac{F_2}{\pi r_2^2} = \frac{F_1}{\pi r_1^2}$$

$$\Rightarrow \quad F_2 = \frac{F_1 r_2^2}{r_1^2} = \frac{(270 \text{ N})(5.1 \times 10^{-3} \text{ m})^2}{(3.5 \times 10^{-3} \text{ m})^2} = 570 \text{ N}$$

28. The velocity attained by the sphere in falling freely from a height h is

$$v = \sqrt{2gh} \qquad \ldots(1)$$

This is the terminal velocity of the sphere in water. Hence by Stokes law, we get

$$v = v_T = \frac{2}{9}\left(\frac{r^2}{\eta}\right)(\rho - \sigma)g$$

$$\Rightarrow \quad v_T = \frac{2}{9}\frac{(1.0 \times 10^{-3})^2 (1.0 \times 10^4 - 1.0 \times 10^3) \times 10}{1.0 \times 10^{-3}}$$

$$\Rightarrow \quad v_T = 20 \text{ ms}^{-1}$$

So, from Equation (1), we get

$$h = \frac{v^2}{2g} = \frac{v_T^2}{2g} = \frac{(20)^2}{2(10)} = 20 \text{ m}$$

29. $$\sigma = \frac{F}{A} = \frac{F}{\pi(R^2 - r^2)}$$

$$\Rightarrow \quad R = \sqrt{\frac{F}{\sigma\pi} + r^2}$$

$$\Rightarrow \quad R = \sqrt{\frac{1.6 \times 10^6}{90 \times 10^6 \times 3.14} + (0.1)^2}$$

$$\Rightarrow \quad R = 0.1251 \text{ m} = 125.1 \text{ mm}$$

So, diameter $d = 2R = 250.2$ mm

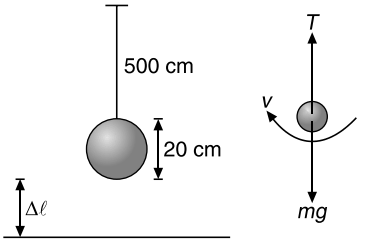
30. According to the Equation of Continuity, we have

$$a_1 v_1 + a_2 v_2 = av$$

$$\Rightarrow \quad (12)(20) + (8)(16) = (16)v$$

$$\Rightarrow \quad v = \frac{240 + 128}{16} = 23 \text{ kmh}^{-1}$$

31. $\Delta l = 521 - 500 - 20 = 1$ cm $= 0.01$ m



$$T - mg = \frac{mv^2}{R}$$

$$\Rightarrow \quad T = m\left(g + \frac{v^2}{R}\right) = m\left(g + \frac{v^2}{l}\right)$$

$$\Rightarrow \quad \Delta l = \frac{Tl}{AY} = \frac{m\left(g + \frac{v^2}{l}\right)l}{\left(\frac{\pi d^2}{4}\right)Y}$$

$$\Rightarrow \quad \Delta l = \frac{mgl + mv^2}{\left(\frac{\pi d^2}{4}\right)Y}$$

$$\Rightarrow \quad v = \sqrt{\frac{\pi d^2 \Delta l Y}{4m} - gl}$$

$$\Rightarrow \quad v = \sqrt{\frac{(3.14)(4 \times 10^{-3})^2 (0.01)(2 \times 10^{11})}{4 \times 25} - 9.8 \times 5}$$

$$\Rightarrow \quad v \approx 31 \text{ ms}^{-1}$$

32. We are given that

$$(\text{Stress})_{\lim} = 0.9 \times 10^8 \text{ Nm}^{-2}, \ Y = 1.4 \times 10^{10} \text{ Nm}^{-2}$$

Length of two bones in a leg is

$$L = 2 \times 50 \text{ cm} = 100 \text{ cm} = 1.0 \text{ m}$$

Area of cross-section of the bone is

$$A = 5 \text{ cm}^2 = 5 \times 10^{-4} \text{ m}^2$$

Since, $\text{Stress} = \frac{F}{A}$, so $F = (\text{Stress})A$

$$\Rightarrow \quad F = (0.9 \times 10^8)(5 \times 10^{-4}) = 4.50 \times 10^4 \text{ N}$$

Further, $Y = \frac{F/A}{\Delta L/L}$

$$\Rightarrow \quad \Delta L = \left(\frac{\text{Stress}}{Y}\right)L$$

$$\Rightarrow \quad \Delta L = \left(\frac{0.9 \times 10^8}{1.4 \times 10^{10}}\right)(1) = 6.43 \times 10^{-3} \text{ m}$$

Elastic potential energy is $U = \frac{1}{2}F\Delta L$

$$\Rightarrow \quad U = \frac{1}{2}(4.50 \times 10^4)(6.43 \times 10^{-3}) = 145 \text{ J}$$

33. $\frac{1}{2}\rho v^2 = \rho g h + \frac{mg}{A}$

where, $h = 1.0 - 0.5 = 0.5$ m,

A = Area of piston $= 0.5$ m^2

$\Rightarrow \quad v = \sqrt{2gh + \frac{2mg}{\rho A}}$

$\Rightarrow \quad v = \sqrt{2\times 9.8\times 0.5 + \frac{2\times 20\times 9.8}{10^3\times 0.5}} = 3.25$ ms^{-1}

Speed with which it hits the surface is

$\Rightarrow \quad v' = \sqrt{v^2 + 2gh'}$

$\Rightarrow \quad v' = \sqrt{(3.25)^2 + (2\times 9.8\times 0.5)}$

$\Rightarrow \quad v' = 4.51$ ms^{-1}

34. Applying Bernoulli's theorem for a horizontal pipe, we get

$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

$\Rightarrow \quad P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$

So, kinetic energy per unit mass is

$\frac{\text{K.E.}}{\text{Mass}} = \frac{1}{2}\left(v_2^2 - v_1^2\right) = \frac{P_1 - P_2}{\rho} = \frac{8}{800}$ Jkg^{-1}

$\Rightarrow \quad \frac{1}{2}\left(v_2^2 - v_1^2\right) = \frac{1}{100}$ Jkg^{-1} $= 10$ mJkg^{-1}

35. Since, $T - mg = ml\omega^2 = ml(2\pi f)^2$

$\Rightarrow \quad T = mg + 4\pi^2 mlf^2$

$\Rightarrow \quad T = 6\times 9.8 + 4\pi^2(6)(0.6)(2)^2$

$\Rightarrow \quad T = 628$ N

Now, $\Delta l = \frac{Tl}{AY}$

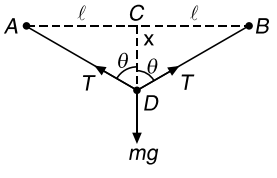
$\Rightarrow \quad \Delta l = \frac{(628)(0.6)}{(0.05\times 10^{-4})(2\times 10^{11})}$

$\Rightarrow \quad \Delta l = 3.8\times 10^{-4}$ m $= 0.38$ mm

36. Given that $m = 100$ g $= 0.100$ kg

Refer figure, let x be the depression at the mid-point i.e., $CD = x$

In figure, $AC = CB = \ell = 0.5$ m



$AD = BD = (l^2 + x^2)^{\frac{1}{2}}$

Increase in length, $\Delta l = AD + DB - AB = 2AD - AB$

$\Rightarrow \quad \Delta l = 2(l^2 + x^2)^{\frac{1}{2}} - 2l = 2l\left(1 + \frac{x^2}{l^2}\right)^{\frac{1}{2}} - 2l$

$\Rightarrow \quad \Delta l = 2l\left(1 + \frac{x^2}{2l^2}\right) - 2l = \frac{x^2}{l}$

Strain $= \frac{x^2}{2l^2}$

If T is the tension in the wire, then

$2T\cos\theta = mg$

$\Rightarrow \quad T = \frac{mg}{2\cos\theta}$

Here, $\cos\theta = \frac{x}{(l^2 + x^2)^{\frac{1}{2}}} = \frac{x}{l\left(1 + \frac{x^2}{l^2}\right)^{\frac{1}{2}}} = \frac{x}{l\left(1 + \frac{1}{2}\frac{x^2}{l^2}\right)}$

Since $x \ll l$, so $\frac{x^2}{2l^2} \ll 1$ and $1 + \frac{x^2}{2l^2} \approx 1$

$\Rightarrow \quad \cos\theta = \frac{x}{l}$

Hence, $T = \frac{mg}{2\left(\frac{x}{l}\right)} = \frac{mgl}{2x}$

Stress $= \frac{T}{A} = \frac{mgl}{2Ax}$

$Y = \frac{\text{stress}}{\text{strain}} = \frac{mgl}{2Ax}\times\frac{2l^2}{x^2} = \frac{mgl^3}{Ax^3}$

$\Rightarrow \quad x = l\left(\frac{mg}{YA}\right)^{\frac{1}{3}} = 0.5\left(\frac{0.1\times 10}{2\times 10^{11}\times 0.5\times 10^{-6}}\right)^{\frac{1}{3}}$

$\Rightarrow \quad x = 1.077\times 10^{-2}$ m $= 10.8$ mm

37. Given that, $\frac{m(g+a)}{A} = \frac{1}{3}\sigma_{\max}$

$\Rightarrow \quad a = \frac{\sigma_{\max} A}{3m} - g$

$\Rightarrow \quad a = \frac{(3\times 10^8)(4\times 10^{-4})}{3\times 900} - 9.8$

$\Rightarrow \quad a = 34.64$ ms^{-2}

38. According to Equation of Continuity, we have

$A_1 v_1 = A_2 v_2$

$\Rightarrow \quad \pi\left(\frac{4}{2}\right)^2 v_1 = \pi\left(\frac{3}{2}\right)^2 v_2$

$\Rightarrow \quad v_2 = \frac{16}{9}v_1$

The pressure at the cross-sections are

$P_1 = 20\times 1\times 980 = 19600$ dynecm^{-2}

and $P_2 = 15\times 1\times 980 = 14700$ dynecm^{-2}

Using Bernoulli's theorem, we get

$v_2^2 - v_1^2 = \frac{2}{\rho}(P_1 - P_2)$

$\Rightarrow \quad \left(\frac{16v_1}{9}\right) - v_1^2 = \frac{2}{1}(19600 - 14700)$

$\Rightarrow \quad v_1 = 67.4$ cms^{-1}

Now discharge rate is

$Q = A_1 v_1 = \pi(2)^2(67.4) \approx 847$ cm^{3}s^{-1}

$\Rightarrow \quad Q = 847$ cm^{3}s^{-1} $= 847$ cm^{3}s^{-1}

39. Since, $\Delta l = \dfrac{Fl}{AY}$

Here, $F = \text{Upthrust} = V_i\rho_l g$, $A = \dfrac{\pi d^2}{4}$

$\Rightarrow \quad \Delta l = \dfrac{4V_i\rho_l g l}{\pi d^2 Y}$

$\Rightarrow \quad \Delta l = \dfrac{(4)(10^{-3})(800)(9.8)(3)}{(3.14)(0.4\times10^{-3})^2(8\times10^{10})}$

$\Rightarrow \quad \Delta\ell = 2.34\times10^{-3}\ \text{m} = 2.34\ \text{mm}$

40. The volume of the blood given to the patient per second is given by Poiseuille Equation according to which

$$Q = \frac{V}{t} = \frac{\pi}{8}\left(\frac{r^4}{\eta}\right)\left(\frac{\Delta P}{l}\right) \qquad \ldots(1)$$

Given that, length of the needle is $l = 5$ cm , radius of the needle is $r = 0.03$ cm , viscosity of blood is, $\eta = 0.017$ poise , density of blood is $\rho = 1.02\ \text{gcm}^{-3}$ and height at which blood bag hangs is $h = 85$ cm . So, the pressure difference is

$$\Delta P = h\rho g = (85)(1.02)(980)\ \text{dynecm}^{-2}$$

$\Rightarrow \quad Q = \dfrac{V}{t} = \dfrac{\pi}{8}\left(\dfrac{(0.03)^4}{0.017}\right)\left(\dfrac{85\times1.02\times980}{5}\right)$

$\Rightarrow \quad Q = 0.32\ \text{cm}^3\text{s}^{-1}$

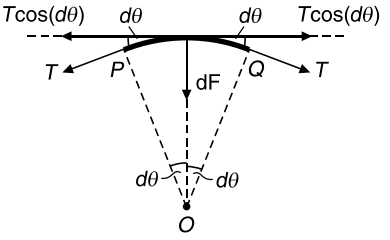
So, time for which transfusion takes place is

$$t = \frac{\text{Total Volume of Blood}}{\text{Rate of Flow of Blood}} = \frac{V}{Q} = \frac{500\ \text{cm}^3}{0.32\ \text{cm}^3\text{s}^{-1}}$$

$\Rightarrow \quad t = 1562.5\ \text{s} \approx 26\ \text{min}$

41. The component of tension force radially inwards provides the necessary centripetal force dF to the element to revolve in a circle, so

$$2T\sin d\theta = (dm)R\omega^2$$



For small angles, $\sin d\theta \approx d\theta$

$\Rightarrow \quad 2T(d\theta) = (2Rd\theta)(\pi r^2)(\rho)(R)(2\pi f)$

$\Rightarrow \quad T = \left(4\pi^3 f^2R^2r^2\rho\right)$

Now $\Delta l = \dfrac{Tl}{AY}$

$\Rightarrow \quad \dfrac{\Delta l}{l} = \dfrac{T}{AY} = \dfrac{T}{\pi r^2 Y}$

Also, $l = 2\pi R$

$\Rightarrow \quad \Delta l = 2\pi(\Delta R)$

$\Rightarrow \quad \dfrac{\Delta l}{l} = \dfrac{\Delta R}{R} = \dfrac{T}{\pi r^2 Y}$

$\Rightarrow \quad \dfrac{\Delta l}{l} = \dfrac{4\pi^3 f^2R^2r^2\rho}{\pi r^2 Y} = 4\pi^2\left(\dfrac{f^2R^2\rho}{Y}\right)$

$\Rightarrow \quad \dfrac{\Delta l}{l} = 40\left(\dfrac{f^2R^2\rho}{Y}\right)$

$\Rightarrow \quad a = 40$

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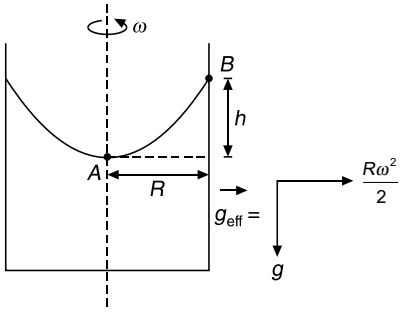
1. Applying pressure equation between A and B, we get



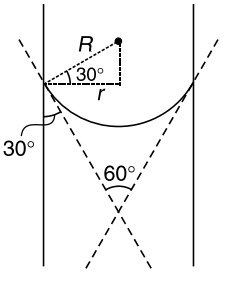
$$P_0 + \rho\frac{R\omega^2}{2}R - \rho gh = P_0$$

$\Rightarrow \quad \dfrac{\rho R^2\omega^2}{2} = \rho gh$

$\Rightarrow \quad h = \dfrac{R^2\omega^2}{2g} = (5)^2\dfrac{\omega^2}{2g} = \dfrac{25\omega^2}{2g}$

Hence, the correct answer is (A).

2. Let r be radius of capillary and R be radius of meniscus



From figure, we see that angle of contact $\theta = 30°$

$\Rightarrow \quad \dfrac{r}{R} = \cos 30°$

Height of liquid in capillary is

$$h = \frac{2T\cos\theta}{r\rho g}$$

$\Rightarrow \quad h = \dfrac{2\times0.05\times\left(\sqrt{3}/2\right)}{0.15\times10^{-3}\times667\times10}$

$\Rightarrow \quad h = 0.087$ m

Hence, the correct answer is (C).

3. Since, $\Delta P_1 = 0.01 = \dfrac{4T}{R_1}$ and $\Delta P_2 = 0.02 = \dfrac{4T}{R_2}$

$\Rightarrow \quad \dfrac{R_1}{R_2} = 2$

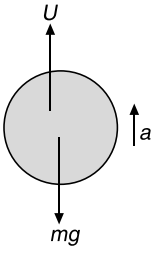
$\Rightarrow \quad \dfrac{V_1}{V_2} = \dfrac{R_1^3}{R_2^3} = \dfrac{8R^3}{R_2^3} = 8$

Hence, the correct answer is (A).

4. Since, $V = \dfrac{4\pi}{3} r^3 = \dfrac{4\pi}{3} \times (1)^3 = 4.19 \text{ cm}^3$

$a = 9.8 \text{ cms}^{-2}$

Since, $U - mg = ma$



$\Rightarrow \quad m = \dfrac{U}{g+a} = \dfrac{(V\rho_\omega g)}{g+a} = \dfrac{V\rho_\omega}{1+\dfrac{a}{g}}$

$\Rightarrow \quad m = \dfrac{(4.19)\times 1}{1+\dfrac{9.8}{980}} = \dfrac{4.19}{1.01} = 4.15$ g

Hence, the correct answer is (C).

5. Since, $B = -\dfrac{\Delta P}{\Delta V/V}$

$\Rightarrow \quad \left|\dfrac{\Delta V}{V}\right| = \dfrac{\Delta P}{B} = \dfrac{4\times 10^9}{8\times 10^{10}} = \dfrac{1}{20}$

$\Rightarrow \quad \dfrac{\Delta l}{l} = \dfrac{1}{3} \times \dfrac{\Delta V}{V} = \dfrac{1}{60}$

Percentage change is $\dfrac{\Delta l}{l} \times 100\% = \dfrac{100}{60}\% = 1.67\%$

Hence, the correct answer is (B).

6. Since, $\dfrac{4}{3}\pi\left(R^3 - r^3\right)\rho_m g = \dfrac{4}{3}\pi R^3 \rho_w g$

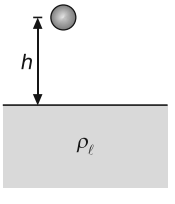
$\Rightarrow \quad 1 - \left(\dfrac{r}{R}\right)^3 = \dfrac{8}{27}$

$\Rightarrow \quad \dfrac{r}{R} = \left(\dfrac{19}{27}\right)^{1/3} = \dfrac{19^{1/3}}{3} = 0.88 \approx \dfrac{8}{9}$

Hence, the correct answer is (B).

7. After falling through h, the velocity be equal to terminal velocity, so

$$\sqrt{2gh} = \frac{2}{9}\frac{r^2 g}{\eta}(\rho_l - \rho)$$



$\Rightarrow \quad h = \dfrac{2}{81}\dfrac{r^4 g(\rho_l - \rho)^2}{\eta^2}$

$\Rightarrow \quad h \propto r^4$

Hence, the correct answer is (B).

8. Applying Bernoulli's Equation, we get

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$\Rightarrow \quad P + \dfrac{1}{2}\rho v^2 = \dfrac{P}{2} + \dfrac{1}{2}\rho V^2$

$\Rightarrow \quad \dfrac{P}{\rho} + v^2 = V^2$

$\Rightarrow \quad V = \sqrt{\dfrac{P}{\rho} + v^2}$

Hence, the correct answer is (B).

9. Capillary rise

$$h = \frac{2T\cos\theta}{r\rho g}$$

$\Rightarrow \quad T = \dfrac{\rho g r h}{2\cos\theta}$

$\Rightarrow \quad T = \dfrac{(900)(10)\left(15\times 10^{-5}\right)\left(15\times 10^{-2}\right)}{2}$

$\Rightarrow \quad T = 1012.5\times 10^{-4} \text{ Nm}^{-1}$

$\Rightarrow \quad T = 101.25\times 10^{-3} = 101.25 \text{ mNm}^{-1}$

In question closest integer is asked

So closest integer is 101.00

Hence, the correct answer is 101.

10. Since, $A_1 v_1 = A_2 v_2$

$\Rightarrow \quad \dfrac{v_{\min}}{v_{\max}} = \dfrac{A_{\min}}{A_{\max}} = \left(\dfrac{4.8}{6.4}\right)^2 = \dfrac{9}{16}$

Hence, the correct answer is (D).

11. In case of minimum density of liquid, sphere will be floating while completely submerged, so

$mg = U$

$\Rightarrow \quad mg = \left(\displaystyle\int_0^R \rho\left(4\pi r^2 dr\right)\right) g = U$

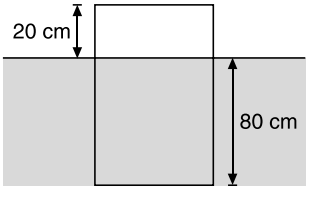
$$\rho_0 g \int_0^R \left(1 - \frac{r^2}{R^2}\right) 4\pi r^2 dr = \frac{4}{3}\pi R^3 \rho_l g$$

$\Rightarrow \quad \rho_l = \dfrac{2\rho_0}{5}$

Hence, the correct answer is (D).

12. For equilibrium, $mg = U$

$\Rightarrow \quad m = \rho_0 A(80)$ …(1)

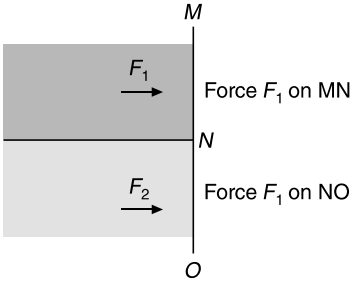


Similarly, $m = \rho A(79)$ …(2)

Dividing, we get $\dfrac{\rho_0}{\rho} = \dfrac{80}{79} = 1.01$

Hence, the correct answer is (A).

13. Since, $F_1 = \left(\frac{\rho g h}{2}\right)A$



$\Rightarrow \quad F_2 = \left(\rho g h + \frac{2\rho g h}{2}\right)A$

$\Rightarrow \quad \frac{F_1}{F_2} = \frac{1}{4}$

Hence, the correct answer is (A).

14. Rate of flow of water is $Q = A_A V_A = A_B V_B$

$\Rightarrow \quad (40)V_A = (20)V_B$

$\Rightarrow \quad V_B = 2V_A \qquad \ldots(1)$

According to Bernoulli's theorem, we have

$$P_A + \frac{1}{2}\rho V_A^2 = P_B + \frac{1}{2}\rho V_B^2$$

$\Rightarrow \quad P_A - P_B = \frac{1}{2}\rho\left(V_B^2 - V_A^2\right)$

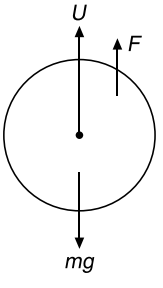
$\Rightarrow \quad 700 = \frac{1}{2}\times 1000\left(4V_A^2 - V_A^2\right)$

$\Rightarrow \quad V_A = 0.68 \text{ ms}^{-1} = 68 \text{ cms}^{-1}$

Rate of flow $Q = A_A V_A = (40)(68) = 2720 \text{ cm}^3\text{s}^{-1}$

Hence, the correct answer is (C).

15. FBD of droplet is shown in Figure.



Since, $U + F = mg$

where, $U = \left(\frac{2}{3}\pi R^3\right)\rho g$, $F = T(2\pi R)$, $m = d\left(\frac{4}{3}\pi R^3\right)$

$\Rightarrow \quad \left(\frac{2}{3}\pi R^3\right)\rho g + T(2\pi R) = d\left(\frac{4}{3}\pi R^3\right)g$

$\Rightarrow \quad T(2\pi R) = \left(\frac{2}{3}\pi R^3\right)g(2d - \rho)$

$\Rightarrow \quad R = \sqrt{\frac{3T}{(2d-\rho)g}}$

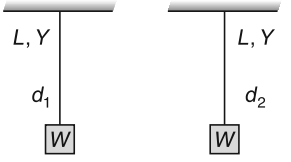
Hence, the correct answer is (B).

16. Since, $\frac{\text{Energy stored}}{\text{Volume}} = \frac{1}{2}\frac{(\text{Stress})^2}{Y}$

$\Rightarrow \quad \frac{u_1}{u_2} = \frac{1}{4}$

$\Rightarrow \quad 4u_1 = u_2$

$\Rightarrow \quad 4\frac{1}{2Y}\left(\frac{W4}{\pi d_1^2}\right)^2 = \frac{1}{2Y}\left(\frac{W4}{\pi d_2^2}\right)^2$



$\Rightarrow \quad 4 = \left(\frac{d_1}{d_2}\right)^4$

$\Rightarrow \quad \frac{d_1}{d_2} = \sqrt{2} : 1$

Hence, the correct answer is (D).

17. Since, $T = m\omega^2 l$

Breaking stress is $\frac{T}{A} = \frac{m\omega^2 l}{A}$

$\Rightarrow \quad \omega^2 = \frac{4.8\times 10^7 \times (10^{-2}\times 10^{-4})}{10\times 0.3} = 16$

$\Rightarrow \quad \omega = 4$

Hence, the correct answer is 4.00.

18. Flow rate of water $(Q) = 100$ lit/min

$\Rightarrow \quad Q = \frac{100\times 10^{-3}}{60} = \frac{5}{3}\times 10^{-3} \text{ m}^3$

Since $Q = Av$

So, velocity of flow is $v = \frac{Q}{A} = \frac{5\times 10^{-3}}{3\times\pi\times(5\times 10^{-2})^2}$

$\Rightarrow \quad v = \frac{10}{15\pi} = \frac{2}{3\pi} \text{ ms}^{-1} = 0.2 \text{ ms}^{-1}$

Reynold's number $n_R = \frac{Dv\rho}{\eta}$

$\Rightarrow \quad n_R = \frac{(10\times 10^{-2})\times\frac{2}{3\pi}\times 1000}{1} \simeq 2\times 10^4$

Order of n_R is 10^4

Hence, the correct answer is (B).

19. $h \propto \frac{1}{r}$ and $M \propto \pi r^2 h$

So, $M \propto r$

Hence, the correct answer is (A).

20. Since, $V\sigma g = \frac{4}{5}v\rho_\omega g \qquad \ldots(1)$

When oil is poured in it, then

$$V\sigma g = \frac{v}{2}\rho_\omega g + \frac{v}{2}\rho_0 g$$

$\Rightarrow \quad \left(\frac{\rho_\omega}{2} + \frac{\rho_{oil}}{2}\right) = \frac{4}{5}\rho_\omega$

$\Rightarrow \quad \frac{\rho_{oil}}{2} = \rho_\omega\left(\frac{4}{5} - \frac{1}{2}\right) = \frac{3}{10}\rho_\omega$

$\Rightarrow \quad \rho_{oil} = \frac{3}{5}\rho_{\omega} = 0.6\rho_{\omega}$

Hence, the correct answer is (D).

21. Since, $h = \frac{2T\cos\theta}{r\rho g}$

$\Rightarrow \quad r = \frac{2T\cos\theta}{h\rho g}$

$\Rightarrow \quad \frac{r_{Hg}}{r_{\text{water}}} = \frac{r_1}{r_2} = \left(\frac{T_{Hg}}{T_W}\right)\left(\frac{\rho_W}{\rho_{Hg}}\right)\left(\frac{\cos\theta_{Hg}}{\cos\theta_W}\right)$

$\Rightarrow \quad \frac{r_1}{r_2} = 7.5 \times \frac{1}{13.6} \times \frac{1}{\sqrt{2}} = 0.4 = \frac{2}{5}$

Hence, the correct answer is (D).

22. Given $(50)^3 \times \frac{30}{100} \times (1) \times g = M_{\text{cube}}g$ …(1)

Let m mass should be placed

Hence $(50)^3 \times (1) \times g = (M_{cube} + m)g$ …(2)

From (1) and (2), we get

$\Rightarrow \quad mg = (50)^3 \times g(1 - 0.3) = 125 \times 0.7 \times 10^3$ g

$\Rightarrow \quad m = 87.5$ kg

Hence, the correct answer is (C).

23. Using Bernoullie's equation, we get

$v_2 = \sqrt{v_1^2 + 2gh}$

According to equation of continuity, we have

$A_1V_1 = A_2V_2$

$\Rightarrow \quad (1)(1) = (A_2)\left(\sqrt{(1)^2 + 2 \times 10 \times \frac{15}{100}}\right)$

$\Rightarrow \quad A_2 = \frac{1}{2}$ cm$^2 = 5 \times 10^{-5}$ m^2

Hence, the correct answer is (B).

24. Since, $\Delta P = P_2 - P_1 = \rho g \Delta H$

$\Rightarrow \quad 3.03 \times 10^6 = 10^3 \times 10 \times \Delta H$

$\Rightarrow \quad \Delta H \approx 300$ m

Hence, the correct answer is (C).

25. $\frac{4}{3}\pi R^3 = 27\left(\frac{4}{3}\pi r^3\right)$

$\Rightarrow \quad R^3 = 27r^3$

$\Rightarrow \quad R = 3r$

Since $v \propto r^2$

$\Rightarrow \quad \frac{v_1}{v_2} = \left(\frac{R}{r}\right)^2 = 9$

Hence, the correct answer is (B).

26. Volume Flow Rate $Q = \frac{0.74}{60}$ m^{3}s^{-1}

Speed of efflux $v = \frac{0.74 \times 10^4}{60 \times \pi \times 4}$ ms^{-1} $= \sqrt{2gh}$

$\Rightarrow \quad 9.82 = \sqrt{2 \times 10 \times h}$

$\Rightarrow \quad h = 4.8$ m

Hence, the correct answer is (C).

27. $-\frac{dh}{dt} = \sqrt{2gh}$

$\Rightarrow \quad Q = a\sqrt{2gh} = Q$

$\Rightarrow \quad 10^{-4} = 10^{-4}\sqrt{2gh} = 10^{-4}$

$\Rightarrow \quad h = \frac{1}{2g}$

$\Rightarrow \quad h = 5.1$ cm

Hence, the correct answer is (A).

28. If area be A, then thrust force is $F = Av^2\rho$

$\Rightarrow \quad F = \frac{1}{2}\rho Av^2 + \frac{1}{4}\rho Av^2$

So, pressure P is given by

$P = \frac{3\rho Av^2}{4A} = \frac{3\rho}{4}v^2$

Hence, the correct answer is (A).

29. Area of wire is $A = \pi r^2$

Since $Y = \frac{F/A}{\Delta\ell/\ell_0}$

$\Rightarrow \quad \frac{Mg}{\pi r^2} = \frac{\Delta\ell}{\ell_0}Y$

$\Rightarrow \quad \frac{Mg}{\pi r^2} = \left(\frac{4 \times 10^{-3}}{2}\right)Y$ …(1)

Mass of load of volume V is given by

$M = V(8\rho_w)$

Now when load is immersed in liquid, then

$\frac{8V\rho_w g - 2V\rho_w g}{\pi r^2} = \frac{\Delta\ell'}{\ell_0}Y$ …(2)

$\Rightarrow \quad \frac{6V\rho_w g}{\pi r^2} = \frac{\Delta\ell'}{\ell_0}Y$

$\Rightarrow \quad \frac{\Delta\ell'}{4 \times 10^{-3}} = \frac{6V\rho_w g}{8V\rho_w g}$

$\Rightarrow \quad \Delta\ell' = \frac{6}{8} \times 4 \times 10^{-3}$ m

$\Rightarrow \quad \Delta\ell' = 3 \times 10^{-3}$ m $= 3$ mm

Hence, the correct answer is (D).

30. $P - P_0 = \frac{4S}{R}$ when S is surface tension

$\Rightarrow \quad P = \frac{4S}{R} + P_0$

$\Rightarrow \quad P = 4S\left(\frac{4\pi}{3v}\right)^{\frac{1}{3}} + P_0$

$\Rightarrow \quad P = 4S\left(\frac{4\pi}{3kt}\right)^{\frac{1}{3}} + P_0$

Also, $V = \frac{4}{3}\pi R^3$

$\Rightarrow \quad R = \left(\frac{3V}{4\pi}\right)^{1/3} = R$

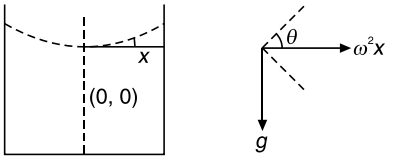
Given that, $V = kt$

So, correct form is $P = m\left(\frac{1}{t^{1/3}}\right) + c$

*No given option is correct.

31. Given that $\omega = 4\pi \text{ rads}^{-1}$

Since, $\tan\theta = \frac{dy}{dx} = \frac{\omega^2 x}{g}$



$\Rightarrow \quad y = \int_0^h dy = \int_0^x \frac{\omega^2 x}{g} dx = \frac{\omega^2 x^2}{2g}\Big|_0^{5\times10^{-2}}$

$\Rightarrow \quad y = \frac{16\pi^2 \times 25 \times 10^{-4}}{2 \times 10} = 1.9 \text{ cm} \approx 2.0 \text{ cm}$

Hence, the correct answer is (D).

32. Bulk modulus $= \frac{\text{Volumetric stress}}{\text{Volumetric strain}}$

$$K = \left|\frac{\Delta P}{\frac{\Delta V}{V}}\right| = \frac{\frac{F}{a}}{\frac{dV}{V}}$$

Here, $F = mg$, $\frac{dV}{V} = 3\frac{dr}{r}$

$\Rightarrow \quad K = \frac{\frac{mg}{a}}{3\frac{dr}{r}}$

$\Rightarrow \quad \frac{dr}{r} = \frac{mg}{3Ka}$

Hence, the correct answer is (C).

33. At depth h, $\Delta P = \frac{4T}{r}$

$\Rightarrow \quad P_1 = P_0 + \rho g h + \frac{4T}{r}$

At the surface of lake, $\Delta P' = \frac{4T}{5r/4} = \frac{16T}{5r}$

$\Rightarrow \quad P_2 = P_0 + \frac{16T}{5r}$

Also, $P_1V_1 = P_2V_2$

$\Rightarrow \quad \frac{P_1}{P_2} = \frac{r_2^3}{r_1^3}$

$\Rightarrow \quad \frac{P_0 + \rho g h + (4T/r)}{P_0 + (16T/5r)} = \frac{(5r/4)^3}{r^3}$

$\Rightarrow \quad \frac{125}{64} = \frac{10 + h}{10}$

$\Rightarrow \quad h = 9.5 \text{ cm}$

(Excess pressure is very small so we can neglect it).

Hence, the correct answer is (C).

34. For a given material, shear modulus is constant, so

$$\left(\frac{F}{A_1}\right)\left(\frac{L_1}{\Delta x_1}\right) = \left(\frac{F}{A_2}\right)\left(\frac{L_2}{\Delta x_2}\right)$$

Since, $L_2 = 2L_1$, $A_2 = L_2^2 = 4L_1^2 = 4A_1$, $\Delta x_1 = 0.5$ cm

$\Rightarrow \quad \frac{1}{A_1} \times \frac{L_1}{0.5} = \frac{1}{4A_1} \times \frac{2L_1}{\Delta x_2}$

$\Rightarrow \quad \Delta x_2 = 0.25 \text{ cm}$

Hence, the correct answer is (A).

35. Excess pressure inside the inner bubble,

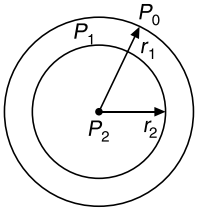
$$P_2 - P_1 = \frac{4T}{r_2} \qquad \ldots(1)$$

Excess pressure inside the outer bubble,

$$P_1 - P_0 = \frac{4T}{r_1} \qquad \ldots(2)$$

From equation (1) and (2), we get

$$P_2 - P_0 = 4T\left(\frac{1}{r_2} + \frac{1}{r_1}\right) = \frac{4T}{r}$$



where, r is required radius of a soap bubble. So

$$r = \frac{r_2 r_1}{r_1 + r_2} = \frac{4 \times 6}{4 + 6} = \frac{24}{10} = 2.4 \text{ cm}$$

Hence, the correct answer is (C).

36. Given that, $\frac{V_f}{V_i} = 9^3$ $\qquad \{\because V \propto \ell^3\}$

Since density remains same, so

mass $\propto$ volume

$\Rightarrow \quad \frac{m_f}{m_i} = 9^3$

$\Rightarrow \quad \frac{(\text{Area})_f}{(\text{Area})_i} = 9^2$

$\Rightarrow \quad \text{Stress} = \frac{\text{Weight of man}}{\text{Area}}$

$\Rightarrow \quad \frac{\sigma_2}{\sigma_1} = \left(\frac{m_f}{m_i}\right)\left(\frac{A_i}{A_f}\right)$

$\Rightarrow \quad \frac{\sigma_2}{\sigma_1} = \frac{9^3}{9^2} = 9$

Hence, the correct answer is (A).

37. Rate of flow of liquid (Q) through narrow tube is

$$Q = \frac{dv}{dt} = \frac{\pi \Pr^4}{8\eta\ell}$$

Since both the given tubes are connected in series so rate of flow of liquid is same.

$$\Rightarrow \quad \frac{\pi P_1 r_1^4}{8\eta\ell_1} = \frac{\pi P_2 r_2^4}{8\eta\ell_2}$$

$$\Rightarrow \quad r_2^4 = \left(\frac{P_1}{P_2}\right)\left(\frac{\ell_2}{\ell_1}\right) r_1^4$$

Given $P_2 = 4P_1$, $\ell_2 = \dfrac{\ell_1}{4}$

$$\Rightarrow \quad r_2^4 = \left(\frac{P_1}{4P_1}\right)\left(\frac{\ell_1}{4\ell_1}\right) r_1^4 = \frac{r_1^4}{16} = \left(\frac{r_1}{2}\right)^4$$

$$\Rightarrow \quad r_2 = \frac{r_1}{2}$$

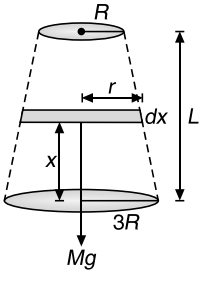
Hence, the correct answer is (B).

38. Consider a uniform cross-section of wire of length dx and radius r at a vertical distance of x from the lower end. Here,

$$r = 3R - \frac{2R}{L}x$$

Extension in wire of length dx is

$$d\ell = \frac{F dx}{AY} = \frac{Mg dx}{\pi\left(3R - \dfrac{2R}{L}x\right)^2 Y}$$



Hence total extension in wire is

$$\Delta L = \int d\ell = \int_0^L \frac{Mg dx}{\pi\left(3R - \dfrac{2R}{L}x\right)^2 Y}$$

$$\Rightarrow \quad \Delta L = \frac{Mg}{\pi Y}\int_0^L \frac{dx}{\left(3R - \dfrac{2R}{L}x\right)^2} = \frac{MgL}{3\pi R^2 Y}$$

So, extended length of wire is

$$L_f = L + \frac{MgL}{3\pi R^2 Y} = L\left(1 + \frac{Mg}{3\pi R^2 Y}\right)$$

Hence, the correct answer is (C).

39. Let v_1 and v_2 be the velocities of water when it leaks out through the hole and when it hits the ground respectively. Then, applying Bernoulli's theorem, we get

$$v_1^2 + 2gh = v_2^2$$

Also, $v_1 = \sqrt{2gH}$ …(1)

$$\Rightarrow \quad 2gH + 2gh = v_2^2 \qquad \ldots(2)$$

According to continuity Equation, $a_1 v_1 = a_2 v_2$

$$\Rightarrow \quad \pi r^2\sqrt{2gH} = \pi x^2\sqrt{2g(H+h)} \qquad \{\text{using (1) and (2)}\}$$

$$\Rightarrow \quad x^2 = r^2\sqrt{\frac{H}{H+h}}$$

$$\Rightarrow \quad x = r\left(\frac{H}{H+h}\right)^{\frac{1}{4}}$$

Hence, the correct answer is (A).

40. Equation of motion for the point mass

$$ma = mg - kv \qquad \ldots(1)$$

$$\Rightarrow \quad \frac{dv}{dt} = \frac{mg - kv}{m}$$

$$\Rightarrow \quad \frac{dv}{mg - kv} = \frac{dt}{m}$$

Integrating $\displaystyle\int_0^v \frac{dv}{mg - kv} = \frac{1}{m}\int_0^t dt$

$$\Rightarrow \quad -\frac{1}{k}\left[\ell n(mg - kv)\right]_0^v = \frac{t}{m}$$

$$\Rightarrow \quad \ell n\left(\frac{mg - kv}{mg}\right) = \frac{-k}{m}t$$

$$\Rightarrow \quad 1 - \frac{kv}{mg} = e^{\frac{-kt}{m}}$$

$$\Rightarrow \quad \frac{kv}{mg} = 1 - e^{-\frac{kt}{m}}$$

$$\Rightarrow \quad v = \frac{mg}{k}\left(1 - e^{\frac{-kt}{m}}\right) \qquad \ldots(2)$$

Substituting (2) in (1), we get

$$ma = mg - k \times \frac{mg}{k}\left(1 - e^{-\frac{kt}{m}}\right)$$

$$\Rightarrow \quad a = ge^{\frac{-kt}{m}}$$

Hence, the correct answer is (C).

41. Bulk modulus, $B = \dfrac{\text{Normal stress}}{\text{Volumetric strain}}$

Pressure, $P = \dfrac{N}{A} = \dfrac{N}{(2\pi a)b}$

Volumetric strain $\dfrac{\Delta V}{V} = \dfrac{2\pi a \Delta a \times b}{\pi a^2 \times b} = \dfrac{2\Delta a}{a}$

$$\Rightarrow \quad B = \frac{N}{2\pi ab} \times \frac{a}{2\Delta a}$$

$$\Rightarrow \quad N = (4\pi b \Delta a)B$$

So, required force equals the frictional force

$$\Rightarrow \quad F_{\text{required}} = f = \mu N = (4\pi\mu B b)\Delta a$$

Hence, the correct answer is (D).

42. Given that time $t = 5 \text{ min} = 300 \text{ s}$, volume, $V = 15 \text{ ltr} = 15 \times 10^{-3} \text{ m}^3$ and diameter, $D = \dfrac{2}{\sqrt{\pi}}$ cm

So, cross sectional area of tap is

$$A = \pi\left(\frac{1}{\sqrt{\pi}} \times 10^{-2}\right)^2 = 10^{-4}\text{m}^2$$

Velocity of water is

$$v = \frac{Q}{A} = \frac{V}{At} = \frac{15\times10^{-3}}{1\times10^{-4}\times300} = 0.5 \text{ ms}^{-1}$$

$$\Rightarrow \quad R_w = \frac{\rho v D}{\eta} = \frac{10^3\times0.5\times\frac{2}{\sqrt{\pi}}\times10^{-2}}{10^{-3}} = 5642 \approx 5500$$

Hence, the correct answer is (A).

43. For cylindrical shape, excess pressure is given by $\Delta P = \frac{T}{R}$

*None of the given options is correct.

44. For air trapped in the tube, we have

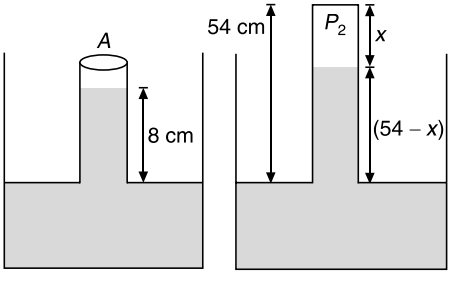
$$P_1V_1 = P_2V_2$$

where, $P_1 = P_{\text{atm}} = 76(\rho_m g)$ and

$$P_2 = P_{\text{atm}} - (54 - x)\rho_m g$$

$$\Rightarrow \quad P_2 = 76\rho_m g - 54\rho_m g + x\rho_m g = (22 + x)\rho_m g$$

where, ρ_m =Density of mercury



Also, $V_1 = 8A$ and $V_2 = Ax$

$$\Rightarrow \quad (76\rho_m g)(8A) = (22 + x)\rho_m g Ax$$

$$\Rightarrow \quad 608 = 22x + x^2$$

$$\Rightarrow \quad x^2 + 22x - 608 = 0$$

$$\Rightarrow \quad x^2 + 38x - 16x - 608 = 0$$

$$\Rightarrow \quad x(x + 38) - 16(x + 38) = 0$$

$$\Rightarrow \quad (x - 16)(x + 38) = 0$$

$$\Rightarrow \quad x = 16 \text{ cm}$$

Hence, the correct answer is (A).

45. $(R\sin\alpha)d_2 + (R\cos\alpha)d_2 + R(1 - \cos\alpha)d_1 = R(1 - \sin\alpha)d_1$

$$\Rightarrow \quad (\sin\alpha + \cos\alpha)d_2 = (\cos\alpha - \sin\alpha)d_1$$

$$\Rightarrow \quad \frac{d_1}{d_2} = \frac{\sin\alpha + \cos\alpha}{\cos\alpha - \sin\alpha}$$

$$\Rightarrow \quad \frac{d_1}{d_2} = \frac{1 + \tan\alpha}{1 - \tan\alpha}$$

Hence, the correct answer is (C).

46. The bubble will detach when
(Buoyant Force) ≥ (Surface Tension Force)

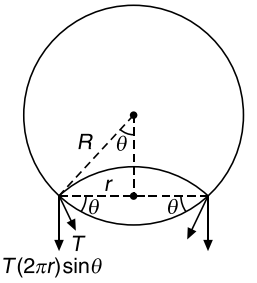
$$\Rightarrow \quad \left(\frac{4}{3}\pi R^3\right)\rho_w g \geq T(2\pi r)\sin\theta$$

Since, $\sin\theta = \frac{r}{R}$

$$\Rightarrow \quad \left(\frac{4}{3}\pi R^3\right)\rho_w g \geq \frac{2\pi r^2 T}{R}$$

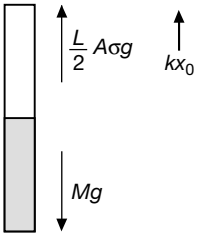
$$\Rightarrow \quad r^2 = \frac{2R^4\rho_w g}{3T}$$

$$\Rightarrow \quad r = R^2\sqrt{\frac{2\rho_w g}{3T}}$$



Hence, the correct answer is (A).

47. For equilibrium, we have



$$Mg = \frac{L}{2}A\sigma g + kx_0$$

$$\Rightarrow \quad x_0 = \frac{Mg}{k}\left(1 - \frac{LA\sigma}{2M}\right)$$

Hence, the correct answer is (C).

48. The force due to the surface tension will balance the weight, so

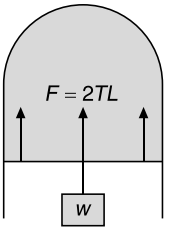
$$F = w$$

$$\Rightarrow \quad 2TL = w$$

$$\Rightarrow \quad T = \frac{w}{2L}$$

Substituting the given values, we get

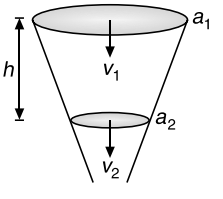
$$T = \frac{1.5\times10^{-2} \text{ N}}{2\times30\times10^{-2} \text{ m}} = 0.025 \text{ Nm}^{-1}$$



Hence, the correct answer is (C).

49. According to equation of motion,

$$v_2 = \sqrt{v_1^2 + 2gh} = \sqrt{(0.4)^2 + 2\times10\times0.2} \approx 2 \text{ ms}^{-1}$$



According to equation of continuity

$$a_1v_1 = a_2v_2$$

$$\pi\times\left(\frac{8\times10^{-3}}{2}\right)^2\times0.4 = \pi\times\left(\frac{d_2}{2}\right)^2\times2$$

$$d_2 = 3.6\times10^{-3} \text{ m}$$

Hence, the correct answer is (D).

50. Here, surface tension, $T = 0.03\ \text{Nm}^{-1}$

$r_1 = 3\ \text{cm} = 3\times10^{-2}\ \text{m}$, $r_2 = 5\ \text{cm} = 5\times10^{-2}\ \text{m}$

Since bubble has two surfaces, so, initial surface area of the bubble

$$A_i = 2\times4\pi r_1^2 = 2\times4\pi\times(3\times10^{-2})^2 = 72\pi\times10^{-4}\ \text{m}^2$$

Final surface area of the bubble is

$$A_f = 2\times4\pi r_2^2 = 2\times4\pi(5\times10^{-2})^2 = 200\pi\times10^{-4}\ \text{m}^2$$

Increase in surface is

$$\Delta A = 200\pi\times10^{-4} - 72\pi\times10^{-4} = 128\pi\times10^{-4}\ \text{m}^2$$

Since work done W is

$$W = T\Delta A$$

$$\Rightarrow\quad W = 0.03\times128\times\pi\times10^{-4} = 3.84\pi\times10^{-4}$$

$$\Rightarrow\quad W = 4\pi\times10^{-4}\ \text{J} = 0.4\pi\ \text{mJ}$$

Hence, the correct answer is (D).

51. Given that $U = \dfrac{a}{x^{12}} - \dfrac{b}{x^6}$

So, force $F = -\dfrac{dU}{dx} = -\dfrac{d}{dx}\left(\dfrac{a}{x^{12}} - \dfrac{b}{x^6}\right)$

$$\Rightarrow\quad F = -\left[\frac{-12a}{x^{13}} + \frac{6b}{x^7}\right] = \left[\frac{12a}{x^{13}} - \frac{6b}{x^7}\right]$$

At equilibrium, $F = 0$

$$\Rightarrow\quad \frac{12a}{x^{13}} - \frac{6b}{x^7} = 0$$

$$\Rightarrow\quad x^6 = \frac{2a}{b}$$

$$U_{\text{at equilibrium}} = U_{\text{eq}} = \frac{a}{(2a/b)^2} - \frac{b}{(2a/b)}$$

$$\Rightarrow\quad U_{\text{eq}} = \frac{ab^2}{4a^2} - \frac{b^2}{2a} = \frac{b^2}{4a} - \frac{b^2}{2a} = -\frac{b^2}{4a}$$

Since, $U_\infty = U\big|_{x\to\infty} = 0$

$$\Rightarrow\quad D = \left[U_\infty - U_{\text{eq}}\right] = \left[0 - \left(-\frac{b^2}{4a}\right)\right] = \frac{b^2}{4a}$$

Hence, the correct answer is (D).

52. Since $\rho_{\text{oil}} < \rho_{\text{water}}$, so oil should be above the water. Also, $\rho > \rho_{\text{oil}}$, hence the ball will sink in the oil but $\rho < \rho_{\text{water}}$ so ball will float in the water.

Hence, the correct answer is (C).

53. For the same material, Young's modulus is the same and it is given that the volume is the same and the area of cross-section for the wire ℓ_1 is A and that of ℓ_2 is $3A$.

$$V = V_1 = V_2$$

$$\Rightarrow\quad V = A\times\ell_1 = 3A\times\ell_2$$

$$\Rightarrow\quad \ell_2 = \frac{\ell_1}{3}$$

$$Y = \frac{F/A}{\Delta\ell/\ell}$$

So, $F_1 = YA\dfrac{\Delta\ell_1}{\ell_1}$ and $F_2 = Y(3A)\dfrac{\Delta\ell_2}{\ell_2}$

For the same extension $\Delta\ell_1 = \Delta\ell_2 = \Delta x$

$$\Rightarrow\quad F_2 = Y(3A)\frac{\Delta x}{\ell_1/3} = 9\left(\frac{YA\Delta x}{\ell_1}\right) = 9F_1$$

$$\Rightarrow\quad F_2 = 9F$$

Hence, the correct answer is (D).

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Single Correct Choice Type Problems

1. Let water level fall by x m due to oil, so we have

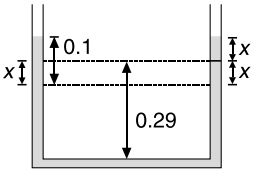
$$(800)(g)(0.1) = (1000)(g)(2x)$$

$$\Rightarrow\quad x = \frac{8}{200} = \frac{1}{25} = 0.04\ \text{m}$$

So, $h_2 = 0.04 + 0.29 = 0.33$ m and

$$h_1 = 0.29 - 0.04 + 0.1 = 0.35$$

$$\Rightarrow\quad \frac{h_1}{h_2} = \frac{35}{33}$$



Hence, the correct answer is (B).

2. Using geometry, we get

$$\frac{b}{R} = \cos\left(\theta + \frac{\alpha}{2}\right)$$

$$\Rightarrow\quad R = \frac{b}{\cos\left(\theta + \dfrac{\alpha}{2}\right)}$$

Applying pressure equation along path $MNTK$

$$p_0 - \frac{2S}{R} + h\rho g = p_0$$

Substituting the value of R, we get

$$h = \frac{2S}{R\rho g} = \frac{2S}{b\rho g}\cos\left(\theta + \frac{\alpha}{2}\right)$$

Hence, the correct answer is (D).

3. Since, $\Delta l = \dfrac{FL}{AY} = \dfrac{FL}{(\pi r^2)Y}$

$$\Rightarrow\quad \Delta l \propto \frac{L}{r^2}$$

$$\Rightarrow\quad \frac{\Delta l_1}{\Delta l_2} = \frac{L/R^2}{2L/(2R)^2} = 2$$

Hence, the correct answer is (C).

4. Let V_1 be the total material volume of shell, V_2 be the total inside volume of shell and x be the fraction of V_2 volume filled with water.

For floating condition, we have

Total weight = Upthrust

$$\Rightarrow \quad V_1\rho_c g + (xV_2)(1)g = \left(\frac{V_1+V_2}{2}\right)(1)g$$

$$\Rightarrow \quad x = 0.5 + (0.5-\rho_c)\frac{V_1}{V_2}$$

From here we can see that

$x > 0.5$ if $\rho_c < 0.5$

Hence, the correct answer is (A).

5. Since, $\Delta p_1 = \frac{4T}{r_1}$ and $\Delta p_2 = \frac{4T}{r_2}$

$\Rightarrow \quad r_1 < r_2$

$\Rightarrow \quad \Delta p_1 > \Delta p_2$

So, air will flow from 1 to 2 and volume of bubble at end 1 will decrease.

Hence, the correct answer is (B).

6. Force from right hand side liquid on left hand side liquid

(i) due to surface tension is $F_1 = 2RT$, towards right

(ii) due to liquid pressure is

$$F_2 = \int_{x=0}^{x=h} (p_0 + \rho g h)(2Rx)\,dx$$

$$\Rightarrow \quad F_2 = \left(2p_0Rh + R\rho g h^2\right), \text{ towards left}$$

So, net force is

$$|F_2 - F_1| = \left|2p_0Rh + R\rho g h^2 - 2RT\right|$$

Hence, the correct answer is (B).

7. Applying continuity equation at 1 and 2, we get

$$A_1v_1 = A_2v_2 \qquad \ldots(1)$$

Further applying Bernoulli's equation at these two points, we get

$$P_0 + \rho g h + \frac{1}{2}\rho v_1^2 = P_0 + 0 + \frac{1}{2}\rho v_2^2 \qquad \ldots(2)$$

Solving equations (1) and (2), we have

$$v_2^2 = \frac{2gh}{1-\left(A_2^2/A_1^2\right)}$$

Substituting the values, we have

$$v_2^2 = \frac{2\times 10\times 2.475}{1-(0.1)^2} = 50 \text{ m}^2\text{s}^{-2}$$

Hence, the correct answer is (A).

8. From the definition of bulk modulus,

Since, $B = \left|\frac{dP}{dV/V}\right|$

Substituting the values, we get

$$B = \frac{(1.165-1.01)\times 10^5}{(10/100)}\text{Pa} = 1.55\times 10^5\,\text{Pa}$$

Hence, the correct answer is (D).

9. From the graph $\ell = 10^{-4}$ m, $F = 20$ N

$A = 10^{-6}$ m^2, $L = 1$ m

$$\Rightarrow \quad Y = \frac{FL}{A\ell}$$

$$\Rightarrow \quad Y = \frac{20\times 1}{10^{-6}\times 10^{-4}} = 20\times 10^{10}$$

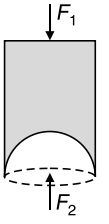
$$\Rightarrow \quad Y = 2\times 10^{11} \text{ Nm}^{-2}$$

Hence, the correct answer is (A).

10. As the block moves up with the fall of coin, ℓ decreases, similarly h will also decrease because when the coin is in water, it displaces water equal to its own volume only.

Hence, the correct answer is (D).

11. Net force $F_2 - F_1$ equals the upthrust (U)



$$\Rightarrow \quad F_2 - F_1 = U$$

$$\Rightarrow \quad F_2 = F_1 + U$$

$$\Rightarrow \quad F_2 = \rho g h\left(\pi R^2\right) + V\rho g$$

$$\Rightarrow \quad F_2 = \rho g\left(V + \pi R^2 h\right)$$

Hence, the correct answer is (D).

12. Velocity of efflux when the hole is at depth h, $v = \sqrt{2gh}$

Rate of flow of water from square hole

$$Q_1 = a_1v_1 = L^2\sqrt{2gy}$$

Rate of flow of water from circular hole

$$Q_2 = a_2v_2 = \pi R^2\sqrt{2g(4y)}$$

According to problem $Q_1 = Q_2$

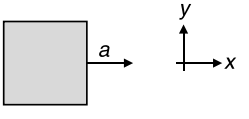
$$\Rightarrow \quad L^2\sqrt{2gy} = \pi R^2\sqrt{2g(4y)}$$

$$\Rightarrow \quad R = \frac{L}{\sqrt{2\pi}}$$

Hence, the correct answer is (A).

13. When a fluid (gas or liquid) is accelerated along positive x-direction, then pressure decreases along positive x-direction. Change in pressure has following differential equation.

$$\frac{dP}{dx} = -\rho a$$



where, ρ is the density of the fluid.

Therefore, pressure is lower in front side.

Hence, the correct answer is (B).

14. $a_1v_1 = a_2v_2$

where $v_2^2 = v_1^2 + 2gh$

$$v_2^2 = 1 + 2\times 10\times 0.15$$

$$\Rightarrow \quad v_2 = 2 \text{ ms}^{-1}$$

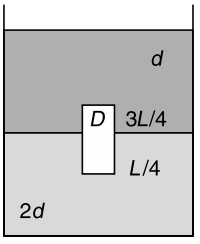
$$\Rightarrow \quad 10^{-4}\times 1 = a_2\times 2$$

$$\Rightarrow \quad a_2 = 5\times 10^{-5} \text{ m}^{-2}$$

Hence, the correct answer is (C).

15. Considering vertical equilibrium of cylinder, we get

$$W = U_{\text{total}}$$



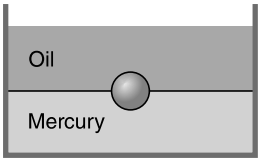
$$\Rightarrow \quad \left(\frac{A}{5}\right)(L)gD = \left(\frac{A}{5}\right)\left(\frac{3L}{4}\right)gd + \left(\frac{A}{5}\right)\left(\frac{L}{4}\right)g(2d)$$

$$\Rightarrow \quad D = \left(\frac{3}{4}\right)d + \left(\frac{1}{4}\right)(2d)$$

$$\Rightarrow \quad D = \frac{5}{4}d$$

Hence, the correct answer is (A).

16. As the sphere floats in the liquid. Therefore, its weight will be equal to the upthrust force on it.



Weight of sphere is

$$W = \frac{4}{3}\pi R^3 \rho g \qquad \ldots(1)$$

Upthrust due to oil and mercury is

$$\frac{2}{3}\pi R^3 \times \sigma_{oil} g + \frac{2}{3}\pi R^3 \sigma_{Hg} g \qquad \ldots(2)$$

Equating (1) and (2)

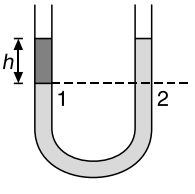
$$\frac{4}{3}\pi R^3 \rho g = \frac{2}{3}\pi R^3 0.8g + \frac{2}{3}\pi R^3 \times 13.6g$$

$$\Rightarrow \quad 2\rho = 0.8 + 13.6 = 14.4$$

$$\Rightarrow \quad \rho = 7.2$$

Hence, the correct answer is (C).

17. Since, $P_1 = P_2$



$$\Rightarrow \quad P_0 + \rho_1 gh = P_0 + \rho_{II} gh$$

$$\Rightarrow \quad \rho_I = \rho_{II}$$

Hence, the correct answer is (B).

18. For a freely falling system $g_{\text{eff}} = 0$ and since, upthrust U is

$$U = V_{\text{imm}} \rho_{\text{liq}} g_{\text{eff}}$$

$$\Rightarrow \quad U = 0$$

Hence, the correct answer is (A).

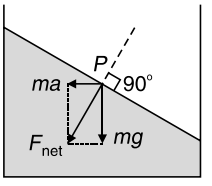
19. Since $\Delta\ell = \dfrac{F\ell}{AY} = \dfrac{F\ell}{(\pi d^2/4)Y}$

$$\Rightarrow \quad \Delta\ell \propto \frac{\ell}{d^2}$$

$\dfrac{\ell}{d^2}$ is maximum in (A)

Hence, the correct answer is (A).

20. Net force on the free surface of the liquid in equilibrium (from accelerated frame) should be perpendicular to it. Forces on a water particle P on the free surfaces have been shown in the figure. In the figure ma is the pseudo force.



Hence, the correct answer is (C).

Multiple Correct Choice Type Problems

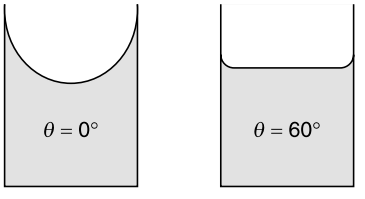
1. For A, we have $h = \dfrac{2T\cos(60°)}{\rho g r}$

$$\Rightarrow \quad h = \frac{2 \times 0.075 \times 1}{10^3 \times 10 \times 2 \times 10^{-4} \times 2} \times 100 \text{ cm} = 3.75 \text{ cm}$$

For B, we have $h = \dfrac{2T\cos(0°)}{\rho g r} = 7.5$ cm

For C, angle of contact will adjust so as to make $h = 5$ cm

For D, the shape of meniscus will be different in the two cases



So, correction is different for both cases.

Hence, (A), (B) and (D) are correct.

2. For floating, $W = U$

$$\Rightarrow \quad (\rho_1 + \rho_2)Vg = (\sigma_1 + \sigma_2)Vg$$

Since string is taut, so, $\rho_1 < \sigma_1$ and $\rho_2 > \sigma_2$

$$|\vec{v}_P| = \frac{2r^2 g}{2\eta_2}(\sigma_2 - \rho_1), \text{ upward terminal velocity}$$

$$|\vec{v}_Q| = \frac{2r^2 g}{9\eta_1}(\rho_2 - \sigma_1), \text{ downward terminal velocity}$$

$$\Rightarrow \quad \left|\frac{\vec{v}_P}{\vec{v}_Q}\right| = \frac{\eta_1}{\eta_2}$$

Further, $\vec{v}_P \cdot \vec{v}_Q$ will be negative as they are opposite to each other.

Hence, (A) and (D) are correct.

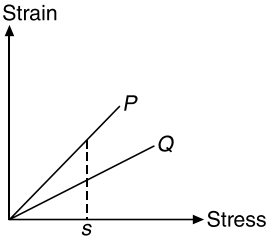
3. Since, $Y = \dfrac{\text{Stress}}{\text{Strain}}$

$\Rightarrow \quad Y \propto \dfrac{1}{\text{Strain}}$

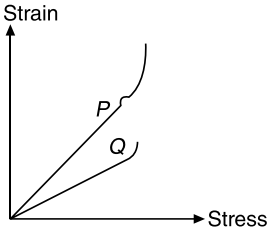
For same stress, say σ

$(\text{Strain})_Q < (\text{Strain})_P$

$\Rightarrow \quad Y_Q > Y_P$



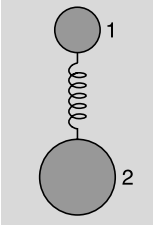
So, P is more ductile than Q. Further, from the given figure we can also see that breaking stress of P is more than Q. So, it has more tensile strength.



Hence, (A) and (B) are correct.

4. On small sphere i.e., sphere 1

$$\frac{4}{3}\pi R^3(\rho)g + kx = \frac{4}{3}\pi R^3(2\rho)g \qquad \ldots(1)$$



On large sphere i.e., sphere 2

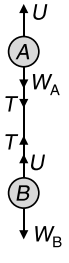
$$\frac{4}{3}\pi R^3(3\rho)g = \frac{4}{3}\pi R^3(2\rho)g + kx \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$x = \frac{4\pi R^3 \rho g}{3k}$$

Hence, (A) and (D) are correct.

5. Upthrust $U = Vd_F g$



For equilibrium of A, $U = T + W_A$

$\Rightarrow \quad Vd_F g = T + W_A$

$\Rightarrow \quad Vd_F g = T + Vd_A g \qquad \ldots(1)$

For equilibrium of B, $T + U = W_B$

$$T + Vd_F g = Vd_B g \qquad \ldots(2)$$

Adding equations (1) and (2), we get

$$2d_F = d_A + d_B$$

From equation (1), we see that $d_F > d_A \qquad \{\because\ T > 0\}$

From equation (2), we see that, $d_B > d_F$

Hence, (A), (B) and (D) are correct.

6. Liquid will apply an upthrust on m. An equal force will be exerted (from Newton's third law) on the liquid. Hence, A will read less than 2 kg and B more than 5 kg.

Hence, (B) and (C) are correct.

Reasoning Based Questions

1. From continuity equation, $Av = \text{constant}$

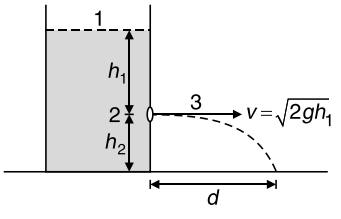
$\Rightarrow \quad A \propto \dfrac{1}{v}$

Hence, the correct answer is (A).

Matrix Match/Column Match Type Questions

1. A → (p); B → (p); C → (p); D → (s)

$$d = 2\sqrt{h_1 h_2} = \sqrt{4h_1 h_2}$$



This is independent of the value of g.

For A, $g_{\text{eff}} > g$ $d = \sqrt{4h_1 h_2} = 1.2$ m

For B, $g_{\text{eff}} < g$ $d = \sqrt{4h_1 h_2} = 1.2$ m

For C, $g_{\text{eff}} = g$ $d = \sqrt{4h_1 h_2} = 1.2$ m

For D, $g_{\text{eff}} = 0$

No water leaks out of jar. As there will be no pressure difference between top of the container and any other point.

$p_1 = p_2 = p_3 = p_0$

Linked Comprehension Type Questions

1. From continuity equation, we have

$$A_1 v_1 = A_2 v_2$$

where, $A_1 = 400A_2$

Also, $r_1 = 20r_2$ and $A = \pi r^2$

$\Rightarrow \quad v_2 = \left(\dfrac{A_1}{A_2}\right) v_1 = 400v_1$

$\Rightarrow \quad v_2 = 400(5)\ \text{mms}^{-1} = 2000\ \text{mms}^{-1} = 2\ \text{ms}^{-1}$

Hence, the correct answer is (C).

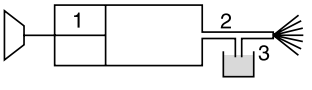
2. According to Bernoulli equation, we have

$$p_1 - p_2 = \frac{1}{2}\rho_a v_a^2 \quad \ldots(1)$$

$$p_3 - p_2 = \frac{1}{2}\rho_l v_l^2 \quad \ldots(2)$$

Since, $p_3 = p_1$

$$\Rightarrow \quad \frac{1}{2}\rho_l v_l^2 = \frac{1}{2}\rho_a v_a^2$$



$$\Rightarrow \quad v_l = \sqrt{\frac{\rho_a}{\rho_l}}\, v_a$$

$$\Rightarrow \quad \text{Volume flow rate} \propto \sqrt{\frac{\rho_a}{\rho_l}}$$

Hence, the correct answer is (A).

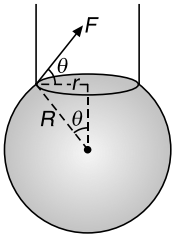
3. Vertical force due to surface tension

$$F_V = F\sin\theta = [(T)(2\pi r)]\left(\frac{r}{R}\right)$$

$$\Rightarrow \quad F_V = \frac{2\pi r^2 T}{R}$$

Hence, the correct answer is (C).

4. Since, $F_V = \dfrac{2\pi r^2 T}{R} = mg = \dfrac{4}{3}\pi R^3 \rho g$



$$\Rightarrow \quad R^4 = \frac{3r^2 T}{2\rho g} = \frac{3\times(5\times10^{-4})^2(0.11)}{2\times10^3\times10}$$

$$\Rightarrow \quad R^4 = 4.125\times10^{-12}\ \text{m}^4$$

$$\Rightarrow \quad R = 1.425\times10^{-3}\ \text{m} \approx 1.4\times10^{-3}\ \text{m}$$

Hence, the correct answer is (A).

5. Surface energy is $E = (4\pi R^2)T$

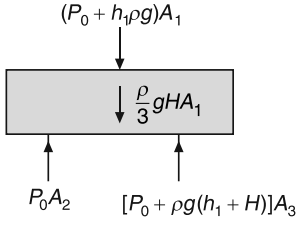
$$\Rightarrow \quad E = (4\pi)(1.4\times10^{-3})^2(0.11)$$

$$\Rightarrow \quad E = 2.7\times10^{-6}\ \text{J}$$

Hence, the correct answer is (B).

6. Let A_1 = Area of cross-section of cylinder = $4\pi r^2$

A_2 = Area of base of cylinder in air = πr^2



and A_3 = Area of base of cylinder in water = $A_1 - A_2 = 3\pi r^2$

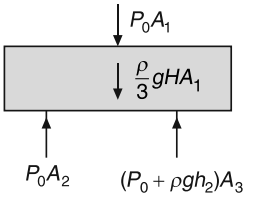
Drawing free body diagram of cylinder:

Equating the net downward forces and net upward forces, we get

$$h_1 = \frac{5}{3}H$$

Hence, the correct answer is (C).

7. Again, equating the forces, we get



Hence, the correct answer is (A).

8. For $h_2 < \dfrac{4h}{9}$, buoyant force will further decrease.

Hence, the cylinder remains at its original position.

No solution is required.

Hence, the correct answer is (A).

Integer/Numerical Answer Type Questions

1. Taking the velocity w.r.t. train, from equation of continuity, we get

$$4S_t v_t = (4S_t - S_t)v$$

$$\Rightarrow \quad v = \frac{4}{3}v_t$$

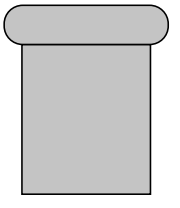
According to Bernoulli's equation, we get

$$\Rightarrow \quad p_0 + \frac{1}{2}\rho v_t^2 = p + \frac{1}{2}\rho\left(\frac{16}{9}v_t^2\right)$$

$$\Rightarrow \quad p_0 - p = \frac{7\rho v_t^2}{18} = \frac{7}{2(9)}\rho v_t^2$$

$$\Rightarrow \quad N = 9$$

2. Water surface will break, when

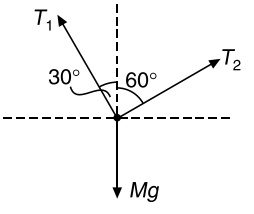


$$P_0 + h\rho g \geq P_0 + \frac{T}{h/2}$$

$$\Rightarrow \quad h^2 \geq \frac{2T}{\rho g}$$

$$\Rightarrow \quad h \geq \sqrt{\frac{2T}{\rho g}}$$

3. Since $\sin\theta$, $T_1\sin(30^\circ) = T_2\sin(60^\circ)$



$$\Rightarrow \quad \frac{T_1}{2} = \frac{T_2\sqrt{3}}{2}$$

$$\Rightarrow \quad T_1 = \sqrt{3}T_2$$

Since $\Delta\ell = \dfrac{F\ell}{AY}$

$$\Rightarrow \quad \frac{\Delta\ell_2}{\Delta\ell_1} = \left(\frac{T_2\ell_2}{A_2Y_2}\right)\left(\frac{A_1Y_1}{T_1\ell_1}\right)$$

$$\Rightarrow \quad \frac{\Delta\ell_2}{\Delta\ell_1} = \frac{\sqrt{3}T_2}{T_1}\times\frac{2}{1} = 2$$

4. From mass conservation, we have

$$\frac{4}{3}\pi R^3\rho = K\frac{4}{3}\pi r^3\rho$$

$$\Rightarrow \quad R = K^{\frac{1}{3}} r$$

$$\Rightarrow \quad \Delta U = T\Delta A = T\left(K4\pi r^2 - 4\pi R^2\right)$$

$$\Rightarrow \quad \Delta U = T\left(K4\pi R^2 K^{-\frac{2}{3}} - 4\pi R^2\right)$$

$$\Rightarrow \quad \Delta U = T\left[4\pi R^2 T\left(\mathrm{K}^{\frac{1}{3}} - 1\right)\right]$$

Putting the values, we get

$$10^{-3} = \frac{10^{-1}}{4\pi} \times 4\pi \times 10^{-4}\left[K^{\frac{1}{3}} - 1\right]$$

$$\Rightarrow \quad 100 = K^{\frac{1}{3}} - 1$$

$$\Rightarrow \quad K^{\frac{1}{3}} \cong 100 = 10^2$$

Given that $K = 10^{\alpha}$

$$\Rightarrow \quad 10^{\frac{\alpha}{3}} = 10^2$$

$$\Rightarrow \quad \frac{\alpha}{3} = 2$$

$$\Rightarrow \quad \alpha = 6$$

5. Terminal velocity is given by

$$v_T = \frac{2}{9}\frac{r^2}{\eta}(d - \rho)g$$

$$\Rightarrow \quad \frac{v_P}{v_Q} = \frac{r_P^2}{r_Q^2} \times \frac{\eta_Q}{\eta_P} \times \frac{(d - \rho_P)}{(d - \rho_Q)}$$

$$\Rightarrow \quad \frac{v_P}{v_Q} = \left(\frac{1}{0.5}\right) \times \left(\frac{2}{3}\right) \times \frac{(8 - 0.8)}{(8 - 1.6)}$$

$$\Rightarrow \quad \frac{v_P}{v_Q} = 4 \times \frac{2}{3} \times \frac{7.2}{6.4} = 3$$

6. Although not given in the question, but we will have to assume that temperatures of A and B are same.

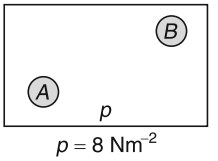
$$\frac{n_B}{n_A} = \frac{p_B V_B / RT}{p_A V_A / RT} = \frac{p_B V_B}{p_A V_A}$$

If S be the surface tension of bubble, then

$$p_A = p + \frac{45}{r_A},\ p_B = p + \frac{4S}{r}$$

$$\Rightarrow \quad V_A = \frac{4}{3}\pi r_A^3,\ V_B = \frac{4}{3}\pi r_B^3$$

$$\Rightarrow \quad \frac{n_B}{n_A} = \frac{\left(p = \frac{4S}{r_B}\right)\left(\frac{4}{3}\pi r_B^3\right)}{\left(p + \frac{4S}{r_A}\right)\left(\frac{4}{3}\pi r_A^3\right)}$$



Substituting the values, we get

$$\frac{n_B}{n_A} = 6$$

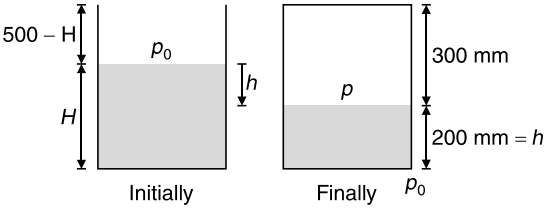
7. Assuming that the temperature of enclosed air is constant i.e., pV = constant

$$\Rightarrow \quad p_0 V_0 = pV$$

$$p = p_0 - \rho g h \qquad \ldots(1)$$

$$p_0[A(500 - H)] = p[A(300)] \qquad \ldots(2)$$

where, $p = p_0 - \rho g h$ and $h = H - 200$



Solving these two equations, we get $H = 206$ mm

So, level fall is

$$h = H - 200 = (206 - 200)\ \text{mm} = 6\ \text{mm}$$

Test Your Concepts-I (Based on Thermometry and Thermal Expansion)

1. (a) When the temperature decreases, the length of the ruler also decreases through thermal contraction. Below 20 °C, each centimetre division is actually somewhat shorter than 1 cm, so the steel ruler gives readings that are too long.

(b) At 40 °C, the increase in length of the ruler is

$$\Delta\ell = \ell\alpha\Delta T$$

$$\Rightarrow \quad \Delta\ell = (20)(1.2\times10^{-5})(40-20)$$

$$\Rightarrow \quad \Delta\ell = 0.48\times10^{-2} \text{ cm}$$

So, the actual length of the ruler is,

$$\ell' = \ell + \Delta\ell = 20.0048 \text{ cm}$$

2. (a) The 88.42 cm mark on aluminium rod is actually at a greater distance from the zero position than indicated because of the increase in temperature $\Delta T = 30$ °C. This increased length is

$$\Delta\ell = \alpha_{A\ell}\ell_{A\ell}\Delta T = (2.55\times10^{-5})(88.42)(30)$$

$$\Rightarrow \quad \Delta\ell = 0.068 \text{ cm}$$

(b) At 35 °C the measuring rod at 88.42 cm mark is actually $(88.42 + 0.068)$ cm $= 88.49$ cm long, as calculated from part (a). It thus measures 88.49 cm length of steel at 35 °C.

3. At 30 °C, the copper rod will be of length $\ell(1+\alpha_c\Delta\theta)$, while adjacent 'centimetre' marks on the steel tape will be separated by a distance of $(1)(1+\alpha_s\Delta\theta)$. Therefore, the number of centimetres reading on the tape is

$$\text{Reading (in cm)} = \frac{\ell(1+\alpha_c\Delta\theta)}{(1)(1+\alpha_s\Delta\theta)}$$

$$\Rightarrow \quad \text{Reading (in cm)} = \frac{(90)(1+1.7\times10^{-5}\times20)}{(1)(1+1.2\times10^{-5}\times20)}$$

$$\Rightarrow \quad \text{Reading (in cm)} = 90\left(\frac{1+3.4\times10^{-4}}{1+2.4\times10^{-4}}\right)$$

Since $\dfrac{1}{1+x} = (1+x)^{-1} \cong 1-x$ for $x \ll 1$, so we get

$$90\left(\frac{1+3.4\times10^{-4}}{1+2.4\times10^{-4}}\right) = 90(1+3.4\times10^{-4})(1-2.4\times10^{-4})$$

So, reading (in cm) is $\approx 90(1+3.4\times10^{-4}-2.4\times10^{-4})$

$$\Rightarrow \quad \text{Reading (in cm)} = 90 + 0.009 \approx 90.01 \text{ cm}$$

So, the tape will read 90.01 cm

4. (i) Let V_C be the volume of the container and V_L be the volume of the liquid. According to the condition given, we have

$$\frac{V_L}{V_C} = \text{constant (i.e., independent of temperature)}$$

$$\Rightarrow \quad \frac{\Delta V_L}{V_L} = \frac{\Delta V_C}{V_C}$$

$$\Rightarrow \quad \gamma\Delta T = \gamma_C\Delta T = (3\alpha)\Delta T$$

For container, coefficient of volume expansion is $\gamma_C = 3\alpha$. Hence

$$\gamma = 3\alpha$$

(ii) If one heating, H does not change, then

$$H = \frac{V}{A} = \text{constant}$$

$$\Rightarrow \quad V \propto A$$

$$\Rightarrow \quad \frac{\Delta V}{V} = \frac{\Delta A}{A}$$

$$\Rightarrow \quad \gamma\Delta T = \beta_C\Delta T$$

$$\Rightarrow \quad \gamma = 2\alpha \qquad \{\because\ \beta_C = 2\alpha\}$$

(iii) This is exactly same as part (i), so $\gamma = 3\alpha$

5. At higher temperatures actual reading is more than the scale reading. The error in the reading is given by

$$\Delta\ell = (\text{scale reading})(\alpha)(\Delta T)$$

$$\Rightarrow \quad \Delta\ell = (60)(1.2\times10^{-5})(27-15) = 0.00864 \text{ cm}$$

6. Volume expansion of mercury with respect to bulb is

$$\Delta V = V_0(\gamma - 3\alpha)t$$

Find area of cross section of capillary is

$$A_f = A_0(1+2\alpha t)$$

Height to which mercury rises in capillary is

$$h = \frac{\Delta V}{A_f} = \frac{V_0(\gamma-3\alpha)t}{A_0(1+2\alpha t)}$$

7. The time period of second's pendulum is 2 second. As the temperature increases, the length and hence time period increases and so the clock becomes slow and loses the time. The change in time period is given by

$$\Delta t = \frac{1}{2}t\alpha\Delta T = \left(\frac{1}{2}\right)(2)(1.2\times10^{-5})(30-20)$$

$$\Rightarrow \quad \Delta t = 1.2\times10^{-4} \text{ sec}$$

So, new time period is,

$$t' = t + \Delta t = (2 + 1.2\times10^{-4})$$

$$\Rightarrow \quad t' = 2.00012 \text{ sec}$$

So, time lost in one week, i.e., 7 days is

$$\Delta t' = \frac{(1.2\times10^{-4})}{(2.00012)}(7\times24\times3600)$$

$$\Rightarrow \quad \Delta t' = 36.28 \text{ s}$$

8. Due to drop in temperature, tension increment in the clamped wire is $F = YA\alpha\Delta T$

$$\Rightarrow \quad F = 2.1\times10^{12}\times0.5\times10^{-2}\times1.1\times10^{-5}\times20$$

$$\Rightarrow \quad F = 2.31\times10^{-6} \text{ dyne}$$

9. Since, thermal stress is given by

$$\frac{F}{A} = Y\alpha\Delta T$$

$$\Rightarrow \quad F = YA\alpha\Delta T = (2\times10^{11})(0.8\times10^{-4})(10^{-5})(10)$$

$$\Rightarrow \quad F = 1.6\times10^{3}\ \text{N}$$

10. Temperature on Kelvin scale are

$$T_K = T_C + 273$$

For $T_C = 4000\ ^\circ\text{C}$, we have

$$T_K = 4000 + 273 = 4273\ \text{K}$$

and for $T_C = 1.5\times10^{7}\ ^\circ\text{C}$, we have

$$T_K = (1.5\times10^{7} + 273)\ \text{K}$$

Percentage error is given by

$$\%\text{error} = \left(\frac{T_K - T_C}{T_C}\right)\times100\% = \frac{273}{T_C}\times100\%$$

For $T_c = 4000\ ^\circ\text{C}$

$$\%\ \text{Error} = \frac{273}{4000}\times100 = 6.825\%$$

For $T_C = 1.5\times10^{7}\ ^\circ\text{C}$

$$\%\ \text{error} = \frac{273}{1.5\times10^{7}}\times100 = 1.82\times10^{-3}\%$$

11. Angle $B'AC = \frac{1}{2}\left[\pi - \left(\frac{\pi}{3} - \theta\right)\right] = \frac{\pi}{3} + \frac{\theta}{2} = \beta$ (say)

Since, $AB' = L_0(1+100\alpha)$

Also, $AB'\cos\beta = \frac{L_0}{2} = \frac{AC}{2}$

$$\Rightarrow \quad L_0(1+100\alpha)\cos\left(\frac{\pi}{3}+\frac{\theta}{2}\right) = \frac{L_0}{2}$$

Using approximation, $\sin\left(\frac{\theta}{2}\right) \approx \frac{\theta}{2}$ and $\cos\left(\frac{\theta}{2}\right) \approx 1$, we get

$$\alpha = \frac{\sqrt{3}}{200}\theta$$

12. For an ideal gas, we have $PV = nRT$

$$\Rightarrow \quad d(PV) = d(nRT)$$

Since, P is constant, we get $PdV = nRdT$

$$\Rightarrow \quad \frac{dV}{dT} = \frac{nR}{P}$$

Now, by definition, $\gamma = \left(\frac{dV}{V}\right)\frac{1}{dT}$

$$\Rightarrow \quad \gamma = \frac{nR}{PV} = \frac{nR}{nRT} = \frac{1}{T}$$

13. Since $\frac{T_C - 32}{212 - 32} = \frac{T_C - 0}{100 - 0}$ …(1)

For same value on both the scales, we have

$$T_F = T_C = T_0$$

Substituting in equation (1), we get

$$T_0 - 32 = 1.8T_0$$

$$\Rightarrow \quad -0.8T_0 = 32$$

$$\Rightarrow \quad T_0 = -40\ ^\circ\text{F}$$

14. (a) Since $L = \text{I}\omega = \left(\frac{1}{2}MR^2\right)\omega$

$$\Rightarrow \quad L = \frac{1}{2}(0.5)(0.030)^2(60) = 0.0135\ \text{kgm}^2\text{s}^{-1}$$

(b) When the cylinder is heated without any mechanical contact, then no external force and hence torque acts on the cylinder, so angular momentum is conserved. Hence,

$$\text{I}\omega = \text{constant}$$

$$\Rightarrow \quad \Delta(\text{I}\omega) = 0$$

$$\Rightarrow \quad \frac{\Delta\omega}{\omega} = -\frac{\Delta\text{I}}{\text{I}} = -\frac{2\Delta R}{R} \qquad \left\{\because\ \text{I} = \frac{1}{2}MR^2\right\}$$

Since, $\frac{\Delta R}{R} = \alpha\Delta T$

$$\Rightarrow \quad \frac{\Delta\omega}{\omega} = -2(\alpha\Delta T) = -2\times2\times10^{-5}\times80$$

$$\Rightarrow \quad \frac{\Delta\omega}{\omega} = -3.2\times10^{-3}$$

15. (a) Since, $\frac{T-0}{100-0} = \frac{15.0-12.45}{21.30-12.45} = \frac{2.65}{8.85}$

$$\Rightarrow \quad T = \frac{2.65}{8.85}\times100 = 29.94\ ^\circ\text{C}$$

(b) Since, $\frac{T-0}{100-0} = \frac{22.95-12.45}{21.30-12.45} = \frac{10.5}{8.85}$

$$\Rightarrow \quad T = \frac{10.5}{8.85}\times100 = 118.64\ ^\circ\text{C}$$

16. By definition, we have

$$\alpha(T) = \frac{dL}{L_0}\frac{1}{dT}$$

$$\Rightarrow \quad dL = L_0\alpha(T)dT$$

$$\Rightarrow \quad \int_{L_0}^{L} dL = L_0\int_{T_0}^{T}\alpha(T)dT$$

$$\Rightarrow \quad L - L_0 = L_0\int_{T_0}^{T}\alpha(T)dT$$

$$\Rightarrow \quad L = L_0\left[1 + \int_{T_0}^{T}\alpha(T)dT\right]$$

17. Since, at all temperatures, we have $\ell_s - \ell_c = 5$ cm …(1)

$$\Rightarrow \quad \Delta\ell_s - \Delta\ell_c = 0$$

$$\Rightarrow \quad \Delta\ell_s = \Delta\ell_c$$

By definition, we have $\Delta\ell_s = \ell_s\alpha_s\Delta T$ and $\Delta\ell_c = \ell_c\alpha_c\Delta T$

$$\Rightarrow \quad \ell_s\alpha_s = \ell_c\alpha_c$$

$$\Rightarrow \quad \frac{\ell_s}{\ell_c} = \frac{\alpha_c}{\alpha_s} = \frac{17}{11} \qquad \ldots(2)$$

So, from (1) and (2), we get

$\ell_s = 14.16$ cm and $\ell_c = 9.16$ cm

18. Since $\gamma = \dfrac{\Delta V}{V}\dfrac{1}{\Delta T} = \left(\dfrac{0.12}{100}\right)\dfrac{1}{20}$

$\Rightarrow \quad \gamma = 6\times10^{-5}\,(°C)^{-1}$

Further, we know that $\dfrac{\alpha}{1} = \dfrac{\beta}{2} = \dfrac{\gamma}{3}$

$\Rightarrow \quad \alpha = \dfrac{\gamma}{3} = 2\times10^{-5}\,(°C)^{-1}$

19. Let the distance of centre of mass from suspension point O be h, then h = constant

$\Rightarrow \quad h^2 = l_2^2 - \dfrac{l_1^2}{4} = \text{constant}$

$\Rightarrow \quad \Delta\left(l_2^2\right) - \Delta\left(\dfrac{l_1^2}{4}\right) = 0$

$\Rightarrow \quad 2l_2\Delta l_2 - \dfrac{1}{4}2l_1\Delta l_1 = 0$

Since, by definition, $\Delta l = l\alpha\Delta T$

$\Rightarrow \quad \Delta l_2 = l_2\alpha_2\Delta T$ and $\Delta l_1 = l_1\alpha_1\Delta T$

$\Rightarrow \quad 2l_2\left(l_2\alpha_2\Delta T\right) = \dfrac{1}{4}\left(2l_1\right)\left(l_1\alpha_1\Delta T\right)$

$\Rightarrow \quad l_2^2\alpha_2 = \dfrac{1}{4}l_1^2\alpha_1$

$\Rightarrow \quad \dfrac{l_1^2}{l_2^2} = 4\left(\dfrac{\alpha_2}{\alpha_1}\right)$

$\Rightarrow \quad l_2 = \dfrac{l_1}{2}\sqrt{\dfrac{\alpha_1}{\alpha_2}} = \dfrac{5}{2}\sqrt{\dfrac{18}{9}} = 5\sqrt{2} \approx 7$ cm

20. Since, $D_T = D_0(1+\alpha\Delta T)$

$\Rightarrow \quad D_T = 2.54\left(1+2.3\times10^{-5}\times100\right) = 2.54(1.0023)$

$\Rightarrow \quad D_T = 2.5458$ cm

21. Since, $T_K = T_C + 273$ and $T_F = \dfrac{9}{5}T_C + 32$

$\Rightarrow \quad T_C = T_K - 273.15$ and $T_F = \dfrac{9}{5}T_C + 32$

For Neon

$T_C = T_K - 273.15$

$\Rightarrow \quad T_C = 24.57 - 273.15 = -248.58$ °C

and $T_F = \dfrac{9}{5}T_C + 32 = \dfrac{9}{5}(-248.58) + 32$

$\Rightarrow \quad T_F = -447.44 + 32 = -415.44$ °F

For Carbon dioxide

$T_C = T_K - 273.15 = 216.55 - 273.15$

$\Rightarrow \quad T_C = -56.60$ °C

and $T_F = \dfrac{9}{5}T_C + 32 = \dfrac{9}{5}(-56.60) + 32$

$\Rightarrow \quad T_F = -101.88 + 32 = -69.88$ °F

22. **Method-I**

At any temperature t, the lengths of the two bars are

$\ell_1' = \ell_1(1+\alpha_1 t),\ \ell_2' = \ell_2(1+\alpha_2 t)$ …(1)

From the given conditions,

$\ell_2' - \ell_1' = \ell$ and $\ell_2 - \ell_1 = \ell$ …(2)

From equations (1) and (2),

$\ell_2' - \ell_1' = (\ell_2 - \ell_1) + \ell_2\alpha_2 t - \ell_1\alpha_1 t$

$\ell_2\alpha_2 = \ell_1\alpha_1$ …(3)

$\dfrac{\ell_2}{\ell_1} = \dfrac{\alpha_1}{\alpha_2}$ …(4)

From equations (2) and (4),

$\ell_1 = \dfrac{\ell\alpha_2}{\alpha_2 - \alpha_1},\ \ell_2 = \dfrac{\ell\alpha_1}{\alpha_2 - \alpha_1}$

Method-II

Since, the separation between the mirrors, ℓ, is constant. So,

$\Delta\ell = \Delta(\ell_2 - \ell_1) = 0$

$\Rightarrow \quad \Delta\ell_2 - \Delta\ell_1 = 0$

$\Rightarrow \quad \Delta\ell_2 = \Delta\ell_1$

$\Rightarrow \quad \ell_1\alpha_1\Delta T = \ell_2\alpha_2\Delta T$

$\Rightarrow \quad \ell_1\alpha_1 = \ell_2\alpha_2$ …(5)

Since, $\ell = \ell_2 - \ell_1$ …(6)

So, from (5) and (6), on solving, we get

$\ell_1 = \dfrac{\ell\alpha_2}{\alpha_2 - \alpha_1}$ and $\ell_2 = \dfrac{\ell\alpha_1}{\alpha_2 - \alpha_1}$

Test Your Concepts-II (Based on Calorimetry)

1. Let Q be the heat required to convert 200 g of water at 40 °C into 100 °C, then

$Q = mc\Delta T = (200)(1)(100 - 40) = 12000$ cal

Now, suppose m_0 mass of steam converts into water to liberate this much amount of heat, then

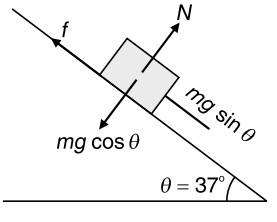
$m_0 = \dfrac{Q}{L} = \dfrac{12000}{539} = 22.26$ g

Since this is less than 30 g, so the temperature of the mixture is 100 °C

Mass of steam in the mixture = 30 – 22.26 = 7.74 g

and mass of water in the mixture = 200 + 22.26 = 222.26 g

2. When the copper cube slides down a rough inclined plane at constant speed, then $f = mg\sin\theta = mg\sin(37°)$



CHAPTER 2

$$(\text{Heat gained}) = (\text{Work done against friction})$$

$$\Rightarrow \quad mc\Delta T = f\Delta r$$

$$\Rightarrow \quad mc\Delta T = mg\sin(37°)\left(\frac{60}{100}\right)$$

$$\Rightarrow \quad 420\Delta T = (10)\left(\frac{3}{5}\right)\left(\frac{3}{5}\right)$$

$$\Rightarrow \quad \Delta T = 8.6\times10^{-3}\ °\text{C}$$

3. After time t, the mass of water inside pitcher is given by $m = (1000 - 0.1t)$ gram
In a further time dt, mass of water vapourised is $dm = (0.1)dt$ and let the temperature of water inside pitcher fall by dT.
So, we get $(1000 - 0.1t)(1)(-dT) = (0.1dt)(540)$

$$\Rightarrow \quad \int_0^t \frac{54dt}{1000-0.1t} = -\int_{40}^{30} dT$$

$$\Rightarrow \quad \int_0^t \frac{540dt}{10000-t} = -\int_{40}^{30} dT$$

$$\Rightarrow \quad -540\ln(10000-t)\Big|_0^t = 10$$

$$\Rightarrow \quad \ln\left(\frac{10000-t}{10000}\right) = -\frac{1}{54}$$

$$\Rightarrow \quad 10000 - t = 10000e^{-1/54}$$

$$\Rightarrow \quad t = 10000\left(1-e^{-1/54}\right)\ \text{s} = 10000(1-0.98)\ \text{s}$$

$$\Rightarrow \quad t = (10000)(0.02) = 200\ \text{s}$$

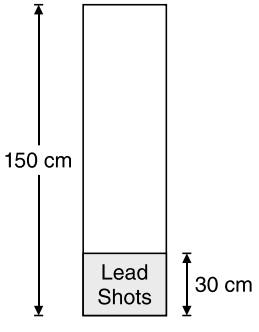
4. When inverted, each lead shot falls through an effective height of $(150 - 30)$ cm $= 120$ cm. So,

$$h = 120\ \text{cm} = \frac{120}{100}\ \text{cm}$$

Let the tube has to be inverted n times, then

$$n(mgh) = mc_{\text{lead}}\Delta T$$

$$\Rightarrow \quad n = \frac{c_{\text{lead}}\Delta T}{gh}$$



where $c_{\text{lead}} = (0.031)(4200)\ \text{Jkg}^{-1}\text{K}^{-1}$ and $\Delta T = 2\ °\text{C}$

$$\Rightarrow \quad n = \frac{(0.031)(4200)(2)}{(9.8)\left(\frac{120}{100}\right)} = 22.1$$

So, the number of times the tube has to be inverted end to end is 23.

5. Since $\frac{1}{2}\left(\frac{1}{2}mv^2\right) = mc_{\text{bullet}}\Delta T$

and $1\ \text{calg}^{-1}(°\text{C})^{-1} = 4200\ \text{Jkg}^{-1}(°\text{C})^{-1} = 4200\ \text{Jkg}^{-1}\text{K}^{-1}$

$$\Rightarrow \quad \frac{(100)^2}{4} = 0.030\times4200\times\Delta T$$

$$\Rightarrow \quad \Delta T \cong 20\ \text{K or}\ \Delta T \cong 20\ °\text{C}$$

6. Heat supplied is

$$Q = (22)(0.5)(8) + (22)(80) + (22)(1)(16)$$

$$\Rightarrow \quad Q = 88 + 1760 + 352 = 2200\ \text{cal}$$

Heat capacity of body is

$$\text{Heat Capacity} = \frac{2200\ \text{cal}}{44\ °\text{C}} = 50\ \text{cal}(°\text{C})^{-1}$$

Water equivalent of the body is

$$w = \frac{\text{Heat capacity of the body}}{\text{Specific heat of water}}$$

$$\Rightarrow \quad w = \frac{50\ \text{cal°C}^{-1}}{1\ \text{calg}^{-1}\text{°C}^{-1}} = 50\ \text{g}$$

7. $$c = \frac{1}{m}\frac{\Delta Q}{\Delta T} = \left(\frac{1}{0.1}\right)\left(\frac{400}{20}\right) = 200\ \text{Jkg}^{-1}(°\text{C})^{-1}$$

8. The mixture has 1 kg ice at 0 °C and 1 kg steam at 100 °C.
Heat required to completely melt ice is

$$Q_1 = 1\times3.36\times10^5\ \text{J}$$

Heat required to raise temperature of water from 0 °C to 100 °C is

$$Q_2 = 1\times4200\times100 = 4.2\times10^5\ \text{J}$$

So, total heat required to melt 1 kg ice at 0 °C to water at 0 °C and then take it to a temperature of 100 °C is

$$Q_{\text{required}} = Q_1 + Q_2 = 7.56\times10^5\ \text{J}$$

If this much entire heat is provided by condensation of steam, then mass m of steam condensed (to give water) would be

$$m = \frac{Q_1+Q_2}{L_v} = \frac{7.56\times10^5}{2.27\times10^6} = 0.333\ \text{kg}$$

$$\Rightarrow \quad m_{\text{water}} = 1333\ \text{g and}\ m_{\text{steam}} = 667\ \text{g}$$

Hence, final mixture is at 100 °C. The final mixture has 667 g steam and 1333 g water.

9. Let t °C be the temperature of the mixture. Then, by Law of Calorimetry, we have

$$\begin{pmatrix}\text{Heat given by}\\ 10\ \text{g of water}\end{pmatrix} = \begin{pmatrix}\text{Heat taken by}\\ 5\ \text{g of water}\end{pmatrix}$$

$$\Rightarrow \quad m_1c_{\text{water}}|\Delta t_1| = m_2c_{\text{water}}|\Delta t_2|$$

$$\Rightarrow \quad (10)(70-t) = 5(t-30)$$

$$\Rightarrow \quad t = 36.67\ °\text{C}$$

10. By Law of Calorimetry, heat supplied by the block equals heat gained by calorimeter and liquid. Assuming water equivalent of the calorimeter to be w, we get
For 200 g of liquid, we have

$$(110)(0.1)(82)=(8)\left[w(1)+(200)c_L\right] \quad \ldots(1)$$

For 400 g of liquid, we have

$$(110)(0.1)(85.5)=(4.5)\left[w(1)+(400)c_L\right] \quad \ldots(2)$$

Multiply equation (2) by 8, equation (1) by 4.5 and subtracting, we get

$$110\times0.1(85.5\times8-82\times4.5)=c_L(400\times4.5\times8-200\times4.5\times8)$$

$$\Rightarrow \quad c_L=\frac{3150}{7200}=0.481 \text{ calg}^{-1}{}^\circ\text{C}^{-1}$$

From equation (1), we get

$$w=\frac{902-769.6}{8}=16.55 \text{ g}$$

11. Let s_A, s_B and s_C be the specific heats of liquid A, B and C. When A and B are mixed, then

$$\begin{pmatrix}\text{Heat Lost}\\ \text{by } B\end{pmatrix}=\begin{pmatrix}\text{Heat Gained}\\ \text{by } A\end{pmatrix}$$

$$\Rightarrow \quad ms_B(19-16)=ms_A(16-12)$$

$$\Rightarrow \quad 3s_B=4s_A \quad \ldots(1)$$

When B and C are mixed, then

$$\begin{pmatrix}\text{Heat Lost}\\ \text{by } C\end{pmatrix}=\begin{pmatrix}\text{Heat Gained}\\ \text{by } B\end{pmatrix}$$

$$\Rightarrow \quad ms_C(28-23)=ms_B(23-19)$$

$$\Rightarrow \quad 5s_C=4s_B \quad \ldots(2)$$

When A and C are mixed, let the equilibrium temperature be T, then

$$\begin{pmatrix}\text{Heat Lost}\\ \text{by } C\end{pmatrix}=\begin{pmatrix}\text{Heat Gained}\\ \text{by } A\end{pmatrix}$$

$$\Rightarrow \quad ms_C(28-T)=ms_A(T-12) \quad \ldots(3)$$

From (1) and (2), we get $s_C=\frac{16}{15}s_A$

$$\Rightarrow \quad m\left(\frac{16}{15}s_A\right)(28-T)=ms_A(T-12)$$

$$\Rightarrow \quad 16(28-T)=15(T-12)$$

$$\Rightarrow \quad 448-16T=15T-180$$

$$\Rightarrow \quad 31T=628$$

$$\Rightarrow \quad T=20.25 \text{ °C}$$

12. 75% of heat is retained by lead and due to this its temperature first rises from 27 °C to 327 °C and then at 327 °C it melts. So, we have

$$\frac{3}{4}\left(\frac{1}{2}mv^2\right)=mc_{\text{lead}}\Delta T+mL_{\text{lead}}$$

$$\Rightarrow \quad \frac{3}{8}v^2=(0.03)(4200)(300)+(6)(4200)$$

$$\Rightarrow \quad v=410 \text{ ms}^{-1}$$

13. Let x kg of water be frozen, then amount of heat released is

$$Q_1=\left(x\times4200\times50+x\times3.36\times10^5\right) \text{ J}$$

$$\Rightarrow \quad Q_1=\left(5.46\times10^5x\right) \text{ J}$$

The heat required to vapourise $(1-x)$ kg of water from 50 °C is

$$Q_2=(1-x)\times22.5\times10^5 \text{ J}$$

This heat Q_2 must be provided by the first part of water. So, we have $Q_2=Q_1$

$$\Rightarrow \quad (1-x)\left(22.5\times10^5\right)=5.46\times10^5x$$

$$\Rightarrow \quad 22.5-22.5x=5.46x$$

$$\Rightarrow \quad x=\frac{22.5}{27.96}=0.805 \text{ kg}$$

14. (a) Given $\frac{dT}{dt}=0.5 \text{ °Cs}^{-1}$

Since, $dQ=mcdT$ i.e., $\frac{dQ}{dt}=mc\left(\frac{dT}{dt}\right)$

$$\Rightarrow \quad \frac{dQ}{dt}=\left(\frac{180}{1000}\right)(0.10)(4200)(0.5) \text{ Js}^{-1}$$

$$\Rightarrow \quad \frac{dQ}{dt}=37.8 \text{ W}$$

So, rate of working of drill in watt is 37.8

(b) Since $P=\tau\omega$ and $1 \text{ rpm}=\frac{2\pi}{60} \text{ rads}^{-1}$

$$\Rightarrow \quad 37.8=\tau\left(180\times\frac{2\pi}{60}\right)=2 \text{ Nm}$$

15. Since $\frac{1}{2}\left(\frac{1}{2}mv^2\right)=mc_{\text{bullet}}\Delta T$

$$\Rightarrow \quad \frac{v^2}{4}=c_{\text{bullet}}\Delta T$$

where $c_{\text{bullet}}=(0.03)(4200) \text{ Jkg}^{-1}\text{K}^{-1}$

$$\Rightarrow \quad \frac{(300)^2}{4}=(0.03)(4200)\Delta T$$

$$\Rightarrow \quad \Delta T=178 \text{ °C}=178 \text{ K}$$

16. Heat supplied in 4 min is

$$Q=100\times4\times60=24000 \text{ cal}$$

$$\Rightarrow \quad 100\times0.215\times(T+20)+200\times0.5\times20+200\times80+200\times1\times T=2400$$

$$\Rightarrow \quad 221.5T=2400-16000-2000=6000$$

$$\Rightarrow \quad T=27.08 \text{ °C}$$

17. $dQ=mcdT$

$$\Rightarrow \quad \frac{dQ}{dt}=mc\left(\frac{dT}{dt}\right)$$

If heat lost per second to the surroundings is q, then

$$135=\left(\frac{5}{1000}\right)c_w(5)+q \quad \ldots(1)$$

Also, $240=\left(\frac{10}{1000}\right)c_w(5)+q \quad \ldots(2)$

From (1) and (2), we get

$$105=\left(\frac{25}{1000}\right)c_w$$

$$\Rightarrow \quad c_w=4200 \text{ Jkg}^{-1}\text{K}^{-1}$$

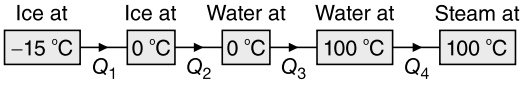
CHAPTER 2

18. $Q_1 = mc_{ice}\left(T_f - T_i\right) = (8)(0.5)[0-(-15)] = 60$ cal

$Q_2 = mL_f = (8)(80) = 640$ cal

$Q_3 = mc_{\text{water}}\left(T_f - T_i\right) = (8)(1)(100-0) = 800$ cal

$Q_4 = mL_v = (8)(540) = 4320$ cal



So, net heat required is

$Q = Q_1 + Q_2 + Q_3 + Q_4 = 5820$ cal

19. Heat required to raise temperature of ice to 100 °C is

$Q_1 = m_1L_1 + m_1c_1\Delta T_1 = 5\times 80 + 5\times 1\times 100$

$\Rightarrow \quad Q_1 = 400 + 500 + 900 = 1800$ cal

Heat liberated on condensation of steam is

$Q_2 = m_2L_2 = 5\times 536 = 2680$ cal

Since, $Q_2 > Q_1$

This simply means that the entire steam is not condensed and hence temperature of mixture will remain at 100 °C

20. Let calorimeter contains x gram of ice and $(200-x)$ gram of water.

By Law of Calorimetry, the heat lost by $330-(100+200) = 30$ g of steam equals the heat gained by 100 g calorimeter plus heat gained by x gram ice plus heat gained by $(200-x)$ gram of water

$\Rightarrow \quad 30L_{steam} + 30s_{water}(100-50) = 100s_{Cu}(50) +$

$xL_{ice} + xs_{water}(50) + (200-x)s_{water}(50)$

$\Rightarrow \quad x = 88.4$ g

So, the desired ratio is

$$\frac{x}{200-x} = \frac{88.4}{200-88.4} \approx 0.8$$

Test Your Concepts-III (Based on Kinetic Theory of Gases and Ideal Gas Equation)

1. Initially let each bulb contain n moles of the gas, so two bulbs contains $2n$ moles. If the volume of each bulb is V then, at NTP, i.e., $P = 76$ cm of Hg and $T = 273$ K, we have

$$\frac{76\times V}{273} = nR \qquad \ldots(1)$$

Further, $\frac{PV}{373} = n_1R$ and $\frac{PV}{273} = n_2R$

$$\Rightarrow \quad PV\left(\frac{1}{373}+\frac{1}{273}\right) = (n_1+n_2)R = 2nR \qquad \ldots(2)$$

From (1) and (2), we get

$P = 87.76$ cm of Hg

2. We use $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

$$\Rightarrow \quad \frac{v_1}{v_2} = \sqrt{\frac{T_1M_2}{T_2M_1}}$$

$$\Rightarrow \quad v_2 = v_1\sqrt{\frac{T_2M_1}{T_1M_2}} = 300\sqrt{\frac{2T_1M_1}{T_1(M_1/2)}}$$

$\Rightarrow \quad v_2 = 600$ ms^{-1}

3. Each electron exerts an impulse of m_0v when it strikes and sticks to the surface. This times the number striking in unit time divided by area is pressure. Hence,

$$P = \frac{10^{15}}{\left(10^{-3}\right)^2}m_0v = 10^{21}m_0v$$

Substituting the given values, we get

$$P = \left(9.1\times 10^{-31}\right)\left(8\times 10^{7}\right)\left(10^{21}\right) = 0.073 \text{ Nm}^{-2}$$

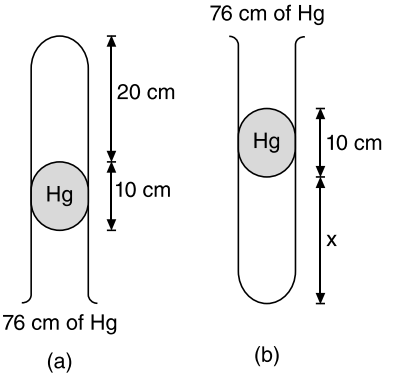
4. Since, initial number of moles equals the final number of moles of the gas, so we have

$n_1 + n_2 = n$

$$\Rightarrow \quad \frac{p_1V}{T_1} + \frac{p_2V}{T_2} = \frac{p_f(2V)}{T}$$

$$\Rightarrow \quad p_f = \frac{T}{2}\left[\frac{p_1}{T_1}+\frac{p_2}{T_2}\right]$$

5. The initial and final states of the tube are shown in Figure (a) and (b) respectively.



In the initial state, pressure just below the mercury pallet (at the bottom of pallet) is the atmospheric pressure i.e., 76 cm of Hg and at top of the pallet, 10 cm above the bottom is

$76 - 10 = 66$ cm of Hg

Finally, when the tube is inverted with its open end upwards, then pressure at the top of pallet is atmospheric pressure i.e., 76 cm of Hg but at the bottom of the pallet, the pressure will be

$76 + 10 = 86$ cm of Hg

If the length of air column is x, then applying Boyle's Law, we get

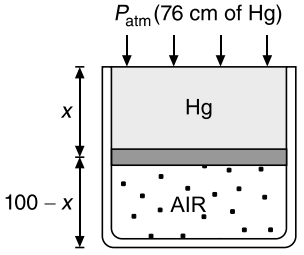
$P_1V_1 = P_2V_2$

$\Rightarrow \quad (66)(20A) = (86)(xA)$

where, A is the cross-sectional area of tube

$\Rightarrow \quad x = 15.35$ cm

6. When mercury is poured on the top of the piston, due to increase in pressure, the volume of air will decrease according to Boyle's Law. Since atmospheric pressure is equivalent to the pressure due to a mercury column of height 76 cm, so if final mercury column of height x is poured on the piston, then gas pressure in equilibrium is given by



$$P_f = (76 + x) \text{ cm of Hg}$$

If A be the area of cross section of cylinder, then we have according to Boyle's Law

$$P_1V_1 = P_2V_2$$

$$\Rightarrow \quad (76 \text{ cm})(100A) = (76 + x)(100 - x)A$$

$$\Rightarrow \quad 7600 = 7600 + 24x - x^2$$

$$\Rightarrow \quad x = 24 \text{ cm}$$

7. Moment of inertia, $I = 2(mr^2)$

where $m = \dfrac{70 \times 10^{-3}}{2 \times 6.02 \times 10^{23}} = 5.81 \times 10^{-26}$ kg

and $r = \dfrac{2 \times 10^{-10}}{2} = 1 \times 10^{-10}$ m

$$\Rightarrow \quad I = 2(5.81 \times 10^{-26})(1 \times 10^{-10})^2 = 1.16 \times 10^{-45} \text{ kgm}^2$$

Since, $K_R = \dfrac{1}{2} I\omega^2 = \dfrac{1}{2} \times (1.16 \times 10^{-45}) \times (2 \times 10^{12})^2$

$$\Rightarrow \quad K_R = 2.32 \times 10^{-21} \text{ J}$$

Conceptual Note(s)

At $T = 300$ K, rotational K.E. should be equal to $\dfrac{1}{2}kT$

$$\Rightarrow \quad K_R = \frac{1}{2} \times (1.38 \times 10^{-23}) \times (300) = 2.07 \times 10^{-21} \text{ J}$$

Approximately the same as calculated above.

8. Done in Theory.

9. Since, initial number of moles equals the final number of moles of the gas, so we have

$$n_1 = n_2$$

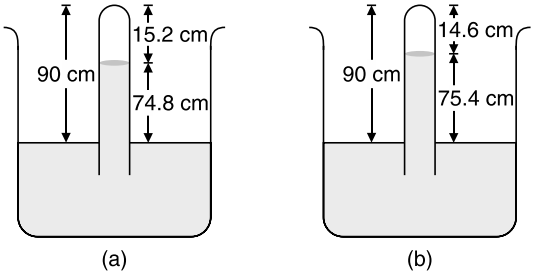
$$\Rightarrow \quad \frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

$$\Rightarrow \quad \frac{2 \times 10^5 \times V}{293} = \frac{p \times 1.02V}{313}$$

$$\Rightarrow \quad p = 2.095 \times 10^5 \text{ Nm}^{-2}$$

10. Let A be the area of cross section of the tube at 30 °C, volume of air be $V_1 = 15.2$ A cm^3. The pressure of air will be $P_1 = 76 - 74.8 = 1.2$ cm of Hg.

At 10 °C, $V_2 = 14.6$ A cm^3



and $P_2 = (P - 75.4)$ cm of Hg

where, P is the true atmospheric pressure

Since, $\dfrac{P_1V_1}{T_1} = \dfrac{P_2V_2}{T_2}$

$$\Rightarrow \quad P = 76.57 \text{ cm of Hg}$$

11. Friction force balances the force due to mercury pressure that corresponds to

$$76 - 5 - 69.47 = 1.53 \text{ cm of } Hg$$

Hence, $f_r = PA = P(\pi r^2)$

$$\Rightarrow \quad f_r = (0.0153)(13600)(10)\left[3.14(2 \times 10^{-3})^2\right]$$

$$\Rightarrow \quad f_r = 0.026 \text{ N}$$

12. Let the initial pressure of the gas be P_1 and the final temperature be P_2. Then from Ideal Gas Equation, we have

$$P_1V = n_1RT$$

$$P_2V = n_2RT \text{ where, } T = 273 \text{ K}$$

$$\Rightarrow \quad (P_1 - P_2)V = (n_1 - n_2)RT = \left(\frac{m_1 - m_2}{M}\right)RT$$

$$\Rightarrow \quad (\Delta P)V = \frac{\Delta m}{M}RT \qquad \ldots(1)$$

In the initial condition (at STP), i.e., at $T = 273$ K, we have

$$\frac{RT}{M} = \frac{P_0}{\rho} \qquad \ldots(2)$$

From equations (1) and (2), we get

$$\Delta m = \frac{\rho V \Delta P}{P_0} = \frac{1.2 \times 0.4 \times 0.24}{1} = 0.1152 \text{ kg}$$

$$\Rightarrow \quad \Delta m = 115.2 \text{ g}$$

13. Since, $P_{H_2} = \left(\dfrac{30}{2}\right) \times \dfrac{8.314 \times 300}{30 \times 10^{-3}}$

$$\Rightarrow \quad P_{H_2} = 12.471 \times 10^5 \text{ Pa}$$

Also, $P_{O_2} = \dfrac{160}{32} \times \dfrac{8.314 \times 300}{10 \times 10^{-3}}$

$$\Rightarrow \quad P_{O_2} = 12.471 \times 10^5 \text{ Pa}$$

Similarly, $P_{N_2} = \dfrac{70}{28} \times \dfrac{8.314 \times 300}{20 \times 10^{-3}}$

$$\Rightarrow \quad P_{N_2} = 3.118 \times 10^5 \text{ Pa}$$

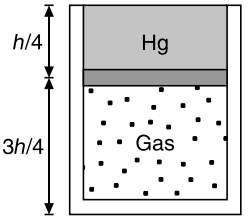
Hence, $P_{\text{left}} = P_{H_2} = 12.471 \times 10^5$ Pa

$$P_{\text{middle}} = P_{H_2} + P_{O_2} + P_{N_2} = 28.06 \times 10^5 \text{ Pa}$$

and $P_{\text{right}} = P_{H_2} + P_{N_2} = 15.589 \times 10^5$ Pa

CHAPTER 2

14. The final state of gas after half of mercury overflows is shown in Figure.



Let this temperature be T, then from ideal gas equation applied for initial and final state of gas, we get

$$\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2} \qquad \ldots(1)$$

The initial pressure, volume and temperature of gas are

$$P_1=76+\frac{h}{2}=76+76=152 \text{ cm of Hg}$$

$$V_1=A\left(\frac{h}{2}\right)=76A \text{ and } T_1=300 \text{ K}$$

Similarly after heating, the pressure, volume and temperature of gas are

$$P_2=76+\frac{h}{4}=76+38=114 \text{ cm of Hg}$$

$$V_2=A\left(\frac{3h}{4}\right)=114A \text{ and } T_2=?$$

$$\Rightarrow \quad \frac{(152)(76A)}{300}=\frac{(114)(114A)}{T_2} \qquad \{\text{from (1)}\}$$

$$\Rightarrow \quad T_2=337.5 \text{ K}$$

15. Since, $P=P_0e^{-\left(\frac{gM}{RT}\right)h}$ {when T =constant}

Since, $PV=RT$

$$\Rightarrow \quad P\left(\frac{M}{\rho}\right)=RT$$

$$\Rightarrow \quad \rho=\frac{PM}{RT}$$

$$\Rightarrow \quad \rho\propto P$$

$$\Rightarrow \quad \rho=\rho_0e^{-\left(\frac{gM}{RT}\right)h}$$

$$\Rightarrow \quad h=-\left(\frac{RT}{gM}\right)\log_e\left(\frac{\rho}{\rho_0}\right)$$

(a) When, $\frac{\rho_0}{\rho}=e$ $\{\because \rho_0>\rho\}$

then, $h=-\frac{RT}{Mg}\log_e\left(e^{-1}\right)=\frac{RT}{Mg}$

$$\Rightarrow \quad h=\frac{(8.31)(273)}{(28\times10^{-3})(9.8)}=8.26\times10^3 \text{ m}$$

$$\Rightarrow \quad h=8.26 \text{ km}$$

(b) When, $\frac{\rho}{\rho_0}=0.98$

$$\Rightarrow \quad h=-\frac{RT}{Mg}\log_e(0.98)\approx0.09 \text{ km}$$

16. Centre of gravity of gas is given by $h_c=\frac{\int ydm}{\int dm}$

where dm is mass of a layer of width dy at a height y. If A be the base area of the vessel, then $dm=(Ady)\rho$

Since for uniform g and T, we have $\rho=\rho_0e^{-(Mg/RT)y}$

where ρ_0 is density at base of the vessel

$$\Rightarrow \quad h_c=\frac{\int_0^\infty A\left(\rho_0e^{-(Mg/RT)y}\right)ydy}{\int_0^\infty A\left(\rho_0{}^{-(Mg/RT)y}\right)dy}$$

$$\Rightarrow \quad h_c=\frac{\left[y\frac{e^{-(Mg/RT)y}}{(-Mg/RT)}-\frac{e^{-(Mg/RT)y}}{(-Mg/RT)^2}\right]_0^\infty}{\left[\frac{e^{-(Mg/RT)y}}{-Mg/RT}\right]_0^\infty}$$

$$\Rightarrow \quad h_c=\frac{0-(RT/Mg)^2}{(-RT/Mg)}=\frac{RT}{Mg}$$

17. Since, $\frac{v_{rms}}{v_{av}}=\sqrt{\frac{3\pi}{8}}$

$$\Rightarrow \quad v_{rms}=\left(\sqrt{\frac{3\pi}{8}}\right)v_{av}=\left(\sqrt{\frac{3\pi}{8}}\right)(4.25\times10^2)$$

$$\Rightarrow \quad v_{rms}=4.613\times10^2 \text{ ms}^{-1}$$

Average kinetic energy is

$$\frac{1}{2}mv_{\text{rms}}^2=\frac{1}{2}\times(5.28\times10^{-26})(4.613\times10^2)^2$$

$$\Rightarrow \quad \langle KE\rangle=5.62\times10^{-21} \text{ J}$$

18. If vessel volume is V then partial pressures of N_2 and CO_2 are

$$P_{N_2}=\left(\frac{7}{28}\right)\frac{(8.314)(290)}{V}$$

and $P_{CO_2}=\left(\frac{11}{44}\right)\frac{(8.314)(290)}{V}$

Given that $P_1+P_2=10^5$

$$\Rightarrow \quad \frac{8.314}{V}\left(\frac{290}{4}+\frac{290}{4}\right)=10^5$$

$$\Rightarrow \quad V=1.205\times10^{-2} \text{ m}^3$$

So, density of mixture is

$$\rho=\frac{m_1+m_2}{V}=\frac{18\times10^{-3}}{1.205\times10^{-2}}=1.494 \text{ kgm}^{-3}$$

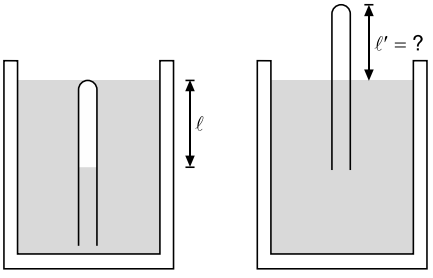
19. $P=P_{N_2}+P_N+P_{He}$

$$\Rightarrow \quad P=\left(\frac{1.4\times0.7}{28}+\frac{1.4\times0.3}{14}+\frac{0.4}{4}\right)\frac{RT}{V}$$

$$\Rightarrow \quad P=\frac{(0.035+0.03+0.1)(8.31)(1500)}{5\times10^{-3}}$$

$$\Rightarrow \quad P=4.1\times10^5 \text{ Nm}^{-2}$$

20. Since, $P_1V_1 = P_2V_2$



$\Rightarrow \quad (76+10)\times 10 = 76\times l'$

$\Rightarrow \quad l' = 11.315 \text{ cm}$

Mass of air is $m = \dfrac{PVM}{RT}$

$\Rightarrow \quad m = \dfrac{(1.013\times 10^5)\left(\dfrac{11.315}{100}\times 10^{-4}\right)(29\times 10^{-3})}{8.314\times 300}$

$\Rightarrow \quad m = 13.32 \text{ mg}$

21. Let v_i for $i = 1, 2, \ldots\ldots N$ represent the various speeds. Then,

$$v_{av} = \frac{1}{N}\sum_{i=1}^{N} v_i \text{ and } v_{rms}^2 = \frac{1}{N}\sum_{i=1}^{N} v_i^2$$

Let us consider the expression $A = \dfrac{1}{N}\sum_{i=1}^{N}(v_i - v_{av})^2$

where, A is inherently a non-negative quantity.

$\Rightarrow \quad A = \dfrac{1}{N}\sum_{i=1}^{N} v_i^2 - \dfrac{2v_{av}}{N}\sum_{i=1}^{N} v_i + \dfrac{1}{N}\left(Nv_{av}^2\right)$

$\Rightarrow \quad A = v_{rms}^2 - 2v_{av}^2 + v_{av}^2$

$\Rightarrow \quad A = v_{rms}^2 - v_{av}^2$

Since, $A \geq 0$

$\Rightarrow \quad v_{rms}^2 \geq v_{av}^2$

$\Rightarrow \quad v_{rms} \geq v_{av}$

22. At a distance x from open end of the cylinder, we consider an elemental layer of gas width dx. If pressure difference across this layer is dp, then we have

$$dF = Adp = (dm)x\omega^2 = (\rho A dx)(x\omega^2)$$

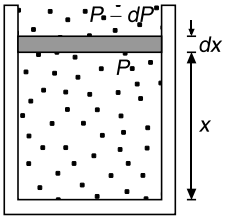
$\Rightarrow \quad dp = \left(\dfrac{pM}{RT}\right)(x\omega^2)dx \qquad \left\{\because \rho = \dfrac{PM}{RT}\right\}$

$\Rightarrow \quad \displaystyle\int_{p_0}^{p}\frac{dp}{p} = \frac{M\omega^2}{RT}\int_0^r x dx$

$\Rightarrow \quad \ln\left(\dfrac{p}{p_0}\right) = \dfrac{M\omega^2 r^2}{2RT}$

$\Rightarrow \quad p = p_0 e^{\frac{M\omega^2 r^2}{2RT}}$

23. Consider a gas layer of width dx at a height x above the bottom of container as shown in Figure.



If P is the pressure just below this layer, then just above this layer, we can consider pressure to be $P - dP$, where dP is the pressure due to the gas layer of width dx and is given by

$$dP = -(dx)\rho g \qquad \ldots(1)$$

where, ρ is the density of gas which is constant.

Since, $P = \dfrac{\rho RT}{M}$

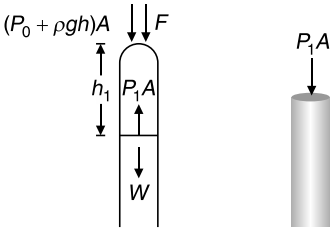
$\Rightarrow \quad dP = \dfrac{\rho R}{M}dT \qquad \ldots(2)$

From equations (1) and (2), we get

$$\frac{\rho R}{M}dT = -(dx)\rho g$$

$\Rightarrow \quad \text{Temperature Gradient} = \dfrac{dT}{dx} = -\dfrac{Mg}{R}$

24. In equilibrium, net downward force on the tube is F, $(P_0 + \rho gh)A$, W and upward force is P_1A, where P_1 is the pressure of air in the tube after submergence.



$\Rightarrow \quad F + (P_0 + \rho gh)A + W = P_1A$

$\Rightarrow \quad F = P_1A - (P_0 + \rho gh)A - W \qquad \ldots(1)$

Since temperature is constant, so according to Boyle's Law, we have

$$P_0(\ell A) = P_1(h_1 A)$$

$\Rightarrow \quad P_0\ell = P_1h_1 \qquad \ldots(2)$

Further, $P_1 = P_0 + \rho g(h + h_1) \qquad \ldots(3)$

Solving the above three equations, for F, P_1 and h_1, we get

$$F = 8.5\times 10^{-2} \text{ N}$$

25. (a) For a layer of width dy at a height y, we use

$$dp = -(dy)\rho g = -(dy)\left(\frac{pM}{RT}\right)g$$

$\Rightarrow \quad \displaystyle\int_{p_0}^{p}\frac{dp}{p} = -\frac{Mg}{RT_0}\int_0^h \frac{dy}{1-ay}$

$\Rightarrow \quad \ln\dfrac{p}{p_0} = \dfrac{Mg}{RT_0}\ln(1-ah)$

$\Rightarrow \quad p = p_0(1-ah)^{Mg/aRT_0}$

Above relation is valid if $h < \dfrac{1}{a}$

CHAPTER 2

(b) Similarly, we have

$$dp = -(dy)\rho g = -(dy)\left(\frac{pM}{RT}\right)g$$

$$\Rightarrow \int_{p_0}^{p}\frac{dp}{p} = -\frac{Mg}{RT_0}\int_0^h \frac{dy}{(1+ay)}$$

$$\Rightarrow \ln\frac{p}{p_0} = -\frac{Mg}{aRT_0}\ln(1+ah)$$

$$\Rightarrow p = \frac{p_0}{(1+ah)^{Mg/aRT_0}}$$

Test Your Concepts-IV (Based on Internal Energy, Degrees of Freedom and Molar Specific Heats for Ideal Gases)

1. Since $U_{\text{mixture}} = U_1 + U_2$

$$\Rightarrow (n_1+n_2)\left(\frac{3}{2}RT\right) = n_1\left(\frac{3}{2}RT_1\right) + n_2\left(\frac{3}{2}RT_2\right)$$

$$\Rightarrow T = \frac{n_1T_1 + n_2T_2}{n_1+n_2}$$

2. Let the gas have f degrees of freedom, then three degrees of freedom are associated with translational motion and the remaining $f-3$ degrees of freedom are associated with rotational motion. So, we have

$$\frac{K_T}{K_R} = \frac{3(nRT/2)}{(f-3)(nRT/2)} = \frac{3}{2}$$

$$\Rightarrow \frac{3}{f-3} = \frac{3}{2}$$

$$\Rightarrow 6 = 3f - 9$$

$$\Rightarrow f = 5$$

$$\Rightarrow U = \left(\frac{nf}{2}\right)RT = \left(1\times\frac{5}{2}\right)(8.3)(100) = 2075 \text{ J}$$

3. (a) Since, $TKE = U = n\left(\frac{3}{2}RT\right)$

where, $n = 2$ mol and $T = 293$ K, so

$$TKE = \frac{3}{2}(2)(8.31)(293) = 7.3\times10^3 \text{ J}$$

(b) The average kinetic energy per molecule is $\frac{3}{2}k_BT$

$$\Rightarrow \frac{1}{2}m\langle v^2\rangle = \frac{1}{2}mv_{rms}^2 = \frac{3}{2}k_BT$$

$$\Rightarrow \frac{1}{2}m\langle v^2\rangle = \frac{3}{2}(1.38\times10^{-23})(293)$$

$$\Rightarrow \frac{1}{2}m\langle v^2\rangle = 6.07\times10^{-21} \text{ J}$$

4. (a) An oxygen molecule has three translational degrees of freedom, thus the average translational kinetic energy of an oxygen molecule at 27 °C is

$$E_T = \frac{3}{2}k_BT = \frac{3}{2}\times1.38\times10^{-23}\times300$$

$$\Rightarrow E_T = 6.21\times10^{-21} \text{ Jmole}^{-1}$$

(b) An oxygen molecule has total five degrees of freedom, hence its total kinetic energy is

$$E_T = \frac{5}{2}k_BT = \frac{5}{2}\times1.38\times10^{-23}\times300$$

$$\Rightarrow E_T = 10.35\times10^{-21} \text{ Jmole}^{-1}$$

(c) Total kinetic energy of one mole of oxygen is its internal energy, which can be given as

$$U = \frac{5}{2}nRT = \frac{5}{2}\times1\times8.314\times300$$

$$\Rightarrow U = 6235.5 \text{ Jmole}^{-1}$$

5. Helium is a monoatomic gas so, internal energy of n moles of the gas is $U = \frac{3}{2}nRT$

$$\Rightarrow \Delta U = \frac{3}{2}nR(\Delta T) = \left(\frac{3}{2}\right)(3)(8.31)(2) = 74.8 \text{ J}$$

6. Since, the number of moles remains constant, so, we have

$$\Sigma n_{\text{initial}} = \Sigma n_{\text{final}}$$

From ideal gas equation, we get

$$\frac{p_1V}{T_1} + \frac{p_2V}{T_2} = \frac{p_1'V}{T_f} + \frac{p_2'V}{T_f} \quad \ldots(1)$$

Applying conservation of energy, we get

$$\frac{f}{2}p_1V + \frac{f}{2}p_2V = \frac{f}{2}p_1'V + \frac{f}{2}p_2'V \quad \ldots(2)$$

$$\Rightarrow p_1' + p_2' = p_1 + p_2$$

From equations (1) and (2), we get

$$\frac{p_1}{T_1} + \frac{p_2}{T_2} = \frac{p_1+p_2}{T_f}$$

$$\Rightarrow T_f = \frac{T_1T_2(p_1+p_2)}{p_1T_2 + p_2T_1}$$

For left and right part of gas, we have

$$\frac{p_1}{T_1} = \frac{p_1'}{T_f} \text{ and } \frac{p_2}{T_2} = \frac{p_2'}{T_f}$$

$$\Rightarrow p_1' = \frac{p_1T_2(p_1+p_2)}{p_1T_2 + p_2T_1} \text{ and } p_2' = \frac{p_2T_1(p_1+p_2)}{p_1T_2 + p_2T_1}$$

Heat supplied from the left part is $Q = U_i - U_f$

$$\Rightarrow Q = \frac{f}{2}p_1V - \frac{f}{2}p_1'V = \frac{fV}{2}\left(p_1 - \frac{p_1T_2(p_1+p_2)}{p_1T_2 + p_2T_1}\right)$$

$$\Rightarrow Q = \frac{fV}{2}\left(\frac{p_1p_2T_2 - p_1p_2T_2}{p_1T_2 + p_2T_1}\right)$$

Since the gas is monatomic, so $f = 3$

$$\Rightarrow Q = \frac{3p_1p_2V(T_2-T_1)}{2(p_1T_2 + p_2T_1)}$$

7. Since, $U_{\text{mixture}} = U_1 + U_2$

$$\Rightarrow (n_1+n_2)C_{V_{mix}}T = n_1C_{V_1}T + n_2C_{V_2}T$$

$$\Rightarrow C_{V_{mix}} = \frac{n_1C_{V_1} + n_2C_{V_2}}{n_1+n_2}$$

Since, for O_2, $C_{V_1} = \frac{5R}{2}$ and for CO_2, $C_{V_2} = \frac{7R}{2}$. So, we get

$$C_{V_{mix}} = \frac{(2)\left(\frac{5R}{2}\right)+3\left(\frac{7R}{2}\right)}{2+3} = \left(\frac{31}{12}\right)R$$

Since, $C_{P_{mix}} - C_{V_{mix}} = R$

$$\Rightarrow \quad C_{P_{mix}} = \left(\frac{31}{12}+1\right)R = \left(\frac{43}{12}\right)R$$

So, $\gamma_{mix} = \frac{C_{P_{mix}}}{C_{V_{mix}}} = \frac{43}{31} \approx 1.4$

8. Rotational kinetic energy of a diatomic molecule is

$$E = 2\left(\frac{1}{2}k_BT\right) = \frac{1}{2}I\omega_{rms}^2$$

$$\Rightarrow \quad \omega_{rms} = \sqrt{\frac{2k_BT}{I}}$$

$$\Rightarrow \quad \omega_{rms} = \sqrt{\frac{2\times1.38\times10^{-23}\times300}{2.1\times10^{-39}}} \approx 2\times10^9 \text{ rads}^{-1}$$

9. The disordered kinetic energy of the gas converts into the internal energy of its molecules, so

$$\frac{1}{2}mv^2 = nC_V\Delta T = \left(\frac{m}{M}\right)\left(\frac{R}{\gamma-1}\right)\Delta T \qquad \left\{\because C_v = \frac{R}{\gamma-1}\right\}$$

$$\Rightarrow \quad v = \sqrt{\frac{2R\Delta T}{M(\gamma-1)}} \qquad \ldots(1)$$

Kinetic Energy of the system is $K = K_{\text{gas jar}} + K_{\text{molecules}}$

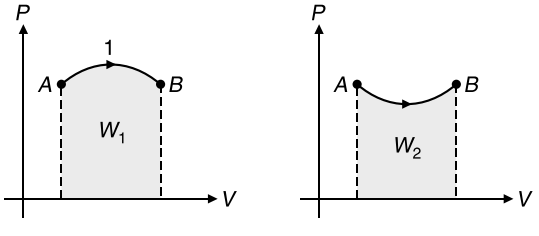
$$\Rightarrow \quad K = \frac{1}{2}Mv^2 + \frac{1}{2}(N_Am)v^2 = \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2$$

$$\Rightarrow \quad K = \frac{1}{2}(2M)v^2 = Mv^2$$

$$\Rightarrow \quad K = \frac{2R\Delta T}{\gamma-1} \qquad \{\text{from (1)}\}$$

Test Your Concepts-V (Based on Work Done and First Law of Thermodynamics)

1. From FLTD, we have $Q_1 = W_1 + \Delta U_1$ and $Q_2 = W_2 + \Delta U_2$
Since, U is a state function so ΔU depends only on the initial and final positions. Therefore, $\Delta U_1 = \Delta U_2$



Now, since the area under 1 is greater than area under 2. So $W_1 > W_2$ and hence $Q_1 > Q_2$

2. $$W = \int_{V_1}^{V_2} PdV = P(V_2 - V_1) = P\left(\frac{\alpha(2T_0)^2}{P} - \frac{\alpha(T_0)^2}{P}\right) = 3\alpha T_0^2$$

3. For a cyclic process, no change in internal energy takes place, so heat supplied is equal to the work done by the gas in one complete cycle. Hence

$$Q = \Delta U + W = 0 + W = W$$

Also, work done W by the gas equals the positive area enclosed by the ellipse, so

$$Q = \text{Area of ellipse} = \pi ab$$

where, a and b are semi major and semi-minor axis of the ellipse. From the figure, we have

$$2a = 2\times10^5 \text{ Nm}^{-2} \text{ and } 2b = 200\times10^{-6} \text{ m}^3$$

$$\Rightarrow \quad Q = W = \pi\times1.0\times10^5\times100\times10^{-6}$$

$$\Rightarrow \quad Q = 3.14\times10 = 31.4 \text{ J}$$

4. Since work done equals the area enclosed between PV curve and the V axis, so we have

$$W_{AC} = \frac{1}{2}(540)(0.3) + (340)(1) = 421 \text{ kJ}$$

Similarly, we have

$$W_{AD} = 421 + \frac{1}{2}(440)(0.6) = 553 \text{ kJ}$$

5. According to FLTD, we have $Q = \Delta U + W$
where, $\Delta U = U_f - U_i$

For Process 1

$Q = 35$ J, $W = -15$ J

$$\Rightarrow \quad 35 = \Delta U - 15$$

$$\Rightarrow \quad \Delta U = U_f - U_i = 50 \text{ J}$$

$$\Rightarrow \quad -10 - U_i = 50 \text{ J}$$

$$\Rightarrow \quad U_i = -60 \text{ J}$$

For Process 2

$Q = -15$ J, $\Delta U = -20$ J

$$\Rightarrow \quad -15 = -20 + W$$

$$\Rightarrow \quad W = 5 \text{ J}$$

Since, $\Delta U = U_f - U_i = -20$ J

$$\Rightarrow \quad 60 - U_i = -20 \text{ J}$$

$$\Rightarrow \quad U_i = 80 \text{ J}$$

For Process 3

$W = -20$ J, $\Delta U = 40$ J

$$\Rightarrow \quad Q = 40 - 20 = 20 \text{ J}$$

Since, $\Delta U = U_f - U_i = 40$ J

$$\Rightarrow \quad U_f - 80 = 40 \text{ J}$$

$$\Rightarrow \quad U_f = 120 \text{ J}$$

Process	Q	W	U_i	U_f	ΔU
1.	35	–15	–60	–10	50
2.	–15	5	80	60	–20
3.	20	–20	80	120	40

CHAPTER 2

6. According to FLTD, we have $Q = \Delta U + W = (U_F - U_I) + W$ where, U_I and U_F are the internal energies in the initial and the final state.

For path IAF, $Q = 55$ J and $W = 25$ J.

$\Rightarrow \quad \Delta U = U_F - U_I = Q - W = 55 - 25 = 30$ J

Now, we know that the internal energy is path independent i.e., it depends only on the initial and final states of the system. *So, internal energy between I and F irrespective of the path followed by the system between I and F is* 30 J

(a) For path IBF, $Q = 35$ J and $\Delta U = 30$ J

$\Rightarrow \quad W = Q - \Delta U = 35 - 30 = 5$ J

i.e., work is being done by the system.

(b) For path FI, we have

$W = -15$ J, $\Delta U = U_I - U_F = -30$ J

$\Rightarrow \quad Q = W + \Delta U = -15 - 30 = -45$ J

i.e., heat is lost by the system.

(c) Given, $U_I = 10$ J and $\Delta U = U_F - U_I$

$\Rightarrow \quad U_F = \Delta U + U_I = 30 + 10 = 40$ J

(d) The process BF is isochoric, i.e., the volume is constant, so $W = 0$.

$\Rightarrow \quad Q = \Delta U_{BF} = U_F - U_B = 40 - 20 = 20$ J

The process IB is isobaric (constant pressure). Therefore,

$Q = Q_{IBF} - Q_{BF} = 35 - 20 = 15$ J

7. Change in internal energy of gas

$$\Delta U = \frac{3}{2}(P_B V_B - P_A V_A)$$

$$\Rightarrow \quad \Delta U = \frac{3}{2}(8 \times 10^5 \times 0.8 - 4 \times 10^5 \times 0.5)$$

$$\Rightarrow \quad \Delta U = 6.6 \times 10^5 \text{ J}$$

Work done by gas is equal to area under the PV diagram, so we have

$$W = \frac{1}{2}(12 \times 10^5)(0.3) = 1.8 \times 10^5 \text{ J}$$

By FLTD, we have $Q = \Delta U + W$

$$\Rightarrow \quad Q = 8.4 \times 10^5 \text{ J}$$

8. Work done is given by

$$W = \underbrace{(\text{Area } 021)}_{\text{Clockwise}} - \underbrace{(\text{Area } 043)}_{\text{Counter Clockwise}}$$

$$W = \frac{1}{2}(V_2 - V_1)(P_0 - P_1) - \frac{1}{2}(V_4 - V_3)(P_2 - P_0)$$

Since by similarity, we have

$$\frac{V_4 - V_3}{V_2 - V_1} = \frac{P_2 - P_0}{P_0 - P_1}$$

$$\Rightarrow \quad V_4 - V_3 = \left(\frac{P_2 - P_0}{P_0 - P_1}\right)(V_2 - V_1)$$

$$\Rightarrow \quad V_4 - V_3 = \left(\frac{10^5}{2 \times 10^5}\right)(100 \times 10^{-3}) \text{ m}^3$$

$$\Rightarrow \quad V_4 - V_3 = 50 \times 10^{-3} \text{ m}^3$$

$$W = \frac{1}{2}(100 \times 10^{-3})(2 \times 10^5) - \frac{1}{2}(50 \times 10^{-3})(10^5)$$

$$\Rightarrow \quad W = 10^4 - 25 \times 10^2 = 7500 \text{ J}$$

9. Work done on the gas is the area bound by the semi-circle given by

$$W = \frac{\pi(1)(1)}{2} = \frac{\pi}{2} \text{ atmlt}$$

10. Work done are

$W_{A \to B} = 8 \times (3 \times 10^5) \times 10^{-3} = 2400$ J

$W_{B \to C} = 0$

$W_{C \to D} = -2 \times 3 \times 10^5 \times 10^{-3} = -600$ J

$W_{D \to A} = 0$

Total work is

$W_{\text{cycle}} = 2400 - 600 = 1800$ J

Test Your Concepts-VI (Based on Isochoric, Isobaric and Isothermal Processes)

1. Since helium is a monatomic gas. Therefore,

$$C_V = \frac{3R}{2} \text{ and } C_P = \frac{5R}{2}$$

(a) At constant volume, we have

$$Q = \Delta U = nC_V \Delta T = (2)\left(\frac{3R}{2}\right)(100)$$

$\Rightarrow \quad Q = 300R$

(b) At constant pressure,

$$Q = nC_P \Delta T = (2)\left(\frac{5R}{2}\right)(100)$$

$\Rightarrow \quad Q = 500R$

(c) At constant pressure, from FLTD, we have

$$W = Q - \Delta U = nC_P \Delta T - nC_V \Delta T$$

$\Rightarrow \quad W = nR\Delta T = (2)(R)(100)$

$\Rightarrow \quad W = 200R$

2. **Process** AB is an isothermal process with $T =$ constant and $P_B > P_A$.

For P-V graph: $P \propto \frac{1}{V}$ i.e., P-V graph is a hyperbola with $P_B > P_A$ and $V_B < V_A$.

For V-T graph: $T =$constant. Therefore, V-T graph is a straight line parallel to V-axis with $V_B < V_A$.

For ρ-T graph: Since $PV = RT$ where $V = \frac{M}{\rho}$

$$\Rightarrow \quad \rho = \frac{PM}{RT}$$

$$\Rightarrow \quad \rho \propto P$$

As T is constant. Therefore, ρ-T graph is a straight line parallel to ρ-*axis* with $\rho_B > \rho_A$ as $P_B > P_A$.

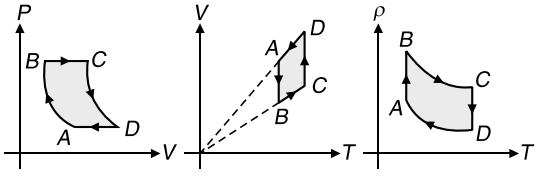
Process BC is an isobaric process with $P =$ constant and $T_C > T_B$.

For P-V graph: As P is constant. Therefore, P-V graph is a straight line parallel to V-axis with $V_C > V_B$ (because $V \propto T$ in an isobaric process).

For V-T graph: In isobaric process $V \propto T$, i.e., V-T graph is a straight line passing through origin, with $T_C > T_B$ and $V_C > V_B$.

For ρ-T graph: $\rho \propto \frac{1}{T}$ (when P =constant), i.e., ρ-T graph is a hyperbola with $T_C > T_B$ and $\rho_C < \rho_B$.

Similarly, we can repeat for the processes CD and DA. As they are opposite to AB and BC respectively. The corresponding three graphs are shown below.



3. One mole of gas occupies a volume of 22400 cm^3 at 0 °C and at pressure of 1 atm. So, the initial volume of the gas is

$$V_1 = 22400 \text{ cm}^3 = 22400 \times 10^{-6} \text{ m}^3$$

$$\Rightarrow \quad V_1 = 0.0224 \text{ m}^3$$

Since pressure is constant equal to atmospheric pressure $P_0 = 1 \text{ atm} = 10^5 \text{ Nm}^{-2}$, so the final volume V_2 is

$$V_2 = V_1\left(\frac{T_2}{T_1}\right) \qquad \left\{\because \frac{V_1}{T_1} = \frac{V_2}{T_2}\right\}$$

where, $T_1 = 0$ °C $= 273$ K, $T_2 = 100$ °C $= 373$ K

$$\Rightarrow \quad V_2 = 0.0224\left(\frac{373}{273}\right) \text{m}^3 = 0.0306 \text{ m}^3$$

Work done in the process is

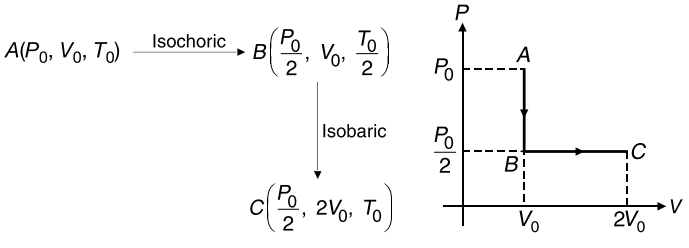
$$W = P(V_2 - V_1) \qquad \{\because P = \text{constant}\}$$

$$\Rightarrow \quad W = 10^5(0.0306 - 0.0224) = 10^5(0.0082)$$

$$\Rightarrow \quad W = 820 \text{ J}$$

i.e., work is done by the gas because the gas is expanding.

4. The most important thing to be observed here is that temperature of final state C (i.e., T_0) is equal to the temperature of initial state A (i.e., T_0). So, internal energy $\Delta U_{AC} = 0$



Now for the complete process $Q = \Delta U + W$

$$\Rightarrow \quad Q_{net} = W_{net} = W_{isochoric} + W_{isobaric}$$

But $W_{isochoric} = 0$

and $W_{isobaric} = \frac{P_0}{2}(\Delta V_0) = \frac{P_0 V_0}{2} = \frac{nRT_0}{2}$

$$\Rightarrow \quad W_{net} = \frac{(2)(8.3)(300)}{2} = 2490 \text{ J}$$

5. $W = P\Delta V = nR\Delta T = 1 \times 8.31 \times 72 \cong 600$ J

$\Delta U = Q - W = (500 \times 4.2 - 6 \times 10^2) = 1500$ J

$$\Rightarrow \quad \Delta U = \frac{nR\Delta T}{\gamma - 1}$$

$$\Rightarrow \quad 1500 = \frac{600}{\gamma - 1}$$

$$\Rightarrow \quad \gamma = 1.4$$

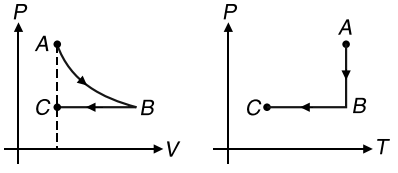
6. At constant pressure work done is

$$W = nR\Delta T = 2 \text{ J}$$

So, heat supplied to diatomic gas is

$$\Delta Q = \frac{7}{2}nR\Delta T = \frac{7}{2} \times 2 = 7 \text{ J}$$

7. (a)



(b) For the process AB (Isothermal),

$$\frac{P_0 V_0}{T_0} = \frac{P_B(2V_0)}{T_0}$$

$$\Rightarrow \quad P_B = \frac{P_0}{2}$$

Since, for isothermal process, $\Delta U = 0$

$$\Rightarrow \quad Q = W + \Delta U = nRT\log_e\left(\frac{V_B}{V_A}\right)$$

$$\Rightarrow \quad Q = 3RT_0\log_e(2)$$

For the process BC (Isobaric), $\frac{2V_0}{T_0} = \frac{V_0}{T_C}$

$$\Rightarrow \quad T_C = \frac{T_0}{2}$$

For isobaric process,

$$W = nR\Delta T = 3R\left(\frac{T_0}{2} - T_0\right) = -\frac{3}{2}RT_0$$

$$Q = nC_P\Delta T = -\frac{21}{4}RT_0$$

$$\Rightarrow \quad W_{Total} = 3RT_0\log_e(2) - \frac{3}{2}RT_0$$

(c) $Q_{Total} = 3RT_0\log_e(2) - \frac{21}{4}RT_0$

8. Let spring be displaced by x towards left, then

$$P_0(A\ell_0) = (P_0 + \Delta P)A(\ell_0 - x) \qquad \{\because P_1V_1 = P_2V_2\}$$

$$\Rightarrow \quad \Delta P = \frac{P_0 x}{\ell_0} \qquad \{\text{as } x \text{ is small}\}$$

When the spring is displaced by x, then spring force $= -kx$

Net, force against displacement $= -\frac{2P_0xA}{\ell_0} - kx = -M\omega^2 x$

$$T = 2\pi\sqrt{\frac{M\ell_0}{2P_0A + k\ell_0}}$$

CHAPTER 2

So, net restoring force is

$$F_{\text{net}} = \begin{pmatrix}\text{Force due to}\\ \text{Excess Pressure}\end{pmatrix} + \begin{pmatrix}\text{Spring}\\ \text{Force}\end{pmatrix}$$

$$\Rightarrow \quad F_{\text{net}} = -(\Delta P)A - kx$$

$$\Rightarrow \quad F_{\text{net}} = -\left(\frac{2P_0x}{\ell_0}\right)A - kx$$

$$\Rightarrow \quad M\ddot{x} + \left(\frac{2P_0A}{\ell_0} + k\right)x = 0 \qquad \{\because F_{\text{net}} = M\ddot{x}\}$$

Comparing with standard equation of SHM, i.e., $\ddot{x} + \omega^2 x = 0$, we get

$$\omega = \sqrt{\frac{1}{M}\left(\frac{2P_0A}{\ell_0} + k\right)}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{M\ell_0}{2P_0A + k\ell_0}}$$

9. Given, mass of piston $M = 5$ kg, cross-sectional area of piston $A = 5\times10^{-3}$ m^2, $g = 10$ ms^{-2} and atmospheric pressure $P_0 = 10^5$ Nm^{-2}.

Initial pressure of gas in the cylinder is $P = P_0 + \frac{Mg}{A}$

$$\Rightarrow \quad P = 10^5 + \frac{5\times10}{5\times10^{-6}} = 1.1\times10^5 \text{ Nm}^{-3}$$

When the piston rises by $x = 0.1$ m at constant pressure, then work done is $W = P_0\Delta V = P_0A\Delta x$

$$\Rightarrow \quad W = (10^5)(5\times10^{-3})(0.1) = 50 \text{ J}$$

At constant pressure, heat supplied to gas is

$$Q = \Delta U + W = \Delta U + 50$$

Now the piston is clamped, so volume of the gas remains constant during cooling. Hence work done during cooling is zero and $\Delta U' = -\Delta U$ (because the gas is cooled back to initial temperature). So by FLTD, we have

$$Q' = \Delta U' + W' = -\Delta U + 0 = -\Delta U$$

i.e., the heat energy lost during the isochoric cooling process is $Q_{\text{lost}} = |Q'| = \Delta U$

The difference between heat energy added during heating and heat energy lost during cooling is

$$\Delta Q = Q - |Q'| = (\Delta U + 50) - \Delta U = 50 \text{ J}$$

10. Let initial pressure is P_0 and pressure after first stroke is P_1. Then

$$P_1(V + \Delta V) = P_0V$$

$$\Rightarrow \quad P_1 = \frac{P_0}{\left(1 + \frac{\Delta V}{V}\right)}$$

Similarly, if P_n be the pressure after n strokes, then

$$P_n = \frac{P_0}{\left(1 + \frac{\Delta V}{V}\right)^n}$$

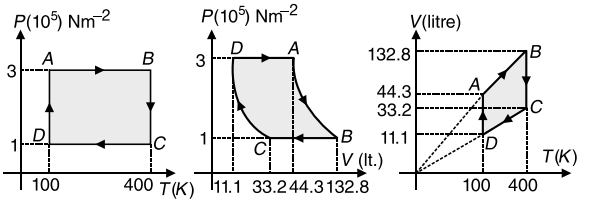
Since, we want that $P_n = \frac{P_0}{x}$

$$\Rightarrow \quad \frac{P_n}{P_0} = \frac{1}{x} = \frac{1}{\left(1 + \frac{\Delta V}{V}\right)^n}$$

Taking log both sides and solving, we get

$$n = \frac{\log_e(x)}{\log_e\left(1 + \frac{\Delta V}{V}\right)}$$

11.



12. Heat absorbed in an isobaric process is

$$\Delta Q = nC_P\Delta T = \frac{n\gamma R}{\gamma - 1}\Delta T = \frac{\gamma P}{\gamma - 1}(V_2 - V_1)$$

$$\Rightarrow \quad \Delta Q = \frac{1.67}{0.67}\times10^5\times10^{-4} = 24.92 \text{ J}$$

13. Since, $(PV)_A = (PV)_C = 3P_0V_0$

$$\Rightarrow \quad T_A = T_C$$

$$\Rightarrow \quad \Delta T = 0$$

$$\Rightarrow \quad \Delta U_{ABC} = 0 \qquad \{\because \Delta U = nC_V\Delta T\}$$

So, $Q_{ABC} = W_{ABC}$ =Area under the graph

$$\Rightarrow \quad Q_{ABC} = W_{ABC} = -2P_0V_0 \qquad \{\because \text{of compression}\}$$

So, heat is released during the process.

Since, $PV = nRT$

$$\Rightarrow \quad \frac{PM}{\rho} = nRT$$

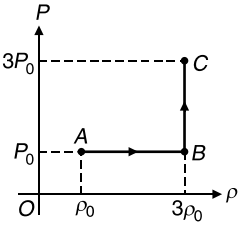
$$\Rightarrow \quad P = \left(\frac{nRT}{M}\right)\rho$$

For AB, $P = P_0$ =constant i.e., $V \propto T$

$$\Rightarrow \quad \rho \propto \frac{1}{T}$$

Also, volume decreases from $3V_0$ to V_0, so temperature decreases from $3T_0$ to T_0 and hence ρ increases from ρ_0 to $3\rho_0$.

For B to C, V =constant and P increases from P_0 to $3P_0$.



14. As the piston is displaced slowly the change in kinetic energy is zero. From Work Energy Theorem, we have

$$W_{\text{atm}} + W_{\text{gas}} + W_{\text{ext}} = \Delta KE = 0 \qquad \ldots(1)$$

where, $W_{atm} = -\int_{V_0}^{nV_0} P_0 dV = -P_0V_0(n-1)$

Also, $W_{gas} = \int_{V_0}^{nV_0} PdV = RT\int_{V_0}^{nV_0}\frac{dV}{V}$ $\{\because PV = nRT\}$

$\Rightarrow \quad W_{gas} = RT\log_e n$

From (1), we get $W_{ext} = -W_{atm} - W_{gas}$

$\Rightarrow \quad W_{ext} = -\left(-P_0V_0(n-1) + RT\log_e n\right)$

$\Rightarrow \quad W_{ext} = RT\left((n-1) - \log_e(n)\right)$

15. (a) At constant pressure $W = P\Delta V = nR\Delta T$ (as $PV = nRT$)

$\Rightarrow \quad W = nR\Delta T = 2\times 2\times(35-30) = 20$ cal

(b) $\Delta U = Q_p - W$

$\Rightarrow \quad \Delta U = 70 - 20 = 50$ cal

Test Your Concepts-VII (Based on Adiabatic Process)

1. In adiabatic process $\frac{dP}{dV} = -\gamma\left(\frac{P_0}{V_0}\right)$

$\Rightarrow \quad dP = -\gamma\left(\frac{P_0}{V_0}\right)dV$

When the piston is displaced towards right by a small distance x, then $dV = Ax$

So, net restoring force is

$$F = (2dP)A = -2\gamma\left(\frac{P_0}{V_0}\right)A^2x$$

Since, $F \propto -x$

Hence the motion of piston is simple harmonic in nature

So, acceleration a is given by

$$a = \frac{F}{m} = -\left(\frac{2\gamma P_0A^2}{V_0m}\right)x$$

$\Rightarrow \quad T = 2\pi\sqrt{\left|\frac{x}{a}\right|} = 2\pi\sqrt{\frac{mV_0}{2\gamma A^2P_0}}$

2. In case of isothermal expansion, the work done is given by

$$W_1 = W_{isot} = nRT\ln\left(\frac{V_2}{V_1}\right)$$

where, $T = 27\ ^\circ\text{C} = 273 + 27 = 300\ ^\circ\text{K}$, $n = 1$, $\frac{V_2}{V_1} = 2$ and $R = \frac{25}{3}$ Jmol^{-1}K^{-1}

$\Rightarrow \quad W_1 = (25/3)\times 300\times\ln(2)$

$\Rightarrow \quad W_1 = (1)(25/3)(300)(0.7) = 1750$ J

Now the gas is compressed adiabatically to its original volume. Initially at the beginning of adiabatic compression, the temperature of the gas is 300 K and at the end of adiabatic compression, the temperature becomes T_2 because the temperature is changed. Now, the initial volume of the gas is $2V_1$ and after compression it again becomes the original volume i.e., V_1. For an adiabatic process

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$\Rightarrow \quad 300(2V_1)^{\gamma-1} = T_2(V_i)^{\gamma-1}$

$\Rightarrow \quad 300(2)^{1.4-1} = T_2(1)^{1.4-1}$

$\Rightarrow \quad T_2 = 300(2)^{0.4} = (300)(1.3) = 390$ K

Work done during adiabatic process is

$$W_2 = W_{ad} = \frac{nR}{1-\gamma}(T_2 - T_1)$$

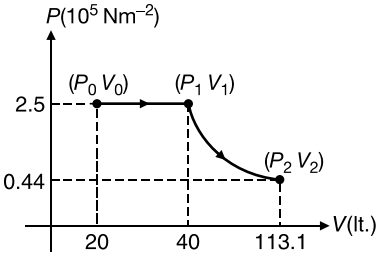
$\Rightarrow \quad W_2 = \frac{(1)(25/3)}{(-0.4)}(390-300) = -1875$ J

Total work done is given by

$$W = W_1 + W_2 = 1750 - 1875 = -125 \text{ J}$$

i.e., the work is done on the gas.

3. (a)



(b) $P_0 = \frac{nRT_0}{V_0} = \frac{2\times 8.31\times 300}{20\times 10^{-3}}$

$\Rightarrow \quad P_0 = 2.5\times 10^5$ Nm^{-2}

$\Rightarrow \quad P_1 = P_0 = 2.5\times 10^5$ Nm^{-2}

Also, $V_1 = 2V = 40\times 10^{-3}$ m^3

Process *AB*: $V \propto T$

$\Rightarrow \quad T_1 = 2T_0 = 600$ K

Process *BC*: Using $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$, we get

$V_2 = 2\sqrt{2}V_1 = 113.1\times 10^{-3}$ m^3

and $P_2 = \frac{nRT_2}{V_2} = \frac{(2)(8.31)(300)}{113.1\times 10^{-3}}$

$P_2 = 0.44\times 10^5$ Nm^{-2}

(c) $W_{total} = W_1 + W_2$

$$W_{total} = P_0(V_1 - V_0) + \frac{nR}{\gamma-1}(T_1 - T_2)$$

Substituting the values, we get

$$W_{total} = 12470 \text{ J}$$

4. Sudden compression implies adiabatic process, so

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$\Rightarrow \quad 300(V_1)^{0.5} = T_2\left(\frac{V_1}{2}\right)^{0.5}$

$\Rightarrow \quad T_2 = 300\sqrt{2} = 423$ K

At the end of adiabatic process, pressure is

$$P_2 = P_1\left(\frac{V_1}{V_1/2}\right)^{1.5} = 2\sqrt{2}P_1 = 2\sqrt{2}\times 10^5 \text{ Pa}$$

CHAPTER 2

Now, for cooling the gas isobarically to 300 K from an initial temperature of $300\sqrt{2}$ K, we have

$$\frac{V_2}{T_2}=\frac{V_3}{T_3}$$

$$\Rightarrow \quad V_3=V_2\left(\frac{T_3}{T_2}\right)=\left(\frac{V_1}{2}\right)\left(\frac{300}{300\sqrt{2}}\right)=\frac{V_1}{2\sqrt{2}}$$

Now the gas is isothermally expanded to its original volume, so we have $V_4=V_1$

Since, $P_3V_3=P_4P_4$

$$\Rightarrow \quad \left(2\sqrt{2}P_1\right)\left(\frac{V_1}{2\sqrt{2}}\right)=P_4V$$

$$\Rightarrow \quad P_4=P_1$$

Work done by gas in first process is

$$W_{\text{adia}}=\frac{P_2V_2-P_1V_1}{1-\gamma}=\frac{\left(2\sqrt{2}P_1\right)\left(\frac{V_1}{2}\right)-P_1V_1}{(-0.5)}$$

$$\Rightarrow \quad W_{\text{adia}}=-\left(\frac{\sqrt{2}-1}{0.5}\right)(10^5)(10^{-3})$$

$$\Rightarrow \quad W_{\text{adia}}=-82\text{ J}$$

Also, $W_{\text{isob}}=P\Delta V=P_2(V_3-V_2)$

$$\Rightarrow \quad W_{\text{isob}}=\left(2\sqrt{2}P_1\right)\left(\frac{V_1}{2\sqrt{2}}-\frac{V_1}{2}\right)$$

$$\Rightarrow \quad W_{\text{isob}}=P_1V_1\left(1-\sqrt{2}\right)$$

$$\Rightarrow \quad W_{\text{isob}}=\left(1-\sqrt{2}\right)(10^5)(10^{-3})=-41\text{ J}$$

Now, $W_{\text{isot}}=nRT_3\ln\left(\frac{V_4}{V_3}\right)=P_3V_3\ln\left(\frac{V_4}{V_3}\right)$

$$\Rightarrow \quad W_{\text{isot}}=\left(2\sqrt{2}P_1\right)\left(\frac{V_1}{2\sqrt{2}}\right)\ln\left(\frac{V_1}{V_1/2\sqrt{2}}\right)$$

$$\Rightarrow \quad W_{\text{isot}}=P_1V_1\ln\left(2\sqrt{2}\right)=P_1V_1\ln(2)^{1.5}$$

$$\Rightarrow \quad W_{\text{isot}}=1.5(10^5)(10^{-3})(0.7)=105\text{ J}$$

Now, $W_{\text{total}}=W_{\text{adia}}+W_{\text{isob}}+W_{\text{isot}}$

$$\Rightarrow \quad W_{\text{total}}=-82-41+105=-18\text{ J}$$

5. For Isothermal process, $\Delta U_1=0$

$$\Rightarrow \quad Q_1=W_1=RT\log_e\left(\frac{V_f}{V_i}\right)=RT\log_e(2)$$

$$\Rightarrow \quad Q_1=300R\log_e(2) \qquad \ldots(1)$$

For Isochoric process, $W_2=0$

$$\Rightarrow \quad Q_2=\Delta U_2=C_V\Delta T=\frac{3}{2}R\Delta T \qquad \ldots(2)$$

Since it is given that $Q_1=Q_2$

So, from (1) and (2), we get

$$\Delta T=\left(\frac{2}{3}\right)(300)(\log_e 2)=138.6\text{ K}$$

That is, final temperature of second vessel is

$$(300+138.6)K=438.6\text{ K}$$

Let P_0 and T_0 be the final pressure and temperature of the mixture and since, same mass of the same gas is being mixed, so we have

$$T_0=\frac{300+438.6}{2}=369.3\text{ K}$$

$$\Rightarrow \quad P_0=\frac{nRT_0}{V}=\frac{2\times8.3\times369.3}{3\times8.3\times10^{-3}}=2.46\times10^5\text{ Nm}^{-2}$$

6. For an adiabatic process, we have

$$P^{1-\gamma}T^{\gamma}=\text{constant}$$

$$\Rightarrow \quad T_2=T_1\left(\frac{P_1}{P_2}\right)^{\frac{1-\gamma}{\gamma}}=300\left(\frac{1}{10}\right)^{-\frac{2}{7}}=300(10)^{2/7}$$

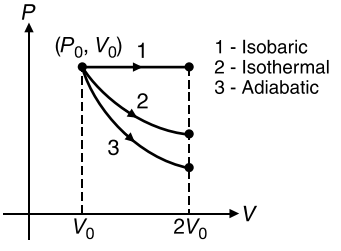
$$\Rightarrow \quad T_2=300(1.4)^2=300(1.96)=588\text{ K}$$

Also, $W_{\text{ad}}=\frac{nR}{1-\gamma}(T_2-T_1)$

$$\Rightarrow \quad W_{\text{ad}}=\frac{(1)(8.3)}{1-1.4}(588-300)=-5976\text{ J}$$

Negative sign implies that work is being done on the system.

7. (a)



Area under graph 3 is least. Therefore, work done is least in adiabatic process

(b) For 1, an Isobaric process, pressure remains constant, so $V\propto T$, so when V_0 becomes $2V_0$, T_0 also becomes $2T_0$.

For 2, an Isothermal process, T is constant while pressure goes from P_0 to $\frac{P_0}{2}$.

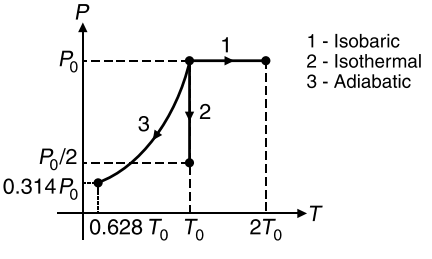
For 3, an Adiabatic equation, the P-T equation is given by

$$PT^{\left(\frac{\gamma}{1-\gamma}\right)}=\text{constant}$$

For a monatomic gas, $\gamma=\frac{5}{3}$

$$\Rightarrow \quad P_0V_0^{5/3}=P(2V_0)^{5/3}$$

$$\Rightarrow \quad P=0.314P_0$$



Also, for an adiabatic process, we have

$$PV^{\gamma}=\text{constant and } TV^{\gamma-1}=\text{constant}$$

$$\Rightarrow \quad \frac{T}{PV}=\text{constant}$$

$\Rightarrow \quad T \propto PV$

$\Rightarrow \quad \frac{T}{T_0} = \left(\frac{P}{P_0}\right)\left(\frac{V}{V_0}\right)$

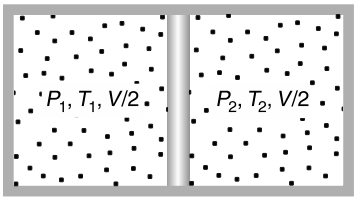
$\Rightarrow \quad T = (0.314)(2)T_0 = 0.628T_0$

8. For an adiabatic process, $Q = 0$ i.e., $W = -\Delta U$

Given that $\Delta U = -100$ J, so $W = 100$ J

9. (a) Let T be the final temperature in both the compartments, then from First Law of Thermodynamics applied to the cylinder, we have

$Q = \Delta U + W$



Since the cylinder wall is adiabatic and fixed, so

$Q = W = 0$

$\Rightarrow \quad \Delta U = 0$

So, there is no change in internal energy of the system

$\Rightarrow \quad n_1 C_V (T - T_1) = n_2 C_V (T_2 - T)$

$\Rightarrow \quad \frac{P_1 V}{2RT_1}(T - T_1) = \frac{P_2 V}{2RT_2}(T_2 - T)$

Solving, we get $T = \frac{(P_1 + P_2)T_1 T_2}{P_1 T_2 + P_2 T_1}$

For the left compartment, $\frac{P_1}{T_1} = \frac{P_1'}{T}$

where, P_1' is the final pressure in the left compartment

$\Rightarrow \quad P_1' = \frac{P_1 (P_1 + P_2)T_2}{P_1 T_2 + P_2 T_1}$

Similarly, we get $P_2' = \frac{P_2 (P_1 + P_2)T_1}{P_1 T_2 + P_2 T_1}$

(b) Since, $P_2 T_1 > P_1 T_2$, it follows that $P_2' > P_1'$. Therefore, the piston moves to the left. Let ΔV be the change in volume of any compartment, then on applying the Ideal Gas Equation, we get

$$\frac{P_f\left(\frac{V}{2} - \Delta V\right)}{T_f} = \frac{P_1 V}{2T_1} \qquad \ldots(1)$$

and $$\frac{P_f\left(\frac{V}{2} + \Delta V\right)}{T_f} = \frac{P_2 V}{2T_2} \qquad \ldots(2)$$

On dividing equation (2) by (1), we get

$$\frac{\left(\frac{V}{2} + \Delta V\right)}{\left(\frac{V}{2} - \Delta V\right)} = \frac{P_2 T_1}{P_1 T_2}$$

So, volume of left compartment is

$$V_L = \frac{V}{2} - \Delta V = V\left(\frac{P_1 T_2}{P_2 T_1 + P_1 T_2}\right)$$

and volume of right compartment is

$$V_R = \frac{V}{2} + \Delta V = V\left(\frac{P_2 T_1}{P_2 T_1 + P_1 T_2}\right)$$

10. Since, $P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$

$\Rightarrow \quad \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma}$

$\Rightarrow \quad \frac{P_2}{P_1} = \left(\frac{16}{12}\right)^{3/2} = \left(\frac{4}{3}\right)^{3/2} = 1.533$

Also, $T_1 V_1^{\gamma - 1} = T_2 V_2^{\gamma - 1}$

$\Rightarrow \quad \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1} = \left(\frac{16}{12}\right)^{\frac{1}{2}} = \left(\frac{4}{3}\right)^{\frac{1}{2}} = 1.153$

11. For an adiabatic process, we have

$TV^{\gamma - 1} = \text{constant}$

Differentiating with respect to T,

$$V^{\gamma - 1}(1) + (T)(\gamma - 1)V^{\gamma - 2}\frac{dV}{dT} = 0$$

$\Rightarrow \quad \frac{dV}{dT} = -\frac{V}{T(\gamma - 1)}$

But from the graph, we have slope $= \tan(\pi - \theta)$

$\Rightarrow \quad \left|\frac{dV}{dT}\right|_{T_0,\, V_0} = \tan(\pi - \theta) = -\tan\theta$

$\Rightarrow \quad \frac{V_0}{T_0(\gamma - 1)} = \tan\theta$

$\Rightarrow \quad \gamma - 1 = \frac{V_0}{T_0 \tan\theta}$

$\Rightarrow \quad \gamma = \frac{V_0}{T_0 \tan\theta} - 1$

$\Rightarrow \quad C_P = \frac{\gamma R}{\gamma - 1} = \frac{(V_0 - T_0 \tan\theta)R}{(T_0 \tan\theta)V_0} T_0 \tan\theta$

$\Rightarrow \quad C_P = \left(1 - \frac{T_0}{V_0}\tan\theta\right)R$

and $C_V = \frac{R}{\gamma - 1} = \frac{RT_0 \tan\theta}{V_0}$

Test Your Concepts-VIII (Based on Polytropic Process)

1. Given equation is $VT^2 = \text{constant}$

Since, $PV = nRT$

$\Rightarrow \quad PV^{3/2} = \text{constant}$

Comparing with $PV^x = \text{constant}$, we get $x = \frac{3}{2}$

Since, $C = C_V + \frac{R}{1 - x} = \frac{R}{\gamma - 1} + \frac{R}{1 - x}$ and $\gamma = \frac{7}{5}$

CHAPTER 2

$$\Rightarrow \quad C=\frac{R}{\frac{7}{5}-1}+\frac{R}{1-\frac{3}{2}}=\frac{5}{2}R-2R=\frac{R}{2}$$

Now, $Q=nC\Delta T$

$$\Rightarrow \quad Q=(3)\left(\frac{R}{2}\right)(2T-T)=\frac{3}{2}RT$$

2. (a) Heat supplied to raise the temperature of one mole of gas from T_0 to ηT_0

$$Q=\int CdT=\int_{T_0}^{\eta T_0}\frac{\alpha}{T}dT=\alpha\ln(\eta)$$

Increase in internal energy of gas is

$$\Delta U=C_V(\eta T_0-T_0)=\frac{RT_0}{\gamma-1}(\eta-1)$$

From FLTD, work done is

$$W=Q-\Delta U=\alpha\ln(\eta)-\frac{RT_0(\eta-1)}{(\gamma-1)}$$

(b) Since, $C=C_V+\frac{PdV}{ndT}$

$$\Rightarrow \quad \frac{\alpha}{T}=\frac{R}{\gamma-1}+\frac{RT}{V}\frac{dV}{dT}$$

$$\Rightarrow \quad \int\frac{dV}{V}=\int\frac{\alpha}{RT^2}dT-\frac{1}{\gamma-1}\int\frac{dT}{T}$$

$$\Rightarrow \quad \ln V=-\frac{\alpha}{R}\frac{1}{T}-\frac{1}{\gamma-1}\ln T+C$$

$$\Rightarrow \quad (\gamma-1)\ln V+\ln T=-\frac{\alpha(\gamma-1)}{RT}+C$$

$$\Rightarrow \quad \ln(TV^{\gamma-1})=-\frac{\alpha(\gamma-1)}{RT}+C$$

$$\Rightarrow \quad TV^{\gamma-1}e^{\frac{\alpha(\gamma-1)}{RT}}=\text{constant}$$

$$\Rightarrow \quad PV^{\gamma}e^{\frac{\alpha(\gamma-1)}{PV}}=\text{constant}$$

3. Equation of the process is $P=P_0-\alpha V^2$

Using ideal gas equation $PV=RT$, we get

$$(P_0-\alpha V^2)V=RT$$

$$\Rightarrow \quad T=\frac{P_0V}{R}-\frac{\alpha V^3}{R}$$

For maximum value of T, we have $\frac{dT}{dV}=0$

$$\Rightarrow \quad \frac{dT}{dV}=\frac{P_0}{R}-\frac{3\alpha V^2}{R}=0$$

$$\Rightarrow \quad V=\sqrt{\frac{P_0}{3\alpha}}$$

$$\Rightarrow \quad T_{\max}=\frac{2P_0}{3R}\sqrt{\frac{P_0}{3\alpha}}$$

4. Equation for first thermodynamic process is

$$T=T_0e^{\alpha V} \qquad \ldots(1)$$

Since, we know that for any process

$$C=\frac{1}{n}\left(\frac{dQ}{dT}\right)=\frac{1}{n}\left(\frac{dU+dW}{dT}\right)$$

$$\Rightarrow \quad C=\frac{1}{n}\left(\frac{dU}{dT}\right)+\frac{P}{n}\left(\frac{dV}{dT}\right)$$

$$\Rightarrow \quad C=C_V+\frac{P}{n}\left(\frac{dV}{dT}\right) \qquad \ldots(2)$$

From gas law, the pressure and volume relation is

$$P=\frac{nRT}{V}=\frac{nR(T_0e^{\alpha V})}{V} \qquad \ldots(3)$$

Differentiating equation (1) w.r.t. V, we get

$$\frac{dT}{dV}=T_0\alpha e^{\alpha V}$$

$$\Rightarrow \quad \frac{dV}{dT}=\frac{1}{T_0\alpha e^{\alpha V}} \qquad \ldots(4)$$

Substituting equations (3) and (4) in equation (2), we get

$$C=C_V+\left(\frac{nRT_0e^{\alpha V}}{nV}\right)\left(\frac{1}{T_0\alpha e^{\alpha V}}\right)$$

$$\Rightarrow \quad C=C_V+\frac{R}{\alpha V}$$

Since, the molar specific heat of the gas undergoing this given process is not constant and is a function of volume, so this process is a non-polytropic process.

Equation for second thermodynamic process is

$$P=P_0e^{\alpha V} \qquad \ldots(5)$$

$$\Rightarrow \quad \frac{nRT}{V}=P_0e^{\alpha V}$$

$$\Rightarrow \quad T=\frac{P_0}{nR}(Ve^{\alpha V}) \qquad \ldots(6)$$

Differentiate equation (6), w.r.t. V, we get

$$\frac{dT}{dV}=\frac{P_0}{nR}(e^{\alpha V}+V\alpha e^{\alpha V})=\frac{P_0e^{\alpha V}}{nR}(1+\alpha V) \qquad \ldots(7)$$

Substituting equations (7) and (5) in equation (2), we get

$$C=C_V+\left(\frac{P_0e^{\alpha V}}{n}\right)\left(\frac{nR}{P_0e^{\alpha V}(1+\alpha V)}\right)$$

$$\Rightarrow \quad C=C_V+\frac{R}{1+\alpha V}$$

Again, we note that the molar specific heat of the gas undergoing this given process is not constant and is a function of volume, so this process is a non-polytropic process.

5. Given that $TV=\text{constant}$

Compare with $TV^{x-1}=\text{constant}$, we get $x-1=1$

$$\Rightarrow \quad x=2$$

$$\Rightarrow \quad C=C_V+\frac{R}{1-x}=C_V+\frac{R}{1-2}$$

$$\Rightarrow \quad C=C_V-R$$

Since, $Q=nC\Delta T=(1)C\Delta T$

$$\Rightarrow \quad Q=(C_V-R)\Delta T=C_V\Delta T-R\Delta T$$

$$\Rightarrow \quad Q = \frac{R}{\gamma-1}\Delta T - R\Delta T = R\Delta T\left(\frac{1}{\gamma-1}-1\right)$$

$$\Rightarrow \quad Q = \frac{R\Delta T(1-\gamma+1)}{\gamma-1} = \frac{\gamma R\Delta T}{1-\gamma}$$

6. Since the rate of collisions with wall of vessel

$$N_C = \frac{1}{6}n_0 v_{rms} = \text{constant}$$

where, n_0 is the number density of molecules.

$$\Rightarrow \quad \frac{1}{6}\left(\frac{N}{V}\right)\sqrt{\frac{3RT}{m}} = \text{constant}$$

$$\Rightarrow \quad \frac{\sqrt{T}}{V} = \text{constant}$$

$$\Rightarrow \quad T = kV^2$$

$$\Rightarrow \quad \frac{dT}{dV} = 2kV$$

Molar specific of the gas is given by

$$C = C_V + \frac{PdV}{ndT}, \text{ where } C_V = \frac{5R}{2}$$

$$\Rightarrow \quad C = \frac{5R}{2} + \left(\frac{RT}{V}\right)\frac{dV}{dT} = \frac{3R}{2} + \frac{R(kV^2)}{V}\frac{1}{2kV}$$

$$\Rightarrow \quad C = \frac{5R}{2} + \frac{R}{2} = 3R$$

7. Let the initial volume of the gas is V and it is expanded to volume ηV. The work done in the process is given by

$$W = \int_V^{\eta V} PdV = \int_V^{\eta V} \alpha V dV = \frac{\alpha V^2}{2}(\eta^2 - 1)$$

The pressure of the gas as a function of volume is $P = \alpha V$. The initial and final values of pressure would be $V_i = \alpha V$ and $V_f = \eta\alpha V$ respectively.

From Ideal Gas Equation, we get

$$T_i = \frac{P_i V_i}{nR} = \frac{\alpha V^2}{nR} \text{ and } T_f = \frac{P_f V_f}{nR} = \frac{\eta^2\alpha V^2}{nR}$$

Since, $\Delta U = nC_V\Delta T = \frac{nR}{\gamma-1}\Delta T = \frac{nR}{\gamma-1}(T_f - T_i)$

$$\Rightarrow \quad \Delta U = \left(\frac{nR}{\gamma-1}\right)\left(\frac{\alpha V^2}{nR}\right)(\eta^2-1) = \frac{\alpha V^2}{\gamma-1}(\eta^2-1)$$

Since, $P = \alpha V$

$$\Rightarrow \quad PV^{-1} = \text{constant}$$

Compare with $PV^x = \text{constant}$, we get $x = -1$

$$\Rightarrow \quad C = C_V + \frac{R}{1-x} = C_V + \frac{R}{2}$$

8. For a given polytropic process $PV^x = \text{constant}$ and molar specific heat is given by

$$C = C_V + \frac{R}{1-x} = \frac{R}{\gamma-1} + \frac{R}{1-x} \qquad \ldots(1)$$

The given equation can be re-written as

$$PV^{-\alpha} = k = \text{constant}$$

So, comparison of the two equations gives

$$x = -\alpha$$

According to the problem, we should have $C = 0$

$$\Rightarrow \quad C = \frac{R}{\gamma-1} + \frac{R}{1-(-\alpha)} = 0$$

$$\Rightarrow \quad \frac{R}{1-\gamma} = \frac{R}{1+\alpha}$$

$$\Rightarrow \quad \alpha = -\gamma$$

We could have given this answer directly, because for an adiabatic process, $Q = 0$ and hence $C = 0$.

9. (a) Given that $PT^{-1/2} = \text{constant}$ …(1)

For a polytropic process, we have

$$PT^{\frac{x}{1-x}} = \text{constant} \qquad \ldots(2)$$

Comparing equations (1) and (2), we get

$$\frac{x}{1-x} = -\frac{1}{2}$$

$$\Rightarrow \quad -2x = 1 - x$$

$$\Rightarrow \quad x = -1$$

Work done by a gas in a polytropic process is

$$W_{\text{poly}} = \frac{nR}{1-x}(T_2 - T_1), \text{ where } x = -1$$

$$\Rightarrow \quad W_{\text{poly}} = \frac{nR}{1-(-1)}(T_2 - T_1) = \frac{nR}{2}\Delta T$$

$$\Rightarrow \quad W_{\text{poly}} = \frac{(2)(25/3)}{2}(60) = 500 \text{ J}$$

(b) Since $C = C_V + \frac{R}{1-x}$ where, $(C_V)_{\text{monatomic}} = \frac{3R}{2}$

$$\Rightarrow \quad C = \frac{3R}{2} + \frac{R}{1-(-1)} = 2R$$

Test Your Concepts-IX (Based on Cyclic Process, Heat Engine and Refrigerator)

1. For process AB we use

$$PT = \text{constant}$$

$$\Rightarrow \quad \left(\frac{RT}{V}\right)T = \text{constant}$$

$$\Rightarrow \quad T^2 \propto V$$

$$\Rightarrow \quad T = k\sqrt{V} \text{ OR } P\sqrt{V} = \text{constant}$$

$$\Rightarrow \quad PV^{\frac{1}{2}} = \text{constant i.e., } x = \frac{1}{2}$$

Thus, molar specific heat of gas is

$$C = C_V + \frac{R}{1-x} = C_V + 2R$$

$$\Rightarrow \quad C = \frac{3R}{2} + 2R = \frac{7R}{2}$$

Work done by gas in process AB is

$$W_{\text{poly}} = \frac{nR}{1-x}(T_2 - T_1)$$

CHAPTER 2

$$\Rightarrow \quad W_{\text{poly}} = \frac{2R}{1-(1/2)}(T_0 - 2T_0) = -4RT_0$$

$$\Rightarrow \quad W_{\text{poly}} = -4RT_0 = -1200R$$

$$Q_{AB} = 2C(T_0 - 2T_0) = -2\left(\frac{7R}{2}\right)T_0$$

$$\Rightarrow \quad Q_{AB} = -7RT_0 = -2100R$$

$$Q_{BC} = 2C_P(2T_0 - T_0) = 2\left(\frac{5R}{2}\right)T_0 = 1500R$$

$$Q_{CA} = 2R(2T_0)\ln(2)$$

$$\Rightarrow \quad Q_{CA} = 1200R \times 0.693 = 831.6R$$

2. Done in Theory

3. Let P and V be the initial pressure and volume respectively, then for process $1 \to 2$

$$\frac{P_2}{T_2} = \frac{P}{T_1} \qquad \{\because \text{volume remains constant}\}$$

$$\Rightarrow \quad P_2 = P\frac{T_2}{T_1} \qquad \ldots(1)$$

For process $2 \to 3$

$$\frac{V_3}{T_3} = \frac{V}{T_2}$$

$$\Rightarrow \quad V_3 = V\frac{T_3}{T_2} \qquad \ldots(2)$$

For process $3 \to 4$ (temperature at points 3 and 4 is the same), so

$$V_4 = V\frac{T_2}{T_1} \qquad \ldots(3)$$

Since work done by the gas is area of the trapezium 12341

$$\Rightarrow \quad W = \frac{1}{2}\left[(V_3 - V_2) + (V_4 - V_1)\right](P_2 - P_1)$$

$$\Rightarrow \quad W = \frac{1}{2}\left(V\frac{T_3}{T_2} + V\frac{T_2}{T_1} - 2V\right)\left(P\frac{T_2}{T_1} - P\right)$$

$$\Rightarrow \quad W = \frac{1}{2}PV\left(\frac{T_3}{T_2} + \frac{T_2}{T_1} - 2\right)(T_2 - T_1)$$

But $PV = nRT_1$

$$\Rightarrow \quad W = \frac{1}{2}nR\left(\frac{T_3}{T_2} + \frac{T_2}{T_1} - 2\right)(T_2 - T_1)$$

4. For $A \to B$, $V \propto T$

$$\Rightarrow \quad P = \text{constant}$$

So, V and T both are increasing.
Hence we get a straight line parallel to V axis as P is constant.

For $B \to C$, V is constant

$$\Rightarrow \quad P \propto T$$

So, T is decreasing and P is also decreasing.
Hence we get a straight line parallel to P axis, as V is constant.

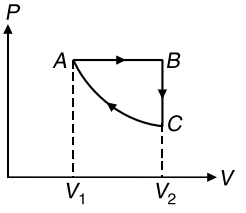
For $C \to A$, T is constant

$$\Rightarrow \quad P \propto \frac{1}{V}$$ So, if V is decreasing, then P should increase

Hence, we get a rectangular hyperbola because

$$PV = \text{constant}$$

The corresponding graph is shown in figure given below.



5. For process CA, volume remains constant, so no work is done in this process.
For process AB, since $T \propto V$ so gas pressure remains constant and hence work done by the gas in this process is

$$W_{AB} = nR(T_B - T_A)$$

$$\Rightarrow \quad W_{AB} = 2 \times 8.314 \times (500 - 300)$$

$$\Rightarrow \quad W_{AB} = 2 \times 8.314 \times 200 = 3325.6 \text{ J}$$

For the cyclic process, we have $\Delta U = 0$, so

$$Q = W = W_{AB} + W_{BC} + W_{CA}$$

According to the problem a total 1200 J of heat is rejected by the gas, so we have

$$Q = -1200 \text{ J}$$

$$\Rightarrow \quad -1200 = Q_{AB} + W_{BC} + W_{CA}$$

$$\Rightarrow \quad -1200 = 3325.6 + W_{BC} + 0$$

$$\Rightarrow \quad W_{BC} = -4525.6 \text{ J}$$

6. For isothermal process AB, we have

$$Q_{AB} = 2R(500)\ln(2)$$

For isothermal process CD, we have

$$Q_{CD} = -2R(300)\ln(2)$$

For isochoric process BC, we have

$$Q_{BC} = 2C_V(-200) = -400C_V$$

Similarly, for isochoric process DA, we have

$$Q_{DA} = 2C_V(200) = 400C_V$$

Total heat supplied to gas is

$$Q = Q_{AB} + Q_{BC} + Q_{CD} + Q_{DA}$$

$$\Rightarrow \quad Q = 400R\ln 2 = (400)(8.314)(0.693)$$

$$\Rightarrow \quad Q = 2304.64 \text{ J}$$

7. Since the process is cyclic, i.e., $\Delta U = 0$, so

$$Q = W_{AB} + W_{BC} + W_{CA} \qquad \ldots(1)$$

For process AB, volume of gas remains constant, so work done is zero.

$$\Rightarrow \quad W_{AB} = 0$$

For process BC, temperature is constant at 500 K and volume of gas changes from $V_1 = 2 \text{ m}^3$ to $V_2 = 5 \text{ m}^3$, so we have

$$W_{BC} = nRT\ln\left(\frac{V_2}{V_1}\right) = (1)R(500)\ln\left(\frac{5}{2}\right)$$

For process CA, we observe that $V \propto T$ and the line passes through the origin, so the pressure is constant

$$W_{CA} = P\Delta V = nR\Delta T = nR(T_2 - T_1)$$

$\Rightarrow \quad W_{CA} = (1)R(300-500) = -200R$

This is negative as gas is being compressed from volume 5 m^3 to 2 m^3 or work is done on the gas.

Substituting these values in equation (1), we get

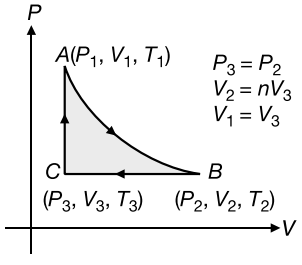
$$Q = 0 + 500R\ln\left(\frac{5}{2}\right) - 200R$$

$\Rightarrow \quad Q = 8.3[500(0.9) - 200] = 2075$ J

8. $Q_1 = 0$ {adiabatic}

$Q_2 = C_P(T_3 - T_2)$ {isobaric}

Further, $\frac{V_2}{T_2} = \frac{V_3}{T_3}$



$\Rightarrow \quad \frac{nV}{T_2} = \frac{V}{T_3}$

$\Rightarrow \quad T_3 = \frac{T_2}{n}$

Since, $C_P = \left(\frac{\gamma}{\gamma-1}\right)R$

$\Rightarrow \quad Q_2 = \frac{R\gamma}{\gamma-1}\left(\frac{T_2}{n} - T_2\right) = \frac{R\gamma T_2}{n(\gamma-1)}(1-n)$

So, heat rejected in isobaric process is

$$Q_2 = -\frac{R\gamma T_2}{n(\gamma-1)}(n-1)$$

At constant volume,

$$Q_3 = C_V(T_1 - T_3) = \frac{R}{\gamma-1}(T_1 - T_3)$$

$\Rightarrow \quad Q_3 = \frac{R}{\gamma-1}\left(T_1 - \frac{T_2}{n}\right)$

Further, $T_1V_1^{\gamma-1} = T_2(nV_1)^{\gamma-1}$

$\Rightarrow \quad T_1 = T_2 n^{\gamma-1}$

$\Rightarrow \quad Q_3 = \frac{RT_2}{\gamma-1}\left(n^{\gamma-1} - \frac{1}{n}\right) = \frac{RT_2}{n(\gamma-1)}(n^\gamma - 1)$

So, $W_{\text{Total}} = Q_1 + Q_2 + Q_3$

$$W_{\text{Total}} = \frac{RT_2}{n(\gamma-1)}\left[(n^\gamma - 1) - \gamma(n-1)\right]$$

Now, efficiency η is $\eta = \frac{W_{\text{Total}}}{Q_{+ive}} = \frac{W_{\text{Total}}}{Q_3}$

$\Rightarrow \quad \eta = \frac{(n^\gamma - 1) - \gamma(n-1)}{(n^\gamma - 1)}$

$\Rightarrow \quad \eta = 1 - \gamma\left(\frac{n-1}{n^\gamma - 1}\right)$

9. Given that $U_A = 0$, $U_B = 50$ J and $Q_{BC} = 90$ J.

Also, $P_A = P_B = 10$ Nm^{-2}, $P_C = 30$ Nm^{-2}

$V_A = 1$ m^3 and $V_B = V_C = 3$ m^3.

For process BC, volume of gas remains constant, so work done by gas in this process is zero i.e., $W_{BC} = 0$

Since heat absorbed by the gas is $Q_{BC} = 90$ J, so from FLTD, we have

$$\Delta U_{BC} = U_C - U_B = Q_{BC} - W_{BC} = 90 \text{ J} - 0$$

$\Rightarrow \quad \Delta U_{BC} = U_C - U_B = 90$ J

$\Rightarrow \quad U_C = \Delta U_{BC} + U_B = 90 \text{ J} + 50 \text{ J} = 140$ J

For process AB, we have

$$\Delta U_{AB} = U_B - U_A = 50 - 0 = 50 \text{ J}$$

$$W_{AB} = P\Delta V = P_A(V_B - V_A) = 10(3-1) = 20 \text{ J}$$

Thus, heat absorbed by the system is

$$Q_{AB} = (\Delta U)_{AB} + W_{AB} = 50 + 20 = 70 \text{ J}$$

For process CA

$$\Delta U_{CA} = U_A - U_C = 0 - 140 = -140 \text{ J}$$

Work done is equal to the negative area under the curve CA (a compression process)

$$W_{CA} = -(\text{Area } ACED)$$

$\Rightarrow \quad W_{CA} = -\left[\frac{1}{2}(10+30)(3-1)\right] = -40$ J

So, by FLTD, we have

$$Q_{CA} = \Delta U_{CA} + W_{CA} = -140 - 40 = -180 \text{ J}$$

Negative sign shows that heat is being rejected in the process CA.

Net work done in the counter clockwise cyclic process $ABCA$ is equal to the negative of the area of triangle ABC i.e.,

$$W_{ABCA} = -(\text{Area } \Delta ABC)$$

$\Rightarrow \quad W_{ABCA} = -\frac{1}{2} \times 2 \times 20 = -20$ J

Negative sign indicates that the work is being done on the system.

10. If T is the temperature of state A we use

$$T_A = T;\ T_B = T_D = nT;\ T_C = n^2T$$

Total work done by gas in cycle for n_1 moles of gas

$$W = (P_2 - P_1)(V_2 - V_1)$$

$$n_1R(n^2T - nT - nT + T) = n_1RT(n-1)^2$$

Heat supplied to gas is $Q_{\text{supp}} = \Delta Q_{AB} + \Delta Q_{BC}$

$\Rightarrow \quad Q_{\text{supp}} = n_1\left(\frac{R}{\gamma-1}\right)T(n-1) + n_1\left(\frac{\gamma R}{\gamma-1}\right)T(n^2-n)$

$\Rightarrow \quad Q_{\text{supp}} = \frac{n_1RT}{\gamma-1}(n-1+\gamma n^2 - \gamma n)$

$\Rightarrow \quad Q_{\text{supp}} = \frac{n_1RT}{\gamma-1}(n-1)(1+\gamma n)$

Cycle efficiency is

$$\eta = \frac{W}{Q_{\text{supp}}} = \frac{n_1RT(n-1)^2}{\frac{n_1RT}{\gamma-1}(n-1)(1+\gamma n)}$$

$\Rightarrow \quad \eta = \left(\frac{\gamma-1}{1+\gamma n}\right)(n-1)$

CHAPTER 2

Test Your Concepts-X (Based on Conduction)

1. (a) Thermal resistance of aluminium cube $R_1 = \frac{\ell}{kA}$

$$\Rightarrow \quad R_1 = \frac{(3\times10^{-2})}{(237)(3\times10^{-2})^2} = 0.14 \text{ KW}^{-1}$$

and thermal resistance of copper cube $R_2 = \frac{\ell}{kA}$

$$\Rightarrow \quad R_2 = \frac{(3\times10^{-2})}{(401)(3\times10^{-2})^2} = 0.08 \text{ KW}^{-1}$$

As these two resistances are in parallel, their equivalent resistance is given by

$$R = \frac{R_1R_2}{R_1+R_2}$$

$$\Rightarrow \quad R = \frac{(0.14)(0.08)}{(0.14)+(0.08)}$$

$$\Rightarrow \quad R = 0.05 \text{ KW}^{-1}$$

So, thermal current

$$H = \frac{\text{Temperature difference}}{\text{Thermal resistance}}$$

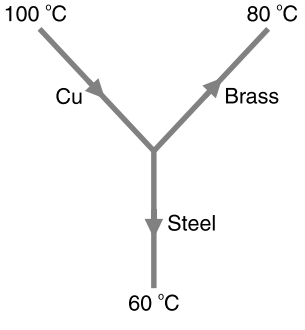
$$\Rightarrow \quad H = \frac{(100-20)}{0.05}$$

$$\Rightarrow \quad H = 1.6\times10^3 \text{ watt}$$

(b) In parallel thermal current distributes in the inverse ratio of resistance, so

$$\frac{H_{Cu}}{H_{Al}} = \frac{R_{Al}}{R_{Cu}} = \frac{R_1}{R_2} = \frac{0.14}{0.08} = 1.75$$

2. (a) Since $H_{Cu} = H_{\text{Steel}} + H_{\text{Brass}}$



If T be the temperature of the junction, then

$$\frac{k_{Cu}A(100-T)}{46} = \frac{k_BA(T-80)}{13} + \frac{k_SA(T-60)}{12}$$

$$\Rightarrow \quad \frac{0.92}{46}(100-T) = \frac{0.26}{13}(T-80) + \frac{0.12}{12}(T-60°)$$

$$\Rightarrow \quad 200-2T = 2T-160+T-60$$

$$\Rightarrow \quad 5T = 420$$

$$\Rightarrow \quad T = 84 \text{ °C}$$

(b) $H_{Cu} = \frac{k_{Cu}A(100-T)}{\ell_{Cu}}$

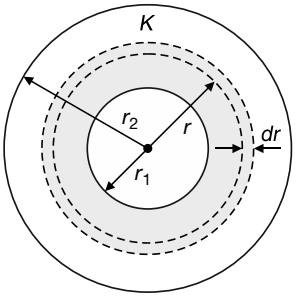
$$\Rightarrow \quad H_{Cu} = \frac{(0.92)(4)(100-84)}{46}$$

$$\Rightarrow \quad H_{Cu} = 1.28 \text{ cals}^{-1}$$

3. Consider a differential cylindrical element

$$H = \frac{dQ}{dt} = kA\frac{dT}{dr} = (2\pi kr\ell)\frac{dT}{dr}$$

$$\frac{H}{2\pi k\ell}\int_{r_1}^{r_2}\frac{dr}{r} = \int_0^{50} dT$$



$$\Rightarrow \quad \frac{H}{2\pi k\ell}\log_e\left(\frac{r_2}{r_1}\right) = 50$$

$$\Rightarrow \quad H = \frac{100\pi k\ell}{\log_e\left(\frac{r_2}{r_1}\right)}$$

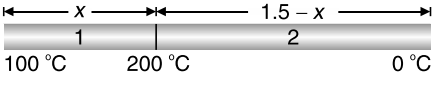
Since, $Q = Ht = mL$

$$\Rightarrow \quad t = \frac{mL}{H} = \frac{mL\log_e\left(\frac{r_2}{r_1}\right)}{100\pi k\ell}$$

4. Since, $\frac{dQ}{dt} = kA\frac{\Delta T}{\ell}$

and $\frac{dQ}{dt} = L\left(\frac{dm}{dt}\right)$ $\{\because dQ = (dm)L\}$

$$\Rightarrow \quad \left(\frac{dm}{dt}\right) = \left(\frac{kA}{\ell}\right)\left(\frac{\Delta T}{L}\right)$$



Let the desired point be at a distance x from water at 100 °C. Since

$$\left(\begin{matrix}\text{Rate of Melting}\\ \text{of Ice}\end{matrix}\right) = \left(\begin{matrix}\text{Rate of Production}\\ \text{of Steam}\end{matrix}\right)$$

$$\Rightarrow \quad \left(\frac{dm}{dt}\right)_1 = \left(\frac{dm}{dt}\right)_2$$

$$\Rightarrow \quad \left(\frac{\Delta T}{L\ell}\right)_1 = \left(\frac{\Delta T}{L\ell}\right)_2 \{\text{as } A_1 = A_2 \text{ and } k_1 = k_2\}$$

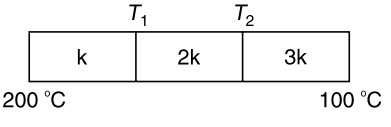
$$\Rightarrow \quad \frac{200-100}{(540)(x)} = \frac{100-0}{(80)(1.5-x)}$$

Solving this, we get

$x = 0.1034 \text{ m} = 10.34 \text{ cm}$

So, the temperature of 200 °C must be maintained at a distance 10.34 cm from water at 100 °C.

5. Thermal resistance $\propto \frac{1}{k}$



$$\Rightarrow \quad R_1 : R_2 : R_3 = \frac{1}{1} : \frac{1}{2} : \frac{1}{3} = 6 : 3 : 2$$

So if, $R_1 = 6R$, $R_2 = 3R$ and $R_3 = 2R$

then, $H = \dfrac{\Delta T}{R_{net}} = \dfrac{200 - 100}{6R + 3R + 2R} = \dfrac{100}{11R}$

Now, $200 - T_1 = HR_1$

$$\Rightarrow \quad T_1 = 200 - \left(\frac{100}{11R}\right)(6R) = 145.5\ ^\circ\text{C}$$

Also, $T_2 - 100 = HR_3$

$$\Rightarrow \quad T_2 = 100 + HR_3$$

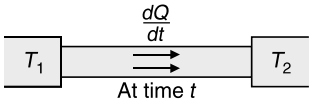
$$\Rightarrow \quad T_2 = 100 + \left(\frac{100}{11R}\right)(2R)$$

$$\Rightarrow \quad T_2 = 118.2\ ^\circ\text{C}$$

6. Let T_1 and $T_2 (< T_1)$ be the temperatures of the two chunks at time t and T be their temperature difference. Then

$$T = T_1 - T_2$$

$$\Rightarrow \quad \left(-\frac{dT}{dt}\right) = \left(-\frac{dT_1}{dt}\right) + \left(\frac{dT_2}{dt}\right) \qquad \ldots(1)$$



Since, $\dfrac{dQ}{dt} = \dfrac{kAT}{\ell} = C_1\left(-\dfrac{dT_1}{dt}\right) = C_2\left(\dfrac{dT_2}{dt}\right) \quad \{\because T_1 - T_2 = T\}$

$$\Rightarrow \quad \left(-\frac{dT_1}{dt}\right) = \left(\frac{kA}{\ell C_1}\right)T$$

and $\left(\dfrac{dT_2}{dt}\right) = \left(\dfrac{kA}{\ell C_2}\right)T$

Substituting these values in Equation (1), we get

$$\left(-\frac{dT}{dt}\right) = \frac{kA}{\ell}\left(\frac{C_1 + C_2}{C_1 C_2}\right)T = \alpha T \text{ where } \alpha = \frac{kA(C_1 + C_2)}{\ell C_1 C_2}$$

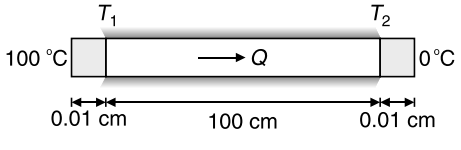
$$\Rightarrow \quad \int_{(\Delta T)_0}^{T} -\frac{dT}{T} = \alpha \int_0^t dt$$

$$\Rightarrow \quad T = (\Delta T)_0\, e^{-\alpha t}$$

7. Since, $\dfrac{dQ}{dt} = \dfrac{0.0014 \times A \times (100 - T_1)}{0.01} \qquad \ldots(1)$

$$\frac{dQ}{dt} = \frac{1.04 \times A \times (T_1 - T_2)}{100} \qquad \ldots(2)$$

and $\dfrac{dQ}{dt} = \dfrac{0.0014 \times A \times (T_1 - 0)}{0.01} \qquad \ldots(3)$



From Equations (1) and (3), we get

$$100 - T_1 = T_2 \qquad \ldots(4)$$

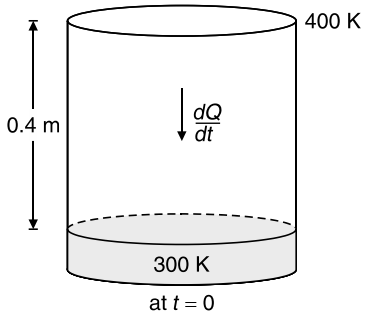
Solving above four equation, we get

$$T_1 = 93.5\ ^\circ\text{C} \text{ and } T_2 = 6.5\ ^\circ\text{C}$$

So, temperature gradient in the bar

$$\frac{\Delta T}{\ell} = \frac{T_1 - T_2}{100} = 0.87\ (^\circ\text{C})\text{cm}^{-1}$$

8. Since, $\dfrac{dQ}{dt} = mc\dfrac{dT}{dt} = \dfrac{KA(T_1 - T_2)}{\ell}$ we have



$$mc\left(\frac{dT}{dt}\right) = \frac{KA(400 - T)}{0.4}$$

$$\Rightarrow \quad (0.4)(600)\frac{dT}{dt} = \frac{(10)(0.04)(400 - T)}{0.4}$$

$$\Rightarrow \quad \left(\frac{dT}{400 - T}\right) = \frac{1}{240}dt$$

$$\Rightarrow \quad \int_0^t dt = 240 \int_{300}^{350} \frac{dT}{400 - T}$$

Solving this, we get $t \approx 166$ s

9. (a) Since, $Q = nC_P\Delta T \qquad \{\because P = \text{constant}\}$

$$\Rightarrow \quad \frac{kA(T_S - T)}{a}dt = \left(\frac{P_0V_0}{RT_0}\right)\left(\frac{5R}{2}\right)dT$$

$$\Rightarrow \quad \frac{kA(T_S - T)}{a}dt = \frac{5P_0V_0}{2T_0}dT$$

$$\Rightarrow \quad \frac{kA}{a}\int_0^t dt = \frac{5P_0V_0}{2T_0}\int_{T_0}^{T}\frac{dT}{T_S - T}$$

$$\Rightarrow \quad T = T_S + (T_S - T_0)e^{-\alpha t}$$

where, $\alpha = \dfrac{2T_0kA}{5P_0V_0a}$

(b) Since $P = \text{constant}$, so $\dfrac{V_i}{T_i} = \dfrac{V_f}{T_f}$

$$\Rightarrow \quad \frac{h_0A}{T_0} = \frac{hA}{T}$$

$$\Rightarrow \quad h = \left(\frac{T}{T_0}\right)h_0$$

CHAPTER 2

10. Let θ be the temperature difference at time t, then

$$\frac{dQ}{dt}=\frac{kA\theta}{\ell} \qquad \ldots(1)$$

Since, $\frac{dQ}{dt}=nC_{V_1}\left(-\frac{d\theta_1}{dt}\right)=nC_{V_2}\left(\frac{d\theta_2}{dt}\right)$ …(2)

where, $C_{V_1}=\frac{3R}{2}$ and $C_{V_2}=\frac{5R}{2}$

Since, $\left(-\frac{d\theta_1}{dt}\right)=\frac{2kA\theta}{3nR\ell}$

$$\left(\frac{d\theta_2}{dt}\right)=\frac{2kA\theta}{5nR\ell}$$

Also, $\theta=\theta_1-\theta_2$

$$\Rightarrow \quad \left(-\frac{d\theta}{dt}\right)=\left(-\frac{d\theta_1}{dt}\right)+\left(\frac{d\theta_2}{dt}\right)$$

$$\Rightarrow \quad \left(-\frac{d\theta}{dt}\right)=\frac{16kA\theta}{15nR\ell}$$

$$\Rightarrow \quad dt=-\frac{15nR\ell}{16kA}\int_{T_1-T_2}^{\frac{T_1-T_2}{2}}\frac{d\theta}{\theta}$$

$$\Rightarrow \quad t=\frac{15nR\ell}{16kA}\log_e(2)$$

Test Your Concepts-XI (Based on Radiation)

1. Since, $\left(\begin{matrix}\text{Rate of}\\ \text{Cooling}\end{matrix}\right)\propto\left(\begin{matrix}\text{Average Excess}\\ \text{Temperature}\end{matrix}\right)$

$$\Rightarrow \quad \frac{T_i-T_f}{t}=k\left(\frac{T_i+T_f}{2}-T_0\right)$$

$$\Rightarrow \quad \frac{80-50}{5}=k\left(\frac{80+50}{2}-20\right) \qquad \ldots(1)$$

Similarly

$$\frac{60-30}{t}=k\left(\frac{60+30}{2}-20\right) \qquad \ldots(2)$$

Dividing (1) by (2), we get

$$\Rightarrow \quad \frac{t}{5}=\frac{65-20}{45-20}$$

$$\Rightarrow \quad t=\frac{45}{25}\times 5=9 \text{ minute}$$

2. Since temperature of surrounding is very low as compared to that of heating element, so we can ignore the amount of radiant energy absorbed by the filament from its surroundings. So, according to Stefan's Law, we get

$$P=\sigma AT^4$$

$$\Rightarrow \quad 1000=5.67\times10^{-8}\times0.02\times T^4$$

$$\Rightarrow \quad T^4=8.82\times10^{11}$$

$$\Rightarrow \quad T=969.3 \text{ K}$$

3. Since the curved surface of the rod is insulated, so in steady state, the heat conducted from end A to end B of the rod is lost from the end B of rod by radiation.
The energy received through conduction at end B of rod per unit time is

$$u_1=\left|\frac{dQ}{dt}\right|=\frac{kA\left(T_A-T_B\right)}{l} \qquad \ldots(1)$$

where, T_A, T_B are absolute temperatures of end A and B, l is the length and k is thermal conductivity of rod.
Energy radiated per unit time from end B is

$$u_2=\left|\frac{dQ}{dt}\right|=\sigma A\left(T_B^4-T_0^4\right) \qquad \ldots(2)$$

where T_0 is room temperature, σ is Stefan's constant.
In the steady state, we have $u_1=u_2$

$$\Rightarrow \quad \frac{k\left(T_A-T_B\right)}{l}=\sigma\left(T_B^4-T_0^4\right)$$

$$\Rightarrow \quad k=\frac{\sigma l\left(T_B^4-T_0^4\right)}{T_A-T_B} \qquad \ldots(3)$$

Substituting the values, we get

$$k=36.6 \text{ Wm}^{-1}\text{K}^{-1}$$

4. Since, $-\frac{dQ}{dt}=mc\left(-\frac{dT}{dt}\right)=\sigma AT^4$

$$\int_{800\text{ K}}^{300\text{ K}}-\frac{dT}{T^4}=\frac{\sigma A}{mc}\int_0^t dt$$

Substituting $A=4\pi r^2$, $m=\frac{4}{3}\pi r^3\rho$, so integrating the above equation, we get

$$t=1.01\times10^5 \text{ s}$$

5. Since, $-\frac{dQ}{dt}=\sigma e(\pi d^2)T^4$

$$\Rightarrow \quad \rho\left(\frac{4}{3}\pi\frac{d^3}{8}\right)c\frac{dT}{dt}=-\sigma e\pi d^2T^4$$

$$\Rightarrow \quad \frac{\rho dc}{6\sigma e}=\int_{T_0}^{T_0/\eta}\frac{dT}{T^4}=-\int_0^t dt$$

$$\Rightarrow \quad t=\frac{\rho dc}{18\sigma e}\left(\frac{\eta^3}{T_0^3}-\frac{1}{T_0^3}\right)=\frac{\rho dc\left(\eta^3-1\right)}{18\sigma eT_0^3}$$

6. The rate of loss of heat by sphere is given by

$$\frac{dQ}{dt}=\sigma A\left(T^4-T_0^4\right)$$

where, $A=4\pi r^2$ is the surface area of the sphere of radius $r=10 \text{ cm}=0.1 \text{ m}$, $T=327\ ^\circ\text{C}=600 \text{ K}$ and $T_0=27\ ^\circ\text{C}=300 \text{ K}$, So, we get

$$\frac{dQ}{dt}=\left(5.67\times10^{-8}\right)4\pi(0.1)^2\left[(600)^4-(300)^4\right]$$

$$\Rightarrow \quad \frac{dQ}{dt}=866 \text{ Js}^{-1}$$

Since, $dQ = msdT$, where dT is the fall in temperature in time dt, hence $\frac{dQ}{dt} = ms\frac{dT}{dt}$

$\Rightarrow \quad 866 = 10 \times 420 \times \frac{dT}{dt}$

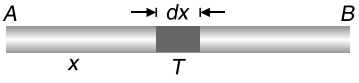
$\Rightarrow \quad \frac{dT}{dt} = \frac{866}{4200} = 0.206\ ^\circ\text{Cs}^{-1}$

7. Since, $\lambda_m T_B = b$

$\Rightarrow \quad T_B = \frac{b}{\lambda_m}$

$\Rightarrow \quad T_B = \frac{2.9 \times 10^{-3}}{14.5 \times 10^{-7}}$

$\Rightarrow \quad T_B = 2000\ \text{K}$



Since, $\frac{dQ}{dt} = P = -\frac{kAdT}{dx}$

$\Rightarrow \quad P = -\frac{k_0(1+ax)AdT}{dx}$

$\Rightarrow \quad -\int_{\ell_0}^{x} \frac{dx}{1+ax} = \frac{k_0 A}{P}\int_{T_B}^{T} dT$

Solving this equation, we get

$$T = \frac{P}{k_0 aA}\log_e\left(\frac{1+a\ell_0}{1+ax}\right) + T_B$$

Substituting the values, we get

$$T = \frac{4560}{11.4 \times 0.2}\log_e\left(\frac{1+0.2\times 5}{1+0.2x}\right) + 2000$$

$\Rightarrow \quad T = 2000\left[\log_e\left(\frac{2}{1+0.2x}\right)+1\right]$

$\Rightarrow \quad T = 2000\left[\log_e\left(\frac{20}{10+2x}\right)+1\right]$

$\Rightarrow \quad T = 2000\left[\log_e\left(\frac{10}{5+x}\right)+1\right]$

8. In this case, rate of heat lost by the block is

$$\frac{dQ}{dt} = \sigma A\left(T^4 - T_5^4\right)$$

where, A is the total surface area of cube given by $6a^2 = 150\ \text{cm}^2 = 150 \times 10^{-4}\ \text{m}^2$

$$\left|\frac{dQ}{dt}\right| = (5.67\times 10^{-8})\left(\frac{150}{10^4}\right)\left((500)^4 - (300)^4\right)$$

If $\frac{dT}{dt}$ is rate of cooling of block, then

$$\left|\frac{dQ}{dt}\right| = ms\frac{dT}{dt}$$

$\Rightarrow \quad \frac{dT}{dt} = \frac{1}{ms}\frac{dQ}{dt} = \left(\frac{1}{1\times 400}\right)38.556$

$\Rightarrow \quad \frac{dT}{dt} = 0.115\ ^\circ\text{Cs}^{-1}$

9. If emissivity of body is e, the power required equals the rate of emission of radiation.

$$P = e\sigma A\left(T^4 - T_5^4\right) \qquad \ldots(1)$$

For a black body

$$P_B = \sigma A\left(T^4 - T_5^4\right) \qquad \ldots(2)$$

Dividing, we get $\frac{P}{P_B} = e$

$\Rightarrow \quad e = \frac{210}{700} = 0.3$

10. Since temperature difference of water with its surrounding is not large, so according to Newton's Law of Cooling, we have

$$\frac{T_i - T_f}{t} = k\left(\frac{T_i + T_f}{2} - T_0\right), \text{ where } k = \frac{4\sigma A T_0^3}{mc}$$

So, $k = \frac{4(5.67\times 10^{-8})6(10/100)^2(300)^3}{1\times 4200}$

$\Rightarrow \quad k = 8.75 \times 10^{-5}$

$\Rightarrow \quad \frac{30-29}{t} = 8.75\times 10^{-5}(29.5-27)$

$\Rightarrow \quad t = \frac{1}{(8.75\times 10^{-5})(2.5)} = 4571.43\ \text{s} = 76.2\ \text{min}$

11. Using average from of Newton's Law of Cooling, we use For water, we have

$$\frac{40-35}{5} = \frac{k}{0.1\times 4200}\left(\frac{40+35}{2} - T_S\right) \qquad \ldots(1)$$

For liquid, we have

$$\frac{40-35}{2} = \frac{k}{m\times 2100}\left(\frac{40+35}{2} - T_S\right) \qquad \ldots(2)$$

Dividing (1) by (2), we get

$$\frac{2}{5} = \frac{m\times 2100}{0.1\times 4200}$$

$\Rightarrow \quad m = \frac{2\times 420}{5\times 2100} = 0.08\ \text{kg} = 80\ \text{gm}$

As the volume of liquid is same that of water 100 cm^3, then density of liquid is

$$\rho = \frac{m}{V} = \frac{80\times 10^{-3}}{100\times 10^{-6}} = 800\ \text{kgm}^{-3}$$

12. In the bulb filament given, the energy radiated per sec per square metre of its surface area is given as

$$E = \frac{P}{A} = \frac{114.75}{0.25\times 10^{-4}} = 459\times 10^4\ \text{Js}^{-1}\text{m}^2$$

If T is the temperature of the filament then according to Stefan's law, we have

$$E = \sigma T^4$$

$\Rightarrow \quad 459\times 10^4 = \left(\frac{17}{3}\times 10^{-8}\right)T^4$

CHAPTER 2

$$\Rightarrow \quad T^4 = \frac{459\times10^4}{(17/3)\times10^{-8}} = 81\times10^{12}$$

$$\Rightarrow \quad T = (81\times10^{12})^{\frac{1}{4}} = 3000 \text{ K}$$

Single Correct Choice Type Questions

1. If temperature of the source is increased by α Kand temperature of the sink is decreased by the same amount i.e. α K then efficiency increases more in the second case.

Since $\eta_0 = 1-\frac{T_2}{T_1}$

According to the problem $\eta_1 = 1-\left(\frac{T_2-\alpha}{T_1}\right)$

$$\Rightarrow \quad \eta_1 = \left(1-\frac{T_2}{T_1}\right)+\frac{\alpha}{T_1}$$

$$\Rightarrow \quad \eta_1 = \eta_0 + \frac{\alpha}{T_1} = \frac{\eta_0 T_1+\alpha}{T_1}$$

$$\Rightarrow \quad \eta_1 > \eta_0$$

Further, $\eta_2 = 1-\frac{T_2}{T_1+\alpha} = \frac{T_1+\alpha-T_2}{T_1+\alpha}$

$$\Rightarrow \quad \eta_2 = \frac{\eta_0 T_1+\alpha}{T_1+\alpha} \qquad \{\because\ T_1-T_2=\eta_0 T_1\}$$

$$\Rightarrow \quad \eta_2 < \eta_1$$

$$\Rightarrow \quad \eta_1 > \eta_2 > \eta_0$$

Hence, the correct answer is (B).

2. $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$$\Rightarrow \quad 300 = T_2\left(\frac{8}{27}\right)^{\frac{5}{3}-1}$$

$$\Rightarrow \quad T_2 = 300\times\left(\frac{27}{8}\right)^{\frac{2}{3}} = 300\times\frac{9}{4} = 675 \text{ K}$$

$$\Rightarrow \quad \Delta T = 375\ ^\circ\text{C}$$

Hence, the correct answer is (B).

3. At $V=V_0$, $P=\frac{P_0}{2}$

$$\Rightarrow \quad T_i = \frac{PV}{nR} = \frac{\left(\frac{P_0}{2}\right)(V_0)}{R} = \frac{P_0V_0}{2R} \qquad \{\because\ n=1\}$$

and at $V=2V_0$, $P=\frac{4P_0}{5}$

$$\Rightarrow \quad T_f = \frac{PV}{nR} = \frac{(2V_0)\left(\frac{4P_0}{5}\right)}{R} = \frac{8P_0V_0}{5R}$$

$$\Rightarrow \quad \Delta T = T_f - T_i = \left(\frac{8}{5}-\frac{1}{2}\right)\frac{P_0V_0}{R} = \frac{11P_0V_0}{10R}$$

Hence, the correct answer is (B).

4. $-mc\frac{dT}{dt} = \sigma T^4$

$$\Rightarrow \quad -mcT^{-4}dT = \sigma\, dt$$

$$\Rightarrow \quad -\int_{T_1}^{T_2} T^{-4}dT = \frac{\sigma}{mc}\int_0^t dt$$

$$\Rightarrow \quad -\left[\frac{T^{-3}}{-3}\Big|_{T_1}^{T_2}\right] = \frac{\sigma}{mc}t$$

$$\Rightarrow \quad t = \frac{mc}{3\sigma}\left(\frac{1}{T_2^3}-\frac{1}{T_1^3}\right)$$

Hence, the correct answer is (C).

5. $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$ and $v_{\text{sound}} = \sqrt{\frac{\gamma RT}{M}}$, $v_{\text{rms}} = \sqrt{2}\ v_{\text{sound}}$

i.e., $\gamma = \frac{3}{2}$ =ratio of $\frac{C_P}{C_V}$ for the mixture

$$C_V = \frac{n_1C_{V_1}+n_2C_{V_2}}{n_1+n_2}$$

and $C_P = \frac{n_1C_{P_1}+n_2C_{P_2}}{n_1+n_2}$

$$\Rightarrow \quad \gamma = \frac{C_P}{C_V} = \frac{n_1C_{P_1}+n_2C_{P_2}}{n_1C_{V_1}+n_2C_{V_2}}$$

$$\Rightarrow \quad \frac{3}{2} = \frac{(2)\left(\frac{5}{2}R\right)+(n)\left(\frac{7}{2}R\right)}{2\left(\frac{3}{2}R\right)+(n)\left(\frac{5}{2}R\right)}$$

$$\Rightarrow \quad \frac{3}{2} = \frac{10+7n}{6+5n}$$

$$\Rightarrow \quad n = 2$$

Hence, the correct answer is (C).

6. For gas A, we have

$$PV = n_ART \qquad \ldots(1)$$

When another gas B is also added, then

$$3PV = (n_A+n_B)RT \qquad \ldots(2)$$

$$\Rightarrow \quad n_A+n_B = 3n_A \qquad \{\text{from (1) and (2)}\}$$

$$\Rightarrow \quad n_B = 2n_A$$

Since masses of both the gases are same, so

$$n_BM_B = n_AM_A \qquad \left\{\because\ n=\frac{m}{M}\right\}$$

Hence, molecular weights are in the ratio

$$\frac{M_A}{M_B} = \frac{n_B}{n_A} = 2$$

Hence, the correct answer is (B).

7. $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

$$\Rightarrow \quad v_{\text{rms}} \propto \sqrt{T}$$

v_{rms} is to reduce to time i.e., temperature of the gas will have no reduce four times or

$$\frac{T'}{T} = \frac{1}{4}$$

During adiabatic process

$$TV^{\gamma-1} = T'V'^{\gamma-1}$$

$$\Rightarrow \quad \frac{V'}{V} = \left(\frac{T}{T'}\right)^{\frac{1}{\gamma-1}} = (4)^{\frac{1}{1.5-1}} = (4)^2 = 16$$

$$\Rightarrow \quad V' = 16 \text{ V}$$

Hence, the correct answer is (B).

8. $$-\frac{1}{A}\frac{dQ}{dt} = \sigma(T^4 - 0^4)$$

$$\Rightarrow \quad -mc\frac{dT}{dt} = \sigma A T^4$$

$$\Rightarrow \quad T^{-4}dT = -\frac{\sigma A}{mc}dt$$

$$\Rightarrow \quad \int_{200\text{ K}}^{100\text{ K}} T^{-4}dT = -\frac{\sigma A}{mc}\int_0^t dt$$

$$\Rightarrow \quad \frac{T^{-3}}{-3}\Big|_{200}^{100} = -\frac{\sigma A}{mc}t$$

$$\Rightarrow \quad \frac{1}{(100)^3} - \frac{1}{(200)^3} = \frac{\sigma(4\pi r^2)}{\frac{4}{3}\pi r^3 \rho c}t$$

$$\Rightarrow \quad t = \frac{7}{72}\frac{r\rho c}{\sigma} \times 10^{-6} \text{ s}$$

$$\Rightarrow \quad t = \frac{7}{72}\frac{r\rho c}{\sigma}\,\mu s$$

Hence, the correct answer is (B).

9. As two arms of U-tube are maintained at different temperatures, densities in two arms will be different but pressure at the bottom is same. So, we have

$$h_1\left(\frac{\rho_0}{1+\gamma T_1}\right)g = h_2\left(\frac{\rho_0}{1+\gamma T_2}\right)g$$

$$\Rightarrow \quad 49\left(\frac{\rho_0}{1+50\gamma}\right) = 50\left(\frac{\rho_0}{1+60\gamma}\right)$$

$$\Rightarrow \quad 49 + 49(60\gamma) = 50 + 50(50\gamma)$$

$$\Rightarrow \quad \gamma(2940 - 2500) = 1$$

$$\Rightarrow \quad \gamma = \frac{1}{440} \approx 2.3 \times 10^{-3} \text{ °C}^{-1}$$

Hence, the correct answer is (C).

10. Mass of $P = 2(\text{Mass of } Q)$

$$\Rightarrow \quad \frac{4}{3}\pi r_P^3 \rho = 2\left(\frac{4}{3}\pi r_Q^3 \rho\right)$$

$$\Rightarrow \quad r_P = 2^{\frac{1}{3}} r_Q$$

Since $\frac{1}{A}\frac{dQ}{dt} \propto (T - T_0)$

$$\Rightarrow \quad \frac{1}{A}mc\frac{dT}{dt} \propto (T - T_0)$$

$$\Rightarrow \quad \frac{dT}{dt} \propto \frac{A}{m}$$

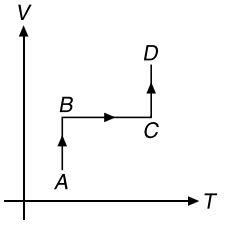
$$\Rightarrow \quad \frac{dT}{dt} \propto \frac{4\pi r^2}{\frac{4}{3}\pi r^3 \rho}$$

$$\Rightarrow \quad \frac{dT}{dt} \propto \frac{1}{r}$$

$$\Rightarrow \quad \frac{\text{Rate of cooling of P}}{\text{Rate of cooling of Q}} = \frac{r_Q}{r_P} = \frac{1}{2^{1/3}}$$

Hence, the correct answer is (C).

11. Process AB is isothermal, so its V-T curve will remain a vertical straight line. Process BC is isochoric (because it is a straight line passing through origin) hence its V-T curve will be a horizontal straight line. Process CD is again isothermal, so it will be represented by a vertical straight line on V-T curve and hence the V-T curve is shown in Figure.



Hence, the correct answer is (D).

12. $$\frac{mL}{t} = \frac{kA(T_1 - T_2)}{d}$$

$$\Rightarrow \quad T_1 - T_2 = \frac{mLd}{KAt}$$

$$\Rightarrow \quad T_1 - T_2 = \left(\frac{1}{60}\right)\frac{(540)(0.2)}{(0.5)(300)}$$

$$\Rightarrow \quad T_1 - T_2 = 0.012 \text{ °C}$$

Hence, the correct answer is (D).

13. From First Law of Thermodynamics,

$$dQ = dU + dW$$

$$\Rightarrow \quad dQ = dU \text{ if } dW = 0$$

Since $dQ < 0$ therefore, $dU < 0$

$$\Rightarrow \quad U_{\text{final}} < U_{\text{initial}}$$

$\Rightarrow$ Temperature will decrease.

Conceptual Note(s)

Internal energy U of an ideal gas depends only on the temperature of the gas. Internal energy of n moles of an ideal gas is given by

$$U = \left(\frac{nf}{2}\right)RT$$

$$U \propto T$$

Here, f is the degree of freedom of the gas.

Hence, the correct answer is (A).

14. Heat released when 5 kg of water cools from 20 °C to 0 °C is

$$Q_1 = m_{\text{water}}c_{\text{water}}\Delta T = (5)(1)(20) = 100 \text{ kcal}$$

Heat required to raise temperature of 2 kg of ice from –20 °C to 0 °C is

$$Q_2 = m_{\text{ice}}c_{\text{ice}}\Delta T = (2)(0.5)(20) = 20 \text{ kcal}$$

CHAPTER 2

Heat required to melt 2 kg of ice at 0 °C is

$$Q_3 = m_{ice}L_{ice} = (2)(80) = 160 \text{ kcal}$$

Since $Q_1 > Q_2$, so temperature of ice will reach 0 °C. However we observe that $Q_1 < Q_2 + Q_3$, therefore the complete ice will not melt and final mixture will have both ice and water. The amount of ice melted m is

$$m = \frac{\text{Available Heat}}{\text{Latent Heat}} = \frac{(100-20)\text{ kcal}}{80\text{ calg}^{-1}} = 1 \text{ kg}$$

So in equilibrium, the vessel will have water given by

$$m'_{water} = 5 + 1 = 6 \text{ kg}$$

Hence, the correct answer is (C).

15. Let the initial pressure and volume be P and V respectively.

$$\Rightarrow \quad P_f = P + \frac{P}{100} = \frac{101}{100}P$$

For Boyle's Law

$$P_iV_i = P_fV_f$$

$$\Rightarrow \quad V_f = \frac{100}{101}V$$

$$\Rightarrow \quad \Delta V = \frac{1}{101}V$$

$$\Rightarrow \quad \frac{\Delta V}{V} \times 100 = \frac{100}{101}\%$$

Hence, the correct answer is (B).

16. Work done$(W) = fx = \mu mg(vt)$

Since half the heat produced is absorbed by body, so

$$\frac{1}{2}\mu mg(vt) = mc\Delta T$$

$$\Rightarrow \quad \frac{1}{2}\mu g(vt) = c\Delta T$$

$$\Rightarrow \quad \frac{1}{2}(0.5)(9.8)(2000) = (0.1 \times 4200)\Delta T$$

$$\Rightarrow \quad \Delta T = 11.6 \text{ °C}$$

Hence, the correct answer is (D).

17. To double rms speed, temperature must be raised to four times the initial temperature, i.e.

$$T_f = 4 \times 300 = 1200 \text{ K}.$$

Heat required for doing this is

$$Q = \Delta U = n_1(C_V)_1\Delta T + n_2(C_V)_2\Delta T_2$$

$$\Rightarrow \quad Q = \left[(1)(3R/2) + (1)(5R/2)\right](1200 - 300)$$

$$\Rightarrow \quad Q = 3600R$$

Hence, the correct answer is (D).

18. $U = nC_vT = n\left(\frac{3}{2}R\right)T$

$$\Rightarrow \quad \frac{U}{V} = \frac{3nRT}{2V} = \frac{3}{2}P \qquad \left\{\because \frac{nRT}{V} = P\right\}$$

Hence, the correct answer is (B).

19. Let M be the mass of the sphere and R its radius before increasing the temperature. Then From Conservation of Angular Momentum:

$$I\omega = I_0\omega_0$$

$$\Rightarrow \quad \omega = \left(\frac{I_0}{I}\right)\omega_0$$

$$\Rightarrow \quad \omega = \frac{\frac{2}{5}MR^2\omega_0}{\frac{2}{5}MR^2(1 + 2\alpha\Delta T)} = \frac{\omega_0}{1 + 2\alpha\Delta T}$$

$$\Rightarrow \quad \omega = \frac{\omega_0}{1 + 2(20 \times 10^{-5})(100)} = 0.996\omega_0$$

Hence, the correct answer is (C).

20. Since $\beta = \frac{277}{300 - 277}$

$$\Rightarrow \quad \beta \simeq 12$$

Further $\beta = \frac{Q_2}{W} = 12$

$$\Rightarrow \quad Q_2 = 12(1 \text{ J})$$

$$\Rightarrow \quad Q_2 = 12 \text{ J}$$

Since by Law of Conservation of Energy, we have

$$Q_1 = Q_2 + W$$

$$\Rightarrow \quad Q_1 = 12 + 1$$

$$\Rightarrow \quad Q_1 = 13 \text{ J}$$

Hence, the correct answer is (D).

21. In a cyclic process, $\Delta U = 0$ i.e. U = constant.

Free expansion is an irreversible process.

In a cyclic process, $\Delta U = 0$, so $W = Q \neq 0$.

Efficiency of Carnot engine is 100% if temperature of sink is 0 K (not 0 °C).

Hence, the correct answer is (C).

22. Since $Q = \Delta U = U_f - U_i$, where

$$\Delta U = \left(U_{\substack{4 \text{ mole} \\ \text{monatomic}}} + U_{\substack{2 \text{ mole} \\ \text{diatomic}}}\right) - U_{\substack{4 \text{ mole} \\ \text{diatomic}}}$$

$$\Rightarrow \quad \Delta U = \left(4 \times \frac{3}{2}RT + 2 \times \frac{5}{2}RT\right) - \left(4 \times \frac{5}{2}RT\right)$$

$$\Rightarrow \quad \Delta U = (6RT + 5RT) - (10RT) = RT$$

Conceptual Note(s)

(a) Two moles of diatomic gas become 4 moles of a monatomic gas when get dissociated into atoms.

(b) Internal energy of n moles of an ideal gas of degrees of freedom f is given by

$$U = \left(\frac{nf}{2}\right)RT$$

$f = 3$ for a monatomic gas and $f = 5$ for diatomic gas

Hence, the correct answer is (B).

23. $\left(\begin{array}{c}\text{Heat} \\ \text{conducted} \\ \text{in at } A \\ \text{per second}\end{array}\right) + \left(\begin{array}{c}\text{Heat} \\ \text{generated} \\ \text{by source} \\ \text{per second}\end{array}\right) = \left(\begin{array}{c}\text{Heat} \\ \text{conducted} \\ \text{out at } B \\ \text{per second}\end{array}\right)$

$$\frac{(0.5)(12)(100-T)}{8}+36=\frac{(0.5)(12)(T-4)}{8}$$

Solving the above equation for T, we have

$$T=76\ ^{\circ}\text{C}$$

Hence, the correct answer is (C).

24. Since, $V \propto T$, the process is isobaric

$$\Rightarrow \quad \frac{Q}{W}=\frac{nC_P\Delta T}{nR\Delta T}=\frac{C_P}{R}=\frac{5}{2}$$

Hence, the correct answer is (A).

25. $$\frac{Q}{t}=kA\frac{\Delta T}{\ell}$$

$$\Rightarrow \quad k \propto \frac{1}{\left(\frac{\Delta T}{\ell}\right)}$$

$$\Rightarrow \quad k \propto \frac{1}{\text{Temperature Gradient}}$$

Since, $K_C > K_M > K_G$

$$\Rightarrow \quad X_C < X_M < X_G$$

Hence, the correct answer is (C).

26. Let initially n be the total number of moles of O_2. Then after dissociation

$$C_v(\text{mixture})=\frac{(0.6)(2)(n)C_{v_1}+(0.4)(n)C_{v_2}}{(0.6)(2)n+(0.4)n}$$

$$\Rightarrow \quad (C_V)_{\text{mix}}=\frac{\left(1.2\times\frac{3}{2}\times R\right)+\left(0.4\times\frac{5}{2}\times R\right)}{1.6}$$

$$\Rightarrow \quad (C_V)_{\text{mix}}=\frac{1.8R+R}{1.6}=\frac{7}{4}R$$

Hence, the correct answer is (C).

27. Since gas is undergoing a process given by

$$PT=\text{constant}$$

$$\Rightarrow \quad P_0T_0=\frac{P_0}{2}T$$

$$\Rightarrow \quad T=2T_0$$

Applying Ideal Gas Equation, we get

$$P_0V_0=2RT_0$$

So, change in internal energy of the gas is

$$\Delta U=nC_V\Delta T=2\left(\frac{3R}{2}\right)(T-T_0)=3RT_0=\frac{3P_0V_0}{2}$$

Hence, the correct answer is (B).

28. $$P=\frac{\mu RT}{V}$$

where, μ is the number of molecules of the gas

$$R=k_BN_A$$

where N_A is Avogadro's Number.

$$\Rightarrow \quad P=\frac{\mu k_BN_AT}{V}=\left(\frac{\mu N_A}{V}\right)k_BT$$

$\frac{\mu N_A}{V}$ = number of molecules per unit volume (n)

$$\Rightarrow \quad P=nk_BT$$

Hence, the correct answer is (B).

29. $$H_1+H_2=\frac{KA(T_1-T_2)}{3\ell}+\frac{KA(T_1-T_2)}{\ell}=\frac{4}{3\ell}KA(T_1-T_2)$$

In later case

$$H_2=2H-H_1=\frac{7KA}{3\ell}(T_1-T_2)=\frac{K'A}{\ell}(T_1-T_2)$$

$$\Rightarrow \quad K'=\frac{7}{3}K$$

Hence, the correct answer is (A).

30. $$Q=\int_0^{20}mS\,dT=\int_0^{20}m(AT^3)dT=4\times10^4\ \text{mA}$$

Hence, the correct answer is (A).

31. Since the given graph is a straight line, so we have

$$P=P_0+CT, \text{ where } C \text{ is a constant}$$

Applying Ideal Gas Equation $PV=nRT$, we get

$$V \propto \frac{T}{P_0+CT}$$

$$\Rightarrow \quad \frac{1}{V} \propto \left(\frac{P_0}{T}+C\right)$$

So, as T increases, $\frac{1}{V}$ decreases. Hence, volume increases.

Hence, the correct answer is (C).

32. Since, $$kA\left(\frac{\sqrt{3}T-T_C}{l}\right)=kA\left(\frac{T_C-T}{l}\right)$$

$$\Rightarrow \quad \sqrt{3}T-T_C=T_C-T$$

$$\Rightarrow \quad (\sqrt{3}+1)T=2T_C$$

$$\Rightarrow \quad \frac{T_C}{T}=\frac{T_C}{T_B}=\frac{\sqrt{3}+1}{2}$$

Hence, the correct answer is (A).

33. $PV=$ constant

$$\Rightarrow \quad T=\text{constant}$$

Now, $\rho=\frac{PM}{RT}$

$$\Rightarrow \quad \rho \propto P \text{ for } T=\text{constant}$$

Hence, P-ρ graph is a straight line passing through origin.
Hence, the correct answer is (C).

34. By Laws of Calorimetry

$$\begin{pmatrix}\text{Total heat lost}\\ \text{by steam}\end{pmatrix}=\begin{pmatrix}\text{Total heat gained}\\ \text{by water + ice}\end{pmatrix}$$

Let m gram of steam be required for the purpose. Then,

$$Q_{\text{available}}=\underbrace{540m}_{\substack{\text{heat liberated due}\\ \text{to condensation of}\\ m \text{ gram steam at}\\ 100\ ^{\circ}\text{C to water}}}+\underbrace{m(1)(100-5)}_{\substack{\text{heat liberated due}\\ \text{to decrease in}\\ \text{temperature from}\\ 100\ ^{\circ}\text{C to 5 }^{\circ}\text{C}}}$$

CHAPTER 2

and $Q_{\text{required}} = \underbrace{100(1)(5-0)}_{\substack{\text{heat required to}\\ \text{raise temperature}\\ \text{of 100 g of water}\\ \text{from 0 °C to 5 °C}}} + \underbrace{10(80)+10(1)(5-0)}_{\substack{\text{heat required to convert 10 g}\\ \text{of ice to 10 g of water at 0 °C}\\ \text{and then raise temperature of}\\ \text{10 g water from 0 °C to 5 °C}}}$

By Law of Calorimetry, $Q_{\text{available}} = Q_{\text{required}}$

$\Rightarrow \quad m = 2.1\ g$

Hence, the correct answer is (A).

35. By FLTD, $Q = \Delta U + W$. Since ΔU is same for both as it depends upon initial and final state, so $\Delta U_1 = \Delta U_2$. Also, work done for process A is more than that for B because area under the P-V graph for the process A is more than that for process B and hence $Q_1 > Q_2$.
Hence, the correct answer is (D).

36. $\frac{\Delta T}{\ell} = \frac{80-0}{1} = 80\ °\text{Cm}^{-1}$

$\Delta T_1 = (0.60)(80) = 48\ °\text{C}$

Decrease in temperature after a distance of 60 cm is 48 °C and hence actual temperature is $80 - 48 = 32$ °C
Hence, the correct answer is (B).

37. **On Mixing A and C**
Heat lost by A = Heat gained by C

$\Rightarrow \quad (4)s_A(60-55) = (3)s_C(55-50)$

$\Rightarrow \quad 4s_A = 3s_C \qquad \ldots(1)$

On Mixing A and B
Heat lost by A =Heat gained by B

$2s_A(60-57) = 3s_B(57-55)$

$\Rightarrow \quad 6s_A = 6s_B$

$\Rightarrow \quad s_A = s_B \qquad \ldots(2)$

On mixing equal masses of B and C, let equilibrium temperature be T.
Heat lost by B = Heat gained by C

$ms_B(55-T) = ms_C(T-50) \qquad \ldots(3)$

$\Rightarrow \quad \frac{s_B}{s_C} = \frac{3}{4} \qquad \{\because \text{of (1) and (2)}\}$

So, equation (3) becomes $\frac{3}{4} = \frac{T-50}{55-T}$

$\Rightarrow \quad 165 - 3T = 4T - 200$

$\Rightarrow \quad 7T = 365$

$\Rightarrow \quad T = \frac{365}{7} = 52.1$

Hence, the correct answer is (A).

38. For the given process $Q = \Delta U + W$

$\Rightarrow \quad 200 - 100 = \Delta U - 50$

$\Rightarrow \quad \Delta U = 150$ kJ

So for the adiabatic process which restores original state, ΔU must be –150 kJ and hence work done in the process will be +150 kJ
Hence, the correct answer is (C).

39. $W_{BA} = -30$ J, $Q_{BA} = 0$

$\Rightarrow \quad \Delta U_{BA} = -W_{BA} = 30$ J

Now, $\Delta U_{AB} = -U_{BA} = -30$ J
Hence, the correct answer is (B).

40. Since air bubble doubles in radius. Hence its volume becomes 8 times. Further since the bubble rises slowly, hence the rising process can be regarded as an isothermal process. So according to Boyle's Law.

$(PV)_{\text{at depth } h} = (PV)_{\text{at surface}}$

If P_a be the atmospheric pressure then

$(P_a + h\rho g)V = P_a(8V) \qquad \{\text{where } P_a = H\rho g\}$

$\Rightarrow \quad P_a + h\rho g = 8P_a$

$\Rightarrow \quad h\rho g = 7P_a = 7(H\rho g)$

$\Rightarrow \quad h = 7H$

Hence, the correct answer is (C).

41. Since, $PV = nRT$, so $T \propto PV$

$\Rightarrow \quad T_1 : T_2 : T_3 = 1 : 4 : 4$

Also, average molecular speed i.e. $v_{av} = v \propto \sqrt{T}$

$\Rightarrow \quad v_1 : v_2 : v_3 = 1 : 2 : 2$

Hence, the correct answer is (A).

42. $\frac{PV}{T} = nR = \left(\frac{m}{M}\right)R$

$\Rightarrow \quad \frac{PV}{T} = \left(\frac{R}{M}\right)m$

i.e., $\frac{PV}{T}$ versus m graph is a straight line passing through origin with slope $\frac{R}{M}$ i.e., the slope depends on molecular mass of the gas M and is different for different gases.
Hence, the correct answer is (C).

43. Rate of flow of heat $\frac{dQ}{dt}$ or H is equal throughout the rod.
Temperature difference is given by

Since, $\Delta T = HR_{\text{Th}}$

where, $R_{\text{Th}} = \frac{\ell}{KA}$

$\Rightarrow \quad R_{\text{Th}} \propto \frac{1}{A}$

Area across CD is less. Therefore, temperature difference across CD will be more.
Hence, the correct answer is (C).

44. By FLTD, $Q = \Delta U + W$
Since U is a state function i.e. ΔU is same for all processes carried between same points.
For process $a \to c \to e$, net work done is negative
For process $a \to d \to e$, net work done is zero.
For process $a \to b \to e$, net work done is positive. Therefore Q is minimum for the process $a \to c \to e$.

Hence, the correct answer is (D).

45. $\eta = 1 - \left(\frac{T_2}{T_1}\right)$ and $\beta = \frac{T_2}{T_1 - T_2}$

$$\Rightarrow \quad \eta\beta = \frac{T_2}{T_1}$$

$$\Rightarrow \quad \eta\beta = 1-\eta$$

$$\Rightarrow \quad \eta(\beta+1) = 1$$

$$\Rightarrow \quad \eta = \frac{1}{\beta+1}$$

Hence, the correct answer is (C).

46. η is maximum, when β is minimum i.e., 1

$$\Rightarrow \quad \eta_{max} = \frac{1}{2} = 50\%$$

Hence, the correct answer is (B).

47. Since, $PV = nRT$ i.e. $T \propto PV$

$$\Rightarrow \quad T_1 : T_2 = 2\times 2 : 1\times 1 = 4:1$$

Also, $v_{rms} \propto \sqrt{T}$

$$\Rightarrow \quad \frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{4}{1}} = 2$$

Hence, the correct answer is (C).

48. For the two sheets if H = is rate of heat transfer, then

$$H_1 = H_2$$

$$\Rightarrow \quad \frac{\theta_1-\theta}{R_1} = \frac{\theta-\theta_2}{R_2}$$

Solving this we get, $\theta = \dfrac{\theta_1 R_2 + \theta_2 R_1}{R_1+R_2}$

Hence, the correct answer is (D).

49. Since $P_1V = n_1RT_1$ and $P_2V = n_2RT_2$

After joining the vessels we have

$$P(2V) = (n_1+n_2)RT$$

$$\Rightarrow \quad P(2V) = \left(\frac{P_1V}{RT_1} + \frac{P_2V}{RT_2}\right)RT$$

$$\Rightarrow \quad P = \frac{1}{2}\left(\frac{P_1}{T_1} + \frac{P_2}{T_2}\right)T$$

Hence, the correct answer is (B).

50. Gas in cylinder A will undergo an isobaric process and gas in cylinder B will undergo an isochoric process. Since,

$$Q_A = Q_B$$

$$\Rightarrow \quad nC_P\Delta T_A = nC_V\Delta T_B$$

$$\Rightarrow \quad \left(\frac{5}{2}R\right)(15) = \left(\frac{3}{2}R\right)\Delta T_B$$

$$\Rightarrow \quad \Delta T_B = 25 \text{ K}$$

Hence, the correct answer is (B).

51. $\rho = \dfrac{PM}{RT}$ and $\rho \propto \dfrac{1}{V}$

During AB, ρ and hence V is constant.

Therefore, work done is zero.

During BC, $P \propto \rho$ i.e., T and hence, U is constant.

Hence, the correct answer is (D).

52. $P \propto AT^4$

Hence, the correct answer is (C).

53. $C = \dfrac{dQ}{dT} = 0$, so process is adiabatic, i.e., $n = \gamma = \dfrac{C_P}{C_V}$

Hence, the correct answer is (A).

54. Since the wall is conducting, temperature of both the gases is same. For rms speed of molecule of A to be equal to mean speed of molecules of B, we have

$$\sqrt{\frac{3RT}{M_A}} = \sqrt{\frac{8RT}{\pi M_B}} \quad \Rightarrow \quad \frac{M_A}{M_B} = \frac{3\pi}{8}$$

Hence, the correct answer is (D).

55. **Method I:** Heat transferred to the gas is

$$Q = \Delta U + W, \text{ where } \Delta U = (1)C_V(T_2 - T_1)$$

$$\Rightarrow \quad \Delta U = (1)C_V[T_0 + \alpha V_2 - T_0 - \alpha V_1]$$

$$\Rightarrow \quad \Delta U = \alpha C_V(V_2 - V_1) \qquad \ldots(1)$$

Since, $dW = PdV = \dfrac{RT}{V}dV = R\left(\dfrac{T_0+\alpha V}{V}\right)dV$

$$\Rightarrow \quad W = \int dW = R\left[T_0\int_{V_1}^{V_2}\frac{dV}{V} + \alpha\int_{V_1}^{V_2}dV\right]$$

$$\Rightarrow \quad W = RT_0\ell n\left(\frac{V_2}{V_1}\right) + \alpha R(V_2 - V_1) \qquad \ldots(2)$$

So, $Q = \alpha C_V(V_2 - V_1) + RT_0\ell n\left(\dfrac{V_2}{V_1}\right) + \alpha R(V_2 - V_1)$

$$\Rightarrow \quad Q = \alpha(V_2 - V_1)(C_V + R) + RT_0\ell n\left(\frac{V_2}{V_1}\right)$$

$$\Rightarrow \quad Q = RT_0\ell n\left(\frac{V_2}{V_1}\right) + \alpha C_P(V_2 - V_1)$$

Method II: Since $T = T_0 + \alpha V$

(a) Think $\alpha = 0$, then

$T = T_0$ = constant

(Isothermal process)

$\Rightarrow \quad \Delta U = 0$

$\Rightarrow \quad Q_1 = RT_0\ell n\left(\dfrac{V_2}{V_1}\right)$

(b) Think $T_0 = 0$, then

$T = \alpha V$

where, α is a constant

(Isobarric process)

$\Rightarrow \quad Q_2 = (1)C_P(T_2 - T_1)$

Since, $T_2 = T_0 + \alpha V_2$ and

$T_1 = T_0 + \alpha V_1$

$\Rightarrow \quad Q_2 = \alpha C_P(V_2 - V_1)$

SUPERIMPOSING THE TWO, WE GET

$$Q = RT_0\ell n\left(\frac{V_2}{V_1}\right) + \alpha C_P(V_2 - V_1)$$

Hence, the correct answer is (C).

CHAPTER 2

56. Process A is an isobaric expansion process, as V increases, T also increases.

Process B is an isothermal process, so temperature is constant.

Process C is an adiabatic expansion process, so as V increases, T decreases.

Process D is an isochoric process in which P is decreasing, so T is also decreasing.

Hence, the correct answer is (C).

57. When the temperature of the gas is same as that of the wall there is no exchange of energy. Hence, molecules return with same average speed whether the collision is elastic or inelastic. Hence, the temperature and pressure would not change.

Hence, the correct answer is (C).

58. Since $T = KV^2$, so $dT = 2KVdV$

$$\Rightarrow \quad dV = \frac{dT}{2KV}$$

Further, $P = \dfrac{nRT}{V}$

$$\Rightarrow \quad W = \int PdV = \int\left(\frac{nRT}{V}\right)\left(\frac{dT}{2KV}\right)$$

$$\Rightarrow \quad W = \int_{T_0}^{4T_0} \frac{nR}{2}dT = \frac{3nRT_0}{2} \qquad \{\because\ KV^2 = T\}$$

Hence, the correct answer is (C).

59. In this problem two concepts are used:

(i) When a solid floats in a liquid, then fraction of volume submerged is

$$k = \frac{\rho_{\text{solid}}}{\rho_{\text{liquid}}}$$

This result comes from the fact that

Weight = Upthrust

$$V\rho_{\text{solid}}g = V_{\text{submerged}}\rho_{\text{liquide}}$$

$$\Rightarrow \quad \frac{V_{\text{submerged}}}{V} = \frac{\rho_{\text{solid}}}{\rho_{\text{liquid}}}$$

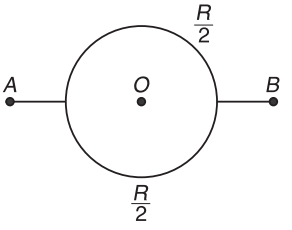
(ii) $\dfrac{\rho_{T\ ^\circ\text{C}}}{\rho_{0\ ^\circ\text{C}}} = \dfrac{1}{1+\gamma T}$

$$k_1 = \left(\frac{\rho_{Fe}}{\rho_{Hg}}\right),\ k_2 = \left(\frac{\rho_{\text{Fe}}}{\rho_{Hg}}\right)_{60\ ^\circ\text{C}} \text{ and } \rho = \text{density}$$

$$\Rightarrow \quad \frac{k_1}{k_2} = \frac{(\rho_{Fe})_{0\ ^\circ\text{C}}}{(\rho_{Hg})_{0\ ^\circ\text{C}}} \times \left(\frac{\rho_{Hg}}{\rho_{Fe}}\right) = \frac{(1+60\gamma_{Fe})}{(1+60\gamma_{Hg})}$$

Hence, the correct answer is (A).

60. When $\theta = 180^\circ$, then thermal resistance of each branch is $\dfrac{R}{2}$. So, net thermal resistance is $\dfrac{R}{4}$. Further



$$\text{Thermal Current } = 1.2 = \frac{\Delta T}{R_{\text{net}}} = \frac{\Delta T}{\left(\frac{R}{4}\right)}$$

$$\Rightarrow \quad \Delta T = 0.3R \qquad \ldots(1)$$

When $\theta = 90^\circ$, then thermal resistance of one branch is $\dfrac{R}{4}$ and that of other is $\dfrac{3R}{4}$ and both are in parallel.

$$\Rightarrow \quad R_{\text{net}} = \frac{\left(\frac{R}{4}\right)\left(\frac{3R}{4}\right)}{\frac{R}{4}+\frac{3R}{4}} = \frac{3R}{16}$$

$$\Rightarrow \quad I' = \frac{\Delta T}{\left(\frac{3R}{16}\right)} = \frac{(0.3R)16}{3R} = 1.6 \text{ W}$$

Hence, the correct answer is (C).

61. Net rate of absorption of heat by water is

$$\Rightarrow \quad 840\Delta t = mc\Delta T = 2\times 4200\times(77-27)$$

$$\Rightarrow \quad \Delta t = \frac{2\times 4200\times 50}{840} = 500 \text{ s} = 8 \text{ min } 20 \text{ s}$$

Hence, the correct answer is (A).

62. $C_P = \left(\dfrac{\gamma}{\gamma-1}\right)R$

$$dQ = nC_P dT \qquad \ldots(1)$$

and $dU = nC_V dT$

Using FLTD, we get

$$dW = dQ - dU = n\left(C_P - C_V\right)dT = nRdT \qquad \ldots(2)$$

Given $dQ = 100$ J

$$\Rightarrow \quad ndT = \frac{100}{C_P} \qquad \{\text{from equation (1)}\}$$

Substituting this in equation (2), we get

$$dW = R\left(\frac{100}{C_P}\right) = R\left(\frac{\gamma-1}{\gamma R}\right)100$$

$$\Rightarrow \quad dW = \left(\frac{1.4-1}{1.4}\right)\times 100 = 28.57 \text{ J}$$

Hence, the correct answer is (D).

63. For the process shown, $32P_0V_0 = RT_A$ and $8P_0V_0 = RT_B$

$$\Rightarrow \quad T_B - T_A = (8-32)\frac{P_0V_0}{R} = -\frac{24P_0V_0}{R}$$

$\Rightarrow \quad \Delta U = 1.5R(T_B - T_A) = -36P_0V_0$

Also $W = \frac{1}{2}(32P_0 + P_0)(8V_0 - V_0) = 115.5(P_0V_0)$

Applying FLTD, we get

$Q = \Delta U + W = -36P_0V_0 + 115.5(P_0V_0) = 79.5P_0V_0$

The molar specific heat C of the gas is

$$C = \frac{Q}{\Delta T} = \frac{79.5(P_0V_0)}{-(24P_0V_0/R)} = -3.3R$$

Hence, the correct answer is (B).

64. In Ingen Hausz experiment

$k \propto (\text{length})^2$

$$\Rightarrow \quad \frac{k_1}{k_2} = \left(\frac{5}{10}\right)^2 = \frac{1}{4}$$

Hence, the correct answer is (B).

65. Since, $\left(P + \frac{a}{V^2}\right)(V - b) = RT$

$$\Rightarrow \quad P + \frac{a}{V^2} = \frac{RT}{V - b}$$

$$\Rightarrow \quad P = \frac{RT}{V - b} - \frac{a}{V^2}$$

Further, $W = \int dW = \int_{V_1}^{V_2} PdV$

$$\Rightarrow \quad W = RT\int_{V_1}^{V_2} \frac{dV}{V - b} - a\int_{V_1}^{V_2} V^{-2}dV$$

$$\Rightarrow \quad W = RT\ell n\left(\frac{V_2 - b}{V_1 - b}\right) + a\left(\frac{1}{V_2} - \frac{1}{V_1}\right)$$

Hence, the correct answer is (D).

66. Thermal expansion for both is the same, but in the hollow sphere the air inside it also expands, thus exerting an extra thrust on the walls of the hollow sphere.
Hence, the correct answer is (C).

67. For Process 1, $W = 0$, so process is isochoric.
For Process 2, the positive work done increases linearly with temperature, so process is isobaric.
For Process 3, the gas heats a linearly with compression i.e. negative work is done and hence process is adiabatic.
Hence, the correct answer is (D).

68. We observe that W_{AB} is negative (volume is decreasing) and W_{BC} is positive (volume is increasing) and since $|W_{BC}| > |W_{AB}|$, so net work done is positive and area between semicircle which is equal to $\frac{\pi}{2}$ atmlt.

Hence, the correct answer is (B).

69. $W_{AB} = -P_0V_0$

$W_{BC} = 0$ and $W_{CD} = 4P_0V_0$

$\Rightarrow \quad W_{ABCD} = -P_0V_0 + 0 + 4P_0V_0 = 3P_0V_0$

Hence, the correct answer is (C).

70. Net rate of absorption of heat by water is

$$\frac{\Delta Q}{\Delta t} = 1000 - 160 = 840 \text{ Js}^{-1}$$

$\Rightarrow \quad 840\Delta t = mc\Delta T = 2 \times 4200 \times (77 - 27)$

$$\Rightarrow \quad \Delta t = \frac{2 \times 4200 \times 50}{840} = 500 \text{ s} = 8 \text{ min } 20 \text{ s}$$

Hence, the correct answer is (A).

71. Since, $\eta = \frac{W}{Q_1} = 1 - \frac{300}{600} = 0.5$

$$\Rightarrow \quad \frac{800}{Q_1} = 0.5$$

$\Rightarrow \quad Q_1 = 1600$ J

Hence, the correct answer is (C).

72. Since, radius or volume of all the four spheres are equal, the ratio of their masses will be $2:3:5:1$.

Heat capacity = (mass) × (specific heat)

So, ratio of heat capacities will be $6:18:10:4$. The sphere having the maximum heat capacity will show the fastest rate of cooling.
Hence, the correct answer is (B).

73. Efficiency of heat engine,

$$\eta = 1 - \frac{\text{Total Heat Rejected}}{\text{Total Heat Absorbed}}$$

$$\Rightarrow \quad \eta = 1 - \frac{100 + 50}{500 + 150} = \frac{500}{650} = 0.77$$

Hence, the correct answer is (A).

74. Internal energy of n moles of an ideal gas having f degrees of freedom at temperature T is given by $U = \frac{f}{2}nRT$

Since, $U_1 = U_2$

$\Rightarrow \quad f_1n_1T_1 = f_2n_2T_2$

$$\Rightarrow \quad \frac{n_1}{n_2} = \frac{f_2T_2}{f_1T_1} = \frac{(3)(2)}{(5)(1)} = \frac{6}{5}$$

where f_2 =degrees of freedom of $He = 3$

and f_1 =degrees of freedom of $He = 5$

Hence, the correct answer is (C).

75. **For** $A \to B$

$V \propto T$ (i.e., $P = \text{constant} = P_1$ (say)) i.e. A to B is Isobaric expansion, as T is increasing. Since $PV = RT$

$$\Rightarrow \quad \frac{V}{T} = (\text{slope})_{AB} = \frac{R}{P_1} \qquad \ldots(1)$$

For $B \to C$

$T = \text{constant}$ (say T_1) and V is increasing i.e. P must be decreasing. So $B \to C$ is Isothermal expansion.

For $C \to D$

Again $V \propto T$ with greater slope for this process (i.e. $P = \text{constant} = P_2$ (say)) i.e. $C \to D$ is Isobaric compression as T is decreasing. Since $PV = RT$

$$\Rightarrow \quad \frac{V}{T} = (\text{slope})_{CD} = \frac{R}{P_2} \qquad \ldots(2)$$

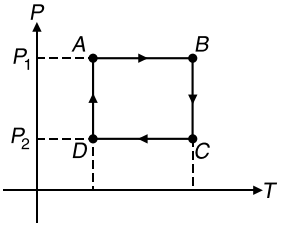
CHAPTER 2

For $D \to A$

T = constant (say T_2) and V is decreasing, so P must be increasing. So $D \to A$ is Isothermal compression.

Also $(\text{slope})_{CD} > (\text{slope})_{AB}$

$\Rightarrow \quad P_1 > P_2$



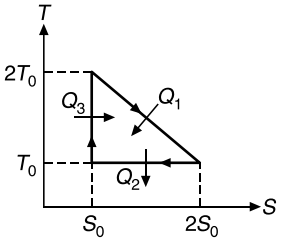
Hence, the correct answer is (A).

76. Change in entropy is given by $dS = \dfrac{dQ}{T}$

Therefore, heat exchange Q in a process is

$$Q = \int dQ = \int TdS = \text{Area under } T\text{-}S \text{ graph}$$

$$\Rightarrow \quad Q_1 = T_0S_0 + \frac{1}{2}T_0S_0 = \frac{3}{2}T_0S_0$$



$$\Rightarrow \quad Q_2 = -T_0S_0 \text{ and } Q_3 = 0$$

So, total work done in this cyclic process is

$$W = \Sigma Q = Q_1 + Q_2 + Q_3 \qquad \{\because \Delta U_{\text{cyclic}} = 0\}$$

$$\Rightarrow \quad W = \frac{3}{2}T_0S_0 - T_0S_0 = \frac{1}{2}T_0S_0$$

$$\Rightarrow \quad \eta = \frac{W}{Q_1} = \frac{T_0S_0/2}{3T_0S_0/2} = \frac{1}{3}$$

Hence, the correct answer is (A).

77. Since $\gamma = 1 + \dfrac{2}{f}$, so for $f = 6$, we get

$$\gamma = \frac{4}{3}$$

Also, $dW = 25$ J

By First Law of Thermodynamics $dQ = dU + dW$

where, $dQ = nC_P dT$, $dU = nC_V dT$, $dW = PdV = nRdT$

Also, $\dfrac{dW}{dQ} = \dfrac{dQ - dU}{dQ}$ {Using First Law}

$$\Rightarrow \quad \frac{dW}{dQ} = 1 - \frac{dU}{dQ}$$

$$\Rightarrow \quad \frac{dW}{dQ} = 1 - \frac{nC_V dT}{nC_P dT}$$

$$\Rightarrow \quad \frac{dW}{dQ} = 1 - \frac{C_V}{C_P} = \left(1 - \frac{1}{\gamma}\right)$$

$$\Rightarrow \quad dQ = 25 \times 4 = 100 \text{ J}$$

Hence, the correct answer is (B).

78. Since, $\dfrac{(Q/t)_1}{4} = \dfrac{k_1}{k_2}\dfrac{r_1^2}{r_2^2}\dfrac{\ell_2}{\ell_1}$

$$\Rightarrow \quad \left(\frac{Q}{t}\right)_1 = \left\{\frac{1}{2}\left(\frac{1}{4}\right)2\right\}4 = 1 \text{ cals}^{-1}$$

Hence, the correct answer is (A).

79. Since, $\dfrac{50-40}{120-40} = \dfrac{Y-(-30)}{130-(-30)}$

$$\Rightarrow \quad \frac{10}{80} = \frac{Y+30}{160}$$

$$\Rightarrow \quad \frac{1}{8} = \frac{Y+30}{160}$$

$$\Rightarrow \quad Y + 30 = 20$$

$$\Rightarrow \quad Y = -10°$$

Hence, the correct answer is (C).

80. For AB, an isothermal process, pressure is increasing, so volume is decreasing and hence

$$Q = W = \text{NEGATIVE}$$

Also for AB, $\Delta U = 0$ and for BC, ΔU = NEGATIVE

Hence, the correct answer is (B).

81. For adiabatic process $P_1V_1^\gamma = P_2V_2^\gamma$

$$\Rightarrow \quad P_1\left(\frac{m}{\rho_1}\right)^\gamma = P_2\left(\frac{m}{\rho_2}\right)^\gamma$$

$$\Rightarrow \quad \frac{P_1}{\rho_1^\gamma} = \frac{P_2}{\rho_2^\gamma}$$

$$\Rightarrow \quad \frac{P_1}{P_2} = \left(\frac{\rho_1}{\rho_2}\right)^\gamma = \left(\frac{1}{32}\right)^{7/5}$$

$$\Rightarrow \quad \frac{P_1}{P_2} = \frac{1}{2^7} = \frac{1}{128}$$

Hence, the correct answer is (B).

82. $PV = nRT$ i.e., $n = \dfrac{PV}{RT}$

$$\Rightarrow \quad n \propto \frac{PV}{T}$$

Number of molecules are directly proportional to the number of moles of the gas. (number of molecules = nN)
Here, P and T are doubled while volume is halved. Therefore, number of moles and hence, number of molecules will become half.

Hence, the correct answer is (B).

83. Condition for a balanced Wheatstone Bridge.

$$\frac{k_1}{k_2} = \frac{k_3}{k_4}$$

$$\Rightarrow \quad k_1k_4 = k_3k_2$$

Hence, the correct answer is (A).

84. Coefficient of performance is $\beta = \dfrac{Q_2}{W} = 6$

$\Rightarrow \quad Q_2 = 6 \times 4 \times 10^6 = 24 \times 10^6$ J

So, mass of water converted to ice is

$$m = \frac{Q_2}{L} = \frac{24 \times 10^6}{80 \times 4.2 \times 1000} \approx 72 \text{ kg}$$

Hence, the correct answer is (D).

85. Heat rejected by 100 g of water at 80 °C when its temperature becomes 0 °C is

$$Q = ms\Delta\theta = (100)(1)(80) = 8000 \text{ cal}$$

But this heat can melt

$$m = \frac{Q}{L} = \frac{8000}{80} = 100 \text{ g of ice only}$$

Hence, the temperature of the mixture is 0 °C.
Hence, the correct answer is (A).

86. We have seen in the above question that only 100 gm ice melts. Therefore, mass of water in the mixture is

$$100 + 100 = 200 \text{ g}$$

and mass of ice $= 120 - 100 = 20$ g and temperature of the mixture is 0 °C.
Hence, the correct answer is (C).

87. Since, $\Delta Q = nC_v\Delta T$

$\Rightarrow \quad 25R = (1)(C_V)(310 - 300)$

$\Rightarrow \quad C_V = \dfrac{5}{2}R$ i.e., gas is diatomic

$\Rightarrow \quad \gamma = 1.4$

Now work done in adiabatic process

$$W = \frac{nR(T_i - T_f)}{\gamma - 1} = \frac{(1)(R)(310 - 300)}{1.4 - 1} = 2.5R$$

Alternate Solution:

$$Q = nC_V\Delta T = C_V(310 - 300)$$

$\Rightarrow \quad Q = 10C_V$ ...(1)

In adiabatic process, $Q = 0$

$\Rightarrow \quad W = -\Delta U = -nC_V(T_f - T_i) = C_V(T_i - T_f)$

$\Rightarrow \quad W = C_V(310 - 300)$

$\Rightarrow \quad W = 10C_V$ ...(2)

From equations (1) and (2), we see that

$$W = Q = 25R$$

Hence, the correct answer is (C).

88. $\dfrac{PV}{T} = \dfrac{P}{\rho T} = \text{constant}$

$$\Rightarrow \quad \frac{\rho_2}{\rho_1} = \left(\frac{T_1}{T_2}\right)\left(\frac{P_2}{P_1}\right) = \left(\frac{T_0}{3T_0}\right)\left(\frac{2.5P_0}{P_0}\right) = \frac{5}{6}$$

Hence, the correct answer is (A).

89. Since, $\lambda_m T = \text{constant}$, so $T = \dfrac{4}{3}T_0$

Also, $P \propto T^4$ (from Stefan's Law)

$$\Rightarrow \quad P_{\text{new}} = \frac{256}{81}P$$

Hence, the correct answer is (A).

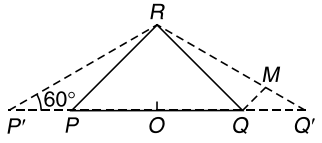
90. $W_{BCOB} = -\text{Area of triangle } BCO = -\dfrac{P_0V_0}{2}$

$W_{AODA} = +\text{Area of triangle } AOD = +\dfrac{P_0V_0}{2}$

$\Rightarrow \quad W_{\text{net}} = 0$

Hence, the correct answer is (D).

91. Since distance OR remains unchanged, the expanded triangle on heating is shown above.



$MQ' = QQ'\cos 60°$

$MQ' \approx L\alpha_2\Delta T$ and $QQ' = 0.5L\alpha_1\Delta T$

Therefore $L\alpha_2\Delta T = 0.5L\alpha_1\Delta T\cos 60°$

$\Rightarrow \quad 4\alpha_2 = \alpha_1$

Hence, the correct answer is (D).

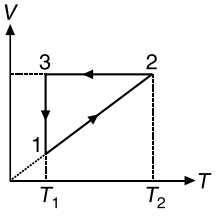
92. $C = \dfrac{dQ}{ndT} = \dfrac{2dU}{ndT} = \dfrac{2nC_vdT}{ndT} = 2C_v = 3R$

Hence, the correct answer is (B).

93. Due to thermal expansion, all x, r and d would increase.
Hence, the correct answer is (C).

94. Energy released in isochoric process $2 \to 3$ is

$$Q_{23} = nC_V(T_3 - T_2) = nC_V(T_1 - T_2)$$



Work done during isobaric process $1 \to 2$ is

$$W_{12} = nR(T_2 - T_1)$$

Hence, $\dfrac{Q_{23}}{W_{12}} = -\dfrac{C_V}{R} = -\dfrac{5R/2}{R} = -2.5$

Therefore, ratio of energy released in process $2 \to 3$ to work done in process $1 \to 2$ is 2.5
Hence, the correct answer is (B).

95. Since, $\alpha = \dfrac{\gamma}{3}$

$$\text{Energy Density} = \frac{(\text{Stress})^2}{2Y} = \frac{1}{2}Y(\text{Strain})^2$$

CHAPTER 2

$$\text{Strain} = \frac{\Delta\ell}{\ell} = \alpha\Delta T = \alpha T = \frac{\gamma T}{3}$$

$$\Rightarrow \quad \text{Stress} = Y(\text{Strain}) = \frac{Y\gamma T}{3} \qquad \left\{\because Y = \frac{\text{stress}}{\text{strain}}\right\}$$

$$\Rightarrow \quad U = \frac{1}{2}\frac{Y^2\gamma^2T^2}{9Y} = \frac{1}{18}\gamma^2T^2Y$$

Hence, the correct answer is (D).

96. Let, m gram of water whose temperature is T_0 $(>30\ ^\circ\text{C})$ and specific heat is 1 calg^{-1} °C added to 20 g of water at 30 °C and let T be the final temperature of the mixture.

$$\begin{pmatrix}\text{Heat given by the}\\ m\text{ g of water}\end{pmatrix} = \begin{pmatrix}\text{heat gained by}\\ 20\text{ g of water}\end{pmatrix}$$

$$\Rightarrow \quad m(1)(T_0 - T) = (20)(1)(T - 30)$$

Solving this we get, $T = \dfrac{600 + mT_0}{20 + m}$

The right hand side is maximum for OPTION (D). Therefore, the correct answer is (D).

Hence, the correct answer is (D).

97. When temperature is increased all a, b, x and ℓ should increase.

Hence, the correct answer is (D).

98. $$\left(\frac{Q}{t}\right)_C = \left(\frac{Q}{t}\right)_A + \left(\frac{Q}{t}\right)_B$$

$$\Rightarrow \quad k_CA\frac{\Delta T}{\ell} = k_AA\frac{\Delta T}{\ell} + k_BA\frac{\Delta T}{\ell}$$

$$\Rightarrow \quad k_C = k_A + k_B$$

Hence, the correct answer is (A).

99. Let length of each portion of cylinder be L. Since final pressure on both sides of the movable piston must be same, therefore

$$\frac{V_A}{T_A} = \frac{V_B}{T_B}$$

$$\Rightarrow \quad \frac{L+5}{373} = \frac{L-5}{273}$$

$$\Rightarrow \quad L = 32.3 \text{ cm}$$

So, length of the cylinder is

$$L_{\text{cylinder}} = 2L = 64.6 \text{ cm}$$

Hence, the correct answer is (D).

100. $$k_1A\frac{(T_1 - T)}{d_1} = k_2A\frac{(T - T_2)}{d_2}$$

$$\Rightarrow \quad T = \frac{k_1T_1d_2 + k_2T_2d_1}{k_1d_2 + k_2d_1}$$

Hence, the correct answer is (B).

101. Since, $W_1 = P_i(V_f - V_i) = P_iV_i\left(\dfrac{V_f}{V_i} - 1\right)$

$$\Rightarrow \quad W_1 = nRT(2-1) = nRT$$

and $W_2 = nRT\log_e\left(\dfrac{V_f}{V_i}\right) = nRT\log_e(2) = W_1\log_e(2)$

Hence, the correct answer is (A).

102. At 30 °C, the copper rod will be of length $L_0(1+\alpha_c\Delta\theta)$ while adjacent centimetre marks on the steel tape will be separated by a distance of $(1\text{ cm})(1+\alpha_s\Delta\theta)$. Therefore, the number of centimetres read on the tape will be

$$\frac{L_0(1+\alpha_c\Delta\theta)}{(1\text{ cm})(1+\alpha_s\Delta\theta)} = \frac{(90)[1+1.7\times10^{-5}](20)}{(1)[1+1.2\times10^{-5}](20)} \approx 90.01 \text{ cm}$$

Hence, the correct answer is (D).

103. Heat exchanged, $\Delta Q = nC_V\Delta T = \dfrac{N}{N_A}C_V\Delta T$

where N is number of molecules of the gas and N_A is Avogadro's number.

In the given situation, let T be the final temperature of the mixture. Then, by Law of Calorimetry, we have

$$\text{Heat lost} = \text{Heat gained}$$

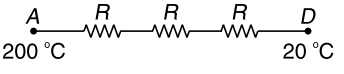
$$\Rightarrow \quad \frac{N_1C_V(T_1 - T)}{N_A} = \frac{N_2C_V(T - T_2)}{N_A}$$

$$\Rightarrow \quad N_1T_1 - N_1T = N_2T - N_2T_2$$

$$\Rightarrow \quad T = \frac{N_1T_1 + N_2T_2}{N_1 + N_2}$$

Hence, the correct answer is (C).

104. Equivalent electrical circuit will be as shown in figure. Temperature difference between A and D is 180 °C which is equally distributed in all the rods. Therefore, temperature difference between A and B will be 60 °C or temperature of B should be 140 °C.



Hence, the correct answer is (C).

105. From A to B, the process is isobaric i.e., $V \propto T$. From B to C and D to A, the process is isothermal but temperature from B to C is higher than that from D to A. From C to D, the process is isochoric.

Hence, the correct answer is (B).

106. Given that, PT = constant …(1)

According to Ideal Gas Equation, we have $PV = nRT$

$$\Rightarrow \quad T = \frac{PV}{nR}$$

So, equation (1) becomes $P\left(\dfrac{PV}{nR}\right) = \text{constant}$

$$\Rightarrow \quad P^2V = \text{constant}$$

This is best represented by graph in OPTION (C).

Hence, the correct answer is (C).

107. Temperature decays exponentially with time.

Hence, the correct answer is (A).

108. 1 g of water equals 1 cc of water

Volume of liquid is $V_\ell = 1 \text{ cc} = 10^{-6}$ m^3

Volume of vapours is $V_v = 1671 \text{ cc} = 1671\times10^{-6}$ m^3

$$\Rightarrow \quad \Delta V = V_v - V_\ell = 1670\times10^{-6} \text{ m}^3$$

$$\Rightarrow \quad W = P\Delta V = 10^5(1670\times10^{-6}) = 167 \text{ J}$$

$\Rightarrow \quad W = \dfrac{167}{4.18}\ \text{cal} \simeq 40\ \text{cal}$

Further $Q = mL$

$\Rightarrow \quad Q = (1\ g)(540\ \text{calg}^{-1}) = 540\ \text{cal}$

According to First Law of Thermodynamics

$Q = \Delta U + W$

$\Rightarrow \quad 540 = \Delta U + 40$

$\Rightarrow \quad \Delta U = 500\ \text{cal}$

Hence, the correct answer is (B).

109. $Y = \dfrac{\text{Stress}}{\text{Strain}} = \dfrac{F/A}{\Delta L/L_0}$

$\Rightarrow \quad Y = \dfrac{F/A}{\alpha \Delta T} \qquad \{\because \Delta L/L_0 = \alpha \Delta T\}$

$\Rightarrow \quad F = YA\alpha\Delta T$

Hence, the correct answer is (A).

110. Since, $PV = nRT$

For isobaric process, $V \propto T$, hence slope of the graph is given by

$\dfrac{dV}{dT} = \dfrac{nR}{P} \propto \dfrac{1}{P}$ (if mass remains constant)

Hence, the correct answer is (C).

111. Rate of melting of ice $\propto$ rate of heat transfer $\left(\dfrac{dQ}{dt}\right)$

Further $\dfrac{dQ}{dt} = \dfrac{\text{temperature difference}}{\left(\dfrac{\ell}{KA}\right)}$

$\Rightarrow \quad \dfrac{dQ}{dt} \propto \dfrac{(\text{temperature difference})}{\ell} A$

If temperature difference, A and ℓ are all doubled then $\dfrac{dQ}{dt}$ and hence, rate of melting of ice will be doubled.

Hence, the correct answer is (D).

112. Let the piston get compressed by x. Then for mechanical equilibrium, we have $P'A = P_0A + kx$

$\Rightarrow \quad P' = P_0 + \dfrac{kx}{A}$

$\Rightarrow \quad 3 \times 10^5 = 1 \times 10^5 + \dfrac{10^4 x}{0.005}$

$\Rightarrow \quad x = 0.1\ \text{m}$

So, work done is

$W = P_0\Delta V + \dfrac{1}{2}kx^2$

$\Rightarrow \quad W = (10^5)(0.005 \times 0.1) + \dfrac{1}{2}(10^4)(0.1)^2$

$\Rightarrow \quad W = 50 + 50 = 100\ \text{J}$

Hence, the correct answer is (A).

113. $\left(\begin{matrix}\text{Rate of}\\ \text{Cooling}\end{matrix}\right) \propto \left(\begin{matrix}\text{Average Excess}\\ \text{Temperature}\end{matrix}\right)$

$\Rightarrow \quad \dfrac{\Delta T}{\Delta t} \propto \left(\dfrac{T_i + T_f}{2} - T_{\text{surr}}\right)$

Lets denote T_{surr} by T_0, then

$\dfrac{T_f - T_i}{t} \propto \left(\dfrac{T_i + T_f}{2} - T_{\text{surr}}\right)$

$\Rightarrow \quad \dfrac{60 - 50}{10} \propto \left(\dfrac{60 + 50}{2} - T_0\right)$

$\Rightarrow \quad 1 \propto 55 - T_0 \qquad \ldots(1)$

Also, $\dfrac{50 - 42}{10} \propto \left(\dfrac{50 + 42}{2} - T_0\right)$

$0.8 \propto 46 - T_0 \qquad \ldots(2)$

$\Rightarrow \quad \dfrac{0.8}{1} = \dfrac{46 - T_0}{55 - T_0}$

$\Rightarrow \quad \dfrac{4}{5} = \dfrac{46 - T_0}{55 - T_0}$

$\Rightarrow \quad 220 - 4T_0 = 230 - 5T_0$

$\Rightarrow \quad T_0 = 10\ ^\circ\text{C}$

Hence, the correct answer is (B).

114. $r_0 \propto (600)^4 - (300)^4$

$r \propto (900)^4 - (300)^4$

$\Rightarrow \quad \dfrac{r}{r_0} = \dfrac{(900)^4 - (300)^4}{(600)^4 - (300)^4}$

$\Rightarrow \quad r = r_0\left(\dfrac{9^4 - 3^4}{6^4 - 3^4}\right)$

$\Rightarrow \quad r = r_0\left(\dfrac{3^4 - 1}{2^4 - 1}\right)$

$\Rightarrow \quad r = r_0\left(\dfrac{80}{15}\right)$

$\Rightarrow \quad r = \dfrac{16}{3} r_0$

Hence, the correct answer is (A).

115. Molar heat capacity $C = \dfrac{\Delta Q}{\Delta T}$

$\Rightarrow \quad C = \dfrac{\Delta U + \Delta W}{dT}$

ΔU is same in both the paths but

$\Delta W_2 > \Delta W_1$

$\Rightarrow \quad C_2 > C_1$

$\Rightarrow \quad \dfrac{C_1}{C_2} < 1$

Hence, the correct answer is (B).

116. After first stroke, we have $PV = P_1(V + v)$

$\Rightarrow \quad P_1 = P\left(\dfrac{V}{V + v}\right)$

After second stroke, we have $P_1V = P_2(V + v)$

$\Rightarrow \quad P_2 = P_1\left(\dfrac{V}{V + v}\right) = P\left(\dfrac{V}{V + v}\right)^2$

and so on for n strokes, we have

$P_n = P_{n-1}\left(\dfrac{V}{V + v}\right) = P\left(\dfrac{V}{V + v}\right)^n$

Hence, the correct answer is (D).

CHAPTER 2

117. Let A_1 be the cross sectional area of silica cylinder, A_2 be the cross-sectional area of the mercury in the glass cylinder, then $(A_1 + A_2)$ is cross sectional area of glass cylinder and h is the height of silica cylinder, then

$$h(A_1 + A_2)(2\alpha)\Delta T = hA_2\gamma\Delta T$$

$$\Rightarrow \quad \frac{hA_1}{hA_2} = \frac{V_{\text{silica}}}{V_{\text{mercury}}} = \frac{\gamma}{2\alpha} - 1$$

Hence, the correct answer is (C).

118. $$\frac{50 - 49.9}{5} \propto \left(\frac{50 + 49.9}{2} - 30\right) \text{ and} \qquad \ldots(1)$$

$$\frac{40 - 39.9}{t} \propto \left(\frac{40 + 39.9}{2} - 30\right) \qquad \ldots(2)$$

$$\Rightarrow \quad \frac{t}{5} = \frac{99.9 - 60}{79.9 - 60} = \frac{39.9}{19.9} \simeq 2$$

$$\Rightarrow \quad t = 10 \text{ s}$$

Hence, the correct answer is (C).

119. $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

Room temperature $T \approx 300$ K

$$\Rightarrow \quad 1930 = \sqrt{\frac{3 \times 8.31 \times 10^3 \times 300}{M}}$$

$$\Rightarrow \quad M = 2 \text{ g mol}^{-1}$$

So, the gas is H_2

Hence, the correct answer is (A).

120. For processes ab and cd, $P \propto T$ so $V = \text{constant}$ and hence processes ab and cd are isochoric, work done in these processes is zero. So, net work done in cyclic process is

$$W = W_{bc} + W_{da}$$

$$\Rightarrow \quad W = nR(T_c - T_b) + nR(T_a - T_d) \qquad \{\text{isobaric}\}$$

$$\Rightarrow \quad W = (2)(8.31)(1600 - 800) + (2)(8.31)(400 - 800)$$

$$\Rightarrow \quad W = 6.6 \text{ kJ}$$

Hence, the correct answer is (C).

121. Since all P, V and T are changing simultaneously and the work is being done at the expense of the internal energy of the gas so this must be an adiabatic process because for an adiabatic process $dQ = 0$ and by First Law of Thermodynamics

$$dU = -dW$$

Hence $C = \frac{dQ}{dT} = 0$

Hence, the correct answer is (A).

122. $$\frac{dQ}{dt} \propto A$$

$$\Rightarrow \quad \frac{\left(\frac{dQ}{dt}\right)_1}{\left(\frac{dQ}{dt}\right)_2} = \frac{R_1^2}{R_2^2}$$

Hence, the correct answer is (C).

123. $$mc\left(\frac{dT}{dt}\right) \propto A$$

$$\Rightarrow \quad \frac{dT}{dt} \propto \frac{A}{mc}$$

$$\Rightarrow \quad \frac{\left(\frac{dT}{dt}\right)_1}{\left(\frac{dT}{dt}\right)_2} = \frac{A_1}{A_2}\frac{m_2}{m_1} = \frac{4\pi R_1^2}{4\pi R_2^2}\left(\frac{\frac{4}{3}\pi R_2^3\rho}{\frac{4}{3}\pi R_1^3\rho}\right) = \frac{R_2}{R_1}$$

Hence, the correct answer is (B).

124. When cooled isochorically (i.e. $V = \text{constant}$)

$$\frac{P}{T} = \frac{P/n}{T'}$$

$$\Rightarrow \quad T' = \frac{T}{n}$$

Heat exchanged is $Q_1 = C_V\Delta T$

$$\Rightarrow \quad Q_1 = C_V\left(\frac{T}{n} - T\right) = -C_VT\left(1 - \frac{1}{n}\right)$$

When heated isobarically (i.e. $P = \text{constant}$) heat exchanged is

$$Q_2 = C_P\left(T - \frac{T}{n}\right) = C_PT\left(1 - \frac{1}{n}\right)$$

Net heat exchanged is

$$Q = Q_2 + Q_1 = C_PT\left(1 - \frac{1}{n}\right) - C_VT\left(1 - \frac{1}{n}\right)$$

$$\Rightarrow \quad Q = (C_P - C_V)T\left(1 - \frac{1}{n}\right) = RT\left(1 - \frac{1}{n}\right)$$

Hence, the correct answer is (A).

125. Let M be the molecular mass of the gas. Then

So, $M(C_P - C_V) = R$

$$\Rightarrow \quad M(525 - 315) = 8.31$$

$$\Rightarrow \quad M = 0.0392$$

Now let ρ be the density of the gas at NTP. Then it implies that So, mass of 22.4 L or $(22.4 \times 10^{-3})m^3$ of gas should be $(\rho \times 22.4 \times 10^{-3})$ kg which should be equal to its molecular mass i.e.,

$$\rho \times 22.4 \times 10^{-3} = 0.0392$$

$$\Rightarrow \quad \rho = 1.75 \text{ kgm}^{-3}$$

Hence, the correct answer is (D).

126. For equilibrium $F_1 - F_2 = 0$

$$\Rightarrow \quad Y_1\alpha_1A\Delta T - Y_2\alpha_2A\Delta T = 0$$

$$\Rightarrow \quad Y_1\alpha_1 = Y_2\alpha_2$$

Hence, the correct answer is (C).

127. Two molecules have a speed of 1.6 kms^{-1}, so most probable speed is also 1.6 kms^{-1}.

Hence, the correct answer is (C).

128. We use $(1 + \alpha_s) \times R = L(1 + \alpha_C)$

$$\Rightarrow \quad R = L\left[\frac{1 + \alpha_C}{1 + \alpha_S}\right]$$

Hence, the correct answer is (C).

129. From First Law of Thermodynamics $\Delta U = Q - W$

Here $Q = mL = L$ $\{\because\ m = 1\}$

and $W = P_0(V_f - V_i) = P_0(V_2 - V_1)$

$\Rightarrow \quad \Delta U = L - P_0(V_2 - V_1)$

Hence, the correct answer is (C).

130. $C_V(\text{mixture}) = \dfrac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2}$

Here, $C_{V_1} = \dfrac{5}{2}R,\ n_1 = \dfrac{10}{2} = 5$ {for H_2}

$C_{V_2} = \dfrac{3}{2}R,\ n_2 = \dfrac{40}{4} = 10$ {for He}

$\Rightarrow \quad C_V(\text{mixture}) = \dfrac{11}{6}R$

$\Rightarrow \quad Q = nC_V\Delta T = 2750$ cal

Hence, the correct answer is (B).

131. $dQ = dU + dW$

$\Rightarrow \quad dQ = dU - \dfrac{2dU}{3} = \dfrac{dU}{3} = \dfrac{1}{3}nC_v dT$

$\Rightarrow \quad dQ = \dfrac{1}{3}n\left(\dfrac{3}{2}RdT\right) = \dfrac{nRdT}{2}$

$\Rightarrow \quad C = \dfrac{1}{n}\dfrac{dQ}{dT} = \dfrac{R}{2}$

which is not equal to 0, infinity or $\dfrac{5R}{2}$

Hence, the correct answer is (D).

132. Given that $P\sqrt{T} = \text{constant}$ …(1)

Also, for a polytropic process, we have

$P^{1-x}T^x = \text{constant}$

$\Rightarrow \quad PT^{\frac{x}{1-x}} = \text{constant}$ …(2)

Comparing equations (1) and (2), we get $\dfrac{x}{1-x} = \dfrac{1}{2}$

$\Rightarrow \quad 2x = 1 - x$

$\Rightarrow \quad x = \dfrac{1}{3}$

$\Rightarrow \quad C = \dfrac{R}{\gamma - 1} + \dfrac{R}{1-x} = \dfrac{R}{(5/3)-1} + \dfrac{R}{1-(1/3)} = 3R$

Hence, the correct answer is (C).

133. $PV = nRT$

$\Rightarrow \quad PdV = nRdT$ {$\because\ P$ =constant}

$\Rightarrow \quad dV = \left(\dfrac{nR}{P}\right)dT$

$\Rightarrow \quad dV = \left(\dfrac{V}{T}\right)dT$ …(1)

Further $dV = \gamma V dT$ …(2)

From equations (1) and (2), we get $\gamma = \dfrac{1}{T}$

Hence, the correct answer is (C).

134. $PV^{3/2} = \text{constant}$

$\Rightarrow \quad \dfrac{PV^{2/3}}{PV} = \dfrac{\text{constant}}{RT}$

$\Rightarrow \quad \dfrac{1}{V^{1/3}} = \dfrac{\text{constant}}{RT}$

$\Rightarrow \quad V \propto T^3$

Temperature increases with increase in volume.

Hence, the correct answer is (A).

135. $K_C A\left(\dfrac{\Delta T_{Cu}}{\ell}\right) = K_B A\left(\dfrac{\Delta T_B}{\ell}\right)$

$\Rightarrow \quad \dfrac{\Delta T_{Cu}}{\Delta T_B} = \dfrac{K_B}{K_C} < 1$

$\Rightarrow \quad \Delta T_{Cu} < \Delta T_B$

i.e. fall in temperature for copper is less than brass and hence temperature at junction is greater than 50 °C.

Hence, the correct answer is (A).

136. For the isochoric process ab, $W = 0$, so we have

$Q = \Delta U = nC_V\Delta T = 5000$

$\Rightarrow \quad n\left(\dfrac{5R}{2}\right)(800 - 300) = 5000$

$\Rightarrow \quad nR = 4$

Now, for isobaric process ca, we have

$Q_{ca} = nC_P\Delta T = n\left(\dfrac{7R}{2}\right)(T_a - T_c)$

Since process bc is isothermal, so $T_b = T_c$ and hence $\Delta U = 0$.

For process ca, P = constant

$\Rightarrow \quad Q_{ca} = n\left(\dfrac{7R}{2}\right)(T_a - T_b) = (4)\left(\dfrac{7}{2}\right)(300 - 800)$

$\Rightarrow \quad Q_{ca} = -7000$ J

Hence, the correct answer is (C).

137. $\gamma = \dfrac{\Delta V}{V\Delta T} = \dfrac{0.12}{100} \times \dfrac{1}{20} = 6 \times 10^{-5}(^\circ\text{C})^{-1}$

$\Rightarrow \quad \alpha = \dfrac{\gamma}{3} = 2 \times 10^{-5}(^\circ\text{C})^{-1}$

Hence, the correct answer is (A).

138. Since, $\dfrac{E_2}{E_1} = \left(\dfrac{727 + 273}{227 + 273}\right)^4 = \left(\dfrac{1000}{500}\right)^4 = 16$

$\Rightarrow \quad E_2 = 16 \times 5 = 80$ cals^{-1}cm^{-2}

Hence, the correct answer is (B).

139. $dQ = +35$ J and $dW = -15$ J

By First Law, we have $dQ = dU + dW$

$\Rightarrow \quad 35 = dU - 15$

$\Rightarrow \quad dU = +50$ J

Hence, the correct answer is (D).

140. Internal energy of n mole of gas having f degrees of freedom is $U = n\left(\dfrac{1}{2}RT\right)f$.

For 1 mole of helium at 600 K, $f = 3$

$\Rightarrow \quad U_1 = (1)\left[\dfrac{1}{2} \times R \times 600\right](3) = 900R$

CHAPTER 2

For 56 g of nitrogen at 300 K, $f = 5$

$$\Rightarrow \quad U_2 = \left(\frac{56}{28}\right)\left[\frac{1}{2}\times R\times 300\right](5) = 1500R$$

For 32 g of oxygen at 300 K, $f = 5$ and

$$U_3 = \left(\frac{32}{32}\right)\left[\frac{1}{2}\times R\times 300\right](5) = 750R$$

For 12×10^{23} molecules of argon at 450 K, $f = 3$

$$\Rightarrow \quad U_4 = \left(\frac{12\times10^{23}}{6\times10^{23}}\right)\left[\frac{1}{2}R\times 450\right](3) = 1350R$$

Hence U is maximum for 56 g of nitrogen at 300 K

Hence, the correct answer is (B).

141. Since, $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$$\Rightarrow \quad T_1(AL_1)^{\frac{5}{3}-1} = T_2(AL_2)^{\frac{5}{3}-1}$$

$$\Rightarrow \quad \frac{T_1}{T_2} = \left(\frac{L_2}{L_1}\right)^{\frac{2}{3}}$$

Hence, the correct answer is (D).

142. $$\frac{PV}{kT} = \frac{nRT}{\left(\frac{R}{N}\right)T} = nN = (\text{Number of molecules in the gas})$$

Hence, the correct answer is (C).

143. V-T graph is a straight line passing through origin

Hence, $V \propto T$ or $P =$ constant

$$\Rightarrow \quad dQ = nC_P dT \text{ and } dU = nC_V dT$$

Since, $dW = dQ - dU = n(C_P - C_V)dT$

$$\Rightarrow \quad \frac{dQ}{dW} = \frac{nC_P dT}{n(C_P - C_V)dT}$$

$$\Rightarrow \quad \frac{dQ}{dW} = \frac{C_P}{C_P - C_V} = \frac{1}{1-\frac{C_V}{C_P}}$$

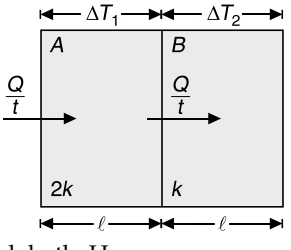
Since, $\frac{C_V}{C_P} = \frac{3}{5}$ for helium gas

$$\Rightarrow \quad \frac{dQ}{dW} = \frac{1}{1-3/5} = \frac{5}{2}$$

Hence, the correct answer is (B).

144. $\Delta T_1 + \Delta T_2 = 36$ …(1)

Also, the slabs are in series. So, same thermal current $\left(\frac{Q}{t}\right)$



Flows through both. Hence

$$2kA\left(\frac{\Delta T_1}{\ell}\right) = kA\left(\frac{\Delta T_2}{\ell}\right)$$

$$\Rightarrow \quad \Delta T_2 = 2\Delta T_1 \quad \text{…(2)}$$

$$\Rightarrow \quad \Delta T_1 = 12\ ^\circ\text{C and } \Delta T_2 = 24\ ^\circ\text{C}$$

Hence, the correct answer is (B).

145. Apparent weight (w_a)=actual weight (w) – upthrust (F)

Here, $F = V\rho_w g$ $\{\rho_w =$ density of water$\}$

i.e., $F_{0\ ^\circ\text{C}} = V_{0\ ^\circ\text{C}}(\rho_w)_{50\ ^\circ\text{C}}\, g$

and $F_{50\ ^\circ\text{C}} = V_{50\ ^\circ\text{C}}(\rho_w)_{50\ ^\circ\text{C}}\, g$

$$\Rightarrow \quad \frac{F_{50\ ^\circ\text{C}}}{F_{0\ ^\circ\text{C}}} = \frac{V_{50\ ^\circ\text{C}}}{V_{0\ ^\circ\text{C}}}\cdot\frac{(\rho_w)_{50\ ^\circ\text{C}}}{(\rho_w)_{0\ ^\circ\text{C}}} = \frac{(1+\gamma_m\Delta T)}{(1+\gamma_w\Delta T)} \quad \{\Delta T = 50\ ^\circ\text{C}\}$$

Given that $\gamma_m < \gamma_w$, so $F_{50\ ^\circ\text{C}} < F_{0\ ^\circ\text{C}}$

Hence apparent weight at 50 °C will be more.

Hence, the correct answer is (B).

146. Since $A_t = A_0[1+\beta\Delta T]$

Let diameter at 0 °C is d_0 cm and at dry ice (–60 °C) is d cm

$$\frac{\pi(10.02)^2}{4} = \frac{\pi d_0^2}{4}\left[1+(4\times10^{-5})(40-0)\right] \quad \text{…(1)}$$

$$\Rightarrow \quad \frac{\pi d^2}{4} = \frac{\pi d_0^2}{4}\left[1+(4\times10^{-5})(-60-0)\right] \quad \text{…(2)}$$

Dividing (1) by (2), we get

$$\frac{(10.02)^2}{d^2} = \frac{1.0016}{0.9976}$$

$$\Rightarrow \quad d = 9.99997\ \text{cm} \approx 10\ \text{cm}$$

Hence, the correct answer is (D).

147. By First Law of Thermodynamics

$$Q_1 + (-W_1) = Q_2 + (-W_2) = U_B - U_A$$

Hence, the correct answer is (D).

148. Cooling must lead to a decrease in volume of the system which is possible only if the piston moves to the right.

Hence, the correct answer is (B).

149. For both graphs, work done is same because area under the curve is same.

However, according to Ideal Gas Equation, $PV = nRT$, we see that the increase in temperature is more in CASE-1. Therefore, increase in internal energy is also more in CASE-1. Hence heat added is more in CASE-1.

Hence, the correct answer is (A).

150. For a cyclic process $\Delta U = 0$, so $Q = W$

$$\Rightarrow \quad (100-20) = 20 + W_2$$

$$\Rightarrow \quad W_2 = 60\ \text{J}$$

Hence, the correct answer is (A).

151. Let h be the distance of the apex from the middle of the base. Further by Pythagoras theorem

$$L_2^2 - \frac{L_1^2}{4} = h^2 = \text{constant}$$

$$\Rightarrow \quad \Delta(h^2) = \Delta\left(L_2^2\right) - \Delta\left(\frac{L_1^2}{4}\right)$$

$$\Rightarrow \quad 0 = 2L_2\Delta L_2 - \frac{1}{4}(2L_1\Delta L_1)$$

Since, $\alpha_1 = \frac{\Delta L_1}{L_1}\frac{1}{\Delta T}$ and $\alpha_2 = \frac{\Delta L_2}{L_2}\frac{1}{\Delta T}$

$$\Rightarrow \quad L_2(L_2\alpha_2\Delta T) = \frac{1}{4}L_1(L_1\alpha_1\Delta T)$$

$$\Rightarrow \quad \frac{L_1}{L_2} = 2\sqrt{\frac{\alpha_2}{\alpha_1}}$$

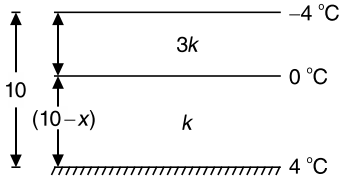
Hence, the correct answer is (D).

152. 1 g of steam at 100 °C liberates 540 cal to convert to 1 g of water at 100 °C.
Further 1 g of ice at 0 °C melts and converts to water at 0 °C. For this purpose 80 cal of heat is required by it.
This 1 g of water at 0 °C requires 100 cal of heat to convert to 1 g of water at 100 °C.
i.e., a total of $(80+100)$ cal = 180 cal of heat is required by ice at 0 °C to convert to water at 100 °C which is much less than the heat liberated by steam at 100 °C to convert to water at 100 °C i.e. 540 cal. Hence equilibrium temperature will just be 100 °C.
Hence, the correct answer is (D).

153. Even if 3 g of ice at 0 °C is taken then total heat required by 3 g of ice at 0 °C to convert to 3 g of water at 100 °C equals $3(180\text{ cal}) = 540$ cal, which is the amount of heat liberated by steam at 100 °C to convert to water at 100 °C.
Hence, the correct answer is (C).

154. A refrigerator is reverse of heat engine and heat pump is same as refrigerator. The coefficient of performance of a refrigerator cannot be infinity and a heat engine cannot convert the heat input fully to work done.
Hence, the correct answer is (B).

155. $kA\left(\frac{4-0}{10-x}\right) = 3kA\left[\frac{0-(-4)}{x}\right]$

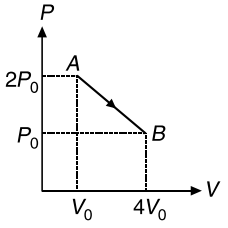


$$\Rightarrow \quad x = 7.5 \text{ m}$$

Hence, the correct answer is (A).

156. Temperature of liquid oxygen will first increase in the same phase. Then, phase change (liquid to gas) will take place. During which temperature will remain constant. After that temperature of oxygen n gaseous state will further increase.
Hence, the correct answer is (C).

157. The process carried on an ideal monatomic gas is shown in Figure.



Applying Ideal Gas Equation at points A and B, we get

$$2P_0V_0 = nRT_A \text{ and } 4P_0V_0 = nRT_B$$

$$\Rightarrow \quad \Delta U = n(1.5R)(T_B - T_A) = 3P_0V_0$$

Also $W = \frac{1}{2}(2P_0 + P_0)(4V_0 - V_0) = 4.5P_0V_0$

$$\Rightarrow \quad Q = \Delta U + W = 3P_0V_0 + 4.5P_0V_0 = 7.5P_0V_0$$

$$\Rightarrow \quad Q = 7.5\times(0.5\times10^6)\times0.1 = 375\times10^3 \text{ J}$$

Hence, the correct answer is (B).

158. For an adiabatic process PV^γ = constant …(1)

Also, density $\rho = \frac{m}{V}$ i.e., $\rho \propto V^{-1}$

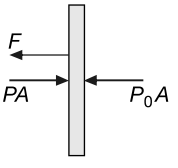
So, equation (1) can be written as $P\rho^{-\gamma}$ = constant
Hence, the correct answer is (B).

159. Volume of the gas is constant V = constant

$$\Rightarrow \quad P \propto T$$

i.e., pressure will be doubled if temperature is doubled

$$\Rightarrow \quad P = 2P_0$$



Now let F be the tension in the wire. Then equilibrium of any one piston gives

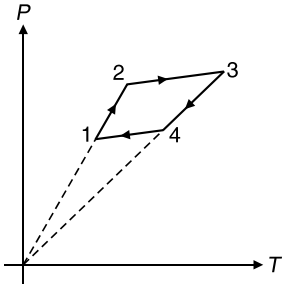
$$F = (P - P_0)A = (2P_0 - P_0)A = P_0A$$

Hence, the correct answer is (B).

160. Processes $1\to2$ and $3\to4$ are isochoric, so work done is zero.

$$\Rightarrow \quad W_{\text{total}} = W_{23} + W_{41}$$

$$\Rightarrow \quad W_{\text{total}} = P_2(V_3 - V_2) + P_4(V_1 - V_4)$$



$$\Rightarrow \quad W_{\text{total}} = nR(T_3 - T_2) + nR(T_1 - T_4)$$

$$\Rightarrow \quad W_{\text{total}} = nR(T_3 - T_2 + T_1 - T_4)$$

$$\Rightarrow \quad W_{\text{total}} = 800nR = 2400R$$

Hence, the correct answer is (C).

CHAPTER 2

161. $PV = nRT$, where $V = \frac{m}{\rho}$

$$\Rightarrow \quad P\left(\frac{m}{\rho}\right) = \left(\frac{m}{M}\right)RT$$

$$\Rightarrow \quad \rho = \left(\frac{M}{RT}\right)P$$

Therefore, slope of ρ-P graph is

$$\frac{M}{RT} \propto \frac{1}{T}$$

So, for lesser slope, temperature has to be more.

$$\Rightarrow \quad T_B > T_A$$

Hence, the correct answer is (B).

162. Let T be the temperature of the mixture. Then

$$U = U_1 + U_2$$

$$\Rightarrow \quad \frac{f}{2}(n_1 + n_2)RT = \frac{f}{2}(n_1)RT_0 + \frac{f}{2}(n_2)(R)(2T_0)$$

$$\Rightarrow \quad (2+4)T = 2T_0 + 8T_0 \qquad (n_1 = 2,\ n_2 = 4)$$

$$\Rightarrow \quad T = \frac{5}{3}T_0$$

Hence, the correct answer is (A).

163. To 80 cm mark on the aluminium rod is really at a greater distance from the zero position than indicated because of the increase in temperature $\Delta T = 40\ ^\circ\text{C}$. The increased length is

$$\Delta L = \alpha_{A\ell} L_{A\ell} \Delta T = (2.5 \times 10^{-5})(80)(40) = 0.08 \text{ cm}$$

So, the correct length of the line is

$$L = 80 + 0.08 = 80.08 \text{ cm}$$

Hence, the correct answer is (A).

164. Since, $\frac{(Q/t)_1}{(Q/t)_2} = \left(\frac{r_1}{r_2}\right)^2 \frac{\ell_2}{\ell_1}$

$$\Rightarrow \quad \text{Ratio} = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

Hence, the correct answer is (C).

165. Due to Kirchhoff's Laws
Hence, the correct answer is (A).

166. Since, $dP = g\rho dy$

$$\Rightarrow \quad \int_{P_1}^{P_2} \frac{dP}{P} = \frac{Mg}{RT}$$

Also, $H = \int_0^H dy$

$$\Rightarrow \quad H = \frac{RT}{Mg} \ln \frac{P_1}{P_2}$$

Hence, the correct answer is (A).

167. Since $\frac{Q}{t} = kA\frac{\Delta T}{\ell}$, so $t \propto \frac{\ell}{A}$

$$\Rightarrow \quad \frac{t_1}{t_2} = \frac{\ell_1}{\ell_2}\frac{A_2}{A_1} = 4$$

$$\Rightarrow \quad t_2 = \frac{t_1}{4} = \frac{4}{4} = 1 \text{ min}$$

Hence, the correct answer is (A).

168. $dU = C_v dT = \left(\frac{5}{2}R\right)dT$

$$\Rightarrow \quad dT = \frac{2(dU)}{5R} \qquad \ldots(1)$$

From First Law of Thermodynamics, we have

$$dU = dQ - dW$$

$$\Rightarrow \quad dU = Q - \frac{Q}{4} = \frac{3Q}{4}$$

Now molar heat capacity is $C = \frac{dQ}{dT} = \frac{Q}{2(dU)}$

$$\Rightarrow \quad C = \frac{5RQ}{2\left(\frac{3Q}{4}\right)} = \frac{10}{3}R$$

Hence, the correct answer is (C).

169. Since volume is constant, so we have

$$\frac{P_2}{T_2} = \frac{P_1}{T_1}$$

When temperature of gas is doubled, then we get

$$P_2 = 2P_1 = 2P_0$$

Now, tension F in the wire is equal to net force on the piston given by

$$F = (2P_0 - P_0)A = (1.013 \times 10^5)(1) = 10^5 \text{ N}$$

Hence, the correct answer is (B).

170. Since, $\frac{dL}{L_0} = \alpha(T)dT$

$$\Rightarrow \quad \int_{L_0}^{L} dL = L_0 \int_{T_0}^{T} \alpha(T)dT$$

$$\Rightarrow \quad L - L_0 = L_0 \int_{T_0}^{T} \alpha(T)dT$$

$$\Rightarrow \quad L = L_0\left[1 - \int_{T}^{T_0} \alpha(T)dT\right]$$

Hence, the correct answer is (B).

171. Since $t \propto x_2^2 - x_1^2$

$$\Rightarrow \quad t = \beta\left(x_2^2 - x_1^2\right)$$

where β is just a constant of proportionality

$$\Rightarrow \quad \frac{t_2}{t_1} = \frac{4-1}{1-0}$$

$$\Rightarrow \quad t_2 = 21 \text{ hour} > 14 \text{ hour}$$

Hence, the correct answer is (D).

172. Since all have same k (as they are made of same material)

So, the rod which has maximum $\frac{Q}{t}$ will conduct the most. Here

$$\frac{Q}{t} \propto \frac{r^2}{\ell}$$

Hence, the correct answer is (A).

173. Since both the slabs are in series and hence $\frac{Q}{t}$ is same through both. Hence, required ratio is 1 : 1.

Hence, the correct answer is (D).

174. Average KE of each molecule is $E_{av} = E = \frac{3}{2}k_BT$

If N_1 is the number of molecules of hydrogen and N_2 is the number of molecules of oxygen, then

Total KE of hydrogen is $E_1 = N_1\left(\frac{3}{2}k_BT\right)$

Total KE of oxygen is $E_2 = N_2\left(\frac{3}{2}k_BT\right)$

$$\Rightarrow \quad \frac{E_1}{E_2} = \frac{N_1}{N_2} = 2$$

Hence, the correct answer is (B).

175. Under steady state condition, heat released to the room = heat dissipated out of the room.

Let θ be the temperature of heater. Then

$$\theta - 20 = \alpha[20 - (-20)] \qquad \ldots(1)$$

and $\theta - 10 = \alpha[10 - (-40)]$ $\ldots(2)$

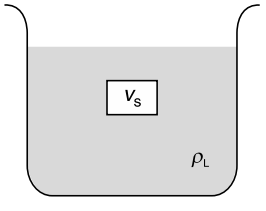
Solving equations (1) and (2), we get

$\theta = 60\ ^\circ C$

Hence, the correct answer is (D).

176. Upthrust, $W_0 = V_s\rho_L g$

where, V_s is volume of solid at 0 °C, ρ_L is density of liquid at 0 °C. As the temperature increases, V_s increases and ρ_L decreases.



Since, $W_0 \propto V_s\rho_L$

$$\Rightarrow \quad \frac{W}{W_0} = \frac{V_s'\rho_L'}{V_s\rho_L} = \frac{(V_s + \Delta V_s)}{V_s} \times \frac{\rho_L(1+\gamma_L\Delta T)^{-1}}{\rho_L}$$

$$\Rightarrow \quad \frac{W}{W_0} = (1+\gamma_s\Delta T)(1-\gamma_L\Delta T) \qquad \text{(Using binomial)}$$

$$\Rightarrow \quad W = W_0(1+\gamma_s\Delta T - \gamma_L\Delta T) = W_0[1+(\gamma_S - \gamma_L)\Delta T]$$

Hence, the correct answer is (B).

177. Since, $\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$

$$\Rightarrow \quad Q_2 = \left(\frac{T_2}{T_1}\right)Q_1 = \left(\frac{300}{400}\right)600 = 450 \text{ cal}$$

Hence, the correct answer is (D).

178. Since $PV = nRT$

$$\Rightarrow \quad \text{Slope} = \frac{P}{T} = \frac{nR}{V}$$

The more the volume, the lesser the slope and vice-versa.

Hence, the correct answer is (B).

179. Given that, $\eta = 40\%$ and $T_2 = 47 + 273 = 320$ K

Since $\eta = 1 - \frac{T_2}{T_1}$, so $\frac{40}{100} = 1 - \frac{320}{T_1}$

$$\Rightarrow \quad \frac{320}{T_1} = 0.6$$

$$\Rightarrow \quad T_1 = \frac{3200}{6} = 533 \text{ K} = 260\ ^\circ C$$

Hence, the correct answer is (A).

180. Since $P_1 = P_2$

$$\Rightarrow \quad \sigma A_1T_1^4 = \sigma A_2T_2^4$$

$$\Rightarrow \quad 4\pi r_1^2T_1^4 = 4\pi r_2^2T_2^4$$

$$\Rightarrow \quad \frac{r_1}{r_2} = \left(\frac{T_2}{T_1}\right)^2$$

Hence, the correct answer is (B).

181. Since, $\Delta U = nC_v\Delta T$

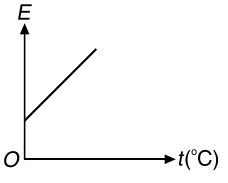
$$\Rightarrow \quad \Delta U = n\left(\frac{R}{\gamma - 1}\right)(T_f - T_i) = \frac{nRT_f - nRT_i}{\gamma - 1}$$

$$\Rightarrow \quad \Delta U = \frac{P(2V) - P(V)}{\gamma - 1} = \frac{PV}{\gamma - 1}$$

Hence, the correct answer is (C).

182. Mean KE of each molecule, $E_K = \frac{3}{2}k_BT = \frac{3}{2}k_B(t + 273)$

At 0 °C or 273 K, the molecules will possess some kinetic energy. So, the graph does not start from origin.



Hence, the correct answer is (C).

183. Let R be radius of sphere, V its volume and ρ its density, then $\Delta R = R\alpha\Delta T$. So, percentage change is

$$\frac{\Delta R}{R} \times 100 = 100\alpha\Delta T \qquad \ldots(1)$$

Also, $\Delta V = V\gamma\Delta T = V(3\alpha)\Delta T$, so percentage change is

$$\frac{\Delta V}{V} \times 100 = 300\alpha\Delta T \qquad \ldots(2)$$

Since, $\rho' = \frac{\rho}{1+\gamma\Delta T} = \frac{\rho}{1+3\alpha\Delta T}$

$$\Rightarrow \quad \Delta\rho = \rho - \rho'$$

CHAPTER 2

$$\Rightarrow \quad \Delta\rho = \rho\left(1 - \frac{1}{1+3\alpha\Delta T}\right) = \frac{(3\alpha\Delta T)\rho}{1+3\alpha\Delta T}$$

So, percentage change in density is $\frac{\Delta\rho}{\rho}\times 100$

$$\Rightarrow \quad \frac{\Delta\rho}{\rho}\times 100 = \frac{300\alpha\Delta T}{1+3\alpha\Delta T} \qquad \ldots(3)$$

From equations (1), (2) and (3), we see that percentage change is maximum in volume.

Hence, the correct answer is (B).

184. Since, $dQ = nC_p dT$

$$\Rightarrow \quad C_p = \frac{dQ}{ndT}$$

$$\Rightarrow \quad C_P = \frac{70}{(2)(5)} = \frac{7}{2}(R)$$

i.e., the gas is diatomic or it may be H_2.

Hence, the correct answer is (A).

185. We use $\Delta l = \alpha l \Delta T$

For rod A, $0.075 = 20\alpha_A(100-0)$

$$\Rightarrow \quad \alpha_A = 3.75\times 10^{-5}\ ^\circ\text{C}^{-1}$$

For rod B, $0.045 = 20\alpha_B(100-0)$

$$\Rightarrow \quad \alpha_B = 2.25\times 10^{-5}\ ^\circ\text{C}^{-1}$$

Let portion of metal A is l cm and that of metal B is $(20-l)$ cm

$$\Delta l_A + \Delta l_B = 0.06 \text{ cm}$$

$$\Rightarrow \quad \alpha_A l(100) + \alpha_B(20-l)(100) = 0.06$$

$$\Rightarrow \quad 3.75\times 10^{-5} l + 4.5\times 10^{-4} - 2.25\times 10^{-5} l = 6\times 10^{-4}$$

$$\Rightarrow \quad 1.5\times 10^{-5} l = 1.5\times 10^{-4}$$

$$\Rightarrow \quad l = 10 \text{ cm}$$

Hence, the correct answer is (B).

186. Since, $\frac{0.1}{t_1} \propto \left(\frac{50+49.9}{2} - 30\right)$

and $\frac{0.1}{t_2} \propto \left(\frac{40+39.9}{2} - 30\right)$

$$\Rightarrow \quad t_2 = 10 \text{ s}$$

Hence, the correct answer is (B).

187. Let temperature of the junction be T, then in steady state, thermal current through steel equals the thermal current through copper, so we have

$$K_{\text{steel}}(2A)\left(\frac{300-T}{15}\right) = K_{\text{copper}}(A)\left(\frac{T-0}{10}\right)$$

$$(50.2)(2)\left(\frac{300-T}{15}\right) = 385\left(\frac{T}{10}\right)$$

$$\Rightarrow \quad T \approx 44\ ^\circ\text{C}$$

Hence, the correct answer is (A).

188. Let m gram of steam be required for the purpose.
By Law of Calorimetry

$$\begin{pmatrix}\text{Total heat lost}\\ \text{by steam to become}\\ \text{water at } 0\ ^\circ\text{C}\end{pmatrix} = \begin{pmatrix}\text{Total heat gained}\\ \text{by ice to become}\\ \text{water at } 0\ ^\circ\text{C}\end{pmatrix}$$

$$\Rightarrow \quad 540m + m(1)(100-0) = 3200(0.5)(10) + 3200(80)$$

$$\Rightarrow \quad 640m = 272000$$

$$\Rightarrow \quad m = 425\ g$$

Hence, the correct answer is (C).

189. Energy received per second by the planet is $\frac{P}{4\pi d^2}(\pi R^2)$

where P is power radiated by the sun and R is the radius of the planet.

Further, energy radiated per second by the planet (according to Stefan's Law) is $\sigma(4\pi R^2)T_0^4$. For thermal equilibrium to exist, we get

$$\frac{P}{4\pi d^2}(\pi R^2) = \sigma(4\pi R^2)T_0^4$$

$$\Rightarrow \quad T_0^4 \propto d^{-2}$$

$$\Rightarrow \quad T_0 \propto d^{-1/2} \qquad \{\text{OPTION (D)}\}$$

Hence, the correct answer is (D).

190. Since $V = kT^{2/3}$, so $TV^{-3/2} = \text{constant}$

For a polytropic process $TV^{x-1} = \text{constant}$

$$\Rightarrow \quad x - 1 = -\frac{3}{2}$$

$$\Rightarrow \quad x = -\frac{1}{2}$$

Work done in a polytropic process is

$$W = \frac{nR}{1-x}(T_2 - T_1)\text{, where } x = -\frac{1}{2}$$

$$\Rightarrow \quad W = \frac{nR}{1+(1/2)}(30) = 20R$$

Hence, the correct answer is (B).

191. From Kinetic Theory of Gases, $p = \frac{2}{3}\left(\frac{E}{V}\right)$

$$\Rightarrow \quad E = \frac{3}{2}pV$$

Hence, the correct answer is (D).

192. Efficiency of generator is,

$$\eta = 0.9 = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{P_{\text{out}}}{1000}$$

$$\Rightarrow \quad P_{\text{out}} = 900 \text{ W}$$

$$\Rightarrow \quad 900 = mc\Delta T + mL$$

$$\Rightarrow \quad 900 = m(4200)(100-20) + m(2250\times 10^3)$$

$$\Rightarrow \quad m = \frac{900}{336000 + 2250000} = 0.00035 \text{ kg}$$

$$\Rightarrow \quad m = 0.35 \text{ g}$$

Hence, the correct answer is (C).

193. Internal energy of an ideal gas depends on temperature only and change in internal energy.

$\Delta U = nC_V\Delta T$ is applicable for all processes.

Hence, the correct answer is (B).

194. $W = \int PdV = a\int_{V_1}^{V_2}\frac{dV}{V} + b\int_{V_1}^{V_2} dV$

$\Rightarrow \quad W = a\log_e\left(\frac{V_2}{V_1}\right) + b(V_2 - V_1)$

Hence, the correct answer is (D).

195. According to *FLTD*, we have

$\Delta U = Q - W$

Also, work done W equals area under the curve

$\Rightarrow \quad W = \begin{pmatrix}\text{Area of Half}\\ \text{Ellipse i.e. } \pi ab/2\end{pmatrix} + \begin{pmatrix}\text{Area of}\\ \text{Rectangle}\end{pmatrix}$

$\Rightarrow \quad W = \left[\frac{\pi}{2}\times(7-4.2)\times10^{-3}\times(500-300)\times10^3\right] + \left[(7-1.4)\times10^{-3}\times300\times10^3\right]$ J

$\Rightarrow \quad W = 2.56\times10^3$ J

$\Rightarrow \quad \Delta U = 5.76\times10^3 - 2.56\times10^3 = 3.2\times10^3$ J

Hence, the correct answer is (A).

196. Since $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

T is doubled and M is halved. Therefore, r.m.s. speed will become two times or 600 ms^{-1}.

Hence, the correct answer is (C).

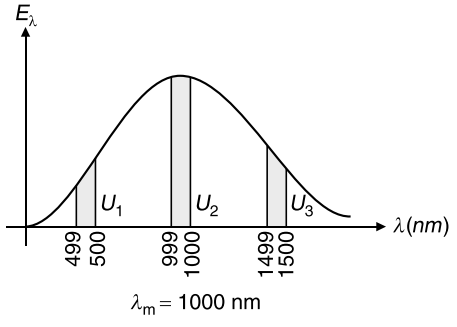
197. Wein's Displacement Law is

$\lambda_m T = b$ {b = Wein's constant}

$\Rightarrow \quad \lambda_m = \frac{b}{T} = \frac{2.88\times10^6 \text{ nm-K}}{2880 \text{ K}}$

$\Rightarrow \quad \lambda = 1000$ nm

Energy distribution with wavelength will be as follows

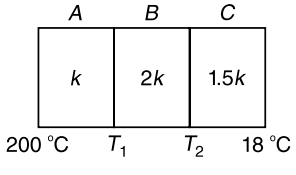


From the graph it is clear that

$U_2 > U_1$ {In fact U_2 is maximum}

Hence, the correct answer is (D).

198. $\frac{kA(200-T_1)}{\ell} = \frac{2kA(T_1-T_2)}{\ell} = \frac{1.5kA(T_2-18)}{\ell}$



$\Rightarrow \quad 200 - T_1 = 2T_1 - 2T_2$

$\Rightarrow \quad 3T_1 - 2T_2 = 200$ …(1)

Further, $200 - T_1 = \frac{3}{2}(T_2 - 18)$

$\Rightarrow \quad 2T_1 + 3T_2 = 454$ …(2)

Solving (1) and (2), we get

$T_1 = 116\ ^\circ C$

Hence, the correct answer is (D).

199. The change in internal energy for any process is

$\Delta U = nC_V\Delta T$

(Do not develop a MISCONCEPTION that $\Delta U = nC_V\Delta T$ only for an isochoric process)

Hence, the correct answer is (D).

200. Let the initial pressure of the three samples be P_A, P_B and P_C, then

$P_A(V)^{3/2} = (2V)^{3/2}P$

$\Rightarrow \quad P_B = P$

$P_C(V) = P(2V)$

$\Rightarrow \quad P_A : P_B : P_C = (2)^{3/2} : 1 : 2 = 2\sqrt{2} : 1 : 2$

Hence, the correct answer is (B).

201. Since, we know that $\frac{\Delta Q}{\Delta t} = \frac{KA(T_1 - T_2)}{L}$

In case of sphere A, we get

$$\frac{\frac{4}{3}\pi(20)^3\times\rho_{\text{ice}}\times L_{\text{ice}}}{80\times60} = \frac{K_A\times4\pi\times(20)^2(T_1-T_2)}{0.2} \quad \text{…(1)}$$

In case of sphere B, we get

$$\frac{\frac{4}{3}\pi(30)^3\times\rho_{\text{ice}}\times L_{\text{ice}}}{t\times60} = \frac{K_B\times4\pi\times(30)^2(\theta_1-\theta_2)}{0.1} \quad \text{…(2)}$$

Dividing (1) by (2), we get

$\frac{(20/80)}{(30/t)} = \frac{K_A}{2K_B} = \frac{1}{4}$

$\Rightarrow \quad t = 30$ minutes

Hence, the correct answer is (A).

202. $n = \frac{PV}{RT} = \frac{2\times10^5\times10^{-3}}{8.3\times300} = 0.080$

$\Rightarrow \quad n = \frac{m}{M} = 0.080$

$\Rightarrow \quad m = (0.080)(4\text{ g}) = 0.32$ g

Hence, the correct answer is (B).

203. Since, $(3L)\alpha_{\text{eff}}\Delta\theta = L\alpha\Delta\theta + 2L(2\alpha)(\Delta\theta)$

$\Rightarrow \quad \alpha_{\text{eff}} = \frac{5}{3}\alpha$

Hence, the correct answer is (C).

CHAPTER 2

204. By Law of Calorimetry, heat lost by steam must be equal to heat gained by water. Since, find temperature of water is 60 °C, so we have

$$(8)L_{\text{steam}} + (8)c_{\text{water}}(100-60) = (80)c_{\text{water}}(60-10)$$

$$\Rightarrow \quad (8)L_{\text{steam}} + (8)(1)(100-60) = (80)(1)(60-10)$$

$$\Rightarrow \quad (8)L_{\text{steam}} = 4000 - 320 = 3680$$

$$\Rightarrow \quad L_{\text{steam}} = 460 \text{ calg}^{-1}$$

Hence, the correct answer is (A).

205. Conceptual

Hence, the correct answer is (B).

206. $\lambda_m T = \text{constant}$

$$\Rightarrow \quad 510T_1 = 350T_2$$

$$\Rightarrow \quad \frac{T_1}{T_2} = 0.69$$

Hence, the correct answer is (B).

207. $$\%\text{age} = \frac{dU}{dQ} \times 100 = \frac{nC_V dT}{nC_P dT} \times 100$$

$$\Rightarrow \quad \%\text{age} = \frac{C_V}{C_P} \times 100 = \frac{3}{5} \times 100 = 60\%$$

Remaining $\%\text{age} = (100-60)\% = 40\%$

Hence, the correct answer is (C).

208. The ratio of rms speeds of molecules of gas A to that of molecules of gas B is

$$\frac{v_A}{v_B} = \sqrt{\frac{M_B}{M_A}} = \sqrt{3}$$

Since $v_A^2 = v_x^2 + v_y^2 + v_z^2$ and by postulates of KTG, we have $v_x = v_y = v_z$

$$\Rightarrow \quad v_x^2 = v_y^2 = v_z^2 = \frac{v_A^2}{3} = \frac{1}{3}\left(3v_B^2\right)$$

$$\Rightarrow \quad v_x^2 : v_B^2 = 1:1$$

$$\Rightarrow \quad \frac{v^2}{V^2} = 1$$

Hence, the correct answer is (D).

209. Since, $P \propto V$ $\quad \{\because P = aV\}$

Therefore, pressure and volume both are doubled or temperature becomes four times (as $T \propto PV$) or 1200 K. So, change in temperature is 900 K.

Hence, the correct answer is (A).

210. Heat released when 20 g of water cools from 20 °C to 0 °C is

$$Q_1 = m_{\text{water}} c_{\text{water}} \Delta T = (20)(1)(20) = 400 \text{ cal}$$

Heat required to raise temperature of 20 g of ice from –10 °C to 0 °C is

$$Q_2 = m_{\text{ice}} c_{\text{ice}} \Delta T = (20)(0.5)(10) = 100 \text{ cal}$$

Heat required to melt 20 g of ice at 0 °C is

$$Q_3 = m_{\text{ice}} L_{\text{ice}} = (20)(80) = 1600 \text{ cal}$$

Since $Q_1 > Q_2$, so temperature of ice will reach 0 °C. However, we observe that $Q_1 < Q_2 + Q_3$, therefore the complete ice will not melt and final mixture will have both ice and water. The amount of ice melted m is

$$m = \frac{\text{Available Heat}}{\text{Latent Heat}} = \frac{400-100}{80} = 3.75 \text{ g}$$

So in equilibrium, the calorimeter will have the mass of ice and water given by

$$m'_{\text{ice}} = 20 - 3.75 = 16.25 \text{ g}$$

$$m'_{\text{water}} = 20 + 3.75 = 23.75 \text{ g}$$

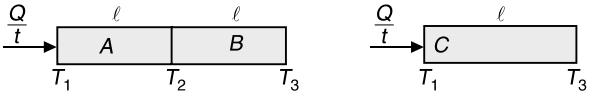
Hence, the correct answer is (A).

211. $$\eta = 1 - \frac{300}{600} = 0.5$$

Hence, the correct answer is (C).

212. Since $$\frac{Q}{t} = k_{\text{eq}} A\left(\frac{T_1 - T_3}{\ell}\right)$$

$$\Rightarrow \quad \frac{Q}{t} = \frac{2k_A k_B}{k_A + k_B} A\left(\frac{T_1 - T_3}{2\ell}\right) \qquad \ldots(1)$$



Also, $$\frac{Q}{t} = k_C A\left(\frac{T_1 - T_3}{\ell}\right) \qquad \ldots(2)$$

From (1) and (2), we get $k_C = \dfrac{k_A k_B}{k_A + k_B}$

Hence, the correct answer is (B).

213. For Isobaric Process

$$W_{\text{isob}} = P(V_2 - V_1) = P(2V - V) = PV$$

For Isothermal Process

$$W_{\text{isot}} = nRT \ln\left(\frac{V_2}{V_1}\right) = nRT \ln(2) = PV \ln(2)$$

For Adiabatic Process

$$W_{\text{ad}} = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$$

Since $PV^\gamma = P_2(2V)^\gamma$, so $P_2 = P/2^\gamma$

$$W_{\text{ad}} = \frac{PV - \left(P/2^\gamma\right)(2V)}{\gamma - 1} = \frac{PV\left(1 - 2^{1-\gamma}\right)}{\gamma - 1}$$

Hence, the correct answer is (C).

214. Coefficient of performance

$$\beta = \frac{\text{Heat Extracted } (Q_2) \text{ From Sink}}{\text{External Work Input } (W)} = \frac{T_2}{T_1 - T_2}$$

$$\Rightarrow \quad \frac{Q_2}{W} = \frac{(1 \text{ g})\left(80 \text{ calg}^{-1}\right)}{W} = \frac{273}{27}$$

$$\Rightarrow \quad W = 7.9 \text{ cal}$$

Hence, the correct answer is (A).

215. When the vessel is suddenly stopped the ordered motion of gas is converted into its disordered motion i.e., internal energy of the gas is increased. Hence,

$$\Delta U = \frac{1}{2}mv^2 \qquad \{\because\ m = nM\}$$

$$\Rightarrow\quad nC_v\Delta T = \frac{1}{2}(nM)v^2$$

where n is the number of moles of the gas.

$$\Rightarrow\quad \Delta T = \frac{Mv^2}{2C_v}$$

$$\Rightarrow\quad \Delta T = \frac{Mv^2}{2\left(\frac{R}{\gamma-1}\right)} \qquad \left\{\because\ C_v = \frac{R}{\gamma-1}\right\}$$

$$\Rightarrow\quad \Delta T = \frac{Mv^2(\gamma-1)}{2R}$$

Hence, the correct answer is (B).

216. Since $dQ = mcdT$

$$\Rightarrow\quad Q = \int dQ = m\alpha\int_1^3 T^3dT$$

$$\Rightarrow\quad Q = (4\alpha)\frac{T^4}{4}\Big|_1^3 = \alpha(3^4 - 1^4) = 80\alpha$$

Hence, the correct answer is (C).

217. Given that, $U = a + bPV = a + b(nRT) \qquad \{\because\ PV = nRT\}$

Since, $C_V = \frac{1}{n}\frac{dU}{dT} = bR$

$$\Rightarrow\quad C_P = C_V + R = bR + R$$

$$\Rightarrow\quad \gamma = \frac{C_P}{C_V} = \frac{b+1}{b}$$

Hence, the correct answer is (A).

218. Solar power received by earth is 1400 Wm^{-2}

Solar power received by 0.2 m^2 area of earth is

$$P = \left(1400\ \text{Wm}^{-2}\right)\left(0.2\ \text{m}^2\right) = 280\ \text{W}$$

Mass of ice = 280 g = 0.28 kg

Heat required to melt ice is

$$Q = (0.28)(3.3\times10^5) \qquad \{\because\ Q = mL\}$$

$$\Rightarrow\quad Q = 9.24\times10^4\ \text{J}$$

$$\Rightarrow\quad t = \frac{9.28\times10^4}{280} = 331\ s \approx 330\ s$$

Hence, the correct answer is (B).

219. Since $C_P - C_V = R$, so $\frac{C_P}{C_V} - 1 = \frac{R}{C_V}$

$$\Rightarrow\quad C_V = \frac{R}{\gamma-1} \text{ and } C_P = \frac{\gamma R}{\gamma-1}$$

Hence, the correct answer is (B).

220. At constant pressure, $\frac{V_1}{T_1} = \frac{V_2}{T_2}$

$$\Rightarrow\quad \frac{1}{400} = \frac{V_2}{300}$$

$$\Rightarrow\quad V_2 = 0.75\ \text{L}$$

$$\Rightarrow\quad \Delta V = 0.75 - 1 = -0.25\ \text{L} = -250\ \text{mL}$$

Hence, the correct answer is (D).

221. Since, $PV = nRT$

$$\Rightarrow\quad (2P)V = n'RT$$

$$\Rightarrow\quad n' = 2n$$

Hence, the correct answer is (B).

222. Since $\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$, so $\frac{300}{150} = \frac{500}{T_2}$

$$\Rightarrow\quad T_2 = 250\ \text{K}$$

Hence, the correct answer is (C).

223. Rate of flow of heat through the cork is

$$\frac{Q}{t} = \frac{KA(T_1 - T_2)}{L}$$

$$\Rightarrow\quad \frac{Q}{t} = \frac{0.008\times100\times(30-0)}{4} = 6\ \text{cals}^{-1}$$

Therefore, time taken to melt 200 g of ice is

$$t = \frac{mL_{\text{ice}}}{Q/t} = \frac{(200)(80)}{6} = \frac{8000}{3}\ \text{s}$$

$$\Rightarrow\quad t \approx 45\ \text{min}$$

Hence, the correct answer is (A).

224. According to Newton's Law of Cooling Rate of Cooling $R \propto$ Temperature Difference

Hence, $R = K(\theta - \theta_0)$

This is an equation of a straight line.

Hence, the correct answer is (B).

225. Work done in the cycle is

$$W = (\text{Area of rectangle } ABCD)$$

$$\Rightarrow\quad W = (2P_0 - P_0)(2V_0 - V_0) = P_0V_0$$

Heat added in the process A to B is

$$Q_{AB} = nC_V\Delta T = n\left(\frac{3R}{2}\right)\Delta T = \frac{3}{2}(2P_0V_0 - P_0V_0)$$

$$\Rightarrow\quad Q_{AB} = \frac{3}{2}P_0V_0$$

Heat added in the process B to C is

$$Q_{BC} = nC_P\Delta T = n\left(\frac{5R}{2}\right)\Delta T = \frac{5}{2}(4P_0V_0 - 2P_0V_0)$$

$$\Rightarrow\quad Q_{BC} = 5P_0V_0$$

Efficiency η is

$$\eta = \frac{W}{Q_{\text{input}}} = \frac{P_0V_0}{\frac{3}{2}P_0V_0 + 5P_0V_0} = \frac{2}{13}$$

$$\Rightarrow\quad \%\ \eta = \frac{200}{13} = 15.4\%$$

Hence, the correct answer is (A).

226. 100 °C $\equiv 150^o$ of the thermometer

$$\Rightarrow\quad 1\ °\text{C} = 1.5° \text{ of the thermometer}$$

$$\Rightarrow\quad 34\ °\text{C} = 51\ °\text{C of the thermometer}$$

CHAPTER 2

So, thermometer will read $-20° + 51° = 31°$

Hence, the correct answer is (A).

227. Since $E = \sigma(673)^4$ and $2E = \sigma T^4$

$\Rightarrow \quad T = 2^{\frac{1}{4}}(673) = 1.18(673) \approx 800$ K

Hence, the correct answer is (D).

228. Since, the gas is expanding isobarically, so $Q = nC_P\Delta T$

$\Rightarrow \quad 3.5RT_0 = 10\left(\frac{7R}{2}\right)(T - T_0)$

$\Rightarrow \quad T_0 = 10T - 10T_0$

$\Rightarrow \quad T = 1.1T_0$

According to Charle's Law, we have $\frac{V}{1.1T_0} = \frac{V_0}{T_0}$

$\Rightarrow \quad V = 1.1V_0$

Hence, the correct answer is (A).

229. $P \propto T$ (at constant volume)

Hence, the correct answer is (D).

230. Since, $v_{O_2} = \frac{1}{2}v_{H_2}$

$\Rightarrow \quad \sqrt{\frac{3RT}{32}} = \frac{1}{2}\sqrt{\frac{3R(273)}{2}}$

$\Rightarrow \quad \frac{T}{32} = \frac{273}{8}$

$\Rightarrow \quad T = 4(273) = 1092$ K $= 819$ °C

Hence, the correct answer is (D).

231. A cooking pot must conduct more. Also, it must absorb less heat itself. Hence a good cooking pot has low specific heat and high conductivity.

Hence, the correct answer is (D).

232. Since $\frac{Q}{t} = kA\left(\frac{dT}{dx}\right)$, so $\frac{mL}{t} = kA\left(\frac{dT}{dx}\right)$

$\Rightarrow \quad k \propto \frac{1}{t}$ i.e., $\frac{k_1}{k_2} = \frac{t_2}{t_1}$

Hence, the correct answer is (B).

233. Since $Q_{AB} = nC_P\Delta T = n\frac{5}{2}R(2T_0 - T_0)$ and

$Q_{BC} = W_{BC} = NR \cdot 2T_0 \log_e 2$

$\Rightarrow \quad \frac{Q_{AB}}{W_{BC}} = \frac{5}{4\log_e 2}$

Hence, the correct answer is (C).

234. Let the length of the part of metal A be L (in cm). Then length of part of metal B will be $(20 - L)$ cm. Since the change in length is proportional to original length, so we have

$\frac{0.02}{20}L + \frac{0.04}{20}(20 - L) = 0.025$

$\Rightarrow \quad 0.02L + 0.8 - 0.04L = 0.5$

$\Rightarrow \quad L = 15$ cm

Hence, the correct answer is (C).

235. The molar specific heat of an ideal gas $C = \frac{dQ}{dT}$ can have any value lying between $-\infty$ and ∞

Hence, the correct answer is (D).

236. Adiabatic constant for the mixture is

$$\gamma_{mix} = \frac{n_1(C_P)_1 + n_2(C_P)_2}{n_1(C_V)_1 + n_2(C_V)_2}$$

$$\Rightarrow \quad \gamma_{mix} = \frac{(0.125)\left(\frac{7}{2}R\right) + (0.5)\left(\frac{5}{2}R\right)}{(0.125)\left(\frac{5}{2}R\right) + (0.5)\left(\frac{3}{2}R\right)}$$

$$\Rightarrow \quad \gamma_{mix} = \frac{0.4375 + 1.25}{0.3125 + 0.75} = \frac{27}{17}$$

For an adiabatic process, we have

$$T_2V_2^{\gamma-1} = T_1V_1^{\gamma-1}$$

$$\Rightarrow \quad T_2 = 390\left(\frac{2}{3}\right)^{\frac{10}{17}} \text{ K}$$

Hence, the correct answer is (A).

237. Since the length of the liquid column does not alter in the tube, hence we have superficial expansion and so $\gamma = 2\alpha$, instead of $\gamma = 3\alpha$.

Hence, the correct answer is (B).

238. Process A-B is an isothermal process i.e., $P \propto \frac{1}{V}$ and since, $\rho \propto \frac{1}{V}$, ρ-V graph will be a rectangular hyperbola. Pressure is increasing. Therefore, volume will decrease and hence, density will increase.

Process B-C is an isochoric process. Therefore,

V =constant and since $\rho = \frac{m}{V}$, density is also constant i.e., ρ-V graph is a dot.

Process C-D is inverse of A-B and D-A is inverse of B-C.

Hence, the correct answer is (B).

239. Let T_{faulty} be the reading of faulty thermometer and T_C be the correct reading on Centigrade scale, then

$$\frac{T_{faulty} - 10}{90 - 10} = \frac{T_C - 0}{100 - 0}$$

Given that $T_{faulty} = T_C = T$ (say)

$$\Rightarrow \quad \frac{T - 10}{90 - 10} = \frac{T - 0}{100 - 0}$$

$$\Rightarrow \quad 100(T - 10) = 80T$$

$$\Rightarrow \quad 20T = 1000$$

$$\Rightarrow \quad T = 50 \text{ °C}$$

Hence, the correct answer is (C).

240. Using $\frac{T_1 - T_2}{t} = \alpha\left(\frac{T_1 + T_2}{2} - \theta_0\right)$

In the first case

$$\frac{75 - 65}{2} = \alpha\left(\frac{75 + 65}{2} - 30\right) \qquad \ldots(1)$$

In the second case

$$\frac{55-45}{t}=\alpha\left(\frac{55+45}{2}-30\right) \qquad \ldots(2)$$

Dividing equations (1) by (2), we get $\frac{t}{2}=2$

$\Rightarrow \quad t=4$ min

Hence, the correct answer is (C).

241. Since, $H=(1)\left(\frac{3R}{2}\right)(10)=15R \qquad \{\because H=nC_V\Delta T\}$

Also, $H'=(2)\left(\frac{7R}{2}\right)(5)=35R \qquad \{\because H'=nC_P\Delta T'\}$

$\Rightarrow \quad H'=\frac{7}{3}(15R)=\frac{7}{3}H$

Hence, the correct answer is (D).

242. Let (P_1,V_1,T_1) be the initial pressure, volume and temperature of the gas. When the gas is first expanded isothermally, then V increases and P decreases, such that

$$(P_1,V_1,T_1)\xrightarrow{\text{isothermal}}(P_2,V_2,T_1)$$

When the gas is compressed adiabatically to its original volume, then V decreases and hence P and T both increase, such that

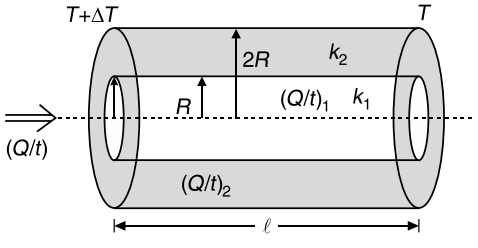
$$(P_2,V_2,T_1)\xrightarrow{\text{adiabatic}}(P_3,V_1,T_2)$$

To restore the gas to its original state, the simplest operation is

$$(P_3,V_1,T_2)\xrightarrow{\text{isochoric}}(P_1,V_1,T_1)$$

This simplest operation should decrease both pressure and temperature and hence it can be an isochoric cooling process.
Hence, the correct answer is (B).

243. Since $\frac{Q}{t}=\left(\frac{Q}{t}\right)_1+\left(\frac{Q}{t}\right)_2$



$\Rightarrow \quad k_{eq}\pi(2R)^2\frac{\Delta T}{\ell}=k_1(\pi R^2)\frac{\Delta T}{\ell}+k_2\left[\pi(2R)^2-\pi R^2\right]\frac{\Delta T}{\ell}$

$\Rightarrow \quad 4k_{eq}=k_1+3k_2$

$\Rightarrow \quad k_{eq}=\frac{k_1+3k_2}{4}$

Hence, the correct answer is (C).

244. $dQ=mcdT=0.1(100T+500)dT$

$\Rightarrow \quad Q=\int_{20}^{400}(10T+50)dT=\left[10\left(\frac{T^2}{2}\right)+50T\right]\Big|_{20}^{40}$

$\Rightarrow \quad Q=5(1600-400)+50(40-20)=7000$ J

Hence, the correct answer is (C).

245. P-V diagram of the gas is a straight line passing through origin. Hence, $P\propto V$

$\Rightarrow \quad PV^{-1}=$ constant

Molar heat capacity in process $PV^x=$ constant is

$$C=\frac{R}{\gamma-1}+\frac{R}{1-x}, \text{ where } \gamma=1.4 \text{ for diatomic gas}$$

$\Rightarrow \quad C=\frac{R}{1.4-1}+\frac{R}{1+1}=3R$

Hence, the correct answer is (C).

246. Given that, $L_i-L_b=0.1$ m $\qquad \ldots(1)$

$\Rightarrow \quad \Delta L_b-\Delta L_i=0$

$\Rightarrow \quad L_b\alpha_b=L_i\alpha_i$

$\Rightarrow \quad \frac{L_b}{L_i}=\frac{\alpha_i}{\alpha_b}=\frac{12}{18}=\frac{2}{3} \qquad \ldots(2)$

Solving (1) and (2), we get

$L_i=0.3$ m and $L_b=0.2$ m

Hence, the correct answer is (C).

247. Since, $Q=nC\Delta T \qquad \ldots(1)$

By FLTD, we have

$$\Delta U=Q-W=Q-\frac{Q}{4}=\frac{3Q}{4}$$

$\Rightarrow \quad \frac{3Q}{4}=nC_V\Delta T \qquad \ldots(2)$

Dividing (1) by (2), we get

$C=\frac{4}{3}C_V=\frac{4}{3}\left(\frac{5R}{2}\right)=\frac{10R}{3}$

Hence, the correct answer is (C).

248. Process 2 is an isothermal process

Hence, $\Delta U_2=0$

Process 1 is an isobaric $(P_1=$ constant$)$ expansion

Hence, temperature of the gas will increase

$\Rightarrow \quad \Delta U_1=$ positive

Process 3 is an adiabatic expansion. Hence, temperature will decrease

$\Rightarrow \quad \Delta U_3=$ negative

Therefore, $\Delta U_1>\Delta U_2>\Delta U_3$ is the correct option

Hence, the correct answer is (A).

249. Conceptual

Hence, the correct answer is (C).

250. Since, $PV=nRT=\frac{m}{M}RT$, where m is mass of the gas of molar mass M. The slope of T-P graph is

$$\frac{dT}{dP}=\tan\theta=\frac{MV}{mR}\propto\frac{1}{m}$$

Hence, the correct answer is (B).

CHAPTER 2

251. A is free to move, therefore, heat will be supplied at constant pressure.

$$dQ_A = nC_P dT_A \quad \ldots(1)$$

B is held fixed, therefore, heat will be supplied at constant volume.

$$dQ_B = nC_V dT_B \quad \ldots(2)$$

But $dQ_A = dQ_B$ {given}

$$\Rightarrow \quad nC_P dT_A = nC_V dT_B$$

$$\Rightarrow \quad dT_B = \left(\frac{C_P}{C_V}\right) dT_A$$

$$\Rightarrow \quad dT_B = \gamma(dT_A)$$

Since, $\gamma_{\text{diatomic}} = 1.4$ and $dT_A = 30$ K

$$\Rightarrow \quad dT_B = (1.4)(30\text{ K}) = 42\text{ K}$$

Hence, the correct answer is (D).

252. Specific heat $C = \dfrac{\Delta Q}{\Delta T} = \dfrac{1}{\Delta T}(\Delta U + W) = C_V + \dfrac{W}{\Delta T}$

For the given process, $W = 4V_0\left(\dfrac{9P_0}{2}\right) = 18P_0V_0$

Also, $\Delta T = T_2 - T_1 = \dfrac{(6P_0)(5V_0)}{R} - \dfrac{(3P_0)V_0}{R} = \dfrac{27P_0V_0}{R}$

and $C_V = \dfrac{3}{2}R$

$$\Rightarrow \quad C = \frac{3R}{2} + \frac{2R}{3} = \frac{13R}{6}$$

Hence, the correct answer is (B).

253. Since work is being done at the expense of internal energy so it must be an adiabatic process i.e., $dQ = 0$

$$\Rightarrow \quad n = \gamma$$

Hence, the correct answer is (C).

254. Since V = constant, so $W = 0$

$$\Rightarrow \quad Q = nC_V\Delta T$$

Now, $PV = nRT$

$$\Rightarrow \quad V\Delta P = nR\Delta T \quad \{V = \text{constant}\}$$

$$\Rightarrow \quad \Delta T = \left(\frac{V}{nR}\right)\Delta P$$

For both processes ΔP and number of moles are same. Since V is more for process CD, hence ΔT is more for process CD. So, $Q_{CD} > Q_{AB}$

Hence, the correct answer is (C).

255. Since, no heat flows through AB, so

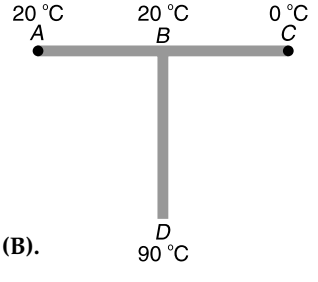
$$\theta_B = 20\ ^\circ\text{C}$$

Since, $H_{DB} = H_{BC}$

$$\Rightarrow \quad \frac{(90^\circ - 20^\circ)}{\ell_{BD}/KA} = \frac{(20^\circ - 0^\circ)}{\ell_{BC}/KA}$$

$$\Rightarrow \quad \frac{\ell_{BD}}{\ell_{BC}} = \frac{7}{2}$$

Hence, the correct answer is (B).



256. Since $\rho = \dfrac{PM}{RT}$, so $\rho \propto \dfrac{P}{T}$

Also, $\left(\dfrac{P}{T}\right)_A = \dfrac{P_0}{T_0}$ and $\left(\dfrac{P}{T}\right)_B = \left(\dfrac{3}{2}\right)\dfrac{P_0}{T_0}$

$$\Rightarrow \quad \left(\frac{P}{T}\right)_B = \frac{3}{2}\left(\frac{P}{T}\right)_A$$

$$\Rightarrow \quad \rho_B = \frac{3}{2}\rho_A = \frac{3}{2}\rho_0$$

Hence, the correct answer is (B).

257. According to Ideal Gas Equation, we have $PV = nRT$

$$\Rightarrow \quad n = \frac{PV}{RT}$$

Given that P, V, T are same, so, $n_1 = n_2 = n_3$

Also $v_{\text{rms}} = \sqrt{\dfrac{3RT}{M}}$

Since, $M_{\text{neon}} < M_{\text{chlorine}} < M_{\text{uranium hexafluoride}}$

$$\Rightarrow \quad v_1 > v_2 > v_3$$

Hence, the correct answer is (D).

258. Since, $\dfrac{\Delta L}{L} = \alpha\Delta T$

$$\Rightarrow \quad -\frac{1.1\times10^{-3}}{10} = 11\times10^{-6}\Delta T$$

$$\Rightarrow \quad \Delta T = T_f - T_i = -10$$

$$\Rightarrow \quad T_f = 27 - 10 = 17\ ^\circ\text{C}$$

Hence, the correct answer is (C).

259. In isochoric process V = constant and $P \propto T$

Therefore, P-T graph is a straight line passing through origin. But since

$$P = \left(\frac{nR}{V}\right)T$$

Slope of the straight line $\propto \dfrac{1}{V}$

$$(\text{Slope})_{12} < (\text{Slope})_{34}$$

$$\Rightarrow \quad V_2 > V_3$$

Also $V_1 = V_2$ and $V_3 = V_4$

Hence, the correct answer is (A).

260. **METHOD I**

Let resistance at 0 °C be R_0. Then

$$2 = R_0(1 + \alpha(20)) \quad \ldots(1)$$

$$4 = R_0(1 + \alpha(100)) \quad \ldots(2)$$

Dividing (2) by (1), we get

$$2 = \frac{1 + 100\alpha}{1 + 20\alpha}$$

$$\Rightarrow \quad 2 + 40\alpha = 1 + 100\alpha$$

$$\Rightarrow \quad \alpha = \frac{1}{60}\ ^\circ\text{C}^{-1}$$

If resistance is 2.2 Ω at T °C, then we have

$$2.2 = R_0(1+\alpha T) \qquad \ldots(3)$$

Dividing equation (3) by (1), we get

$$\frac{2.2}{2} = \frac{1+\alpha T}{1+20T} = \frac{1+\frac{T}{60}}{1+\frac{20}{60}}$$

$$\Rightarrow \quad (1.1)\left(\frac{4}{3}\right) = 1+\frac{T}{60}$$

$$\Rightarrow \quad T = 60(1.463-1) \approx 28\ ^\circ\text{C}$$

METHOD II

$$\frac{T-20}{100-20} = \frac{R_T - R_{20}}{R_{100}-R_{20}} = \frac{2.2-2}{4-2}$$

$$\Rightarrow \quad T = \left(\frac{0.2}{2}\times 80\right)+20 = 28\ ^\circ\text{C}$$

Hence, the correct answer is (D).

261. Conceptual

Hence, the correct answer is (B).

262. $(\text{Rate of Loss of Heat}) \propto (\text{Temperature Difference})$

$$\Rightarrow \quad \frac{dQ}{dt} \propto T - T_0$$

$$\Rightarrow \quad 10 = \beta(50-20)$$

$$\Rightarrow \quad \beta = \frac{1}{3} \quad (\text{where } \beta \text{ is just a constant of proportionality})$$

At average temperature of 35 °C we have

$$\frac{dQ}{dt} = \beta(35-20) = \frac{1}{3}(15) = 5\ \text{Js}^{-1}$$

So, heat lost in a time of one minute (= 60 second) is

$$Q_{\text{total}} = 5\times 60 = 300\ \text{J}$$

By definition, heat capacity is

$$c = \frac{Q_{\text{total}}}{\Delta T} = \frac{300}{0.2} = 1500\ \text{J}(^\circ\text{C})^{-1}$$

Hence, the correct answer is (B).

263. $\text{KE} = n\left(\frac{3}{2}RT\right)$

$$\Rightarrow \quad n_1\left(\frac{3}{2}RT_1\right) = n_2\left(\frac{3}{2}RT_2\right)$$

Since $n_2 = 2n_1$, so $T_1 = 2T_2$

Also $PV = nRT$ and since, V = constant, so $P \propto nT$

$$\Rightarrow \quad \frac{P_1}{P_2} = \left(\frac{n_1}{n_2}\right)\left(\frac{T_1}{T_2}\right) = \left(\frac{1}{2}\right)(2) = 1$$

Hence, the correct answer is (B).

264. Since $dS = \frac{dQ}{T}$

$$\Rightarrow \quad dS = \frac{dU}{T} + \frac{PdV}{T}$$

$$\Rightarrow \quad dS = nC_V\frac{dT}{T} + \frac{nRT}{V}\frac{dV}{T}$$

$$\Rightarrow \quad dS = nC_V\frac{dT}{T} + nR\frac{dV}{V}$$

$$\Rightarrow \quad \int_{S_1}^{S_2} dS = nC_V\int_{T_1}^{T_2}\frac{dT}{T} + nR\int_{V_1}^{V_2}\frac{dV}{V}$$

$$\Rightarrow \quad S_2 - S_1 = \Delta S = nC_V\ell n\left(\frac{T_2}{T_1}\right) + nR\ell n\left(\frac{V_2}{V_1}\right)$$

Hence, the correct answer is (A).

265. Since $dU = 0$, so by First Law of Thermodynamics

$$dQ_{\text{Cyclic}} = dW_{\text{Cyclic}}$$

Process $B \to C$ is isochoric, so

$$\Rightarrow \quad dW_{B\to C} = 0 \qquad \{\because dV = 0\}$$

$$\Rightarrow \quad 5 = dW_{A\to B} + dW_{B\to C} + dW_{C\to A}$$

$$\Rightarrow \quad 5 = 10(2-1) + 0 + dW_{C\to A}$$

$$\Rightarrow \quad dW_{C\to A} = -5\ \text{J}$$

Hence, the correct answer is (A).

266. The volume of the liquid that overflows is $\Delta V = V\gamma_a\Delta T$

$$\Rightarrow \quad \frac{V}{100} = V\gamma_a(80)$$

$$\Rightarrow \quad \gamma_a = \frac{1}{8000} = 1.25\times 10^{-4}\ ^\circ\text{C}^{-1}$$

Hence, the correct answer is (A).

267. $W_{AB} = 0$ as V = constant

$$\Rightarrow \quad Q_{AB} = \Delta U_{AB} = 50\ \text{J} \qquad \{\text{given}\}$$

$$U_A = 1500\ \text{J}$$

$$\Rightarrow \quad U_B = (1500+50)\ \text{J} = 1550\ \text{J}$$

$$W_{BC} = -\Delta U_{BC} = -40\ \text{J} \qquad \{\text{given}\}$$

$$\Rightarrow \quad \Delta U_{BC} = 40\ \text{J}$$

$$\Rightarrow \quad U_C = (1550+40)J = 1590\ \text{J}$$

Hence, the correct answer is (A).

268. Since, $Q = mc_{\text{body}}\Delta T$, where $c_{\text{body}} = 1\ \text{calg}^{-1}{}^\circ\text{C}^{-1}$ and $\Delta T = (103-98.4)\ ^\circ\text{F} = 4.6\ ^\circ\text{F}$

Also we know that $\Delta C = \frac{5}{9}\Delta F$

So, a temperature increment of 4.6 °F equals a temperature increment of $(4.6)\left(\frac{5}{9}\ ^\circ\text{C}\right) = 2.56\ ^\circ\text{C}$

$$\Rightarrow \quad Q = (60000\ \text{g})\left(1\ \text{calg}^{-1}(^\circ\text{C})^{-1}\right)(2.56\ ^\circ\text{C})$$

$$\Rightarrow \quad Q = 153600\ \text{cal} \approx 154\ \text{kcal}$$

Hence, the correct answer is (B).

269. Given that, $\left(\frac{C_P}{C_V}\right)_{\text{mix}} = \frac{3}{2}$

Since $C_{P_{\text{mix}}} = \frac{n_1C_{P_1} + n_2C_{P_2}}{n_1+n_2}$ and $C_{V_{\text{mix}}} = \frac{n_1C_{V_1} + n_2C_{V_2}}{n_1+n_2}$

CHAPTER 2

$$\Rightarrow \quad \frac{n_1C_{P_1}+n_2C_{P_2}}{n_1C_{V_1}+n_2C_{V_2}}=\frac{3}{2}$$

$$\Rightarrow \quad \frac{n_1(5R/2)+n_2(7R/2)}{n_1(3R/2)+n_2(5R/2)}=\frac{3}{2}$$

$$\Rightarrow \quad 10n_1+14n_2=9n_1+15n_2$$

$$\Rightarrow \quad n_1=n_2$$

Hence, the correct answer is (C).

270. Since $Q=nC_p\Delta T=\frac{7}{2}nR\Delta T \quad \left\{\because C_p=\frac{7}{2}R\right\}$

and $\Delta U=nC_v\Delta T=\frac{5}{2}nR\Delta T \quad \left\{\because C_V=\frac{5}{2}R\right\}$

$$\Rightarrow \quad W=Q-\Delta U=nR\Delta T$$

$$\Rightarrow \quad Q:\Delta U:\Delta W=7:5:2$$

Hence, the correct answer is (C).

271. For the first process, we have $Q_1=\Delta U+W_1$

$$\Rightarrow \quad 8\times10^5=\Delta U+6.5\times10^5$$

$$\Rightarrow \quad \Delta U=1.5\times10^5 \text{ J}$$

For the second process, ΔU is same, therefore,

$$Q_2=\Delta U_2+W_2=\Delta U+W_2$$

$$\Rightarrow \quad 10^5=1.5\times10^5+W_2$$

$$\Rightarrow \quad W_2=-0.5\times10^5 \text{ J}$$

Hence, the correct answer is (A).

272. Since, $\rho=\frac{PM}{RT}$

Density ρ remains constant, when $\frac{P}{T}$ or volume remains constant

In graph (i) volume is decreasing hence, density is increasing while in graphs (ii) and (iii) volume is increasing hence, density is decreasing. Note that volume would had been constant in case the straight line in graph (iii) had passed through origin.

Hence, the correct answer is (C).

273. Since $C_P=\frac{3}{2}R$

$$\Rightarrow \quad C_V=\frac{1}{2}R \qquad \{\because C_P-C_V=R\}$$

$$\Rightarrow \quad \gamma=3$$

Since $\gamma=1+\frac{2}{f}$

$$\Rightarrow \quad f=1$$

Possible for a monatomic gas molecule moving in a straight fixed line (one degrees of freedom).

Hence, the correct answer is (B).

274. $\text{Thermal Resistance} = \text{R} = \frac{\text{Temperature Difference}}{\text{Thermal Current}}$

$$\Rightarrow \quad [\text{Thermal Resistance}]=\frac{K}{\left(\frac{ML^2T^{-2}}{T}\right)}$$

$$\Rightarrow \quad [R]=M^{-1}L^{-2}T^3K$$

Hence, the correct answer is (B).

275. Efficiency of an ideal heat engine is $\eta=1-\frac{T_2}{T_1}$

$$\Rightarrow \quad \frac{60}{100}=1-\frac{300}{T_2}$$

$$\Rightarrow \quad T_2=750 \text{ K}=477 \text{ °C}$$

Hence, the correct answer is (C).

276. Since, $P=\frac{\alpha T^2}{V} \qquad (P=\text{constant})$

$$\Rightarrow \quad V=\frac{\alpha T^2}{P}$$

$$\Rightarrow \quad dV=\left(\frac{2\alpha T}{P}\right)dT$$

$$\Rightarrow \quad W=\int_{T_0}^{2T_0}PdV=\int_{T_0}^{2T_0}(P)\left(\frac{2\alpha T}{P}\right)dT=3\alpha T_0^2$$

Hence, the correct answer is (D).

277. For an adiabatic process

$$dQ=0=dU+dW$$

$$\Rightarrow \quad 0=dU+PdV$$

From the given equation

$$dU=3(PdV+VdP)$$

$$\Rightarrow \quad 0=3(PdV+VdP)+PdV$$

$$\Rightarrow \quad 4P(dV)+3V(dP)=0$$

$$\Rightarrow \quad 4\left(\frac{dV}{V}\right)=-3\left(\frac{dP}{P}\right)$$

On integrating, we get

$$\ln(V^4)+\ln(P^3)=\text{constant}$$

$$\Rightarrow \quad PV^{4/3}=\text{constant}$$

$$\Rightarrow \quad \gamma=\frac{4}{3}$$

i.e., gas is polyatomic.

Hence, the correct answer is (C).

278. For a cyclic process, $\Delta U=0$ and hence $Q=W$

$$\Rightarrow \quad Q_{ab}+Q_{bc}+Q_{ca}=W_{ab}+W_{bc}+W_{ca}$$

Since the process ab is isochoric, so $W_{ab}=0$

Also, it is given that $W_{bc}=-50$ J

$$\Rightarrow \quad 100+0+Q_{ca}=0+(-50)+W_{ca}$$

$$\Rightarrow \quad Q_{ca}-W_{ca}=-150 \text{ J}$$

$$\Rightarrow \quad \Delta U_{ca}=Q_{ca}-W_{ca}=-150 \text{ J}$$

Hence, the correct answer is (B).

279. Volume of the gas is constant

$$\Rightarrow \quad \frac{T_2}{T_1}=\frac{P_2}{P_1}$$

$$\Rightarrow \quad \left(\frac{T_1+1}{T_1}\right)=\frac{1.01P_1}{P_1}$$

$$\Rightarrow \quad 0.01T_1=1$$

$\Rightarrow \quad T_1 = 100 \text{ K}$

Hence, the correct answer is (A).

280. Since, $P_1V = n_1RT_1$ and $P_2V = n_2RT_2$

$\Rightarrow \quad \frac{n_1R}{V} = \frac{P_1}{T_1}$ and $\frac{n_2R}{V} = \frac{P_2}{T_2}$

When the vessels are joined, then we have

$P(V+V) = (n_1+n_2)RT$

$\Rightarrow \quad \frac{P}{T} = \frac{(n_1+n_2)R}{2V} = \frac{1}{2}\left(\frac{P_1}{T_1} + \frac{P_2}{T_2}\right)$

Hence, the correct answer is (A).

281. $\frac{W}{Q} = \frac{Q - dU}{Q}$ $\{\because$ From First Law $dQ = dU + dW\}$

$\Rightarrow \quad \frac{W}{Q} = 1 - \frac{dU}{dQ} = 1 - \frac{nC_V dT}{nC_P dT} = 1 - \frac{C_V}{C_P}$

$\Rightarrow \quad \frac{dW}{dQ} = 1 - \frac{1}{\gamma}$

Hence, the correct answer is (B).

282. Since, $P = \frac{1}{3}\frac{mn}{V}v_{\text{rms}}^2$ …(1)

$\Rightarrow \quad P \propto m \propto v_{\text{rms}}^2$

Now, m is halved and v_{rms} is doubled, so P will become two times.

In equation (1) m is the mass of one gas molecule and n the total number of gas molecules.

Hence, the correct answer is (B).

283. Pressure is exerted on the wall due to collisions of molecules with the walls of the container. It is independent of the velocity of container and the frame of reference.

Hence, the correct answer is (A).

284. Since, $U - 0 = nC_V(T - 0)$

$\Rightarrow \quad U = nC_VT$

$\Rightarrow \quad U = \frac{nRT}{\gamma - 1} = \frac{PV}{\gamma - 1}$ $\{\because PV = nRT\}$

Hence, the correct answer is (C).

285. According to Stefan's Law $E = e\sigma T^4$

where e is the emissivity of body and $0 < e < 1$

$\Rightarrow \quad E = \frac{300}{60 \times 5 \times 10^{-4}} = 10000 \text{ Wm}^{-2}$

$\Rightarrow \quad 10^4 = e(5.67 \times 10^{-8})(1000)^4$

$\Rightarrow \quad 10^{-8} = e(5.67)$

$\Rightarrow \quad e = \frac{1}{5.67} \approx 0.18$

Hence, the correct answer is (B).

286. In process ABC, work done is

$W = P_0(2V_0 - V_0) = P_0V_0$

According to Ideal Gas Equation, we have

$\frac{P_AV_A}{T_A} = \frac{P_CV_C}{T_C}$

$\Rightarrow \quad \frac{P_0V_0}{T_0} = \frac{(0.5P_0)(2V_0)}{T_C}$

$\Rightarrow \quad T_C = T_A = T_0$

$\Rightarrow \quad \Delta U = nC_V\Delta T = (1)\left(\frac{3R}{2}\right)(T_C - T_A) = 0$

and $Q = \Delta U + W = 0 + P_0V_0 = RT_0$

Hence, the correct answer is (C).

287. For an Isotherm T = constant

$\Rightarrow \quad PV$ = constant

$\Rightarrow \quad PV$ vs V is a straight line. {OPTION **(A)**}

Similarly, S vs T is a straight line. {OPTION **(B)**}

V and P plotted together must give a rectangular hyperbola (PV = constant). P vs T will again be a straight line. {OPTION **(D)**}

Hence, the correct answer is (C).

288. $C_P - C_V = 2 \text{ cal mol}^{-1}(^\circ\text{C})^{-1}$

Hence, the correct answer is (A).

289. For ideal monatomic gas $C_P = \frac{5}{2}R$

$\Rightarrow \quad R = \frac{2}{5}C_P = 0.4C_P$

$\Rightarrow \quad n = 0.4$

Hence, the correct answer is (C).

290. $v_{\text{mps}} = \sqrt{\frac{2RT}{M}}$

$\Rightarrow \quad 1600 = \sqrt{\frac{2 \times 8.31 \times 300}{M}}$

$\Rightarrow \quad M \approx 2 \times 10^{-3} \text{ kg} = 2 \text{ g}$

Hence the gas is H_2

Hence, the correct answer is (C).

291. When, external work done equals heat supplied, then

$dU = 0$

$\Rightarrow \quad T$ = constant i.e., Isothermal process

Hence, the correct answer is (C).

292. $F = YA\alpha\Delta T = E(1)(\alpha)(1) = E\alpha$

Hence, the correct answer is (A).

293. Initial and final temperatures are same in the both the graphs. Therefore, ΔU is same. Also area under P-V graphs is same. Therefore, work done is same and hence heat exchanged is also same.

Hence, the correct answer is (A).

294. Area of the shell is $A = 4\pi R^2$

Rate of flow of heat is $P = k(4\pi R^2)\frac{T}{d}$

CHAPTER 2

$\Rightarrow \quad d = \dfrac{4\pi R^2 Tk}{P}$

Hence, the correct answer is (B).

295. $\dfrac{E_2}{E_1} = \dfrac{327+273}{27+273} = 2$

Hence, the correct answer is (D).

Multiple Correct Choice Type Questions

1. Potential energy of an ideal gas is zero because intermolecular forces between them is zero. Degree of freedom of an ideal diatomic gas is 5. In this 2 is rotational and 3 is translational. According to Law of Equipartition of Energy internal energy is equally distributed in all degrees of freedom.

Therefore, rotational kinetic energy $= \left(\dfrac{2}{2+3}\right) \times 100 = 40$ J

and translational kinetic energy $= \left(\dfrac{3}{2+3}\right) \times 100 = 60$ J.

Hence, (A), (B) and (C) are correct.

2. Internal energy (U) depends only on the initial and final status. Hence, ΔU will be same in all the three paths. In all the three paths work done by the gas is positive (volume is increasing) and the product PV or temperature. T is increasing. Therefore, internal energy is also increasing so, from First Law of Thermodynamics heat will be absorbed by the gas. Further area under P-V graph is maximum in path 1 while ΔU is same for all three paths. Therefore, heat absorbed by the gas is maximum in path 1. For temperature of the gas we can see the product PV which first increases in path 1 but whether it is decreasing or increasing later on we cannot say anything about it unless the exact values are known to us.
Hence, (A), (B) and (C) are correct.

3. $P(\text{in watt}) = \dfrac{dQ}{dt}(\text{in joule per second})$

$\Rightarrow \quad P(\text{in watt}) = mc\left(\dfrac{dT}{dt}\right)$

$\Rightarrow \quad P = \left(\dfrac{180}{1000}\right)(0.10)(4200)(0.5)$

$\Rightarrow \quad P = 37.8$ watt

Further $P = \tau\omega$

$\Rightarrow \quad 37.8 = \tau\left(180 \times \dfrac{2\pi}{60}\right)$

$\Rightarrow \quad \tau = 2$ Nm

Hence, (A) and (C) are correct.

4. Since $C_V = \dfrac{dU}{dT}$ {at constant volume}

$C_P = \left(\dfrac{dQ}{dT}\right)_P$ {at constant pressure}

$C = \dfrac{dU + dW}{dT}$ {for any process other than an adiabatic process}

$\Rightarrow \quad C = \dfrac{dU}{dT} + \dfrac{PdV}{dT}$ $\{\because \text{work done } dW = PdV\}$

Hence, (A), (B) and (C) are correct.

5. In 1 mole of all the gases there are N (Avagadro's number) number of molecules which are independent of the nature of gas. Hence, OPTION (A) is correct. The translational kinetic energy of 1 mole of any gas at temperature T is $\dfrac{3}{2RT}$. Therefore, OPTION (C) is also correct.
Hence, (A) and (C) are correct.

6. Let C_A and C_B be their heat capacities. Both the spheres have the same surface area. At any given temperature θ they lose heat at the same rate.

$\Rightarrow \quad C_A\left(-\dfrac{d\theta}{dt}\right)_A = C_B\left(-\dfrac{d\theta}{dt}\right)_B$

$\Rightarrow \quad \dfrac{\left(-\dfrac{d\theta}{dt}\right)_A}{\left(-\dfrac{d\theta}{dt}\right)_B} = \dfrac{C_B}{C_A} = \text{constant}$

i.e., rate of cooling of B is fast, but their ratio is constant.
Hence, (C) and (D) are correct.

7. Speed of a gas molecules, $v \propto \sqrt{T}$
Change in momentum of a molecule on hitting a wall and then rebounding is $\Delta p \propto v \propto \sqrt{T}$
Since, pressure of a gas is equal to change in momentum of a molecule colliding with the wall (Δp) multiplied by number of collisions per unit area per second (n). So, for
OPTION (A): If temperature and volume are doubled, pressure remains same. Therefore, n decreases since Δp increases. Thus, OPTION (A) is wrong.
OPTION (B): If temperature and volume are halved, n increases since Δp decreases. Therefore OPTION (B) is correct.
OPTION (C): If pressure and temperature are doubled, n increases because pressure doubles whereas Δp increases only $\sqrt{2}$ times.
Hence, (B) and (C) are correct.

8. $\dfrac{Q}{t} = KA\left(\dfrac{T_S - T_R}{\ell}\right)$

T_S = Temperature of steam

T_R = Temperature of room

T_0 increase $\dfrac{Q}{t}$ either A is increased or ℓ is decreased or T_R is decreased.
Hence, (A) and (D) are correct.

9. H =rate of heat flow $= \dfrac{900}{\dfrac{\ell_1}{K_i A} + \dfrac{\ell_0}{K_0 A}}$

Now $1000 - T = H\left(\dfrac{\ell_1}{K_i A}\right)$

$\Rightarrow \quad T = 1000 - \left(\dfrac{900}{\dfrac{\ell_1}{K_i A} + \dfrac{\ell_0}{K_0 A}}\right)\dfrac{\ell_1}{K_i A}$

$$\Rightarrow \quad T = 1000 - \frac{900}{1 + \frac{\ell_0}{K_0}\frac{K_i}{\ell_1}}$$

We also observe that T can be decreased by increasing thermal conductivity of outer layer (K_0) and thickness of inner layer (ℓ_1).

Hence, (A) and (D) are correct.

10. Since, 1 kg ice at 0 °C needs 80000 cal to get converted to water at 0 °C and 1.5 kg water at 45 °C gives out 67500 cal to get converted to water at 0 °C.

So, entire ice cannot be converted to water and the equilibrium temperature will be 0 °C. So mixture contains both ice and water. Amount of ice left unmelted due to insufficiency of available heat is

$$m_{ice} = \frac{80000 - 67500}{80} = 156.25 \text{ g}$$

Hence, (A) and (B) are correct.

11. $L_1 - L_2 = \text{constant}$

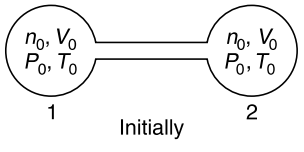
$\Rightarrow \quad \Delta L_1 - \Delta L_2 = 0$

$\Rightarrow \quad \Delta L_2 = \Delta L_1$

$\Rightarrow \quad L_2\alpha_2 = L_1\alpha_1$

Hence, (B) and (C) are correct.

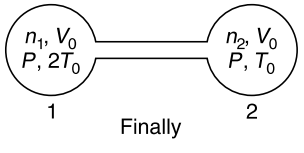
12. $P_0V_0 = n_0RT_0$ (for container 1) …(1)



$P_0V_0 = n_0RT_0$ (for container 2) …(2)

Total number of moles $= N = n_0 + n_0 = 2n_0$

Since, even on heating the total number of moles is conserved, hence



$n_1 + n_2 = 2n_0$ …(3)

Further if P be the common pressure, then

$PV_0 = n_1R(2T_0)$ {in container 1}

$\Rightarrow \quad n_1 = \frac{PV_0}{2RT_0}$ …(4)

Also, $PV_0 = n_2RT_0$ {in container 2}

$\Rightarrow \quad n_2 = \frac{PV_0}{RT_0}$ …(5)

Since, $n_1 + n_2 = 2n_0$

$\Rightarrow \quad \frac{PV_0}{2RT_0} + \frac{PV_0}{RT_0} = 2\frac{P_0V_0}{RT_0}$ $\{\because \text{ of (1)}\}$

$\Rightarrow \quad P = \frac{4}{3}P_0$

Also, $n_1 = \frac{4}{3}\frac{P_0V_0}{2RT_0} = \frac{2}{3}\frac{P_0V_0}{RT_0}$

Hence, (B) and (C) are correct.

13. During expansion an isotherm lies above an adiabat, so greater work is done in isothermal process than in adiabatic process. {OPTION **(A)**}

Let both expand to a final volume V, then

$P_0V_0 = P_{iso}V \quad$ and $\quad P_0V_0^{\gamma} = P_{adi}V^{\gamma}$

$\Rightarrow \quad P_{iso} = P_0\left(\frac{V_0}{V}\right) \quad$ and $\quad P_{adi} = P_0\left(\frac{V_0}{V}\right)^{\gamma}$

Since $\frac{V_0}{V} < 1$ and $\gamma > 1$

$\Rightarrow \quad P_{iso} > P_{adi}$ {OPTION **(B)**}

Finally, since we know that adiabatic expansion always leads to cooling, so we have final temperature of adiabatic process less than the constant temperature of isothermal process. {OPTION **(C)**}

Hence, (A), (B) and (C) are correct.

14. Refer to SOLUTION of PROBLEM 13.

Hence, (A), (B) and (C) are correct.

15. Since, $Q_1 = W + Q_2$

$\Rightarrow \quad W = Q_1 - Q_2 > 0$

$\Rightarrow \quad Q_1 > Q_2 > 0$ OR $Q_1 < Q_2 < 0$

Hence, (A) and (C) are correct.

16. Process DA is isothermal

$\Rightarrow \quad T_D = T_A$

Further $T \propto PV$

From the graph we can see that $(PV)_C > (PV)_D$

$\Rightarrow \quad T_C > T_D$

Further process BC is an adiabatic expansion. So, $T_D > T_C$. Therefore, maximum temperature is at B and minimum temperature at D or A. Hence,

$T_B = 4T_0$ and $T_D = T_A = T_0$

In process *CD*:

$W = 0$

$\Rightarrow \quad Q = \Delta U$ $\{\because \text{ of FLTD}\}$

Volume of the gas is constant and pressure is decreasing. Therefore, temperature and hence, internal energy will decrease i.e., Q is negative or heat is released by the gas in the process CD.

In process *AB*:

$Q = nC_p\Delta T$ {process is isobaric}

Pressure is constant and volume is increasing. Therefore, temperature will also increase or Q is positive. Thus, heat is supplied to the gas only in process AB

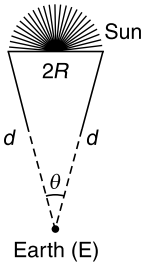
and $Q = n\left(\frac{5R}{2}\right)(T_B - T_A)$

CHAPTER 2

$$\Rightarrow \quad Q = n\left(\frac{5R}{2}\right)(4T_0 - T_0) = \frac{15}{2}nRT_0$$

Hence, (A), (B) and (C) are correct.

17. Power radiated by the sun is $P = (4\pi R^2)\sigma T^4$



Further,

$$\text{Solar constant} = \left(\frac{\text{Energy received per}}{\text{second per unit area}}\right) = \frac{P}{4\pi d^2} = S$$

$$\Rightarrow \quad P = 4\pi d^2 S$$

For equilibrium $4\pi R^2\sigma T^4 = 4\pi d^2 S$

$$\Rightarrow \quad S = \frac{1}{4}\sigma T^4\left(\frac{2R}{d}\right)^2 = \frac{1}{4}\sigma T^4\theta^2 \qquad \left\{\because \frac{2R}{d} = \theta\right\}$$

$$\Rightarrow \quad S \propto T^4$$

$$\Rightarrow \quad S \propto \theta^2$$

Hence, (B) and (D) are correct.

18. Conceptual
Hence, (B) and (D) are correct.

19. A refrigerator is reverse of heat engine and heat pump is same as refrigerator. The coefficient of performance of a refrigerator cannot be infinity and a heat engine cannot convert the heat input fully to work done.
Hence, (C) and (D) are correct.

20. Water equivalent (w) of a body of mass m, gram specific heat c is just the product of m and c i.e., $w = mc$. Similarly, heat capacity of a body of mass m, gram specific heat c is also product of m and c. The major difference between both is that w is expressed in the units of mass and heat capacity is expressed in the units of energy (calorie or joule).
Hence, (A) and (C) are correct.

21. Since, $p^2V = \text{constant}$

$$\Rightarrow \quad \left(\frac{RT}{V}\right)^2 V = \text{constant}$$

$$\Rightarrow \quad T^2 \propto V$$

So, when volume becomes $3V_0$, then temperature also becomes $\sqrt{3}T_0$. Since temperature has increased, so internal energy of the gas will increase.
Hence, (A) and (B) are correct.

22. P-T graph is a straight line passing through origin. Therefore,

$$V = \text{constant}$$

So, work done on the gas is zero.

Further density of the gas $\rho = \frac{m}{V} \propto \frac{1}{V}$

Volume of the gas is constant. Therefore, density of gas is also constant.

$$PV = nRT$$

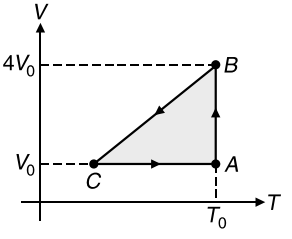
$$\Rightarrow \quad P = \left(\frac{nR}{V}\right)T$$

i.e., slope of P-T line $\propto n$

Hence, (A), (B) and (D) are correct.

23. For n moles of gas with f degrees of freedom at temperature T, we have

$$U = \frac{f}{2}nRT$$



Since, temperature T is same at A and B, so

$$U_A = U_B$$

Also, $W_{AB} = nRT_0\log_e\left(\frac{V_f}{V_i}\right) = nRT_0\log_e\left(\frac{4V_0}{V_0}\right)$

$$W_{AB} = nRT_0\log_e(4) = P_0V_0\log_e(4)$$

Hence, (A) and (B) are correct.

24. Conceptual
Hence, (A), (B), (C) and (D) are correct.

25. Since work done is area under the graph, OPTION (A) is incorrect.
Since net work done in the cyclic clockwise process is positive and for cyclic process $Q = W$, so OPTION (B) is correct.
Since $PV = nRT$, so temperature is maximum at C and hence OPTION (C) is incorrect.
Process C to A is isochoric and pressure is falling. Therefore, temperature also falls i.e. ΔU and Q are negative. Therefore OPTION (D) is correct.
Hence, (B) and (D) are correct.

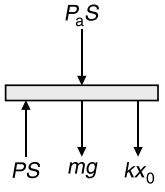
26. Vibrational kinetic energy of a monatomic gas = 0 at all temperatures. So, $C_V = \frac{3}{2}R$ for a monatomic gas even at high temperatures also.
In case of a diatomic gas

$C_V = \frac{5}{2}R$ at low temperatures, while

$C_V > \frac{5}{2}R$ at high temperature due to vibrational kinetic energy

Hence, (A) and (D) are correct.

27. Equilibrium of piston gives



$$PS = P_aS + mg + kx_0$$

where, P is the final pressure of the gas

$$\Rightarrow \quad P = P_a + \frac{mg}{S} + \frac{kx_0}{S}$$

Work done by the gas (W) equals the sum of work done against atmospheric pressure (W_{pr}), elastic potential energy storied in the spring (ΔU_{sp}) and the increase in gravitational potential energy (ΔU_g) of the piston.

$$\Rightarrow \quad W = W_{pr} + \Delta U_{sp} + \Delta U_g$$

where, $W_{pr} = P_0\Delta V$, $\Delta U_{sp} = \frac{1}{2}kx_0^2$, $\Delta U_g = mvx_0$

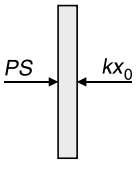
$$\Rightarrow \quad W = P_0\Delta V + \frac{1}{2}kx_0^2 + mgx_0$$

$$\Rightarrow \quad W = P_aSx_0 + \frac{1}{2}kx_0^2 + mgx_0$$

This is also the decrease in internal energy of the gas. Because the gas in thermally insulated and this work is done in the expense of internal energy of the gas.

Hence, (A) and (C) are correct.

28. Equilibrium of piston gives



$$PS = kx_0$$

$$\Rightarrow \quad P = \frac{kx_0}{S}$$

Since, the chamber is thermally insulated

$$\Delta Q = 0$$

$$\Rightarrow \quad \begin{pmatrix}\text{Elastic Potential Energy}\\ \text{of Spring}\end{pmatrix} = \begin{pmatrix}\text{Work Done}\\ \text{by Gas}\end{pmatrix}$$

$$\Rightarrow \quad \text{Work done by gas} = \frac{1}{2}kx_0^2$$

This work is done in the expense of internal energy of the gas. Therefore, internal energy of the gas is deceased by $\frac{1}{2}kx_0^2$.

Internal energy of an ideal gas depends on its temperature only. Internal energy of the gas is decreasing. Therefore, temperature of the gas will decrease.

Hence, (A), (B), (C) and (D) are correct.

29. Since $TP^{-2/5} = \text{constant}$, so comparing it with equation of a polytropic process in which

$$TP^{\frac{1-x}{x}} = \text{constant}$$

$$\Rightarrow \quad \frac{1-x}{x} = -\frac{2}{5}$$

$$\Rightarrow \quad 5 - 5x = -2x$$

$$\Rightarrow \quad x = \frac{5}{3} = \gamma_{\text{monatomic}}$$

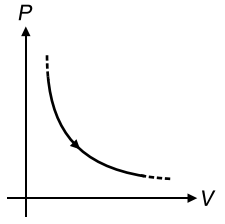
So, this represents an adiabatic process, for which $Q = 0$ i.e., $C = 0$

So, $\Delta U = nC_V\Delta T = (2)\left(\frac{3R}{2}\right)(2T) = 6RT$

For an adiabatic process, $Q = -\Delta U = -6RT$

Hence, (B), (C) and (D) are correct.

30. (A) It is incorrect because isothermal process is graphically represented like as shown.

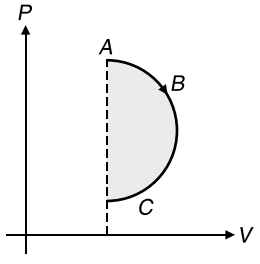


(B) For $B \to C \to D$, $\Delta U = -\text{ve}$ and $W = -\text{ve}$

$\Rightarrow \quad Q = -\text{ve}$

(C) W =shaded area = +ve

(D) Since the P-V cycle is clockwise, so $W = +\text{ve}$



Hence, (B) and (D) are correct.

31. Since $U_c - U_a = (1)\left(\frac{3R}{2}\right)(T_c - T_a)$

$$\Rightarrow \quad U_c - U_a = \left(\frac{3R}{2}\right)\left(\frac{8P_0V_0}{R} - \frac{P_0V_0}{R}\right) = \left(\frac{21}{2}\right)P_0V_0$$

$$\Rightarrow \quad U_c - U_a = 10.5RT_0$$

and similarly it can be done for others.

Hence, (A), (B), (C) and (D) are correct.

32. Internal energy of n moles of an ideal gas is

$$U = \left(\frac{f}{2}\right)(nRT)\ f = \text{degrees of freedom}$$

i.e., U depends on number of moles n and temperature T.

Similarly, $PV = nRT$

i.e., PV depends on n and T

But $\frac{P}{\rho} = \frac{RT}{M}$

i.e., the ratio $\frac{P}{\rho}$ depends on T only

Similarly, $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

i.e., v_{rms} also depends on T only.

Hence, (C) and (D) are correct.

CHAPTER 2

33. Heat required to raise the temperature of a solid to its melting point is

$$Q_1 = mc\Delta T = (0.1)(0.5\times10^{-3})(100-0) = 5000 \text{ cal}$$

Therefore, rate of supply of heat is 5000 calmin^{-1}

Since the solid takes one minute to melt, heat required for making the solid is $Q_2 = 5000\times1 = 5000$ cal

Heat required to raise the temperature of liquid formed to boiling point is $Q_3 = 5000\times4 = 20000$ cal

If c is the specific heat of the liquid, then

$$20000 = (0.1)c(300-100)$$

$\Rightarrow$ $c = 1000 \text{ calkg}^{-1}{}^\circ\text{C}^{-1} = 1 \text{ kcalkg}^{-1}{}^\circ\text{C}^{-1}$

Hence, (A), (B) and (C) are correct.

34. Since $\gamma = \dfrac{C_P}{C_V} = 1+\dfrac{2}{f}$

$\Rightarrow$ $\dfrac{2}{f} = \dfrac{C_P - C_V}{C_V}$

$\Rightarrow$ $f = \dfrac{2C_V}{R}$ $\{\because C_P - C_V = R\}$

$\Rightarrow$ $f = 2\left(\dfrac{C_P}{R}-1\right)$

Hence, (A) and (B) are correct.

35. When heat is supplied to the gas under Isobarric, Isochorric, Adiabatic conditions then

$$\Delta U = nC_v(T_2 - T_1) \quad \{\text{OPTION } \textbf{(A)}\}$$

For an Adiabatic process $dQ = 0$

$\Rightarrow$ $dW = -dU$

Work is being done at the expense of internal energy and both have equal value. {OPTION **(B)**}

For an Isothermal process $\Delta T = 0$

$\Rightarrow$ $\Delta U = 0$ {OPTION **(C)**}

For an Adiabatic process, since $dQ = 0$

$\Rightarrow$ $C = \dfrac{1}{n}\dfrac{dQ}{dT} = 0$ {OPTION **(D)**}

Hence, (A), (B), (C) and (D) are correct.

36. $C = 0$

$\Rightarrow$ $dQ = 0$ $\left\{\because C = \dfrac{1}{n}\dfrac{dQ}{dT}\right\}$

$\Rightarrow$ Adiabatic process

Hence, (A) and (B) are correct.

37. T = constant

$\Rightarrow$ PV =constant {Boyle's Law}

$\Rightarrow$ $P \propto \dfrac{1}{V}$

Pressure of the gas is increasing. Therefore, volume should decrease. Work done by the gas is negative or work done on the gas will be positive. Further temperature of the gas is constant. Therefore, internal energy will remain constant.

Hence, (A), (C) and (D) are correct.

38. **Process *AB*:** P = constant

$\Rightarrow$ $V \propto T$

$\Rightarrow$ $T_B = 2T_A = 2T_0$

$\Rightarrow$ $W_{AB} = P_0V_0 = RT_0$

Since, $Q_{AB} = C_P\Delta T = \left(\dfrac{5}{2}R\right)(2T_0 - T_0)$

$\Rightarrow$ $Q_{AB} = \dfrac{5}{2}RT_0$

So, from $FLTD$, we have

$$Q_{AB} = \Delta U_{AB} + W_{AB}$$

$\Rightarrow$ $\Delta U_{AB} = Q_{AB} - W_{AB} = \dfrac{3}{2}RT_0$

Process *BC*: V = constant

$\Rightarrow$ $P \propto T$

$\Rightarrow$ $P_C = 2P_B$

$\Rightarrow$ $T_C = 2T_B = 4T_0$

Since, $W_{BC} = 0$

$\Rightarrow$ $Q_{BC} = \Delta U_{BC} = C_V\Delta T$

$\Rightarrow$ $Q_{BC} = \left(\dfrac{3}{2}R\right)(4T_0 - 2T_0) = 3RT_0$

$\Rightarrow$ $W_{\text{net}} = RT_0$, $Q_{\text{net}} = \dfrac{11}{2}RT_0$

and $\Delta U_{\text{net}} = \dfrac{9}{2}RT_0$

Hence, (A), and (C) are correct.

39. Conceptual

Hence, (A), and (B) are correct.

40. For isobaric process, we have

$$W = nR\Delta T \text{ and } Q = nC_P\Delta T$$

From the graph, we see that

$$\text{Slope} = \frac{W}{Q} = \frac{R}{C_P}$$

For Graph 1

$$\frac{W}{Q} = \frac{20}{80} = \frac{1}{4} = \frac{R}{C_P}$$

$\Rightarrow$ $C_P = 4R$

Therefore, gas is polyatomic

For Graph 2

$$\frac{W}{Q} = \frac{32}{80} = \frac{2}{5} = \frac{R}{C_P}$$

$\Rightarrow$ $C_P = \dfrac{5R}{2}$

Therefore, gas is monatomic

For Graph 3

$$\frac{W}{Q} = \frac{80}{80} = 1 = \frac{R}{C_P}$$

$\Rightarrow$ $C_P = R$

which is not possible because $C_V \neq 0$ for isobaric process

Hence, the correct answer is (D).

41. $P \propto AT^4$ $\{\because\ T_0 \ll T\}$

i.e., $P \propto r^2$ $\{\because A = 4\pi r^2\}$

Rate of cooling $\propto \frac{1}{r}$

Hence, (B), (C) and (D) are correct.

42. Rate of cooling $= R = -\frac{dT}{dt}$

Since, $\frac{1}{A}\frac{dQ}{dt} = \sigma\left(T^4 - T_0^4\right)$ (By Stefan's Law)

$\Rightarrow \quad -\frac{mc}{A}\frac{dT}{dt} = \sigma\left(T^4 - T_0^4\right)$ $\{\because\ dQ = mcdT\}$

$\Rightarrow \quad -\frac{dT}{dt} = R = \frac{\sigma A}{mc}\left(T^4 - T_0^4\right)$

For T comparable to T_0 we have

$$T^4 - T_0^4 \simeq 4T_0^3\left(T - T_0\right)$$

$\Rightarrow \quad R = \frac{4\sigma A T_0^3}{mc}\left(T - T_0\right)$

$\Rightarrow \quad R \propto \frac{A}{m} \propto \frac{1}{r}$ $\left\{\because A = 4\pi r^2 \text{ and } m = \frac{4}{3}\pi r^3\rho\right\}$

and $R \propto \left(T - T_0\right)$

Hence, (A) and (C) are correct.

43. $\frac{dQ}{dt}$ = Thermal Current I $= kA\left(\frac{dT}{d\ell}\right)$

Hence, (A) and (B) are correct.

44. Thermal Stress $= \frac{\text{Tension}}{\text{Area}} = Y\alpha\Delta T$

$\Rightarrow \quad$ Tension $= YA\alpha\Delta T$

Hence, (C) and (D) are correct.

45. During process A to B, pressure and volume both are decreasing. Therefore, temperature and hence, internal energy of the gas will decrease $(T \propto PV)$ or $\Delta U_{A\to B}$ = negative. Further $\Delta W_{A\to B}$ is also negative as the volume is increasing. Hence, temperature should increase or $\Delta U_{B\to C}$ = positive. During C to A volume is constant while pressure is increasing. Therefore, temperature and hence, internal energy of the gas should increase or $\Delta U_{C\to A}$ =positive. During process CAB volume of the gas is decreasing. hence, work done by the gas is negative.

Hence, (A), (B) and (D) are correct.

46. $m = nM$

n =number of moles, M =molecular mass of gas

$\Rightarrow \quad \frac{mkT}{V} = \frac{nMkT}{V} = \frac{nM\left(\frac{R}{N}\right)T}{V}$ $\left\{\because k = \frac{R}{N}\right\}$

Since, $PV = nRT$

$\Rightarrow \quad \frac{mkT}{V} = \frac{PVM}{NV} = \frac{PM}{N}$

i.e., the given quantity depends on P and M of the gas.

Hence, (C) and (D) are correct.

47. For adiabat bc

$$T_1V_b^{\gamma-1} = T_2V_c^{\gamma-1} \quad \ldots(1)$$

For adiabat da

$$T_2V_d^{\gamma-1} = T_1V_a^{\gamma-1} \quad \ldots(2)$$

Multiplying both (1) and (2)

$\Rightarrow \quad T_1T_2\left(V_bV_d\right)^{\gamma-1} = T_1T_2\left(V_aV_c\right)^{\gamma-1}$

$\Rightarrow \quad V_bV_d = V_aV_c$

Since adiabatic expansion leads to cooling,

so $\quad T_1 > T_2$

Hence, (B) and (C) are correct.

48. Since, net change in internal energy in one cycle of a cyclic process is zero, therefore,

$$\Delta U_1 + \Delta U_2 = 0$$

Also, for the complete cycle, $Q = W$

Hence, (A) and (C) are correct.

49. $\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{2}{3} = \frac{1}{3} = 0.33$

Further $\eta = \frac{W}{Q_1}$

$\Rightarrow \quad \frac{1}{3} = \frac{W}{1000 \text{ cal}}$

$\Rightarrow \quad W = \frac{1000 \times 4.2}{3}$

$\Rightarrow \quad W = 1400 \text{ J}$

Hence, (B) and (D) are correct.

50. Conceptual

Hence, (C) and (D) are correct.

51. Since for any cyclic process $dU = 0$

$\Rightarrow \quad U =$ constant {OPTION **(A)**}

For an Isothermal process $dT = 0$ i.e., $dU = 0$

$\Rightarrow \quad U =$ constant {OPTION **(B)**}

Further according to First Law of Thermodynamics

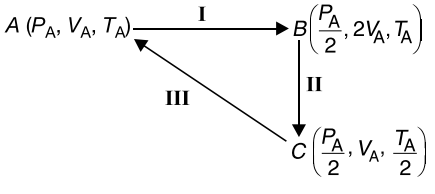
$$dQ = dU + dW$$

For constant U, $dU = 0$

$\Rightarrow \quad dQ = dW$ {OPTION **(D)**}

Hence, (A), (B) and (D) are correct.

52. Let the process start from initial pressure P_A, volume V_A and temperature T_A.



CHAPTER 2

I: Isothermal expansion $(PV = \text{constant})$ at temperature T_A to twice the initial volume V_A

II: Compression at constant pressure $\frac{P_A}{2}$ to original volume V_A $(\text{i.e. } V \propto T)$

III: Isochoric process (at volume V_A) to initial conditions $(\text{i.e. } P \propto T)$

Hence, (A) and (C) are correct.

53. Since work done is area under the graph, OPTION (A) is incorrect.
Since net work done in the cyclic clockwise process is positive and for cyclic process $Q = W$, so OPTION (B) is correct.
Since $PV = nRT$, so temperature is maximum at C and hence OPTION (C) is incorrect.
Process C to A is isochoric and pressure is falling. Therefore, temperature also falls i.e. ΔU and Q are negative. Therefore OPTION (D) is correct.
Hence, (B) and (D) are correct.

54. On heating, a bimetallic strip always bends towards the metal with low α and the reverse process takes place when the bimetallic strip is cooled.
Hence, (B) and (C) are correct.

55. Container is thermally insulated, so we have

$$Q = 0$$

The gas expands against vacuum, then

$$W = 0$$

From First Law of Thermodynamics, we have

$$Q = \Delta U + W$$

$$\Rightarrow \quad \Delta U = 0$$

i.e., internal energy and hence, temperature of the gas is constant

$$\Rightarrow \quad P \propto \frac{1}{V} \qquad \{\text{Boyle's Law}\}$$

Hence, (A), (B), (C) and (D) are correct.

56. Since $T = KV^2$, so we have $TV^{-2} = \text{constant}$
Comparing this with equation of a polytropic process i.e., $TV^{x-1} = \text{constant}$, we get

$$x - 1 = -2$$

$$\Rightarrow \quad x = -1$$

So, molar specific heat of the gas is

$$C = C_V + \frac{R}{1-x} = \frac{3R}{2} + \frac{R}{2} = 2R$$

$$\Rightarrow \quad Q = nC\Delta T = n(2R)(3T_0) = 6nRT_0$$

$$\Rightarrow \quad W = \frac{nR\Delta T}{1-x} = \frac{nR(3T_0)}{2} = \frac{3}{2}nRT_0$$

Hence, (B) and (C) are correct.

57. Since no mechanical contact is there, so angular momentum is conserved.

$$\Rightarrow \quad I\omega = \text{constant}$$

$$\Rightarrow \quad \Delta(I\omega) = 0$$

$$\Rightarrow \quad I\Delta\omega + \omega\Delta I = 0$$

$$\Rightarrow \quad \frac{\Delta\omega}{\omega} = -\frac{\Delta I}{I}$$

$$\Rightarrow \quad \frac{\Delta\omega}{\omega_0} = -\frac{\Delta I}{I} = -2\alpha\Delta T \qquad \left\{\because \frac{\Delta I}{I} = \frac{2\Delta R}{R} = 2\alpha\Delta T\right\}$$

Hence, (A), (B) and (C) are correct.

58. For isothermal process

$$T = \text{constant}$$

Since, $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

$$\Rightarrow \quad V_{\text{rms}} \text{ is also constant}$$

Hence, (B) and (C) are correct.

59. During free expansion, gas is allowed to expand in vacuum. This happens so quickly that there is no heat transferred. Since the gas does not displace anything, so no work is also done by the gas and hence by FLTD, change in internal energy of the gas is also zero. So, there is no change in temperature of the gas.
Hence, (A) and (C) are correct.

60. $W_{\text{cyclic}} = nR(T_1 - T_2)\log_e \alpha$

$$\eta = \frac{W}{Q_1} = 1 - \left(\frac{T_2}{T_1}\right)$$

$$\Delta U_{\text{cyclic}} = 0$$

Hence, (A), (B) and (C) are correct.

61. During expansion an isotherm lies above an adiabat

Since, $(\text{Slope})_{\text{adiabat}} = \gamma(\text{Slope})_{\text{isotherm}}$

$$\Rightarrow \quad m_2 = \frac{C_p}{C_V}(m_1)$$

$$\Rightarrow \quad m_2 C_V = m_1 C_P$$

Since $\gamma > 1$

$$\Rightarrow \quad m_2 > m_1$$

Hence, (C) and (D) are correct.

Reasoning Based Questions

1. Since, for an adiabatic process, $TV^{\gamma-1} = \text{constant}$

$$\Rightarrow \quad d(TV^{\gamma-1}) = 0$$

$$\Rightarrow \quad T(\gamma-1)V^{\gamma}dv + V^{\gamma-1}dT = 0$$

$$\Rightarrow \quad \frac{T}{V}(\gamma-1)dV + dT = 0$$

$$\Rightarrow \quad \frac{dT}{dV} = -\frac{T}{V}(\gamma-1)$$

Slope is negative, so $\frac{dT}{dV} < 0$

Hence, the correct answer is (C).

2. Since, $C = \frac{\Delta Q}{n\Delta T}$

In adiabatic process, $\Delta Q = 0$

$$\Rightarrow \quad C = 0$$

In isothermal process, $\Delta T = 0$

$\Rightarrow \quad C \to \infty$

Hence, the correct answer is (A).

3. Statement-1 is true and Statement-2 is false.
We can change the temperature of a body without giving (or taking) heat to (or from) it. For example, in an adiabatic compression temperature rises and in an adiabatic expansion temperature falls, although no heat is given (or taken) from the system in the respective changes.
Hence, the correct answer is (C).

4. Coefficient of thermal conductivity is the property of the material and changes only when the material changes.
Hence, the correct answer is (D).

5. Since, V-T graph is a rectangular hyperbola, so

$VT = \text{constant}$

$\Rightarrow \quad PV^2 = \text{constant}$

$\Rightarrow \quad \dfrac{P}{T^2} = \text{constant}$

Hence, the correct answer is (B).

6. A gas have may values of specific heat capacity depending on the process used.
Hence, the correct answer is (D).

7. Statement-1 is incorrect, because two isothermal curves can never intersect each other but Statement-2 is correct.
Hence, the correct answer is (D).

8. In a constant pressure process, the heat supplied has two jobs to do

(i) increase the internal energy of the gas by ΔU and

(ii) to push the piston to maintain the pressure.

Hence $C_P > C_V$
Hence, the correct answer is (A).

9. If V increases temperature of gas decreases.
Hence, the correct answer is (A).

10. Coefficient of volume expansion $\delta = \dfrac{1}{V}\dfrac{dV}{dT}$

Since, $PV = nRT$

$\Rightarrow \quad V = \left(\dfrac{nR}{P}\right)T$

$\Rightarrow \quad \dfrac{dV}{dT} = \dfrac{nR}{P}$

$\Rightarrow \quad \delta = \dfrac{1}{V}\dfrac{dV}{dT} = \dfrac{nR}{PV} = \dfrac{nR}{nRT} = \dfrac{1}{T}$

Hence, the correct answer is (D).

11. Conceptual and discussed in Theory.
Hence, the correct answer is (A).

12. Upon increasing pressure, the ice melts so as to decrease the volume and thereby pressure. To freeze the melted ice one needs to decrease the temperature down and hence the melting point decreases.
Hence, the correct answer is (A).

13. Internal energy increases in the melting process.
Linked Comprehension
Hence, the correct answer is (D).

Linked Comprehension Type Questions

1. Since $W = \dfrac{Q}{2}$

From $FLTD$, we have $dQ = dU + dW$

$\Rightarrow \quad dQ = dU + \dfrac{Q}{2}$

$\Rightarrow \quad dU = dW = \dfrac{Q}{2}$

$\Rightarrow \quad C_V dT = PdV$

$\Rightarrow \quad \left(\dfrac{R}{\gamma - 1}\right)dT = PdV$

Since, for a monatomic gas $\gamma = \dfrac{5}{3}$, so on integrating, we get

$P^3V = \text{constant}$

Hence, the correct answer is (C).

2. Also, $C = C_V + \dfrac{PdV}{dT} = 3R$
Hence, the correct answer is (C).

3. Since, $PV = nRT$

$\left(\dfrac{nRT}{V}\right)^3 V = \text{constant}$

$\Rightarrow \quad T^3 \propto V^2$

$\Rightarrow \quad \dfrac{T_0^3}{T^3} = \dfrac{V_0^3}{V^3}$

$\Rightarrow \quad V = 8V_0 \qquad \{\because\ T = 4T_0\}$

Hence, the correct answer is (B).

4. AB is an isobaric process. In $B \to C$ magnitude of $\dfrac{dT}{dv}$ increases and $O \to A$ is an isochoric process.
Hence (C) OPTION is correct.
Hence, the correct answer is (C).

5. For a cyclic process, we have

$\Delta U = 0$

$\Rightarrow \quad W_{\text{net}} = Q_{\text{net}} = -1000 \text{ J}$

$\Rightarrow \quad W_{AB} + W_{BC} + W_{CA} = -1000 \text{ J}$

$\Rightarrow \quad (4)(R)(200) + W_{BC} + 0 = -1000 \text{ J}$

$\Rightarrow \quad W_{BC} = -7640 \text{ J}$

Hence, the correct answer is (C).

6. The volume of gas at state C is given by

$V_C = V_A = \dfrac{nRT_A}{P_A}$

$\Rightarrow \quad V_C = 0.0332 \text{ m}^3$

Hence, the correct answer is (A).

7. The force due to pressure of liquid is Buoyant force. Besides this, gravity and viscous force act on the bubble.
Hence, the correct answer is (D).

CHAPTER 2

8. Since, no heat exchange takes place, so the gas expands adiabatically.

$$P^{1-\gamma}T^{\gamma} = \text{constant}$$

$$\Rightarrow \quad T_2 = T_1\left(\frac{P_1}{P_2}\right)^{\frac{1-\gamma}{\gamma}}$$

where, $T_1 = T_0$, $P_1 = P_0 + \rho_\ell g H$, $P_2 = P_0 + \rho_\ell g(H-y)$ and $\gamma = \frac{5}{3}$

$$\Rightarrow \quad T_2 = T_0\left(\frac{P_0 + \rho_\ell g H}{P_0 + \rho_\ell g(H-y)}\right)^{\frac{1-\frac{5}{3}}{\frac{5}{3}}}$$

$$\Rightarrow \quad T_2 = T_0\left(\frac{P_0 + \rho_\ell g(H-y)}{P_0 + \rho_\ell g H}\right)^{\frac{2}{5}}$$

Hence, the correct answer is (B).

9. Buoyant Force $F_B = V\rho_\ell g$

Since, $V = \frac{nRT}{P} = \frac{nRT_2}{\left(P_0 + \rho_\ell g(H-y)\right)}$

Substituting the value of T_2, V, we get

$$F_B = \frac{\rho_\ell nRgT_0}{(P_0 + \rho_\ell g H)^{\frac{2}{5}}\left(P_0 + \rho_\ell g(H-y)\right)^{\frac{3}{5}}}$$

Hence, the correct answer is (B).

10. Since, $\alpha_t > \alpha_b$, therefore, tube tries to expand more than the bolt when assembly is heated. But the tube is tightened by the bolt, therefore, its expansion cannot be more than that of the bolt. Due to this a compressive stress is developed in the tube and a tensile stress in bolt.
Let initial length of the assembly be ℓ and let its elongation be $\Delta\ell$. Elongation of tube if it were free to expand would be $\ell\alpha_t\Delta T$. But its actual elongation is $\Delta\ell$, therefore elongation prevented in it is equal to $(\ell\alpha_t\Delta T - \Delta\ell)$
So, compressive strain in tube is

$$E_t = \frac{(\ell\alpha_t\Delta T - \Delta\ell)}{\ell} = 5\times10^{-5}$$

Hence, the correct answer is (B).

11. Compressive stress in tube

$$\sigma_t = E_t Y_t = 5\times10^6 \text{ Nm}^{-2} \qquad \left\{\because Y = \frac{\text{Stress}}{\text{Strain}}\right\}$$

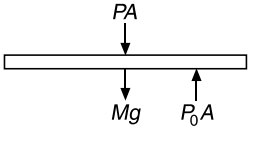
Hence, the correct answer is (A).

12. Tensile stress in bolt $\sigma_b = \frac{Y_b(\Delta\ell - \ell\alpha_b\Delta T)}{\ell} = 10^7 \text{ Nm}^{-2}$

Hence, the correct answer is (B).

13. Since it is open from top, pressure will be P_0.
Hence, the correct answer is (A).

14. Let P be the pressure in equilibrium.



Then, $PA = P_0A - Mg$

$$\Rightarrow \quad P = P_0 - \frac{Mg}{A} = P_0 - \frac{Mg}{\pi R} \qquad \ldots(1)$$

Applying $P_1V_1 = P_2V_2$

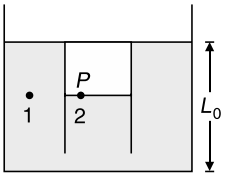
$$\Rightarrow \quad P_0(2AL) = (P)(AL')$$

$$\Rightarrow \quad L' = \frac{2P_0L}{P} = \left(\frac{P_0}{P_0 - \frac{Mg}{\pi R^2}}\right)(2L)$$

$$L' = \left(\frac{P_0\pi R^2}{\pi R^2P_0 - Mg}\right)(2L)$$

Hence, the correct answer is (D).

15. Since we observe, $P_1 = P_2$



$$\Rightarrow \quad P_0 + \rho g(L_0 - H) = P \qquad \ldots(1)$$

Now, applying $P_1V_1 = P_2V_2$ for the air inside the cylinder, we have

$$P_0(L_0) = P(L_0 - H)$$

$$\Rightarrow \quad P = \frac{P_0L_0}{L_0 - H}$$

Substituting in equation (1), we have

$$P_0 + \rho g(L_0 - H) = \frac{P_0L_0}{L_0 - H}$$

$$\Rightarrow \quad \rho g(L_0 - H)^2 + P_0(L_0 - H) - P_0L_0 = 0$$

Hence, the correct answer is (C).

16. The given network forms a Wheatstone's bridge. There will be no heat flow through bd, if the bridge is balanced.

$$\Rightarrow \quad \frac{R_{ab}}{R_{bc}} = \frac{R_{ad}}{R_{dc}}$$

Since, $R \propto \ell$

$$\Rightarrow \quad \frac{\ell}{3\ell} = \frac{2\ell}{x}$$

$$\Rightarrow \quad x = 6\ell$$

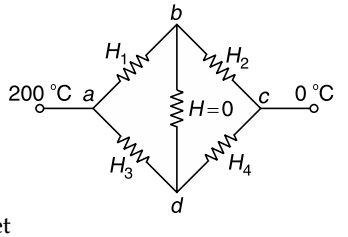
Hence, the correct answer is (C).

17. Since, $H_{bd} = 0$

$$\Rightarrow \quad H_1 = H_2$$

$$\Rightarrow \quad H_3 = H_4$$

$$\Rightarrow \quad \frac{200 - T_b}{\left(\frac{\ell}{KA}\right)} = \frac{T_b - 0}{\left(\frac{3\ell}{KA}\right)}$$



Solving this equation, we get

$$T_b = 150 \text{ °C}$$

Hence, the correct answer is (D).

18. **The correct answer is (D).**

19. **The correct answer is (C).**

Combined solution to 18 and 19

Let m be the mass of ice and q be the rate by which the heat is supplied to it. If T_i and T_f be the initial and final temperature of the ice, then

$$Q_1 = q\times 1 = m\times 0.5\times(-T_i)$$

$$Q_2 = q\times 4 = m\times 80$$

and $Q_3 = q\times 2 = m\times 1\times T_f$

Solving these three equations, we get

$$T_f = 40\ ^\circ\text{C}$$

and $T_i = -40\ ^\circ\text{C}$

20. Since, $C_P - C_V = R$

$$\Rightarrow \quad C_V = \frac{R}{\gamma - 1}$$

So, $C_{V_A} = \dfrac{R}{\left(\frac{5}{3}-1\right)} = \dfrac{3R}{2}$

$$\Rightarrow \quad C_{P_A} = \frac{5R}{2}$$

Similarly, $C_{V_B} = \dfrac{R}{\left(\frac{7}{5}-1\right)} = \dfrac{5R}{2}$

$$\Rightarrow \quad C_{P_B} = \frac{7R}{2}$$

Also, $\gamma_{\text{mixture}} = \dfrac{19}{13} = \dfrac{n_A C_{P_A} + n_B C_{P_B}}{n_A C_{V_A} + n_B C_{V_B}}$

where $n_A = 1$

$$\Rightarrow \quad n_B = 2 \text{ mole}$$

Hence, the correct answer is (D).

21. Since, Bulk Modulus of the gas under adiabatic conditions is

$$B = \gamma P$$

Compressibility, $K = \dfrac{1}{B} = \dfrac{1}{\gamma P}$

So, change in adiabatic compressibility is

$$\Delta K = K_2 - K_1$$

$$\Rightarrow \quad \Delta K = \frac{1}{\gamma P_2} - \frac{1}{\gamma P_1} = \frac{1}{\gamma}\left(\frac{1}{P_2} - \frac{1}{P_1}\right)$$

Since, the process is adiabatic, so we have

$$P_2 V_2^{\gamma} = P_1 V_2^{\gamma}$$

$$\Rightarrow \quad P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma} = P_1\left(\frac{V_1}{V_1/5}\right)^{\gamma} = P_1 5^{\gamma}$$

$$\Rightarrow \quad \Delta K = \frac{1}{\gamma}\left(\frac{1}{P_1 5^{\gamma}} - \frac{1}{P_1}\right) = \frac{1}{\gamma P_1}\left(\frac{1}{5^{\gamma}} - 1\right)$$

where, $P_1 = \dfrac{(n_A + n_B)RT}{V}$

$$\Rightarrow \quad P_1 = \frac{(1+2)\times 8.31\times T}{V} = \frac{24.93T}{V}$$

$$\Rightarrow \quad \Delta K = \frac{1}{\left(\frac{19}{13}\right)(24.93)\left(\frac{T}{V}\right)}\left(\frac{1}{5^{\frac{19}{13}}} - 1\right)$$

$$\Rightarrow \quad \Delta K = -(0.025)\frac{V}{T}$$

Hence, the correct answer is (A).

22. **The correct answer is (B).**

23. **The correct answer is (A).**

24. **The correct answer is (B).**

Combined solution to 22, 23 and 24

Let P_1 and P_2 be the initial pressure in lower chamber of gas and upper chamber of gas, then

$$P_2 = P_0 + \frac{mg}{A} = 2P_0 \text{ and } V_2 = A\times 12\times 10^{-2}\ \text{m}^3$$

If P_2' and V_2' are final pressure and volume in upper chamber, then we have

$$V_2' = A\times(28 - l)\times 10^{-2}\ \text{m}^3$$

Since, $P_2 V_2 = P_2' V_2'$

$$\Rightarrow \quad P_2' = \frac{P_2 V_2}{P_2'} = \frac{24P_0}{28 - l}$$

New consider lower chamber, then

$$P_1 = P_0 + \frac{2mg}{A} = 3P_0 \text{ and } V_1 = A\times 8\times 10^{-2}\ \text{m}^3$$

Also, $P_1' = P_2' + \dfrac{mg}{A} = P_0\left(\dfrac{52 - l}{28 - l}\right)$

and $V_1' = A\times(8 + l)\times 10^{-2}\ \text{m}^3$

Since, $P_1 V_1 = P_1' V_1'$

$$\Rightarrow \quad (3P_0 A)(8\times 10^{-2}) = P_0\left(\frac{52 - l}{28 - l}\right)A\left[(8 + l)\times 10^{-2}\right]$$

$$\Rightarrow \quad 24 = \left(\frac{52 - l}{28 - l}\right)(8 + l)$$

$$\Rightarrow \quad l = 4 \text{ cm}$$

So, $P_1' = 2P_0 = 2\times 10^5\ \text{Am}^{-2}$

and $P_2' = \dfrac{24P_0}{28 - l} = P_0 = 1\times 10^5\ \text{Nm}^{-2}$

$$\Rightarrow \quad \frac{V_2'}{V_1'} = \frac{28 - l}{8 + l} = \frac{24}{12} = 2$$

25. W = Area under the curve $= \dfrac{3}{2}P_1 V_1$

Since, $P_1 V_1 = nRT_1$

$$\Rightarrow \quad \frac{W}{nRT_1} = \frac{\frac{3}{2}P_1 V_1}{P_1 V_1} = \frac{3}{2}$$

Hence, the correct answer is (A).

26. From $FLTD$, we have

$$Q = dU + W$$

CHAPTER 2

Since, $dU = nC_v dT$

For final state, we have

$$P_2V_2 = (2P_1)(2V_1) = 4P_1V_1 = nR(4T_1)$$

Hence final temperature is $4T_1$

$$\Rightarrow \quad dU = n\left(\frac{3R}{2}\right)(3T_1) = \frac{9}{2}nRT_1$$

$$\Rightarrow \quad Q = \frac{3}{2}nRT_1 + \frac{9}{2}nRT_1 = 6nRT_1$$

$$\Rightarrow \quad \frac{Q}{nRT_1} = 6$$

Hence, the correct answer is (D).

27. Since, $Q = 6nRT_1$ …(1)

Also, we know that

$$Q = nC\Delta T$$

$$\Rightarrow \quad Q = nC(3T_1) \qquad \ldots(2)$$

Equating (1) and (2), we get

$$6nRT_1 = nC(3T_1)$$

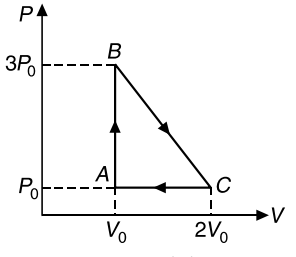
$$\Rightarrow \quad \frac{C}{R} = 2$$

Hence, the correct answer is (B).

28. $ABCA$ is a clockwise cyclic process, so work done by the gas is

$$W = \text{Area of triangle } ABC = \frac{1}{2}(\text{base})(\text{height})$$

$$\Rightarrow \quad W = \frac{1}{2}(2V_0 - V_0)(3P_0 - P_0) = P_0V_0$$



Hence, the correct answer is (B).

29. Number of moles $n = 1$ and gas is monatomic, therefore

$$C_V = \frac{3}{2}R \text{ and } C_P = \frac{5}{2}R$$

$$\Rightarrow \quad \frac{C_V}{R} = \frac{3}{2} \text{ and } \frac{C_P}{R} = \frac{5}{2}$$

Heat rejected in path CA: (process is isobaric)

$$\Rightarrow \quad C \to D:$$

$$\Rightarrow \quad dQ_{CA} = \frac{C_P}{R}(P_fV_f - P_iV_i)$$

Substituting the values

$$dQ_{CA} = \frac{5}{2}(P_0V_0 - 2P_0V_0) = -\frac{5}{2}P_0V_0$$

Hence, the correct answer is (C).

30. Heat rejected in the process AB (process is isochoric) is

$$dQ_{AB} = C_V dT = C_V(T_f - T_i)$$

Also, $Q_{CD} = -900R$ (released)

$$\Rightarrow \quad dU = 0$$

$$dQ_{AB} = 3P_0V_0$$

Heat absorbed in the process AB is $3P_0V_0$

Hence, the correct answer is (B).

31. Let dQ_{BC} be the heat absorbed in the process BC:

Total heat absorbed,

$$dQ = dQ_{CA} + dQ_{AB} + dQ_{BC}$$

$$dQ = \left(-\frac{5}{2}P_0V_0\right) + 3(P_0V_0) + dQ_{BC}$$

$$dQ = dQ_{BC} + \frac{P_0V_0}{2}$$

Change in internal energy, $dU = 0$

$$\Rightarrow \quad dQ = dW$$

$$\Rightarrow \quad dQ_{BC} + \frac{P_0V_0}{2} = P_0V_0$$

$$\Rightarrow \quad dQ_{BC} = \frac{P_0V_0}{2}$$

Heat absorbed in the process BC is $\frac{P_0V_0}{2}$

Hence, the correct answer is (C).

32. Maximum temperature of the gas will be somewhere between B and C. Line BC is a straight line. Therefore, P-V equation for the process BC can be written as

$$P = -mV + c \quad (y = mx + c)$$

Here, $m = \frac{2P_o}{V_o}$ and $c = 5P_o$

$$\Rightarrow \quad P = -\left(\frac{2P_o}{V_o}\right)V + 5P_o$$

Multiplying the equation by V,

$$PV = -\left(\frac{2P_o}{V_o}\right)V^2 + 5P_oV \qquad (PV = RT \text{ for } n = 1)$$

$$+$$

$$RT = -\left(\frac{2P_o}{V_o}\right)V^2 + 5P_oV$$

$$\text{or} \quad T = \frac{1}{R}\left[5P_oV - \frac{2P_o}{V_o}V^2\right] \qquad \ldots(1)$$

For T to be maximum, $\frac{dT}{dV} = 0$

$$\Rightarrow \quad 5P_o - \frac{4P_o}{V_o}V = 0$$

$$\Rightarrow \quad V = \frac{5V_o}{4}$$

i.e. at $V=\frac{5V_o}{4}$, (on line BC), temperature of the gas is maximum. From equation (1) this maximum temperature will be

$$T_{\max}=\frac{1}{R}\left[5P_o\left(\frac{5V_o}{4}\right)-\frac{2P_o}{V_o}\left(\frac{5V_o}{4}\right)^2\right]$$

$$\Rightarrow \quad T_{\max}=\frac{25}{8}\frac{P_oV_o}{R}$$

Hence, the correct answer is (A).

33. $W=\int PdV=\alpha\int_{V_0}^{nV_0}VdV=\frac{\alpha V_0^2}{2}(n^2-1)$

Hence, the correct answer is (A).

34. Since, $dU=C_VdT=\frac{R}{\gamma-1}dT$

$$\Rightarrow \quad \Delta U=\frac{R}{\gamma-1}\int dT$$

Since, $PV=RT=\alpha V^2$

$$\Rightarrow \quad RdT=2\alpha VdV$$

$$\Rightarrow \quad \Delta U=\frac{2\alpha}{(\gamma-1)}\int_{V_0}^{nV_0}VdV=\frac{\alpha}{\gamma-1}V_0^2(n^2-1)$$

Hence, the correct answer is (B).

35. Since, $Q=\Delta U+W$

and $Q=(1)C\Delta T$

$$\Rightarrow \quad C=\left(\frac{\gamma+1}{\gamma-1}\right)\frac{R}{2}$$

Hence, the correct answer is (B).

36. At constant pressure, we have

$$W=P\Delta V=P_0(2V_0-V_0)=P_0V_0$$

$$\Rightarrow \quad \Delta U=3P_0(2V_0)-3P_0(V_0)=3P_0V_0$$

So, from FLTD, we get

$$Q=\Delta U+W=4P_0V_0$$

Hence, the correct answer is (D).

37. Also, for n moles of gas with f degrees of freedom, at temperature T, we have

$$U=\frac{f}{2}nRT=\frac{f}{2}(PV)$$

Since $U=3PV$

So, comparing, we get

$$\frac{f}{2}=3$$

$$\Rightarrow \quad f=6$$

Since, $\gamma=1+\frac{2}{f}$

$$\Rightarrow \quad \gamma=1+\frac{2}{6}$$

$$\Rightarrow \quad \gamma=\frac{4}{3}=1.33$$

Hence, the correct answer is (A).

38. Process $AB: PT=\text{constant}$

Since, $PV=(1)RT$

$$\Rightarrow \quad \left(\frac{RT}{V}\right)T=\text{constant}$$

$$\Rightarrow \quad \frac{RT^2}{V}=\text{constant} \qquad \ldots(1)$$

Since, $W=\int_A^B PdV=\int_A^B\frac{\text{constant}}{T}dV$

Also, $\frac{P_A}{P_B}=\frac{T_B}{T_A}$ $\{\because PT=\text{constant}\}$

$$\Rightarrow \quad \frac{1}{3}=\frac{T_B}{T_A}$$

$$\Rightarrow \quad T_B=\frac{300}{3}=100$$

From (1), we get

$$\frac{dV}{dT}=\frac{2RT}{\text{constant}}$$

$$\Rightarrow \quad W=\int_{300}^{100}\frac{(\text{constant})}{T}\frac{2RT}{(\text{constant})}dt$$

$$\Rightarrow \quad W=2R(100-300)$$

$$\Rightarrow \quad W_{AB}=-400\text{ R}$$

Hence, the correct answer is (B).

39. Since, the process CA is isochoric

So, $\frac{P}{T}=\text{constant}$

$$\Rightarrow \quad \frac{T_A}{T_C}=\frac{P_A}{P_C}$$

$$\Rightarrow \quad \frac{T_A}{T_C}=\frac{P_A}{P_B}=\frac{1}{3}$$

$$\Rightarrow \quad T_C=3T_A=900\text{ K}$$

Since, $\Delta U=nC_V\Delta T$

$$\Rightarrow \quad \Delta U=(1)\frac{3}{2}R\times(T_A-T_C)$$

$$\Rightarrow \quad \Delta U=\frac{3}{2}R\times(300-900)$$

$$\Rightarrow \quad \Delta U=-\frac{3}{2}R\times600=-900R$$

Hence, the correct answer is (D).

40. Since, process BC is isobaric, so we have

$$Q=nC_P\Delta T$$

$$\Rightarrow \quad Q=(1)\frac{5}{2}R(T_C-T_B)=\frac{5}{2}R(900-100)$$

CHAPTER 2

$\Rightarrow \quad Q = \frac{5}{2}R(800) = 2000R$

Hence, the correct answer is (C).

41. Let $T_B = T_0$ and $V_C = V_D = V_0$

Then, $T_D = T_A = 4T_0$ and $V_A = (8\sqrt{2})V_0$

Now, $V_B = \left(\frac{V_A}{T_A}\right)T_B = 2\sqrt{2}V_0$

and $T_C = T_B\left(\frac{V_B}{V_C}\right)^{\gamma-1} = 2T_0$

$\Rightarrow \quad T_C = 2T_B$

Hence, the correct answer is (B).

42. Since, we have

$W = W_{AB} + W_{BC} + W_{CD} + W_{DA}$

$\Rightarrow \quad W = R(T_B - T_A) + nC_V(T_B - T_C) + 0 + RT_D\log_e\left(\frac{V_A}{V_D}\right)$

$\Rightarrow \quad W = -3RT_0 - \frac{3}{2}RT_0 + 0 + 14RT_0\log_e 2$

$\Rightarrow \quad W = 5.2RT_0$

Hence, the correct answer is (A).

43. $Q_{\text{absorbed}} = Q_{CD} + Q_{DA}$

$\Rightarrow \quad Q_{\text{absorbed}} = 3RT_0 + 14RT_0\log_e 2 = 12.7T_0$

So, efficiency, $\eta = \frac{W}{Q_{\text{absorbed}}} \times 100 = 41\%$

Hence, the correct answer is (B).

44. $W_{ab} = W_{cd} = 0$ as V = constant

$W_{da} = nRT_a\log_e\left(\frac{V_a}{V_d}\right)$

$W_{bc} = nRT_b\log_e\left(\frac{V_c}{V_b}\right) = -nRT_b\log_e\left(\frac{V_a}{V_d}\right)$

$\Rightarrow \quad W_{\text{net}} = nR\log_e\left(\frac{V_a}{V_d}\right)(T_a - T_b)$

Since $T_b - T_a > 0$ and $\frac{V_a}{V_d} > 1$

$\Rightarrow \quad W_{\text{net}} = \text{NEGATIVE}$

Hence, the correct answer is (C).

45. $d \to a : T$ = constant

$\Rightarrow \quad \Delta U = 0$

$\Rightarrow \quad Q = W$

Since, volume of the gas is increasing. Hence, W is positive, so Q is positive.

$a \to b : V$ = constant

$\Rightarrow \quad W = 0$

$\Rightarrow \quad Q = \Delta U$

Since, temperature is increasing. Therefore, ΔU will increase, so Q is positive.

Hence, in processes $d \to a$ and $a \to b$ heat is supplied to the gas while in processes $b \to c$ and $c \to d$ heat is taken from the gas.

Hence, the correct answer is (B).

46. From ideal gas equation, we have

$PV = nRT$

$\Rightarrow \quad P = \frac{nRT}{V} = \frac{nR(T_0 + aV^3)}{V}$

$\Rightarrow \quad P = \frac{nRT_0}{V} + nRaV^2$

For P to be minimum, we have $\frac{dP}{dV} = 0$

$\Rightarrow \quad -\frac{nRT_0}{V^2} + 2anRV = 0$

$\Rightarrow \quad \frac{T_0}{V^2} = 2aV$

$\Rightarrow \quad V^3 = \frac{T_0}{2a}$

$\Rightarrow \quad V = \left(\frac{T_0}{2a}\right)^{\frac{1}{3}}$

Hence, the correct answer is (A).

47. From ideal gas equation, we get

$P = \frac{nRT}{V} = \frac{nR(T_0 + aV^3)}{V}$

$\Rightarrow \quad P = \frac{nRT_0}{V} + anRV^2$

$\Rightarrow \quad P = \frac{nRT_0}{T_0^{\frac{1}{3}}}(2a)^{\frac{1}{3}} + anR\frac{T_0^{\frac{2}{3}}}{(2a)^{\frac{2}{3}}}$

For $n = 1$, we get

$\Rightarrow \quad P_{\min} = RT_0^{\frac{2}{3}}(2)^{\frac{1}{3}}(a)^{\frac{1}{3}} + \frac{a^{\frac{1}{3}}RT_0^{\frac{2}{3}}}{2^{\frac{2}{3}}}$

$\Rightarrow \quad P_{\min} = \frac{3RT_0^{\frac{2}{3}}a^{\frac{1}{3}}}{2^{\frac{2}{3}}} = \frac{3}{2}\left(a^{\frac{1}{3}}RT_0^{\frac{2}{3}}\right)(2)^{\frac{1}{3}}$

$\Rightarrow \quad P_{\min} = \frac{3R}{2}\left(2aT_0^2\right)^{1/3}$

Hence, the correct answer is (D).

48. In adiabatic process $TV^{\gamma-1} = \text{constant}$

$\Rightarrow \quad TV^{\gamma-1} = \left(\frac{T}{2}\right)(5.66V)^{\gamma-1}$

$\Rightarrow \quad (5.66V)^{\gamma-1} = 2$

$\Rightarrow \quad (\gamma-1)\ln(5.66) = \ln(2)$

$\Rightarrow \quad \gamma - 1 = 0.4$

$\Rightarrow \quad \gamma = 1.4$

Since, $\gamma = 1 + \frac{2}{f}$

$\Rightarrow \quad f = 5 \qquad \{\text{for } \gamma = 1.4\}$

Hence, the correct answer is (D).

49. Using $PV^{\gamma} = \text{constant}$

$\Rightarrow \quad PV^{1.4} = P_f (5.66V)^{1.4}$

$\Rightarrow \quad P_f = 0.09P$

Since, work done in adiabatic process is given by

$$W = \frac{P_i V_i - P_f V_f}{\gamma - 1} = \frac{(PV) - (0.09P)(5.66V)}{1.4 - 1}$$

$\Rightarrow \quad W = 1.23PV$

Hence, the correct answer is (B).

50. Since, $U = U_0 + 2PV$

$\Rightarrow \quad \Delta U = 2\Delta(PV) = 2R\Delta T$

$\Rightarrow \quad C_V = 2R$

$\Rightarrow \quad C_P = 3R$

Hence, the correct answer is (B).

51. $W = RT\log_e\left(\frac{2V_0}{V_0}\right) = PV\log_e(2) = \frac{U_1 - U_0}{2}\log_e(2)$

Hence, the correct answer is (D).

52. $C_V = 2R$, which lies between C_V value of a monatomic and a diatomic gas.

Hence, the correct answer is (C).

53. Let at any instant t temperature is T. The net rate at which heat is absorbed by the gas is

$$\frac{dQ}{dt} = q - \frac{KA(T - T_0)}{\ell} \qquad \ldots(1)$$

Since, $dQ = nC_P dT = n\left(\frac{7R}{2}\right)dT \qquad \ldots(2)$

$$\Rightarrow \quad n\left(\frac{7R}{2}\right)\frac{dT}{dt} = q - \frac{KA(T - T_0)}{\ell}$$

$$\Rightarrow \quad \int_{T_0}^{T} \frac{dT}{q\ell - KA(T - T_0)} = \frac{2}{7nR\ell}\int_0^t dt$$

Solving, we get

$$T = T_0 + \frac{q\ell}{KA}\left(1 - e^{-\frac{2KAt}{7nR\ell}}\right)$$

Hence, the correct answer is (A).

54. At maximum temperature, we have

$$\frac{dQ}{dt} = 0$$

So, from (1), we get

$$q = \frac{KA}{\ell}(T_{\max} - T_0)$$

$$\Rightarrow \quad T_{\max} = T_0 + \frac{q\ell}{KA}$$

Hence, the correct answer is (B).

55. Since, the process is isobaric, so we have

$$\frac{V_0}{T_0} = \frac{V_{\max}}{T_{\max}}$$

$$\Rightarrow \quad \frac{V_{\max}}{V_0} = \frac{T_{\max}}{T_0}$$

Hence, the correct answer is (D).

56. In process $a \to c$, we observe that

$V \propto T$ i.e., $P = \text{constant}$

$$\Rightarrow \quad Q = nC_p\Delta T = 2\left(\frac{5}{2}R\right)(600 - 300) = 1500R$$

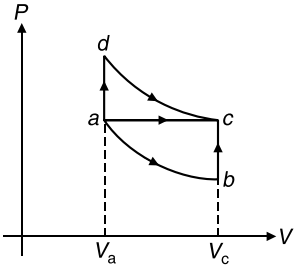
$$\Delta U = nC_V\Delta T = 2\left(\frac{3}{2}R\right)(600 - 300) = 900R$$

From FLTD, we have $Q = \Delta U + W$

$$\Rightarrow \quad W = Q - \Delta U = 600R$$

Hence, the correct answer is (C).

57. The corresponding P-V graph is shown in figure.



Since, area under the P-V graph gives the work done.

Hence, $W_{adc} > W_{ac} > W_{abc}$

$$\Rightarrow \quad W_3 > W_2 > W_1$$

Hence, the correct answer is (D).

58. **The correct answer is (C).**

59. **The correct answer is (B).**

60. **The correct answer is (D).**

Combined solution to 58, 59 and 60

For AB: $P = \text{constant}$

$$\Rightarrow \quad V \propto T$$

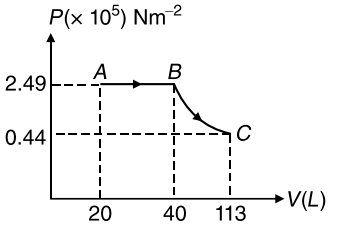
So, if V is doubled, T also becomes two times.

$$T_A = 300\,K$$

$$\Rightarrow \quad T_B = 600\,K$$

$$V_A = 20\,L$$

$$\Rightarrow \quad V_B = 40\,L$$



$$P_A = \frac{nRT_A}{V_A} = \frac{(2)(8.31)(300)}{20 \times 10^{-3}} = 2.49 \times 10^5 \text{ Nm}^{-2}$$

Process 2: Process is adiabatic. So, applying $T^{\gamma}P^{1-\gamma} = \text{constant}$.

$$\Rightarrow \quad \left(\frac{600}{300}\right)^{5/3} = \left(\frac{2.49 \times 10^5}{P_C}\right)^{\frac{5}{3}-1} = \left(\frac{2.49 \times 10^5}{P_C}\right)^{2/3}$$

CHAPTER 2

$$\Rightarrow \quad P_C = \left(2.49\times10^5\right)\left(\frac{300}{600}\right)^{5/2}$$

$$\Rightarrow \quad P_C = 0.44\times10^5 \text{ Nm}^{-2} = 4.4\times10^5 \text{ Nm}^{-2}$$

Similarly using $PV^\gamma = \text{constant}$ we can find that $V_C = 113$ litre

W_{AB} = Area under P-V graph

$$\Rightarrow \quad W_{AB} = \left(2.49\times10^5\right)(40-20)\times10^{-3} = 4980 \text{ J}$$

Also, $W_{BC} = -\Delta U = nC_V\left(T_B - T_C\right)$

$$\Rightarrow \quad W_{BC} = (2)\left(\frac{3}{2}\times8.31\right)(600-300) = 7479 \text{ J}$$

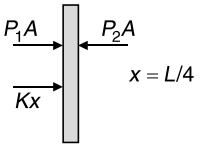
$$\Rightarrow \quad W_{\text{total}} = 12459 \text{ J}$$

61. Since, $P_1A + Kx = P_2A$

$$\Rightarrow \quad \frac{nRT}{(L-x)} + Kx = \frac{nR(T+\Delta T)}{L+x} \qquad \{\because PV = nRT\}$$

$$\Rightarrow \quad \frac{nRT}{\left(\frac{3L}{4}\right)} + K\frac{L}{4} = \frac{nR(T+\Delta T)}{\left(\frac{5L}{4}\right)}$$

$$\Rightarrow \quad \Delta T = \frac{2T}{3} + \frac{5}{16}\frac{KL^2}{nR}$$



Hence, the correct answer is (C).

62. Since, $\Delta U = nC_V\Delta T$

$$\Rightarrow \quad \Delta U = n\left(\frac{3R}{2}\right)\left(\frac{2T}{3} + \frac{5}{16}\frac{KL^2}{nR}\right)$$

$$\Rightarrow \quad \Delta U = nRT + \left(\frac{15}{32}\right)KL^2$$

Hence, the correct answer is (A).

63. Using $FLTD$, we get

$$Q = \Delta U + W$$

where $W = \frac{1}{2}Kx^2 = \frac{1}{2}K\left(\frac{L}{4}\right)^2$

$$\Rightarrow \quad W = \frac{1}{2}\frac{KL^2}{16} = \frac{KL^2}{32}$$

$$\Rightarrow \quad Q = \Delta U + W = nRT + \frac{KL^2}{2}$$

Hence, the correct answer is (B).

64. Since, $T = T_0 + \alpha V^3$

For an Ideal Gas, we have

$$PV = nRT$$

$$\Rightarrow \quad T = \frac{PV}{nR}$$

$$\Rightarrow \quad \frac{PV}{nR} = T_0 + \alpha V^3$$

$$\Rightarrow \quad P = \frac{nRT_0}{V} + (nR\alpha)V^2$$

Now P is minimum, when $\frac{dP}{dV} = 0$

$$\Rightarrow \quad -\frac{nRT_0}{V^2} + nR\alpha(2V) = 0$$

$$\Rightarrow \quad \frac{T_0}{V^2} = 2\alpha V$$

$$\Rightarrow \quad V^3 = \frac{T_0}{2\alpha}$$

$$\Rightarrow \quad V = \left(\frac{T_0}{2\alpha}\right)^{\frac{1}{3}}$$

Hence, the correct answer is (C).

65. Since, $C = C_V + \frac{P}{n}\left(\frac{dV}{dT}\right)$

$$\Rightarrow \quad C = C_V + \frac{P}{n(3\alpha V^2)} \qquad \left\{\because \frac{dV}{dT} = 3\alpha V^2\right\}$$

But $P = \frac{nRT_0}{V} + (nR\alpha)V^2$

$$\Rightarrow \quad C = C_V + \left(\frac{nRT_0}{V} + (nR\alpha)V^2\right)\frac{1}{3n\alpha V^2}$$

$$\Rightarrow \quad C = C_V + \frac{nRT_0}{V(3n\alpha V^2)} + \frac{nR\alpha V^2}{3n\alpha V^2}$$

$$\Rightarrow \quad C = C_V + \frac{R}{3} + \frac{RT_0}{3\alpha V^3}$$

Hence, the correct answer is (A).

66. Since, $P = \frac{nRT_0}{V} + (nR\alpha)V^2$

This P is minimum, when

$$V = \left(\frac{T_0}{2\alpha}\right)^{\frac{1}{3}}$$

$$\Rightarrow \quad P_{\min} = \frac{nRT_0}{\left(\frac{T_0}{2\alpha}\right)^{\frac{1}{3}}} + nR\alpha\left(\frac{T_0}{2\alpha}\right)^{\frac{2}{3}}$$

$$\Rightarrow \quad P_{\min} = 2^{\frac{1}{3}}nRT_0^{\frac{2}{3}}\alpha^{\frac{1}{3}} + 2^{\frac{2}{3}}nRT_0^{\frac{2}{3}}\alpha^{\frac{1}{3}}$$

$$\Rightarrow \quad P_{\min} = 2^{\frac{1}{3}}nRT_0^{\frac{2}{3}}\alpha^{\frac{1}{3}}\left(1+2^{\frac{1}{3}}\right)$$

$$\Rightarrow \quad \frac{P_{\min}}{nRT_0^{\frac{2}{3}}\alpha^{\frac{1}{3}}} = 2^{\frac{1}{3}}\left(1+2^{\frac{1}{3}}\right)$$

$$\Rightarrow \quad \frac{P_{\min}}{nRT_0^{\frac{2}{3}}\alpha^{\frac{1}{3}}} = \left(2^{\frac{1}{3}}+2^{\frac{2}{3}}\right)$$

Hence, the correct answer is (B).

67. $E = \int_0^\infty E_\lambda d\lambda$

The area under the curve represents the total intensity (energy per unit area per second) radiated at a particular temperature.
Hence, the correct answer is (C).

68. According to the Stefan's Law, the intensity of each wavelength increases by increasing the temperature and the area under the curve increases in fourth power of temperature.

According to Wein's Law,

$$\lambda_m \propto \frac{1}{T}$$

Hence, the correct answer is (D).

69. While answering, we must keep in mind that the curve at higher temperature

(i) lies above the curve at lower temperature.
(ii) never intersects with the curve at lower temperature.
(iii) the maximum occurs at a lower wavelength.

Hence, the correct answer is (B).

70. Initially, we have $U_i = n_1\left(\frac{3}{2}k_BT_1\right)+n_1\left(\frac{3}{2}k_BT_2\right)$

$$\Rightarrow \quad U_i = \frac{3}{2}k_B(n_1T_1+n_2T_2)$$

Hence, the correct answer is (B).

71. Finally, we have $U_i = \frac{3}{2}k_BT(n_1+n_2)$

Hence, the correct answer is (B).

72. Heat lost by one equals the heat gained by the other gas, so we have

$$\frac{n_1}{N_A}\left(\frac{3R}{2}\right)(T_1-T)=\frac{n_2}{N_A}\left(\frac{3R}{2}\right)(T-T_2)$$

$$\Rightarrow \quad T=\frac{n_1T_1+n_2T_2}{n_1+n_2}$$

Hence, the correct answer is (D).

73. $C=\frac{R}{\gamma-1}+\frac{R}{1-x}=C_V+\frac{R}{1-x}$

Since it is given that $P \propto V$

$$\Rightarrow \quad PV^{-1} = \text{constant}$$

$$\Rightarrow \quad x = -1$$

$$\Rightarrow \quad C = C_V+\frac{R}{2}=\frac{5R}{2}+\frac{R}{2}=3R$$

Hence, the correct answer is (A).

74. $v_{\text{rms}} \propto \sqrt{T}$

Since, rms speed has doubled. Therefore, temperature must have become four times.

$$\Rightarrow \quad Q = nC\Delta T$$

$$\Rightarrow \quad Q = nC(4T_0-T_0)$$

$$\Rightarrow \quad Q = 3n(3RT_0)$$

$$\Rightarrow \quad Q = 9P_0V_0 \qquad \{\because\ nRT_0 = P_0V_0\}$$

Hence, the correct answer is (C).

75. Since, $T_i = T_f$, therefore net change in internal energy, $dU = 0$.

Hence, the correct answer is (C).

76. Given number of moles $n = 2$

Process AB and CD are isobaric.

Hence, $Q_{AB} = -Q_{CD}$

[Because $(\Delta T)_{AB} = +100\,K$

whereas $(\Delta T)_{CD} = -100\,K$ and $Q_{isobaric} = nC_P\Delta T$]

or $\quad Q_{AB}+Q_{CD}=0$

Process BC is isothermal $(dU=0)$

$$\Rightarrow \quad Q_{BC} = W_{BC} = nRT_B\ln\left(\frac{P_B}{P_C}\right)$$

$V_A = 10\ \text{m}^3$, $Q_{BC} = 4608\ \text{J}$

Similarly, process DA is also isothermal hence

$$Q_{DA} = W_{DA} = nRT_o\ln\left(\frac{P_D}{P_A}\right)$$

$$\Rightarrow \quad Q_{DA} = (2)(8.31)(300)\ln\left(\frac{1}{2}\right)$$

$$Q_{DA} = 3456\ \text{J}$$

(a) Net heat in the process

$$Q = Q_{AB}+Q_{BC}+Q_{CD}+Q_{DA} = (4608-3456)\ \text{J}$$

$$Q = 1152\ \text{J}$$

(b) From First Law of Thermodynamics, $dU = 0$ (in complete cycle) $dQ = dW$

Hence, net work done in the cycle

$$W = Q = 1152\ \text{J}$$

$$W = 1152\ \text{J}$$

Hence, the correct answer is (D).

77. Since entropy is a state function, so $S_f = S_i$ and hence dS for one complete cycle is zero.

Hence, the correct answer is (D).

Matrix Match/Column Match Type Questions

1. A → (q, t); B → (r); C → (s); D → (p)

For a cyclic process, $\Delta U = 0$

By $FLTD$, we get $Q = W$

For Isobaric process, $W = nR\Delta T$

For Isochoric process $W = 0$, so $Q = \Delta U$

Since, adiabatic expansion is accompanied by decrease in temperature, so a decrease in internal energy.

2. A → (p, s); B → (p, r); C → (q, s); D → (q, r)

Process $a \to b$:

$$T = \text{constant}$$

$$\Rightarrow \quad \Delta T = 0$$

$$\Rightarrow \quad \Delta U = 0$$

$$\Rightarrow \quad P \propto \frac{1}{V}$$

ρ is increasing. Therefore, V is decreasing. Since, $V \propto \frac{1}{\rho}$ or P will increase.

Process $b \to c$:

$$\rho = \text{constant}$$

$$\Rightarrow \quad V = \text{constant}$$

Therefore, $P \propto T$

Since, temperature is increasing. Hence pressure should also increase.

CHAPTER 2

3. A → (p, s); B → (q, r); C → (p, r, s); D → (p, s)

4. A → (r); B → (s); C → (p); D → (q)
Since, Isothermal Bulk modulus is

$$B_{iso} = P = \frac{RT}{V} \text{ and}$$

Adiabatic Bulk modulus is

$$B_{ad} = \gamma P = \frac{5RT}{3V} \qquad \left\{\because \gamma_{monatomic} = \frac{5}{3}\right\}$$

$$(\text{Slope})_{isot} = \left(\frac{dP}{dV}\right)_{isot} = -\frac{P}{V} = -\frac{RT}{V^2}$$

$$(\text{Slope})_{ad} = \left(\frac{dP}{dV}\right)_{ad} = -\gamma\left(\frac{P}{V}\right) = -\frac{5P}{3V}$$

5. A → (r); B → (p); C → (s); D → (q)

6. A → (p, s); B → (r, s); C → (p, t); D → (q, t)
$M \to N$ **is an Isochoric Process**
$\Rightarrow \quad V = \text{constant i.e., } P \propto T$
$\Rightarrow \quad W_{isoc} = 0$
Since, P is increasing, so T is also increasing and hence $\Delta U > 0$
$N \to O$ **is an Isobaric Process**
$\Rightarrow \quad P = \text{constant i.e., } V \propto T$
Since, V is increasing, so T is also increasing and hence $\Delta U > 0$
Also, $W > 0$ for isobaric expansion.
$O \to P$ **is an Isochoric Process**
$\Rightarrow \quad V = \text{constant i.e., } P \propto T$
$\Rightarrow \quad W_{isoc} = 0$
Since, P is decreasing, so T is also decreasing and hence $\Delta U < 0$
$P \to M$ **is an Isobaric Process**
$\Rightarrow \quad P = \text{constant i.e., } V \propto T$
Since, V is decreasing, so T is also decreasing and hence $\Delta U < 0$
Furthermore, for isobaric compression $W < 0$

7. A → (u); B → (p, q, t); C → (s); D → (r)

Since $W_{ad} = \frac{R}{1-\gamma}(T_f - T_i)$ for 1 mole of gas

$W_{isochoric} = 0$

$\Delta U_{isothermal} = 0$

$C_{adiabatic} = 0$

$W_{isobaric} = (1)R\Delta T = R(T_f - T_i)$

Since, $U = nC_V T = \frac{nRT}{\gamma - 1} \qquad \left\{\because C_V = \frac{R}{\gamma - 1}\right\}$

$\Rightarrow \quad U = \frac{PV}{\gamma - 1}$

8. Conceptual
A → (s); B → (r); C → (q); D → (p)

9. A → (s); B → (r); C → (q); D → (p)

For an ideal monatomic gas $PV^{\gamma} = \text{constant}$, where $\gamma = \frac{5}{3}$

$\Rightarrow \quad PV^{5/3} = \text{constant(for an Adiabatic process)}$

$\Rightarrow \quad C_{adiabatic} = 0$

Since, for a polytropic process, $TV^{x-1} = \text{constant}$

$\Rightarrow \quad x - 1 = 2$

$\Rightarrow \quad x = 3$

Also, $C = C_V + \frac{R}{1-x}$

$\Rightarrow \quad C = C_V - \frac{R}{2}$

$\Rightarrow \quad C = \frac{3R}{2} - \frac{R}{2} \qquad \left\{\because C_V = \frac{3R}{2}\right\}$

$\Rightarrow \quad C = R$

When $V = \text{constant}$, then $C = C_V = \frac{3R}{2}$

When $P = \text{constant}$, then $C = C_P = \frac{5R}{2}$

10. Conceptual
A → (p, q, r); B → (p, r, s); C → (p, q, r); D → (p, q, r)

11. A → (s); B → (r); C → (q); D → (t)
When A **and** B **are mixed**

$$(m)(c)(T - 20) = (2m)(c)(40 - T)$$

$\Rightarrow \quad T = \frac{100}{3} = 33.3\ ^\circ\text{C}$

When A **and** C **are mixed**

$$(m)(c)(T - 20) = (3m)(c)(60 - T)$$

$\Rightarrow \quad T = 50\ ^\circ\text{C}$

When B **and** C **are mixed**

$$(2m)(c)(T - 40) = (3m)(c)(60 - T)$$

$\Rightarrow \quad T = 52\ ^\circ\text{C}$

When A, B **and** C **are mixed**

$$\begin{pmatrix}\text{Heat Gained by}\\ A + B\end{pmatrix} = \begin{pmatrix}\text{Heat Lost by}\\ \text{C}\end{pmatrix}$$

$\Rightarrow \quad mc(T - 20) + (2m)c(T - 40) = (3m)c(60 - T)$

$\Rightarrow \quad (T - 20) + 2(T - 40) = 3(60 - T)$

$\Rightarrow \quad 3T - 100 = 180 - 3T$

$\Rightarrow \quad 6T = 280$

$\Rightarrow \quad T = 46.67\ ^\circ\text{C}$

Integer/Numerical Answer Type Questions

1. Let m be the mass of the container
Initial temperature of container is

$$T_i = (227 + 273) = 500\ \text{K}$$

And final temperature of container is

$$T_f = (27 + 273) = 300\ \text{K}$$

Now, $\begin{pmatrix}\text{Heat Gained by}\\ \text{the Ice Cube}\end{pmatrix} = \begin{pmatrix}\text{Heat Lost by}\\ \text{the Container}\end{pmatrix}$

$$m_{ice}L_{ice} + m_{ice}c_{water}\Delta T = -m\int_{T_i}^{T_f} cdT$$

$$\Rightarrow \quad (0.1)\left(8\times10^4\right)+(0.1)\left(10^3\right)(27) = -m\int_{500}^{300}(A+BT)dT$$

$$\Rightarrow \quad 10700 = -m\left(AT+\frac{BT^2}{2}\right)\Big|_{500}^{300}$$

After substituting the values of A and B and the proper limits, we get

$$m = 0.495 \text{ kg}$$

$$\Rightarrow \quad m = 495 \text{ g}$$

2. Thermal resistance $R = \dfrac{\ell}{kA}$

$$\Rightarrow \quad \frac{R_x}{R_y} = \frac{k_y}{k_x} \qquad \{\because \ell_x = \ell_y \text{ and } A_x = A_y\}$$

$$\Rightarrow \quad \frac{R_x}{R_y} = \frac{0.46}{0.92} = \frac{1}{2}$$

If $R_x = R$ then $R_y = 2R$

$CEDB$ forms a balanced Wheatstone bridge, i.e., $T_C = T_D$ and no heat flows through CD, so

$$\frac{1}{R_{BE}} = \frac{1}{R+R} + \frac{1}{2R+2R}$$

$$\Rightarrow \quad R_{BE} = \frac{4}{3}R$$

The total resistance between A and E will be,

$$R_{AE} = R_{AB} + R_{BE} = 2R + \frac{4}{3}R = \frac{10}{3}R$$

Now, heat current between A and E is

$$H = \frac{(\Delta T)_{AE}}{R_{AE}} = \frac{(60-10)}{\left(\frac{10}{3}\right)R} = \frac{15}{R}$$

If T_B is the temperature at B, then

$$H_{AB} = \frac{(\Delta T)_{AB}}{R_{AB}}$$

$$\Rightarrow \quad \frac{15}{R} = \frac{60-T_B}{2R}$$

$$\Rightarrow \quad T_B = 30 \text{ °C}$$

Further, $H_{AB} = H_{BC} + H_{BD}$

$$\Rightarrow \quad \frac{15}{R} = \frac{30-T_C}{R} + \frac{30-T_D}{2R} \qquad \{T_C = T_D = T(\text{say})\}$$

$$\Rightarrow \quad 15 = (30-T) + \frac{(30-T)}{2}$$

Solving this, we get

$$T = 20 \text{ °C}$$

$$\Rightarrow \quad T_C = T_D = 20 \text{ °C}$$

3. For an adiabatic process, we have

$$P_f = \left(\frac{V_i}{V_f}\right)^{\gamma} P_i \qquad \ldots(1)$$

Further, $C_V = \dfrac{3R}{2}$

$$\Rightarrow C_P = C_V + R = \frac{5R}{2}$$

$$\Rightarrow \quad \gamma = \frac{C_P}{C_V} = \frac{5}{3}$$

$$\Rightarrow \quad P_f = \left(\frac{6}{2}\right)^{5/3}(10)^5 = 6.24\times10^5 \text{ Nm}^{-2}$$

Now, work done in adiabatic process is given by

$$W = \frac{P_iV_i - P_fV_f}{\gamma - 1}$$

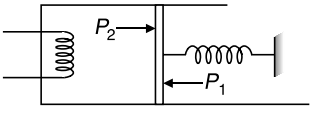
$$\Rightarrow \quad W = \frac{10^5\times6\times10^{-3} - 6.24\times10^5\times2\times10^{-3}}{\left(\frac{5}{3}\right)-1}$$

$$\Rightarrow \quad W = -972 \text{ J}$$

$$\Rightarrow \quad \text{Work done on the gas is } 972 \text{ J}$$

4. (a) Since, $\dfrac{P_1V_1}{T_1} = \dfrac{P_2V_2}{T_2}$

$$\Rightarrow \quad T_2 = \left(\frac{P_2V_2T_1}{P_1V_1}\right) \qquad \ldots(1)$$



Here, $P_1 = 1\times10^5 \text{ Nm}^{-2}$, $V_1 = 2.4\times10^{-3} \text{ m}^3$, $T_1 = 300 \text{ K}$

Since the piston is light and moves out slowly, so we have

$$P_2A = P_1A + kx$$

$$\Rightarrow \quad P_2 = P_1 + \frac{kx}{A} = (1\times10^5) + \left(\frac{8000\times0.1}{8\times10^{-3}}\right)$$

$$\Rightarrow \quad P_2 = 2\times10^5 \text{ Nm}^{-2}$$

and $V_2 = V_1 + Ax = 2.4\times10^{-3} + 8\times10^{-3}\times0.1$

$$\Rightarrow \quad V_2 = 3.2\times10^{-3} \text{ m}^3$$

Substituting in equation (1), we get

$$T_2 = 800 \text{ K}$$

(b) Heat supplied by the heater, using the FLTD is

$$Q = W + \Delta U$$

where, $\Delta U = nC_V\Delta T = \left(\dfrac{P_1V_1}{RT_1}\right)\left(\dfrac{3}{2}R\right)(800-300)$

$$\Rightarrow \quad \Delta U = \frac{(1\times10^5)(2.4\times10^{-3})(1.5)(500)}{(300)}$$

CHAPTER 2

$\Rightarrow \quad \Delta U = 600$ J

Further, $W = \frac{1}{2}kx^2 + P_1\Delta V$

$\Rightarrow \quad W = \frac{1}{2}\times(8000)(0.1)^2 + (1\times10^5)(0.1)(8\times10^{-3})$

$\Rightarrow \quad W = (40+80)$ J

$\Rightarrow \quad W = 120$ J

$\Rightarrow \quad Q = 600 + 120 = 720$ J

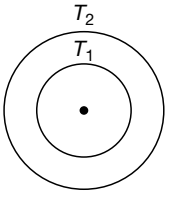
5. Initially when only one sphere is there

$$P_{\text{source}} = \sigma A T_0^4 \qquad \{\because (T_0 = 1000 \text{ K})\}$$

After enveloping, let temperature of the inner sphere is T_1 and that of the outer sphere is T_2

$$\sigma A T_1^4 = P_{\text{source}} = \sigma A T_0^4$$

$\therefore \quad T_1 = T_0 = 1000$ K



Further, $\sigma A T_2^4 - \sigma A T_1^4 = P_{\text{source}} = \sigma A T_0^4$

$\Rightarrow \quad T_2^4 = 2T_0^4$

$\Rightarrow \quad T_2 = (2)^{1/4}(1000)$

$\Rightarrow \quad T_2 = (1.19)(1000)$

$\Rightarrow \quad T_2 = 1190$ K

6. For the given case $\Delta T = 30$ °C

Expansion of girders is $\Delta l = l\alpha\Delta T$

$\Rightarrow \quad \Delta l = 12\times1.1\times10^{-5}\times30 = 0.00396$ m

$\Rightarrow \quad \Delta l = 3.96$ mm

7. For a cyclic process, $\Delta U = 0$

$\Rightarrow \quad Q_{\text{cycle}} = W_{\text{cycle}}$ …(1)

where, $W_{\text{cycle}} = W_{A\to B} + W_{B\to C} + W_{C\to A}$ …(2)

Process $C \to A$ is isochoric, hence $W_{C\to A} = 0$

Process $A \to B$ is isobaric, as its T-V graph is a straight line passing through origin.

In an isobaric process, work done is

$$W_{A\to B} = P(V_B - V_A) = nR(T_B - T_A)$$

$\Rightarrow \quad W_{A\to B} = 1\times8.3(400-300) = 830$ J …(3)

$W_{A\to B}$ is positive, so expansion of gas takes place.

From equations (1), (2) and (3), we get

$$-1000 = 830 + W_{B\to C} + 0$$

$\Rightarrow \quad W_{B\to C} = -1830$ J

Negative work done implies that compression of the gas takes place or work is done on the system/gas.

8. Since $t = 2\pi\sqrt{\frac{\ell}{g}}$

For a pendulum to give correct time, its period must be 2 s.

$\Rightarrow \quad 2 = 2\pi\sqrt{\frac{\ell_0}{9.8}}$ …(1)

$\Rightarrow \quad 2 = 2\pi\sqrt{\frac{\ell_0[1+12\times10^{-6}(T-32)]}{9.788}}$ …(2)

Equating (1) and (2), we get

$$\frac{\ell_0}{9.8} = \frac{\ell_0}{9.788}[1+12\times10^{-6}(T-32)]$$

$\Rightarrow \quad T - 32 = -102$

$\Rightarrow \quad T = -102 + 32$

$\Rightarrow \quad T = -70$ °C

9. Since process is a cyclic process, so we have

So, we have

$\Delta U = 0$ {Answer to Part (c)}

Applying FLTD, we get

$$Q_{\text{net}} = W_{\text{net}} = W_{AB} + W_{BC} + W_{CD} + W_{DA}$$

Now $W_{AB} = nR\Delta T = (2)R(100) = 400$ cal

$W_{BC} = (2)R(400)\log_e(2) = 1120$ cal

$W_{CD} = nR\Delta T = (2)R(-100) = -400$ cal

$W_{DA} = (2)R(300)\log_e\left(\frac{1}{2}\right) = -840$ cal

So, $W_{\text{net}} = 400 + 1120 - 400 - 840$

$\Rightarrow \quad W_{\text{net}} = 280$ cal {Answer to Part (b)}

$\Rightarrow \quad Q_{\text{net}} = 280$ cal {Answer to Part (a)}

10. Thermal expansion in rod is $\Delta l = l\alpha\Delta T$

Due to clamps, the elastic strain developed in the rod is

$$\left|\frac{\Delta l}{l}\right| = \alpha\Delta T = (1.2\times10^{-5})(30) = 36\times10^{-5}$$

11. Since, in steady state, rate of heat flow is

$$H = \frac{4\pi K r_1 r_2 \Delta T}{r_2 - r_1} = \frac{(4)(\pi)(0.002)(11)(9)(100-0)}{(11-9)}$$

$\Rightarrow \quad H = \frac{dQ}{dt} = 124.4$ cals^{-1}

This rate should be equal to $L\left(\frac{dm}{dt}\right)$

$\Rightarrow \quad \left(\frac{dm}{dt}\right) = \frac{\frac{dQ}{dt}}{L} = \frac{124.4}{80} = 1.555$ gs^{-1}

Total mass of ice is $m = \rho_{\text{ice}}(4\pi r_1^2)$

$\Rightarrow \quad m = (0.9)(4)(\pi)(9)^2 = 916$ g

So, time taken for the ice to melt completely is

$$t = \frac{m}{\left(\frac{dm}{dt}\right)} = \frac{916}{1.555} = 589 \text{ sec.}$$

12. (a) $P = \alpha\sqrt{T}$

$\Rightarrow \quad PT^{-\frac{1}{2}} = \text{constant}$

$\Rightarrow \quad P^{\frac{1}{2}}V^{-\frac{1}{2}} = \text{constant}$

$\Rightarrow \quad PV^{-1} = \text{constant}$

Compare with $PV^x = \text{constant}$, we get

$x = -1$

$\Rightarrow \quad W = \left(\frac{R}{1-x}\right)(\Delta T)$

$\Rightarrow \quad W = \frac{R}{2}(50) = 25R$

$\Rightarrow \quad * = 25$

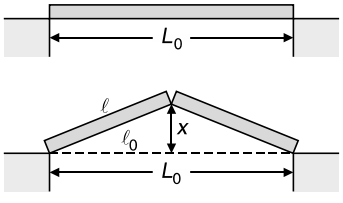
(b) Since, $C = C_V + \frac{R}{1-x} = \frac{3}{2}R + \frac{R}{2} = 2R$

$\Rightarrow \quad C = 2R$

$\Rightarrow \quad * = 2$

13. Consider one-half of the bar, with original length

$\ell_0 = \frac{L_0}{2}$



The length after temperature increase is

$\ell = \ell_0 + \alpha \ell_0 \Delta T$

From Pythagoras theorem, we have

$x^2 = \ell^2 - \ell_0^2 = \ell_0^2(1+\alpha\Delta T)^2 - \ell_0^2$

$\Rightarrow \quad x^2 = \ell_0^2(1+2\alpha\Delta T) - \ell_0^2 = 2\ell_0^2\alpha\Delta T$

$\Rightarrow \quad x = \ell_0\sqrt{2\alpha\Delta T}$

$\Rightarrow \quad x = \frac{4}{2}\sqrt{2(25\times10^{-6})(32)}$

$\Rightarrow \quad x = 2\times40\times10^{-3}$

$\Rightarrow \quad x = 80 \text{ mm}$

14. Since it is given that, initially the separator is in equilibrium, so pressures on both sides of gas are equal say P_i. If A be the area of cross-section of cylinder, then number of moles of gas in left and right part are n_1 and n_2 given by

$n_1 = \frac{P_i(10A)}{R(100)}$ and $n_2 = \frac{P_i(20A)}{R(400)}$

Finally, when separator is displaced to right through a distance x, then we have

$n_1 = \frac{P_f(10+x)A}{RT_f}$ and $n_2 = \frac{P_f(20-x)A}{RT_f}$

where, P_f and T_f be the final pressure and temperature on both sides after a long time.

Equating the ratio of number of moles $\frac{n_1}{n_2}$ in initial and final state, we get

$\frac{n_1}{n_2} = \frac{(10A/100)}{(20A/400)} = \frac{(10+x)A}{(20-x)A}$

$\Rightarrow \quad 2(20-x) = 10+x$

$\Rightarrow \quad x = 10 \text{ cm}$

Hence in final state, when gases in both parts are in thermal equilibrium, the piston is displaced 10 cm to the right from its initial position.

15. When the temperature is increased, volume of the cube will increase while density of liquid will decrease. The depth upto which the cube is submerged in the liquid remains the same, hence the upthrust will not change.

$F = F'$

$\Rightarrow \quad V_i\rho_L g = V_i'\rho'_L g$ {where V_i is the volume immersed}

$\Rightarrow \quad (Ah_i)(\rho_L)(g) = A(1+2\alpha_s\Delta T)(h_i)\left(\frac{\rho_L}{1+\gamma_l\Delta T}\right)g$

Solving this equation, we get $\gamma_\ell = 2\alpha_s$

$\Rightarrow \quad \frac{\gamma_\ell}{\alpha_s} = 2$

16. $v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = v_e = 11.2 \text{ kms}^{-1} = 11.2\times10^3 \text{ ms}^{-1}$

$\Rightarrow \quad \sqrt{\frac{3RT}{M}} = 11.2\times10^3$

$\Rightarrow \quad \frac{3(8.314)T}{\left(\frac{2}{1000}\right)} = (11.2\times10^3)^2$

$\Rightarrow \quad T \approx 20000 \text{ K}$

17. $n = 1,\ \gamma = \frac{7}{5}$

$\Rightarrow \quad \gamma - 1 = \frac{2}{5}$

$Q_{AB} = Q_{CD} = 0$

Let $T_A = T_1$ and $T_D = T_2$ with $T_2 > T_1$

For the process AB, we have

$TV^{\gamma-1} = \text{constant}$

$\Rightarrow \quad TV^{\frac{2}{5}} = \text{constant}$

$\Rightarrow \quad T_A(10V_0)^{\frac{2}{5}} = T_B(V_0)^{\frac{2}{5}}$

$\Rightarrow \quad T_B = T_1(10)^{0.4} \approx 2.5T_1$

Similarly, $T_C = 2.5T_2$

Now, $Q_{BC} = C_V\Delta T$

$\Rightarrow \quad Q_{BC} = \left(\frac{5}{2}R\right)(T_C - T_B) = (6.25)(T_2 - T_1)$ and

$Q_{DA} = -2.5(T_2 - T_1)$

So, $W_{\text{net}} = Q_{\text{net}} = 3.75(T_2 - T_1)$ $\{\because \Delta U = 0\}$

$\Rightarrow \quad \eta = \frac{W_{\text{net}}}{\text{heat absorbed}}\times100 = \frac{3.75}{6.25}\times100 = 60\%$

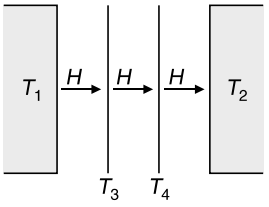
18. Under steady state conditions the net rate of heat flow per unit area is the same everywhere.

$H = \sigma(T_1^4 - T_3^4)$

CHAPTER 2

$$H=\sigma\left(T_3^4-T_4^4\right)$$

$$H=\sigma\left(T_4^4-T_2^4\right)$$



Adding these three equations, we get

$$3H=\sigma\left(T_1^4-T_2^4\right)=H_0$$

Here $H_0=\sigma\left(T_1^4-T_2^4\right)$ is the rate of heat flow per unit area in the absence of the shields.

So, the desired fraction is

$$f=\frac{H}{H_0}=\frac{1}{3}$$

$\Rightarrow \quad x=3.$

19. Piston A is free to move, so gas will expand isobarically and piston B is fixed, so the process is isochoric.

Given that, $Q_A=Q_B$

$\Rightarrow \quad nC_P\Delta T_A=nC_V\Delta T_B$

$\Rightarrow \quad \left(\frac{7R}{2}\right)(30)=\left(\frac{5R}{2}\right)\Delta T_B$

$\Rightarrow \quad \Delta T_B=42\text{ K}$

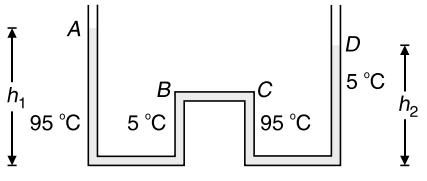
20. Density of a liquid varies with temperature as

$$\rho_{t\,^\circ C}=\left(\frac{\rho_{0\,^\circ C}}{1+\gamma t}\right)$$

where, γ is the coefficient of volume expansion of temperature.

In the figure,

$h_1=52.8$ cm, $h=49$ cm, $h_2=51$ cm



Now, (Pressure at B) = (Pressure at C)

$\Rightarrow \quad P_o+h_1\rho_{95^\circ}g-h\rho_{5^\circ}g=P_0+h_2\rho_{5^\circ}g-h\rho_{95^\circ}g$

$\Rightarrow \quad \rho_{95^\circ}\left(h_1+h\right)=\rho_{5^\circ}\left(h_2+h\right)$

$\Rightarrow \quad \frac{\rho_{95^\circ}}{\rho_{5^\circ}}=\frac{h_2+h}{h_1+h}$

$\Rightarrow \quad \frac{\left(\frac{\rho_{0^\circ}}{1+95\gamma}\right)}{\left(\frac{\rho_{0^\circ}}{1+5\gamma}\right)}=\frac{h_2+h}{h_1+h}$

$\Rightarrow \quad \frac{1+5\gamma}{1+95\gamma}=\frac{51+49}{52.8+49}=\frac{100}{101.8}$

Solving this equation, we get

$$\gamma=2\times10^{-4}\,(^\circ\text{C})^{-1}$$

$\Rightarrow \quad x=2$

21. For an isobaric process, we have

$$W_{\text{isob}}=P\Delta V=nR\Delta T$$

$\Rightarrow \quad W_{\text{isob}}=(0.2)(8.3)(100)=166\text{ J}$

22. Thermal resistance of the rod,

$$R=\frac{\ell}{kA}=\frac{1}{(401)\left(4\times10^{-4}\right)}=6.23\text{ KW}^{-1}$$

$\Rightarrow \quad$ Heat current $H=\frac{\text{Temperature difference}}{\text{Thermal resistance}}$

$\Rightarrow \quad H=\frac{(100-0)}{6.23}=16$ watt

Heat transferred in 1 hr. is $Q=Ht$ $\qquad \left\{\because H=\frac{Q}{t}\right\}$

$\Rightarrow \quad Q=(16)(3600)=57600\text{ J}$

Now, if m mass of ice melts in 1 hr., then $Q=mL$

$$m=\frac{Q}{L}=\frac{57600}{3.35\times10^5}$$

$\Rightarrow \quad m=0.172\text{ kg}=172\text{ g}$

23. Force by rod on wall is $F=YA\alpha\Delta T$

$\Rightarrow \quad F=\left(2\times10^{11}\right)\left(2\times10^{-6}\right)\left(1.2\times10^{-5}\right)(80)=384\text{ N}$

24. In both the cases, the weight of the body is balanced by the buoyant force acting on it. At $t_0=0\ ^\circ\text{C}$, we have buoyant force given by

$$F_b=\left(\delta V_0\right)\rho_0 g \qquad \ldots(1)$$

where δ is the fraction of volume submerged in the liquid, V_0 is the volume of body and ρ_0 is the density of liquid at $t_0=0\ ^\circ\text{C}$.

At $t=50\ ^\circ\text{C}$, the volume of the body now becomes $V=V_0\left(1+\gamma_s t\right)$

Further, the density of the liquid at $t=50\ ^\circ\text{C}$ is $\rho_1=\frac{\rho_0}{1+\gamma_\ell t}$

So, the buoyant force is now given by

$$F_b=\frac{V_0\rho_0 g\left(1+\gamma_s t\right)}{1+\gamma_\ell t} \qquad \ldots(2)$$

From (1) and (2), we get $\%\delta=96\%$

25. For an adiabatic process $TV^{\gamma-1}=\text{constant}$

$\Rightarrow \quad TV^{\gamma-1}=\left(\frac{T}{2}\right)(5.66V)^{\gamma-1}$

$\Rightarrow \quad (5.66V)^{\gamma-1}=2$

$\Rightarrow \quad (\gamma-1)\log_e(5.66)=\log_e(2)$

$\Rightarrow \quad \gamma-1=0.4$

$\Rightarrow \quad \gamma=1.4$

Since, $\gamma=1+\frac{2}{f}=1.4$

$\Rightarrow \quad \frac{2}{f} = 0.4$

$\Rightarrow \quad f = \frac{2}{0.4} = 5$

26. $W = \text{Area Enclosed} = (10)(4-1)(3-1)\times 10^5 \text{ Nm}^{-2}$

$\Rightarrow \quad W = 6\times 10^6 \text{ J} = 6 \text{ MJ}$

If the net work is done by the gas, then direction of traversal around the cycle is clockwise.

27. Let m be the mass of the steam required to raise the temperature of 100 g of water from 24 °C to 90 °C. Then

$$\begin{pmatrix}\text{Heat Lost}\\ \text{by Steam}\end{pmatrix} = \begin{pmatrix}\text{Heat Gained}\\ \text{by Water}\end{pmatrix}$$

$\Rightarrow \quad m(L + c\Delta T_1) = 100c\Delta T_2$

$\Rightarrow \quad m = \frac{(100)(c)(\Delta T_2)}{L + c(\Delta T_1)}$

where, c =specific heat of water = 1 calg^{-1} °C

L = Latent heat of vaporization = 540 calg^{-1}

$\Delta T_1 = (100 - 90) = 10$ °C

and $\Delta T_2 = (90 - 24) = 66$ °C

Substituting the values, we get

$$m = \frac{(100)(1)(66)}{(540)+(1)(10)} = 12g$$

$\Rightarrow \quad m = 12g$

28. Since the boiling process is an isobaric process, so

$$W_{\text{isob}} = P\Delta V = (10^5)(1671-1)\times 10^{-6} \text{ Nm}$$

$\Rightarrow \quad W_{\text{isob}} = 167 \text{ J} \cong 40 \text{ cal}$

Now, according to FLTD we have $Q = \Delta U + W$

where $Q = mL = (1)(540) = 540$ cal

$\Rightarrow \quad 540 = \Delta U + 40$

$\Rightarrow \quad \Delta U = 500$ cal

29. A constant pressure, $V \propto T$

$\Rightarrow \quad \frac{V_2}{V_1} = \frac{T_2}{T_1}$

$\Rightarrow \quad \frac{Ah_2}{Ah_1} = \frac{T_2}{T_1}$

$\Rightarrow \quad h_2 = h_1\left(\frac{T_2}{T_1}\right) = (1)\left(\frac{400}{300}\right)m = \frac{4}{3}m$

As there is no heat loss, process is adiabatic.
For adiabatic process,

$$T_f V_f^{\gamma-1} = T_i V_i^{\gamma-1}$$

$\Rightarrow \quad T_f = T_i\left(\frac{V_i}{V_f}\right)^{\gamma-1} = (400)\left(\frac{h_i}{h_f}\right)^{1.4-1} = 400\left(\frac{4}{3}\right)^{0.4}$

$\Rightarrow \quad T_f = (400)(1.12) = 448$ K

30. If after time t, temperature of water inside the pitcher drops to 15 °C, then by law of calorimetry, we have

$$\left(10 - \frac{0.2}{1000}t\right)(4200)(5) = \left(\frac{0.2t}{1000}\right)(2.27\times 10^6)$$

Here $\frac{0.2t}{1000} \ll 10$, so we have $210000 = 454t$

$\Rightarrow \quad t \approx 462$ s

31. (a) $U \propto \sqrt{V}$

Since, $U \propto T$

$\Rightarrow \quad T \propto V^{\frac{1}{2}}$

$\Rightarrow \quad TV^{-\frac{1}{2}} = \text{constant}$

$\Rightarrow \quad PV^{\frac{1}{2}} = \text{constant}$

Comparing with $PV^x = \text{constant}$, we get $x = \frac{1}{2}$

Since, molar specific heat is

$\Rightarrow \quad C = \frac{R}{\gamma-1} + \frac{R}{1-x} = \frac{R}{\frac{7}{5}-1} + \frac{R}{1-\frac{1}{2}}$

$\Rightarrow \quad C = \frac{5}{2}R + 2R = \frac{9R}{2}$

$\Rightarrow \quad * = 9$

(b) Further $Q = nC\Delta T$ and $\Delta U = nC_V\Delta T$

$\Rightarrow \quad W = Q - \Delta U = n(C - C_V)\Delta T$

$\Rightarrow \quad \frac{W}{\Delta U} = \frac{C - C_V}{C_V}$

$\Rightarrow \quad W = \left(\frac{C - C_V}{C_V}\right)\Delta U = \left(\frac{\frac{9}{2} - \frac{5}{2}}{\frac{5}{2}}\right)(100) = 80$ J

32. According to Newton's Law of Cooling

$$\left(\frac{T_1 - T_2}{t}\right) = \alpha\left[\left(\frac{T_1 + T_2}{2}\right) - T_0\right]$$

For the given conditions,

$$\frac{60-40}{10} = \alpha\left(\frac{60+40}{2} - 10\right) \qquad \ldots(1)$$

Let T be the temperature after next 10 minutes. Then,

$$\frac{40-T}{10} = \alpha\left(\frac{40+T}{2} - 10\right) \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$T = 28$ °C

33. Number of moles (n) is $n = \frac{m}{M}$

Since the masses of two gases is same, therefore the ratio of number of moles is

$$\frac{n_1}{n_2} = \frac{M_2}{M_1} = \frac{28}{32} = \frac{7}{8}$$

CHAPTER 2

Also, pressure and temperature of both gases is same, so from Ideal Gas Equation, we get $V \propto n$ therefore their volumes are in the ratio.

$\Rightarrow \quad \frac{V_1}{V_2} = \frac{n_1}{n_2} = \frac{7}{8} = \frac{360-\theta}{\theta}$

$\Rightarrow \quad \theta = 192°$

34. Since, $mgh = mc\Delta T + mL$

$\Rightarrow \quad L = gh - c\Delta T$

$\Rightarrow \quad L = (10)(6\times10^3) - (125)(200-40)$

$\Rightarrow \quad L = 60000 - 20000$

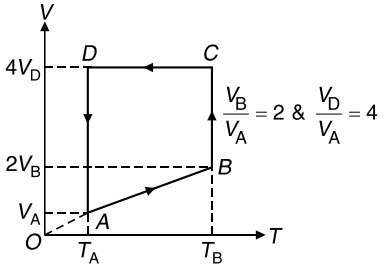
$\Rightarrow \quad L = 40000 \text{ Jkg}^{-1}$

$\Rightarrow \quad L = 40 \text{ kJkg}^{-1}$

35. Number of moles, $n = 2$

$C_V = \frac{3}{2}R$ and $C_P = \frac{5}{2}R$ (Monatomic)

$T_A = 27\ ^{\circ}\text{C} = 300\ \text{K}$



Let $V_A = V_o$ then $V_B = 2V_o$ and $V_D = V_C = 4V_o$

(a) Process $A \to B$, we have

$V \propto T$

$\Rightarrow \quad \frac{T_B}{T_A} = \frac{V_B}{V_A}$

$\Rightarrow \quad T_B = T_A\left(\frac{V_B}{V_A}\right) = (300)(2) = 600\ \text{K}$

$\Rightarrow \quad T_B = 600\ \text{K}$

(b) Process $A \to B$, we have $V \propto T$

$\Rightarrow \quad P = \text{constant}$

$\Rightarrow \quad Q_{AB} = nC_P dT = nC_P(T_B - T_A)$

$\Rightarrow \quad Q_{AB} = (2)\left(\frac{5}{2}R\right)(600-300) = 1500R$, absorbed

$\Rightarrow \quad Q_{AB} = 3000$ cal, absorbed

Process $B \to C$, we have $T = \text{constant}$

$\Rightarrow \quad dU = 0$

$\Rightarrow \quad Q_{BC} = W_{BC} = nRT_B \log_e\left(\frac{V_C}{V_B}\right)$

$\Rightarrow \quad Q_{BC} = (2)(R)(600)\log_e\left(\frac{4V_o}{2V_o}\right)$

$\Rightarrow \quad Q_{BC} = (1200R)\log_e(2) = (1200)(2)(0.7)$

$\Rightarrow \quad Q_{BC} \approx 1680$ cal, absorbed

Process $C \to D$, we have $V = \text{constant}$

$\Rightarrow \quad Q_{CD} = nC_V dT = nC_V(T_D - T_C)$

$\Rightarrow \quad Q_{CD} = n\left(\frac{3}{2}R\right)(T_A - T_B) \quad \{\because T_D = T_A \text{ and } T_C = T_B\}$

$\Rightarrow \quad Q_{CD} = (2)\left(\frac{3}{2}R\right)(300-600)$

$\Rightarrow \quad Q_{CD} = -900R$ (released)

$\Rightarrow \quad Q_{CD} = -1800$ cal, released

Process $D \to A$, we have $T = \text{constant}$

$\Rightarrow \quad dU = 0$

$\Rightarrow \quad Q_{DA} = W_{DA} = nRT_D \log_e\left(\frac{V_A}{V_D}\right)$

$\Rightarrow \quad Q_{DA} = (2)(R)(300)\log_e\left(\frac{V_0}{4V_0}\right)$

Since, $\log_e\left(\frac{1}{4}\right) = -2\log_e(2)$

$\Rightarrow \quad Q_{DA} = 600R\log_e\left(\frac{1}{4}\right)$

$\Rightarrow \quad Q_{DA} \approx -1680$ cal, released

(c) In the complete cycle, $dU = 0$

Therefore, from $FLTD$, we have

$W_{\text{net}} = Q_{AB} + Q_{BC} + Q_{CD} + Q_{DA}$

$\Rightarrow \quad W_{\text{net}} = 3000 + 1680 - 1800 - 1680$

$\Rightarrow \quad W_{\text{net}} = W_{\text{total}} = 1200$ cal

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1. Since, $U = \frac{n_1 f_1 RT}{2} + \frac{n_2 f_2 RT}{2}$

$\Rightarrow \quad U = \frac{(3)(5)}{2}RT + \frac{(5)(3)}{2}RT = 15RT$

Hence, the correct answer is (B).

2. The mean free path of molecules of an ideal gas is

$\lambda = \frac{V}{\sqrt{2}\pi N d^2} = \frac{RT}{\sqrt{2}\pi N P d^2}$

where, V is the volume of container, N is number of molecules and d is the diameter of the molecule.

Since volume of container does not change (closed container) i.e., $\frac{T}{P} = \text{constant}$, so mean free path is unchanged. Hence with increasing temp, mean free path does not change if gas is confined to a closed container. Also, average collision time is

Average collision time

$\tau = \frac{\text{Mean Free Path}}{\text{Average Speed}} = \frac{\lambda}{v_{av}}$

Since, average speed $v_{av} \propto \sqrt{T}$

So, average collision time $\tau \propto \frac{1}{\sqrt{T}}$

Hence with increase in temperature the average collision time decreases.

Hence, the correct answer is (D).

3. Given that, $\frac{\Delta L}{L} = 0.02\%$

Since, $\Delta L = L\alpha\Delta T$

$\Rightarrow \quad \frac{\Delta L}{L} = \alpha\Delta T = 0.02\%$

Also, $\beta = 2\alpha$

$\Rightarrow \quad \beta\Delta T = 2\alpha\Delta T = 0.04\%$

Since, $\rho = \frac{M}{V} = \frac{M}{AL}$

$\Rightarrow \quad \frac{\Delta\rho}{\rho} = \frac{\Delta M}{M} - \frac{\Delta A}{A} - \frac{\Delta L}{L}$

Due to heating, mass remains constant, so we have

$$\left|\frac{\Delta\rho}{\rho}\right| = \frac{\Delta A}{A} + \frac{\Delta L}{L} = \beta\Delta T + \alpha\Delta T$$

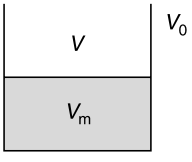
$\Rightarrow \quad \left|\frac{\Delta\rho}{\rho}\right| = 0.04\% + 0.02\% = 0.06\%$

Hence, the correct answer is (B).

4. Unfilled volume V of the beaker is constant, so

$$V = V_0 - V_m = \text{constant}$$

After increasing temperature, V remains same.



$\Rightarrow \quad \Delta V = 0$

$\Rightarrow \quad \Delta V_0 - \Delta V_m = 0$

$\Rightarrow \quad V_0\gamma_b\Delta T = V_m\gamma_m\Delta T$

$\Rightarrow \quad V_m = \frac{V_0\gamma_b}{\gamma_m} = \frac{(500)(6\times10^{-6})}{(1.5\times10^{-4})} = 20 \text{ cc}$

Hence, the correct answer is 20.

5. At constant pressure, $nC_P(50) = 160$

At constant volume, $nC_V(100) = 240$

$\Rightarrow \quad \frac{C_P}{2C_V} = \frac{160}{240} = \frac{\gamma}{2}$

$\Rightarrow \quad \gamma = 1 + \frac{2}{f} = \frac{4}{3}$

$\Rightarrow \quad f = \frac{2}{\gamma - 1} = 6$

Hence, the correct answer is (C).

6. Initially, we have

$$\underbrace{\text{Calorimeter}}_{20\text{ g},\,25\,^\circ\text{C}},\ \underbrace{\text{Water}}_{20\text{ g},\,25\,^\circ\text{C}},\ \underbrace{\text{Steam}}_{m,\,100\,^\circ\text{C}}$$

Finally, due to condensation of steam, we have

$$\underbrace{\text{Calorimeter}}_{20\text{ g},\,31\,^\circ\text{C}},\ \underbrace{\text{Water}}_{20\text{ g},\,31\,^\circ\text{C}},\ \underbrace{\text{Water}}_{m,\,100\,^\circ\text{C}}$$

By law of calorimetry, we have

$$(180+20)(1)(31-25) = 540m + m(1)(100-31)$$

$\Rightarrow \quad m = 1.97 \text{ g} \approx 2 \text{ g}$

Hence, the correct answer is (B).

7. Since, $\gamma = \frac{C_P}{C_V} = 1 + \frac{2}{f}$

where, f the is degrees of freedom of a gas.

(A) For monatomic gas molecule, we have

$$f = 3,\ \gamma = 1 + \frac{2}{3} = \frac{5}{3}$$

(B) For diatomic rigid molecules, we have

$$f = 5,\ \gamma = 1 + \frac{2}{3} = \frac{7}{5}$$

(C) For diatomic non-rigid molecules, we have

$$f = 7,\ \gamma = 1 + \frac{2}{7} = \frac{9}{7}$$

(D) For triatomic rigid molecules, we have

$$f = 6,\ \gamma = 1 + \frac{2}{6} = \frac{4}{3}$$

Hence, the correct answer is (A).

8. Here the water will provide heat for ice to melt therefore

$$m_w c_w \Delta T = m_{\text{ice}} L_{\text{ice}}$$

$\Rightarrow \quad m_{\text{ice}} = \frac{0.2\times4200\times25}{3.4\times10^5} = 0.0617 \text{ kg}$

$\Rightarrow \quad m_{\text{ice}} = 61.7 \text{ g}$

Remaining ice will remain unmelted

Hence, the correct answer is (A).

9. As work done on gas and heat supplied to the gas are zero, total internal energy of gases remains same

$$U_1 + U_2 = U_1' + U_2'$$

$\Rightarrow \quad (0.1)C_V(200) + (0.05)C_V(400) = (0.1+0.05)C_V T$

$\Rightarrow \quad T = \frac{800}{3} \text{ K} = 266.67 \text{ K}$

Or, we could have simply said that internal energy lost by one gas equals the internal energy gained by other i.e.,

$$(0.05)C_V(400-T) = (0.1)C_V(T-200)$$

$\Rightarrow \quad 400 - T = 2(T-200)$

$\Rightarrow \quad 3T = 800$

$\Rightarrow \quad T = \frac{800}{3} = 266.67 \text{ K}$

Hence, the correct answer is 266.67.

10. For an ideal gas, we have $PV = nRT$

$\Rightarrow \quad \Delta(PV) = \Delta(nRT)$

Since volume is constant, so we have

$$V\Delta P = nR\Delta T$$

$\Rightarrow \quad \frac{\Delta T}{\Delta P} = \frac{V}{nR} = \frac{T}{P} \qquad \{\because PV = nRT\}$

CHAPTER 2

$$\Rightarrow \quad \left|\frac{\Delta T}{\Delta P}\right| = \left|\frac{300}{2}\right| = 150$$

Hence, the correct answer is 150.

11. Since, $\frac{1}{2}\left(\frac{1}{2}mv^2\right) = mc\Delta T$

$$\Rightarrow \quad \Delta T = \frac{v^2}{4c} = \frac{(210)^2}{4(0.030\times 4200)} = 87.5\ ^\circ\text{C}$$

Hence, the correct answer is (B).

12. Mean kinetic energy of a molecule at T is

$$\frac{3}{2}k_B T = 4\times 10^{-14} \qquad \ldots(1)$$

Since $n = \frac{PV}{RT}$ and if N be the number of molecules. then we have $n = \frac{N}{N_A}$

$$\Rightarrow \quad N = \left(\frac{PV}{RT}\right)N_A = \frac{PV}{k_B T} = \frac{2\times 13.6\times 980\times 4}{(8/3)\times 10^{-14}}$$

$$\Rightarrow \quad N = 3.99\times 10^{18}$$

Hence, the correct answer is (C).

13. Since, $L = L_1 + L_2$

$$\Rightarrow \quad \Delta L = \Delta L_1 + \Delta L_2$$

$$\Rightarrow \quad (L_1 + L_2)\alpha_{\text{eq}}\Delta T = L_1\alpha_1\Delta T + L_2\alpha_2\Delta T$$

$$\Rightarrow \quad \alpha_{\text{eq}} = \frac{L_1\alpha_1 + L_2\alpha_2}{L_1 + L_2}$$

Hence, the correct answer is (D).

14. Since $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$ and $v_{N_2} = v_{H_2}$

$$\Rightarrow \quad \sqrt{\frac{3RT_{N_2}}{M_{N_2}}} = \sqrt{\frac{3RT_{H_2}}{M_{H_2}}}$$

$$\Rightarrow \quad \frac{573}{28} = \frac{T_{H_2}}{2}$$

$$\Rightarrow \quad T_{H_2} = 40.93\ \text{K} \approx 41\ \text{K}$$

Hence, the correct answer is 41.

15. Total degree of freedom $= 3 + 2 = 5$

$$\Rightarrow \quad U = \frac{nfRT}{2} = \frac{5RT}{2} \qquad \{\because n = 1\}$$

$$\Rightarrow \quad \gamma = \frac{C_P}{C_V} = 1 + \frac{2}{f} = 1 + \frac{2}{5} = \frac{7}{5}$$

Hence, the correct answer is (D).

16. For an ideal gas, we have $PV = nRT$

So, $P_1V_1 = nR(250)$ $\qquad \ldots(1)$

and $P_2(2V_1) = \left(\frac{5n}{4}\right)R(2000)$ $\qquad \ldots(2)$

Dividing equation (2) by (1), we get

$$\frac{2P_2}{P_1} = \frac{5\times 2000}{4\times 250}$$

$$\Rightarrow \quad \frac{P_2}{P_1} = 5$$

Hence, the correct answer is (5).

17. Average collision time

$$\tau = \frac{\text{Mean Free Path}}{\text{Average Speed}} = \frac{\lambda}{v_{av}}$$

Since, average speed $v_{av} \propto \sqrt{T}$

So, average collision time $\tau \propto \frac{1}{\sqrt{T}}$

Hence, the correct answer is (C).

18. Since, $\eta = \frac{\text{Work done}}{\text{Heat Input}} = \frac{W}{\Sigma Q_{\oplus}}$

$$\Rightarrow \quad \frac{1}{2} = \eta = \frac{1915 - 40 + 125 - Q}{1915 + 125}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{2000 - Q}{2040}$$

$$\Rightarrow \quad 2040 = 4000 - 2Q$$

$$\Rightarrow \quad 2Q = 1960$$

$$\Rightarrow \quad Q = 980\ \text{J}$$

Hence, the correct answer is (C).

19. Bursting of helium balloon is an irreversible and adiabatic process.

Hence, the correct answer is (B).

20. Since $\frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$, where, $Q_2 = mL_{\text{ice}} = 8000$ cal

Also, $T_2 = 273$ K and $T_1 = 300$ K

$$\Rightarrow \quad \frac{8000}{W} = \frac{273}{300 - 273} = \frac{273}{27}$$

$$\Rightarrow \quad W = 791.2\ \text{cal}$$

Also $Q_1 = Q_2 + W$, so we get

$$Q_1 = 791.2 + 8000 = 8791.2\ \text{cal} \approx 8791\ \text{cal}$$

Hence, the correct answer is 8791.

21. For an adiabatic process,

$$\Delta Q = 0$$

For an isothermal process, carried on an ideal gas

$$\Delta U = 0$$

For an isochoric process volume remains constant, so

$$W = 0$$

For an isobaric process, $\Delta U \neq 0$, $\Delta W \neq 0$, $\Delta Q \neq 0$

Hence, the correct answer is (A).

22. Since $\Delta U = nC_V\Delta T$, so internal energy change remains the same in all the cases. Along path AB volume is increasing, so $W > 0$. Along path AD volume is decreasing, so $W < 0$ and along AC volume is constant, so $W = 0$

Hence, the correct answer is (A).

23. In adiabatic process

$$PV^{\gamma} = \text{constant}$$

$$\Rightarrow \quad P\left(\frac{m}{\rho}\right)^{\gamma} = \text{constant}$$

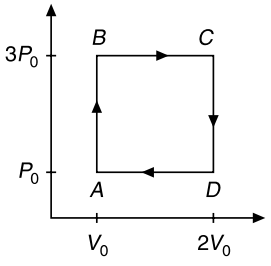
As mass is constant

$$P \propto \rho^{\gamma}$$

$$\Rightarrow \quad \frac{P_f}{P_i} = \left(\frac{\rho_f}{P_i}\right)^{\gamma} = (32)^{\frac{7}{5}} = 2^7 = 128$$

Hence, the correct answer is (D).

24. Since, $W_{ABCDA} = 2P_0V_0$



Also, $Q_{\text{in}} = Q_{AB} + Q_{BC}$

Along path AB, volume is constant, so

$$Q_{AB} = nC_V(T_B - T_A) = n\left(\frac{3R}{2}\right)(T_B - T_A)$$

$$\Rightarrow \quad Q_{AB} = \frac{3}{2}(P_BV_B - P_AV_A)$$

$$\Rightarrow \quad Q_{AB} = \frac{3}{2}(3P_0V_0 - P_0V_0) = 3P_0V_0$$

Along path BC, pressure is constant, so

$$Q_{BC} = nC_P(T_C - T_B) = n\left(\frac{5R}{2}\right)(T_C - T_B)$$

$$\Rightarrow \quad Q_{BC} = \frac{5}{2}(P_CV_C - P_BV_B)$$

$$\Rightarrow \quad Q_{BC} = \frac{5}{2}(6P_0V_0 - 3P_0V_0) = \frac{15}{2}P_0V_0$$

The percentage efficiency is given by

$$\eta = \frac{W}{Q_{\text{in}}} \times 100 = \frac{2P_0V_0}{3P_0V_0 + \frac{15}{2}P_0V_0} \times 100$$

$$\Rightarrow \quad \eta = \frac{400}{21} = 19.04 \approx 19$$

Hence, the correct answer is 19.

25. According to Newton's Law of Cooling, we have the rate of cooling to be proportional to the average excess temperature, so

$$\frac{50-40}{300} = \beta\left(\frac{50+40}{2} - 20\right) \qquad \ldots(1)$$

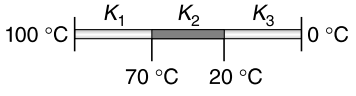
Similarly, $\frac{40-T}{300} = \beta\left(\frac{40+T}{2} - 20\right)$ $\qquad \ldots(2)$

Solving equations (1) and (2), we get

$$T = \frac{100}{3}\ ^{\circ}\text{C} \approx 33\ ^{\circ}\text{C}$$

Hence, the correct answer is (A).

26. Rods are identical have same length (l) and area of cross section (A), so for series combination of rods heat current is same, hence



$$\left(\frac{Q}{t}\right)_{AB} = \left(\frac{Q}{t}\right)_{BC} = \left(\frac{Q}{t}\right)_{CD} = \text{Heat current}$$

$$\Rightarrow \quad \frac{(100-70)K_1A}{l} = \frac{(70-20)K_2A}{l} = \frac{(20-0)K_3A}{l}$$

$$\Rightarrow \quad 30K_1 = 50K_2 = 20K_3$$

So, $3K_1 = 2K_3$

$$\Rightarrow \quad \frac{K_1}{K_3} = \frac{2}{3} = 2:3$$

Also, $5K_2 = 2K_3$

$$\Rightarrow \quad \frac{K_2}{K_3} = \frac{2}{5} = 2:5$$

Hence, the correct answer is (A).

27. Since, $\eta = \frac{1}{1+\beta}$

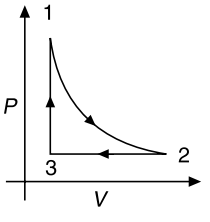
$$\Rightarrow \quad \frac{1}{10} = \frac{1}{1+\beta}$$

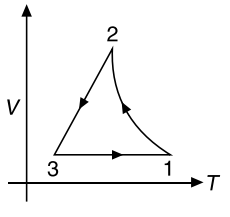
$$\Rightarrow \quad \beta = \frac{Q_2}{W} = 9$$

$$\Rightarrow \quad Q_2 = 9W = 90\ \text{J}$$

Hence, the correct answer is (C).

28. In process 2 to 3 pressure is constant and in process 3 to 1 volume is constant which is correct only in option 4. So, correct graph is





Hence, the correct answer is (D).

29. Since $PV^{\gamma} = \text{constant}$

$$\Rightarrow \quad T^{\gamma-1} = C$$

$$\Rightarrow \quad 300 \times V^{\frac{7}{5}-1} = T_2\left(\frac{V}{16}\right)^{\frac{7}{5}-1}$$

$$\Rightarrow \quad T_2 = 300 \times 2^{4\times\frac{2}{5}}$$

CHAPTER 2

In an isobaric process, we have

$$V=\frac{nRT}{P}$$

$$\Rightarrow \quad V_2=kT_2 \qquad \ldots(1)$$

$$\Rightarrow \quad 2V_2=KT_f \qquad \ldots(2)$$

$$\Rightarrow \quad T_f=2\times 300\times 2^{\frac{8}{5}}=1818.85$$

Hence, the correct answer is 1818.15.

30. **Since,** $T_i=T=273+20=293$ K

If $V_i=V$, then $V_f=\frac{V}{10}$

For adiabatic process, we have $TV^{\gamma-1}=$ constant

$$T_1V_1^{\gamma-1}=T_2V_2^{\gamma-1}$$

$$\Rightarrow \quad TV^{\frac{7}{5}-1}=T_2\left(\frac{V}{10}\right)^{\frac{7}{5}-1}$$

$$\Rightarrow \quad T_2=T(10)^{\frac{2}{5}}=293(10)^{\frac{2}{5}} \text{ K}$$

$$\Rightarrow \quad \Delta U=nC_V\Delta T=5\left(\frac{5R}{2}\right)\left(10^{\frac{2}{5}}-1\right)T$$

$$\Rightarrow \quad \Delta U=\frac{25(8.314)(293)(2.5-1)}{2}\approx 46 \text{ kJ}$$

Hence, the correct answer is 46.

31. Since, $(C_V)_{\text{mix}}=\frac{n_1C_{V_1}+n_2C_{V_2}}{n_1+n_2}=\frac{2(3R/2)+3(3R)}{2+3}$

$$\Rightarrow \quad (C_V)_{\text{mix}}=\frac{12R}{5}$$

$$\Rightarrow \quad (C_P)_{\text{mix}}=(C_V)_{\text{mix}}+R=\frac{12R}{5}+R=\frac{17R}{5}$$

$$\Rightarrow \quad \gamma_{\text{mix}}=\frac{(C_P)_{\text{mix}}}{(C_V)_{\text{mix}}}=\frac{17}{12}\approx 1.42$$

Hence, the correct answer is (B).

32. Since, $\gamma=\alpha_x+\alpha_y+\alpha_z$

$$\Rightarrow \quad \gamma=5\times 10^{-5}+5\times 10^{-6}+5\times 10^{-6}$$

$$\Rightarrow \quad \gamma=(50+5+5)\times 10^{-6}=60\times 10^{-6} \ ^\circ\text{C}^{-1}$$

$$\Rightarrow \quad C=60$$

Hence, the correct answer is 60.

33. Applying Law of Calorimetry, we get

$$540M+M(1)60=(200)(80)+200(1)(40-0)$$

$$\Rightarrow \quad M=40 \text{ g}$$

Hence, the correct answer is 40.

34. Since, $(C_V)_{\text{mix}}=\frac{n_1C_{V_1}+n_2C_{V_2}}{n_1+n_2}=\frac{n(3R/2)+2n(5R/2)}{n+2n}$

$$\Rightarrow \quad (C_V)_{\text{mix}}=\frac{13R}{6}$$

$$\Rightarrow \quad (C_P)_{\text{mix}}=(C_V)_{\text{mix}}+R=\frac{13R}{6}+R=\frac{19R}{6}$$

$$\Rightarrow \quad \gamma_{\text{mix}}=\frac{(C_P)_{\text{mix}}}{(C_V)_{\text{mix}}}=\frac{19}{13}$$

Hence, the correct answer is (B).

35. According to table and applying law of calorimetry

$$1T_1+2T_2=(1+2)60^\circ=180 \qquad \ldots(1)$$

$$1T_2+2T_3=(1+2)30^\circ=90 \qquad \ldots(2)$$

$$2T_1+1T_3=(1+2)60=180 \qquad \ldots(3)$$

Adding $(1)+(2)+(3)$

$$3(T_1+T_2+T_3)=450$$

$$\Rightarrow \quad T_1+T_2+T_3=150^\circ$$

Also, we have

$$1T_1+1T_2+1T_3=(1+1+1)T$$

$$\Rightarrow \quad 150=3T$$

$$\Rightarrow \quad T=\theta=50 \ ^\circ\text{C}$$

Hence, the correct answer is 50.

36. Degree of freedom of a diatomic molecule if vibration is absent is 5

Degree of freedom of a diatomic molecule if vibration is present is 7

$$\Rightarrow \quad \frac{C_V^A}{C_V^B}=\frac{f_AR/2}{f_BR/2}=\frac{f_A}{f_B}=\frac{5}{7}$$

Hence, the correct answer is (B).

37. Since, $\lambda=\frac{1}{\sqrt{2}\pi n_0d^2}$, where $n_0=\frac{N}{V}$

$$\Rightarrow \quad \tau=\frac{\lambda}{v_{\text{rms}}}=\frac{1}{\sqrt{2}\pi n_0d^2v_{\text{rms}}}=\frac{1}{\sqrt{2}\pi n_0d^2}\sqrt{\frac{M}{3RT}}$$

$$\Rightarrow \quad \frac{\tau_1}{\tau_2}=\sqrt{\frac{M_1}{M_2}}\frac{d_2^2}{d_1^2}$$

$$\Rightarrow \quad \frac{\tau_1}{\tau_2}=\sqrt{\frac{40}{140}}\frac{(0.1)^2}{(0.07)^2}$$

$$\Rightarrow \quad \frac{\tau_1}{\tau_2}=1.09$$

Nearest possible answer is 1.83

Hence, the correct answer is (C).

38. Let amount of water evaporated be m gram.

$$\Rightarrow \quad mL_v=(150-m)L_s$$

$$\Rightarrow \quad m(2.1\times 10^6)=(150-m)(3.36\times 10^5)$$

$$\Rightarrow \quad m=\frac{150}{7.25}\approx 20 \text{ g}$$

Hence, the correct answer is (C).

39. Between the isothermal and the adiabatic processes, P-V graph for adiabatic is steeper.

Hence, the correct answer is (C).

40. Since, $v \propto \sqrt{T}$

$\Rightarrow \quad \frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$

$\Rightarrow \quad \frac{200}{v_2} = \sqrt{\frac{400}{500}}$

$\Rightarrow \quad v_2 = 200\sqrt{\frac{5}{4}} = 100\sqrt{5}\ \text{ms}^{-1}$

Hence, the correct answer is (A).

41. Since, $\Delta W_A > \Delta W_B$ and $\Delta U_A = \Delta U_B = U_f - U_i$

So from FLTD, we have

$\Delta Q = \Delta U + \Delta W$

$\Rightarrow \quad \Delta Q_A > \Delta Q_B$

Hence, the correct answer is (A).

42. Since, $v_{\text{rms}} = \overline{v} = \sqrt{\frac{3RT}{M}}$

$\Rightarrow \quad \overline{v} = \sqrt{\frac{3RT}{mN_A}} = \sqrt{\frac{3k_BT}{m}}$

$\Rightarrow \quad T = \frac{m\overline{v}^2}{3k_B}$

Hence, the correct answer is (D).

43. According to Stefan's Law, we have

$$mc\left(-\frac{dT}{dt}\right) = e\sigma A\left(T^4 - T_0^4\right)$$

$\Rightarrow \quad -\frac{dT}{dt} = \frac{e\sigma A\left(T^4 - T_0^4\right)}{\rho Vc} \qquad \{\because m = \rho V\}$

$\Rightarrow \quad -\frac{dT}{dt} \propto \frac{1}{\rho c}$

Since, $\rho_A c_A < \rho_B c_B$

$\Rightarrow \quad \left(-\frac{dT}{dt}\right)_A > \left(-\frac{dT}{dt}\right)_B$

So, A cools down at faster rate

Hence, the correct answer is (B).

44. For gas A, $C_P = 29$, $C_V = 22$

For gas B, $C_P = 30$, $C_V = 21$

$\Rightarrow \quad \gamma_A = \frac{C_{P_A}}{C_{V_A}} = \frac{29}{22} = 1.31$

When A has vibrational degree of freedom, then

$$\gamma_A = \frac{9}{7} \simeq 1.29$$

$$\gamma_B = \frac{C_{P_B}}{C_{V_B}} = \frac{30}{21} = 1.42$$

So, B has no vibrational degree of freedom

Hence, the correct answer is (B).

45. $H = \frac{3KA}{d}(\theta_2 - \theta) = \frac{KA}{3d}(\theta - \theta_1)$

$\Rightarrow \quad \theta = \frac{9\theta_2}{10} + \frac{\theta_1}{10}$

Hence, the correct answer is (B).

46. For isobaric process, work done $W = nR\Delta T$ and heat given $Q = nC_P\Delta T$, so we have

$$\frac{W}{Q} = \frac{R}{C_P} = \frac{R}{C_V + R}$$

47. Number of moles of He at STP is $n = \frac{67.2}{22.4} = 3$

Since the volume is constant, so process is isochoric and hence, we have

$$Q = nC_V\Delta T = 3\left(\frac{3R}{2}\right)(20) = 90R = 90(8.31)$$

$\Rightarrow \quad Q \approx 748\ \text{J}$

Hence, the correct answer is (D).

48. When $V_1 = V_0$, then $P_1 = P_0\left(1 - \frac{1}{2}\right) = \frac{P_0}{2}$

When $V_2 = 2V_0$, then $P_2 = P_0\left[1 - \frac{1}{2}\left(\frac{1}{4}\right)\right] = \frac{7P_0}{8}$

Since, $T = \frac{PV}{nR}$

$\Rightarrow \quad \Delta T = \left|\frac{P_1V_1}{nR} - \frac{P_2V_2}{nR}\right|$

$\Rightarrow \quad \Delta T = \left|\left(\frac{(P_1V_1 - P_2V_2)}{nR}\right)\right| = \frac{1}{nR}\left|\frac{P_0V_0}{2} - \frac{7P_0V_0}{4}\right|$

$\Rightarrow \quad \Delta T = \frac{5P_0V_0}{4nR}$

Hence, the correct answer is (B).

49. Heat supplied at constant volume $Q = nC_V\Delta T$ and heat supplied at constant pressure is $Q_1 = nC_p\Delta T$

$\Rightarrow \quad \frac{Q_1}{Q} = \frac{C_p}{C_v}$

$\Rightarrow \quad Q_1 = \gamma Q = \frac{7Q}{5}$

Hence, the correct answer is (C).

50. For the process ca, $\Delta U_{ca} = -180$ J

For process bc, isochoric $W_{bc} = 0$

$\Rightarrow \quad \Delta U = 60$ J

Heat absorbed along ab is $Q_{ab} = 250$ J

Since $\Delta U_{\text{cycle}} = 0$, so $\Delta U_{ab} = 120$ J

So by FLTD, $W_{a\to b} = 130$ J

Total work done from $(a \to b \to c)$

$W = W_{ab} + W_{bc} = 130$ J

Hence, the correct answer is (A).

51. Applying law of calorimetry, we get

$$5M_1 + M_1L = 50M_2$$

CHAPTER 2

$$\Rightarrow \quad L = \frac{50M_2}{M_1} - 5$$

Hence, the correct answer is (A).

52. Since, $(2+3)C_V = 2\left(\frac{3R}{2}\right) + 3\left(\frac{5R}{2}\right)$

$$\Rightarrow \quad C_V = \frac{21R}{10}$$

Hence, the correct answer is (D).

53. Since, $\eta = \frac{1}{6} = 1 - \frac{T_C}{T_H}$

$$\Rightarrow \quad \frac{T_C}{T_H} = \frac{5}{6} \qquad \ldots(1)$$

Now $\eta' = 2\eta = \frac{1}{3} = 1 - \frac{(T_C - 62)}{T_H}$

$$\Rightarrow \quad \frac{T_C - 62}{T_H} = \frac{2}{3} \qquad \ldots(2)$$

Substituting equation (1) in (2), we get

$$\frac{T_C - 62}{T_C} = \frac{2\times 6}{3\times 5} = \frac{4}{5}$$

$$\Rightarrow \quad T_C = 310 \text{ K} = 37 \text{ °C and } T_H = 372 \text{ K} = 99 \text{ °C}$$

Hence, the correct answer is (A).

54. Since, $W_{\text{isob}} = nR\Delta T = 10$ J

and $Q = nC_P\Delta T = n\left(\frac{7R}{2}\right)\Delta T = \frac{7}{2}(nR\Delta T)$

$$\Rightarrow \quad \Delta Q = \frac{7}{2}(10) = 35 \text{ J}$$

Hence, the correct answer is (B).

55. Since, $v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = 11.2\times 10^3 \text{ ms}^{-1}$

$$\Rightarrow \quad T = \frac{M}{3R}\times(11.2\times 10^3)^2 = \frac{2\times 10^{-3}}{3\times 8.3}\times 125.44\times 10^6$$

$$\Rightarrow \quad T \approx 10^4 \text{ K}$$

Hence, the correct answer is (A).

56. Given that $V = 25\times 10^{-3}$ m^3, $n = 1$ mole of O_2

$T = 300$ K, $v_{\text{rms}} = 200$ ms^{-1}, $d = 0.3$ nm

Since, $\lambda = \frac{1}{\sqrt{2}\pi n_0 d^2}$, where $n_0 = \frac{N}{V} = \frac{N_A}{V}$

$$\Rightarrow \quad \tau = \frac{\lambda}{v_{\text{rms}}} = \frac{1}{\sqrt{2}\pi n_0 d^2 v_{\text{rms}}}$$

The average collision rate (per second) is

$$\frac{1}{\tau} = \frac{v_{\text{rms}}}{\lambda} = \sqrt{2}\pi\left(\frac{N_A}{V}\right)d^2 v_{\text{rms}}$$

$$\Rightarrow \quad \frac{1}{\tau} = \sqrt{2}(3.14)\left(\frac{6.023\times 10^{23}}{25\times 10^{-3}}\right)(0.09\times 10^{-18})(200)$$

Average number of collisions $\approx 10^{10}$

Hence, the correct answer is (B).

57. Since, $n = n_0 e^{-\alpha r^4}$

$$\Rightarrow \quad N = \int dN = \int_0^\infty \left(n_0 e^{-\alpha r^4}\right)4\pi r^2 dr$$

$$\Rightarrow \quad N = 4\pi n_0 \int_0^\infty r^2 e^{-\alpha r^4} dr$$

Substitute $\sqrt{\alpha}r^2 = z$, we get

$$\sqrt{\alpha}(2rdr) = dz$$

$$\Rightarrow \quad N = \frac{4\pi n_0}{2\sqrt{\alpha}}\int_0^\infty \frac{z^{1/2}e^{-z^2}}{\alpha^{1/4}}dz = \frac{4\pi n_0}{2\alpha^{3/4}}\int z^{1/2}e^{-z^2}dz$$

$$\Rightarrow \quad N \propto n_0\alpha^{-\frac{3}{4}}$$

Hence, the correct answer is (A).

58. For path ACB, applying FLTD, we get

$$Q_{ACB} = W_{ACB} + U_{ACB}$$

$$\Rightarrow \quad 60 = 30 + U_{ACB}$$

$$\Rightarrow \quad U_{ACB} = U_{ADB} = 30 \text{ J}$$

Similarly, for path ADB, we get from FLTD

$$Q_{ADB} = W_{ADB} + U_{ADB} = (10+30) \text{ J}$$

$$\Rightarrow \quad Q_{ADB} = 40 \text{ J}$$

Hence, the correct answer is (D).

59. Applying Kirchhoff's Junction Law (KJL) for heat currents at P and Q respectively, we get

At P, $\frac{T_A - T_P}{L/2} = \frac{T_P - T_Q}{L} + \frac{T_P - T_Q}{3L/2}$

$$\Rightarrow \quad 2(T_A - T_P) = \frac{5}{3}(T_P - T_Q) \qquad \ldots(1)$$

At Q, $\frac{T_P - T_Q}{L} + \frac{T_P - T_Q}{3L/2} = \frac{T_Q - T_B}{L/2}$

$$\Rightarrow \quad 2(T_Q - T_B) = \frac{5}{3}(T_P - T_Q) \qquad \ldots(2)$$

From equations (1) and (2), we get

$$2(T_A - T_B) = 2(T_P - T_Q) + \frac{10}{3}(T_P - T_Q)$$

$$\Rightarrow \quad 2\times 120 = \frac{16}{3}(T_P - T_Q)$$

$$\Rightarrow \quad T_P - T_Q = \frac{2\times 120\times 3}{16} = 45 \text{ °C}$$

Hence, the correct answer is (B).

60. $\frac{(v_{\text{rms}})_{He}}{(v_{\text{rms}})_{Ar}} = \sqrt{\frac{M_{Ar}}{M_{He}}} = \sqrt{\frac{40}{4}} = \sqrt{10} = 3.16$

Hence, the correct answer is (C).

61. For no change in length of the rod, we have

$$\Delta L_{\text{Thermal}} - \Delta L_{\text{mechanical}} = 0$$

$$\Rightarrow \quad L_0\alpha\Delta T = \frac{FL_0}{AY}$$

$\Rightarrow \quad Y = \dfrac{F}{A\alpha\Delta T}$

Hence, the correct answer is (B).

62. Since, $\dfrac{Q_1}{T_1} = \dfrac{Q_2}{T_2} = \dfrac{Q_3}{T_3}$

Also, $W_1 = Q_1 - Q_2$ and $W_2 = Q_2 - Q_3$

Given $W_1 = W_2$

$\Rightarrow \quad Q_1 - Q_2 = Q_2 - Q_3$

$\Rightarrow \quad 2Q_2 = Q_1 + Q_3$

$\Rightarrow \quad 2T_2 = T_1 + T_3$

$\Rightarrow \quad T_2 = \dfrac{T_1 + T_3}{2} = 500 \text{ K}$

Hence, the correct answer is (D).

63. Since, $\dfrac{v_2}{v_1} = 2 = \sqrt{\dfrac{T_2}{T_1}}$

$\Rightarrow \quad T_2 = 1200 \text{ K}$

For a closed vessel, volume is constant, so

$Q = nC_V(T_2 - T_1) = \left(\dfrac{15}{28}\right)\left(\dfrac{5R}{2}\right)(900) = 10 \text{ kJ}$

Hence, the correct answer is (C).

64. Since, $\eta = 1 - \dfrac{T_2}{T_1} = 1 - \dfrac{T_3}{T_2} = 1 - \dfrac{T_4}{T_3}$

$\Rightarrow \quad \dfrac{T_2}{T_1} = \dfrac{T_3}{T_2} = \dfrac{T_4}{T_3}$ …(1)

$\Rightarrow \quad T_2 = \sqrt{T_1 T_3}$ and $T_3 = \sqrt{T_2 T_4}$

Substituting value of T_2 in equation (1), we get

$\Rightarrow \quad \dfrac{\sqrt{T_1 T_3}}{T_1} = \dfrac{T_4}{T_3}$

$\Rightarrow \quad \dfrac{T_1 T_3}{T_1^2} = \dfrac{T_4^2}{T_3^2}$

$\Rightarrow \quad T_3^3 = T_1 T_4^2$

$\Rightarrow \quad T_3 = \left(T_1 T_4^2\right)^{1/3}$

Similarly, we get $T_2 = \left(T_1^2 T_4\right)^{\frac{1}{3}}$

Hence, the correct answer is (D).

65. Since, $W_{\text{isob}} = P\Delta V = nR\Delta T$

$\Rightarrow \quad W_{\text{isob}} = \dfrac{1}{2}(8.31)(70) \approx 291 \text{ J}$

Hence, the correct answer is (A).

66. Applying law of calorimetry, we get

$$192c(100 - 21.5) = (128)(0.394)(21.5 - 8.4) + (240)(4.18)(21.5 - 8.4)$$

$\Rightarrow \quad c = \dfrac{660.65 + 13142}{15072} \approx 916 \text{ Jkg}^{-1}\text{K}^{-1}$

Hence, the correct answer is (A).

67. Since, $V = \dfrac{m}{\rho} = \dfrac{2}{8} = \dfrac{1}{4} \text{ m}^3$

From ideal gas equation, we have $PV = nRT$

$\Rightarrow \quad 4 \times 10^4 \times \dfrac{1}{4} = nRT$

$\Rightarrow \quad nRT = 10^4$

Internal Energy $U = \dfrac{3}{2}nRT = \dfrac{3}{2} \times 10^4 = 1.5 \times 10^4 \text{ J}$

So internal energy is of order of 10^4 J

Hence, the correct answer is (C).

68. $\dfrac{Q}{A} = \dfrac{K(T_1 - T_2)}{L} = \dfrac{0.1 \times 900}{1} = 90 \text{ Wm}^{-2}$

Hence, the correct answer is (D).

69. $U = 3\left(\dfrac{5R}{2}\right)T + 5\left(\dfrac{3R}{2}\right)T$

$\Rightarrow \quad U = 15RT$

Hence, the correct answer is (C).

70. For adiabatic process, $TV^{\gamma - 1} = \text{constant}$

$\Rightarrow \quad x = \gamma - 1$

$\Rightarrow \quad x = \dfrac{2}{5}$

Hence, the correct answer is (A).

71. Let m gram of ice be added, then heat lost by water is

$Q_{\text{lost}} = 50(1)(40 - 0) = 2000 \text{ cal}$

Similarly, heat gained by ice is

$Q_{\text{gained}} = m(0.5)(20) + (m - 20)(80)$

By law of calorimetry, we have

$Q_{\text{lost}} = Q_{\text{gained}}$

$\Rightarrow \quad m(0.5)(20) + (m - 20)(80) = 2000$

$\Rightarrow \quad 90m = 3600$

$\Rightarrow \quad m = 40 \text{ g}$

Hence, the correct answer is (B).

72. Let final temperature be T. Since, heat lost equals the heat gained, so we have

$(0.1)(500 - T)(400) = (800 + 0.5 \times 4200)(T - 30)$

$\Rightarrow \quad (500 - T) = \dfrac{2900}{40}(T - 30)$

$\Rightarrow \quad T = 36.39 \text{ °C}$

$\%$ increase $= \dfrac{36.39 - 30}{30} \approx 20\%$

Hence, the correct answer is (B).

73. Since, $100c_1(100 - 90) = 50c_2(90 - 75)$

$\Rightarrow \quad 20S_1 = 15S_2$

$\Rightarrow \quad 4S_1 = 3S_2$

Let final temperature be T, then we have

$100c_1(100 - T) = 50c_2(T - 50)$

$\Rightarrow \quad 75c_2(100 - T) = 50c_2(T - 50)$

$\Rightarrow \quad 3(100 - T) = 2T - 100$

CHAPTER 2

$\Rightarrow \quad T = 80\ °C$

Hence, the correct answer is (B).

74. Since, $\Delta\ell = \ell\alpha\Delta T$

$\Rightarrow \quad \alpha_A(180-30) = \alpha_B(T-30)$

$\Rightarrow \quad 4(180-30) = 3(T-30)$

$\Rightarrow \quad T = 230\ °C$

Hence, the correct answer is (B).

75. Since, for a polytropic process, $TV^{x-1} = \text{constant}$

$\Rightarrow \quad x-1=1$

$\Rightarrow \quad x=2$

$\Rightarrow \quad C = C_V + \dfrac{R}{1-x} = \dfrac{3R}{2} + \dfrac{R}{1-2} = \dfrac{R}{2}$

So, $Q = nC\Delta T = \dfrac{R}{2}\Delta T$

Hence, the correct answer is (B).

76. Since, $\dfrac{(x_0/2)-(x_0/3)}{x_0-(x_0/3)} = \dfrac{T-0}{100}$

$\Rightarrow \quad \dfrac{x_0/6}{2x_0/3} = \dfrac{T}{100}$

$\Rightarrow \quad \dfrac{T}{100} = \dfrac{1}{4}$

$\Rightarrow \quad T = 25\ °C$

Hence, the correct answer is (D).

77. Work done, W = Area under PV graph

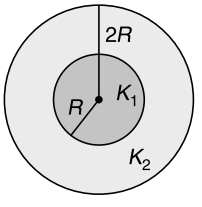
$\Rightarrow \quad W = \dfrac{1}{2}\times 4\times 5 = 10\ \text{J}$

Hence, the correct answer is (B).

78. Since $K_{eq} = \dfrac{K_1A_1 + K_2A_2}{A_1+A_2}$

$\Rightarrow \quad K_{eq} = \dfrac{K_1\pi R^2 + K_2 3\pi R^2}{4\pi R^2}$

$\Rightarrow \quad K_{eq} = \dfrac{K_1 + 3K_2}{4}$



Hence, the correct answer is (B).

79. Since $U = \dfrac{3}{2}nRT$, for a monatomic gas

$\Rightarrow \quad U = \dfrac{3}{2}PV = \dfrac{3}{2}\times 3\times 10^6 \times 2$

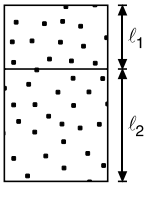
$\Rightarrow \quad U = 9\times 10^6\ \text{J}$

Hence, the correct answer is (B).

80. Since, $(P_2 - P_1)A = mg$

$\Rightarrow \quad \left(\dfrac{nRT}{A\ell_2} - \dfrac{nRT}{A\ell_1}\right)A = mg$

$\Rightarrow \quad m = \dfrac{nRT}{g}\left(\dfrac{\ell_1-\ell_2}{\ell_1\ell_2}\right)$



Hence, the correct answer is (A).

81. Average collision time

$$\tau = \frac{\text{Mean Free Path}}{\text{Average Speed}} = \frac{\lambda}{v_{av}}$$

The mean free path of molecules of an ideal gas is

$$\lambda = \frac{1}{\sqrt{2}\pi n_0 d^2} = \frac{V}{\sqrt{2}\pi N d^2} = \frac{RT}{\sqrt{2}\pi N P d^2}$$

where, V is the volume of container, N is number of molecules and d is the diameter of the molecule.

$$\tau = \frac{\lambda}{v_{av}} = \frac{RT}{\sqrt{2}\pi N P d^2 v_{av}}$$

Since, average speed $v_{av} \propto \sqrt{T}$

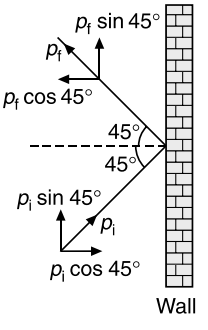
$\Rightarrow \quad \tau \propto \dfrac{\sqrt{T}}{P}$

$\Rightarrow \quad \dfrac{\tau_1}{\tau_2} = \sqrt{\dfrac{5}{3}}\times\dfrac{1}{2}\times 6\times 10^{-8} = 3.87\times 10^{-8}\ \text{s}$

$\Rightarrow \quad \dfrac{\tau_1}{\tau_2} = 4\times 10^{-8}\ \text{s}$

Hence, the correct answer is (D).

82. As $p_i = p_f$



Net force on the wall,

$$F = \frac{dp}{dt} = 2np_f\cos 45° = 2nmv\cos 45°$$

Here, n is the number of hydrogen molecules striking per second.

Pressure $P = \dfrac{F}{A} = \dfrac{2nmv\cos 45°}{\text{Area}}$

$\Rightarrow \quad P = \dfrac{2\times 10^{23}\times 3.32\times 10^{-27}\times 10^3\times\left(\dfrac{1}{\sqrt{2}}\right)}{2\times 10^{-4}}$

$\Rightarrow \quad P = 2.35\times 10^3\ \text{Nm}^{-2}$

Hence, the correct answer is (A).

83. Since, $\dfrac{3}{2}k_BT = \dfrac{1}{2}mv^2$

$\Rightarrow \quad v = \sqrt{\dfrac{3k_BT}{m}} = \sqrt{\dfrac{3\times 1.4\times 10^{-23}\times 300}{7\times 10^{-27}}}$

$\Rightarrow \quad v = \sqrt{1.8}\times 10^3\ \text{ms}^{-1} \approx 1.3\times 10^3\ \text{ms}^{-1}$

Hence, the correct answer is (A).

84. Given $\gamma_{mix} = \frac{3}{2}$, so $(C_V)_{mix} = \frac{R}{\gamma_{mix} - 1} = 2R$

$\Rightarrow \quad (C_V)_{mix} = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2}$

$\Rightarrow \quad 2R = \frac{2(3R/2) + n(5R/2)}{2 + n}$

$\Rightarrow \quad 8 + 4n = 6 + 5n$

$\Rightarrow \quad n = 2$

Hence, the correct answer is (C).

85. For an adiabatic process $TV^{\gamma-1} = \text{constant}$

$\Rightarrow \quad T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

$\Rightarrow \quad T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1}$

Here, $T_1 = 27\ ^\circ\text{C} = 300\ \text{K}$, $V_1 = V$, $V_2 = 2V$, $\gamma = \frac{5}{3}$

$\Rightarrow \quad T_2 = 300\left(\frac{V}{2V}\right)^{\left(\frac{5}{3}-1\right)} = 300\left(\frac{1}{2}\right)^{\frac{2}{3}} \approx 189\ \text{K}$

Change in internal energy, $\Delta U = nC_V \Delta T$

$\Rightarrow \quad \Delta U = n\left(\frac{fR}{2}\right)(T_2 - T_1) = 2 \times \frac{3}{2} \times \frac{25}{3}(189 - 300)$

$\Rightarrow \quad \Delta U = -25 \times 111 = -2775\ \text{J} = -2.7\ \text{kJ}$

Hence, the correct answer is (C).

86. Work done on gas $W_{isot} = nRT \ln\left(\frac{P_f}{P_i}\right)$

$\Rightarrow \quad W_{isot} = R(300)\ell\text{n}(2) = 300R\ell n2$

Hence, the correct answer is (B).

87. For a refrigerator, $\frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$

$\Rightarrow \quad \frac{500}{W} = \frac{250}{50}$

$\Rightarrow \quad W = 100\ \text{cal} = (100)(4.2)\ \text{J} = 420\ \text{J}$

Hence, the correct answer is (D).

88. Assuming equal works to be done by both the engines (should have been mentioned in the question for solving it), we have

$$T = \frac{T_1 + T_2}{2} = \frac{600 + 100}{2} = 350\ \text{K}$$

$$\Rightarrow \quad \frac{\eta_A}{\eta_B} = \frac{1 - \frac{350}{600}}{1 - \frac{100}{350}} = \frac{250/600}{250/350} = \frac{350}{600} = \frac{7}{12}$$

Hence, the correct answer is (C).

89. Equation of line BC is given by

$$P = P_0 - \frac{2P_0}{V_0}(V - 2V_0)$$

$$\Rightarrow \quad PV = P_0 V - \frac{2P_0}{V_0}(V - 2V_0)V$$

$$\Rightarrow \quad T = \frac{P_0 V - \frac{2P_0 V^2}{V_0} + 4P_0 V}{1 \times R} \qquad \{\because\ PV = nRT\}$$

$$\Rightarrow \quad T = \frac{P_0}{R}\left(5V - \frac{2V^2}{V_0}\right)$$

For maximum value of T, $\frac{dT}{dV} = 0$

$$\Rightarrow \quad 5 - \frac{4V}{V_0} = 0$$

$$\Rightarrow \quad V = \frac{5}{4}V_0$$

$$\Rightarrow \quad T_{max} = \frac{P_0}{R}\left[5\left(\frac{5V_0}{4}\right) - \frac{2}{V_0}\left(\frac{25}{16}V_0^2\right)\right] = \frac{25}{8}\frac{P_0 V_0}{R}$$

Hence, the correct answer is (B).

90. Let molar heat capacity at constant pressure be X_P and molar heat capacity at constant volume be X_V

$$X_P - X_V = R$$

$$\Rightarrow \quad MC_P - MC_V = R$$

$$\Rightarrow \quad C_P - C_V = \frac{R}{M}$$

For hydrogen, we have $a = \frac{R}{2}$

For N_2, we have $b = \frac{R}{28}$

$$\Rightarrow \quad \frac{a}{b} = 14$$

$$\Rightarrow \quad a = 14b$$

Hence, the correct answer is (C).

91. An ideal gas has molecules with 5 degrees of freedom, then

$$C_V = \frac{5}{2}R \text{ and } C_P = \frac{7}{2}R$$

$$\Rightarrow \quad \frac{C_P}{C_V} = \frac{7R/2}{5R/2} = \frac{7}{5}$$

Hence, the correct answer is (D).

92. Initial kinetic energy of the system $K_i = \frac{5}{2}RTN$

Final kinetic energy of the system

$$K_f = \frac{5}{2}RT(N - n) + \frac{3}{2}RT(2n)$$

$$\Rightarrow \quad \Delta K = K_f - K_i - = nRT\left(3 - \frac{5}{2}\right) = \frac{1}{2}nRT$$

Hence, the correct answer is (C).

93. Let n_1 be initial number of moles, so

$$n_1 = \frac{P_1 V_1}{RT_1} = \frac{10^5 \times 30}{8.3 \times 290} \approx 1.24 \times 10^3$$

Let n_2 be final number of moles, so

$$n_2 = \frac{P_2 V_2}{RT_2} = \frac{10^5 \times 30}{8.3 \times 300} \approx 1.20 \times 10^3$$

CHAPTER 2

Change of number of molecules is

$$\Delta n = n_f - n_i = (n_2 - n_1) \times 6.023 \times 10^{23}$$

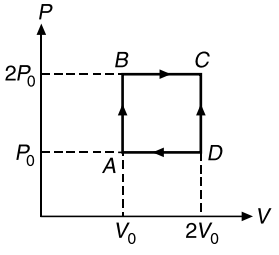
$$\Rightarrow \quad \Delta n \approx -2.5 \times 10^{25}$$

Hence, the correct answer is (D).

94. Work done by engine equals the area enclosed by the PV diagram, so $W = P_0V_0$

Heat given to the system is

$$Q = Q_{AB} + Q_{BC} = nC_V\Delta T_{AB} + nC_P\Delta T_{BC}$$



$$\Rightarrow \quad Q = \frac{3}{2}(nRT_B - nRT_A) + \frac{5}{2}(nRT_C - nRT_B)$$

$$\Rightarrow \quad Q = \frac{3}{2}(2P_0V_0 - P_0V_0) + \frac{5}{2}(4P_0V_0 - 2P_0V_0)$$

$$\Rightarrow \quad Q = \frac{13}{2}P_0V_0$$

Thermal efficiency, $\eta = \dfrac{W}{\Sigma Q_{\oplus}} = \dfrac{W}{Q_{\text{input}}} = \dfrac{P_0V_0}{13P_0V_0/2}$

$$\Rightarrow \quad \eta = \frac{2}{13} \approx 0.15$$

Hence, the correct answer is (A).

95. Here, $PV = \text{constant}$, so given process is isothermal i.e., temperature is constant. Pressure at point 1 is higher than that at point 2. So, correct OPTION is (C).
Hence, the correct answer is (C).

96. Ideal gas equation, $PV = nRT$
As temperature is constant.

$$PV = \text{constant}$$

$$\Rightarrow \quad P\left(\frac{m}{\rho}\right) = \text{constant}$$

$$\Rightarrow \quad P \propto \rho \text{ (for given } m)$$

Hence, the correct answer is (D).

97. Specific heat of a polytropic process is

$$C = C_V + \frac{R}{1-n}$$

$$\Rightarrow \quad \frac{R}{1-n} + C_V = C$$

$$\Rightarrow \quad \frac{R}{1-n} = C - C_V$$

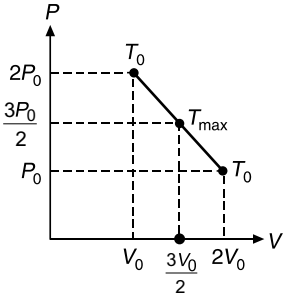
$$\Rightarrow \quad \frac{R}{C - C_V} = 1 - n$$

Since, $R = C_P - C_V$

$$\Rightarrow \quad n = \frac{C - C_P}{C - C_V}$$

Hence, the correct answer is (C).

98. **Method-I**



Since initial and final temperature are equal hence maximum temperature is at middle of line.

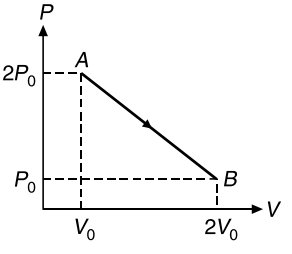
$$PV = nRT$$

$$\Rightarrow \quad T_{\max} = \frac{(3P_0/2)(3V_0/2)}{nR} = \frac{9P_0V_0}{4nR}$$

Method-II
Equation of line

$$P = -\left(\frac{P_0}{V_0}\right)V + 3P_0$$

$$\Rightarrow \quad PV = -\left(\frac{P_0}{V_0}\right)V^2 + 3P_0V = x \text{ (say)}$$



For temperature to be maximum, the product $PV = x$ should also be the maximum, so we have $\dfrac{dx}{dV} = 0$.

$$\Rightarrow \quad V = \frac{3V_0}{2} \text{ and } P = \frac{3P_0}{2}$$

$$\Rightarrow \quad T = \frac{PV}{nR} = \frac{9}{4}\frac{P_0V_0}{nR}$$

Hence, the correct answer is (B).

99. For an ideal gas in an isobaric process,

Heat supplied, $Q = nC_p\Delta T$

Work done, $W = P\Delta V = nR\Delta T$

$$\Rightarrow \quad \frac{W}{Q} = \frac{nR\Delta T}{nC_p\Delta T} = \frac{R}{5R/2} = \frac{2}{5}$$

Hence, the correct answer is (A).

100. For isochoric process, $\Delta U = Q = mc\Delta T$

where, $m = 200 \text{ g} = 0.2 \text{ kg}$, $c = 4184 \text{ Jkg}^{-1}\text{K}^{-1}$

$\Delta T = 60 - 40 = 20\ °C = 20\ K$

$\Rightarrow \quad \Delta U = 0.2 \times 4184 \times 20 = 16736\ J = 16.7\ kJ$

Hence, the correct answer is (D).

101. For a refrigerator, we have

$$\frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$$

So, energy consumed by the freezer is given by

$$W = Q_2\left(\frac{T_1}{T_2} - 1\right)$$

where, $T_1 = 27\ °C = 300\ K$, $T_2 = 0\ °C = 273\ K$ and $Q_2 = mL = 5 \times 336 \times 10^3\ J$

$$\Rightarrow \quad W = 5 \times 336 \times 10^3\left(\frac{300}{273} - 1\right) = 1.67 \times 10^5\ J$$

Hence, the correct answer is (D).

102. Since, average time (τ) is given by

$$\tau = \frac{1}{\sqrt{2}\pi n_0 d^2 v_{rms}}, \text{ where } n_0 = \frac{N}{V}$$

Now, since $n_0 \propto \frac{1}{V}$ and $v_{rms} \propto \sqrt{T}$

$$\Rightarrow \quad \tau \propto \frac{V}{\sqrt{T}} \qquad \ldots(1)$$

Now, for an adiabatic process,

$$TV^{\gamma - 1} = \text{constant} \qquad \ldots(2)$$

So, from (1) and (2), we get

$$\tau \propto V^{\frac{\gamma+1}{2}}$$

Hence, the correct answer is (C).

103. Average force applied on the walls by a molecule,

$$F = \frac{2mv}{t}$$

$$\Rightarrow \quad t = \frac{2\ell}{v}$$

$$\Rightarrow \quad t \propto \frac{\ell}{v}$$

$$\Rightarrow \quad F \propto v^2 \qquad \ldots(1)$$

Since, $v^2 \propto T \qquad \ldots(2)$

From (1) and (2), we get

$$F \propto T$$

Hence, the correct answer is (B).

104. For metals, there is no free motion but rather oscillation about mean position. Thus, these have kinetic energy and potential energy, which are almost equal.

$$(\text{P.E.})_{avg} = (\text{K.E.})_{avg} = \frac{3}{2}k_B T$$

Energy per molecule is $E = K.E. + P.E. = 3k_B T$

Energy per mole is $E = 3RT$

Also, $E = MCT$

$$\Rightarrow \quad 3RT = MCT$$

$$\Rightarrow \quad C = \frac{3k_B}{m} = \frac{3R}{M}$$

$$\Rightarrow \quad C = \frac{3 \times 8.314}{27 \times 10^{-3}} \approx 925\ Jkg^{-1}\ K^{-1}$$

Hence, the correct answer is (C).

105. Since, entropy is a state function, so it does not depends upon the path and only depends on the initial and final state of the system. So, we have

$$dS = \frac{dQ}{T} = mc\left(\frac{dT}{T}\right)$$

$$\Rightarrow \quad \Delta S = \int ds = mc\int_{T_i}^{T_f}\frac{dT}{T}$$

$$\Rightarrow \quad \Delta S = mc\log_e\left(\frac{T_f}{T_i}\right)$$

Now, $mc = 1\ J°C^{-1}$

$$\Rightarrow \quad \Delta S = \log_e\left(\frac{527 + 273}{127 + 273}\right) = \log_e(2)$$

Hence, the correct answer is (B).

106. Since, $p = \frac{1}{3}\left(\frac{U}{V}\right)$ and $pV = nRT$

$$\Rightarrow \quad \frac{nRT}{V} \propto \frac{T^4}{3} \qquad \left\{\because\ \frac{U}{V} \propto T^4\right\}$$

$$\Rightarrow \quad VT^3 = \text{constant}$$

$$\Rightarrow \quad \left(\frac{4}{3}\pi R^3\right)T^3 = \text{constant}$$

$$\Rightarrow \quad TR = \text{constant}$$

$$\Rightarrow \quad T \propto \frac{1}{R}$$

Hence, the correct answer is (C).

107. For a cyclic process, $\Delta U = 0$

So, OPTION (A) is incorrect.

For process CA, $\Delta U = (1)\left(\frac{5R}{2}\right)(-200)$

$$\Rightarrow \quad \Delta U_{CA} = -500R$$

So, OPTION (B) is incorrect.

For process AB, $\Delta U = (1)\left(\frac{5R}{2}\right)(400)$

$$\Rightarrow \quad \Delta U_{AB} = 1000R$$

So, OPTION (C) is incorrect.

For process BC, $\Delta U = (1)\left(\frac{5R}{2}\right)(-200)$

$$\Rightarrow \quad \Delta U_{BC} = -500R$$

So, OPTION (D) is correct.

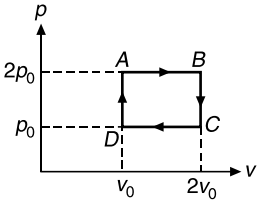
Hence, the correct answer is (D).

108. Heat is extracted from source in AB and DA

In AB, $Q_{AB} = nC_p\Delta T = n\left(\frac{5R}{2}\right)\Delta T$

$$\Rightarrow \quad Q_{AB} = \frac{5}{2}(4p_0v_0 - 2p_0v_0) = 5p_0v_0$$

CHAPTER 2



In DA, $Q_{DA} = nC_V\Delta T = n\left(\frac{3R}{2}\right)\Delta T = \frac{3}{2}p_0v_0$

Total heat extracted from source i.e., absorbed is

$$Q = Q_{AB} + Q_{DA} = \frac{13}{2}p_0v_0$$

Hence, the correct answer is (B).

109. Efficiency of Carnot engine, $\eta = 1 - \frac{T}{T_1}$

where T_1 is the temperature of the source and T_2 is the temperature of the sink?

For 1st case $\eta = 40\%$, $T_1 = 500$ K

$$\Rightarrow \quad \frac{40}{100} = 1 - \frac{T_2}{500}$$

$$\Rightarrow \quad \frac{T_2}{500} = 1 - \frac{40}{100} = \frac{3}{5}$$

$$T_2 = \frac{3}{5}\times 500 = 300 \text{ K}$$

For 2nd case $\eta = 60\%$, $T_2 = 300$ K

$$\Rightarrow \quad \frac{60}{100} = 1 - \frac{300}{T_1}$$

$$\Rightarrow \quad \frac{300}{T_1} = 1 - \frac{60}{100} = \frac{2}{5}$$

$$T_1 = \frac{5}{2}\times 300 = 750 \text{ K}$$

Hence, the correct answer is (B).

110. In case of a cyclic process, work done is equal to the area under the cycle and is taken to be positive if the cycle is clockwise, so work done is

$$W = \text{Area of Rectangle } ABCD = P_0V_0$$

Since, helium gas is a monatomic gas, so

$$C_V = \frac{3R}{2} \text{ and } C_P = \frac{5R}{2}$$

Along the path AB, heat is supplied to the gas at constant volume, so

$$\Delta Q_{AB} = nC_V\Delta T = n\left(\frac{3R}{2}\right)\Delta T = \frac{3}{2}P_0V_0$$

Along the path BC, heat is supplied to the gas at constant pressure, so

$$\Delta Q_{AB} = nC_P\Delta T = n\left(\frac{5R}{2}\right)\Delta T = \frac{5}{2}(2P_0V_0) = 5P_0V_0$$

Along the path CD and DA, heat is rejected by the gas, so efficiency of the cycle is

$$\eta = \frac{\text{Work done by the gas}}{\text{Heat supplied to the gas}}\times 100\%$$

$$\Rightarrow \quad \eta = \frac{P_0V_0}{\frac{3P_0V_0}{2} + 5P_0V_0}\times 100 = \frac{200}{13} = 15.4\%$$

Hence, the correct answer is (D).

111. Kinetic energy of vessel $K = \frac{1}{2}mv^2$

Increase in internal energy is $\Delta U = nC_V\Delta T$

where, n is the number of moles of the gas in vessel. When the vessel is stopped suddenly, its kinetic energy is used to increase temperature of the gas, so

$$\frac{1}{2}mv^2 = \Delta U = nC_V\Delta T = \frac{m}{M}C_V\Delta T \qquad \left\{\because\ n = \frac{m}{M}\right\}$$

$$\Rightarrow \quad \Delta T = \frac{Mv^2}{2C_V}$$

$$\Rightarrow \quad \Delta T = \frac{Mv^2(\gamma - 1)}{2R} \qquad \left\{\because\ C_V = \frac{R}{(\gamma - 1)}\right\}$$

Hence, the correct answer is (D).

112. The efficiency of Carnot engine, $\eta = 1 - \frac{T_2}{T_1}$

$$\Rightarrow \quad \frac{1}{6} = 1 - \frac{T_2}{T_1} \qquad \left(\text{Given, } \eta = \frac{1}{6}\right)$$

$$\Rightarrow \quad \frac{T_2}{T_1} = \frac{5}{6}$$

$$\Rightarrow \quad T_1 = \frac{6T_2}{5} \qquad \ldots(1)$$

As per question, when T_2 is lowered by 62 K, then its efficiency becomes $2\eta = 2\left(\frac{1}{6}\right) = \frac{1}{3}$

$$\Rightarrow \quad \frac{1}{3} = 1 - \frac{T_2 - 62}{T_1}$$

$$\Rightarrow \quad \frac{T_2 - 62}{T_1} = 1 - \frac{1}{3} = \frac{2}{3}$$

$$\Rightarrow \quad \frac{T_2 - 62}{6T_2/5} = \frac{2}{3} \qquad \{\text{using equation (1)}\}$$

$$\Rightarrow \quad \frac{5(T_2 - 62)}{6T_2} = \frac{2}{3}$$

$$\Rightarrow \quad 5T_2 - 310 = 4T_2$$

$$\Rightarrow \quad T_1 = \frac{6\times 310}{5} = 372 \text{ K}$$

Hence, the correct answer is (A).

113. Since $Q = mc\Delta T$

where, $m = 100$ g $= 100\times 10^{-3}$ kg

$c = 4184$ Jkg^{-1}K^{-1} and $\Delta T = (50 - 30) = 20$ °C

$$\Rightarrow \quad Q = 100\times 10^{-3}\times 4184\times 20 = 8.4\times 10^3 \text{ J}$$

By FLTD, we have $Q = \Delta U + W$

Since we are ignoring the slight expansion of water, so we have $W \approx 0$. Hence change in internal energy is

$$U = Q = 8.4\times 10^3 \text{ J} = 8.4 \text{ kJ}$$

Hence, the correct answer is (B).

114. For an adiabatic process $TV^{\gamma-1} = \text{constant}$

$$\Rightarrow \quad T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}, \text{ where } \gamma = \frac{7}{5}$$

$$\Rightarrow \quad T_1 = T_2\left(\frac{V_2}{V_1}\right)^{\frac{2}{5}} = T_2\left(\frac{32V}{V}\right)^{\frac{2}{5}} = T_2(2^5)^{\frac{2}{5}} = 4T_2$$

Efficiency of the engine, $\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{1}{4}$

$$\Rightarrow \quad \eta = \frac{3}{4} = 0.75$$

Hence, the correct answer is (C).

115. For path AB, pressure is constant and $\Delta T = 200$ K

$$\Rightarrow \quad W_{\text{isob}} = P\Delta V = nR\Delta T = 2R(200) = 400R$$

Hence, the correct answer is (C).

116. For path DA, temperature is constant

$$\Rightarrow \quad W_{\text{isot}} = nRT_D \ln\left(\frac{V_A}{V_D}\right) = nRT_D \ln\left(\frac{P_D}{P_A}\right)$$

$$\Rightarrow \quad W = 2R(300)\ln(1/2) = -600R(0.69) = -414R$$

So work done on the gas is $W = +414R$

Hence, the correct answer is (B).

117. Total work done on the gas when taking from A to B is $400R$ and from C to D is equal and opposite, so they cancel each other.

Since $W_{D\to A} = -414R$ and

$$W_{B\to C} = (2)R(500)\ln(2) = 690R$$

$$\Rightarrow \quad W = -414R + 690R = 276R$$

Hence, the correct answer is (B).

118. The internal energy is the energy due to thermal motion, so we have

$$U = \left(\frac{nf}{2}\right)RT = \left(\frac{5n}{2}\right)RT$$

Since $PV = nRT$, $V = \frac{m}{\rho} = \frac{1\text{ kg}}{4\text{ kgm}^{-3}} = \frac{1}{4}\text{ m}^3$ and $P = 8\times10^4\text{ Nm}^{-2}$, so we get

$$U = \frac{5}{2}\times8\times10^4\times\frac{1}{4} = 5\times10^4\text{ J}$$

Hence, the correct answer is (B).

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Single Correct Choice Type Questions

1. Rate of heat transfer through metal rod is

$$\frac{dQ}{dt} = C\frac{dT}{dt} = P(\text{constant}) \qquad \ldots(1)$$

Since temperature variation is given by

$$T = T_0\left(1+\beta t^{\frac{1}{4}}\right) \qquad \ldots(2)$$

$$\Rightarrow \quad \frac{dT}{dt} = \frac{T_0\beta}{4}t^{-\frac{3}{4}}$$

So from equation (1), we get

$$C = \frac{P}{dT/dt} = \frac{4P}{\beta T_0}t^{\frac{3}{4}}$$

Substituting the value of t from equation (2), we get

$$C = \frac{4P(T-T_0)^3}{(\beta T_0)^4}$$

Hence, the correct answer is (C).

2. Heat generated in device in 3 h is

$$W = Pt = 3\times3600\times3\times10^3 = 324\times10^5\text{ J}$$

Heat used to heat water

$$Q = mc\Delta T = 120\times1\times4.2\times10^3\times20\text{ J}$$

$$\Rightarrow \quad Q = 100.8\times10^5\text{ J}$$

So, heat absorbed by coolant is

$$H = 324\times10^5 - 120\times1\times4.2\times10^3\times20\text{ J}$$

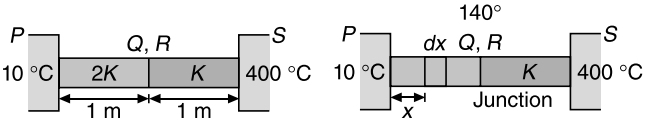
$$\Rightarrow \quad H = P't = (325-100.8)\times10^5\text{ J}$$

$$\Rightarrow \quad P' = \frac{223.2\times10^8}{3600} = 2067\text{ W}$$

Hence, the correct answer is (B).

3. Rate of heat flow from P to Q is

$$\frac{dQ}{dt} = \frac{2KA(T-10)}{1}$$



Rate of heat flow from Q to S is

$$\frac{dQ}{dt} = \frac{KA(4000-T)}{1}$$

At steady state, rate of heat flow is same

$$\frac{2KA(T-10)}{1} = KA(400-T)$$

$$\Rightarrow \quad 2T - 20 = 400 - T$$

$$\Rightarrow \quad 3T = 420$$

$$\Rightarrow \quad T = 140°$$

Temperature of junction is 140 °C

Temperature at a distance x from end P is

$$T_x = (130x + 10°)$$

Change in length of dx is supposed to be dy, then

$$dy = \alpha dx(T_x - 10) \qquad \{\because \Delta l = l\alpha\Delta T\}$$

$$\Rightarrow \quad \int_0^{\Delta y} dy = \int_0^1 \alpha dx(130x + 10 - 10)$$

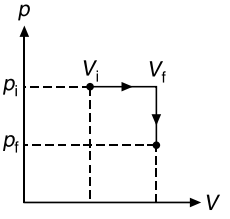
$$\Rightarrow \quad \Delta y = 130\left(\frac{\alpha x^2}{2}\right)\Big|_0^1 = (65)(1.2\times10^{-5}) = 78.0\times10^{-5}\text{ m}$$

$$\Rightarrow \quad \Delta y = 0.78\text{ mm}$$

Hence, the correct answer is (A).

CHAPTER 2

4. For the first process, $p_i V_i^{\gamma} = p_f V_f^{\gamma}$



$$\Rightarrow \quad \frac{p_i}{p_f} = \left(\frac{V_f}{V_i}\right)^{\gamma}$$

$$\Rightarrow \quad 32 = 8^{\gamma}$$

$$\Rightarrow \quad 2^5 = 2^{3\gamma}$$

$$\Rightarrow \quad \gamma = \frac{5}{3} \qquad \ldots(1)$$

For the two-step process, we have

$$W = p_i\left(V_f - V_i\right) = 10^5\left(7\times10^{-3}\right)$$

$$\Rightarrow \quad W = 7\times10^2 \text{ J}$$

Also, $\Delta U = \frac{f}{2}\left(p_f V_f - p_i V_i\right)$

$$\Rightarrow \quad \Delta U = \frac{1}{\gamma - 1}\left(\frac{1}{4}\times10^2 - 10^2\right) = -\frac{9}{8}\times10^2 \text{ J}$$

By FLTD, we have $\quad Q = \Delta U + W$

$$\Rightarrow \quad Q = 7\times10^2 - \frac{9}{8}\times10^2 = \frac{47}{8}\times10^2 \text{ J} = 588 \text{ J}$$

Hence, the correct answer is (C).

5. In CASE-1, $R_I = R_1 + R_2 = \left(\frac{l}{KA}\right) + \left(\frac{l}{2KA}\right) = \frac{3}{2}\left(\frac{l}{KA}\right)$

In CASE-2, $\frac{1}{R_{II}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{KA}{l} + \frac{2KA}{l}$

$$\Rightarrow \quad R_{II} = \frac{l}{3KA} = \frac{R_1}{4.5}$$

Since thermal resistance R_{II} in 4.5 times less than thermal resistance R_1.

$$\Rightarrow \quad t_{II} = \frac{t_1}{4.5} = \frac{9}{4.5} \text{ s} = 2 \text{ s}$$

Hence, the correct answer is (A).

6. From ideal gas equation, we have $\rho = \frac{pM}{RT}$

$$\Rightarrow \quad \rho \propto pM$$

$$\Rightarrow \quad \frac{\rho_1}{\rho_2} = \left(\frac{p_1}{p_2}\right)\left(\frac{M_1}{M_2}\right) = \left(\frac{2}{3}\right)\left(\frac{4}{3}\right) = \frac{8}{9}$$

Hence, the correct answer is (D).

7. $Q = nCp\Delta T = 208 \text{ J}$

Hence, the correct answer is (D).

8. At equilibrium, $(\text{Heat Absorbed}) = (\text{Heat Radiated})$

$$\Rightarrow \quad A(3T)^4 + A(2T)^4 = 2AT_0^4$$

$$\Rightarrow \quad T_0 = \left(\frac{97}{2}\right)^{1/4} T$$

Hence, the correct answer is (C).

9. $\frac{v_{He}}{v_{Ar}} = \sqrt{\frac{M_{Ar}}{M_{He}}} = \sqrt{10} = 3.16$

Hence, the correct answer is (D).

10. Since, $T_1 V_1^{(\gamma-1)} = T_2 V_2^{(\gamma-1)}$

$$\Rightarrow \quad T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$\Rightarrow \quad T_2 = 4T_1 \qquad \left\{\because \gamma = \frac{5}{3}\right\}$$

n = Number of moles of gas $= \frac{5.6 \text{ lt}}{22.4 \text{ lt}} = \frac{1}{4}$

$$\Rightarrow \quad W_{\text{ext}} = \frac{nR}{1-\gamma}\left(T_2 - T_1\right)$$

$$\Rightarrow \quad W_{\text{ext}} = +\frac{9}{8}RT_1$$

Hence, the correct answer is (A).

11. Since $ML = Pt$

$$\Rightarrow \quad L = \frac{Pt}{M}$$

Hence, the correct answer is (C).

12. A real gas behaves like an ideal gas at low pressure and high temperature.
Hence, the correct answer is (D).

13. It cracks due to low thermal conductivity.
Hence, the correct answer is (A).

14. $PT^2 = \text{constant}$

Since, $PV = nRT$

$$\Rightarrow \quad \left(\frac{T}{V}\right)T^2 = \text{constant}$$

$$\Rightarrow \quad V \propto T^3$$

$$\Rightarrow \quad \frac{\Delta V}{V} = \frac{3\Delta T}{T}$$

$$\Rightarrow \quad \frac{\Delta V}{V\Delta T} = \frac{3}{T}$$

Hence, the correct answer is (C).

15. 1 calorie is the heat required to raise the temperature of 1 g of water from 14.5 °C to 15.5 °C at 760 mm of Hg.
Hence, the correct answer is (A).

16. $\lambda_m T = \text{constant}$

From the graph $T_3 > T_2 > T_1$

Temperature of sun will be maximum. Therefore, (C) is the correct OPTION.
Hence, the correct answer is (C).

17. Glass of bulb heats due to filament by radiation.
Hence, the correct answer is (C).

18. Energy gained by water (in 1 s) is

$$\frac{dQ}{dt} = 1000 - 160 = 840 \text{ W}$$

Total heat required to raise the temperature of water from 27 °C to 77 °C is $mc\Delta T$.
Hence, the required time

$$t = \frac{mc\Delta T}{\text{Rate at which energy is gained by water}}$$

$$\Rightarrow \quad t = \frac{(2)(4.2\times10^3)(50)}{840} = 500 \text{ s} = 8 \text{ min } 20 \text{ s}$$

Hence, the correct answer is (A).

19. $Q \propto AT^4$ and $\lambda_m T = \text{constant}$. Hence,

$$Q \propto \frac{A}{(\lambda_m)^4}$$

$$\Rightarrow \quad Q \propto \frac{r^2}{(\lambda_m)^4}$$

$$\Rightarrow \quad Q_A : Q_B : Q_C = \frac{(2)^2}{(3)^4} : \frac{(4)^2}{(4)^4} : \frac{(6)^2}{(5)^4}$$

$$\Rightarrow \quad Q_A : Q_B : Q_C = \frac{4}{81} : \frac{1}{16} : \frac{36}{625}$$

$$\Rightarrow \quad Q_A : Q_B : Q_C = 0.05 : 0.0625 : 0.0576$$

$\Rightarrow$ Q_B is maximum.

Hence, the correct answer is (B).

20. $\frac{dQ}{dt} = L\left(\frac{dm}{dt}\right)$

$$\Rightarrow \quad \frac{\text{Temperature difference}}{\text{Thermal resistance}} = L\left(\frac{dm}{dt}\right)$$

$$\Rightarrow \quad \frac{dm}{dt} \propto \frac{1}{\text{Thermal resistance}}$$

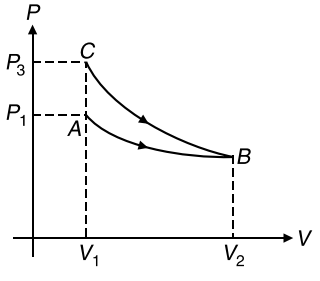
$$q \propto \frac{1}{R}$$

In the first case rode are in parallel and thermal resistance is $\frac{R}{2}$ while in second case rods are in series and thermal resistance is $2R$.

$$\frac{q_1}{q_2} = \frac{2R}{R/2} = \frac{4}{1}$$

Hence, the correct answer is (C).

21. Slope of adiabatic process a given state (P, V, T) is more than the slope of isothermal process. The corresponding P-V graph for the two processes is as shown in figure.



In the graph, AB is isothermal and BC is adiabatic.

W_{AB} =positive (as volume is increasing)

and W_{BC} =negative (as volume is decreasing) plus, $|W_{BC}| > |W_{AB}|$, as area under P-V graph gives the work done.

Hence, $W_{AB} + W_{BC} = W < 0$

From the graph itself, it is clear that $P_3 > P_1$.

Conceptual Note(s)

At point B, slope of adiabatic (process BC) is greater than the slope of isothermal (process AB).

Hence, the correct answer is (C).

22. Temperature of liquid oxygen will first increase in the same phase. Then, phase change (liquid to gas) will take place. During which temperature will remain constant. After that temperature of oxygen n gaseous state will further increase.
Hence, the correct answer is (C).

23. Out of the alternative provided, none appears completely correct.

AB is an isothermal process.

$\left(T = \text{constant}, p \propto \frac{1}{V}\right)$. So, p-V graph should be rectangular hyperbola with p decreasing and V increasing.

BC is an isobaric process.

$(p = \text{constant}, V \propto T)$. Temperature is increasing. Hence, volume should also increase.

CA is an adiabatic process $(pV^\gamma = \text{constant})$. Pressure is increasing. So, volume should decrease. At point A, an isotherm AB and an adiabatic curve AC are meeting. We know that

(slope of an adiabatic graph in p-V diagram) $= \gamma$ (slope of an isothermal graph in the same diagram) with $\gamma > 1$ or $(\text{Slope})_{\text{adiabatic}} > (\text{Slope})_{\text{isothermal}}$

None of the given examples fulfill all the above requirements.
Hence, the correct answer is (None).

24. Heat released by 5 kg of water when its temperature falls from 20 °C to 0 °C is,

$$Q_1 = mc\Delta\theta = (5)(10^3)(20-0) = 10^5 \text{ cal}$$

when 2 kg ice at –20 °C comes to a temperature of 0 °C, it takes an energy.

$$Q_2 = mc\Delta\theta = (2)(500)(20) = 0.2\times10^5 \text{ cal}$$

The remaining heat

$$Q = Q_1 - Q_2 = 0.8\times10^5 \text{ cal}$$

will melt a mass m of the ice, where

$$m = \frac{Q}{L} = \frac{0.8\times10^5}{80\times10^3} = 1 \text{ kg}$$

So, the temperature of the mixture will be 0 °C, mass of water in it is $5+1=6$ kg and mass of ice is $2-1=1$ kg.
Hence, the correct answer is (B).

25. Given $\Delta\ell_1 = \Delta\ell_2$

$$\Rightarrow \quad \ell_1\alpha_a t = \ell_2\alpha_s t$$

$$\Rightarrow \quad \frac{\ell_1}{\ell_2} = \frac{\alpha_s}{\alpha_a}$$

$$\Rightarrow \quad \frac{\ell_1}{\ell_1+\ell_2} = \frac{\alpha_s}{\alpha_a+\alpha_s}$$

Hence, the correct answer is (C).

CHAPTER 2

26. Rate of cooling $\left(-\frac{dT}{dt}\right) \propto$ emissivity (E)

From the graph, $\left(-\frac{dT}{dt}\right)_x > \left(-\frac{dT}{dt}\right)_y$

$\Rightarrow \quad E_x > E_y$

Further emissivity $(E) \propto$ absorptive power (a) (good absorbers are good emitters also)

$\Rightarrow \quad a_x > a_y$

Conceptual Note(s)

Emissivity is a pure ratio (dimensionless) while the emissive power has a unit Js^{-1} or watt.

Hence, the correct answer is (C).

27. Black body radiates maximum number of wavelength and maximum energy if all other conditions (e.g., temperature surface area etc.) are same. So, when the temperature of black body becomes equal to the temperature of the furnace, the black body will radiate maximum energy and it will be brightest of all. Initially it will absorb all the radient energy incident on it, so, it is the darkest one.
Hence, the correct answer is (A).

28. $\beta = -\frac{dV/dP}{V}$ = compressibility of gas

$\Rightarrow \quad \beta = \frac{1}{\text{Bulk modulus of elasticity}}$

$\Rightarrow \quad \beta = \frac{1}{P}$ under isothermal conditions

Thus, β versus P graph will be a rectangular hyperbola.
Hence, the correct answer is (A).

29. $\Delta W_{AB} = P\Delta V = (10)(20-1) = 10$ J

$\Delta W_{BC} = 0$

From First Law of Thermodynamics

$\Delta Q = \Delta W + \Delta U$

$\Delta U = 0$ {$\because$ process $ABCA$ is cyclic}

$\Rightarrow \quad \Delta Q = \Delta W_{AB} + \Delta W_{BC} + \Delta W_{CA}$

$\Rightarrow \quad \Delta W_{CA} = \Delta Q - \Delta W_{AB} - \Delta W_{BC} = 5 - 10 - 0 = -5$ J

Hence, the correct answer is (A).

30. In adiabatic process

Slope of P-V graph, $\frac{dP}{dV} = -\gamma\frac{P}{V}$

Slope $\propto \gamma$ {with negative sign}

From the given graph, $(\text{slope})_2 > (\text{slope})_1$

$\Rightarrow \quad \gamma_2 > \gamma_1$

Therefore, 1 should correspond to $O_2\,(\gamma = 1.4)$ and 2 should correspond to $He\,(\gamma = 1.67)$.
Hence, the correct answer is (B).

31. $dW = 0$

$\Rightarrow$ Isochoric process

$\Rightarrow \quad dQ = dU$ (by First Law of Thermodynamics)

Since $dQ < 0$

$\Rightarrow \quad dU < 0$

There must be a fall in temperature.
Hence, the correct answer is (A).

32. Let θ be the temperature of the junction (say B). Thermal resistance of all the three rods is equal.

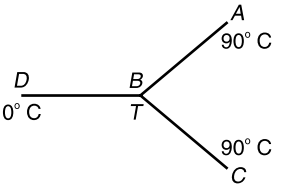
$$\begin{pmatrix}\text{Rate of}\\ \text{heat flow}\\ \text{through}\\ AB\end{pmatrix} + \begin{pmatrix}\text{Rate of}\\ \text{heat flow}\\ \text{through}\\ CB\end{pmatrix} = \begin{pmatrix}\text{Rate of}\\ \text{heat flow}\\ \text{through}\\ BD\end{pmatrix}$$

$\Rightarrow \quad \frac{90-T}{R} + \frac{90-T}{R} = \frac{T-0}{R}$

Here, R = Thermal resistance

$\Rightarrow \quad 3T = 180°$

$\Rightarrow \quad T = 60\ °C$



Conceptual Note(s)

Rate of heat flow

$$(H) = \frac{\text{Temperature difference (TD)}}{\text{Thermal resistnace } (R)}$$

where $R = \frac{\ell}{KA}$

K = Thermal conductivity of the rod.

This is similar to the current flow through a resistance (R) where current (i) = Rate of flow of charge

$$= \frac{\text{Potential difference } (PD)}{\text{Electrical resistance } (R)}$$

Here, $R = \frac{\ell}{\sigma A}$ where σ = Electrical conductivity

Hence, the correct answer is (B).

33. $v \propto \frac{1}{\sqrt{m}}$

Hence, the correct answer is (B).

34. At constant pressure

$V = kT$

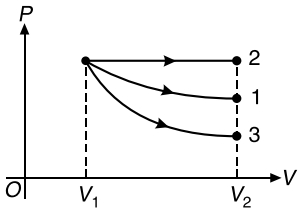
$\Rightarrow \quad \Delta V = k\Delta T$

$\Rightarrow \quad \frac{\Delta V}{V} = \frac{\Delta T}{T}$

$\Rightarrow \quad \delta = \frac{\Delta V}{V\Delta T} = \frac{1}{T}$

$\Rightarrow$ Plot of δ vs T must represent a rectangular hyperbola.
Hence, the correct answer is (C).

35. The work done equals area under the curve



$\Rightarrow \quad W_2 > W_1 > W_3$

Hence, the correct answer is (A).

36. Following steps are involved in this process

(a) Ice at –10 °C will convert to ice at 0 °C and will absorb heat in accordance with formula $dQ = mc_{ice}dT$ i.e. Q vs T must be a straight line with slope $\frac{1}{mc_{ice}} = s_1$ (say) (for T on y-axis and Q on x-axis)

(b) Ice at 0 °C will melt to water at 0 °C (at constant temperature) by absorbing a heat $Q_2 = mL_{ice}$

(c) Water at 0 °C will convert to water at 100 °C and will again absorb heat in accordance with formula $dQ = mc_{water}dT$ i.e. Q vs T is again a straight line with slope $\frac{1}{mc_{water}} = s_2$ (say) $(s_2 < s_1)$

(d) Water at 100 °C will vapourise to steam at 100 °C (at constant temperature) by absorbing heat $Q_4 = mL_{steam}$

Hence, the correct answer is (A).

37. According to Wien's Displacement Law

$\lambda_m T = \text{constant}$

Since, $\lambda_1 < \lambda_3 < \lambda_2$

$\Rightarrow \quad T_1 > T_3 > T_2$

Hence, the correct answer is (B).

38. $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$\Rightarrow \quad T_1(AL_1)^{\frac{5}{3}-1} = T_2(AL_2)^{\frac{5}{3}-1}$

$\Rightarrow \quad \frac{T_1}{T_2} = \left(\frac{L_2}{L_1}\right)^{\frac{2}{3}}$

Hence, the correct answer is (D).

39. $U = U_{Oxygen} + U_{Argon}$

$\Rightarrow \quad U = (2)\left(\frac{5R}{2}\right)T + 4\left(\frac{3R}{2}\right)T$

$\Rightarrow \quad U = 11RT$

Hence, the correct answer is (D).

40. Speed of sound in an ideal gas is given by

$v = \sqrt{\frac{\gamma RT}{M}}$

$\Rightarrow \quad v \propto \sqrt{\frac{\gamma}{M}}$ {T is same for both the gases}

$\Rightarrow \quad \frac{v_{N_2}}{v_{He}} = \sqrt{\frac{\gamma_{N_2}}{\gamma_{He}} \cdot \frac{M_{He}}{M_{N_2}}} = \sqrt{\frac{7/5}{5/3}\left(\frac{4}{28}\right)}$

Since, $\gamma_{N_2} = \frac{7}{5}$ {Diatomic}

and $\gamma_{He} = \frac{5}{3}$ {Monatomic}

$\Rightarrow \quad v = \frac{\sqrt{3}}{5}\ \text{ms}^{-1}$

Hence, the correct answer is (C).

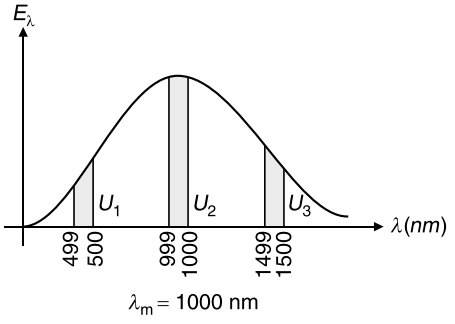
41. Wein's Displacement Law is

$\lambda_m T = b$ {b =Wein's constant}

$\Rightarrow \quad \lambda_m = \frac{b}{T} = \frac{2.88 \times 10^6\ \text{nm-K}}{2880\ \text{K}}$

$\Rightarrow \quad \lambda = 1000\ \text{nm}$

Energy distribution with wavelength will be as follows



From the graph it is clear that

$U_2 > U_1$ {In fact U_2 is maximum}

Hence, the correct answer is (D).

42. $P = \frac{nRT}{V} = \frac{mRT}{MV}$ $\left\{\because\ n = \frac{m}{M}\right\}$

So, at constant volume pressure versus temperature graph is a straight line passing through origin with slope $\frac{mR}{MV}$. As the mass is doubled and volume is halved slope becomes four times. Therefore, pressure versus temperature graph will be shown by the line B.

Hence, the correct answer is (B).

43. **For gas in container** A

$\Delta P = (P_A)_{final} - (P_A)_{initial} = \frac{n_ART}{2V} - \frac{n_ART}{V}$

$\Delta P = -\frac{n_ART}{2V}$ ...(1)

For gas in container B

$1.5\Delta P = (P_B)_{final} - (P_B)_{initial} = \frac{n_BRT}{2V} - \frac{n_BRT}{V}$

$1.5\Delta P = -\frac{n_BRT}{2V}$ ...(2)

From (1) and (2), we get

$n_B = 1.5\, n_A$

CHAPTER 2

$\Rightarrow \quad 2n_B = 3n_A$

$\Rightarrow \quad 2m_B = 3m_A$ {OPTION **(C)**}

Hence, the correct answer is (C).

44. A diatomic molecule has 5 degrees of freedom out of which 3 are translational degrees of freedom and remaining 2 are rotational degrees of freedom. According to Theorem of Equipartition of Energy, the energy associated with each degree of freedom per molecule of a gas in thermal equilibrium is $\frac{1}{2}k_BT$, where k_B is the Boltzmann constant. So, the average rotational kinetic energy associated with each O_2molecule is $2\left(\frac{1}{2}k_BT\right)=k_BT$. Similarly, average rotational kinetic energy associated with each N_2 molecule is $2\left(\frac{1}{2}k_BT\right)=k_BT$. So, the ratio amounts to $1:1$.

{OPTION **(A)**}

However, if we are asked to compare the total rotational kinetic energy associated with 1 mole O_2 gas to that with 2 mole of N_2 gas, then our answer would have been $1:2$.

Hence, the correct answer is (A).

45. Power radiated $\propto$ (surface area) $(T)^4$. The radius is halved, hence, surface area will become $\frac{1}{4}$ times. Temperature is doubled, therefore, T^4 becomes 16 times. New power $=(450)\left(\frac{1}{4}\right)(16)=1800$ W.

Hence, the correct answer is (D).

46. $PV = nRT$

$\Rightarrow \quad P=\frac{nRT}{V}$

$$\frac{P_2}{P_1}=\frac{n_2}{n_1}\frac{T_2}{T_1}$$

$$\frac{P_2}{P_1}=\frac{1}{1}\times\frac{2}{1}$$

$\Rightarrow \quad P_2 = 2P$

Hence, the correct answer is (C).

47. Average translational kinetic energy of an ideal gas molecule is $\frac{3}{2}$ kT which depends on temperature only. Therefore, if temperature is same, translational kinetic energy of O_2 and N_2 both will be equal.

Hence, the correct answer is (C).

48. From Wein's Displacement Law

$\lambda_m T = \text{constant}$

$\Rightarrow \quad T=\frac{1}{\lambda_m}$

$\Rightarrow \quad \frac{T_{\text{sun}}}{T_{\text{north star}}}=\frac{(\lambda_m)_{\text{north star}}}{(\lambda_m)}$

$\Rightarrow \quad \frac{T_{\text{sun}}}{T_{\text{north star}}}=\frac{350}{510}\approx 0.69$

Hence, the correct answer is (B).

49. The average translational KE $=\frac{3}{2}kT$ which is directly proportional to T, while rms speed molecules is given by

$$v_{\text{rms}}=\sqrt{\frac{3RT}{M}} \text{ i.e., } v_{\text{rms}}\propto\sqrt{T}$$

When temperature of gas is increased from 300 Kto 600 K (i.e., 2 times), the average translational KE will increase to 2 times and rms speed to $\sqrt{2}$ or 1.414 times.

So, average translational KE $=2\times6.21\times10^{-21}$ J

$(\text{KE})_{av}=12.42\times10^{-21}$ J and

$v_{\text{rms}}=(1.414)(484)\ \text{ms}^{-1}$

$\Rightarrow \quad v_{\text{rms}}\approx 684\ \text{ms}^{-1}$

Hence, the correct answer is (D).

50. $v_{\text{rms}}=\sqrt{\frac{3RT}{M}}$

i.e., $v_{\text{rms}}\propto\sqrt{T}$

When temperature is increased from 120 K to 480 K (i.e., four times), the root mean square speed will become $\sqrt{4}$ or 2 times i.e., $2v$.

Hence, the correct answer is (B).

51. The rate at which energy leaves the object is $\frac{\Delta Q}{\Delta t}=e\sigma AT^4$

Since, $\Delta Q = mC\Delta T$, we get

$$\frac{\Delta T}{\Delta t}=\frac{e\sigma AT^4}{mC}$$

Also, since $m=\frac{4}{3}\pi r^3\rho$ for a sphere, we get

$$A=\pi r^2=\pi\left(\frac{3m}{4\pi\rho}\right)^{2/3}$$

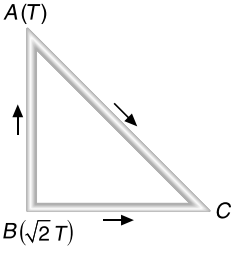
Hence, $\frac{\Delta T}{\Delta t}=\frac{e\sigma T^4}{mC}\left[\pi\left(\frac{3m}{4\pi\rho}\right)^{2/3}\right]=K\left(\frac{1}{m}\right)^{1/3}$

For the given two bodies

$$\frac{(\Delta T/\Delta t)_1}{(\Delta T/\Delta t)_2}=\left(\frac{m_2}{m_1}\right)^{1/3}=\left(\frac{1}{3}\right)^{1/3}$$

Hence, the correct answer is (D).

52. As $T_B > T_A$, heat flows from B to A through both paths



Rate of heat flow in BC =Rate of heat flow in CA

$\Rightarrow \quad \frac{KA\left(\sqrt{2}T-T_C\right)}{\ell}=\frac{KA\left(T_C-T\right)}{\sqrt{2}\ell}$

Solving this we get,

$$T_C = \frac{3T}{\sqrt{2}+1}$$

Hence, the correct answer is (A).

53. The average speed of a gas molecule is $\langle v \rangle = \sqrt{\frac{8k_BT}{\pi m}}$, where m is the mass of the molecule. Since, all the three vessels have same temperature, so average speed of O_2 molecule in the vessel C is also v_1.

Hence, the correct answer is (B).

54. The desired fraction is

$$f = \frac{\Delta U}{\Delta Q} = \frac{nC_V\Delta T}{nC_P\Delta T} = \frac{C_V}{C_P} = \frac{1}{\gamma}$$

$$\Rightarrow \quad f = \frac{5}{7} \qquad \left\{\text{as } \gamma = \frac{7}{5}\right\}$$

Hence, the correct answer is (D).

55. In the process $PV^x = \text{constant}$

Molar heat capacity is given by

$$C = \frac{R}{\gamma - 1} + \frac{R}{1-x}$$

Here the process is $PV^{-1} = \text{constant}$

i.e., $x = -1$

and gas is monatomic. Therefore, $\gamma = \frac{5}{3}$

$$\Rightarrow \quad C = \frac{R}{\frac{5}{3}-1} + \frac{R}{1-(-1)} = 2R$$

$$\Rightarrow \quad \Delta Q = nC\Delta T \qquad \{\because n = 1\}$$

$$\Rightarrow \quad \Delta Q = (1)(2R)(2T_0 - T_0)$$

$$\Rightarrow \quad \Delta Q = 2RT_0$$

Hence, the correct answer is (A).

56. $C_{V_{\text{mixture}}} = \frac{n_1C_{V_1} + n_2C_{V_2}}{n_1 + n_2}$

$$C_{P_{\text{mixture}}} = \frac{n_1C_{P_1} + n_2C_{P_2}}{n_1 + n_2}$$

Hence, the correct answer is (B).

57. Heat required $Q = (1.1 + 0.02) \times 10^2 \times 1 \times (80 - 15)$

$$\Rightarrow \quad Q = 72800 \text{ cal}$$

Therefore, mass of steam condensed (in kg)

$$m = \frac{Q}{L} = \frac{72800}{540} \times 10^{-2} = 0.135 \text{ kg}$$

Hence, the correct answer is (D).

58. Since $70 = (2)C_P(45 - 40)$

$$\Rightarrow \quad C_P = 7 \text{ calmol}^{-1}\text{k}^{-1}$$

$$\Rightarrow \quad C_V = 5 \text{ calmol}^{-1}\text{k}^{-1} \qquad \{\text{Since } C_P - C_V = 2 \text{ calmol}^{-1}\text{k}^{-1}\}$$

$$\Rightarrow \quad Q = (2)(5)(5) = 50 \text{ cal}$$

Hence, the correct answer is (C).

59. $v_{\text{rms}}^2 = \frac{3RT}{M}$

$$\Rightarrow \quad M = \frac{3RT}{v_{\text{rms}}^2} = \frac{3(8.3)(300)}{(1930)^2} \approx 2 \text{ g} \qquad \text{(for } H_2 \text{ gas)}$$

Hence, the correct answer is (A).

60. Since, $\begin{pmatrix}\text{Work done in a}\\ \text{cyclic process}\end{pmatrix} = \begin{pmatrix}\text{Positive area enclosed}\\ \text{by the cycle}\end{pmatrix}$

$$\Rightarrow \quad W = AB \times BC = (2P - P) \times (2V - V) = PV$$

Hence, the correct answer is (A).

Multiple Correct Choice Type Questions

1. Since, $P = \sigma AT^4$

$$\Rightarrow \quad P = 5.67 \times 10^{-8} \times 64 \times 10^{-6} \times (2500)^4$$

$$\Rightarrow \quad P = 141.75 \text{ W}$$

Also, $I = \frac{P}{4\pi r^2} = \frac{141.75}{4\pi \times (100)^2} = 1.13 \times 10^{-3} \text{ Wm}^{-2}$

$$\Rightarrow \quad \Delta E_{\text{eye}} = I \times (\pi r_e^2)$$

$$\Rightarrow \quad \Delta E_{\text{eye}} = I \times (\pi \times 9 \times 10^{-6})$$

$$\Rightarrow \quad \Delta E_{\text{eye}} = 3.2 \times 10^{-8} \text{ W}$$

According to Wein's Law, we have

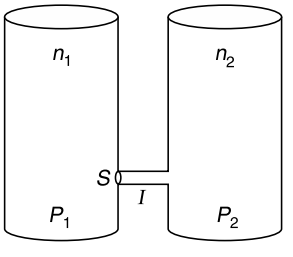
$$\lambda_m = \frac{b}{T} = \frac{2.90 \times 10^{-3}}{2500} = 1160 \text{ nm}$$

$$\Rightarrow \quad N_{\text{photons}} = \frac{\Delta E}{\left(\frac{hc}{\lambda}\right)} = \frac{3.2 \times 10^{-8} \times 1740 \times 10^{-9}}{6.63 \times 10^{-34} \times 3 \times 10^8}$$

$$\Rightarrow \quad N_{\text{photons}} = 2.8 \times 10^{11}$$

Hence, (B), (C) and (D) are correct.

2. Since, $P_1 = n_1k_BT$ and $P_2 = n_2k_BT$



$$\Rightarrow \quad \Delta F = (P_1 - P_2)S$$

$$\Rightarrow \quad \Delta F = (n_1 - n_2)k_BTS = \Delta nk_BTS \qquad \ldots(1)$$

For balancing of force, we have

$$\beta v \times (S\ell)n_1 = \Delta nk_BTS$$

$$\Rightarrow \quad n_1\beta vl = \Delta nk_BT \qquad \ldots(2)$$

$$\Rightarrow \quad v = \frac{\Delta nk_BT}{\beta n_1 I}$$

Number of molecules per second is

$$\frac{\Delta N}{\Delta t} = \frac{(vdt) \times S \times n_1}{dt} = vSn_1$$

CHAPTER 2

$$\Rightarrow \quad \frac{\Delta N}{\Delta t}=\frac{\Delta n k_B T}{\beta n_1 I}\times S n_1=\left(\frac{\Delta n}{I}\right)\left(\frac{k_B T}{\beta}\right)S \qquad \ldots(3)$$

So, Δn decreases with time, so rate of transfer decreases with time.

Hence, (A), (B) and (C) are correct.

3. Since, $\gamma_{\text{mixture}}=\dfrac{5(5R/2)+1(7R/2)}{5(3R/2)+1(5R/2)}=\dfrac{8}{5}=1.6$

For adiabatic process, $PV^{\gamma}=\text{constant}$

$$\Rightarrow \quad P=P_0\left(\frac{V_0}{V}\right)^{\gamma}=(4)^{\frac{8}{5}}P_0=9.2P_0$$

$$\Rightarrow \quad W=\frac{P_0V_0-9.2P_0(V_0/4)}{\gamma-1}=-\frac{13P_0V_0}{6}$$

From ideal gas equation, we have $P_0V_0=6RT_0$

$$\Rightarrow \quad W=-\frac{13P_0V_0}{6}=-13RT_0$$

Final temperature after compression is

$$T=T_0\left(\frac{PV}{P_0V_0}\right)=2.3T_0$$

Also, $(C_V)_{\text{mixture}}=\dfrac{n_1C_{V_1}+n_2C_{V_2}}{n_1+n_2}=\dfrac{5R}{3}$

So, average kinetic energy of gas is

$$U_f=nC_VT=6\left(\frac{5R}{3}\right)(2.3T_0)=23RT_0$$

Hence, (A), (B) and (C) are correct.

4. $|Q_{1\to2}|=nC_PdT=n\left(\dfrac{5R}{2}\right)T_0=\dfrac{5RT_0}{2}$

$$|Q_{3\to4}|=nC_PdT=n\left(\frac{5R}{2}\right)\left(\frac{T_0}{2}\right)=\frac{5RT_0}{4}$$

$$\Rightarrow \quad \frac{|Q_{1\to2}|}{|Q_{3\to4}|}=2$$

Also $|Q_{2\to3}|=\dfrac{3}{2}RT_0$

$$\frac{Q_{1\to2}}{Q_{2\to3}}=\frac{5RT_0/2}{3RT_0/2}=\frac{5}{3}$$

Since, $W=W_{1\to2}+W_{2\to3}+W_{3\to4}+W_{4\to1}$

$$\Rightarrow \quad W=(1)R(2T_0-T_0)+0+(1)R\left(\frac{T_0}{2}-T_0\right)+0$$

$$\Rightarrow \quad W=\frac{RT_0}{2}$$

Hence, (C) and (D) are correct.

5. In Process II, T is constant, so V is increasing. Hence Q is positive i.e., gas absorbs heat. Similarly, in process IV, gas releases heat. For an isobaric process, $V\propto T$.

Hence, (B), (C) and (D) are correct.

6. Assumption $e=1$ i.e., body is a black body, so

$$P=\sigma A\left(T^4-T_0^4\right)$$

$$\Rightarrow \quad P_{\text{rad}}=\sigma AT^4=\sigma 1(T_0+10)^4$$

$$\Rightarrow \quad P_{\text{rad}}=\sigma T_0^4\left(1+\frac{10}{T_0}\right)^4 \qquad [T_0=300\text{ K given}]$$

$$\Rightarrow \quad P_{\text{rad}}=\sigma(300)^4\left(1+\frac{40}{300}\right)\approx 460\times\frac{17}{15}\approx 520\text{ J}$$

$$\Rightarrow \quad P_{\text{net}}=520-460\approx 60\text{ W}$$

So, energy radiated in 1 s = 60 J

Since, $P=\sigma A\left(T^4-T_0^4\right)$

$$\Rightarrow \quad dP=\sigma A\left(0-4T_0^3dT\right) \text{ and } dT=-\Delta T$$

$$\Rightarrow \quad dP=4\sigma AT_0^3\Delta T$$

If surface area decreases, then energy radiation also decreases.

Conceptual Note(s)

While giving answer (B) and (C) it is assumed that energy radiated refers the net radiation. If energy radiated is taken as only emission, then (B) and (C) will not include in answer.

Hence, (B), (C) and (D) are correct.

7. (A) Total internal energy $U=\dfrac{f_1}{2}nRT+\dfrac{f_2}{2}nRT$

$$(U_{\text{ave}})_{\text{per mole}}=\frac{U}{2n}=\frac{1}{4}[5RT+3RT]=2RT$$

(B) $\gamma_{\text{mix}}=\dfrac{n_1C_{p1}+n_2C_{p2}}{n_2C_{V_1}+n_2C_{V_2}}=\dfrac{(1)\frac{7R}{2}+(1)\frac{5R}{2}}{(1)\frac{5R}{2}+(1)\frac{3R}{2}}=\dfrac{3}{2}$

$$\Rightarrow \quad M_{\text{mix}}=\frac{n_1M_1+n_2M_2}{n_1+n_2}=\frac{M_1+M_2}{2}=\frac{2+4}{2}=3$$

Speed of sound $v=\sqrt{\dfrac{\gamma RT}{M}}$

$$\Rightarrow \quad v\propto\sqrt{\frac{\gamma}{M}}$$

$$\Rightarrow \quad \frac{v_{\text{mix}}}{v_{\text{He}}}=\sqrt{\frac{\gamma_{\text{mix}}}{\gamma_{\text{He}}}\times\frac{M_{\text{He}}}{M_{\text{mix}}}}=\sqrt{\frac{3/2}{5/3}\times\frac{4}{3}}=\sqrt{\frac{6}{5}}$$

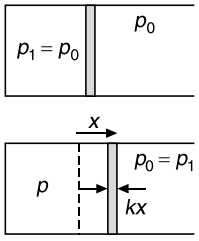
(D) $v_{\text{rms}}=\sqrt{\dfrac{3RT}{M}}$

$$\Rightarrow \quad v_{\text{rms}}\propto\frac{1}{\sqrt{M}}$$

$$\Rightarrow \quad \frac{v_{\text{He}}}{v_H}=\sqrt{\frac{M_H}{M_{\text{He}}}}=\sqrt{\frac{2}{4}}=\frac{1}{\sqrt{2}}$$

Hence, (A), (B) and (D) are correct.

8. Please note that this question can be solved if right hand side hand side chamber is assumed open, so that its pressure remains constant even if the piston shifts towards right.



(A) Since $pV = nRT$, so $p \propto \frac{T}{V}$

When temperature is made three times and volume is doubled, then

$$p_2 = \frac{3}{2}p_1$$

Further $x = \frac{\Delta V}{A} = \frac{V_2 - V_1}{A} = \frac{2V_1 - V_1}{A} = \frac{V_1}{A}$

$$\Rightarrow \quad p_2 = \frac{3p_1}{2} = p_1 + \frac{kx}{A}$$

$$\Rightarrow \quad kx = \frac{p_1 A}{2}$$

So, energy stored in the spring is

$$\frac{1}{2}kx^2 = \frac{1}{2}(kx)x = \frac{p_1 A}{4}x = \frac{p_1 V_1}{4}$$

(B) Since, $\Delta U = nC_V \Delta T = n\left(\frac{3R}{2}\right)\Delta T$

$$\Rightarrow \quad \Delta U = \frac{3}{2}(p_2V_2 - p_1V_1) = \frac{3}{2}\left[\left(\frac{3}{2}p_1\right)(2V_1) - p_1V_1\right]$$

$$\Rightarrow \quad \Delta U = 3p_1V_1$$

(C) When $V_2 = 3V_1$, $T_2 = 4T_1$, then $p_2 = \frac{4p_1}{3}$

$$\Rightarrow \quad p_2 = \frac{4}{3}p_1 = p_1 + \frac{kx}{A}$$

$$\Rightarrow \quad kx = \frac{p_1 A}{3}$$

$$\Rightarrow \quad x = \frac{\Delta V}{A} = \frac{2V_1}{A}$$

Since, $W_{\text{gas}} = \left(p_0 \Delta V + W_{\text{spring}}\right) = \left(p_1 Ax + \frac{1}{2}(kx)x\right)$

$$\Rightarrow \quad W = +\left(p_1 A \frac{2V_1}{A} + \frac{1}{2}\frac{p_1 A}{3}\frac{2V_1}{A}\right)$$

$$\Rightarrow \quad W = 2p_1V_1 + \frac{p_1V_1}{3} = \frac{7p_1V_1}{3}$$

(D) By FLTD, we have $Q = \Delta U + W$

$$\Rightarrow \quad Q = \frac{7p_1V_1}{3} + \frac{3}{2}(p_2V_2 - p_1V_1)$$

$$\Rightarrow \quad Q = \frac{7p_1V_1}{3} + \frac{3}{2}\left(\frac{4}{3}p_1 3V_1 - p_1V_1\right)$$

$$\Rightarrow \quad Q = \frac{7p_1V_1}{3} + \frac{9}{2}p_1V_1 = \frac{41p_1V_1}{6}$$

Hence, (A), (B) and (C) are correct.

9. $dQ = mCdT$

$$\Rightarrow \quad \frac{dQ}{dT} = mC$$

For $0 < T < 100$ graph is approximately linear

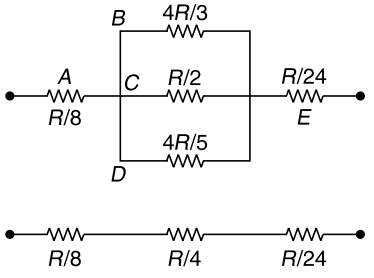
$$\Delta Q = m\int CdT = m(\text{area under } C\text{-}T \text{ graph})$$

For 400-500 K area is more than for 0-100 K, so

$\frac{dQ}{dT}$ increases for $200 < T < 300$ K as C increases in this region.

Hence, (A), (B), (C) and (D) are correct.

10. In steady state: (Heat In) = (Heat Out). So, A is true



OPTION (B) is also true because total heat is flowing through E.

$$(\text{Heat current}) = Q = \frac{\Delta T}{R}$$

Since, heat current (Q) is the same and

R_E is minimum. So, ΔT is minimum

Hence, OPTION (C) is true

So, $Q_B = \frac{\Delta T}{\frac{4R}{3}}$, $Q_C = \frac{\Delta T}{\frac{4R}{2}}$ and $Q_D = \frac{\Delta T}{\frac{4R}{5}}$

$$\Rightarrow \quad Q_B + Q_D = Q_C$$

So, (D) is also true.

Hence, (A), (B), (C) and (D) are correct.

11. For monatomic gas, $C_P = \frac{5}{2}R$, $C_V = \frac{3}{2}R$, $C_P - C_V = R$

For diatomic gas $C_P = \frac{7}{2}R$, $C_V = \frac{5}{2}R$, $C_P - C_V = R$

So, $(C_P - C_V)$ is same for both

$C_P + C_V = 6R$ {for diatomic}

$C_P + C_V = 4R$ {for monatomic}

So, $(C_P + C_V)_{\text{dia}} > (C_P + C_V)_{\text{mono}}$

$\frac{C_P}{C_V} = \frac{7}{5} = 1.4$ {for diatomic}

$\frac{C_P}{C_V} = \frac{5}{3} = 1.66$ {for monatomic}

$(C_P)(C_V) = \frac{35}{4}R^2 = 8.75R^2$ {for diatomic}

CHAPTER 2

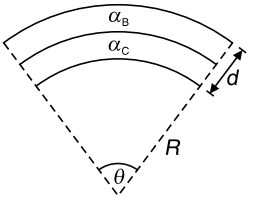
$$(C_P)(C_V)=\frac{15}{4}R^2 \qquad \{\text{for monatomic}\}$$

So, $(C_P\cdot C_V)_{\text{diatomic}}>(C_P\cdot C_V)_{\text{monatomic}}$

Hence, (B) and (D) are correct.

12. Since, the temperature of black body is constant, total heat absorbed = total heat radiated.
Hence, the correct answer is (D).

13. Let ℓ_0 be the initial length of each strip before heating.
Length after heating will be



$$\ell_B=\ell_0(1+\alpha_B\Delta T)=(R+d)\theta$$

and $\ell_C=\ell_0(1+\alpha_C\Delta T)=R\theta$

$$\Rightarrow \quad \frac{R+d}{R}=\left(\frac{1+\alpha_B\Delta T}{1+\alpha_C\Delta T}\right)$$

$$\Rightarrow \quad 1+\frac{d}{R}=1+(\alpha_B-\alpha_C)\Delta T \qquad \{\text{From binomial expansion}\}$$

$$\Rightarrow \quad R=\frac{d}{(\alpha_B-\alpha_C)\Delta T}$$

$$\Rightarrow \quad R\propto\frac{1}{\Delta T} \text{ and } \propto\frac{1}{|\alpha_B-\alpha_C|}$$

Hence, (B) and (D) are correct.

14. There is a decrease in volume during melting on an ice slab at 273 K. Therefore, negative work is done by ice-water system on the atmosphere or positive work is done on the ice-water system by the atmosphere. Hence, OPTION (B) is correct. Secondly heat is absorbed during melting (i.e., dQ is positive) and as we have seen, work done by ice-water system is negative (dW is negative). Therefore, From First Law of Thermodynamics

$$dU=dQ-dW$$

Change in internal energy of ice-water system, dU will be positive or internal energy will increase.
Hence, (B) and (C) are correct.

15. Average speed $=\langle v\rangle=\sqrt{\frac{8RT}{\pi M}}$,

RMS speed $=v_{\text{rms}}=\sqrt{\frac{3RT}{M}}$ and

Most probable speed $=v_P=\sqrt{\frac{2RT}{M}}$

From these we have

$$v_P<\langle v\rangle<v_{\text{rms}}$$

Also, the average kinetic energy of the gaseous molecules is

$$\langle E\rangle=\frac{1}{2}mv_{\text{rms}}^2=\frac{1}{2}m\left(\frac{3}{2}v_P^2\right)=\frac{3}{4}mv_P^2$$

Hence, (C) and (D) are correct.

16. For one mole of an ideal gas $PV=RT$
The coefficient of volume expansion at constant pressure is given by

$$\left(\frac{\Delta V}{\Delta T}\right)_P=\frac{R}{P}=\text{constant} \qquad \{\text{OPTION (A)}\}$$

The average translational kinetic energy per molecule is $\frac{3}{2}$kT and not 3kT. With decrease of pressure, collisions between molecules will be less frequent, hence mean free path will increase.
The average translational kinetic energy of the molecules is independent of their nature, so each component of the gaseous mixture will have the same value of average translational kinetic energy.
Hence, (A) and (C) are correct.

17. Since both bodies emit total radiant power at the same rate.

$$\Rightarrow \quad e_A\sigma(T_A^4)=e_B\sigma(T_B^4)$$

$$\Rightarrow \quad 0.01(T_A^4)=0.81(T_B^4)$$

$$\Rightarrow \quad T_A=3T_B$$

$$\Rightarrow \quad T_B=\frac{1}{3}T_A=\frac{1}{3}(5802\text{ K})$$

$$\Rightarrow \quad T_B=1934\text{ K}$$

$$\lambda_A=\frac{1}{3}\lambda_B \text{ (due to Wien's Law)}$$

Since, $\lambda_B-\lambda_A=1\ \mu$m

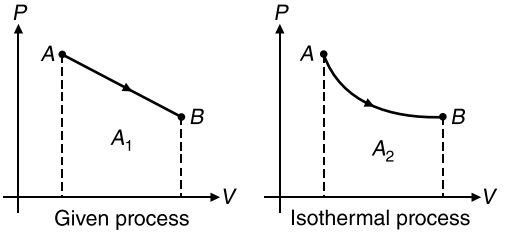
$$\Rightarrow \quad \lambda_B-\frac{\lambda_B}{3}=1\ \mu\text{m}$$

$$\Rightarrow \quad \frac{2}{3}\lambda_B=1\ \mu\text{m}$$

$$\Rightarrow \quad \lambda_B=1.5\ \mu\text{m}$$

Hence, (A) and (B) are correct.

18. (a) Work done = Area under P-V graph



(b) In the given process P-V equation will be a straight line with negative slope and positive intercept i.e., $P=-\alpha V+\beta$ (Here, α and β are positive constant)

$$\Rightarrow \quad PV=-\alpha V^2+\beta V$$

$$\Rightarrow \quad nRT=-\alpha V^2+\beta V$$

$$\Rightarrow \quad T=\frac{1}{nR}(-\alpha V^2+\beta V) \qquad \ldots(1)$$

This is an equation of parabola in T and V.

(c) $\frac{dT}{dV}=0=\beta-2\alpha V$

$$\Rightarrow \quad V=\frac{B}{2\alpha}$$

Now, $\frac{d^2T}{dV^2}=-2\alpha=-\text{ve}$

i.e., T has some maximum value.

Now, $T \propto PV$

and $(PA)_A = (PV)_B$

$\Rightarrow \quad T_A = T_B$

We conclude that temperatures are same at A and B and temperature has a maximum value. Therefore, in going from A to B, T will first increase to a maximum value and then decrease.

Hence, (A), (B) and (D) are correct.

19. (a) $\Delta U = nC_V \Delta T = nC_V (T_2 - T_1)$ in all processes

(b) For an adiabatic process $Q = 0$

$\Rightarrow \quad \Delta U = -W$

$\Rightarrow \quad |\Delta U| = |W|$

(c) In isothermal process $\Delta T = 0$

$\Rightarrow \quad \Delta U = 0 \qquad \{\because \Delta U = nC_V \Delta T\}$

(d) In adiabatic process $Q = 0$

Hence, all options are correct.

Hence, (A), (B), (C) and (D) are correct.

Reasoning Based Questions

1. Total translational kinetic energy

$$K_T = \frac{3}{2}nRT = \frac{3}{2}pV = 1.5pV$$

Hence, the correct answer is (B).

Matrix Match/Column Match Type Questions

1. **A → (q, r, s, u); B → (q, s, r, u); C → (s, r, q, t); D → (q, r, p, u)**

(A) $W_{1\to2\to3} = P_0V_0 = \frac{1}{3}RT_0$

(B) $\Delta U_{1\to2\to3} = \frac{3}{2}\left[\left(\frac{3P_0}{2}\right)(2V_0) - P_0V_0\right] = RT_0$

(C) $Q_{1\to2\to3} = \Delta U + W = \frac{4}{3}RT_0$

(D) $Q_{1\to2} = \frac{1}{3}RT_0 + \frac{3}{2}(2P_0V_0 - P_0V_0) = \frac{5}{6}RT_0$

2. **A → (p, r, t, s); B → (p, t, q, t); C → (p, r, t, p); D → (s, t, q, u)**

Since, $W_{1\to2\to3} = W_{1\to2} + W_{2\to3} = R\left(\frac{T_0}{3}\right)\ln\left(\frac{2V_0}{V_0}\right) + 0$

$\Rightarrow \quad W_{1\to2\to3} = \frac{RT_0}{3}\ln 2$

$$\Delta U_{1\to2\to3} = \Delta U_{1\to2} + \Delta U_{2\to3} = 0 + nC_V\left(T_0 - \frac{T_0}{3}\right)$$

$\Rightarrow \quad \Delta U_{1\to2\to3} = (1)\left(\frac{3R}{2}\right)\left(\frac{2T_0}{3}\right) = RT_0$

Along the path $1 \to 2$, we have $dT = 0$, so

$$Q_{1\to2} = \int TdX = \int T\left(\frac{3R}{2T}dT + \frac{R}{2}dV\right)$$

$$Q_{1\to2} = \int_{V_0}^{2V_0} \frac{RT}{V}dV = 0 + RT\ln 2$$

Similarly, along the path $2 \to 3$, we have $dV = 0$, so

$$Q_{2\to3} = \int TdX = \frac{3R}{2}\int_{\frac{T_0}{3}}^{T_0} dT = \left(\frac{3R}{2}\right)\left(\frac{2T_0}{3}\right) + 0$$

$\Rightarrow \quad Q_{2\to3} = RT_0$

$\Rightarrow \quad Q_{1\to2\to3} = RT\ln 2 + RT_0 = \frac{1}{3}RT_0(3 + \ln 2)$

3. **A → (r); B → (s); C → (p); D → (q)**

Since $\left(\frac{dP}{dV}\right)_{\text{adia}} = \gamma\left(\frac{dP}{dV}\right)_{\text{isot}}$, so process I is adiabatic and hence $Q = 0$

Process II is isobaric, so work done is

$$W_{\text{isob}} = P\Delta V = 3P_0(3V_0 - V_0) = 6P_0V_0$$

Process III is isochoric, so $W_{\text{isoc}} = 0$

Process IV is isothermal, so temperature is constant.

4. Since, $v_{\text{sound}} = \sqrt{\frac{\gamma RT}{M}}$. As the sound wave propagates, the air in a chamber undergoes compression and rarefaction very fastly, hence undergo a adiabatic process. So, curves are steeper than isothermal.

$$\left(\frac{dp}{dV}\right)_{\text{Adi}} = -\gamma\left(\frac{p}{V}\right) \qquad \ldots(1)$$

$$\left(\frac{dp}{dV}\right)_{\text{Iso}} = -\left(\frac{p}{V}\right) \qquad \ldots(2)$$

Graph Q satisfied Equation (1)

Hence, the correct answer is (C).

5. By FLTD, we have $Q = \Delta U + p\Delta V$

Since, $\Delta U \neq 0$, $W \neq 0$, $\Delta Q \neq 0$. The process represents, isobaric process, so

$$W_{\text{gas}} = -p\Delta V = -p(V_2 - V_1) = -pV_2 + pV_1$$

Graph P satisfies isochoric process.

Hence, the correct answer is (B).

6. Work done in isochoric process is zero.

$$W_{12} = 0 \text{ as } \Delta V = 0$$

Graph S represents isochoric process.

Hence, the correct answer is (B).

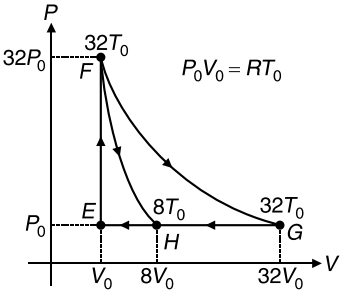
7. **A → (s); B → (r); C → (q); D → (p)**

FG is isothermal and FH is adiabatic, so we have

$$P_0V_G^{\frac{5}{3}} = 32P_0V_0^{\frac{5}{3}} \qquad \{\because \gamma_{\text{monatomic}} = 5/3\}$$

$\Rightarrow \quad V_G = (32)^{\frac{3}{5}}V_0 = 8V_0$

CHAPTER 2



So, $W_{GE} = P_0(V_0 - 32V_0) = -31P_0V_0$

$W_{GH} = P_0(8V_0 - 32V_0) = -24P_0V_0$

$W_{FH} = \dfrac{P_0(8V_0) - 32P_0V_0}{1-(5/3)} = \dfrac{-24P_0V_0}{-2/3} = 36P_0V_0$

$W_{FG} = 32RT_0 \log_e 32 = 32RT_0 \log_e 2^5$

Hence, $W_{FG} = 160RT_0 \log_e 2$

8. **A → (p, r, t); B → (p, r); C → (q, s); D → (r, t)**

In AB temperature and volume are decreasing

In BC temperature decreases, volume does not change

In CD temperature and volume increase

In DA final temperature equals initial temperature. Also, volume decreases

For all processes, we have

$$\Delta U = nC_V\Delta T,\ W = \int PdV,\ Q = \Delta U + W$$

9. **A → (q); B → (p, r); C → (p, s); D → (q, s)**

(A) in free expansion, we have

$W = \Delta U = Q = 0$

For a polytropic process, we have

PV^x = constant if $x < \gamma$, $Q > 0$

$x > \gamma$, $Q < 0$

$x = \gamma$, $Q = 0$

$x = 1$, T = constant

$x > 1$, T = decreases

$x < 1$, T = increases

10. Conceptual

A → (s); B → (q); C → (p, q); D → (q, r)

11. **A → (s); B → (p, r); C → (p); D → (q, s)**

In process $J \to K$ V is constant whereas p is decreasing. Therefore, T should also decrease.

$\Rightarrow$ $W = 0$, ΔU = –ve and $Q < 0$

In process $K \to L$ p is constant while V is increasing. Therefore, temperature should also increase.

$\Rightarrow$ $W > 0$, $\Delta U > 0$ and $Q > 0$

In process $L \to M$ This is inverse of process $J \to K$.

$\Rightarrow$ $W = 0$, $\Delta U > 0$ and $Q > 0$

In process $M \to J$

V is decreasing. Therefore, $W < 0$

$(pV)_J < (pV)_M$

$\Rightarrow$ $T_J < T_M$

$\Rightarrow$ $\Delta U < 0$

Therefore, $Q < 0$

Linked Comprehension Type Questions

1. Let final equilibrium temperature of gases is T

Heat rejected by gas in lower compartment is

$$Q_{\text{lower}} = nC_V\Delta T = 2 \times \frac{3}{2}R(700 - T)$$

Heat received by the gas in above compartment

$$Q_{\text{upper}} = nC_p\Delta T = 2 \times \frac{7}{2}R(T - 400)$$

Equating the two, we get

$2100 - 3T = 7T - 2800$

$\Rightarrow$ $T = 490$ K

Hence, the correct answer is (D).

2. By FLTD, we get

$Q_1 = \Delta U_1 + W_1$ …(1)

Also, $Q_2 = \Delta U_2 + W_2$ …(2)

Since, $Q_1 + Q_2 = 0$

$\Rightarrow$ $(nC_p\Delta T)_1 + (nC_p\Delta T)_2 = 0$

Since, $n_1 = n_2 = 2$

$\Rightarrow$ $\dfrac{5}{2}R(T - 700) + \dfrac{7}{2}R(T - 400) = 0$

Solving, we get $T = 525$ K

From equations (1) and (2), we get

$\Delta W_1 + W_2 = -\Delta U_1 - \Delta U_2$ $\{\because \Delta Q_1 + \Delta Q_2 = 0\}$

$\Rightarrow$ $W_1 + W_2 = -[(nC_V\Delta T)_1 + (nC_V\Delta T)_2]$

$\Rightarrow$ $W = -\left[2\left(\dfrac{3R}{2}\right)(525 - 700) + 2\left(\dfrac{5R}{2}\right)(525 - 400)\right]$

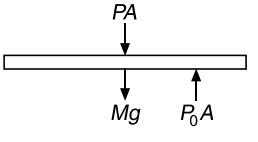
$\Rightarrow$ $W = -100R$

Hence, the correct answer is (D).

3. Since it is open from top, pressure will be P_0.

Hence, the correct answer is (A).

4. Let P be the pressure in equilibrium



Then $PA = P_0A - Mg$

$$P = P_0 - \frac{Mg}{A} = P_0 - \frac{Mg}{\pi R}$$ …(1)

Applying $P_1V_1 = P_2V_2$

$\Rightarrow \quad P_0(2AL) = (P)(AL')$

$\Rightarrow \quad L' = \dfrac{2P_0L}{P} = \left(\dfrac{P_0}{P_0 - \dfrac{Mg}{\pi R^2}}\right)(2L)$

$$L' = \left(\frac{P_0\pi R^2}{\pi R^2P_0 - Mg}\right)(2L)$$

Hence, the correct answer is (D).

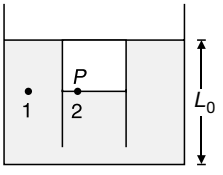
5. Since we observe, $P_1 = P_2$

$\Rightarrow \quad P_0 + \rho g(L_0 - H) = P \qquad \ldots(1)$

Now, applying $P_1V_1 = P_2V_2$ for the air inside the cylinder, we have

$$P_0(L_0) = P(L_0 - H)$$

$\Rightarrow \quad P = \dfrac{P_0L_0}{L_0 - H}$



Substituting in equation (1), we have

$$P_0 + \rho g(L_0 - H) = \frac{P_0L_0}{L_0 - H}$$

$\Rightarrow \quad \rho g(L_0 - H)^2 + P_0(L_0 - H) - P_0L_0 = 0$

Hence, the correct answer is (C).

Integer/Numerical Answer Type Questions

1. In an isothermal process, work done is

$$W_{isot} = RT\ln 4$$

In an adiabatic process, we have

$$T(4V_1)^{\frac{2}{3}} = T'(32V_1)^{\frac{2}{3}}$$

$\Rightarrow \quad T' = \dfrac{T}{4}$

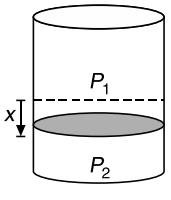
So, $W_{adia} = \dfrac{2R\left(T - \dfrac{T}{4}\right)}{3} = \left(\dfrac{9RT}{8}\right)$

$\Rightarrow \quad \dfrac{W_{isot}}{W_{ad}} = \dfrac{8}{9}\ln 4 = \dfrac{16}{9}\ln 2$

$\Rightarrow \quad f = \dfrac{16}{9} = 1.78$

2. Assuming the gas to be an ideal monoatomic gas

$\Rightarrow \quad C_V = \dfrac{3}{2}R$



Let area of cross section of the cylinder be A.

Initially, we have $PV = nRT$

$\Rightarrow \quad P_0(4A) = 0.1R \times 300$ (For both)

Let partition shifts by x meter and final temperature be T, then we have (for lower portion)

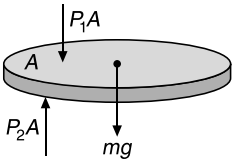
$$P_2A(4 - x) = 0.1RT$$

$\Rightarrow \quad P_2A = \dfrac{0.1RT}{4 - x} = \dfrac{RT}{10(4 - x)}$

Similarly, for upper portion, we have

$$P_1A = \frac{RT}{10(4 + x)}$$

But finally, in equilibrium, we have



$$P_1A + mg = P_2A$$

$\Rightarrow \quad P_2A - P_1A = mg = 83$

$$\frac{RT}{10}\left(\frac{1}{4 - x} - \frac{1}{4 + x}\right) = 83$$

$\Rightarrow \quad \dfrac{RT}{10}\left(\dfrac{2x}{16 - x^2}\right) = 83 \qquad \ldots(1)$

Also, we have

$$0.2C_V \times 300 + mgx = 0.2C_VT$$

$\Rightarrow \quad \left(\dfrac{2}{10}\right)\left(\dfrac{3}{2}R\right)(300) + 83x = \left(\dfrac{2}{10}\right)\left(\dfrac{3}{2}R\right)T$

$\Rightarrow \quad \dfrac{900R}{10} + 83x = 3\left(\dfrac{RT}{10}\right)$

$$\frac{300R}{10} + \frac{83}{3}x = \frac{RT}{10}$$

Using equation (1), we get

$$\left(30R + \frac{83x}{3}\right)\left(\frac{2x}{16 - x^2}\right) = 83$$

$\Rightarrow \quad 83\left(3 + \dfrac{x}{3}\right)\left(\dfrac{2x}{16 - x^2}\right) = 83$

$\Rightarrow \quad \dfrac{(9 + x)}{3}\dfrac{(2x)}{(16 - x^2)} = 1$

$\Rightarrow \quad 18x + 2x^2 = 48 - 3x^2$

$\Rightarrow \quad 5x^2 + 18x - 48 = 0$

$\Rightarrow \quad x = 1.78 \approx 2$

So, the distance from top is 6 m.

3. For an adiabatic process, we have

$$\Delta P = -\left(\frac{rP}{V}\right)\Delta V$$

where, $P = p_0$ and $\Delta V = -(4\pi R^2)a$

$\Rightarrow \quad \Delta P = \left(\dfrac{rp_0}{\frac{4}{3}\pi R^3}\right)(4\pi R^2)a = \dfrac{3rp_0a}{R}$

Work done in the process is

$$W = -(\Delta P)_{av}\,\Delta V$$

Since the change is small, so $(\Delta P)_{av}$ equals the arithmetic mean and hence $(\Delta P)_{av} = \dfrac{\Delta P}{2}$

CHAPTER 2

$$\Rightarrow \quad W=-\left(\frac{\Delta P}{2}\right)\left(-4\pi R^2 a\right)=\left(\frac{3rp_0 a}{2R}\right)\left(4\pi R^2 a\right)$$

$$\Rightarrow \quad W=\left(4\pi p_0 R a^2\right)\left(\frac{3r}{2}\right)$$

$$\Rightarrow \quad W=\left(4\pi p_0 R a^2\right)\left(\frac{3}{2}\times\frac{41}{30}\right)$$

$$\Rightarrow \quad W=\left(4\pi p_0 R a^2\right)(2.05)$$

$$\Rightarrow \quad X=2.05$$

4. In steady state, heat lost per second equals the heat gained per second.

$$\left(\frac{dQ}{dt}\right)_{\text{lost}}=\left(\frac{dQ}{dt}\right)_{\text{gained}} \qquad \ldots(1)$$

Since, $\left(\frac{dQ}{dt}\right)_{\text{gained}}=(700)(0.05)\ \text{W}$

Also, according to Newton's Law of cooling, we have

$$\frac{1}{A}\frac{dQ}{dt}=\left(4\sigma e T_0^3\right)\Delta T$$

$$\Rightarrow \quad \left(\frac{dQ}{dt}\right)_{\text{lost}}=\left(4\sigma e T_0^3\right)A\Delta T \qquad \ldots(2)$$

Given that the constant of proportionality for Newton's Law of cooling is $0.001\ \text{s}^{-1}$

Also, $-\frac{dT}{dt}=\left(\frac{4\sigma eA}{mc}\right)T_0^3\Delta T$

So, constant of proportionality for Newton's Law of cooling is $k=\left(\frac{4\sigma eA}{mc}\right)T_0^3$

$$\Rightarrow \quad 4\sigma e T_0^3 A=k(mc)$$

Substituting in (2), we get

$$\left(\frac{dQ}{dt}\right)_{\text{lost}}=k(mc)\Delta T$$

$$\Rightarrow \quad 700\times 0.05=(0.001)(4200)\Delta T$$

$$\Rightarrow \quad \Delta T=\frac{25}{3}\ \text{K}$$

$$\Rightarrow \quad \Delta T=8.33\ \text{K}$$

5. CASE-1: $5c(50)+5L=c_2(30)$ …(1)

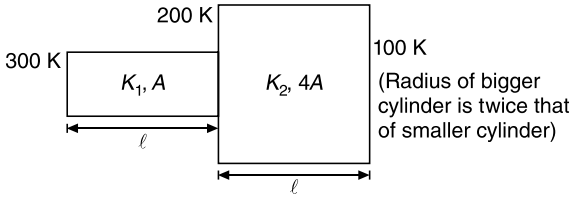
CASE-2: $80c(50-30)=c_2(80-50)$ …(2)

From equations (1) and (2), we get

$$1600c=250c+5L$$

$$\Rightarrow \quad \frac{L}{c}=\frac{1350}{5}=270\ ^\circ\text{C}$$

6. In steady state, the rate of flow of heat through both the conducting cylinders will be equal. So, we have



$$\frac{K_1 A(300-200)}{\ell}=\frac{K_2 4A(200-100)}{\ell}$$

$$\Rightarrow \quad \frac{K_1}{K_2}=4=4.00$$

7. For an adiabatic process, $T_1V_1^{\gamma-1}=T_2V_2^{\gamma-1}$

where $\gamma=\gamma_{\text{monatomic}}=\frac{5}{3}$

$$\Rightarrow \quad T_2=T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1}=100\left(\frac{1}{8}\right)^{\frac{2}{3}}$$

$$\Rightarrow \quad T_2=25\ \text{K}$$

Since, $\Delta U=nC_V\Delta T=1\left(\frac{3R}{2}\right)(T_2-T_1)$

$$\Delta U=1\left(\frac{3}{2}\times 8\right)(25-100)=-900\ \text{J}$$

Hence, decrease in internal energy is 900 J

8. For reading of sensor to be 1, we have

$$\Rightarrow \quad \log_2\frac{P_1}{P_0}=1$$

$$\Rightarrow \quad \frac{P_1}{P_0}=2$$

According to Stefan's law, we have

$$P\propto T^2$$

$$\Rightarrow \quad \frac{P_2}{P_1}=\left(\frac{T_2}{T_1}\right)^4=\left(\frac{2767+273}{487+273}\right)^4=4^4$$

$$\Rightarrow \quad \frac{P_2}{P_1}=\frac{P_2}{2P_0}=4^4$$

$$\Rightarrow \quad \frac{P_2}{P_0}=2\times 4^4$$

So, new reading is

$$\log_2\left(\frac{P_2}{P_0}\right)=\log_2 2+\log_2 4^4$$

$$\log_2\left(\frac{P_2}{P_0}\right)=1+\log_2\left(2^8\right)=9$$

9. Power, $P_{\text{radiated}}=P=\sigma T^4 A=\sigma T^4\left(4\pi R^2\right)$

$$\Rightarrow \quad P\propto T^4R^2 \qquad \ldots(1)$$

If λ is the wavelength at which peak occurs, then according to Wien's law, we have

$$\lambda\propto\frac{1}{T}$$

So, equation (1) becomes

$$P\propto\frac{R^2}{\lambda^4}$$

$$\Rightarrow \quad \lambda\propto\left(\frac{R^2}{P}\right)^{\frac{1}{4}}$$

$$\Rightarrow \quad \frac{\lambda_A}{\lambda_B}=\left(\frac{R_A}{R_B}\right)^{1/2}\left(\frac{P_B}{P_A}\right)^{1/4}=(400)^{1/2}\left(\frac{1}{10^4}\right)^{1/4}=2$$

10. Since, $W_{ibf} = W_{ib} + W_{bf} = 50\text{ J} + 100\text{ J} = 150\text{ J}$

Similarly, $W_{iaf} = W_{ia} + W_{af} = 0 + 200\text{ J} = 200\text{ J}$

Also, $Q_{iaf} = 500\text{ J}$

So, by FLTD, we have $\Delta U_{iaf} = Q_{iaf} - W_{iaf}$

$\Rightarrow \quad \Delta U_{iaf} = 500\text{ J} - 200\text{ J} = 300\text{ J}$

$\Rightarrow \quad U_f - U_i = 300\text{ J}$

$\Rightarrow U_f = U_{iaf} + U_i = 300\text{ J} + 100\text{ J} = 400\text{ J}$

Similarly, $\Delta U_{ib} = U_b - U_i = 200\text{ J} - 100\text{ J} = 100\text{ J}$

By FLTD, we have $Q_{ib} = \Delta U_{ib} + W_{ib}$

$\Rightarrow \quad Q_{ib} = 100\text{ J} + 50\text{ J} = 150\text{ J}$

Similarly by FLTD, we have

$$Q_{ibf} = \Delta U_{ibf} + W_{ibf} = \Delta U_{iaf} + W_{ibf}$$

$$Q_{ibf} = 300\text{ J} + 150\text{ J} = 450\text{ J}$$

So, the required ratio is

$$\frac{Q_{bf}}{Q_{ib}} = \frac{Q_{ibf} - Q_{ib}}{Q_{ib}} = \frac{450 - 150}{150} = 2$$

11. Increase in length is $\Delta l_1 = \dfrac{FL}{AY} = \dfrac{mgL}{\pi r^2 Y}$

On cooling, the decrease in length is $\Delta l_2 = L\alpha\Delta T$

To regain its original length, we must have $\Delta l_1 = \Delta l_2$

$\Rightarrow \quad \dfrac{mgL}{\pi r^2 Y} = L\alpha\Delta T$

$\Rightarrow \quad m = \dfrac{r^2 Y\alpha\Delta T}{g}$

Substituting the values, we get

$$m \approx 3\text{ kg}$$

12. $S = 2100\text{ Jkg}^{-1}\,^\circ\text{C}^{-1}$

$$L = 3.36 \times 10^5\text{ Jkg}^{-1}$$

$$420 = mc\Delta T + (1 \times 10^{-3})L$$

$\Rightarrow \quad 420 = mc(5) + 3.36 \times 10^2$

$\Rightarrow \quad 420 - 336 = m(2100)(5)$

$\Rightarrow \quad m = \dfrac{1000}{125} = 8\text{ g}$

13. Given that, $(\lambda_m)_B = 3(\lambda_m)_A$

$\Rightarrow \quad T_A = 3T_B$

So, $E_1 = 4\pi(6)^2\sigma T_A^4 = 4\pi(6)^2(3T_B)^4$

and $E_2 = 4\pi(18)^2\sigma T_B^4$

$\Rightarrow \quad \dfrac{E_1}{E_2} = 9$

14. For adiabatic process,

$$TV^{\gamma-1} = \text{constant}$$

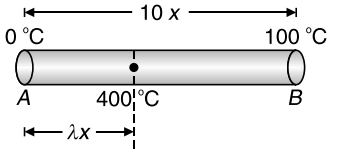
$\Rightarrow \quad T_2 = T_1\left(\dfrac{V_1}{V_2}\right)^{\gamma-1}$

$\Rightarrow \quad T_2 = T_1(32)^{\frac{7}{5}-1}$

$\Rightarrow \quad T_2 = 4T_1$

$\Rightarrow \quad a = 4$

15.



$$\frac{400 - 0}{\frac{\lambda x}{KA}} = \frac{dm_\ell}{dt} \times L_{\text{ice}}$$

Also, $\dfrac{400 - 100}{\frac{10x - \lambda x}{KA}} = \dfrac{dm_w}{dt} \times L_{steam}$

$\Rightarrow \quad \dfrac{400(10 - \lambda)}{\lambda \times 300} = \dfrac{L_{\text{ice}}}{L_{\text{Steam}}} = \dfrac{80}{540} = \dfrac{4}{27}$

$\Rightarrow \quad 9(10 - \lambda) = \lambda$

$\Rightarrow \quad 90 = 10\lambda$

$\Rightarrow \quad \lambda = 9$

CHAPTER 2

CHAPTER 3: SIMPLE HARMONIC MOTION

Test Your Concepts-I (Based on SHM Properties)

1. (a) From graph, we observe that

$A = 0.08 \text{ m} = 8 \text{ cm}$

(b) Since, $T = 4$ s

$\Rightarrow \quad \omega = \dfrac{2\pi}{T} = \dfrac{\pi}{2} = 1.57 \text{ rads}^{-1}$

(c) $\omega = \sqrt{\dfrac{k}{m}}$

$\Rightarrow \quad k = m\omega^2 = (0.8)\left(\dfrac{\pi}{2}\right)^2 = 1.97 \text{ Nm}^{-1}$

(d) Since, $x = 0.08\sin(\omega t) = 0.08\sin\left(\dfrac{\pi t}{2}\right) \qquad \left\{\because \omega = \dfrac{\pi}{2}\right\}$

$\Rightarrow \quad v = \dfrac{dx}{dt} = (0.08)\left(\dfrac{\pi}{2}\right)\cos\left(\dfrac{\pi t}{2}\right)$

At $t = 1$ s, $v = 0 \qquad \left\{\because \cos\left(\dfrac{\pi}{2}\right) = 0\right\}$

(e) At $t = 1.0$ s, $x = 0.08$ m

$\Rightarrow \quad |a| = \omega^2 x = \left(\dfrac{\pi}{2}\right)^2 \times (0.08) = 0.197 \text{ ms}^{-2}$

2. Differentiating with respect to time, we get

$$\frac{dx}{dt} = iA\omega e^{i\omega t}$$

Differentiating again with respect to time, we get

$$\frac{d^2x}{dt^2} = i^2\omega^2\left(Ae^{i\omega t}\right) = -\omega^2 x \qquad \left\{\because x = Ae^{i\omega t}\right\}$$

Since x satisfies, $\dfrac{d^2x}{dt^2} + \omega^2 x = 0$, so $x = Ae^{i\omega t}$ represents equation of an SHM.

3. (a) $F = -2(x-2)^3$

$F = 0$ at $x = 2$

Force is along negative x-direction for $x > 2$ and it is along positive x-direction for $x < 2$. Thus, the motion of the particle is oscillatory (but not simple harmonic) about $x = 2$.

(b) $F = 0$ for $x = 2$, but force is always along negative x-direction for any value of x except at $x = 2$. Thus, the motion of the particle is rectilinear along negative x-direction.

(c) Let, us take $x - 2 = X$, then the given force can be written as,

$F = -2X$

This is the equation of SHM. Hence, the particle oscillates simple harmonically about $X = 0$ or $x = 2$.

4. Amplitude, $A = 6$ cm

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi \text{ rads}^{-1}$$

$\Rightarrow \quad x = A\cos(\omega t) = 6\cos(\pi t)$

Differentiating with respect to time, we get

$v = -6\pi\sin(\pi t)$

Again, differentiating with respect to time,

$a = -6\pi^2\cos(\pi t)$

5. Time taken in SHM from one end to other is

$$\Delta t = \frac{T}{2} = \frac{\pi}{\omega}$$

Mean velocity is $v_{\text{mean}} = \dfrac{\Delta s}{\Delta t} = \dfrac{2A}{\pi/\omega} = \dfrac{2A\omega}{\pi}$

Maximum velocity in SHM is

$v_{\max} = A\omega$

$\Rightarrow \quad \dfrac{v_{\text{mean}}}{v_{\max}} = \dfrac{2}{\pi}$

Mean acceleration from one end to centre is

$$a_{\text{mean}} = \frac{\Delta v}{\Delta t} = \frac{A\omega}{T/4} = \frac{A\omega}{\pi/2\omega} = \frac{2A\omega^2}{\pi}$$

and maximum acceleration is

$a_{\max} = A\omega^2$

$\Rightarrow \quad \dfrac{a_{\text{mean}}}{a_{\max}} = \dfrac{2}{\pi}$

6. (a) $\dfrac{1}{2}kA^2 = (9-5) = 4$ J

$\Rightarrow \quad k = \dfrac{8}{A^2} = \dfrac{8}{(10^{-2})^2} = 8\times10^4 \text{ Nm}^{-1}$

Since, $T = 2\pi\sqrt{\dfrac{m}{k}} = 2\pi\sqrt{\dfrac{2.0}{8\times10^4}}$

$\Rightarrow \quad T = 0.03145 = 3.14\times10^{-2}$ s

(b) $E = -\dfrac{dV}{dx} = -16x$

$\Rightarrow \quad F = mE = -16mx$

$\Rightarrow \quad a = -16x$

$\Rightarrow \quad T = 2\pi\sqrt{\left|\dfrac{x}{a}\right|} = 2\pi\sqrt{\dfrac{1}{16}} = 1.571$ s

7. According to the statement, we have

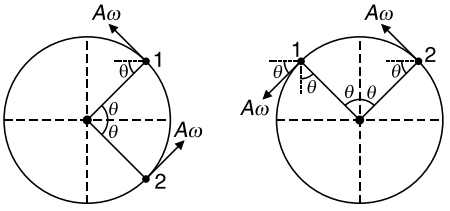
$A\omega\sin\theta = 1.6 \qquad \ldots(1)$

$A\omega\cos\theta = 1.2 \qquad \ldots(2)$

Squaring and adding equations (1) and (2), we get

$A\omega = v_{\max} = 2 \text{ ms}^{-1}$

Further, $\sin\theta = \dfrac{1.6}{A\omega} = \dfrac{1.6}{2} = \dfrac{4}{5}$



So, phase difference between the particles is

$$\Delta\phi = 2\theta = 2\sin^{-1}\left(\frac{4}{5}\right)$$

8. We know in one complete oscillation i.e., in period T, a particle covers a distance $4A$ and in first one quarter of its period it goes from its mean position to its extreme position. Since it starts from mean position, so the distance travelled by the particle in time $\frac{5T}{4}$ is $5A$.

9. (a) Since, $v = \omega\sqrt{A^2 - x^2}$

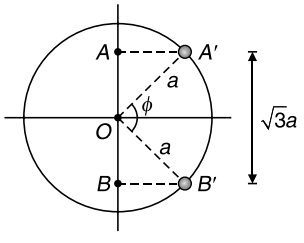
Given $A = 0.100$ m, $x = 0.060$ m, $v = 0.360$ m, we get ω and hence T, because $T = \frac{2\pi}{\omega}$

(b) $v = \omega\sqrt{A^2 - x^2}$

(c) $\mu g = a_{max} = \omega^2 A$

10. The phase difference between SHMs of A and B is given as

$$\phi = 2\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 2\times\frac{\pi}{3} = \frac{2\pi}{3}$$



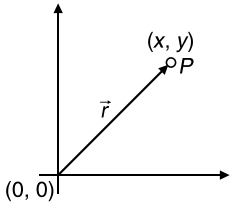
11. Since $\vec{F} = -kx\hat{i} - ky\hat{j}$, so $\vec{F} = 0$ at $(0,\ 0)$

When it is displaced to a point P whose position vector is given by

$$\vec{r} = x\hat{i} + y\hat{j}$$

Then, force on it is given by

$$\vec{F} = -k\left(x\hat{i} + y\hat{j}\right) = -k\vec{r}$$

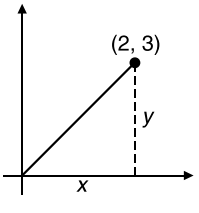


Since, $\vec{F} \propto -\vec{r}$, motion is simple harmonic. At $t = 0$ particle is at $(2,\ 3)$

So, $\frac{y}{x} = \frac{3}{2}$

$\Rightarrow \quad 2y - 3x = 0$

i.e., the particle will oscillate simple harmonically along this line.



12. (a) Since $x = 4\sin(2t)$, so $x_{max} = 4$ m

When, $\sin(2t) = 1$

$\Rightarrow \quad 2t = \frac{\pi}{2}$

$\Rightarrow \quad t = \frac{\pi}{4}$ second

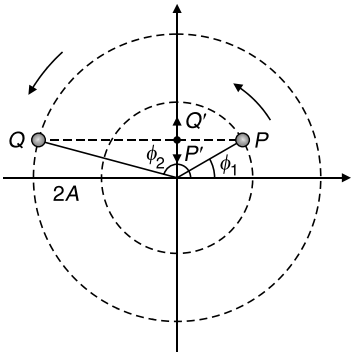
(b) $v = \frac{dx}{dt} = 8\cos(2t)$

$\Rightarrow \quad v(0) = 8\ \text{ms}^{-1}$

(c) $a = \frac{dv}{dt} = -16\sin(2t)$

$\Rightarrow \quad |a_{max}| = 16\ \text{ms}^{-2}\ a(0) = 0$

13. The two corresponding particles of circular motion for the two mentioned particles in SHM are shown in Figure.



Let particle P be going up and particle Q be going down. The respective phase differences of particles P' and Q' are

$$\phi_1 = \sin^{-1}\left(\frac{A/3}{A}\right)$$

and $\phi_2 = \pi - \sin^{-1}\left(\frac{A/3}{2A}\right)$

Hence phase difference between two SHMs is

$$\Delta\phi = \phi_2 - \phi_1 = \pi - \sin^{-1}\left(\frac{1}{6}\right) - \sin^{-1}\left(\frac{1}{3}\right)$$

14. (a) $v = \omega A = \left(\frac{2\pi}{T}\right)A = \left(\frac{2\pi}{1.8}\right)(0.1)$

$\Rightarrow \quad v = 0.35\ \text{ms}^{-1}$

(b) $a = \omega^2 x = \left(\frac{2\pi}{T}\right)^2(0.05)$

$\Rightarrow \quad a = \left(\frac{2\pi}{1.8}\right)^2(0.05) = 0.61\ \text{ms}^{-2}$

(c) $0.05 = 0.1\sin(\omega t)$

where, t is the time taken by the particle to move from equilibrium position to 0.05 m

$\Rightarrow \quad \omega t = \frac{\pi}{6}$

$\Rightarrow \quad \left(\frac{2\pi}{1.8}\right)t = \frac{\pi}{6}$

$\Rightarrow \quad t = 0.15$ s

So, total time $= 2\times 0.15 = 0.3$ s

CHAPTER 3

(d) $kA = mg$

$$\Rightarrow \quad A = \frac{g}{\omega^2} = \frac{gT^2}{4\pi^2}$$

Substituting the values,

$$A = 0.80 \text{ m}$$

15. At mean position, potential energy is minimum and kinetic energy is maximum. Hence,

$$U_{\min} = 50 \text{ J} \qquad \{\text{at mean position}\}$$

and $K_{\max} = E - U_{\min} = 200 - 50$

$$K_{\max} = 150 \text{ J} \qquad \{\text{at mean position}\}$$

At extreme positions, kinetic energy is zero and potential energy is maximum, so

$$U_{\max} = E$$

$$\Rightarrow \quad U_{\max} = 200 \text{ J} \qquad \{\text{at extreme position}\}$$

16. At mean position, kinetic energy is

$$K = \frac{1}{2}m\omega^2 A^2 = 8 \times 10^{-3}$$

$$\Rightarrow \quad \frac{1}{2}(0.1)\omega^2 (0.1)^2 = 8 \times 10^{-3}$$

$$\Rightarrow \quad \omega^2 = 16$$

$$\Rightarrow \quad \omega = 4 \text{ rads}^{-1}$$

So, general equation of SHM is

$$y = A\sin(\omega t + \phi) = 0.1\sin\left(4t + \frac{\pi}{4}\right) \text{ metre}$$

17. $F = -\dfrac{dU}{dx} = -U_0 b \sin(bx)$

For small oscillations, $\sin(bx) \approx bx$

$$\Rightarrow \quad F = -U_0 b^2 x$$

$$\Rightarrow \quad ma = m\ddot{x} = -U_0 b^2 x$$

$$\Rightarrow \quad T = 2\pi\sqrt{\left|\frac{x}{\ddot{x}}\right|} = 2\pi\sqrt{\frac{m}{b^2 U_0}}$$

18. $\mu mg = m|a_{\max}| = m\omega^2 A$

$$\Rightarrow \quad \mu g = (2\pi f)^2 A$$

$$\Rightarrow \quad \mu = \frac{4\pi^2 f^2 A}{g} = \frac{4\pi^2 (2)^2 (5.0 \times 10^{-2})}{9.8} = 0.806$$

19. $\dfrac{1}{2}mv^2 = \dfrac{1}{2}kA^2$

$$\Rightarrow \quad A = \left(\sqrt{\frac{m}{k}}\right)v = \left(\sqrt{\frac{10^{-2}}{124}}\right)(8.00) = 0.072 \text{ m} = 7.2 \text{ cm}$$

20. $T = 8$ s

$$\Rightarrow \quad \omega = \frac{2\pi}{T} = \frac{\pi}{4} \text{ rads}^{-1}$$

$$x = A\sin\left(\frac{\pi}{4}\right)t$$

(a) In first 4 second it travels a distance $2A$. In next 4 second it again travels a distance $2A$.

(b) In first 2 second it travels a distance A and in next 2 second it again travels a distance A.

(c) Distance travelled in first 1 second is

$$x_1 = A\sin\left(\frac{\pi}{4}\right)(1) = \frac{A}{\sqrt{2}}$$

Hence distance travelled in next 1 second is

$$x_2 = A - x_1 = A - \frac{A}{\sqrt{2}}$$

$$\Rightarrow \quad \frac{x_1}{x_2} - \frac{\frac{A}{\sqrt{2}}}{A - \frac{A}{\sqrt{2}}} = \frac{1}{\sqrt{2} - 1} \approx 2.4$$

$$\Rightarrow \quad x_1 = 2.4x_2$$

21. $E = \dfrac{1}{2}kA^2$ and $K = \dfrac{1}{2}k(A^2 - x^2)$

$$\Rightarrow \quad \frac{K}{E} = \frac{A^2 - x^2}{A^2}$$

At $x = \dfrac{A}{2}$, we get $\dfrac{K}{E} = \dfrac{A^2 - \frac{A^2}{4}}{A^2} = \dfrac{3}{4}$

Since, $U = \dfrac{1}{2}kx^2$

Equating $U = K$, we get $x^2 = A^2 - x^2$

$$\Rightarrow \quad x = \frac{A}{\sqrt{2}}$$

22. (a) $y = a(1 - \cos\omega t)$

$$\Rightarrow \quad \frac{d^2y}{dt^2} = a\omega^2 \cos(\omega t)$$

Since, $N - mg = m\dfrac{d^2y}{dt^2}$

$$\Rightarrow \quad N = mg + ma\omega^2 \cos(\omega t)$$

$$\Rightarrow \quad N = m(g + a\omega^2 \cos\omega t)$$

(b) $\left(\dfrac{d^2y}{dt^2}\right) = a\omega^2$

$$\Rightarrow \quad a\omega^2 = g$$

$$\Rightarrow \quad a = \frac{g}{\omega^2} = \frac{980}{(11)^2} = 8.01 \text{ cm}$$

23. (a) $f = \dfrac{\omega}{2\pi} = \dfrac{5\pi}{2\pi} = 2.5$ Hz

$$T = \frac{1}{f} = 0.4 \text{ s}$$

(b) Amplitude = 3.0 m

(c) At $t = 0$, $x = 3\cos 3\pi = -3$ m

and at $t = \dfrac{1}{2}$ s, $x = 3\cos\left(\dfrac{5\pi}{2} + \pi\right) = 0$

Test Your Concepts-II (Based on Spring Mass Systems)

1. (a) The total mechanical energy of the block and spring before the lump of putty is dropped is

$$E_1 = \frac{1}{2}kA_1^2$$

Since, the block is at the equilibrium position, $U = 0$ and the energy is purely kinetic. Let v_1 be the speed of the block at the equilibrium position, we get

$$E_1 = \frac{1}{2}Mv_1^2 = \frac{1}{2}kA_1^2$$

$$\Rightarrow \quad v_1 = \sqrt{\frac{k}{M}}A_1$$

During the process, the momentum of the system in horizontal direction is conserved. Let v_2 be the speed of the combined mass, then

$$(M+m)v_2 = Mv_1$$

$$\Rightarrow \quad v_2 = \frac{M}{M+m}v_1$$

Now, let A_2 be the amplitude afterwards, then

$$E_2 = \frac{1}{2}kA_2^2 = \frac{1}{2}(M+m)v_2^2$$

Substituting the values, we get

$$A_2 = A_1\sqrt{\frac{M}{M+m}}$$

We observe that $E_2 < E_1$ because some energy is lost into heating up the block and putty.

Further, $T_2 = 2\pi\sqrt{\frac{M+m}{k}}$

(b) When the putty drops on the block, the block is instantaneously at rest. All the mechanical energy is stored in the spring as potential energy. Again, the momentum in horizontal direction is conserved during the process. But now it is zero just before and after putty is dropped. So, in this case, adding the extra mass of the putty has no effect on the mechanical energy, i.e.,

$$E_2 = E_1 = \frac{1}{2}kA_1^2$$

and the amplitude is still A_1. So, $A_2 = A_1$

and $T_2 = 2\pi\sqrt{\frac{M+m}{k}}$

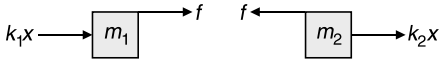
2. (a) $f = \frac{1}{2\pi}\sqrt{\frac{k_{\text{eff}}}{\text{total mass}}} = \frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m_1+m_2}}$

(b) Suppose the system is displaced towards left by a distance x.

Restoring force on m_1 is

$$F = m_1\omega^2 x \qquad \text{(towards right)}$$

$$F = m_1\left(\frac{k_1+k_2}{m_1+m_2}\right)x$$



Friction f on it will be towards right if,

$$k_1x < F$$

$$\Rightarrow \quad k_1x < m_1\left(\frac{k_1+k_2}{m_1+m_2}\right)x$$

$$\Rightarrow \quad \frac{k_1}{k_2} < \frac{m_1}{m_2}$$

(c) $k_1A_m + \mu m_2 g = m_1\left(\frac{k_1+k_2}{m_1+m_2}\right)A_m$

$$\Rightarrow \quad A_m\left(\frac{m_1k_1+m_1k_2}{m_1+m_2} - k_1\right) = \mu m_2 g$$

$$\Rightarrow \quad A_m = \frac{\mu(m_1+m_2)m_2 g}{m_1k_2 - m_2k_1}$$

3. For a spring block system, we have

$$T = 2\pi\sqrt{\frac{M}{k}}$$

where, k is the spring constant of the spring

In the first case, $2 = 2\pi\sqrt{\frac{M}{k}}$ …(1)

In the second case, $3 = 2\pi\sqrt{\frac{M+2}{k}}$ …(2)

Dividing (2) by (1), we get

$$\frac{9}{4} = \frac{M+2}{M} = 1 + \frac{2}{M}$$

$$\Rightarrow \quad M = 1.6 \text{ kg}$$

4. $T = 2\pi\sqrt{\frac{\mu}{k}} = 2\pi\sqrt{\frac{mM}{k(M+m)}}$

5. (a) $v = \omega A = \left(\frac{2\pi}{T}\right)A = \left(\frac{2\pi}{1.8}\right)(0.1)$

$$\Rightarrow \quad v = 0.35 \text{ ms}^{-1}$$

(b) $a = \omega^2 x = \left(\frac{2\pi}{T}\right)^2(0.05)$

$$\Rightarrow \quad a = \left(\frac{2\pi}{1.8}\right)^2(0.05) = 0.61 \text{ ms}^{-2}$$

(c) $0.05 = 0.1\sin(\omega t)$

where, t is the time taken by the particle to move from equilibrium position to 0.05 m

$$\Rightarrow \quad \omega t = \frac{\pi}{6}$$

$$\Rightarrow \quad \left(\frac{2\pi}{1.8}\right)t = \frac{\pi}{6}$$

$$\Rightarrow \quad t = 0.15 \text{ s}$$

So, total time $= 2 \times 0.15 = 0.3$ s

(d) $kA = mg$

$$\Rightarrow \quad A = \frac{g}{\omega^2} = \frac{gT^2}{4\pi^2}$$

Substituting the values,

$$A = 0.80 \text{ m}$$

CHAPTER 3

6. $\frac{1}{2}mv^2 = \frac{1}{2}kA^2$

$\Rightarrow \quad A = \left(\sqrt{\frac{m}{k}}\right)v = \left(\sqrt{\frac{10^{-2}}{124}}\right)(8.00)$

$\Rightarrow \quad A = 0.072 \text{ m} = 7.2 \text{ cm}$

7. (a) $\mu_s m_2 g \geq m_2 |a_{\max}|$

$\Rightarrow \quad \mu_s \geq \frac{\omega^2 A}{g} \geq \frac{kA}{(m_1 + m_2)g}$

(b) A and $E\left(=\frac{1}{2}kA^2\right)$ will remain unchanged ω will decrease as

$\Rightarrow \quad \omega = \sqrt{\frac{k}{m}}$

$\Rightarrow \quad \omega_f = \left(\sqrt{\frac{m_1}{m_1 + m_2}}\right)\omega_i$

T will increase, because $T \propto \frac{1}{\omega}$

$\Rightarrow \quad T_f = \left(\sqrt{\frac{m_1 + m_2}{m_1}}\right)T_i$

8. Let F be the restoring force (extra tension) on block m when displaced by x from its equilibrium position.

$$x = 2x_1 + 2x_2 = 2\left[\frac{2F}{k_1} + \frac{2F}{k_2}\right] = 4F\left(\frac{k_1 + k_2}{k_1 k_2}\right)$$

$\Rightarrow \quad F = -\frac{k_1 k_2}{4(k_1 + k_2)}x$

$\Rightarrow \quad a = -\frac{k_1 k_2}{4m(k_1 + k_2)}x$

$\Rightarrow \quad \omega = \sqrt{\frac{k_1 k_2}{4m(k_1 + k_2)}}$

9. (a) $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$\Rightarrow \quad m = \frac{k}{(2\pi f)^2} = \frac{1800}{(2\pi \times 5.5)^2} = 1.5 \text{ kg}$

(b) $kx = mg$

$\Rightarrow \quad x = \frac{mg}{k} = \frac{(1.5)(9.8)}{1800} = 0.0082 \text{ m} = 0.82 \text{ cm}$

(c) $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1800}{1.5}} = 34.6 \text{ rads}^{-1}$

$\Rightarrow \quad x = A\cos(\omega t) = 2.5\cos(34.6t) \text{ cm}$

$\Rightarrow \quad v = \frac{dx}{dt} = -86.5\sin(34.6t) \text{ cms}^{-1}$

$\Rightarrow \quad a = \frac{dv}{dt} = -29.9\cos(34.6t) \text{ ms}^{-2}$

10. In the present case,

$$x = 2\left(\frac{2F}{k_1}\right) + \left(\frac{F}{k_2}\right)$$

$\Rightarrow \quad F = -\frac{x}{\left(\frac{4}{k_1} + \frac{1}{k_2}\right)}$

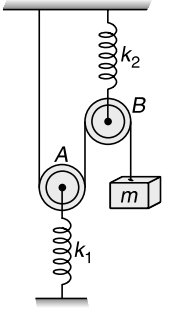
$\Rightarrow \quad a = \frac{-x}{m\left(\frac{4}{k_1} + \frac{1}{k_2}\right)}$

$\Rightarrow \quad T = 2\pi\sqrt{\left|\frac{x}{a}\right|} = 2\pi\sqrt{\frac{m(k_1 + 4k_2)}{k_1 k_2}}$

11. Frequency will remain unchanged, whereas the mean position will change.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

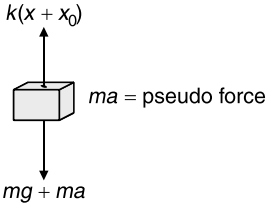
Free body diagram of block w.r.t. ground is shown in figure.



For equilibrium of block, we have

$kx = mg$ …(1)

Free body diagram of block w.r.t. elevator is shown in figure.



For equilibrium of block, we have

$k(x + x_0) = mg + ma$ …(2)

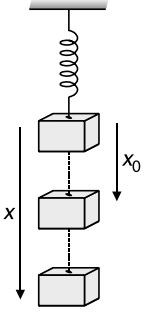
From equations (1) and (2), we get

x_0 = amplitude $= \frac{ma}{k}$

12. (a) $mgx = \frac{1}{2}kx^2$

$\Rightarrow \quad x = \frac{2mg}{k} = \frac{2 \times 1.0 \times 9.8}{150}$

$\Rightarrow \quad x = 0.13 \text{ m} = 13 \text{ cm}$



(b) $mg = kx_0$

$\Rightarrow \quad x_0 = \frac{mg}{k} = \frac{x}{2} = 6.5 \text{ cm}$

(c) $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1.0}{150}} = 0.51 \text{ s}$

(d) $U = \frac{1}{2}k(x - x_0)^2 = \frac{1}{2} \times 150 \times (0.065)^2$

$\Rightarrow \quad U = 0.32 \text{ J}$

(e) By Law of Conservation of Energy, we have

$$mgx_0 = \frac{1}{2}kx_0^2 + \frac{1}{2}mv^2$$

Substituting the values, we get

$$\frac{1}{2}mv^2 = 0.32 \text{ J}$$

$$\Rightarrow \quad v = \sqrt{\frac{0.32\times 2}{m}} = \sqrt{\frac{0.64}{1.0}} = 0.8 \text{ ms}^{-1}$$

$$t = \frac{T}{4} = \frac{0.51}{4} = 0.13 \text{ s}$$

(f) $t = \sqrt{\frac{2x_0}{g}} = \sqrt{\frac{2\times 0.065}{9.8}} = 0.12$ s $\quad \left\{x_0 = \frac{1}{2}gt^2\right\}$

$v = \sqrt{2gx_0} = \sqrt{2\times 9.8\times 0.065} = 1.13 \text{ ms}^{-1}$

13. $kx = mg$

$$\Rightarrow \quad k = \frac{mg}{x} = \frac{2.8\times 9.8}{0.018} = 1524.4 \text{ Nm}^{-1}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$\Rightarrow \quad m = \frac{k}{4\pi^2 f^2} = \frac{1524.4}{4\pi^2(3)^2} = 4.3 \text{ kg}$$

14. When both the blocks stick to each other, then by law of conservation of linear momentum, we get

$$mu + 0 = (m+m)v$$

$$\Rightarrow \quad v = \frac{u}{2}$$

This velocity is the velocity of combined blocks at the mean position, so

$$v = \frac{u}{2} = A\omega$$

where, A is the amplitude of oscillation having

$$\omega = \sqrt{\frac{k}{2m}}$$

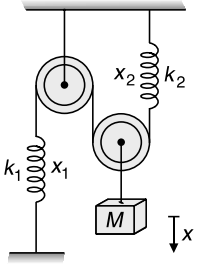
$$\Rightarrow \quad \frac{u}{2} = A\sqrt{\frac{k}{2m}}$$

$$\Rightarrow \quad A = \frac{u}{2}\sqrt{\frac{2m}{k}} = u\sqrt{\frac{m}{2k}}$$

The time taken by combined system to reach extreme position is

$$t = \frac{T}{4} = \frac{2\pi}{4\omega} = \frac{\pi}{2\omega} = \frac{\pi}{2}\sqrt{\frac{k}{2m}}$$

15. Let x be the displacement of the mass from the equilibrium value and let x_1 and x_2 be the deformation of spring k_1 and k_2, respectively from the equilibrium. Then, using constraint relation,



$$\frac{x_1 + x_2}{2} = x$$

$$\Rightarrow \quad x_1 + x_2 = 2x \qquad \ldots(1)$$

Also, tension in string should be same

$$\Rightarrow \quad k_1x_1 = k_2x_2 \qquad \ldots(2)$$

Solving (1) and (2), we get

$$x_1 = \frac{2k_2}{k_1+k_2}x, \; x_2 = \frac{2k_1}{k_1+k_2}x \qquad \ldots(3)$$

The energy of the system is $E = \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_2x_2^2 + \frac{1}{2}mv^2$

$$\Rightarrow \quad E = \frac{2k_1k_2}{k_1+k_2}x^2 + \frac{1}{2}mv^2 \qquad \{\text{from (3)}\}$$

Differentiating w.r.t. time, we get

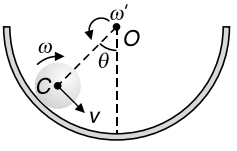
$$\frac{dE}{dt} = \frac{4k_1k_2}{k_1+k_2}x\frac{dx}{dt} + mv\frac{dv}{dt} = 0$$

$$\Rightarrow \quad \frac{d^2x}{dt^2} + \left[\frac{4k_1k_2}{(k_1+k_2)m}\right]x = 0$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{(k_1+k_2)m}{4k_1k_2}}$$

Test Your Concepts-III (Based on Rotational SHM)

1. For pure rolling, $v = R\omega$ and $a = R\alpha$. Let, ω' be the angular velocity of CM of sphere C about O, then



$$\omega' = \frac{v}{4R} = \frac{R\omega}{4R} = \frac{\omega}{4}$$

$$\Rightarrow \quad \frac{d\omega'}{dt} = \frac{1}{4}\frac{d\omega}{dt}$$

$$\Rightarrow \quad \alpha' = \frac{\alpha}{4}$$

Since, for pure rolling $\alpha = \frac{a}{R}$, where

$$a = \frac{g\sin\theta}{1+(I/mR^2)} = \frac{5g\sin\theta}{7} \qquad \left\{\because I = \frac{2}{5}mR^2\right\}$$

$$\Rightarrow \quad \alpha' = \frac{\alpha}{4} = \frac{5g\sin\theta}{28R}$$

For small θ, $\sin\theta \approx \theta$

$$\Rightarrow \quad \alpha' = -\frac{5g}{28R}\theta$$

Negative sign shows restoring nature of α'

$$\Rightarrow \quad \ddot{\theta} + \left(\frac{5g}{28R}\right)\theta = 0$$

$$\Rightarrow \quad T = 2\pi\sqrt{\left|\frac{\theta}{\alpha'}\right|} = 2\pi\sqrt{\frac{28R}{5g}}$$

2. In the displaced position, total energy is

$$E = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}k(2x)^2$$

where, $I = \frac{1}{2}mR^2$ and $\omega = \frac{v}{R}$

$$\Rightarrow \quad E = \frac{3}{4}mv^2 + 2kx^2$$

Since E =constant, so $\frac{dE}{dt} = 0$

CHAPTER 3

$$\Rightarrow \quad \frac{3}{2}mv\frac{dv}{dt}+4kx\frac{dx}{dt}=0$$

Substituting, $\frac{dx}{dt}=v$ and $\frac{dv}{dt}=a$

$$\Rightarrow \quad a=-\left(\frac{8k}{3m}\right)x$$

Comparing with, $a=-\omega^2 x$, we get

$$\omega=\sqrt{\frac{8k}{3m}}=\sqrt{\frac{8(1000)}{3(100/9.8)}}=16.16 \text{ rads}^{-1}$$

$$\Rightarrow \quad \theta=\theta_0\cos(\omega t)=0.4\cos(16.16t)$$

3. Let x_0 be the initial extension in spring at equilibrium, then

$$\Sigma\tau_0=0$$

$$\Rightarrow \quad (kx_0)b=(mg)a \qquad \ldots(1)$$

When mass m is displaced downwards, then

$$m\ddot{x}=-(T-mg) \qquad \ldots(2)$$

For the light (i.e., massless) rod, we have

$$k\left(x_0+\frac{bx}{a}\right)b=Ta \qquad \ldots(3)$$

Substituting value of T from equation (3) in (1), we get

$$m\ddot{x}=-\left[\frac{k}{a}\left(x_0+\frac{bx}{a}\right)b-mg\right]$$

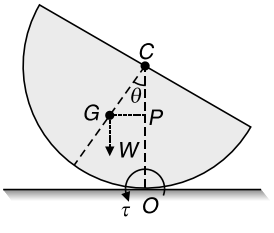
$$\Rightarrow \quad m\ddot{x}=-\left(\frac{kb^2}{a^2}\right)x$$

$$\Rightarrow \quad \ddot{x}+\left(\frac{kb^2}{ma^2}\right)x=0$$

$$\Rightarrow \quad T=2\pi\sqrt{\frac{x}{|\ddot{x}|}}=2\pi\sqrt{\frac{ma^2}{kb^2}}=\frac{2\pi a}{b}\sqrt{\frac{m}{k}}$$

4. Restoring torque is given by

$$\tau=-W(GP)=-W(GC\sin\theta)$$



For small oscillations $\sin\theta\approx\theta$

$$\Rightarrow \quad \tau=I\alpha=-W\left(\frac{4r}{3\pi}\right)\theta \qquad \ldots(1)$$

where, I is the moment of inertia about the point O. So, we have

$$I=\frac{1}{2}mr^2+m\left(r-\frac{4r}{3\pi}\right)^2-m\left(\frac{4r}{3\pi}\right)^2$$

$$\Rightarrow \quad I\alpha=-mg\left(\frac{4r}{3\pi}\right)^2-m\left(\frac{4r}{3\pi}\right)^2$$

Further, $I\alpha=-mg\left(\frac{4r}{3\pi}\right)\theta \qquad \{\because \text{ of } (1)\}$

Since, α is proportional to $-\theta$, motion is simple harmonic in nature.

$$T=2\pi\sqrt{\left|\frac{\theta}{\alpha}\right|}$$

Substituting the proper values, we get

$$T=0.963 \text{ second}$$

5. Let the rod be given a small angular displacement from the mean position, so restoring torque is

$$\tau=-mg(l\theta)+2k(b\theta)b$$

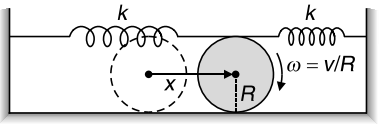
$$\Rightarrow \quad \tau=-\left(mgl+2kb^2\right)\theta$$

$$\Rightarrow \quad \ddot{\theta}+\left(\frac{mgl+2kb^2}{I}\right)\theta=0 \qquad \{\because \tau=I\ddot{\theta}\}$$

where, $I=ml^2$

$$\Rightarrow \quad T=2\pi\sqrt{\frac{\theta}{|\ddot{\theta}|}}=2\pi\sqrt{\frac{ml^2}{mgl+2kb^2}}$$

6. Let cylinder be displaced to right, then after being released, at some instant let it have a speed v, angular speed ω and is at a distance x from the mean position. In this situations, one springs will have extension $2x$ and other will have a compression $2x$. Also, for no slipping between cylinder and ground, we have $v=R\omega$.



The total energy of system at this instant is

$$E=\frac{1}{2}Mv^2\left(1+\frac{1}{2}\right)+2\left[\frac{1}{2}k(2x)^2\right]$$

Since $E=$ constant, So $\frac{dE}{dt}=0$

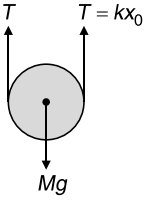
$$\Rightarrow \quad \frac{3M}{4}\left(2v\frac{dv}{dt}\right)+2k(2x)(2v)=0$$

$$\Rightarrow \quad \frac{3M}{4}\ddot{x}+4kx=0$$

$$\Rightarrow \quad \ddot{x}+\left(\frac{16k}{3M}\right)x=0$$

$$\Rightarrow \quad T=2\pi\sqrt{\frac{x}{|\ddot{x}|}}=2\pi\sqrt{\frac{3M}{16k}}$$

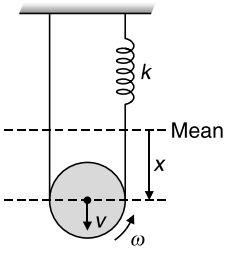
7. Initially in equilibrium, let extension in the spring be x_0, then $T=kx_0$. Also, free body diagram of pulley at equilibrium is shown in Figure.



So, $2T=Mg$

$$\Rightarrow \quad 2kx_0=Mg \qquad \ldots(1)$$

Let the pulley be now displaced slightly through x from its mean position as shown in Figure.
Due to a further pull by x, the spring extends by $x_0 + 2x$. If at this instant it is going down with a speed v and rotating with angular speed ω, then due to non-slipping of string on pulley surface, we have $\omega = \frac{v}{R}$.



Total mechanical energy of system is

$$E = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}k(x_0 + 2x)^2 - Mgx$$

$$\Rightarrow \quad E = \frac{v^2}{2}\left(M + \frac{I}{R^2}\right) + \frac{1}{2}k(x_0 + 2x)^2 - Mgx$$

Since E is constant, so $\frac{dE}{dt} = 0$

$$\Rightarrow \quad \left(M + \frac{I}{R^2}\right)\frac{vdv}{dt} + k(x_0 + 2x)2v - Mgv = 0$$

$$\Rightarrow \quad \left(M + \frac{I}{R^2}\right)\ddot{x} + 2k(x_0 + 2x) - Mg = 0$$

$$\Rightarrow \quad \left(M + \frac{I}{R^2}\right)\ddot{x} + 2kx_0 + 4kx - Mg = 0$$

Since $Mg = 2kx_0$

$$\Rightarrow \quad \ddot{x} + \left(\frac{4k}{M + \frac{I}{R^2}}\right)x = 0$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{M + \frac{I}{R^2}}{4k}}$$

8. For equilibrium of T shaped rod, if h is the initial elongation in spring, then taking torque about O to be zero, we get

$$\left(\frac{2Mg}{3}\right)(a) + \left(\frac{Mg}{3}\right)(2a) = (kh)a$$

$$\Rightarrow \quad 4Mg = 3kh$$

On slightly displacing the rod by θ, the restoring τ is given by

$$\tau = -[k(h + a\theta)a - kh] = -ka^2\theta$$

Since $\tau = I\alpha$, where, I is the moment of inertia about the point O.

$$I = \left[\frac{1}{3}\left(\frac{2M}{3}\right)(2a)^2 + \frac{1}{12}\left(\frac{M}{3}\right)a^2 + \frac{M}{3}(2a)^2\right]$$

$$\Rightarrow \quad I = \left(\frac{8}{9} + \frac{1}{36} + \frac{4}{3}\right)Ma^2 = \frac{81}{36}Ma^2 = \frac{9}{4}Ma^2$$

$$\Rightarrow \quad -ka^2\theta = \left(\frac{9}{4}Ma^2\right)\alpha$$

$$\Rightarrow \quad \alpha = -\left(\frac{4k}{9M}\right)\theta$$

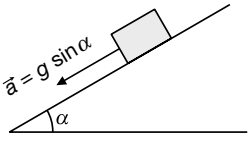
$$\Rightarrow \quad T = 2\pi\sqrt{\frac{\theta}{|\ddot{\theta}|}} = 2\pi\sqrt{\frac{9M}{4k}}$$

Test Your Concepts-IV (Based on Pendulum Systems)

1. $T_0 = 2\pi\sqrt{\frac{\ell}{g}}$ …(1)

When the cart rolls down the slope (without friction), acceleration of the cart.

$$a = g\sin\alpha$$



Now $\vec{g}_{\text{eff}} = \vec{g} - \vec{a}$

We can show that $|\vec{g}_{\text{eff}}| = g\cos\alpha$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{\ell}{g\cos\alpha}} \quad \text{…(2)}$$

From equations (1) and (2), we get

$$T = \frac{T_0}{\sqrt{\cos\alpha}}$$

2. $$\frac{T_A}{T_B} = \frac{2\pi\sqrt{\frac{(md^2/3)}{(mg)(d/2)}}}{2\pi\sqrt{\frac{d}{g}}} = \sqrt{\frac{2}{3}} = 0.816$$

3. Since, $g_{\text{eff}} = \frac{\text{Weight} - \text{Upthrust}}{\text{Mass}}$

$$\Rightarrow \quad g_{\text{eff}} = \frac{V\rho g - V(\rho/3)g}{V_\rho} = \frac{2g}{3}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{\ell}{g_{\text{eff}}}} = 2\pi\sqrt{\frac{0.2\times 3}{2\times 9.8}} = 1.1 \text{ s}$$

4. $g_{\text{eff}} = \sqrt{a^2 + g^2} = \sqrt{(7)^2 + (9.8)^2}$

$$\Rightarrow \quad g_{\text{eff}} = 12.0 \text{ ms}^{-2}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{\ell}{g_{\text{eff}}}} = 2\pi\sqrt{\frac{0.5}{12}} = 12.8 \text{ s}$$

5. Since $\theta = \theta_0\sin(\omega t)$, so time t_0 required to move from equilibrium position to wall for bob is given by

$$\alpha = \beta\sin\left(\sqrt{\frac{g}{l}}t_0\right)$$

$$\Rightarrow \quad t_0 = \sqrt{\frac{l}{g}}\left[\sin^{-1}\left(\frac{\alpha}{\beta}\right)\right]$$

Since collision is elastic, so ball will return at same speed and hence total oscillation time period is

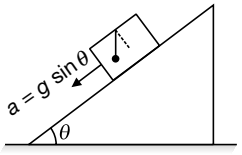
$$T = \pi\sqrt{\frac{l}{g}} + 2t_0$$

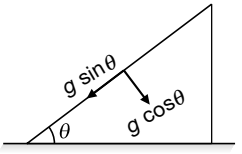
$$\Rightarrow \quad T = \pi\sqrt{\frac{l}{g}} + 2\sqrt{\frac{l}{g}}\sin^{-1}\left(\frac{\alpha}{\beta}\right)$$

$$\Rightarrow \quad T = \sqrt{\frac{l}{g}}\left[\pi + 2\sin^{-1}\left(\frac{\alpha}{\beta}\right)\right]$$

6. Here, point of suspension has an acceleration. $\vec{a} = g\sin\theta$ (down the plane). Further, $\vec{g}$ can be resolved into two components $g\sin\theta$ (along the plane) and $g\cos\theta$ (perpendicular to plane).

CHAPTER 3





Since, $\vec{g}_{\text{eff}} = \vec{g} - \vec{a}$

and $\vec{g}_{\text{eff}} = g\cos\theta$ {perpendicular to plane}

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{\ell}{|\vec{g}_{\text{eff}}|}} = 2\pi\sqrt{\frac{\ell}{g\cos\theta}}$$

Observation: If $\theta = 0°$, $T = 2\pi\sqrt{\frac{\ell}{g}}$

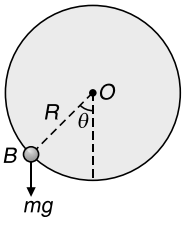
7. $\frac{f_1}{f_2} = \frac{15}{10} = \sqrt{\frac{\ell_2}{\ell_1}}$

$$\Rightarrow \quad \frac{\ell_1}{\ell_2} = \frac{4}{9}$$

8. $\tau = -mgR\sin\theta$

$$\Rightarrow \quad I\alpha \approx -mgR\theta \qquad \{\because \sin\theta \approx \theta\}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\left|\frac{\theta}{\alpha}\right|} = 2\pi\sqrt{\frac{I}{mgR}}$$



where, $I = mR^2 + \frac{1}{2}MR^2$

Substituting the values, we get

$$T = 2\pi\sqrt{\frac{\left(m + \frac{M}{2}\right)R}{mg}}$$

Further, $2\pi\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{\frac{\left(m + \frac{M}{2}\right)R}{mg}}$

$$\Rightarrow \quad \ell = \frac{\left(m + \frac{M}{2}\right)R}{m}$$

9. (a) We are given $\theta_0 = 0.1\pi$ rad, $\phi = \frac{\pi}{6}$ rad and $\omega = 2\pi$ rads^{-1}. Since $\omega^2 = \left(\frac{g}{L}\right)$, we have

$$L = \frac{g}{\omega^2} = \frac{9.8 \text{ ms}^{-2}}{\left(2 \times 3.14 \text{ rads}^{-2}\right)^2} = 0.25 \text{ m}$$

(b) Since $s = L\theta$, the velocity of the bob, $v = \frac{ds}{dt}$, is

$$v = L\frac{d\theta}{dt} = (0.25 \text{ m})(0.1\pi)(2\pi)\cos\left(\frac{\pi}{2} + \frac{\pi}{6}\right)$$

$$\Rightarrow \quad v = -0.123 \text{ ms}^{-1}$$

10. This is the case of a physical pendulum, the time period of which is,

$$T = 2\pi\sqrt{\frac{I}{mgl}}$$

where, I = moment of inertia of the ring about point of suspension

$$\Rightarrow \quad I = mr^2 + mr^2 = 2mr^2$$

and l = distance of point of suspension from centre of gravity

So, $l = r$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{2mr^2}{mgr}} = 2\pi\sqrt{\frac{2r}{g}}$$

Since, angular frequency $\omega = \frac{2\pi}{T}$

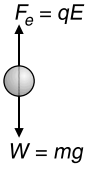
$$\Rightarrow \quad \omega = \sqrt{\frac{g}{2r}}$$

Test Your Concepts-V (Based on SHM in Other Physical Systems, Composition of SHM, Damped Oscillations, Forced Oscillations & Resonance)

1. The two forces acting on the bob are shown in figure. So, effective value of gravity in this case is

$$g_{\text{eff}} = \frac{W - F_e}{m}$$

$$\Rightarrow \quad g_{\text{eff}} = \frac{mg - qE}{m} = g - \frac{qE}{m}$$



Since, $T = 2\pi\sqrt{\frac{\ell}{g_{\text{eff}}}}$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{\ell}{g - \frac{qE}{m}}}$$

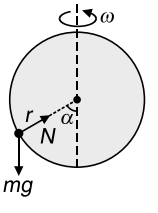
2. Let, ω =angular speed of wire frame, then

In equilibrium, $N\cos\alpha = mg$ …(1)

and $N\sin\alpha = m(r\sin\alpha)\omega^2$ …(2)

From these two equations, we get,

$$\omega^2 = \frac{g}{r\cos\alpha}$$



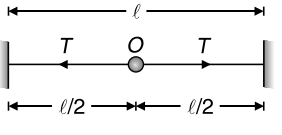
Since, $T = 2\pi\sqrt{\frac{r}{g_{\text{eff}}}}$

where, $g_{\text{eff}} = \sqrt{g^2 + (r\omega^2)^2} = \sqrt{g^2 + \left(\frac{g}{\cos\alpha}\right)^2}$

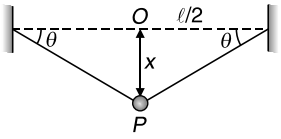
$$\Rightarrow \quad g_{\text{eff}} = \frac{g}{\cos\alpha}\sqrt{1 + \cos^2\alpha}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{r\cos\alpha}{g\left(1 + \cos^2\alpha\right)^{1/2}}}$$

3. The situation is shown in Figure. The ball is in equilibrium at point O as shown in Figure.



Now it is displaced from O by a distance x in horizontal plane to a position P as shown in Figure.



The components of tensions on ball towards the mean position O are $T\sin\theta$ and $T\sin\theta$, hence the restoring force on ball toward mean position is

$$F = -2T\sin\theta$$

Negative sign tells restoring nature of force.

$$\Rightarrow \quad F = -\frac{2Tx}{\sqrt{x^2 + \frac{l^2}{4}}}$$

Since, x is very small compared to ℓ, so we can neglect its square compared to $\frac{l^2}{4}$.

$$\Rightarrow \quad F = -\left(\frac{4T}{l}\right)x$$

If a is the acceleration of ball toward mean position, then

$$a = \frac{F}{m} = -\left(\frac{4T}{ml}\right)x$$

We observe that acceleration is directly proportional to x and is directed towards the mean position, so motion of ball is simple harmonic with angular frequency ω, given by

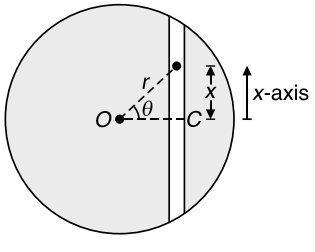
$$\omega = \sqrt{\frac{4T}{ml}}$$

$$\Rightarrow \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{ml}{4T}} = 3.14\sqrt{\frac{(0.04)(1)}{10}} = 0.2 \text{ s}$$

4. For the elastic wire of area A, length l and young's modulus Y, the equivalent spring constant is $k = \frac{YA}{l}$. Since $T = 2\pi\sqrt{\frac{m}{k}}$

$$\Rightarrow \quad f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{YA}{ml}}$$

5. Let at some instant the particle be at radial distance r from centre of earth O. Since the particle is constrained to move along the tunnel, we define its position as distance x from C.



Hence, equation of motion of the particle is,

$$ma_x = F_x$$

The gravitational force on mass m at distance r is,

$$F = \frac{GMmr}{R^3} \qquad \{\text{towards } O\}$$

Therefore, $F_x = -F\sin\theta$

$$\Rightarrow \quad F_x = -\frac{GMmr}{R^3}\left(\frac{x}{r}\right)$$

$$\Rightarrow \quad F_x = -\left(\frac{GMm}{R^3}\right)x$$

Since, $F_x \propto -x$, motion is simple harmonic in nature. Further,

$$ma_x = -\left(\frac{GMm}{R^3}\right)x$$

$$\Rightarrow \quad a_x = -\left(\frac{GM}{R^3}\right)x$$

So, the period of oscillation is,

$$T = 2\pi\sqrt{\left|\frac{x}{a_x}\right|} = 2\pi\sqrt{\frac{R^3}{GM}}$$

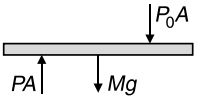
The time taken by particle to go from one end to the other is $\frac{T}{2}$

$$\Rightarrow \quad t = \frac{T}{2} = \pi\sqrt{\frac{R^3}{GM}}$$

6. At equilibrium if gas pressure is P, then we have

$$PA = Mg + P_0A$$

$$\Rightarrow \quad P = \frac{Mg}{A} + P_0$$



Let the piston be displaced down by a distance x and since the system is isolated, so we have

$$PV^\gamma = \text{constant}$$

$$\Rightarrow \quad d\left(PV^\gamma\right) = 0$$

$$\Rightarrow \quad P\left(\gamma V^{\gamma-1}\right)dV + V^\gamma dP = 0$$

$$\Rightarrow \quad dP = -\gamma\left(\frac{dV}{V}\right)P$$

Now the restoring force developed due to this excess pressure is

$$F = AdP = -\gamma\left(\frac{dV}{V}\right)PA$$

where, $dV = Ax$ and $V = V_0$

$$\Rightarrow \quad M\ddot{x} = -\left(\frac{\gamma A^2P}{V_0}\right)x$$

$$\Rightarrow \quad \ddot{x} + \left(\frac{\gamma PA^2}{MV_0}\right)x = 0$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{x}{|\ddot{x}|}} = 2\pi\sqrt{\frac{MV_0}{\gamma PA^2}}$$

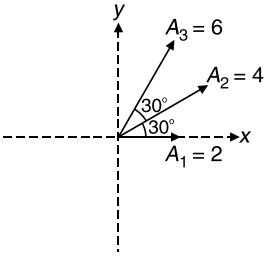
$$\Rightarrow \quad T = 2\pi\sqrt{\frac{MV_0}{\gamma A\left(P_0A + Mg\right)}}$$

7. The resultant equation is,

$$x = A\sin\left(\omega t + \phi\right)$$

$$\Sigma A_x = 2 + 4\cos(30^\circ) + 6\cos(60^\circ) = 8.46$$

CHAPTER 3



and $\Sigma A_y = 4\sin(30°) + 6\cos(30°) = 7.2$

$\Rightarrow \quad A = \sqrt{(\Sigma A_x)^2 + (\Sigma A_y)^2} = \sqrt{(8.46)^2 + (7.2)^2}$

$\Rightarrow \quad A = 11.25$ and $\tan\phi = \dfrac{\Sigma A_y}{\Sigma A_x} = \dfrac{7.2}{8.46} = 0.85$

$\Rightarrow \quad \phi = \tan^{-1}(0.85) = 40.4°$

Thus, the displacement equation of the combined motion is,

$x = 11.25\sin(\omega t + \phi)$, where $\phi = 40.4°$

8. In 25 oscillations amplitude is reduced to half the initial value of 5 cm. So, in 50 oscillations the amplitude becomes $\left(\dfrac{1}{2}\right)^2 = \dfrac{1}{4}$ times the initial value, i.e.,1.25 cm.

9. Since $b \ll \sqrt{km}$, so we have $T = 2\pi\sqrt{\dfrac{m}{k}}$

$\Rightarrow \quad m = \dfrac{T^2 k}{4\pi^2} = \dfrac{(0.5)^2(200)}{4(3.14)^2} \approx 1.25$ kg

10. In one minute, the amplitude becomes half the original value. Since the amplitude decays exponentially, so after three minutes, it should become $\left(\dfrac{1}{2}\right)^3 = \dfrac{1}{8}$ times the original value. Hence,

$x = 8$

Single Correct Choice Type Questions

1. Since, $T = 2\pi\sqrt{\dfrac{M}{K}}$ and $T' = 2\pi\sqrt{\dfrac{M+m}{K}}$

By Conservation of Linear Momentum, we have

$Mv = (M+m)v'$

$\Rightarrow \quad M(A\omega) = (M+m)(A'\omega')$

$\Rightarrow \quad A' = A\sqrt{\dfrac{M}{M+m}}$

Hence, the correct answer is (B).

2. $v_{cm} = \dfrac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \dfrac{1\times 12}{3} = 4$ ms^{-1}

Energy of oscillation $E = \dfrac{1}{2}m_1 v_1^2 - \dfrac{1}{2}(m_1 + m_2)v_{cm}^2$

Since $E = \dfrac{1}{2}kA^2$

$\Rightarrow \quad A = \sqrt{\dfrac{m_1 v_1^2 - (m_1 + m_2)v_{cm}^2}{k}}$

$\Rightarrow \quad A = \sqrt{\dfrac{(1)(0.12)^2 - (3)(0.04)^2}{24}} = \dfrac{2}{100}$ m $= 2$ cm

Hence, the correct answer is (B).

3. Since, $y = x_0\sin^2\omega t = x_0\left(\dfrac{1-\cos 2\omega t}{2}\right)$

$\Rightarrow \quad y = \dfrac{x_0}{2} - \dfrac{x_0}{2}\cos 2\omega t$

$\Rightarrow \quad y - \dfrac{x_0}{2} = -\dfrac{x_0}{2}\cos 2\omega t = \dfrac{x_0}{2}\sin\left(2\omega t - \dfrac{\pi}{2}\right)$

Comparing with $y - y_0 = A\sin(\omega' t + \phi_0)$, we get

$\omega' = 2\omega$

$\Rightarrow \quad \dfrac{2\pi}{T'} = 2\omega$

$\Rightarrow \quad T' = \dfrac{\pi}{\omega}$

So, the function $x = x_0\sin^2(\omega t)$ represents SHM of amplitude $\dfrac{x_0}{2}$ about the mean position at $\dfrac{x_0}{2}$ and having a period of $\dfrac{\pi}{\omega}$

Hence, the correct answer is (B).

4. $A_R = \sqrt{(\sqrt{3})^2 + (1)^2} = 2$

$\Rightarrow \quad \phi = \tan^{-1}\left(\dfrac{\sqrt{3}}{1}\right) = \dfrac{\pi}{3}$

$\Rightarrow \quad y = 2\sin\left(\omega t + \dfrac{\pi}{3}\right)$

$\Rightarrow \quad \dfrac{d^2y}{dt^2} = a = -2\omega^2\sin\left(\omega t + \dfrac{\pi}{3}\right)$

$\Rightarrow \quad |a_{max}| = 2\omega^2 = g$

For this maximum acceleration, the mass just breaks off the plank, so $\omega = \sqrt{\dfrac{g}{2}}$. This will happen for the first time when

$\omega t + \dfrac{\pi}{3} = \dfrac{\pi}{2}$

$\Rightarrow \quad \omega t = \dfrac{\pi}{6}$

$\Rightarrow \quad t = \dfrac{\pi}{6\omega} = \dfrac{\pi}{6}\sqrt{\dfrac{2}{g}} = \dfrac{\sqrt{2}}{6}\left(\dfrac{\pi}{g}\right)$

Hence, the correct answer is (A).

5. Since, $\dfrac{x}{a} = \cos(\omega t)$ and $\dfrac{y}{a} = \cos\left(\omega t + \dfrac{\pi}{6}\right)$

$\Rightarrow \quad \dfrac{y}{a} = \cos(\omega t)\cos\left(\dfrac{\pi}{6}\right) - \sin(\omega t)\sin\left(\dfrac{\pi}{6}\right)$

$\Rightarrow \quad \dfrac{y}{a} = \cos(\omega t)\left(\dfrac{\sqrt{3}}{2}\right) - \sin(\omega t)\left(\dfrac{1}{2}\right)$

$\Rightarrow \quad \dfrac{y}{a} = \dfrac{\sqrt{3}}{2}\dfrac{x}{a} - \dfrac{1}{2}\sqrt{1 - \dfrac{x^2}{a^2}}$

$\Rightarrow \quad \dfrac{1}{2}\sqrt{1 - \dfrac{x^2}{a^2}} = \dfrac{\sqrt{3}}{2}\dfrac{x}{a} - \dfrac{y}{a}$

$$\Rightarrow \quad \frac{1}{4}\left(1-\frac{x^2}{a^2}\right)=\left(\frac{\sqrt{3}}{2}\frac{x}{a}-\frac{y}{a}\right)^2$$

$$\Rightarrow \quad \frac{1}{4}\left(1-\frac{x^2}{a^2}\right)=\frac{3}{4}\frac{x^2}{a^2}+\frac{y^2}{a^2}-\frac{\sqrt{3}xy}{a^2}$$

$$\Rightarrow \quad a^2-x^2=3x^2+4y^2-4\sqrt{3}xy$$

$$\Rightarrow \quad a^2=4x^2+4y^2-4\sqrt{3}xy$$

$$\Rightarrow \quad x^2+y^2-\sqrt{3}xy=\frac{a^2}{4}$$

Hence, the correct answer is (D).

6. Potential energy of the particle is

$$U=x^2-4x+4$$

$$\Rightarrow \quad F=-\frac{dU}{dx}=-(2x-4)=-2x+4$$

So F varies linearly with x and is directed towards the mean position, hence the particle performs SHM. At equilibrium i.e. at the mean position

$$F=0$$

$$\Rightarrow \quad -2x+4=0$$

$$\Rightarrow \quad x=2$$

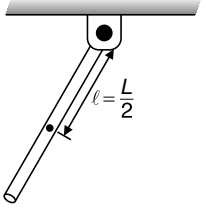
So, particle execute SHM with its equilibrium position at $x=2$

Hence, the correct answer is (C).

7. $T_1=2\pi\sqrt{\frac{L}{g}}$ and $T_2=2\pi\sqrt{\frac{I}{Mgl}}$ (physical pendulum)

$$\Rightarrow \quad T_2=2\pi\sqrt{\frac{ML^2/3}{Mg(L/2)}}=2\pi\sqrt{\frac{2L}{3g}}$$

$$\Rightarrow \quad \frac{T_1}{T_2}=\sqrt{\frac{3}{2}}$$



Hence, the correct answer is (C).

8. Since, $x=A\sin\omega t$

Over the interval $0\le t\le\frac{\pi}{6\omega}$, the particle gets displaced from

$x=0$ to $x=A\sin\left(\omega\times\frac{\pi}{6\omega}\right)=\frac{A}{2}$

So, average speed of particle in the given interval is

$$v_{av}=\frac{\text{distance}}{\text{time}}=\frac{A/2}{\pi/6\omega}=\frac{3A\omega}{\pi}$$

Hence, the correct answer is (C).

9. $U(6\text{ m})=10+(6-2)^2=26\text{ J}$

$U(-2\text{ m})=10+(-2-2)^2=26\text{ J}=U(6\text{ m})$

On negative x-axis particle travels upto $x=-2$ m

Mean position of the particle is $x=2$ m

$$U(2\text{ m})=10\text{ J}$$

$$\Rightarrow \quad K(2\text{ m})=(26-10)\text{ J}=16\text{ J}=K_{\max}$$

Substitute $x-2=X$

$$\Rightarrow \quad U=10+X^2$$

$$\Rightarrow \quad F=-\frac{dU}{dX}=-2X$$

$$\Rightarrow \quad a=\frac{F}{m}=-2X \qquad \{\because\ m=1\text{ kg}\}$$

$$\Rightarrow \quad \omega=\sqrt{2}=\frac{2\pi}{T}$$

$$\Rightarrow \quad T=\sqrt{2}\pi$$

Hence, the correct answer is (D).

10. The amplitude of SHM represented by the equation $y=a\sin(\omega t)+y_0$ is a and mean position is at y_0. So, amplitude of SHM is 2 and mean position is at 1.

Hence, the correct answer is (C).

11. Since, $\omega=\sqrt{\frac{k}{\Sigma m}}=\sqrt{\frac{54}{6}}=3\text{ rads}^{-1}$

Maximum friction between 1 kg and 2 kg blocks can be $0.6\times1\times10=6$ N. Therefore, maximum acceleration of 1 kg block can be $\frac{6}{1}=6\text{ ms}^{-2}$. Maximum force of friction between 2 kg and 3 kg blocks can be $0.4\times3\times10=12$ N. Therefore, maximum acceleration of 1 kg and 2 kg blocks jointly can be $\frac{12}{3}=4\text{ ms}^{-2}$.

So, maximum acceleration of the whole system, so that there is no slipping between any of blocks is 4 ms^{-2}.

Now, $\omega^2A_{\max}=4$

$$\Rightarrow \quad A_{\max}=\frac{4}{\omega^2}=\frac{4}{9}\text{ m}$$

Hence, the correct answer is (C).

12. With increase in k, the angular frequency ω will increase. Hence, $A_{\max}$ will decrease.

Hence, the correct answer is (C).

13. Let $BC=CD=x$, then according to the problem, we have

$$\frac{1}{2}m\omega^2(R^2-x^2)=\frac{1}{4}\left(\frac{1}{2}m\omega^2R^2\right)$$

$$\Rightarrow \quad x=\pm\frac{\sqrt{3}R}{2}$$

$$\Rightarrow \quad BD=2x=\sqrt{3}R$$

Hence, the correct answer is (C).

14. Effective force constant in Case (3) and (4) is

$$k_{\text{eff}}=2k+2k=4k$$

Therefore, $T_1=T_2=2\pi\sqrt{\frac{m}{k}}$

and $T_3=T_4=2\pi\sqrt{\frac{m}{4k}}=\pi\sqrt{\frac{m}{k}}$

Hence, the correct answer is (B).

15. Since, $\frac{1}{k_s}=\frac{1}{k}+\frac{1}{2k}+\frac{1}{4k}+\frac{1}{8k}+\ldots$

$$\Rightarrow \quad \frac{1}{k_s}=\frac{1}{k}\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\right)$$

CHAPTER 3

$$\Rightarrow \quad \frac{1}{k_s}=\frac{1}{k}\left(\frac{1}{1-1/2}\right)=\frac{2}{k}$$

$$\Rightarrow \quad k_s=\frac{k}{2}$$

$$\Rightarrow \quad T=2\pi\sqrt{\frac{m}{k_s}}=2\pi\sqrt{\frac{2m}{k}}$$

Hence, the correct answer is (B).

16. When simple pendulum executes SHM with angular amplitude α and its amplitude is A. Its time period is T. In second case, the angular amplitude of oscillation is $\frac{\alpha}{2}$ and hence its amplitude is $\frac{A}{2}$.

Since time taken by a particle (executing SHM) to reach the position $\frac{A}{2}$ from mean position is $\frac{T}{12}$, so its time period is

$$T'=4\left(\frac{T}{12}\right)=\frac{T}{3}$$

$$\Rightarrow \quad \frac{T}{T'}=\frac{3}{1}$$

Hence, the correct answer is (B).

17. Since, $T=2\pi\sqrt{\frac{m}{k}}T=2\pi\sqrt{\frac{m}{F/x}}$ $\quad\{\because k=\frac{F}{x}\}$

$$\Rightarrow \quad T=2\pi\sqrt{\frac{mx}{mg}}=2\pi\sqrt{\frac{x}{g}} \qquad \{\because F=mg\}$$

$$\Rightarrow \quad \frac{T_1}{T_2}=\sqrt{\frac{x_1}{x_2}}=\sqrt{\frac{5}{10}}=\frac{1}{\sqrt{2}}$$

Hence, the correct answer is (A).

18. Equations can be written as $x_1=A\cos(\omega t)$ and $x_2=A\sin\left(\omega t-\frac{\pi}{6}\right)$, where $\omega=\frac{2\pi}{T}$.

Equating $x_1=x_2$, we get $t=\frac{T}{6}$

Hence, the correct answer is (D).

19. $K_{eff}=\frac{(2K)(2K)}{2K+2K}+K+K$

$$\Rightarrow \quad K_{eff}=3K$$

$$\Rightarrow \quad f=\frac{1}{2\pi}\sqrt{\frac{3K}{M}}$$

Hence, the correct answer is (A).

20. Time taken by particle to move from extreme $x=a$ to $x=\frac{a}{2}$ is $\frac{T}{6}$. So, we have average velocity given by

$$v_{av}=\frac{\text{displacement}}{\text{time interval}}=\frac{a/2}{T/6}=\frac{3a}{T}$$

Hence, the correct answer is (C).

21. At equilibrium position, $\rho=\frac{\rho_0}{2}(\alpha+\beta h_0)$

$$\Rightarrow \quad h_0=\frac{2-\alpha}{\beta}=A, \text{ amplitude of oscillation}$$

Net force at (h_0+x) is $F_{net}=F_b-W$ {upwards}

$$\Rightarrow \quad F_{net}=Vg\left[\frac{\rho_0}{2}\{\alpha+\beta(h_0+x)\}\right]-Vg\rho_0$$

Since, $\rho=\frac{\rho_0}{2}(\alpha+\beta h_0)$, so $F=-\left(\frac{Vg\rho_0\beta}{2}\right)x$

$$F\propto -x$$

So, motion is simple harmonic with acceleration

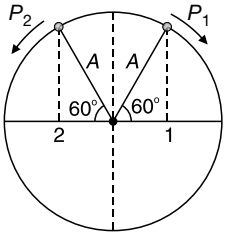
$$a=\frac{F}{m}=\frac{F}{\rho_0 V}=-\left(\frac{\beta g}{2}\right)x$$

$$\Rightarrow \quad \omega=\sqrt{\frac{\beta g}{2}}$$

$$\Rightarrow \quad v_{max}=A\omega=\left(\frac{2-\alpha}{\sqrt{2\beta}}\right)\sqrt{g}$$

Hence, the correct answer is (C).

22. Points on the circle corresponding to particles 1 and 2 are P_1 and P_2 as shown in Figure.



Particles 1 and 2 will collide when P_1 and P_2 will collide i.e., $\theta_1+\theta_2=60°+180°+60°$

$$\Rightarrow \quad \omega t+\omega t=\frac{\pi}{3}+\pi+\frac{\pi}{3}$$

$$\Rightarrow \quad 2\omega t=2\left(\frac{2\pi}{T}\right)t=\frac{5\pi}{3}$$

$$\Rightarrow \quad t=5T/12$$

Hence, the correct answer is (B).

23. Let potential energy at mean position be U_0. At the extreme position, the potential energy equals the total energy (because kinetic energy is zero at extreme). So,

$$U_0+\frac{1}{2}kA^2=200$$

$$\Rightarrow \quad U_0=200-\frac{1}{2}\times 3\times 10^4\times 0.1\times 0.1=50 \text{ J}$$

Hence, potential energy varies between 50 J and 200 J whereas kinetic energy varies between 0 and 150 J.

Hence, the correct answer is (D).

24. For the ball after collision $h=\frac{1}{2}gt^2$, $d=ut$

$$\Rightarrow \quad u=d\sqrt{\frac{g}{2h}}$$

The velocity of recoil of M, $v=\frac{md}{M}\sqrt{\frac{g}{2h}}$

Since $v=A\omega$, so $A=\frac{v}{\omega}=\frac{md}{M}\sqrt{\frac{Mg}{k(2h)}}$

Hence, the correct answer is (A).

25. Since acceleration is $a = -\omega^2 x$

$\Rightarrow \quad \Delta a = -\omega^2 \Delta x$

$\Rightarrow \quad \omega^2 = -\dfrac{\Delta a}{\Delta x} = -\dfrac{27-9}{(-3)-(-1)} = 9$

$\Rightarrow \quad \omega = 3 = \dfrac{2\pi}{T}$

$\Rightarrow \quad T = \dfrac{2\pi}{3}$ s

Hence, the correct answer is (C).

26. Since, $a = 0$, at $x = 2$, so, $x = 2$ is the mean position.

Let $x - 2 = X$

$\Rightarrow \quad a = -\beta x$

$\Rightarrow \quad a \propto -X$

Since, the oscillations are simple harmonic, so time period of the oscillations is given by

$$T = 2\pi\sqrt{\left|\frac{x}{a}\right|} = 2\pi\sqrt{\frac{1}{\beta}}$$

Hence, the correct answer is (B).

27. Since, $k_s = \dfrac{k_1 k_2}{k_1 + k_2} = \dfrac{(k/5)(k/4)}{(k/5)+(k/4)} = \dfrac{k}{9}$

$\Rightarrow \quad T = 2\pi\sqrt{\dfrac{m}{k_s}} = 2\pi\sqrt{\dfrac{9m}{k}} = 6\pi\sqrt{\dfrac{m}{k}}$

$\Rightarrow \quad f = \dfrac{1}{T} = \dfrac{1}{6\pi}\sqrt{\dfrac{k}{m}}$

Hence, the correct answer is (A).

28. Since, $k_P = \dfrac{k}{5} + \dfrac{k}{4} = \dfrac{9k}{20}$

$\Rightarrow \quad T = 2\pi\sqrt{\dfrac{m}{9k/20}} = \dfrac{2\pi}{3}\sqrt{20}\sqrt{\dfrac{m}{k}}$

$\Rightarrow \quad f = \dfrac{1}{T} = \dfrac{3}{2\pi\sqrt{20}}\sqrt{\dfrac{k}{m}}$

Hence, the correct answer is (B).

29. Let the piston be displaced slightly through x. Considering this process to take place gradually, we apply the equation obeying isothermal process.

$\Rightarrow \quad PV = \text{constant}$

$\Rightarrow \quad P\Delta V + V\Delta P = 0$

$\Rightarrow \quad \Delta P = -\dfrac{P\Delta V}{V} = -\dfrac{P(Ax)}{Ah} = -\dfrac{Px}{h}$ …(1)

This excess pressure is responsible for providing the restoring force to the piston of mass M.

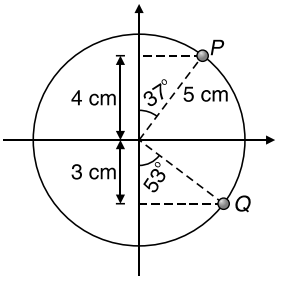
$\Rightarrow \quad F = M\ddot{x} = A\Delta P = -\left(\dfrac{PA}{h}\right)x$ {∵ of (1)}

$\Rightarrow \quad \ddot{x} + \dfrac{PA}{Mh}x = 0$

So, $\omega = \sqrt{\dfrac{PA}{Mh}}$ and hence $T = 2\pi\sqrt{\dfrac{Mh}{PA}}$

Hence, the correct answer is (A).

30. The mapping of SHM on the circle is shown in Figure.



From figure, we see that the angle between the radii vectors when the particle goes from P to Q is 90°. Therefore, time taken is

$$t = \left(\frac{90}{360}\right)T = \frac{20}{4} = 5 \text{ s}$$

Hence, the correct answer is (C).

31. Since, $\phi = \omega_s t - \omega_\ell t$

$\Rightarrow \quad \phi = \dfrac{2\pi}{T}\dfrac{5T}{4} - \dfrac{2\pi}{(5T/4)}\dfrac{5T}{4}$

$\Rightarrow \quad \phi = 2\pi\left(\dfrac{5}{4} - 1\right) = 2\pi\left(\dfrac{1}{4}\right) = \dfrac{\pi}{2}$

Hence, the correct answer is (D).

32. Since, $T = 2\pi\sqrt{\dfrac{m}{k_P}} = 2\pi\sqrt{\dfrac{m}{2k}}$ and $T' = 2\pi\sqrt{\dfrac{m}{k}} = \dfrac{T}{\sqrt{2}}$

Hence, the correct answer is (A).

33. Since, $F = -kx$, so the displacement and force are out of phase $(\Delta\phi = \pi)$ in SHM. Therefore, the correct graph will be (D).

Hence, the correct answer is (D).

34. Phase difference between the two SHMs is 90°. So, resultant amplitude is

$$A_R = \sqrt{2}A$$

$\Rightarrow \quad E = \dfrac{1}{2}m\omega^2 A_R^2 = \dfrac{1}{2}m\omega^2(\sqrt{2}A)^2 = m\omega^2 A^2$

Hence, the correct answer is (B).

35. Potential energy of the particle is

$$U = mV = 8\times10^5 x^2 \text{ erg}$$

If A is the amplitude, then at the extreme position, total energy equals the potential energy and hence we get

$8\times10^7 = 8\times10^5 A^2$

$\Rightarrow \quad A = 10$ cm

Also, $F = -\dfrac{dU}{dx} = -16\times10^5 x$

$\Rightarrow \quad k = \dfrac{F}{x} = 16\times10^5$ dynecm^{-1}

So, angular frequency is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{16\times10^5}{10}} = 400 \text{ rads}^{-1}$$

Position of the particle is given by

$$x = A\sin(\omega t + \phi) = 10\sin(400t + \phi) \text{ cm}$$

Hence, the correct answer is (A).

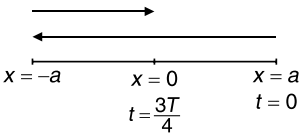
CHAPTER 3

36. Comparing the given equation with $x = a\cos(\omega t)$, we get

$\omega = \dfrac{\pi}{2}$

$\Rightarrow \quad \dfrac{2\pi}{T} = \dfrac{\pi}{2}$

$\Rightarrow \quad T = 4$ s



The given time is $t = 3$ s

$\Rightarrow \quad t = \dfrac{3T}{4}$

At time $t = 0$ particle is at $x = a$ (at extreme position) and at $t = 3$, i.e., $t = \dfrac{3T}{4}$ it will be at mean position $x = 0$. So, distance covered will be $3a$.

Hence, the correct answer is (B).

37. At A, $v = 0$ i.e., particle is at extreme position. At B, v is maximum i.e., particle is at mean position, so acceleration of particle is zero. At C, $v = 0$ and after some time its velocity is negative. Hence, particle is at $x = +A$ i.e., acceleration of particle is maximum in negative directions.
Hence, the correct answer is (C).

38. In case (a), MI of system about point of suspension is

$$I_1 = mL^2 + mL^2 = 2mL^2$$

and centre of mass of system from point of suspension is $d_1 = L$. Therefore, frequency of the system is

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{mgd_1}{I_1}} = \frac{1}{2\pi}\sqrt{\frac{mgL}{2mL^2}} = \frac{1}{2\pi}\sqrt{\frac{g}{2L}}$$

In case (b), MI of system about point of suspension is

$$I_2 = m\left(\frac{L}{2}\right)^2 + mL^2 = \frac{5}{4}mL^2$$

and centre of mass of system from point of suspension is

$$d_2 = \frac{m(L/2) + mL}{m + m} = \frac{3L}{4}$$

Frequency of the system is

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{mgd_2}{I_2}} = \frac{1}{2\pi}\sqrt{\frac{mg\left(\frac{3}{4}L\right)}{\frac{5}{4}mL^2}} = \frac{1}{2\pi}\sqrt{\frac{3g}{5L}}$$

Ratio of frequencies in configuration (b) to (a) is

$$\frac{f_b}{f_a} = \sqrt{\frac{3g}{5L}} \Big/ \sqrt{\frac{g}{2L}} = \sqrt{\frac{6}{5}}$$

Hence, the correct answer is (A).

39. Since, $\omega = \sqrt{\dfrac{k}{m}}$ and general equation of motion is

$x = 2\ell\cos(\omega t)$

$\Rightarrow \quad \ell = 2\ell\cos(\omega t)$

$\Rightarrow \quad \omega t = \dfrac{\pi}{3}$

$\Rightarrow \quad \sqrt{\dfrac{k}{m}}t = \dfrac{\pi}{3}$

$\Rightarrow \quad t = \dfrac{\pi}{3}\sqrt{\dfrac{m}{k}}$

$\Rightarrow \quad T = 2t = \dfrac{2\pi}{3}\sqrt{\dfrac{m}{k}}$

Hence, the correct answer is (B).

40. Starting from rest means starting from extreme, so we have $y = A\cos(\omega t)$

At $t = 1$ s, $y = A\cos\omega$

At $t = 2$ s, $y = A\cos(2\omega)$

and at $t = 0$ s, $y = A$

$\Rightarrow \quad a = A - A\cos\omega$ ...(1)

and $a + b = A - A\cos 2\omega$ ...(2)

$\cos\omega = 1 - \dfrac{a}{A}$ {from equation (1)}

$\Rightarrow \quad a + b = A - A(2\cos^2\omega - 1) = A - A\left[2\left(1 - \dfrac{a}{A}\right)^2 - 1\right]$

Solving this equation, we get

$$A = \frac{2a^2}{3a - b}$$

Hence, the correct answer is (C).

41. If the two limbs of the U-tube make angles θ_1 and θ_2 respectively with the vertical, then the time period of oscillation of the liquid is given by

$$T = 2\pi\sqrt{\frac{m}{S\rho(\cos\theta_1 + \cos\theta_2)g}}$$

In the given case, we have $\theta_1 = 0°$ and $\theta_2 = 60°$, so we get

$$T = 2\pi\sqrt{\frac{m}{S\rho(\cos 0° + \cos 60°)g}} = 2\pi\sqrt{\frac{2m}{3\rho gS}}$$

Hence, the correct answer is (B).

42. $F = Kx + K(x\cos 45)\cos 45 + K(x\cos 45)\cos 45 = 2Kx$

$\Rightarrow \quad K_{eff} = 2K$

$\Rightarrow \quad T = 2\pi\sqrt{m/2K}$

Hence, the correct answer is (B).

43. Since, $\omega_s t - \omega_\ell t = 2\pi$

$\Rightarrow \quad \dfrac{2\pi}{T}t - \dfrac{2\pi}{4T}t = 2\pi$ $\{\because T \propto \sqrt{\ell}\}$

$\Rightarrow \quad t\left(1 - \dfrac{1}{4}\right) = T$

$\Rightarrow \quad t = 4T/3$

Hence, the correct answer is (C).

44. Given that, $E_1 = \dfrac{1}{2}kx^2$ and $E_2 = \dfrac{1}{2}ky^2$

$\Rightarrow \quad E = \dfrac{1}{2}k(x + y)^2 = \dfrac{1}{2}k\left(\sqrt{\dfrac{2E_1}{k}} + \sqrt{\dfrac{2E_2}{k}}\right)^2$

$\Rightarrow \quad E = \left(\sqrt{E_1} + \sqrt{E_2}\right)^2$

Hence, the correct answer is (D).

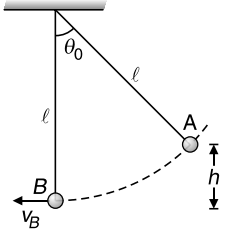
45. If ℓ_0 is the extension of spring in equilibrium, then $K\ell_0 = mg$. If y is downward displacement of mass from equilibrium, then spring is stretched upward by y. So, restoring force is $F = ma = -Ky$ i.e., $a = -\frac{K}{m}y$.

Hence $\omega^2 = \frac{K}{m}$ and time period $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{K}}$

Hence, the correct answer is (B).

46. Maximum tension is in position B and minimum at A. Let ℓ be the length of string and m the mass of bob. Then

$h = \ell(1-\cos\theta_0)$



Applying Law of Conservation of Energy, we get

$\frac{1}{2}mv^2 = mg\ell(1-\cos\theta_0)$

$\Rightarrow \quad v_B^2 = 2gh = 2g\ell(1-\cos\theta_0)$

Also, $T_A = mg\cos\theta_0$ ...(1)

and $T_B - mg = \frac{mv_B^2}{\ell} = 2mg(1-\cos\theta_0)$

$\Rightarrow \quad T_B = mg(3-2\cos\theta_0)$ ...(2)

Given that $T_B = 2T_A$ ...(3)

From equations (1), (2) and (3), we get

$\cos\theta_0 = \frac{3}{4}$

Hence, the correct answer is (B).

47. Given that, $v^2 = \frac{1}{4}(5^2 - x^2)$

Comparing with $v^2 = \omega^2(A^2 - x^2)$, we get

$\omega = \frac{1}{2}$ rads^{-1}

$\Rightarrow \quad T = 4\pi$

Hence, the correct answer is (C).

48. Since, $T = 2\pi\sqrt{\frac{\ell}{g}}$

$\Rightarrow \quad \frac{T'}{T} = \sqrt{\frac{g}{g'}}$ ...(1)

Since, $g' = \frac{g}{\left(1+\frac{h}{R}\right)^2}$

$\Rightarrow \quad \frac{g'}{g} = \frac{1}{\left(1+\frac{h}{R}\right)^2}$

$\Rightarrow \quad \frac{T'}{T} = \left(1+\frac{h}{R}\right)$ {From (1)}

$\Rightarrow \quad T' = T\left(1+\frac{h}{R}\right)$

Since, $T' > T$, the clock will lose the time, so

$\Delta T = T' - T = T\left(\frac{h}{R}\right)$

So, time lost in $t = 1$ day is

$\Delta t = \left(\frac{\Delta T}{T'}\right)t = \frac{t\left(\frac{h}{R}\right)}{\left(1+\frac{h}{R}\right)} \approx t\left(\frac{h}{R}\right)$

$\Rightarrow \quad \Delta t = \frac{(24\times3600)(200)}{6.4\times10^6}\text{ s} = 2.7\text{ s}$

Hence, the correct answer is (B).

49. Two springs in parallel give a force constant of $2k$ which is in series with k.

$k_{\text{net}} = \frac{(2k)(k)}{2k+k} = \frac{2k}{3}$

$\Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\frac{k_{\text{net}}}{m}} = \frac{1}{2\pi}\sqrt{\frac{2k}{3m}}$

Hence, the correct answer is (B).

50. Given that, $y = a\sin(\omega t + \theta)$

At $t = 0$, $y = a\sin\theta = 3$ ...(1)

Also, $v = a\omega\cos(\omega t + \theta)$

At $t = 0$, $v = a\omega\cos\theta = 1.5\pi$

$\Rightarrow \quad a(0.5\pi)\cos\theta = 1.5\pi$

$\Rightarrow \quad a\cos\theta = 3$ ...(2)

From equation (1) and (2), we get

$\theta = 45°$ and $a = 3\sqrt{2}$ cm

Hence, the correct answer is (C).

51. Since, $F = ma = -\frac{dU}{dx} = -8\sin(2x)$

$\Rightarrow \quad a = \frac{F}{m} = -8\sin(2x)$ $\{\because\ m = 1\text{ kg}\}$

For small oscillations $\sin(2x) \approx 2x$, so $a = -16x$

Hence the oscillations are simple harmonic in nature

$\Rightarrow \quad T = 2\pi\sqrt{\left|\frac{x}{a}\right|} = 2\pi\sqrt{\frac{1}{16}} = \frac{\pi}{2}$ s

Hence, the correct answer is (C).

52. $v^2 = \omega^2(a^2 - x^2)$

Since $v_{\max} = a\omega$

$\Rightarrow \quad v^2 = v_{\max}^2\left(1-\frac{x^2}{a^2}\right)$

$\Rightarrow \quad 2500 = 10000\left(1-\frac{x^2}{a^2}\right)$

CHAPTER 3

$$\Rightarrow \quad x = \frac{\sqrt{3}}{2}a = 5\sqrt{3} \text{ cm}$$

Hence, the correct answer is (B).

53. At $x = \frac{\sqrt{3}}{2}A$, kinetic energy increases by $\frac{1}{2}m\omega^2A^2$, so the new total energy is

$$E_{new} = E + \frac{1}{2}m\omega^2A^2$$

$$\Rightarrow \quad E_{new} = \frac{1}{2}m\omega^2A^2 + \frac{1}{2}m\omega^2A^2 = m\omega^2A^2$$

If A_{new} is the new amplitude, then

$$\frac{1}{2}m\omega^2A_{new}^2 = m\omega^2A^2$$

$$\Rightarrow \quad A_{new} = \sqrt{2}A$$

Hence, the correct answer is (C).

54. Block of mass m_2 shorts off carrying some kinetic energy away from the system.

Applying Law of Conservation of Mechanical Energy

$$\begin{pmatrix}\text{Potential Energy}\\ \text{of Spring}\end{pmatrix} = \begin{pmatrix}\text{Maximum Kinetic}\\ \text{Energy of Blocks}\end{pmatrix}$$

$$\Rightarrow \quad \frac{kd^2}{2} = (m_1 + m_2)\frac{v^2}{2} \qquad \{k = \text{force constant of spring}\}$$

$$\Rightarrow \quad v^2 = \frac{kd^2}{m_1 + m_2}$$

With m_1 alone on the spring, we have

$$\begin{pmatrix}\text{Maximum Potential}\\ \text{Energy}\end{pmatrix} = \begin{pmatrix}\text{Maximum Kinetic}\\ \text{Energy of } m_1\end{pmatrix}$$

$$\Rightarrow \quad \frac{1}{2}kA^2 = \frac{1}{2}m_1v^2$$

$$\Rightarrow \quad kA^2 = \frac{km_1d^2}{m_1 + m_2}$$

$$\Rightarrow \quad A = d\sqrt{\frac{m_1}{m_1 + m_2}}$$

Hence, the correct answer is (A).

55. From Trigonometry

$$\cos^2\theta - \sin^2\theta = \cos(2\theta)$$

$$\Rightarrow \quad \cos^2\left(\frac{\pi t}{2}\right) - \sin^2\left(\frac{\pi t}{2}\right) = \cos(\pi t)$$

$$\Rightarrow \quad y = 0.4\cos(\pi t)$$

which indicates that motion is harmonic with amplitude 0.4 m.

Hence, the correct answer is (B).

56. Equivalent spring constant of the combination is

$$\frac{1}{k_{eq}} = \frac{1}{k} + \frac{1}{2k} = \frac{3}{2k}$$

$$\Rightarrow \quad k_{eq} = \frac{2k}{3}$$

The reduced mass is $\mu = \frac{m_1m_2}{m_1 + m_2} = \frac{m}{2}$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{\mu}{k_{eq}}} = 2\pi\sqrt{\frac{m/2}{2k/3}} = 2\pi\sqrt{\frac{3m}{4k}}$$

Hence, the correct answer is (B).

57. Amplitude of motion $A = 2x_0$. Time to cover from extreme position to mean position (i.e., from compressed position to normal position) is $\frac{T}{4}$. Time taken to cover distance x_0 from mean position is calculated using $y = A\sin(\omega t)$

$$\Rightarrow \quad x_0 = 2x_0\sin\left(\frac{2\pi t}{T}\right)$$

$$\Rightarrow \quad \frac{2\pi t}{T} = \frac{\pi}{6}$$

$$\Rightarrow \quad t = \frac{T}{12}$$

So, total time taken to hit the wall

$$t = \frac{T}{4} + \frac{T}{12} = \left(\frac{3+1}{12}\right)T = \frac{T}{3} = \frac{1}{3}\left(2\pi\sqrt{\frac{m}{K}}\right)$$

Hence, the correct answer is (D).

58. Since, $k_{eff} = 2k$, so $T = 2\pi\sqrt{\frac{M}{2k}}$

Period (or k_{eff}) is independent of θ.

Hence, the correct answer is (A).

59. Time period of pendulum is given by

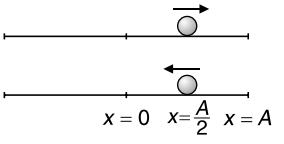
$$T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{1}{\pi^2}} = 2 \text{ s}$$

On the right side, it completes half an oscillation, whereas on the left side, it only goes from mean to half the amplitude and comes back. Therefore, time of oscillation is

$$T' = \frac{T}{2} + 2\left(\frac{T}{12}\right) = \frac{2}{3}T = \frac{4}{3} \text{ s}$$

Hence, the correct answer is (B).

60. The displacement time and velocity-time equations in this situation can be written as



$$x = A\sin(\omega t + \phi)$$

$$\Rightarrow \quad v = A\omega\cos(\omega t + \phi)$$

at $t = 0$, $x = \frac{A}{2}$

$$\Rightarrow \quad \phi = \frac{\pi}{6} \text{ and } \frac{5\pi}{6}$$

If $\phi = \frac{\pi}{6}$, displacement and velocity both are positive at $t = 0$.

When, $\phi = \frac{5\pi}{6}$, displacement is positive but velocity is negative. Displacement-time equations of the two particles can be written as

$$x_1 = A\sin\left(\omega t + \frac{\pi}{6}\right) \text{ and } x_2 = A\sin\left(\omega t + \frac{5\pi}{6}\right)$$

$$\Rightarrow \quad \Delta\phi = \frac{5\pi}{6} - \frac{\pi}{6} = \frac{2\pi}{3}$$

Hence, the correct answer is (D).

61. In SHM $v_{max} = a\omega$

Since, $\langle v \rangle = \dfrac{\text{distance travelled in one oscillation}}{\text{time period}}$

$$\Rightarrow \quad \langle v \rangle = \frac{4a}{T} = \frac{4a}{2\pi} = \frac{2a\omega}{\pi} = \frac{2v_{max}}{\pi}$$

Hence, the correct answer is (D).

62. Since, $T = 2\pi\sqrt{\dfrac{I}{mgd}}$

where $I = I_{cm} + md^2 = mR^2 + mR^2 = 2mR^2$ is the moment of inertial if the ring about the point of suspension and $d = R$ is the distance of separation between the point of suspension and the centre of mass of the disc.

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{2mR^2}{mgR}} = 2\pi\sqrt{\frac{2R}{g}}$$

Hence, the correct answer is (B).

63. Force constant of a spring is inversely proportional to its natural length i.e., $k \propto \dfrac{1}{l}$ i.e., force constant of two halves will become $2k$ each, where k is force constant of complete spring, hence $k_{eff} = 2k + 2k = 4k$.

Since $T \propto \dfrac{1}{\sqrt{k}}$, so $T' = \dfrac{T}{2}$

Hence, the correct answer is (C).

64. $a_{max} = \omega^2 A$ and $v_{max} = A\omega$

If ω is doubled and amplitude is halved, v_{max} remains constant while a_{max} becomes two times.

Hence, the correct answer is (C).

65. Since, $\omega = \dfrac{v}{R} = 4 \text{ rads}^{-1}$

$$\Rightarrow \quad \frac{\text{displacement}}{\text{acceleration}} = \frac{1}{\omega^2} = \frac{1}{16}(\text{second})^2$$

Hence, the correct answer is (A).

66. When the bead is attached at the middle of the spring of length L, effectively the spring behaves as a parallel combination of two springs each of length $\dfrac{L}{2}$ and spring constant $2K$. Therefore, effective spring constant is

$$K_{eff} = 200 + 200 = 400 \text{ Nm}^{-1}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{40/1000}{400}} = \frac{\pi}{50}$$

Hence, the correct answer is (A).

67. Since $T = 2\pi\sqrt{\dfrac{M}{k}}$ and $\dfrac{5T}{4} = 2\pi\sqrt{\dfrac{M+m}{k}}$

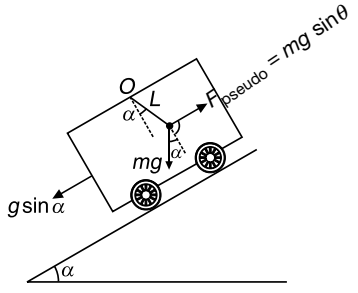
$$\Rightarrow \quad \frac{5}{4} = \sqrt{\frac{M+m}{M}}$$

$$\Rightarrow \quad \frac{25}{16} = 1 + \frac{m}{M}$$

$$\Rightarrow \quad \frac{m}{M} = \frac{9}{16}$$

Hence, the correct answer is (C).

68. Since the vehicle is moving down the frictionless incline, so the acceleration of the vehicle is $g\sin\alpha$ along the incline. So, the bob will experience a pseudo force $mg\sin\theta$ in the backward direction, as shown. The weight of the bob is mg acting vertically downwards. If a_{net} is the net acceleration of the bob then,



$$a_{net} = \sqrt{g^2 + g^2\sin^2\alpha + 2(g)(g\sin\alpha)\cos(90+\alpha)}$$

$$\Rightarrow \quad a_{net} = \sqrt{g^2 + g^2\sin^2\alpha - 2g^2\sin^2\alpha}$$

$$\Rightarrow \quad a_{net} = \sqrt{g^2 - g^2\sin^2\alpha} = g\sqrt{1-\sin^2\alpha}$$

$$\Rightarrow \quad a_{net} = g\cos\alpha$$

Further $T = 2\pi\sqrt{\dfrac{L}{a_{net}}}$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{L}{g\cos\alpha}}$$

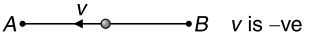
As a short cut if we think $\alpha \to 0$, then

$$T = 2\pi\sqrt{\frac{L}{g}}$$

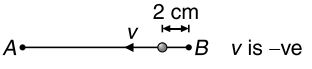
i.e., the time period of a simple pendulum

Hence, the correct answer is (B).

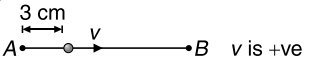
69. Since direction from A to B is positive, therefore for position given in a, we have



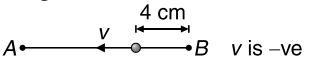
for position given in b, we have



for position given in c, we have



and for position given in d, we have



Hence, the correct answer is (B).

70. If y is downward displacement of mass, then stretching of spring $= 2y$

Tension $T = F = K(2y)$

CHAPTER 3

Also, $mg = 2T$

Restoring force $F' = 2T = 4Ky$

$\Rightarrow \quad ma = -4Ky$

$\Rightarrow \quad m\ddot{y} = -4Ky$

$\Rightarrow \quad \ddot{y} + \left(\frac{4K}{m}\right)y = 0$

$\Rightarrow \quad t = 2\pi\sqrt{\frac{y}{|\ddot{y}|}} = 2\pi\sqrt{\frac{m}{4K}} = \pi\sqrt{\frac{m}{K}}$

Hence, the correct answer is (C).

71. Since, we know that time period of a simple pendulum is proportional to the square root of the length.

$\Rightarrow \quad T \propto \sqrt{\ell}$

$\Rightarrow \quad \frac{T_s}{T_\ell} = \frac{1}{4}$

$\Rightarrow \quad 4T_s = 1\,T_\ell$

So, four oscillations of shorter pendulum equals one oscillation of longer pendulum.

Hence, the correct answer is (B).

72. Since, the sphere floats half immersed in the liquid of density ρ, so we have

$$\begin{pmatrix}\text{Mass of}\\ \text{the sphere}\end{pmatrix} = \begin{pmatrix}\text{Mass of liquid}\\ \text{displaced}\end{pmatrix} = \frac{2}{3}\pi R^3\rho$$

When pushed down by a small distance y, excess upthrust (restoring force) generated is

$$F = \left(\pi R^2 y\right)\rho g$$

Therefore, force constant of SHM is

$$k = \frac{F}{y} = \left(\pi R^2\right)\rho g$$

Hence, frequency of oscillations is

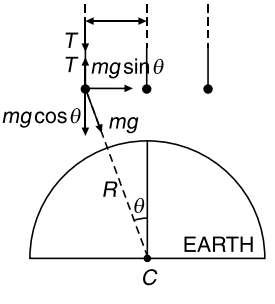
$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{\pi R^2\rho g}{\frac{2}{3}\pi R^3\rho}} = \frac{1}{2\pi}\sqrt{\frac{3g}{2R}}$$

Hence, the correct answer is (B).

73. $T = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{5}{10}} = \sqrt{2}\pi$

Hence, the correct answer is (D).

74. If the length of the pendulum is infinite, the bob would move along the arc of a circle of infinite radius, that is, along a straight line. If the amplitude of oscillation is small compared to the radius of the earth, the bob will always be at a distance R from the centre of the earth.



Restoring force F is given by

$$F = -mg\sin\theta$$

$\Rightarrow \quad m\ddot{x} = -mg\sin\theta$

$\Rightarrow \quad \ddot{x} + \left(\frac{g}{R}\right)x = 0 \qquad \left\{\because \sin\theta \simeq \theta = \frac{x}{R}\right\}$

$\Rightarrow \quad T = 2\pi\sqrt{\frac{x}{|\ddot{x}|}} = 2\pi\sqrt{\frac{R}{g}}$

Hence, the correct answer is (B).

75. At $t = 0$, the particle is at mean position, so displacement of the particle at time t is

$$x = a\sin(\omega t) = a\sin\left(\frac{2\pi}{8}t\right) = a\sin\left(\frac{\pi}{4}t\right)$$

Let x_1 be the displacement of particle from $0 \to 1$ s

and x_2 be the displacement of particle from $0 \to 2$ s

Then, $x_1 = a\sin\frac{\pi}{4} = \frac{a}{\sqrt{2}}$ and $x_2 = a$

Displacement of the particle from 1 s to 2 s is

$$x_2 - x_1 = a\left(1 - \frac{1}{\sqrt{2}}\right)$$

$\Rightarrow \quad \frac{x_1}{x_2 - x_1} = \frac{a/\sqrt{2}}{a\left(1 - \frac{1}{\sqrt{2}}\right)} = \frac{1}{\sqrt{2} - 1} = \sqrt{2} + 1$

Hence, the correct answer is (D).

76. Let speed of each block just before colliding be v. Then on colliding, if v' be the velocity of the combined mass, then by Law of Conservation of Linear Momentum, we have

$$2mv - mv = (2m + m)v'$$

$\Rightarrow \quad v' = \frac{v}{3}$

Since, this maximum velocity of the combined mass is one-third of that when amplitude was R, so the new amplitude will be $\frac{R}{3}$ because the angular frequency remains unchanged.

Hence, the correct answer is (C).

77. At the mean position a particle possesses minimum potential energy and maximum kinetic energy.

Hence, the correct answer is (C).

78. $(\text{K.E.})_{\max} = \frac{1}{2}(2)(100)^2\left(\frac{6}{100}\right)^2$

$\Rightarrow \quad (\text{K.E.})_{\max} = 36\text{ J}$

Hence, the correct answer is (B).

79. $v^2 = \omega^2\left(a^2 - x^2\right)$

$\Rightarrow \quad v^2 + \omega^2 x^2 = \omega^2 a^2$

$\Rightarrow \quad \frac{v^2}{a^2\omega^2} + \frac{x^2}{a^2} = 1$

which is the equation of an ellipse

Hence, the correct answer is (C).

80. Since $T \propto \ell^{\frac{1}{2}}$, so we have $\frac{\Delta T}{T} = \frac{1}{2}\left(\frac{\Delta\ell}{\ell}\right) = 1\%$

$$\Rightarrow \quad \frac{T'-T}{T}=\frac{1}{100}$$

$$\Rightarrow \quad T'=T+0.01T=1.01T$$

Hence, the correct answer is (D).

81. Since, $g'=\frac{GM'}{R'^2}=\frac{G(2M_e)}{(2R_e)^2}=\frac{g}{2}$ $\left\{\because g=\frac{GM_e}{R_e^2}\right\}$

$$\Rightarrow \quad T'=2\pi\sqrt{\frac{\ell}{g'}}=\sqrt{2}\left(2\pi\sqrt{\frac{\ell}{g}}\right)=\sqrt{2}T$$

For a second's pendulum, we have $T=2$ s, so

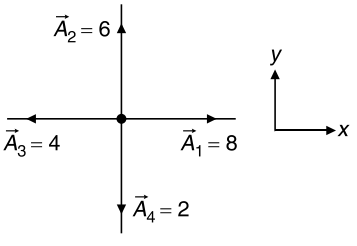
$$\Rightarrow \quad T'=2\sqrt{2}\text{ s}$$

Hence, the correct answer is (D).

82. Initial phase of the block is zero.
Hence, the correct answer is (B).

83. For a spring $T=2\pi\sqrt{\frac{m}{k}}$. Since liquid is non-viscous, hence the time period remains unaltered.
Hence, the correct answer is (C).

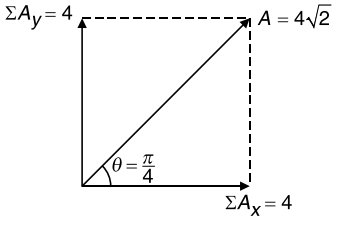
84. The resulting amplitude and corresponding phase difference can be calculated by vector method as follows:



$$\Sigma A_x=8-4=4$$

and $\Sigma A_y=6-2=4$

Therefore, resulting amplitude is $4\sqrt{2}$ and phase difference with x_1 is $\phi=\frac{\pi}{4}$.



Hence, the correct answer is (D).

85. Let two SHMs be represented as

$$x_1=a\sin(2\pi ft)\text{ and }x_2=a\sin(2\pi ft+\phi)$$

Then, separation between the particles is

$$x=x_2-x_1=a\sin(2\pi ft+\phi)-a\sin(2\pi ft)$$

Since, $\sin C-\sin D=2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$$\Rightarrow \quad x=2a\cos\left(2\pi ft+\frac{\phi}{2}\right)\sin\left(\frac{\phi}{2}\right)$$

Given that the maximum separation between the particles is $a\sqrt{2}$, so we have

$$2a\sin\left(\frac{\phi}{2}\right)=a\sqrt{2}$$

$$\Rightarrow \quad \sin\left(\frac{\phi}{2}\right)=\frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \phi=90^\circ$$

Hence, the correct answer is (B).

86. For equilibrium of $(M+m)$, we have

$$x_1=\frac{(M+m)g}{k}$$

and for equilibrium of m, we have

$$x_2=\frac{mg}{k}$$

So, amplitude of oscillation is given by

$$A=x_1-x_2=\frac{Mg}{k}$$

Hence, the correct answer is (A).

87. Since, $F=\left(\frac{YA}{\ell}\right)x$, so effective force constant of wire is $k=\frac{YA}{\ell}$ and hence $T=2\pi\sqrt{\frac{m}{k}}=2\pi\sqrt{\frac{m\ell}{YA}}$.

Hence, the correct answer is (C).

88. Since, $0.5=2\pi\sqrt{\frac{m}{k}}$ and $0.6=2\pi\sqrt{\frac{m+m'}{k}}$

$$\Rightarrow \quad \frac{0.36}{4\pi^2}=\frac{m}{k}+\frac{m'}{k}=\frac{0.25}{4\pi^2}+\frac{m'}{k}$$

$$\Rightarrow \quad \frac{m'}{k}=\frac{0.11}{4\pi^2}$$

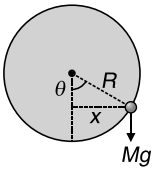
Let x be the additional extension, then

$$m'g=kx$$

$$\Rightarrow \quad x=\frac{m'g}{k}=\frac{(0.11)(9.8)}{4\pi^2}=2.69\text{ cm}$$

Hence, the correct answer is (B).

89. **Method-1 (Using restoring torque method)**
Let the arrangement be given a small angular displacement θ from the mean position as shown in Figure.



The restoring torque acting on the system is

$$\tau=-mgR\sin\theta=-mgR\theta$$

(because for small θ, we have $\sin\theta\approx\theta$)

$$\Rightarrow \quad I\left(\frac{d^2\theta}{dt^2}\right)=-mgR\theta \qquad \ldots(1)$$

CHAPTER 3

where I is the moment of inertia of the system about the axis of rotation, given by

$$I = \frac{1}{2}MR^2 + mR^2 = \frac{1}{2}(2)(5)^2 + 1(5)^2 = 50 \text{ kgm}^2$$

From equation (1), we get

$$\omega = \frac{2\pi}{T} = \sqrt{\frac{mgR}{I}} = \sqrt{\frac{(1)(10)(5)}{50}} = 1 \text{ rads}^{-1}$$

$$\Rightarrow \quad T = (2\pi) \text{ second}$$

Method-2 (Using the concept of Physical Pendulum)

Distance of centre of mass of the system from the axis of oscillation is

$$d = \frac{(2)(0)+(1)(5)}{2+1} = \frac{5}{3} \text{ m}$$

Also, MI of the system about axis of rotation is

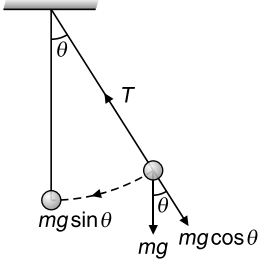
$$I = \frac{1}{2}MR^2 + mR^2 = \frac{1}{2}(2)(5)^2 + 1(5)^2 = 50 \text{ kgm}^2$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{I}{Mgd}} = 2\pi\sqrt{\frac{50}{(3)(10)(5/3)}} = 2\pi \text{ s}$$

Hence, the correct answer is (A).

90. Initially when water starts draining out of the sphere its cg falls down i.e., the effective length of the pendulum increases so that the time period is increased. As the level has considerably fallen the effective cg starts moving towards the centre thus indicating a decrease in the effective length and hence the time period. Finally, when the sphere becomes empty the CG moves to the centre and the new time period will be same as that of the completely filled sphere.
Hence, the correct answer is (D).

91. Free body diagram of pendulum is shown in Figure.



Since $T = mg\cos\theta$, net force is $F_{net} = mg\sin\theta$

For small θ, $\sin\theta \approx \tan\theta \approx \theta$, so acceleration is

$$\Rightarrow \quad a = g\sin\theta \approx g\theta = (10)\left(\frac{0.2}{4}\right) = 0.5 \text{ ms}^{-2}$$

Hence, the correct answer is (C).

92. $T = 2\pi\sqrt{\frac{I}{mgl}}$, where, $I = \frac{3}{2}mR^2$ and $l = R$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{3R}{2g}}$$

Hence, the correct answer is (D).

93. Time period of linear oscillations of a spring mass system is independent of any constant force acting on the block.
Hence, the correct answer is (D).

94. On the planet $s = \frac{1}{2}at^2$ i.e., $8 = \frac{1}{2}a(4)$

$$\Rightarrow \quad a = 4 \text{ ms}^{-2}$$

So, $T = 2\pi\sqrt{\frac{\ell}{a}} = 2\pi\sqrt{\frac{1}{4}} = \pi \text{ s}$

Hence, the correct answer is (B).

95. Energy of oscillation is $E = \alpha A^4$

So, kinetic energy of mass at $x = x$ is

$$K = E - U = \alpha(A^4 - X^4)$$

Given, $K = 3U$

$$\Rightarrow \quad \alpha(A^4 - x^4) = 3\alpha x^4$$

$$\Rightarrow \quad x = \pm\frac{A}{\sqrt{2}}$$

Hence, the correct answer is (B).

96. Beyond point P, length of pendulum becomes $\frac{\ell}{4}$.

Since $T \propto \sqrt{\ell}$, so beyond P time period will become $T = \frac{T}{2}$.
Hence desired time is

$$t = \frac{T}{2} + \frac{T'}{2} = \frac{T}{2} + \frac{T}{4} = \frac{3T}{4}$$

Hence, the correct answer is (B).

97. Mean position is $\frac{6+(-2)}{2} = 2$ cm

Amplitude is $a = \frac{6-(-2)}{2} = 4$ cm

Therefore, equation of SHM is

$$x = 2 + 4\sin(\omega t + \phi)$$

where $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.5} = 4\pi \text{ rads}^{-1}$

Now, at $t = 0$, $x = 2 + 4\sin\phi = 4$ {given}

$$\Rightarrow \quad \sin\phi = \frac{1}{2}$$

$$\Rightarrow \quad \phi = 30°, 150°, \ldots\ldots$$

Also, velocity is $v = 4\omega\cos(\omega t + \phi)$

At $t = 0$, $v = 4\omega\cos\phi =$ positive {given}

So, ϕ must lie in first or fourth quadrant, i.e. $\phi = 30°$

$$\Rightarrow \quad x = 2 + 4\sin(4\pi t + 30°)$$

Also, we see that at $x = 4$ cm, acceleration of particle will be towards mean position, i.e. towards negative x-direction.
Hence, the correct answer is (A).

98. Force constant is $k = \frac{F}{x} = \frac{\pi^2}{16}$

So, the angular frequency of SHM is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{\pi^2/16}{4}} = \frac{\pi}{8} \text{ rads}^{-1}$$

Since, particle passes through mean position at $t = 2$ s, hence velocity of particle is given by

$$v = A\omega\cos[\omega(t-2)]$$

According to the problem, we have

$$4\sqrt{2} = A\left(\frac{\pi}{8}\right)\left|\cos\left[\frac{\pi}{8}(10-2)\right]\right| = \frac{\pi A}{8}|\cos\pi|$$

$$\Rightarrow \quad A = \frac{32\sqrt{2}}{\pi}\ \text{m}$$

Hence, the correct answer is (A).

99. Since, the particle starts from its equilibrium position i.e., mean position, so $x = A\sin(\omega t)$, where A is the amplitude.

At time $t = \frac{T}{12}$, its displacement from the mean position is

$$x = A\sin\left(\frac{2\pi}{T}\frac{T}{12}\right) = \frac{A}{2}$$

Now, $KE = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(A^2 - x^2) = \frac{3}{4}\left(\frac{1}{2}m\omega^2A^2\right)$

and $PE = \frac{1}{2}m\omega^2x^2 = \frac{1}{4}\left(\frac{1}{2}m\omega^2A^2\right)$

$$\Rightarrow \quad \frac{KE}{PE} = \frac{3}{1}$$

Hence, the correct answer is (B).

100. Velocity of particle performing SHM is

$$v = \omega\sqrt{A^2 - x^2}$$

$$v^2 = \omega^2A^2 - \omega^2x^2 \qquad \ldots(1)$$

Acceleration of particle performing SHM is

$$a = -\omega^2x \text{ i.e. } x = -\frac{a}{\omega^2}$$

Substituting the value x in equation (1), we get

$$v^2 = \omega^2A^2 - \omega^2\left(\frac{a^2}{\omega^4}\right) = -\frac{a^2}{\omega^2} + \omega^2A^2$$

Therefore, graph between v^2 and a^2 is a straight line with negative slope and positive intercept.

Hence, the correct answer is (D).

101. When the plank is displaced downwards by x upthrust due to lower liquid will increase while due to upper liquid will decrease. The difference in these two upthrust will become net restoring force. Thus, net restoring force

$$F = -(\text{Extra Upthrust})$$

$$\Rightarrow \quad F = -(Ax)(2\rho - \rho)g = -(\rho Ag)x$$

$$\Rightarrow \quad \text{Acceleration } a = \frac{F}{m} = -\left(\frac{\rho Ag}{m}\right)x$$

$$\Rightarrow \quad T = 2\pi\sqrt{\left|\frac{x}{a}\right|} = 2\pi\sqrt{\frac{m}{\rho Ag}}$$

Hence, the correct answer is (A).

102. The energy is given by $E = \frac{1}{2}mv^2 + mgy$

$$\Rightarrow \quad E = \frac{1}{2}mv^2 + mg\left(\frac{x^2}{40}\right) \qquad \left\{\because\ y = \frac{x^2}{40}\right\}$$

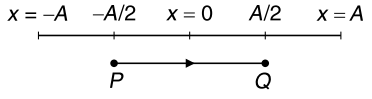
For SHM E = constant, so $\frac{dE}{dt} = 0$

$$\Rightarrow \quad a = -\left(\frac{g}{20}\right)x$$

$$\Rightarrow \quad \omega = \sqrt{\frac{g}{20}}$$

Hence, the correct answer is (C).

103. For minimum time, the particle must move along the path PQ shown in Figure.



Time taken to go from P to Q is

$$t = \frac{T}{12} + \frac{T}{12} = \frac{T}{6} = \frac{2\pi/\omega}{6} = \frac{\pi}{3\omega}$$

Hence, the correct answer is (A).

104. Since $T = 2\pi\sqrt{\frac{L}{g}}$, where L is the length of the liquid in one of the limbs. However, if L is taken to be the length of the liquid column then length of liquid in each limb is $\frac{L}{2}$ and in that case, $T = 2\pi\sqrt{\frac{L}{2g}}$.

Also $M = (AL)d$, so $L = \frac{M}{Ad}$.

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{M}{2Agd}}$$

Hence, the correct answer is (D).

105. Since $\omega = 2\pi f = \sqrt{\frac{k}{m}}$, so $k = (2\pi f)^2 m$

Total energy of oscillation is $E = (0.5 + 0.4) = 0.9$ J

$$\Rightarrow \quad 0.9 = \frac{1}{2}kA^2$$

$$\Rightarrow \quad A = \sqrt{\frac{1.8}{k}} = \sqrt{\frac{1.8}{(2\pi f)^2 m}} = \frac{1}{2\pi f}\sqrt{\frac{1.8}{0.2}}$$

$$\Rightarrow \quad A = \frac{1}{2\pi(25/\pi)}\sqrt{\frac{1.8}{0.2}} = \frac{3}{50}\ \text{m} = 6\ \text{cm}$$

Hence, the correct answer is (A).

106. Mean position will be $x = \frac{mg}{k}$

$$\Rightarrow \quad x = \frac{mg}{k} = \frac{8\times10}{200} = \frac{2}{5}\ \text{m} = 0.4\ \text{m}$$

This is also the amplitude of oscillation, so

$$A = 0.4\ \text{m}$$

Now, $v_{max} = A\omega = A\sqrt{\frac{k}{m}}$

$$\Rightarrow \quad v_{max} = (0.4)\sqrt{\frac{200}{8}} = 2\ \text{ms}^{-1}$$

Hence, the correct answer is (B).

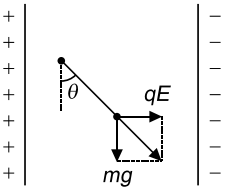
107. The block remains in contact with the spring as long as spring is compressed, i.e. for half the oscillation of system.

Also, $\mu = \dfrac{m_1 m_2}{m_1 + m_2} = \dfrac{2\times 2}{2+2} = 1 \text{ kg}$

$\Rightarrow \quad t = \dfrac{T}{2} = \pi\sqrt{\dfrac{\mu}{k}} = \pi\sqrt{\dfrac{1}{\pi^2}} = 1 \text{ s}$

Hence, the correct answer is (C).

108.



Since, $a_{\text{net}} = \dfrac{1}{m}\sqrt{(qE)^2 + (mg)^2} = \sqrt{g^2 + \left(\dfrac{qE}{m}\right)^2}$

$\Rightarrow \quad T = 2\pi\sqrt{\dfrac{L}{a_{\text{net}}}}$

Hence, the correct answer is (D).

109. Since, $\dfrac{1}{2}kA^2 = (9-5) = 4 \text{ J}$

$\Rightarrow \quad k = \dfrac{8}{A^2} = \dfrac{8}{(0.01)^2} = 8\times 10^4 \text{ Nm}^{-1}$

$\Rightarrow \quad T = 2\pi\sqrt{\dfrac{m}{k}} = 2\pi\sqrt{\dfrac{2}{8\times 10^4}} = \dfrac{\pi}{100} \text{ s}$

Hence, the correct answer is (D).

110. Since, $T' = 2\pi\sqrt{\dfrac{l}{g_{\text{eff}}}} = 2\pi\sqrt{\dfrac{l}{g\left(1 - \dfrac{\rho_{\text{liquid}}}{\rho_{\text{bob}}}\right)}}$

$\Rightarrow \quad T' = T\sqrt{\dfrac{\rho_{\text{bob}}}{\rho_{\text{bob}} - \rho_{\text{liquid}}}}$

Given that $T' = 2T$, we get

$$\rho_{\text{body}} = \frac{4}{3}\rho_{\text{water}} = 1.33 \text{ gcc}^{-1}$$

Hence, the correct answer is (B).

111. Since $E = \dfrac{1}{2}mA^2\omega^2$, so $E \propto (A\omega)^2$

$\Rightarrow \quad (A_1\omega_1)^2 = (A_2\omega_2)^2$

$\Rightarrow \quad A_1\omega_1 = A_2\omega_2$

$\Rightarrow \quad 4\times 10 = 5\times\omega$

$\Rightarrow \quad \omega = 8 \text{ unit}$

Hence, the correct answer is (D).

112. At $t = 0$, $x = \dfrac{A}{2}$

$\Rightarrow \quad \dfrac{A}{2} = A\sin\phi$

$\Rightarrow \quad \sin\phi = \dfrac{1}{2}$

$\Rightarrow \quad \phi = \dfrac{\pi}{6}$ OR $\phi = \dfrac{5\pi}{6}$

Since, $v = \dfrac{dx}{dt} = A\omega\cos(\omega t + \phi)$

At $t = 0$, $v = A\omega\cos(\omega t + \phi)$

At $t = 0$, $v = A\omega\cos\phi$

Now, $\cos\left(\dfrac{\pi}{6}\right) = \dfrac{\sqrt{3}}{2}$

and $\cos\left(\dfrac{5\pi}{6}\right) = -\dfrac{\sqrt{3}}{2}$

Since, v is negative at $t = 0$, so ϕ must be $\dfrac{5\pi}{6}$.

Hence, the correct answer is (D).

113. $T = 2\pi\sqrt{\dfrac{\ell}{g_{\text{eff}}}}$

where, $g_{\text{eff}} = \left(1 - \dfrac{1}{2}\right)g = \dfrac{g}{2}$

$\Rightarrow \quad T' = \sqrt{2}T = 2\sqrt{2} \text{ s}$

Hence, the correct answer is (C).

114. If L be the length of the pendulum, then

$$T_1 = 2\pi\sqrt{\frac{L}{g}},\ T_2 = 2\pi\sqrt{\frac{L}{g+a}},\ T_3 = 2\pi\sqrt{\frac{L}{g-a}}$$

$\Rightarrow \quad \dfrac{1}{T_2^2} + \dfrac{1}{T_3^2} = \dfrac{1}{4\pi^2}\left(\dfrac{g+a}{L}\right) + \dfrac{1}{4\pi^2}\left(\dfrac{g-a}{L}\right)$

$\Rightarrow \quad \dfrac{1}{T_2^2} + \dfrac{1}{T_3^2} = 2\dfrac{1}{4\pi^2}\dfrac{g}{L} = \dfrac{2}{T_1^2}$

$\Rightarrow \quad T_1 = \sqrt{2}\dfrac{T_2 T_3}{\sqrt{T_2^2 + T_3^2}}$

Hence, the correct answer is (C).

115. Net weight W' of pendulum is $W' = W - U$

$$W' = V\rho g - \frac{V\rho g}{10} = \frac{9}{10}W$$

So, $g_{\text{eff}} = \dfrac{9}{10}g$

$\Rightarrow \quad T' = 2\pi\sqrt{\dfrac{l}{g'}} = \sqrt{\dfrac{10}{9}}T$

Hence, the correct answer is (C).

116. Displacement-time equation of the particle will be

$$x = A\cos(\omega t)$$

So, $x_1 = A\cos\omega$, $x_2 = A\cos(2\omega)$ and $x_3 = A\cos(3\omega)$

Now, $\dfrac{x_1 + x_3}{2x_2} = \dfrac{A(\cos\omega + \cos(3\omega))}{2A\cos(2\omega)}$

$\Rightarrow \quad \dfrac{x_1 + x_3}{2x_2} = \dfrac{2A\cos(2\omega)\cos\omega}{2A\cos(2\omega)} = \cos\omega$

$$\Rightarrow \quad \omega = \cos^{-1}\left(\frac{x_1 + x_3}{2x_2}\right) = \frac{2\pi}{T}$$

$$\Rightarrow \quad T = \frac{2\pi}{\theta} \text{ where, } \theta = \cos^{-1}\left(\frac{x_1 + x_3}{2x_2}\right)$$

Hence, the correct answer is (A).

117. At $t = 0$, $x = A\sin\left(\frac{\pi}{6}\right) - A\cos\left(\frac{\pi}{6}\right) = -\text{ve}$

So, the acceleration is positive. Similarly, we can find sign of v. Now $\frac{dx}{dt}$ at $t = 0$

$$\Rightarrow \quad v\Big|_{t=0} = A\cos\left(\frac{\pi}{6}\right) + A\sin\left(\frac{\pi}{6}\right) > 0$$

Hence, the correct answer is (D).

118. Using, $s = \frac{1}{2}at^2 = \frac{1}{2}(g\sin\theta)t^2$

For left wedge, $s = \frac{0.2}{\sin 30^\circ} = 0.4$ m and time of ascent or descent is $0.4 = \frac{1}{2}(10\sin 30^\circ)t_1^2$

$$\Rightarrow \quad t_1 = 0.4 \text{ s}$$

Since the energy is conserved, so the block will also rise to a height of 20 cm on the right wedge. Hence for the right wedge, we have time of ascent or time of descent given by

$$\frac{0.2}{\sin 60^\circ} = \frac{1}{2}(10\sin 60^\circ)t_2^2$$

$$\Rightarrow \quad t_2 = \frac{0.4}{\sqrt{3}} \text{ s}$$

So, total time of oscillation is

$$T = 2(t_1 + t_2) = 0.8\left(1 + \frac{1}{\sqrt{3}}\right) \text{ s}$$

Hence, the correct answer is (B).

119. Since, $(v_M)_{\max} = (v_N)_{\max}$

$$\Rightarrow \quad \omega_M A_M = \omega_N A_N$$

$$\Rightarrow \quad \frac{A_M}{A_N} = \frac{\omega_N}{\omega_M} = \sqrt{\frac{k_2}{k_1}} \qquad \left\{\because \omega = \sqrt{\frac{k}{m}}\right\}$$

Hence, the correct answer is (B).

120. Maximum Acceleration $= A\omega^2$

$$\Rightarrow \quad \left(\frac{\pi}{2}\right)^2 = 1\left(\frac{2\pi}{T}\right)^2$$

$$\Rightarrow \quad T^2 = \frac{4\pi^2}{(\pi^2/4)}$$

$$\Rightarrow \quad T = 4 \text{ s}$$

Hence, the correct answer is (B).

121. $\frac{T_\ell}{T_s} = \frac{11}{10} \qquad \{\because T \propto \sqrt{\ell}\}$

$$\Rightarrow \quad 10T_\ell = 11T_s$$

$$\Rightarrow \quad \begin{pmatrix}10 \text{ oscillations} \\ \text{of LONGER} \\ \text{pendulum}\end{pmatrix} \equiv \begin{pmatrix}11 \text{ oscillations} \\ \text{of SHORTER} \\ \text{pendulum}\end{pmatrix}$$

Hence, the correct answer is (B).

122. Maximum velocity is $v_{\max} = A\omega$

$$\Rightarrow \quad v = A\sqrt{\frac{g}{L}}$$

$$\Rightarrow \quad A = v\sqrt{\frac{L}{g}}$$

Hence, the correct answer is (A).

123. Since, $T = 2\pi\sqrt{\frac{\ell}{g}}$

$$\Rightarrow \quad \frac{\Delta T}{T} = -\frac{1}{2}\left(\frac{\Delta g}{g}\right) = -\frac{1}{2}(0.02) = -0.01$$

$$\Rightarrow \quad T' - T = -0.01T = 0.99T$$

Hence, the correct answer is (C).

124. On reaching the steady state the frequency of the driven damped oscillator equals the frequency of the driver.
Hence, the correct answer is (B).

125. If a pendulum is oscillating inside a container, filled with liquid, placed in a lift accelerating down with retardation a_0, then effective gravity for the pendulum bob is

$$g_{\text{eff}} = (g + a_0)\left(1 - \frac{\rho_{\text{liquid}}}{\rho_{\text{bob}}}\right)$$

$$\Rightarrow \quad T' = 2\pi\sqrt{\frac{L}{(g + a_0)\left(1 - \frac{\rho_{\text{liquid}}}{\rho_{\text{bob}}}\right)}}$$

$$\Rightarrow \quad T' = 2\pi\sqrt{\frac{L}{(g + a_0)\left(1 - \frac{\sigma}{\rho}\right)}}$$

Hence, the correct answer is (C).

126. The given equation is a combination of two equations

$$x = x_1 + x_2$$

where $x_1' = 4\sin(\omega t)$

and $x_2 = 3\sin\left(\omega t + \frac{\pi}{3}\right)$

where, $A_1 = 4$ m, $A_2 = 3$ m and $\phi = \frac{\pi}{3}$

$$\Rightarrow \quad A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

$$\Rightarrow \quad A = \sqrt{(4)^2 + (3)^2 + 2(4)(3)\frac{\cos\pi}{3}}$$

$$\Rightarrow \quad A = \sqrt{37} \approx 6 \text{ cm}$$

Hence, the correct answer is (C).

127. Maximum acceleration in SHM is

$$a_{\max} = \omega^2 A$$

this will be provided to the block by friction.

CHAPTER 3

Hence, $a_{max} = \mu g$

$$\Rightarrow \quad A = \frac{\mu g}{\omega^2} = \frac{\left(\frac{1}{2}\right)(10)}{(10)^2} = 0.05 \text{ m} = 5 \text{ cm}$$

Hence, the correct answer is (B).

128. Since, $y = a\sin\omega t + a\cos\omega t$

The amplitude of oscillation is $A = \sqrt{2}a$

Therefore, mechanical energy is

$$E = \frac{1}{2}m\omega^2 A^2 = \frac{1}{2}m\omega^2\left(\sqrt{2}a\right)^2 = m\omega^2 a^2$$

Hence, the correct answer is (B).

129. Since $T = 2\pi\sqrt{\frac{m}{k}}$

$$\Rightarrow \quad 2 = 2\pi\sqrt{\frac{0.1+0.3}{k}} = 2\pi\sqrt{\frac{0.4}{k}} \qquad \ldots(1)$$

When 0.3 kg is also removed, then

$$T' = 2\pi\sqrt{\frac{0.1}{k}} \qquad \ldots(2)$$

$$\Rightarrow \quad \frac{T'}{2} = \sqrt{\frac{0.1}{0.4}} \qquad \{\text{Dividing (2) by (1)}\}$$

$$\Rightarrow \quad T' = 1 \text{ s}$$

Hence, the correct answer is (A).

130. $T = 2\pi\sqrt{\frac{\ell_{\text{immersed}}}{g}} = 2\pi\sqrt{\frac{0.80}{9.8}} = \frac{4\pi}{7}$

Hence, the correct answer is (A).

131. Time period of a simple pendulum of length L comparable to radius of earth R is

$$T = 2\pi\sqrt{\frac{LR}{(L+R)g}}$$

Since, $L = R$ (given)

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{R\times R}{(R+R)g}} = 2\pi\sqrt{\frac{R}{2g}}$$

Hence, the correct answer is (D).

132. Since, $a = -\omega^2 x$

$$\Rightarrow \quad |a| = \omega^2 x$$

$$\Rightarrow \quad 64 = \omega^2(4)$$

$$\Rightarrow \quad \omega = 4$$

$$\Rightarrow \quad T = \frac{\pi}{2} \text{ s}$$

Hence, the correct answer is (A).

133. Let $x = a\cos(\omega t)$

$$\Rightarrow \quad y = a\cos\left(2\omega t + \frac{\pi}{2}\right)$$

$$\Rightarrow \quad y = -a\sin(2\omega t)$$

$$\Rightarrow \quad y = -2a\sin(\omega t)\cos(\omega t)$$

$$\Rightarrow \quad y = -2a\sqrt{1-\frac{x^2}{a^2}}\frac{x}{a}$$

$$\Rightarrow \quad a^2y^2 = 4x^2\left(a^2 - x^2\right)$$

Hence, the correct answer is (C).

134. Maximum acceleration of the platform is

$$a_{max} = \omega^2 A = \left(2\pi \times \frac{2}{\pi}\right)^2 \times 0.1 = 1.6 \text{ ms}^{-2}$$

Hence, reading of the balance can be

$$N = 60(9.8 \pm 1.6) = (588 \pm 96) \text{ newton}$$

i.e., the reading fluctuates between 492 N and 684 N or between 50 kg and 70 kg approximately.

Hence, the correct answer is (D).

135. Since, $U = \frac{1}{2}m\omega^2 A^2$

$$\Rightarrow \quad 1 = \frac{1}{2}(2)\left(\frac{2\pi}{T}\right)^2(0.4)^2$$

$$\Rightarrow \quad T = \frac{4\pi}{5} \text{ s}$$

Hence, the correct answer is (D).

136. Since, $y_1 = \frac{1}{2}\sin(\omega t) + \frac{\sqrt{3}}{2}\cos(\omega t) = \sin\left(\omega t + \frac{\pi}{3}\right)$

and $y_2 = \sin(\omega t) + \cos(\omega t) = \sqrt{2}\sin\left(\omega t + \frac{\pi}{4}\right)$

$$\Rightarrow \quad \Delta\phi = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$$

Hence, the correct answer is (C).

137. $3 = 2\pi\sqrt{\frac{m}{k}}$ and $3+1 = 2\pi\sqrt{\frac{m+1}{k}}$

$$\Rightarrow \quad m = \frac{9}{7} \text{ kg}$$

Hence, the correct answer is (B).

138. Since the potential energy U is given by

$$U = \frac{1}{2}m\omega^2 x^2$$

$$\Rightarrow \quad \frac{dU}{dx} = \frac{1}{2}m\omega^2 2x = m\omega^2 x$$

Hence, slope of the graph is proportional to $m\omega^2$. Since slope for A is more than that for B and both masses are the same, so $\omega_A > \omega_B$.

Hence, the correct answer is (A).

139. For object not to break contact with the board, maximum acceleration equals g. So, we have

$$A\omega^2 = g$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{A}{g}}$$

Hence, the correct answer is (D).

140. Acceleration of a particle executing SHM is given by

$$a = -\omega^2 x$$

So, from the graph, we get $\omega^2 = \tan 45^\circ = 1$

$\Rightarrow \quad T = \dfrac{2\pi}{\omega} = 2\pi \text{ s}$

Hence, the correct answer is (B).

141. Since. $\dfrac{1}{2}m\omega^2x^2 = \dfrac{1}{2}m\omega^2\left(A^2 - x^2\right)$

$\Rightarrow \quad x^2 = A^2 - x^2$

$\Rightarrow \quad x = \pm\dfrac{A}{\sqrt{2}}$

Hence, the correct answer is (C).

142. Since, $U = a + bx^2$

So, restoring force is $F = m\left(\dfrac{d^2x}{dt^2}\right) = -\dfrac{dU}{dx} = -2bx$

$\Rightarrow \quad \dfrac{d^2x}{dt^2} + \left(\dfrac{2b}{m}\right)x = 0$

$\Rightarrow \quad f = \dfrac{\omega}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{2b}{m}}$

Hence, the correct answer is (C).

143. Mean position of the particle is $\dfrac{mg}{k}$ distance below the unstretched position of spring. Therefore, amplitude of oscillation is $A = \dfrac{mg}{k}$

$\Rightarrow \quad \omega = \sqrt{\dfrac{k}{m}} = 2\pi f = 20\pi \qquad \{\because f = 10 \text{ Hz}\}$

$\Rightarrow \quad \dfrac{m}{k} = \dfrac{1}{400\pi^2}$

Therefore, the maximum speed of particle is

$$v_{\max} = A\omega = \left(\frac{g}{400\pi^2}\right)(20\pi) = \frac{1}{2\pi} \text{ ms}^{-1}$$

Hence, the correct answer is (D).

144. Since, $v^2 = \omega^2\left(a^2 - x^2\right)$

$\Rightarrow \quad v_1^2 = \omega^2\left(a^2 - y_1^2\right) \qquad \ldots(1)$

$\Rightarrow \quad v_2^2 = \omega^2\left(a^2 - y_2^2\right) \qquad \ldots(2)$

From (1) and (2), we get

$$T = 2\pi\sqrt{\frac{y_1^2 - y_2^2}{v_2^2 - v_1^2}}$$

Hence, the correct answer is (D).

145. Since, $T = 2\pi\sqrt{\dfrac{m}{k}}$

$\Rightarrow \quad k = \dfrac{4\pi^2 m}{T^2} = \dfrac{(4\pi^2)(2)}{(2\pi)^2} = 2 \text{ Nm}^{-1}$

Now $mg = kx_0$

$\Rightarrow \quad x_0 = \dfrac{mg}{k} = \dfrac{2 \times g}{2} = g \text{ metre} = 10 \text{ m}$

Hence, the correct answer is (A).

146. Total energy, $E = U_0 + \dfrac{1}{2}kA^2$

$\Rightarrow \quad 9 = 5 + \dfrac{1}{2}k(0.01)^2$

$\Rightarrow \quad k = 8 \times 10^4 \text{ Nm}^{-1}$

$\Rightarrow \quad T = 2\pi\sqrt{\dfrac{m}{k}} = 2\pi\sqrt{\dfrac{2}{8 \times 10^4}} = \dfrac{\pi}{100} \text{ s}$

Hence, the correct answer is (D).

147. $U_{\max} = E_{\max} = E_0$

Hence, the correct answer is (C).

148. By Law of Conservation of Mechanical Energy, we have

$$\frac{1}{2}kA^2 = \frac{1}{2}mv^2$$

$\Rightarrow \quad A = v\sqrt{\dfrac{m}{k}} = 6\sqrt{\dfrac{0.1}{10}}$

$\Rightarrow \quad A = \dfrac{6}{10} = 0.6 \text{ m}$

Hence, the correct answer is (A).

149. Velocity of the body executing SHM is

$$v^2 = \omega^2\left(A^2 - y^2\right)$$

Acceleration of the body is

$$|a| = \omega^2 y$$

$\Rightarrow \quad a^2 = \omega^4 y^2$

Given that, $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{\pi} = 2 \text{ rads}^{-1}$ and $v = a$ at 10 cm from mean position, so we have

$$v^2 = a^2$$

$\Rightarrow \quad \omega^2\left(A^2 - y^2\right) = \omega^4 y^2$

$\Rightarrow \quad A^2 - y^2 = \omega^2 y^2$

$\Rightarrow \quad A^2 = \left(\omega^2 + 1\right)y^2 = \left(2^2 + 1\right) \times 10^2 = 500$

$\Rightarrow \quad A = 10\sqrt{5} \text{ cm}$

Hence, the correct answer is (D).

150. $T = 2\pi\sqrt{\dfrac{\ell}{g}}$

$T' = 2\pi\sqrt{\dfrac{\ell}{g+g}} = \dfrac{T}{\sqrt{2}}$

Hence, the correct answer is (D).

151. Given, $y = 5\cos\left(2\pi t + \dfrac{\pi}{3}\right)$

Speed is maximum at mean position, i.e., when

$$y = 0$$

$\Rightarrow \quad 2\pi t + \dfrac{\pi}{3} = \dfrac{\pi}{2}$

$\Rightarrow \quad t = \dfrac{1}{12} \text{ s}$

Hence, the correct answer is (C).

CHAPTER 3

152. $T = 2\pi\sqrt{\frac{x}{g}}$

$\Rightarrow \quad T = 2\pi\sqrt{\frac{9.8/100}{9.8}}$

$\Rightarrow \quad T = \frac{2\pi}{10}$

Hence, the correct answer is (A).

153. On standing the effective length of the pendulum measured from the point of suspension decreases.
Hence, the correct answer is (A).

Multiple Correct Choice Type Questions

1. For equilibrium, we have
Weight of block = Upthrust on the immersed part of the block

$\Rightarrow \quad mg = (Ad)\rho g$

$\Rightarrow \quad m = A\rho d$

$\Rightarrow \quad d = \frac{m}{A\rho} = \frac{m}{\pi r^2 \rho}$

$\begin{pmatrix}\text{Restoring}\\ \text{force}\end{pmatrix} = \begin{pmatrix}\text{Upthrust experienced by the}\\ \text{additional part immersed}\end{pmatrix}$

$\Rightarrow \quad F = -Ax\rho g$

$\Rightarrow \quad m\ddot{x} = -Ax\rho g$

$\Rightarrow \quad (A\sigma\ell)\ddot{x} = -Ax\rho g \qquad \{\because\ m = (A\ell)\sigma\}$

$\Rightarrow \quad \ddot{x} + \frac{\rho g}{\sigma\ell}x = 0$

$\Rightarrow \quad T = 2\pi\sqrt{\frac{\sigma\ell}{\rho g}}$

Since $m = (\pi r^2)\ell\sigma$

$\Rightarrow \quad \ell\sigma = \frac{m}{\pi r^2}$

$\Rightarrow \quad T = 2\pi\sqrt{\frac{m}{\pi r^2 \rho g}} = \frac{2\pi}{r}\sqrt{\frac{m}{\pi\rho g}}$

Hence, (B) and (D) are correct.

2. $x_{\max} = 3A$, when $\cos(\omega t) = -1$
and $x_{\min} = A$, when $\cos(\omega t) = 1$
Therefore, the particle oscillates between $x = 3A$ to $x = A$
Since, $x = 2A - A\cos(\omega t)$

$\Rightarrow \quad v = \frac{dx}{dt} = A\omega\sin(\omega t)$

Now $v = v_{\max} = A\omega$ at $\omega t = \frac{\pi}{2}$

$\Rightarrow \quad x = 2A$ at $\omega t = \frac{\pi}{2}$

Further,

$T = \frac{2\pi}{\omega}$

$t_{3A\to A} = \frac{T}{2} = \frac{\pi}{\omega}$ and $t_{A\to 2A} = \frac{T}{4} = \frac{\pi}{2\omega}$

Hence, (B), (C) and (D) are correct.

3. Given that $y = 0.05\sin 4\pi(5t + 0.4)$

$\Rightarrow \quad y = 0.05\sin(20\pi t + 1.6\pi)$

$\Rightarrow \quad \omega = 20\pi$

$\Rightarrow \quad T = \frac{2\pi}{\omega} = \frac{2\pi}{20\pi} = 0.1\text{ s}$

Total energy of SHM is

$$E = \frac{1}{2}m\omega^2 A^2 = \frac{1}{2}(0.1)(20\pi)^2(0.05)^2 = 0.05\pi^2 \text{ joule}$$

Maximum acceleration of particle is

$$a_{\max} = \omega^2 A = (20\pi)^2 \times (0.05) = 20\pi^2 \text{ ms}^{-2}$$

Force acting on the particle is zero at the mean position and maximum at the extreme position.
Hence, (A) and (C) are correct.

4. Let the displacement equation of particle is

$$x = a\sin(\omega t)$$

Time period of particle is given by

$$T = (t_{PA} + t_{AP}) + (t_{PB} + t_{BP})$$

$\Rightarrow \quad T = (0.5\text{ s}) + (1.5\text{ s}) = 2\text{ s} = \frac{2\pi}{\omega}$

$\Rightarrow \quad \omega = \pi\text{s}^{-1}$

$\Rightarrow \quad x = a\sin(\pi t)$ …(1)

and $v = (a\pi)\cos\pi t$ …(2)

Let $t_{OP} = t$ then $t_{OAP} = t + \frac{1}{2}$

Since, it is given that

$$3 = a\pi\cos(\pi t) = \left|a\pi\cos\pi\left(t + \frac{1}{2}\right)\right|$$

$\Rightarrow \quad 3 = \left|a\pi\cos\left(\frac{\pi}{2} + \pi t\right)\right| = a\pi\sin(\pi t)$

$\Rightarrow \quad \pi t = \frac{\pi}{4}$

$\Rightarrow \quad a\pi = \frac{3}{\cos\left(\frac{\pi}{4}\right)} = 3\sqrt{2}\text{ ms}^{-1} = v_{\max}$

Also, $x = a\sin(\pi t) = a\sin\left(\frac{\pi}{4}\right) = \frac{a}{\sqrt{2}}$

$\Rightarrow \quad \frac{AP}{BP} = \frac{a - x}{a + x} = \frac{a - \frac{a}{\sqrt{2}}}{a + \frac{a}{\sqrt{2}}} = \frac{\sqrt{2} - 1}{\sqrt{2} + 1}$

Hence, (A) and (C) are correct.

5. Time period of spring block system does not depend on the effective value of g. But in case of simple pendulum

$$T \propto \frac{1}{\sqrt{g}}$$

Hence, (B) and (C) are correct.

6. Relative error in measurement of time $= \frac{1\text{ s}}{40\text{ s}} = \frac{1}{40}$

Time period = 2 s

So, error in measurement of time period is

$$\Delta T = 2\times\frac{1}{40} = \frac{1}{20}\text{ s} = 0.05\text{ s}$$

$$g \propto \frac{1}{T^2}$$

$$\Rightarrow \quad \frac{\Delta g}{g} = \frac{2\Delta T}{T} = \frac{2\times 1}{40} = \frac{1}{20}$$

$$\Rightarrow \quad \frac{\Delta g}{g}\times 100 = 5\%$$

Hence, (A) and (C) are correct.

7. $\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{16}{1}} = 4\text{ rads}^{-1}$

$$\frac{1}{2}mv_{\max}^2 = 8$$

$$\Rightarrow \quad \frac{1}{2}\times 1\times(\omega A)^2 = 8$$

$$\Rightarrow \quad \frac{1}{2}\times 1\times 16\times A^2 = 8$$

$$\Rightarrow \quad A = 1\text{ m}$$

At half the amplitude $x = \frac{A}{2} = 0.5$ m, potential energy stored in the spring will be

$$U = \frac{1}{2}Kx^2 = \frac{1}{2}\times 16\times\left(\frac{1}{2}\right)^2 = 2\text{ J}$$

Only when spring block system is horizontal. At half the amplitude

$$v = \omega\sqrt{A^2 - x^2}$$

$$\Rightarrow \quad v = 4\sqrt{(1)^2 - (0.5)^2} = 2\sqrt{3}\text{ ms}^{-1}$$

$$\Rightarrow \quad KE = \frac{1}{2}mv^2 = \frac{1}{2}\times 1\times\left(2\sqrt{3}\right)^2 = 6\text{ J}$$

Hence, (A), (B) and (C) are correct.

8. From the graph, we see that the mean position is at $x = 20$ cm i.e., where the potential energy is minimum.

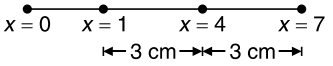
Also, we know that $U = \frac{1}{2}k(\Delta x)^2$, where Δx is the displacement of the particle form the mean position. Also, from the graph, we see that at $x = 14$ cm, the potential energy is 8 J, so we have

$$8 = \frac{1}{2}k(0.2 - 0.14)^2$$

$$\Rightarrow \quad k = 4444.4\text{ Nm}^{-1}$$

Also, a. is correct, because at the extreme positions, i.e. at $x = 20\text{ cm} \pm 6$ cm, potential energy is equal to total energy (because kinetic energy at the extreme position is zero).
Hence, (A), (B) and (D) are correct.

9. The motion of the particle is somewhat like the minimum value of x can be

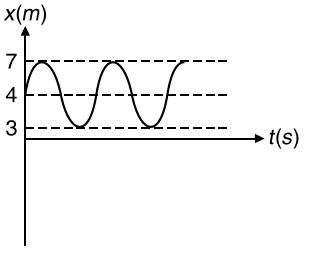


$$4 - 3 = 1\text{ cm}$$

and maximum value of x can be

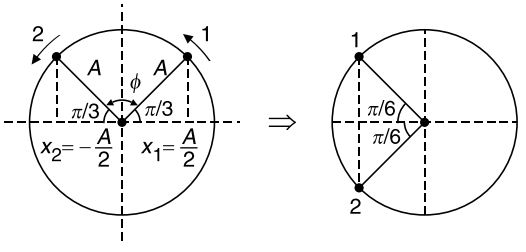
$$4 + 3 = 7\text{ cm}$$

i.e., the particle oscillates simple harmonically about point $x = 4$ cm with amplitude 3 cm. The x-t graph will be as shown.



Hence, (A) and (C) are correct.

10. From the figure we can see that phase difference between them is $\phi = \frac{\pi}{3}$



They will collide when the perpendiculars drawn on the diameter coincide. This will happen when any of the particles rotates an angle $\theta = \frac{\pi}{3} + \frac{\pi}{6} = \frac{\pi}{2}$.

$$\Rightarrow \quad \omega t = \frac{\pi}{2}$$

$$\Rightarrow \quad t = \frac{\pi}{2\omega}$$

Hence, (A) and (C) are correct.

11. The situation is similar as if a block of mass m is suspended from a vertical spring and a constant force mg acts downwards. Therefore, in this case also block will execute SHM with time period

$$T = 2\pi\sqrt{\frac{m}{k}}$$

At compression x, we have

$$F = kx$$

$$\Rightarrow \quad x = \frac{F}{k}$$

This is also the amplitude of oscillations. Hence,

$$A = \frac{F}{k}$$

At mean position speed of the block will be maximum. Applying Work Energy Theorem

CHAPTER 3

$$Fx = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{\frac{2Fx - kx^2}{m}}$$

Hence, (A), (C) and (D) are correct.

12. Since we know that $\vec{v}$ is either parallel or antiparallel to $\vec{r}$, so $\vec{v}\cdot\vec{r}$ can be positive for $\theta = 0°$ (parallel) or negative for $\theta = 180°$ (anti-parallel). Hence (a) is false.

Also, $\vec{v}\times\vec{r} = \vec{0}$, for both $\theta = 0°$ and $180°$. Hence (c) is correct.

Since, $\vec{F} = -k\vec{r}$, so $\vec{F}$ is anti-parallel to $\vec{r}$. Hence $\vec{F}\cdot\vec{r}$ is always negative and $\vec{F}\times\vec{r}$ will always be zero, hence (b) and (d) are correct.

Hence, (B), (C) and (D) are correct.

13. At $t = 0$, kinetic energy is $\frac{1}{4}$th the maximum kinetic energy i.e., speed of the particle is half the maximum speed, so $v = \frac{\pm A\omega}{2}$ at $t = 0$

$$v = \frac{-A\omega}{2} \qquad \text{in OPTION (A)}$$

$$v = \frac{A\omega}{2} \qquad \text{in OPTION (B)}$$

$$v = \frac{-A\omega}{2} \qquad \text{in OPTION (C)}$$

and $v = \frac{A\omega}{2}$ in OPTION (D)

Therefore, all the OPTIONS are correct.

Hence, (A), (B), (C) and (D) are correct.

14. Restoring force $F = -(\rho Ag)x$

$$\Rightarrow \quad F \propto -x$$

This is just like a spring-block system of force constant

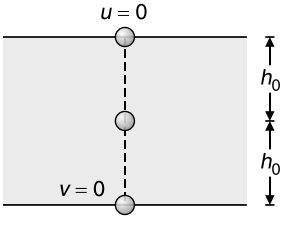
$$K = \rho Ag$$

Hence, (A), (C) and (D) are correct.

15. Net force on the ball will be zero at $\rho = \rho_0$

$$\Rightarrow \quad \alpha h_0 = \rho_0$$

$$\Rightarrow \quad h_0 = \frac{\rho_0}{\alpha}$$



i.e., the mean position is at a depth $h_0 = \frac{\rho_0}{\alpha}$

Net force at a depth $(h_0 + x)$ is given by

$$F = (\rho - \rho_0)Vg \qquad \{\text{upwards}\}$$

$$\Rightarrow \quad F = \alpha xVg \qquad \{\text{upwards}\}$$

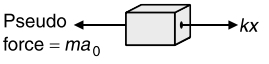
F is proportional to $-x$

Thus, motion of the ball is simple harmonic.

$$h_{max} = 2h_0 = \frac{2\rho_0}{\alpha}$$

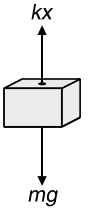
Hence, (A) and (C) are correct.

16. Free body diagram of the truck from non-inertial frame of reference is shown in figure.



This is similar to a situation when a block is suspended from a vertical spring.

Therefore, the block will execute simple harmonically with time period $T = 2\pi\sqrt{\frac{m}{k}}$



Amplitude will be given by

$$A = x = \frac{ma_0}{k} \qquad \{\because\ ma_0 = kx\}$$

Energy of oscillation will be

$$E = \frac{1}{2}kA^2 = \frac{1}{2}k\left(\frac{ma_0}{k}\right)^2 = \frac{m^2a_0^2}{2k}$$

Hence, (A), (B) and (C) are correct.

17. Restoring Force = Extra Upthrust

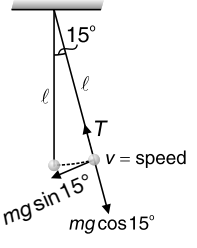
$$\Rightarrow \quad F = -Ax\rho g$$

$$\Rightarrow \quad a = \frac{F}{m} = -\left(\frac{A\rho g}{m}\right)x$$

$$\Rightarrow \quad T = 2\pi\sqrt{\left|\frac{x}{a}\right|} = 2\pi\sqrt{\frac{m}{A\rho g}}$$

Hence, (A), (C) and (D) are correct.

18. Simple pendulum of length 1 m is called second's pendulum whose time period is $T = 2$ s. But this time period is for small oscillations. In this case angular amplitude is $30°$. Therefore, time period will not be 2 s.



At angular displacement $15°$

$$T - mg\cos(15°) = \frac{mv^2}{\ell}$$

$\Rightarrow \quad T > mg\cos(15°)$

and tangential acceleration $a_t = g\sin(15°) =$ rate of change of speed.

Hence, (B) and (C) are correct.

19. Let $x = a\sin\omega t$

Then, $v = \frac{dx}{dt} = a\omega\cos\omega t$

So, potential energy is

$$PE = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2a^2\sin^2\omega t \qquad \{\because\ k = m\omega^2\}$$

$$\Rightarrow \quad U = U_{avg} = \frac{1}{2}m\omega^2a^2(\sin^2\omega t)_{avg}$$

Since, $(\sin^2\omega t)_{avg} = (\cos^2\omega t)_{avg} = \frac{1}{2}$

$$\Rightarrow \quad U = \frac{1}{4}m\omega^2a^2 \qquad \ldots(1)$$

Similarly, $KE = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2a^2\cos^2\omega t$

$$\Rightarrow \quad K = (KE)_{avg} = \frac{1}{2}m\omega^2a^2(\cos^2\omega t)_{avg}$$

$$\Rightarrow \quad K = \frac{1}{4}m\omega^2a^2 \qquad \ldots(2)$$

From (1) and (2), we get

$K = U$

Since total energy (TE) is $TE = PE + KE$

$$\Rightarrow \quad (TE)_{avg} = (PE)_{avg} + (KE)_{avg}$$

$$\Rightarrow \quad E = \frac{1}{2}m\omega^2a^2 = 2U = 2K$$

Hence, (B) and (C) are correct.

20. Given $x = x_0 + a\sin(\omega t)$

$$\Rightarrow \quad v = \frac{dx}{dt} = a\omega\cos(\omega t) \qquad \{\text{for block } A\}$$

at $t = 0$, $x = x_0$ and $v = a\omega$

i.e., block A is at $x = x_0$ (main position) and its velocity is $a\omega$ is positive x-direction. It collides elastically with an identical block B moving towards negative x-direction with velocity v_0. So, the blocks will interchange their velocities.

$\Rightarrow \quad v_A = v_0 \qquad \{\text{in negative } x\text{-direction}\}$

and $v_B = a\omega \qquad \{\text{in positive } x\text{-direction}\}$

Let A be the new amplitude of block A, then by Law of Conservation of Mechanical Energy, we get

$$\frac{1}{2}mv_A^2 = \frac{1}{2}kA^2$$

$$\Rightarrow \quad A = v_A\sqrt{\frac{m}{k}} = v_0\sqrt{\frac{m}{k}}$$

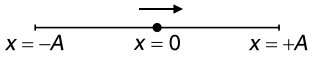
New displacement time equation is

$$x = x_0 - v_0\sqrt{\frac{m}{k}}\sin(\omega t)$$

Hence, (A) and (C) are correct.

21. At 1, acceleration is positive therefore, displacement is negative $(a \propto -x)$.

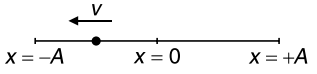
At 2, acceleration of particle is zero but after some time acceleration is negative. Therefore, velocity of particle is positive. This is shown in figure below



At 3, acceleration of particle is maximum. Therefore, potential energy of maximum.

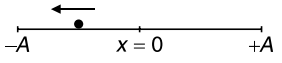
At 4, acceleration of particle is positive and its is increasing in magnitude.

Therefore, speed of particle is decreasing as shown below:



Hence, (A), (B) (C) and (D) are correct.

22. At position 1, velocity of particle is negative but afterwards speed of particle is decreasing. It implies that displacement of particle is negative as shown below



At position 2, velocity of particle is zero, but after some time its velocity is positive i.e., in this position displacement is negative.

At position 3, velocity of particle is positive and still increasing, so acceleration is positive.

At position 4, velocity of particle is maximum i.e., acceleration of particle is zero.

Hence, (B) and (C) are correct.

23. OPTIONS (B) and (C) can be true if we choose zero potential energy at extreme position.

Hence, (A), (B) and (C) are correct.

24. Reading will be maximum when platform accelerates up and minimum when it accelerates down. So,

$$R_{max} = m(g + a\omega^2) \qquad \{\because\ \text{maximum acceleration} = a\omega^2\}$$

$\Rightarrow \quad R_{max} = 60[10 + 8]$

$\Rightarrow \quad R_{max} = 1080$ N

$\Rightarrow$ Reading = 108 kg

Similarly, $R_{min} = m(g = a\omega^2)$

$\Rightarrow \quad R_{min} = 60(10 - 8)$

$\Rightarrow \quad R_{min} = 120$ N

$\Rightarrow$ Reading = 12 kg

Hence, (A) and (C) are correct.

25. Since $U = 5x(x - 4) = 5x^2 - 20x$

Force acting is

$$F = -\frac{dU}{dx} = -10x + 20$$

Since the force depends on position, so it is not constant. Also, F is linearly related with x and is restoring. So, motion is SHM with mean position i.e., equilibrium position i.e., when $F = 0$ at

CHAPTER 3

$$-10x + 20 = 0$$

$\Rightarrow \quad x = 2$ m

Hence, (B), (C) and (D) are correct.

26. Comparing the given equation with standard velocity-displacement equation of SHM i.e.,

$$v = \omega\sqrt{A^2 - x^2}$$

$\Rightarrow \quad v^2 = A^2\omega^2 - \omega^2x^2$

We observe that $\omega = 3$

$\Rightarrow \quad \dfrac{2\pi}{T} = 3$

$\Rightarrow \quad T = \dfrac{2\pi}{3}$ units

Similarly, amplitude of oscillations is

$$A = \frac{v_{\max}}{\omega} = \sqrt{\frac{A^2\omega^2}{\omega^2}}$$

$\Rightarrow \quad A = \sqrt{\dfrac{A^2\omega^2}{\omega^2}} = \sqrt{\dfrac{144}{9}} = \dfrac{12}{3} = 4$ unit

Acceleration of the particle at a distance x from the mean position is given by

$$a = \omega^2x = (9)(3) \qquad \{\because\ x = 3 \text{ unit}\}$$

$\Rightarrow \quad a = 27$ unit

Hence, (A), (B) and (C) are correct.

27. Let, $x_1 = -A\cos(\omega t)$ and $x_2 = A\sin(\omega t)$

Equating $x_1 = x_2$ at the point of crossing, we get

$$-A\cos(\omega t) = A\sin(\omega t)$$

$\Rightarrow \quad \tan(\omega t) = -1$

$\Rightarrow \quad \omega t = \dfrac{3\pi}{4}$

$\Rightarrow \quad \left(\dfrac{2\pi}{T}\right)t = \dfrac{3\pi}{4}$

$\Rightarrow \quad t = \dfrac{3T}{8}$

$\Rightarrow \quad x_2 = A\sin\left(\dfrac{3\pi}{4}\right) = \dfrac{A}{\sqrt{2}}$

Hence, (B) and (D) are correct.

28. Only along y-axis force is restoring in nature. Further oscillations are simple harmonic in nature only for small oscillations.
Hence, (C) and (D) are correct.

29. $T_1 = 2\pi\sqrt{\dfrac{m}{k_1}}$ and $T_2 = 2\pi\sqrt{\dfrac{m}{k_2}}$

$\Rightarrow \quad T_s = 2\pi\sqrt{\dfrac{m}{k_s}},\ k_s = \dfrac{k_1k_2}{k_1 + k_2}$

$\Rightarrow \quad T_p = 2\pi\sqrt{\dfrac{m}{k_p}},\ k_p = k_1 + k_2$

Hence, (A), (B) and (C) are correct.

30. Since, $F = -\dfrac{dU}{dx}$

$\Rightarrow \quad F = -U_0a\sin(ax)$ …(1)

For small oscillations,

$$\sin(ax) \approx ax$$

$\Rightarrow \quad m\ddot{x} = -U_0a(ax)$

$\Rightarrow \quad \ddot{x} + \left(\dfrac{U_0a^2}{m}\right)x = 0$

Compare with $\ddot{x} + \omega^2x = 0$, we get

$$\omega = \sqrt{\frac{U_0a^2}{m}}$$

$\Rightarrow \quad T = 2\pi\sqrt{\dfrac{m}{U_0a^2}}$

The speed of the particle is maximum i.e., kinetic energy is maximum and hence potential energy is minimum i.e., $\dfrac{dU}{dx} = 0$

From (1), we get $\dfrac{dU}{dx} = 0$ at $x = 0$

Hence, (B), (C) and (D) are correct.

31. $U_{\min} = U_{\max} - K_{\max} = 20$ J

Since, $K_{\max} = 40$ J

$\Rightarrow \quad \dfrac{1}{2}mv_{\max}^2 = \dfrac{1}{2}m(A^2\omega^2) = 40$ J

Since, $m\omega^2 = k$

$\Rightarrow \quad K_{\max} = \dfrac{1}{2}(m\omega^2)A^2 = \dfrac{1}{2}kA^2 = 40$ J

$\Rightarrow \quad \dfrac{1}{2}k\left(\dfrac{A}{2}\right)^2 = 10$ J

$\Rightarrow \quad U_{A/2} = U_{\min} + \dfrac{1}{2}k\left(\dfrac{A}{2}\right)^2 = 30$ J

$$K_{A/2} = E - U_{A/2} = 60 - 30 = 30 \text{ J}$$

Hence, (A) and (B) are correct.

Reasoning Based Questions

1. From the definition of SHM a particle executes SHM when it goes to and fro about a mean position under the restoring force. Hence, Statement-1 is true.
The earth completes one revolution around the sun after a fixed interval of 1 year. Therefore, it is a periodic motion but not harmonic Statement-2 is also true.
Hence, the correct answer is (B).

2. In initial position, the ball is completely filled with water, CG of the ball is at the centre. When water flows out of the ball the centre of gravity goes below the centre of the ball. So, the effective length of the ball increases, hence the time period of pendulum also increases. After draining out the water

continuously and when ball is emptied more than half then centre of gravity rises so that effective length decreases and so the time period of pendulum also decreases. Hence, the time period does not remain constant. We also know that time period of a simple pendulum does not depend upon the mass of bob as is obvious from relation $T = 2\pi\sqrt{\frac{\ell}{g}}$.

Hence, the correct answer is (B).

3. From the relation

$$T = 2\pi\sqrt{\frac{m}{k}}, \text{ where } k \text{ is the spring constant.}$$

$$\Rightarrow \quad T \propto \frac{1}{\sqrt{k}} \qquad \ldots(1)$$

As we know that the spring constant of hard spring is large in comparison to that of soft spring, therefore from equation (1) the time period will be less for hard spring in comparison to that of soft spring.

Hence, the correct answer is (B).

4. From the relation, potential energy of SHM is given as

$$\text{PE} = \frac{1}{2}m\omega^2 y^2$$

and $\text{KE} = \frac{1}{2}m\omega^2\left(A^2 - y^2\right)$

where y is the displacement and A is the amplitude. By the condition KE = PE,

$$\frac{1}{2}m\omega^2\left(A^2 - y^2\right) = \frac{1}{2}m\omega^2 y^2$$

$$\Rightarrow \quad m\omega^2 y^2 = \frac{1}{2}m\omega^2 A^2$$

$$\Rightarrow \quad y = \frac{A}{\sqrt{2}} \qquad \ldots(1)$$

Also, we know that total energy is

$$E = \text{KE} + \text{PE}$$

Hence, if PE is maximum then KE will be zero.

Hence, the correct answer is (B).

5. As velocity, $\frac{dy}{dt} = \omega\sqrt{A^2 - y^2}$

and acceleration, $\frac{d^2y}{dt^2} = -\omega^2 y$

When $y = 0$ i.e., at mean position

Velocity, $\frac{dy}{dt} = \omega A$ (maximum)

and acceleration, $\frac{d^2y}{dt^2} = 0$ (minimum)

When $y = \pm A$, i.e., at extreme position

$$v = \frac{dy}{dt} = 0 \text{ (minimum)}$$

and acceleration, $\frac{d^2y}{dt^2} = \mp\omega^2 A$ (maximum)

Hence, acceleration of particle executing SHM is zero (where velocity is maximum) at mean position and maximum at extreme position where velocity is minimum.

Now, let $y = a\sin(\omega t + \phi)$

Then velocity

$$\frac{dy}{dt} = a\omega\cos(\omega t + \phi) = a\omega\sin\left\{\left(\omega t + \frac{\phi}{2}\right) + \frac{\pi}{2}\right\}$$

Thus, displacement and velocity of SHM differ by a phase of $\frac{\pi}{2}$.

Hence, the correct answer is (D).

6. From the relation of time period

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\Rightarrow \quad T \propto \frac{1}{\sqrt{g}}$$

When the satellite is orbiting the earth, the value of g inside it is zero, the time period of pendulum in a satellite will be infinity and it is also clear that time period of pendulum is inversely proportional to square root of acceleration due to gravity g.

Hence, the correct answer is (A).

7. In SHM, the body does a to and fro motion about a fixed point called the mean position. At the extreme position, velocity of body is zero but acceleration is maximum which tends it to move back to mean position. Thus, acceleration is always directed towards mean position.

Also, in SHM, $a \propto -x$ which states that acceleration is in opposite direction of displacement i.e., towards mean position.

Hence, the correct answer is (A).

8. As $T \propto \sqrt{\ell}$

$$\frac{\Delta T}{T} = \frac{1}{2}\left(\frac{\Delta\ell}{\ell}\right) = \frac{1}{2} \times 4\% = 2\%$$

As length increases, the time period will increase by 2% it will not decrease.

Hence, the correct answer is (D).

9. If length of the pendulum is large, g no longer remains vertical but will be directed towards the centre of the earth and expression of the time period is given by

$$T = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{l} + \frac{1}{R}\right)}}$$

Here, R is the radius of earth.

When $\ell \to \infty$, $\frac{1}{\ell} \to 0$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{R}{g}} = 84.6 \text{ min}$$

In general, time period of a simple pendulum

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\Rightarrow \quad T \propto \sqrt{l}$$

Hence, the correct answer is (D).

CHAPTER 3

10. In simple harmonic motion,

$$v = \omega\sqrt{\left(a^2 - y^2\right)}$$

As y changes, velocity v will also change. So, simple harmonic motion is not a uniform motion.
Simple harmonic motion may be defined as the projection of uniform circular motion along any one or two mutually perpendicular diameters of the circle.
Hence, the correct answer is (B).

11. From the relations,

$$\text{PE} = \frac{1}{2}m\omega^2 y^2 \quad \ldots(1)$$

and $$\text{KE} = \frac{1}{2}m\omega^2\left(A^2 - y^2\right) \quad \ldots(2)$$

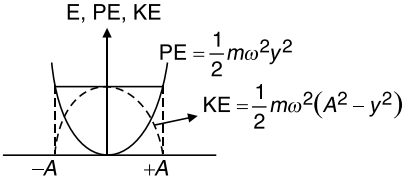


Figure shows the variation of total energy (E), potential energy (PE) and kinetic energy (KE) with displacement y.
Thus, the graph is a parabola.
Statement-2 says "PE and KE do not vary linearly" i.e., this portion is correct in the manner that KE and PE vary non-linearly but then how do they vary is missing. So, Statement-2 is not the correct explanation to Statement-1.
Hence, the correct answer is (B).

12. The period of the liquid executing SHM in a U-tube does not depend upon the density of the liquid. Therefore, time period will be the same, when mercury is filled up to the same height as the water in the U-tube. In fact, the period of the liquid is given by

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where, L = Length to which the liquid is filled in one limb of the U-tube.
Now, as the pendulum oscillates, it drags air along with it. Therefore, its kinetic energy is dissipated in overcoming viscous drag due to air and hence, its amplitude goes on decreasing.
Hence, the correct answer is (D).

13. In nature, many types of dissipating forces such as friction, air resistance etc., are operating. Therefore, as the oscillator oscillates, a part of its energy is used up in overcoming these dissipating forces. Such oscillations are damped oscillations.
Hence, the correct answer is (A).

14. When the earth contracts, its moment of inertia decreases. Therefore, from the relation

$$I\omega = L$$

$$\Rightarrow \quad I\omega = \text{constant}$$

with decrease of moment of inertia, angular velocity of earth increases. Thus, time period $T = \frac{2\pi}{\omega}$ will decrease.

Hence, duration of the day will decrease.
Hence, the correct answer is (C).

15. The ball will not go out of the other end of the hole, because it will execute SHM. On reaching the other end of the hole, its velocity become zero and acceleration of ball will be maximum and will be directed towards the centre of earth. The ball will have a time period of 84.6 minute and hence will be seen at the other end at half the time period i.e., 42.3 minute.
Hence, the correct answer is (A).

16. While executing SHM, from relation,

$$v = \omega\sqrt{A^2 - y^2}$$

or $$v^2 = \omega^2\left(A^2 - y^2\right)$$

$$v^2 = \omega^2 A^2 - \omega^2 y^2$$

Dividing both side by $\omega^2 A^2$

$$\frac{v^2}{\omega^2 A^2} + \frac{y^2}{A^2} = 1 \quad \ldots(1)$$

It is understood that Equation (1) represents the equation of an ellipse so the graph between v (velocity) and y (displacement) is not a parabola.
Again, from Equation (1), as displacement (y) increases, the velocity decreases, becomes zero at maximum displacement and again increases with decrease in displacement. Thus, in SHM, the velocity does not change uniformly.
Hence, the correct answer is (D).

17. The total energy of the harmonic oscillator is given by

$$E = \frac{1}{2}m\omega^2 A^2$$

$$\Rightarrow \quad E \propto A^2$$

Hence, $$\frac{E_2}{E_1} = \left(\frac{2A}{A}\right)^2$$

$$\Rightarrow \quad E_2 = 4E_1 = 4E$$

Also, total energy must become 4 times when amplitude is doubled.
Hence, the correct answer is (D).

18. Time period of a liquid column

$$T = 2\pi\sqrt{\frac{L}{g}} = 2 \times \frac{22}{7} \times \sqrt{\frac{0.3}{9.8}} = 1.1 \text{ s}$$

Hence, the correct answer is (A).

19. From the relation,

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\Rightarrow \quad T^2 = \frac{4\pi^2 l}{g}$$

$\Rightarrow \quad T^2 \propto \dfrac{1}{g}$

Therefore, the graph between T^2 and g is hyperbola.

Hence, the correct answer is (C).

20. When the man falls from the top of a tower, he will be in the state of weightlessness. A spring wound watch runs on the basis of spring action i.e., on the basis of the potential energy stored in the wound spring. Since, acceleration due to gravity does not play any role, the watch will give correct time, when the man falls from the top of a tower.

Hence, the correct answer is (A).

Linked Comprehension Type Questions

1. Given: Mass of each block A and B, $m = 0.1$ kg

Radius of circle, $R = 0.06\,m$

Natural length of spring $\ell_o = 0.06\pi = \pi R$ (half circle) and spring constant, $k = 0.1$ Nm^{-1}.

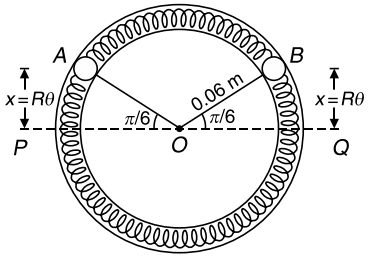
In the stretched position elongation in each spring

$x = R\theta$

Let draw FBD of A:

Spring in lower side is stretched by $2x$ and on upper side compressed by $2x$. Therefore, each spring will exert a force $2kx$ on each block.

Hence, a restoring force, $F = -4kx$ will act on A in the direction shown in Figure.



Restoring force of this force about origin

$\tau = -F.R = -(4kx)R = -(4kR\theta)R$

$\Rightarrow \quad \tau = -4kR^2\theta$ …(1)

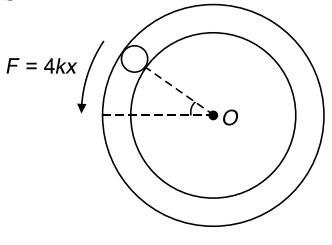
Since $\tau \propto -\theta$, each ball executes angular SHM about origin O.

Equation (1), can be written as

$I\alpha = -4kR^2\theta$

$\Rightarrow \quad (mR^2)\alpha = -4kR^2\theta$

$\Rightarrow \quad \alpha = -\left(\dfrac{4k}{m}\right)\theta$



So, frequency of oscillation

$$f = \frac{1}{2\pi}\sqrt{\left|\frac{\text{Acceleration}}{\text{Displacement}}\right|} = \frac{1}{2\pi}\sqrt{\left|\frac{\alpha}{\theta}\right|}$$

$\Rightarrow \quad f = \dfrac{1}{2\pi}\sqrt{\dfrac{4k}{m}}$

Substituting the values, we have

$$f = \frac{1}{2\pi}\sqrt{\frac{4\times 0.1}{0.1}} = \frac{1}{\pi}\ \text{Hz}$$

Hence, the correct answer is (C).

2. In stretched position, potential energy of the system is

$$\text{PE} = 2\left\{\frac{1}{2}k\right\}(2x)^2 = 4kx^2$$

And in mean position, both the blocks have kinetic energy only. Hence,

$$\text{KE} = 2\left\{\frac{1}{2}mv^2\right\} = mv^2$$

From energy conservation, PE = KE

$\therefore \quad 4kx^2 = mv^2$

$\therefore \quad v = 2x\sqrt{\dfrac{k}{m}} = 2R\theta\sqrt{\dfrac{k}{m}}$

Substituting the values

$$v = 2(0.06)(\pi/6)\sqrt{\frac{0.1}{0.1}}$$

or $v = 0.0628$ ms^{-1}

Hence, the correct answer is (B).

3. Total energy of the system is E equal to potential energy in stretched position.

or KE in mean position

$\Rightarrow \quad E = mv^2 = (0.1)(0.0628)^2$ J

$\Rightarrow \quad E = 39\times 10^{-4}$ J

Hence, the correct answer is (C).

4. $F = -4x + 8$

Let us write $x = (X + 2)$

Then, $F = -4X$

This is the equation of SHM.

Further $F = 0$ as $X = 0$ or $x = 2$ m

Hence, mean position is $x = 2$ m

Hence, the correct answer is (D).

5. $F = -4X$

$\Rightarrow \quad k = 4$ Nm^{-1}

$$A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2\times 18}{4}} = 3\ \text{m}$$

Mean position of the particle is $x = 2$ m

Hence, the extreme points are 5 m and –1 m

Hence, the correct answer is (B).

6. $T_0 = 2\pi\sqrt{\dfrac{m}{2k}}$

$T = 2\pi\sqrt{\dfrac{m}{k}} = \sqrt{2}T_0$

Hence, the correct answer is (B).

CHAPTER 3

7. Since, the velocity of the block remains unchanged, so

$$\omega_0\sqrt{A_0^2-\frac{A_0^2}{4}}=\omega\sqrt{A^2-\frac{A_0^2}{4}}$$

$$\Rightarrow \quad \sqrt{\frac{2k}{m}}\sqrt{\frac{3A_0^2}{4}}=\sqrt{\frac{k}{m}}\sqrt{A^2-\frac{A_0^2}{4}}$$

$$\Rightarrow \quad A=\frac{\sqrt{7}A_0}{2}$$

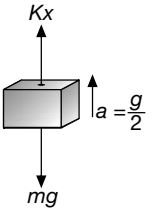
Hence, the correct answer is (C).

8. $v=A\omega=\sqrt{\frac{k}{m}}\frac{\sqrt{7}A_0}{2}=\frac{\sqrt{7}A_0}{2}\times\frac{\sqrt{2}\pi}{T_0}$

$$\Rightarrow \quad v=A\omega=\frac{\sqrt{14}\pi A_0}{2T_0}$$

Hence, the correct answer is (D).

9. In mean position of oscillations, spring will be elongated. Let x be the extension, then



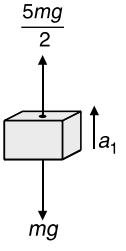
$$kx-mg=ma=\frac{mg}{2}$$

$$\Rightarrow \quad x=\frac{3mg}{2k}$$

Hence, the correct answer is (D).

10. **For a_1:**

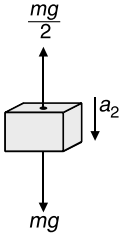
$$a_1=\frac{\frac{5mg}{2}-mg}{m}=\frac{3}{2}g$$



Amplitude $A=\frac{5mg}{2k}=\frac{3mg}{2k}=\frac{mg}{k}$

So, minimum extension $=\frac{3mg}{2k}-\frac{mg}{k}=\frac{mg}{2k}$

For a_2:



$$a_2=\frac{mg-\frac{mg}{2}}{m}=\frac{g}{2}$$

Hence, the correct answer is (B).

11. $p^2 \propto x$

Hence, the correct answer is (D).

12. $p_1=2p_2$

$$\Rightarrow \quad \frac{E_2}{E_1}=\frac{p_2^2}{p_1^2}=\frac{1}{4}$$

$$\Rightarrow \quad E_1=4E_2$$

Hence, the correct answer is (C).

13. Due to the viscous drag offered by the water, the momentum will not be same but less when it returns.

Hence, the correct answer is (B).

14. Since the particle starts from the extreme position, so

$$x=a\cos\omega t$$

$$\Rightarrow \quad \frac{a}{2}=a\cos\omega t$$

$$\Rightarrow \quad \cos\omega t=\frac{1}{2}=\cos\frac{\pi}{3}$$

$$\Rightarrow \quad t=\frac{T}{6}=\frac{0.60}{6}=0.10\text{ s}$$

Hence, the correct answer is (C).

15. $\langle v\rangle=\frac{a/2}{T/6}=\frac{3a}{T}$

$$\Rightarrow \quad \langle v\rangle=\frac{3\times10\times10^{-2}}{0.60}\text{ ms}^{-1}=0.50\text{ ms}^{-1}$$

Hence, the correct answer is (A).

16. Since the particle starts from the mean position, so

$$x=a\sin\omega t$$

$$\Rightarrow \quad \frac{a}{2}=a\sin\omega t$$

$$\Rightarrow \quad \frac{1}{2}=\sin\omega t\sin\frac{\pi}{6}=\sin\omega t$$

$$\Rightarrow \quad \frac{\pi}{6}=\frac{2\pi}{T}t$$

$$\Rightarrow \quad t=\frac{T}{12}=\frac{0.60}{12}\text{ s}=0.05\text{ s}$$

Hence, the correct answer is (D).

17. $\langle v\rangle=\frac{a/2}{T/12}=\frac{6a}{T}$

$$\Rightarrow \quad \langle v\rangle=\frac{6\times10\times10^{-2}}{0.60}\text{ ms}^{-1}=1\text{ ms}^{-1}$$

Hence, the correct answer is (B).

18. Since $F=-4x$, so $k=4\text{ Nm}^{-1}$

$$\Rightarrow \quad a=\frac{F}{m}=-8x$$

$$\Rightarrow \quad \omega=\sqrt{8}=2\sqrt{2}\text{ rads}^{-1}$$

Since the total energy of the particle is 10 J and the amplitude of the oscillations is 2 m, so the potential energy of the particle at the specified instance is

$$U=\frac{1}{2}kA^2=\frac{1}{2}\times4\times4=8\text{ J}$$

So, potential energy at mean position is

$$U=E-\frac{1}{2}kA^2=2\text{ J}$$

Hence, the correct answer is (D).

19. At time $t=0$, $a=16\text{ ms}^{-2}$, so $a=-\omega^2x=-8x$

$$\Rightarrow \quad x=A=2\text{ m}$$

So, the equation of motion is $x = 2\cos\left(2\sqrt{2t}\right)$

Hence, the correct answer is (D).

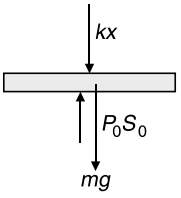
20. $\frac{1}{2}kx^2 = \frac{1}{2}\times 4\times 1 = 2\text{ J}$

Potential energy at mean position is 2 J

Therefore, $U = (2+2) = 4\text{ J}$ and $K = E - U = 6\text{ J}$

Hence, the correct answer is (D).

21.



$kx = P_0S_0 - mg$

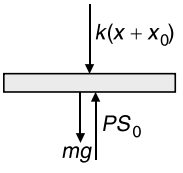
$\Rightarrow \quad x = \frac{P_0S_0 - mg}{k}$

Hence, the correct answer is (C).

22. From the diagram, we observe that

$k(x + x_0) + mg - PS_0 = ma$

For an adiabatic process, we have



$PV^\gamma = \text{constant}$

$\Rightarrow \quad P_0(\ell S_0)^\gamma = P((\ell + x)S_0)^\gamma$

$\Rightarrow \quad P = P_0\left(1 - \frac{\gamma x}{\ell}\right)$

$\Rightarrow \quad kx + kx_0 + mg - P_0S_0 + \frac{P_0\gamma xS_0}{\ell} = ma$

$\Rightarrow \quad \left(k + \frac{\gamma P_0S_0}{\ell}\right)x = ma$

$\Rightarrow \quad a = \underbrace{\left(\frac{k\ell + \gamma P_0S_0}{m\ell}\right)}_{\omega^2}x$

$\Rightarrow \quad \omega = \sqrt{\frac{k\ell + \gamma P_0S_0}{m\ell}}$

Hence, the correct answer is (D).

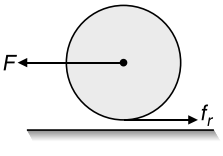
23. $\omega = \sqrt{\frac{\gamma P_1S_0}{m\ell_1}}$

So only restoring force is due to pressure.

Hence, the correct answer is (A).

24. $F - f_r = Ma$

Since, $\tau = f_rR = I\alpha$



$\Rightarrow \quad f_r = \frac{Ia}{R^2}$

Solving these two, we get

$a = \frac{F}{M + \frac{I}{r^2}}$

Also, $F = -2kx$

$\Rightarrow \quad a = \frac{-2kx}{M + \frac{I}{r^2}}$

$\Rightarrow \quad f_r = \frac{I}{R^2}\times\left(\frac{-2kx}{M + \frac{I}{r^2}}\right)$

For disc $I = \frac{1}{2}MR^2$

$\Rightarrow \quad f_r = -\frac{4kx}{3}$

Hence, the correct answer is (D).

25. $a = -\frac{2kx}{M + \frac{I}{r^2}}$

Since, $a = -\omega^2x$, which is comparable to the characteristic equation of SHM, so

$\omega = \sqrt{\frac{|a|}{|x|}}$

$\Rightarrow \quad \omega = \sqrt{\frac{2k}{M + \frac{I}{r^2}}} = \sqrt{\frac{4k}{3M}}$

Hence, the correct answer is (B).

26. $f_r = \frac{Ia}{R^2} \le \mu Mg$

and $\frac{Iv_0\omega}{R^2} \le \mu mg$ $\qquad \{\because a_{\max} = v_0\omega\}$

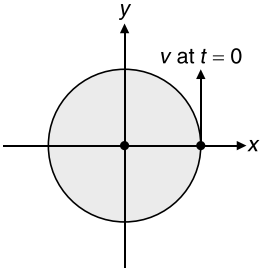
$\Rightarrow \quad \frac{MR^2}{2}\frac{v_0\omega}{R^2} \le \mu mg$

$\Rightarrow \quad v_0 \le 2\mu g\sqrt{\frac{3M}{4k}}$

$\Rightarrow \quad v_0 \le \mu g\sqrt{\frac{3M}{k}}$

Hence, the correct answer is (C).

27.



At any time t, particle is at $\frac{T_0}{\sqrt{2}}$ and $\omega r = v$

$v_x = -v\sin\omega t$

$v_y = v\cos\omega t$

Hence, the correct answer is (D).

CHAPTER 3

28. $F_y = -F_C \sin(\omega t)$

$\Rightarrow \quad F_y = -\dfrac{mv^2}{r}\sin(\omega t)$

Hence, the correct answer is (B).

29. $\theta = \omega t$

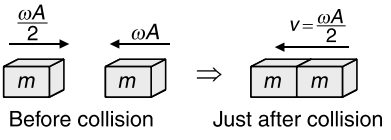
$\theta = \dfrac{v}{r}t$

Hence, the correct answer is (A).

30. They will collide at their mean positions because time period of both are same and that is $2\pi\sqrt{\dfrac{m}{K}}$. After collision combined mass is 2 m and $K_{\text{eff}} = 2K$. Hence, time period remains unchanged.

Hence, the correct answer is (C).

31. Applying Law of Conservation of Linear Momentum, the velocity of combined mass just after collision is $v = \dfrac{\omega A}{4}$.



Since, this is the velocity at mean position, so

$v = \omega' A'$

$\Rightarrow \quad \dfrac{\omega A}{4} = \omega' A'$

$\Rightarrow \quad A' = \dfrac{A}{4}$ because $\omega' = \omega = \sqrt{\dfrac{K}{m}} = \sqrt{\dfrac{2K}{2m}}$

Hence, the correct answer is (A).

32. $E = \dfrac{1}{2}(2m)v^2 = \dfrac{1}{2}(2m)\left(\dfrac{\omega A}{4}\right)^2$

$\Rightarrow \quad E = \dfrac{m\omega^2 A^2}{16} = \dfrac{kA^2}{16} \qquad \left\{\because\ \omega^2 = \dfrac{k}{m} \text{ or } \dfrac{2k}{2m}\right\}$

$\Rightarrow \quad E = \dfrac{1}{2}(2k)\left(\dfrac{A}{4}\right)^2 = \dfrac{kA^2}{16}$

Hence, the correct answer is (D).

33. $x = a\cos(\omega t)$

$\Rightarrow \quad \dfrac{a}{2} = a\cos(\omega t)$

$\Rightarrow \quad \cos^{-1}\left(\dfrac{1}{2}\right) = \dfrac{2\pi}{T}t$

$\Rightarrow \quad t = \dfrac{T}{6} = \dfrac{0.6}{6} = 0.1 \text{ s}$

Hence, the correct answer is (C).

34. $v_{\text{average}} = \dfrac{\text{total displacement}}{\text{total time}} = \dfrac{\frac{a}{2}}{t} = \dfrac{5 \text{ cm}}{0.1} = 0.5 \text{ ms}^{-1}$

Hence, the correct answer is (A).

35. $\omega = \sqrt{\dfrac{K}{m}} = \sqrt{\dfrac{200}{2}}$

$\omega = 10 \text{ sec}^{-1}$

Hence, the correct answer is (B).

36. $x = A\sin(\omega t + \delta)$ …(1)

$v = A\omega\cos(\omega t + \delta)$ …(2)

From (1), $0.05 = A\sin\delta$

From (2), $1 = A\omega\cos\delta$

$0.1 = A\cos\delta \qquad \{\because\ \omega = 10\}$

$\Rightarrow \quad 0.1^2 + 0.05^2 = A^2$

$\Rightarrow \quad A = \sqrt{\dfrac{1}{10^2} + \dfrac{25}{100^2}}$

$\Rightarrow \quad A = \dfrac{1}{100}\sqrt{125} = 0.112 \text{ m}$

$\Rightarrow \quad \sin\delta = \dfrac{0.05}{0.112}$

$\Rightarrow \quad \delta = \sin^{-1}(0.446)$

Hence, the correct answer is (C).

37. $(10t + \delta) = \dfrac{\pi}{2}$

$\Rightarrow \quad 10t = \dfrac{\pi}{2} - 8$

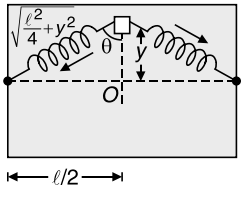
$\Rightarrow \quad t = \dfrac{\pi}{20} - \dfrac{\sin^{-1}(0.446)}{10}$

$\Rightarrow \quad t = 0.157 - \dfrac{2\pi}{360} \times 26.5 \times \dfrac{1}{10}$

$\Rightarrow \quad t = 0.157 - 0.0462 \approx 0.11 \text{ s}$

Hence, the correct answer is (A).

38. Extension in spring, $\sqrt{y^2 + \dfrac{\ell^2}{4}} - \dfrac{\ell}{2}$



So, spring force due to one spring is

$F = k\left(\sqrt{\dfrac{\ell^2}{4} + y^2} - \dfrac{\ell}{2}\right)$

Net spring force due to both the springs is

$F_{\text{net}} = 2F\cos\theta$ along negative Y-axis

$F_{\text{net}} = -2k\left(\sqrt{\dfrac{\ell^2}{4} + y^2} - \dfrac{\ell}{2}\right)\left(\dfrac{y}{\sqrt{\dfrac{\ell^2}{4} + y^2}}\right)$

$\Rightarrow \quad F_{\text{net}} = 2ky\left(\dfrac{\frac{\ell}{2}}{\sqrt{\dfrac{\ell^2}{4} + y^2}} - 1\right)\hat{j}$

Hence, the correct answer is (C).

39. Since, $U = \frac{1}{2}k_{net}(\Delta y)^2$

where $k_{net} = 2k$ and $\Delta y = \sqrt{y^2 + \frac{\ell^2}{4}} - \frac{\ell}{2}$

Hence, the correct answer is (B).

40. Extension in the spring is

$$x = \sqrt{5^2 + (\sqrt{11})^2} - 5 = 1 \text{ m}$$

Then by law of conservation of energy, we have

$$2\left[\frac{1}{2}(100)(1)^2\right] = \frac{1}{2}(2)v^2$$

$\Rightarrow \quad v = 10 \text{ ms}^{-1}$

Hence, the correct answer is (C).

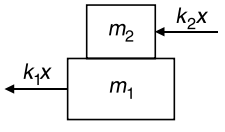
41. Since both the blocks moves together and springs are in parallel, so we have

$$f = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m_1 + m_2}}$$

Hence, the correct answer is (B).

42. As springs are in parallel, so we have

$$a = \frac{F_{net}}{\text{Total mass}} = \frac{(k_1 + k_2)x}{m_1 + m_2}$$



We can also answer the same question by taking help from the free body diagram as shown in figure. So, we have

$$a = \frac{F_{net}}{\text{Total mass}} = \left(\frac{k_1 + k_2}{m_1 + m_2}\right)x$$

Hence, the correct answer is (C).

43. Frictional force on m_2 will act in direction of displacement if

$$k_2x > m_2a$$

$\Rightarrow \quad k_2x > m_2\left(\frac{k_1 + k_2}{m_1 + m_2}\right)x$

$\Rightarrow \quad \frac{m_1}{m_2} > \frac{k_1}{k_2}$

Hence, the correct answer is (A).

44. Again, if amplitude of oscillation is A and a_{max} is the maximum acceleration, then we have

$$a_{max} = A\omega^2$$

Now, for the said condition to be met, we have

$$k_2A - f = m_2(\omega^2 A)$$

$\Rightarrow \quad f = A(k_2 - m_2\omega^2)$

$\Rightarrow \quad A_{max} = \left(\frac{\mu m_2 g}{k_2 - m_2\omega^2}\right)$

where $\omega^2 = \frac{k_1 + k_2}{m_1 + m_2}$

Hence, the correct answer is (A).

45. Particle 1 is oscillating between $(8+3)$ m and $(8-3)$ m i.e., between 11 m and 5 m. While the particle 2 is oscillating between 4 m and –4 m so, they will not collide.

Hence, the correct answer is (D).

46. At time $t = 0$, x_1 is at +8 m and moving towards positive x-axis and x_2 is at +4 m and is moving towards negative x-axis. At this time they are at shortest distance. Next time they will be at minimum distance after time, $t = T = \frac{2\pi}{\omega} = 2$ s. Time period of both is same.

Hence, the correct answer is (B).

47. Maximum KE = Total Energy

Total energy $= 20 + 20 + (5-2)^2$

$\Rightarrow \quad E = 40 + 9 = 49$ J

Hence, the correct answer is (A).

48. $F = -\frac{dU}{dx} = -2(x-2) = 0$

$\Rightarrow \quad x = 2$ m

Hence, the correct answer is (A).

49. $F = 2x - 4 = 2x$

$\Rightarrow \quad F = \omega^2 x$

So, $\omega = \sqrt{2}$

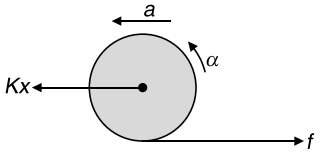
Hence, the correct answer is (A).

50. The forces on the disc at displacement x of the spring are shown in Figure.

For pure rolling to take place, we have

$$a = R\alpha$$

$\Rightarrow \quad \frac{kx - f}{m} = \frac{R(fR)}{\frac{1}{2}mR^2} = \frac{2f}{m}$



$\Rightarrow \quad f = \left(\frac{k}{3}\right)x$

The cylinder is starting from $x = A$, so the x-t equation would be

$$x = A\cos(\omega t)$$

$\Rightarrow \quad f = \frac{kA}{3}\cos(\omega t)$

$\Rightarrow \quad f$-t curve will be a cosine curve.

Hence, the correct answer is (C).

51. Energy of oscillation, $E = \frac{1}{2}kA^2 = \frac{1}{2} \times 10 \times 4 = 20$ J

At mean position this is totally kinetic energy (translational + rotational). In case of pure rolling ratio of rotational and translational kinetic energy is $\frac{1}{2}$ in case of disc.

Hence, $K_T = \frac{1}{2}mv^2 = \frac{2}{3} \times 20 = \frac{40}{3}$

$\Rightarrow \quad v^2 = \frac{80}{3 \times 2} = \frac{40}{3} \text{ m}^2\text{s}^{-2}$

$\Rightarrow \quad v = \sqrt{\frac{40}{3}} \text{ ms}^{-1}$

Hence, the correct answer is (A).

52. $x = a\sin^2\left(\omega t - \frac{\pi}{4}\right)$

$$x = \frac{a}{2}\left[1 - \cos\left\{2\left(\omega t - \frac{\pi}{4}\right)\right\}\right]$$

CHAPTER 3

$\Rightarrow$ Amplitude is $\frac{a}{2}$

Hence, the correct answer is (B).

53. $x = \frac{a}{2}\left[1 - \cos\left(2\omega t - \frac{\pi}{2}\right)\right]$

$\Rightarrow \quad 2\omega = \frac{2\pi}{T}$

$\Rightarrow \quad T = \frac{\pi}{\omega}$

Hence, the correct answer is (A).

54. Conceptual

Hence, the correct answer is (A).

55. $x = a\sin^2\left(\omega t - \frac{\pi}{4}\right)$ …(1)

The plot of x versus ωt follows from equation (1)

Differentiating equation (1)

$$\frac{dx}{dt} = 2a\omega\sin\left(\omega t - \frac{\pi}{4}\right)\cos\left(\omega t - \frac{\pi}{4}\right) \quad \ldots(2)$$

Again, $x = a\sin^2\left(\omega t - \frac{\pi}{4}\right)$

$$ax = a^2\sin^2\left(\omega t - \frac{\pi}{4}\right)$$

$\Rightarrow \quad \sqrt{ax} = a\sin\left(\omega t - \frac{\pi}{4}\right)$

$\Rightarrow \quad \sin\left(\omega t - \frac{\pi}{4}\right) = \sqrt{\frac{x}{a}}$

Also, $\cos\left(\omega t - \frac{\pi}{4}\right) = \sqrt{1 - \frac{x}{a}}$

From equation (2),

$$v_x = 2a\omega\sqrt{\frac{x}{a}}\sqrt{1 - \frac{x}{a}}$$

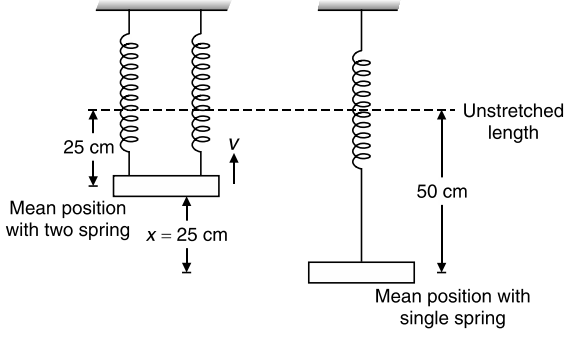
$\Rightarrow \quad v_x^2 = 4a^2\omega^2\frac{x}{a}\left(1 - \frac{x}{a}\right)$

$\Rightarrow \quad v_x^2 = 4\omega^2 x(a - x)$

The plot of v_x versus x follows from this equation.

Hence, the correct answer is (B).

56. Let x_1 be the elongation in equilibrium with two springs and x_2 with single spring.



Then, $2kx_1 = mg$

$\Rightarrow \quad (2)(10)(x_1) = \frac{1}{2}(10)$

$\Rightarrow \quad x_1 = 25$ cm

Further, $kx_2 = mg$

$\Rightarrow \quad (10)(x_2) = \frac{1}{2}(10)$

$\Rightarrow \quad x_2 = 50$ cm

The spring breaks when stretched by 25 cm, so the speed of the block at this instant is

$$v = \omega A = \left(\sqrt{\frac{2k}{m}}\right)(0.25)$$

$\Rightarrow \quad v = \left(\sqrt{\frac{20}{1/2}}\right)(0.25) = \frac{\sqrt{10}}{2}$ ms^{-1}

For single mass, we have

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{20}\ \text{rads}^{-1}$$

Also, the block has the above velocity when it is at a displacement $x = 25$ cm $= 0.25$ m from its mean position. Hence, from the equation

$$v = \omega\sqrt{A^2 - x^2}$$

$\Rightarrow \quad v^2 = \omega^2(A^2 - x^2)$, we have

$\Rightarrow \quad \left(\frac{\sqrt{10}}{2}\right)^2 = 20\left(A_1^2 - (0.25)^2\right)$

$\Rightarrow \quad A_1 = 0.433$ m

$\Rightarrow \quad A_1 = 43.3$ cm

Hence, the correct answer is (C).

57. Let x_3 be the extension with two springs and $\frac{m}{2}$ be the mass.

Then, $2kx_3 = \frac{m}{2}g$

$\Rightarrow \quad 2 \times 10 \times x_3 = \frac{1}{4} \times 10$

$\Rightarrow \quad x_3 = \frac{1}{8}$ m $= 12.5$ cm

$\Rightarrow \quad \omega = \sqrt{\frac{2k}{m/2}} = \sqrt{\frac{20}{1/4}} = \sqrt{80}\ \text{rads}^{-1}$

Since, $v^2 = \omega^2(A^2 - x^2)$

$\Rightarrow \quad \left(\frac{\sqrt{10}}{2}\right)^2 = 80\left(A_2^2 - (0.125)^2\right)$

$\Rightarrow \quad A_2 = 21.6$ cm

Hence, the correct answer is (C).

Matrix Match/Column Match Type Questions

1. A $\rightarrow$ (q); B $\rightarrow$ (r); C $\rightarrow$ (s); D $\rightarrow$ (t)

The given equation, $y = A\sin(\omega t) + A\sin\left(\omega t + \frac{2\pi}{3}\right)$ can also be written as.

$$y = 2A\sin\left(\omega t + \frac{\pi}{3}\right)\cdot\cos\left(\frac{\pi}{3}\right)$$

$$y = A\sin\left(\omega t + \frac{\pi}{3}\right)$$

Now, we can see that this is SHM with amplitude A and initial phase $\frac{\pi}{3}$.

$$v = A\omega\cos\left(\omega t + \frac{\pi}{3}\right)$$

$$\Rightarrow \quad v_{max} = A\omega$$

2. A → (r); B → (p); C → (q); D → (s)

Frequency of oscillation of kinetic energy is $2f$.

Since, $\omega = 12\pi$

$$\Rightarrow \quad 2\pi f = 12\pi$$

$$\Rightarrow \quad 2f = 12$$

$$v = \frac{dx}{dt} = 12\pi\cos(12\pi t)$$

So, v is MAXIMUM, when $12\pi t = 0, \pi, 2\pi, 3\pi, \ldots$

$$\Rightarrow \quad 12\pi t = \pi$$

$$\Rightarrow \quad t = \frac{1}{12}\text{ s}$$

$$U_{max} = \frac{1}{2}kA^2 = \frac{1}{2}(m\omega^2)A^2$$

$$\Rightarrow \quad U_{max} = \frac{1}{2}\left(\frac{1}{4}\right)(144\pi^2)(1)^2 = 18\pi^2$$

Also, $k = m\omega^2$

$$\Rightarrow \quad k = \left(\frac{1}{4}\right)(144\pi^2) = 36\pi^2$$

3. A → (r); B → (q); C → (p); D → (q)

Time period is given by

$$T = 2\pi\sqrt{\frac{m}{k}} = \frac{\pi}{20}\text{ s}$$

Displacement of particle is given by (taking mean position as origin) is

$$x = -A\cos(\omega t) = -10\cos\left(\frac{\pi t}{20}\right)$$

Kinetic Energy is

$$K = \frac{1}{2}m\omega^2A^2\sin^2(\omega t) = K_{max}\sin^2\left(\frac{\pi t}{20}\right)$$

Potential Energy is

$$U = \frac{1}{2}m\omega^2A^2\cos^2(\omega t) = U_{max}\cos^2(\omega t)$$

$$\Rightarrow \quad U = U_{max}\cos^2\left(\frac{\pi t}{20}\right)$$

Now, $x = -5$ cm

$$\Rightarrow \quad t = \frac{T}{6}$$

$$x = 0$$

$$\Rightarrow \quad t = \frac{T}{4}$$

Time taken to travel the first 5 cm distance is

$$t_1 = \frac{T}{6} = \frac{\pi}{120}\text{ sec}$$

Time taken to travel the next 5 cm distance is

$$t_2 = \frac{T}{4} - \frac{T}{6} = \frac{\pi}{240}\text{ sec}$$

$$U = K$$

$$\Rightarrow \quad \omega t = \frac{\pi}{4}$$

$$\Rightarrow \quad t = \frac{\pi}{160}\text{ sec}$$

$$K = \frac{K_{max}}{4}$$

$$\Rightarrow \quad \omega t = \frac{\pi}{12}$$

$$\Rightarrow \quad t = \frac{\pi}{240}\text{ sec}$$

4. A → (r); B → (r); C → (s); D → (s)

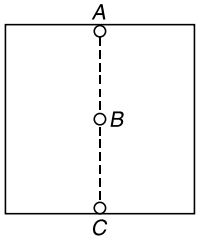
Ball will oscillate simple harmonically. The mean position in at depth

$$\rho = \alpha h$$

$$\Rightarrow \quad h = \frac{\rho}{\alpha} = AB$$

$$h_{max} = \frac{2\rho}{\alpha} = AC$$

Amplitude $= \frac{\rho}{\alpha} = AB$ or BC



From A to B : $\rho > \rho_l$, weight > upthrust

At B, $\rho = \rho_l$, weight = upthrust

From B to C, $\rho_e > \rho$, upthrust > weight.

From A to $C \rightarrow$ upthrust will increase and gravitational potential energy will decrease.

From C to A, upthrust will decrease and gravitational potential energy will increase.

From A to B, speed will increase.

From B to C, speed will decrease.

5. A → (q); B → (p); C → (s); D → (q)

Since $F = 8 - 2x$, so we have $m\omega^2 = 2$

$$\Rightarrow \quad \omega = 1\text{ rads}^{-1}$$

At mean position $F = 0$, so $x = 4$ m

Since particle is released from $x = 7$ m, so the amplitude of motion is

$$A = 7 - 4 = 3\text{ m}$$

Total mechanical energy is

$$E = \frac{1}{2}(m\omega^2)A^2 = 9\text{ J}$$

Velocity of particle at mean position i.e., $x = 4$ m is

$$v_{max} = A\omega = 3\text{ ms}^{-1}$$

The particle will oscillate between $x = 1$ m to $x = 7$ m with mean position at $x = 4$ m, so when the particle is at $x = 2.5$ m, then its distance from the mean position is

$$x = 4 - 2.5 = 1.5 = A/2$$

So, time taken by particle to go from 2.5 m to 4 m is

$$t = \frac{T}{12} = \frac{2\pi}{12} \approx 0.5\text{ s}$$

CHAPTER 3

6. A → (q); B → (s); C → (p); D → (r)
From x-t graph, $T = 8$ s

$$\Rightarrow \quad \omega = \frac{2\pi}{T} = \frac{\pi}{4} \text{ rads}^{-1}$$

Since, $F = -m\omega^2 x$
So, from F-x graph

$$10 = -m\left(\frac{\pi^2}{16}\right)(-1)$$

$$\Rightarrow \quad m = \frac{160}{\pi^2}$$

Also, from x-t graph, we observe that

$$x = 4\sin(\omega t) = 4\sin\left(\frac{\pi t}{4}\right)$$

$$\Rightarrow \quad v = \frac{dx}{dt} = \pi\cos\left(\frac{\pi t}{4}\right)$$

$$\Rightarrow \quad v_{max} = \pi$$

$$\Rightarrow \quad K_{max} = \frac{1}{2}mv_{max}^2 = \frac{1}{2}\left(\frac{160}{\pi^2}\right)(\pi^2) = 80$$

Further, spring constant, $k = m\omega^2$

$$\Rightarrow \quad k = \left(\frac{160}{\pi^2}\right)\left(\frac{\pi^2}{16}\right) = 10$$

7. A → (q); B → (s); C → (p); D → (r)

$\omega = \sqrt{\frac{k}{\mu}}$. Here, μ = reduced mass $= \frac{m_1 m_2}{m_1 + m_2} = \frac{2}{3}$ kg

$$\Rightarrow \quad \omega = \sqrt{\frac{6}{2/3}} = 3 \text{ rads}^{-1}$$

Amplitude 12 cm distributes in the inverse ratio of mass, so we get

$A_1 = 8$ cm and $A_2 = 4$ cm

Now, maximum kinetic energy for 1 kg block is

$$K_1 = \frac{1}{2}m_1(v_1)_{max}^2 = \frac{1}{2}m_1\omega^2 A_1^2$$

$$\Rightarrow \quad K_1 = \frac{1}{2}\times 1\times 9\times(8\times 10^{-2})^2 = 28.8 \text{ mJ}$$

Maximum kinetic energy for 2 kg block is

$$K_2 = \frac{1}{2}\times 2\times 9\times(4\times 10^{-2})^2 = 14.4 \text{ mJ}$$

8. A → (q); B → (r); C → (q); D → (q)
Suppose $x = A\sin(\omega t)$

then $v = \frac{dx}{dt} = \omega A\cos(\omega t)$ and $a = \frac{dv}{dt} = -\omega^2 A\sin(\omega t)$

9. Conceptual
A → (r); B → (p); C → (s); D → (p), (q)

10. A → (q); B → (s); C → (s); D → (s)
Since, $a = -\omega^2 x$, so a-x graph is straight line passing through origin.
If $v = v_0\sin(\omega t)$

then $a = \frac{dv}{dt} = v_0\omega\cos(\omega t) = v_0\omega\sqrt{1-\sin^2\omega t}$

$$\Rightarrow \quad a = \frac{dv}{dt} = v_0\omega\sqrt{1-\frac{v^2}{v_0^2}} = \omega\sqrt{v_0^2 - v^2}$$

So, a-v graph is neither a straight line nor a parabola. Further, acceleration and velocity time graphs are sine or cosine functions.

11. A → (q); B → (t); C → (p); D → (t)
On a satellite and at centre of earth $g' = 0$, so

$T \to \infty$

At pole, value of g is more than the normal value. Hence,

$T > 2$ s

Integer/Numerical Answer Type Questions

1. m = mass of block = 2 kg
In equilibrium, $kx_0 = mg$ …(1)
When displaced further by x,

$$E = \frac{1}{2}kv^2 + \frac{1}{2}I\omega^2 - mgx + \frac{1}{2}k(x + x_0)^2$$

$$\Rightarrow \quad E = \frac{1}{2}mv^2 + \frac{1}{2}(0.6MR^2)\left(\frac{v^3}{R^2}\right) - mgx + \frac{1}{2}k(x + x_0)^2$$

$$\Rightarrow \quad E = \frac{1}{2}(m + 0.6M)v^2 - mgx + \frac{1}{2}k(x + x_0)^2$$

Since, E =constant

$$\Rightarrow \quad \frac{dE}{dt} = 0$$

$$\Rightarrow \quad v(m + 0.6M)\frac{dv}{dt} - mg\left(\frac{dx}{dt}\right) + k(x_0 + x)\frac{dx}{dt} = 0$$

Substituting $\frac{dx}{dt} = v$, $\frac{dv}{dt} = a$ and $kx_0 = mg$

We get $(m + 0.6M)a = -kx$

$$\Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\left|\frac{a}{x}\right|} = \frac{1}{2\pi}\sqrt{\frac{k}{m + 0.6M}}$$

$$\Rightarrow \quad \omega = 2\pi f = \sqrt{\left|\frac{a}{x}\right|} = \sqrt{\frac{k}{m + 0.6M}}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{20}{2 + (0.6)(5)}} = \sqrt{\frac{20}{5}} = 4 \text{ rads}^{-1}$$

2. (a) $mg = kx$

$$\Rightarrow \quad x = \frac{mg}{k} = \frac{1.1\times 9.8}{120} \approx 9.0\times 10^{-2} \text{ m} = 9 \text{ cm}$$

(b) $\frac{1}{2}mv^2 + \frac{1}{2}kx_1^2 = \frac{1}{2}kx_2^2$

where, $x_1 = 9.0\times 10^{-2}$ m = 0.09 m

and $x_2 = 0.2 + 0.09 = 0.29$ m

$$\Rightarrow \quad v = \sqrt{\frac{k}{m}\left(x_2^2 - x_1^2\right)}$$

$$\Rightarrow \quad v = \sqrt{\frac{120}{1.1}(0.29\times 0.29 - 0.09\times 0.09)}$$

$$\Rightarrow \quad v = 2.88 \text{ ms}^{-1} = 3 \text{ ms}^{-1}$$

3. In equilibrium, let the spring is stretched by x_0, then

$$T = mg \quad \ldots(1)$$

Also, $T + mg\sin\theta = kx_0$ …(2)

$\Rightarrow$ $mg + mg\sin\theta = kx_0$ …(3)

Let the spring is further stretched by x then total mechanical energy of the system in this position is,

$$E = \frac{1}{2}(m+m)v^2 + \frac{1}{2}k(x+x_0)^2 - mg(x + x\sin\theta)$$

Since, E =constant

$\Rightarrow$ $\frac{dE}{dt} = 0$

$\Rightarrow$ $0 = 2mv\frac{dv}{dt} + k(x+x_0)\frac{dx}{dt} - mg(1+\sin\theta)\frac{dx}{dt}$

Substituting, $\frac{dx}{dt} = v$, $\frac{dv}{dt} = a$ and $kx_0 = mg(1+\sin\theta)$, we get

$$a = -\frac{k}{2m}x$$

Comparing with $a = -\omega^2 x$, we get

$$\omega = \sqrt{\frac{k}{2m}}$$

$\Rightarrow$ $\omega = \sqrt{\frac{20}{2(2.5)}} = \sqrt{\frac{20}{5}} = 2 \text{ rads}^{-1}$

4. Since, $\vec{g}_{\text{eff}} = \vec{g} - \vec{a}_0$

$\Rightarrow$ $g_{\text{eff}} = \sqrt{a_0^2 + g^2 - 2a_0 g\cos\beta}$

$\Rightarrow$ $\vec{g}_{\text{eff}}\sqrt{\left(\frac{g}{2}\right)^2 + g^2 - 2\times\frac{g}{2}\times g\times\cos 120^\circ}$

$\Rightarrow$ $\vec{g}_{\text{eff}} = \frac{\sqrt{7}}{2}g = 12.96 \text{ ms}^{-2}$

$\Rightarrow$ $T = 2\pi\sqrt{\frac{\ell}{g_{\text{eff}}}} = 2\pi\sqrt{\frac{0.21}{12.96}} = 0.8 \text{ s}$

$\Rightarrow$ $10T = 8 \text{ s}$

5. Applying Law of Conservation of Momentum, velocity of (ball + shell) just after collision is

$$v_1 = \frac{v_0}{2} = 3 \text{ ms}^{-1}$$

At highest point the whole system has same horizontal velocity (say v_2), where

$$v_2 = \frac{(m+m)v_1}{(m+m+M)} = \frac{2\times 3}{6} = 1 \text{ ms}^{-1}$$

Applying Law of Conservation of Energy, we get

$$\frac{1}{2}(2m)v_1^2 = \frac{1}{2}(2m+M)v_2^2 + 2mgh$$

$\Rightarrow$ $\frac{1}{2}\times 2\times 9 = \frac{1}{2}\times 6\times 1 + 2\times 10\times h$

$\Rightarrow$ $h = 0.3 \text{ m}$

Further $h = \ell(\ell - \cos\theta)$

$\Rightarrow$ $0.3 = 1.5(1-\cos\theta)$

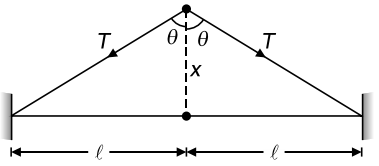
$\Rightarrow$ $\cos\theta = \frac{3}{5}$

$\Rightarrow$ $x = 3$ and $y = 5$

6. Net restoring force $F = -2T\cos\theta$

$\Rightarrow$ $F = -\frac{2Tx}{\sqrt{x^2+\ell^2}}$

If $x \ll \ell$, then $x^2 + \ell^2 \approx \ell^2$



$\Rightarrow$ $F = -\frac{2Tx}{\ell}$

$\Rightarrow$ $a = -\frac{2Tx}{m\ell}$

as $a \propto -x$, motion is simple harmonic.

$\Rightarrow$ $T = 2\pi\sqrt{\left|\frac{x}{a}\right|} = 2x\sqrt{\frac{m\ell}{2T}}$

$\Rightarrow$ $T = 2\pi\sqrt{\frac{40\times 10^{-3}\times 0.5}{2\times 10}} = 0.2 \text{ second}$

$\Rightarrow$ $5T = 1 \text{ s}$

7. In the displaced position.

$$E = \frac{1}{2}kx^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

where $I = \frac{1}{2}mR^2$ and $\omega = \frac{v}{R}$

$\Rightarrow$ $E = \frac{1}{2}kx^2 + \frac{3}{4}mv^2$

Since E =constant

$\Rightarrow$ $\frac{dE}{dt} = 0$

$\Rightarrow$ $0 = kx\frac{dx}{dt} + \frac{3}{2}kv\frac{dv}{dt}$

Substituting $\frac{dx}{dt} = v$ and $\frac{dv}{dt} = a$

$$\frac{3}{2}ma = -kx$$

Since, $a \propto -x$, motion is simple harmonic with period given by

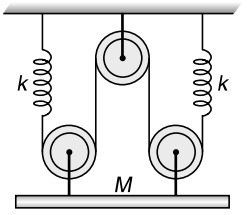
$$T = 2\pi\sqrt{\left|\frac{x}{a}\right|} = 2\pi\sqrt{\frac{3m}{2k}}$$

$\Rightarrow$ $T = 2\pi\sqrt{\frac{3(4)}{2(3)}} = 2\pi\sqrt{2} \text{ s}$

So, $* = 2$

8. If the mass M is displaced by x from its mean position each spring further stretches by $2x$.

CHAPTER 3



Net restoring force is

$F = -8kx$

$\Rightarrow \quad Ma = -8kx$

$\Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\left|\frac{a}{x}\right|}$

$\Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\frac{8k}{M}} = \frac{1}{\pi}\sqrt{\frac{2k}{M}}$

$\Rightarrow \quad x = 2$

9. Since, $kx - mg = ma$

$\Rightarrow \quad x = \frac{m(g+a)}{k} = \frac{(5)(9.8+2.2)}{1200}$

$\Rightarrow \quad x = 5\times10^{-2}$ m $= 5$ cm

10. $mgx = \frac{1}{2}kx^2$

$\Rightarrow \quad \frac{m}{k} = \frac{x}{2g}$

$\Rightarrow \quad T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{x}{2g}} = 2\pi\sqrt{\frac{4.9\times10^{-2}}{2\times9.8}}$

$T = 0.26$ s

$\Rightarrow \quad T = 2\pi\sqrt{\frac{1}{400}}$

$\Rightarrow \quad T = \frac{2\pi}{20}$

$\Rightarrow \quad T = \frac{\pi}{10}$ s

$\Rightarrow \quad * = 10$

11. Since, $2\pi\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{\frac{I}{mgR}} = 2\pi\sqrt{\frac{7mR^2}{5mgR}}$

$\Rightarrow \quad \ell = \frac{7}{5}R$

$\Rightarrow \quad \ell = \frac{7}{5}(5\text{ cm})$

$\Rightarrow \quad \ell = 7$ cm

12. (a) In equilibrium, $k\ell = mg$

$\Rightarrow \quad k = \frac{mg}{\ell}$

Substituting the proper values, we have

$k = \frac{(3)(9.8)}{0.2} = 147$ Nm^{-1}

(b) $T = 2\pi\sqrt{\frac{\ell}{g}}$

$\Rightarrow \quad \omega = \frac{2\pi}{T} = \sqrt{\frac{g}{\ell}} = \sqrt{\frac{9.8}{0.2}} = 7$ rads^{-1}

13. Restoring torque, $\tau = -k\ell\theta - k\frac{\ell}{2}\theta = -\frac{3}{2}k\ell\theta$

$\Rightarrow \quad \left(\frac{m\ell^2}{3}\right)\alpha = -\frac{3}{2}k\ell\theta$

$\Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\left|\frac{\alpha}{\theta}\right|} = \frac{1}{2\pi}\sqrt{\frac{9k}{2m\ell}}$

$\Rightarrow \quad * = 9$

14. (a) $T = 2\pi\sqrt{\frac{m}{k}}$

$\Rightarrow \quad k = \frac{4\pi^2 m}{T^2} = 4\pi^2 mf^2 = 4\pi^2(0.1)(20)^2$

$\Rightarrow \quad k = 1600$ Nm^{-1}

(b) $a_{max} = \omega^2 A = \left(\frac{k}{m}\right)(A) = \left(\frac{1600}{0.1}\right)(0.5\times10^{-2})$

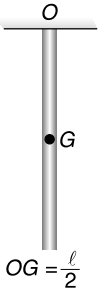
$\Rightarrow \quad a_{max} = 80$ ms^{-2}

(c) $E = \frac{1}{2}m\omega^2A^2 = \frac{1}{2}kA^2$

$\Rightarrow \quad E = \frac{1}{2}\times1600\times(0.5\times10^{-2})^2 = 0.02$ J

$\Rightarrow \quad E = 20\times10^{-3}$ J $= 20$ mJ

15. Since, $T = 2\pi\sqrt{\frac{I_0}{mg(OG)}}$



where, $I_0 = \frac{1}{3}m\ell^2$ and $OG = \frac{\ell}{2}$

$\Rightarrow \quad T = 2\pi\sqrt{\frac{\left(\frac{1}{3}m\ell^2\right)}{(m)(g)\left(\frac{\ell}{2}\right)}} = 2\pi\sqrt{\frac{2\ell}{3g}}$

$\Rightarrow \quad \omega = \frac{2\pi}{T} = \sqrt{\frac{3g}{2\ell}} = \sqrt{\frac{3(9.8)}{2(0.147)}} = 10$ rads^{-1}

16. Since, $kA = ma$

$\Rightarrow \quad A = \frac{ma}{k} = \frac{(1)(2)}{100} = 0.02$ m $= 2$ cm

$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{100}{1}} = 10$ rads^{-1}

17. In equilibrium, $mg\sin\theta = kx_0$

When further displaced by x, then by Law of Conservation of Mechanical Energy, we get

$E = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}k(x+x_0)^2 - mgx\sin\theta$

Since, E =constant

$\Rightarrow \quad \frac{dE}{dt} = 0$

$$\Rightarrow \quad 0 = mv\left(\frac{dv}{dt}\right) + I\omega\left(\frac{d\omega}{dt}\right) + k(x + x_0)\frac{dx}{dt} - mg\sin\theta\frac{dx}{dt}$$

Substituting, $\frac{dv}{dt} = a$, $\omega = \frac{v}{R}$, $I = \frac{1}{2}mR^2$

$\frac{d\omega}{dt} = \alpha = \frac{a}{R}$, $\frac{dx}{dt} = v$ and $kx_0 = mg\sin\theta$, we get

$3ma = -2kx$

$$\Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\left|\frac{a}{x}\right|} = \frac{1}{2\pi}\sqrt{\frac{2k}{3m}}$$

$$\Rightarrow \quad \omega = 2\pi f = \sqrt{\frac{2k}{3m}}$$

Substituting the values, we get

$$\omega = \sqrt{\frac{2(300)}{3(0.5)}} = \sqrt{400} = 20 \text{ rads}^{-1}$$

18. By Law of Conservation of Mechanical Energy, we have

$$\frac{1}{2}kx_0^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx_f^2 - mg(x_0 - x_f)\sin(30°)$$

$$\Rightarrow \quad \frac{1}{2}\times 310\times(0.31)^2 = \frac{1}{2}\times(1.7)v^2 + \frac{1}{2}\times 310\times(0.14)^2 - (1.7)(9.8)(0.31-0.14)\left(\frac{1}{2}\right)$$

$$\Rightarrow \quad 14.9 = 0.85v^2 + 3.0 - 1.4$$

$$\Rightarrow \quad v = 3.95 \text{ ms}^{-1}$$

$$\Rightarrow \quad v = 4 \text{ ms}^{-1}$$

19. Since, $T = 2\pi\sqrt{\frac{m/\rho gS}{\sin\theta_1 + \sin\theta_2}}$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{0.2/(13.6\times10^3\times9.8\times0.5\times10^{-4})}{(\sin 90° + \sin 60°)}}$$

$$\Rightarrow \quad T = 0.8 \text{ s}$$

$$\Rightarrow \quad 10T = 8 \text{ s}$$

20. (a) $g' = g\left(1 - \frac{h}{R}\right)$

$$\Rightarrow \quad \frac{T'}{T} = \sqrt{\frac{g}{g'}} = \left(1 - \frac{h}{R}\right)^{-\frac{1}{2}} \approx 1 + \frac{h}{2R}$$

$$\Rightarrow \quad T' - T = \frac{Th}{2R} = \Delta T$$

Time lost per day $= \frac{\Delta T}{T'}\times t$

$$\Rightarrow \quad \frac{\Delta T}{T'}\times t = \left(\frac{\frac{h}{2R}}{1 + \frac{h}{2R}}\right)(24\times3600) \text{ second}$$

$$\Rightarrow \quad (\text{Time Lost Per Day}) = 54 \text{ second}$$

(b) $g' \approx g\left(1 - \frac{2h}{R}\right)$

Proceeding in the similar manner we can show that time lost per day to be

$$\frac{\Delta T}{T'}\times t = \left(\frac{\frac{h}{R}}{1 + \frac{h}{R}}\right)(24\times3600) \text{ sec}$$

$$\Rightarrow \quad (\text{Time Lost Per Day}) = 107.9 \text{ sec} = 108 \text{ s}$$

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1. (A) $F = ma$ and $a = -\omega^2 x$

At $\frac{3T}{4}$, is $x = 0$, so $a = 0$ i.e., $F = 0$

(B) at $t = T$, $x = A$ i.e., x is maximum, so acceleration is maximum

(C) Since $v = \omega\sqrt{A^2 - x^2}$

$$\Rightarrow \quad v_{max} = A\omega \text{ at } x = 0$$

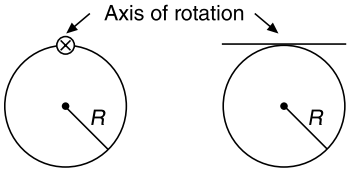
At $t = \frac{T}{4}$, $x = 0$, So $v = v_{max}$

(D) For KE = PE, we have $x = \frac{A}{\sqrt{2}}$

At $t = \frac{T}{2}$, $x = -A$ (So not possible)

Hence, the correct answer is (D).

2. Moment of inertia in case (1) is $I_1 = 2MR^2$



(1) (2)

Moment of inertia in case (2) is $I_2 = \frac{3}{2}MR^2$

$$\Rightarrow \quad T_1 = 2\pi\sqrt{\frac{I_1}{Mgd}} \text{ and } T_2 = 2\pi\sqrt{\frac{I_2}{Mgd}}$$

$$\Rightarrow \quad \frac{T_1}{T_2} = \sqrt{\frac{I_1}{I_2}} = \sqrt{\frac{2MR^2}{\frac{3}{2}MR^2}} = \frac{2}{\sqrt{3}}$$

Hence, the correct answer is (A).

CHAPTER 3

3. An elastic wire can be treated as a spring with spring constant

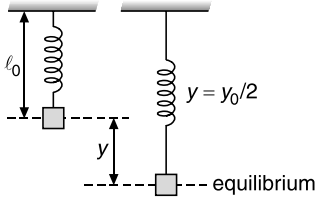
$$k = \frac{YA}{l}$$

Since, $T = 2\pi\sqrt{\frac{m}{k}}$

$$\Rightarrow \quad f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$$

Hence, the correct answer is (A).

4. Since, $y = y_0 \sin^2 \omega t$



$$\Rightarrow \quad y = \frac{y_0}{2}(1 - \cos 2\omega t)$$

$$\Rightarrow \quad y - \frac{y_0}{2} = -\frac{y_0}{2}\cos(2\omega t)$$

So, amplitude is $\frac{y_0}{2}$ and time period is $\frac{2\pi}{2\omega} = \frac{\pi}{\omega} = \frac{T}{2}$ such that

$$mg\left(\frac{y_0}{2}\right) = \frac{1}{2}k\left(\frac{y_0}{2}\right)^2$$

$$\Rightarrow \quad y_0 = \frac{2mg}{k} \qquad \ldots(1)$$

$$\Rightarrow \quad \frac{y_0}{2} = \frac{mg}{k}$$

$$\Rightarrow \quad 2\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2g}{y_0}}$$

From (1), we get

$$\omega = \sqrt{\frac{g}{2y_0}}$$

Hence, the correct answer is (B).

5. Time for 10 oscillations is $\frac{10}{5} = 2$ s

Since, $A = A_0 e^{-kt}$

$$\Rightarrow \quad \frac{1}{2} = e^{-2k}$$

$$\Rightarrow \quad \ell n 2 = 2k$$

Also, $10^{-3} = e^{-kt}$

$$\Rightarrow \quad 3\ell n 10 = kt$$

$$\Rightarrow \quad t = \frac{3\ell n 10}{k} = \frac{3\ell n 10}{\ell n 2} \times 2$$

$$\Rightarrow \quad t = 6 \times \frac{2.3}{0.69} \approx 20 \text{ s}$$

Hence, the correct answer is (D).

6. Since, $T = 2\pi\sqrt{\frac{I}{g_{\text{eff}}}}$

$$\Rightarrow \quad \frac{T'}{T} = \sqrt{\frac{g_{\text{eff}}}{g'_{\text{eff}}}}, \text{ where } g'_{\text{eff}} = g - \frac{g}{16} = \frac{15}{16}$$

$$\Rightarrow \quad \frac{T'}{T} = \sqrt{\frac{16}{15}}$$

Hence, the correct answer is (B).

7. Amplitude at $(t = 0)$ is $A_0 = e^{-0.1 \times 0} = 1$

$$\Rightarrow \quad \text{at } t = t \text{ if } A = \frac{A_0}{2}$$

$$\Rightarrow \quad \frac{1}{2} = e^{-0.1t}$$

$$\Rightarrow \quad t = 10 \ln 2 \simeq 7 \text{ s}$$

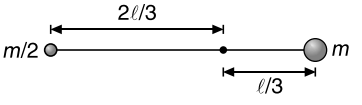
Hence, the correct answer is (A).

8. Since, $W_{\text{man}} = Mg_{\text{eff}}\ell$, where $g_{\text{eff}} = g\left(1 + \theta_0^2\right)$

$$\Rightarrow \quad W_{\text{man}} = Mg\ell\left(1 + \theta_0^2\right)$$

Hence, the correct answer is (B).

9. Since, $I = \frac{m\ell^2}{9} + \frac{m}{2}\left(\frac{4\ell^2}{9}\right) = \frac{m\ell^2}{3}$



PE at (θ_0) is $U = \frac{1}{2}k\theta_0^2$

$$\Rightarrow \quad \frac{1}{2}k\theta_0^2 = \frac{1}{2}\left(\frac{m\ell^2}{3}\right)\omega^2$$

$$\Rightarrow \quad \frac{3k\theta_0^2}{m\ell^2} = \omega^2$$

$$\Rightarrow \quad T = m\omega^2\frac{\ell}{3} = \frac{m\left(3k\theta_0^2\right)}{m\ell^2}\left(\frac{\ell}{3}\right) = \frac{k\theta_0^2}{\ell}$$

Hence, the correct answer is (B).

10. Since, $I_1 = \frac{M(2L)^2}{12} = \frac{ML^2}{3}$

Now, $I_2 = I_1 + 2\left(\frac{mL^2}{4}\right) = \frac{ML^2}{3} + \frac{mL^2}{2}$

Since $T = 2\pi\sqrt{\frac{I}{C}}$

$$\Rightarrow \quad \omega \propto \frac{1}{\sqrt{I}}$$

$$\Rightarrow \quad \frac{\omega_1}{\omega_2} = \frac{1}{0.8} = \sqrt{\frac{\frac{M}{3} + \frac{m}{2}}{M/3}}$$

$$\Rightarrow \quad \frac{m}{M} = 0.375$$

Hence, the correct answer is (C).

11. Since KE = PE

$$\Rightarrow \quad \frac{1}{2}m\omega^2\left(A^2 - x^2\right) = \frac{1}{2}m\omega^2 x^2$$

$$\Rightarrow \quad A^2 - x^2 = x^2$$

$$\Rightarrow \quad x = \pm\frac{A}{\sqrt{2}}$$

Hence, the correct answer is (A).

12. Since, $a = -\omega^2 x = -\omega^2 \times 4$

Also, $v = \omega\sqrt{A^2 - x^2} = \omega \times \sqrt{5^2 - 4^2} = 3\omega$

Given that $|v| = |a|$

$\Rightarrow \quad 3\omega = 4\omega^2$

$\Rightarrow \quad \omega = \frac{3}{4}$ rads^{-1}

$\Rightarrow \quad T = \frac{2\pi}{3/4} = \frac{8\pi}{3}$

Hence, the correct answer is (C).

13. $\frac{KE}{PE} = \frac{\frac{1}{2}kA^2 - \frac{1}{2}kA^2\sin^2\frac{\pi t}{90}}{\frac{1}{2}kA^2\sin^2\frac{\pi t}{90}} = \frac{1}{3}$ $\quad \left\{\because \frac{\pi t}{90} = \frac{7\pi}{3}\right\}$

*No given option is correct.

14. Since, $T = 2\pi\sqrt{\frac{I}{g_{\text{eff}}}}$

$\Rightarrow \quad \omega = \sqrt{\frac{g_{\text{eff}}}{I}}$

$\Rightarrow \quad \frac{\Delta\omega}{\omega} = \frac{1}{2}\frac{\Delta g_{\text{eff}}}{g_{\text{eff}}} = \frac{1}{2}\frac{(2\omega^2 A)}{g}$

$\Rightarrow \quad \frac{\Delta\omega}{\omega} = 10^{-3}$ rads^{-1}

Hence, the correct answer is (A).

15. Since, $T = 2\pi\sqrt{\frac{\ell}{g}}$

$\Rightarrow \quad \frac{T_1}{T_2} = \sqrt{\frac{g_2}{g_1}}$

$\Rightarrow \quad T_2 = 2\sqrt{\frac{g_1}{g_2}}$

Since, $g_2 = \frac{3GM}{(3R)^2} = \frac{g}{3}$ and $g_1 = g$

$\Rightarrow \quad T_2 = 2\sqrt{3}$ s

Hence, the correct answer is (D).

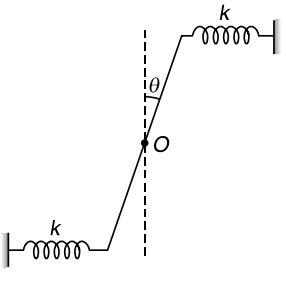
16. Since, $\tau = I\alpha$

$\Rightarrow \quad \frac{M\ell^2}{12}\alpha = 2k\left(\frac{\ell}{2}\right)\left(\frac{\ell}{2}\right)\theta$

$\Rightarrow \quad \frac{M\ell^2}{12}\alpha = \frac{-k\ell^2}{2}\theta$

$\Rightarrow \quad \omega = \sqrt{\frac{6k}{m}}$

$\Rightarrow \quad \nu = \frac{1}{2\pi}\sqrt{\frac{6k}{m}}$



Hence, the correct answer is (D).

17. Since, $y = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$

$\Rightarrow \quad y = 10\sin\left(3\pi t + \frac{\pi}{3}\right)$

So, $A = 10$ cm, $\frac{2\pi}{T} = 3\pi$

$\Rightarrow \quad T = \frac{2}{3}$ s

Hence, the correct answer is (A).

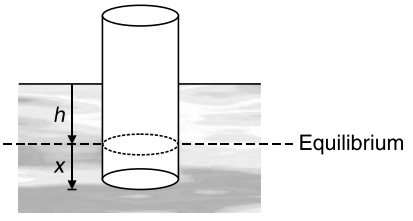
18. In equilibrium, $mg = U = (\rho Ah)g$

When displaced x from equilibrium position, then

$F_{\text{restoring}} = U - mg$

$\Rightarrow \quad F_{\text{restoring}} = \rho A(h + x)g - \rho Ahg$

$\Rightarrow \quad ma = -\rho Agx$



$\Rightarrow \quad a = \ddot{x} = -\left(\frac{\rho Ag}{m}\right)x$

$\Rightarrow \quad \ddot{x} + \omega^2 x = 0$

$\Rightarrow \quad \omega = \sqrt{\frac{10^3 \times A \times g}{310 \times 10^{-3}}} = \sqrt{\frac{10^3 \times \pi \times 6.25 \times 10^{-4} \times 10}{310 \times 10^{-3} \times 100}}$

$\Rightarrow \quad \omega = \sqrt{62.5} \approx 8$ rads^{-1}

*No given option is correct.

19. Frequency of a particle executing SHM is

$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$\Rightarrow \quad k = 4\pi^2 \times \nu^2 \times m$

where, $\nu = 10^{12}$ s^{-1}, $m = \frac{108}{6.02 \times 10^{23}} \times 10^{-3}$ kg, $k = ?$

$\Rightarrow \quad k = 4(3.14)^2(10^{12})^2\left(\frac{108 \times 10^{-3}}{6.02 \times 10^{23}}\right) = 7.1$ Nm^{-1}

Hence, the correct answer is (B).

20. For $A = B$, $a = b$ and $\delta = \frac{\pi}{2}$, we get

$x^2 + y^2 = A^2$ i.e., a circle

For $A \neq B$, $a = b$ and $\delta = 0$, we get

$y = \left(\frac{B}{A}\right)x$

i.e., a straight line

For $A = B$, $a = 2b$ and $\delta = \frac{\pi}{2}$, we get

$x^2 + y^2 = A^2[\cos^2(2bt) + \sin^2 bt]$

For $A \neq B$, $a = b$ and $\delta = \frac{\pi}{2}$, we get

$x = A\sin\left(at + \frac{\pi}{2}\right)$ and $y = B\sin(at)$

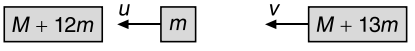
CHAPTER 3

$\Rightarrow \quad \dfrac{x^2}{A^2}+\dfrac{y^2}{B^2}=1$ i.e., an ellipse

Hence, the correct answer is (D).

21. In first collision, momentum mu will be imparted to system. In second collision, when momentum of $(M+m)$ is in opposite direction with momentum of particle mu will make its momentum zero.

On 13^{th} collision,



Applying momentum conservation, we get

$$mu=(M+13m)v$$

$\Rightarrow \quad v=\dfrac{mu}{M+13m}=\dfrac{u}{15}$

Also, $v=A\omega$

$\Rightarrow \quad \dfrac{u}{15}=A\sqrt{\dfrac{k}{M+13m}}$

$\Rightarrow \quad A=\dfrac{1}{15}\sqrt{\dfrac{75}{1}}=\dfrac{1}{\sqrt{3}}$

Hence, the correct answer is (A).

22. Different positions of a particle executing simple harmonic motion is given by

$$a=A\sin\omega t_0,\ b=A\sin 2\omega t_0,\ c=A\sin 3\omega t_0$$

Now, $a+c=A(\sin\omega t_0+\sin 3\omega t_0)=2A\sin 2\omega t_0\cos\omega t_0$

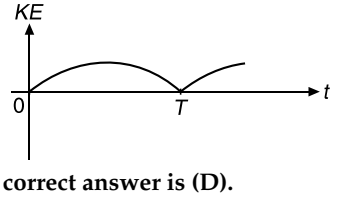
$\Rightarrow \quad \dfrac{a+c}{b}=2\cos\omega t_0$

$\Rightarrow \quad \omega=\dfrac{1}{t_0}\cos^{-1}\left(\dfrac{a+c}{2b}\right)$

$\Rightarrow \quad \nu=\dfrac{1}{2\pi t_0}\cos^{-1}\left(\dfrac{a+c}{2b}\right)$

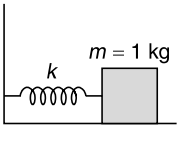
Hence, the correct answer is (D).

23. $\text{KE}=\dfrac{1}{2}m\omega^2A^2\cos^2(\omega t)$



Hence, the correct answer is (D).

24. The 1 kg block attached to a spring vibrates with a frequency of 1 Hz, so



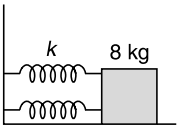
$$\nu=\frac{1}{2\pi}\sqrt{\frac{k}{m}}=1\text{ Hz}$$

$\Rightarrow \quad k=4\pi^2\ \text{Nm}^{-1}$

When two springs are attached in parallel to an 8 kg block, then

$$k_{\text{eq}}=k+k=2k$$

Frequency, $\nu'=\dfrac{1}{2\pi}\sqrt{\dfrac{k_{\text{eq}}}{m'}}$



$\Rightarrow \quad \nu'=\dfrac{1}{2\pi}\sqrt{\dfrac{2k}{8}}=\dfrac{1}{2\pi}\sqrt{\dfrac{2\times 4\pi^2}{8}}=\dfrac{1}{2}\text{ Hz}$

Hence, the correct answer is (B).

25. For simple harmonic motion,

$$\frac{\text{Maximum acceleration}}{\text{Maximum velocity}}=10$$

$\Rightarrow \quad \dfrac{A\omega^2}{A\omega}=10$

$\Rightarrow \quad \omega=10$

At $t=0$, displacement is $x=5$ cm

Since, $x=A\sin(\omega t+\phi)$

$\Rightarrow \quad 5=A\sin\left(0+\dfrac{\pi}{4}\right)$

$\Rightarrow \quad 5=A\sin\dfrac{\pi}{4}$

$\Rightarrow \quad A=5\sqrt{2}$ m

Maximum acceleration is

$$a_{\max}=A\omega^2=10^2\times 5\sqrt{2}=500\sqrt{2}\ \text{ms}^{-2}$$

Hence, the correct answer is (A).

26. Given that, $m=0.1$ kg, $k=640\ \text{Nm}^{-1}$

$$b=10^{-2}\ \text{kgs}^{-1},\ E=\frac{E_0}{2},\ t=?$$

Amplitude of damped oscillation is $A=A_0e^{\frac{-bt}{2m}}$

Total energy of the system is $E=E_0e^{\frac{-bt}{m}}$

$\Rightarrow \quad \dfrac{bt}{m}=\ln\left(\dfrac{E_0}{E}\right)$

$\Rightarrow \quad t=\dfrac{m}{b}\ln\left(\dfrac{E_0}{E}\right)=\dfrac{0.1}{10^{-2}}\ln(2)$

$\Rightarrow \quad t=10\times 0.693=6.93\text{ s}\approx 7\text{ s}$

Hence, the correct answer is (C).

27. Since, $v^2=\omega^2\left(A^2-\left(\dfrac{2A}{3}\right)^2\right)$ ...(1)

where A is initial amplitude and ω is angular frequency. Let new amplitude be A', then

$$(3v)^2=\omega^2\left(A'^2-\left(\frac{2A}{3}\right)^2\right) \qquad \ldots(2)$$

From equation (1) and equation (2), we get

$$\frac{1}{9}=\frac{A^2-\dfrac{4A^2}{9}}{A'^2-\dfrac{4A^2}{9}}$$

$\Rightarrow \quad A'=\dfrac{7A}{3}$

Hence, the correct answer is (A).

28. Since, $T = 2\pi\sqrt{\frac{l}{g}}$ and $T_M = 2\pi\sqrt{\frac{l+\Delta l}{g}}$

$$\Rightarrow \quad \frac{T_M}{T} = \sqrt{\frac{l+\Delta l}{l}}$$

$$\Rightarrow \quad \left(\frac{T_M}{T}\right)^2 = 1 + \frac{\Delta l}{l}$$

Also, $Y = \frac{F/A}{\Delta\ell/\ell} = \frac{Mg/A}{\Delta\ell/\ell}$

$$\Rightarrow \quad \frac{\Delta l}{l} = \frac{Mg}{AY}$$

$$\Rightarrow \quad \left(\frac{T_M}{T}\right)^2 = 1 + \frac{Mg}{AY}$$

$$\Rightarrow \quad \frac{1}{Y} = \frac{A}{Mg}\left[\left(\frac{T_M}{T}\right)^2 - 1\right]$$

Hence, the correct answer is (A).

29. For a simple pendulum in harmonic motion,

(i) at the mean position, KE is maximum and PE is minimum.

(ii) at the extreme position, PE is maximum and KE is minimum.

Hence, the correct answer is (B).

30. Given that, $x = a\sin\omega t$ and $y = a\sin(2\omega t)$

$$\Rightarrow \quad y = 2a\sin\omega t\cos\omega t$$

$$\Rightarrow \quad y = 2x\sqrt{1 - \frac{x^2}{a^2}} \qquad \left\{\because \sin(\omega t) = \frac{x}{a}\right\}$$

$$\Rightarrow \quad y = \frac{2}{a}x\sqrt{(a-x)(a+x)}$$

So, $y = 0$ at $x = 0$ and at $x = \pm a$

Hence, the correct answer is (C).

31. It is given that oscillator is at rest at $t = 0$ i.e., at $t = 0$, $v = 0$.

So, we can check options for $v = \frac{dx}{dt} = 0$ by substituting $t = 0$ in value of v.

$$\Rightarrow \quad v = \int a\,dt = \int \frac{F}{m}dt$$

$$\Rightarrow \quad v = \frac{1}{m}\int \sin t\,dt$$

$$\Rightarrow \quad v = \frac{1}{m}(-\cos t)\Big|_0^t = \frac{1}{m}(1 - \cos t)$$

(A) If $x \propto \sin t + \frac{1}{2}\sin 2t$

$v \propto \cos t + \frac{1}{2}\times 2\cos 2t$ at $t = 0$, $v \propto 1 + 1 = 2 \neq 0$

(B) If $x \propto \sin t + \frac{1}{2}\cos 2t$

$v \propto \cos t + \frac{1}{2}\times 2(-\sin 2t)$ at $t = 0$, $v \propto 1 - 0 \neq 0$

(C) If $x \propto \cos t - \frac{1}{2}\sin 2t$

$v \propto -\sin t - \frac{1}{2}\times 2\cos 2t$ at $t = 0$, $v \propto -1 \neq 0$

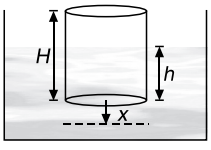
(D) If $x \propto \sin t - \frac{1}{2}\sin 2t$

$v \propto \cos t - \frac{1}{2}\times 2\cos 2t$ at $t = 0$, $v \propto 1 - 1 = 0$

So, in OPTION (D) $v = 0$, at $t = 0$

Hence, the correct answer is (B).

32. Let the block of height H be floating with depth h inside the liquid.



Then at equilibrium

$$M_{\text{Block}}g = F_{\text{up}}$$

$$(AH\rho_B)g = (Ah)\rho_L g \qquad \ldots(1)$$

When block depressed slightly by distance x then

$$F_{\text{Net}} = Mg - F'_{\text{up}} = AH\rho_B g - A(h+x)\rho_L g$$

$$\Rightarrow \quad F_{\text{net}} = -Ax\rho_L g \qquad \{\text{Using equation (1)}\}$$

$$H\rho_B\frac{d^2x}{dt^2} = -x\rho_L g$$

$$\frac{d^2x}{dt^2} = -\frac{\rho_L g}{H\rho_B}x = -\omega^2 x$$

$$\Rightarrow \quad \omega^2 = \frac{\rho_L g}{H\rho_B}$$

For simple pendulum $\omega^2 = \frac{g}{l}$

$$\Rightarrow \quad l = \frac{H\rho_B}{\rho_L} = \frac{650\times 54}{900} = 39 \text{ cm}$$

Hence, the correct answer is (C).

33. Amplitude in damped oscillation is given by $A = A_0 e^{-\beta t}$

Energy, $E \propto A^2$

$$\Rightarrow \quad \sqrt{E} = \sqrt{E_0}e^{-\beta t} \text{ where } E_0 \text{ is initial energy}$$

Here, $E_0 = 45$ J, $T = 1$ s, $E = 15$ J

$$t = nT = 15\times 1 = 15 \text{ s}$$

Then, $\sqrt{15} = \sqrt{45}e^{-\beta\times 15}$

$$3^{-\frac{1}{2}} = e^{-15\beta}$$

Taking log on both sides $-\frac{1}{2}\ln(3) = -15\beta$

$$\beta = \frac{\ln 3}{30}$$

Hence, the correct answer is (A).

34. Since the particle starts from rest i.e., at $t = 0$, $v = 0$, so it must be starting from the extreme position and hence we have

$$x = A\cos(\omega t)$$

where x is displacement of particle from the mean position and A is the amplitude. Now at $t = \tau$, the particle travels a distance a, so

CHAPTER 3

$$x = A - a = A\cos(\omega\tau) \quad \ldots(1)$$

and similarly

$$x = A - 3a = A\cos(2\omega\tau) \quad \ldots(2)$$

$$\Rightarrow \quad a = A[1-\cos(\omega\tau)] \quad \{\text{from (1)}\}$$

$$\Rightarrow \quad 3a = A[1-\cos(2\omega\tau)] \quad \{\text{from (2)}\}$$

$$\Rightarrow \quad \frac{1}{3} = \frac{1-\cos(\omega\tau)}{1-\cos(2\omega\tau)}$$

$$\Rightarrow \quad \frac{1}{3} = \frac{1-\cos(\omega\tau)}{2\sin^2(\omega\tau)}$$

$$\Rightarrow \quad \frac{1}{3} = \frac{1-\cos(\omega\tau)}{2[1-\cos^2(\omega\tau)]}$$

$$\Rightarrow \quad \frac{1}{3} = \frac{1-\cos(\omega\tau)}{2[1+\cos(\omega\tau)][1-\cos(\omega\tau)]}$$

$$\Rightarrow \quad 1+\cos(\omega\tau) = \frac{3}{2}$$

$$\Rightarrow \quad \cos(\omega\tau) = \frac{1}{2} \text{ i.e., } \omega\tau = \frac{\pi}{3}$$

$$\Rightarrow \quad \left(\frac{2\pi}{T}\right)\tau = \frac{\pi}{3} \text{ i.e., } T = 6\tau$$

Hence, the correct answer is (D).

35. The amplitude of a damped oscillator at a given instant of time t is given by $A = A_0 e^{-\frac{bt}{2m}}$

where A_0 is its amplitude in the absence of damping, b is the damping constant?

As per question

After 5 s (i.e., $t = 5$ s), its amplitude becomes

$$0.9A_0 = A_0 e^{-\frac{b(5)}{2m}} = A_0 e^{-\frac{5b}{2m}}$$

$$\Rightarrow \quad 0.9 = e^{-\frac{5b}{2m}} \quad \ldots(1)$$

After 10 more second (i.e., $t = 15$ s), its amplitude becomes

$$\alpha A_0 = A_0 e^{\frac{-b(15)}{2m}} = A_0 e^{-\frac{15b}{2m}}$$

$$\alpha = \left(e^{-\frac{5b}{2m}}\right)^3 = (0.9)^3 \quad \{\text{using (1)}\}$$

$$\Rightarrow \quad \alpha = 0.729$$

Hence, the correct answer is (D).

36. $PV^\gamma = \text{constant}$

Differentiating, we get

$$\frac{dP}{P} + \gamma\frac{dV}{V} = 0$$

$$\Rightarrow \quad dP = -\frac{\gamma P\,dV}{V} = -\frac{\gamma PAx}{V}$$

Taking $P = P_0$, $V = V_0$, we get $F_{\text{restoring}} = -\frac{\gamma PA^2x}{V}$

$$\Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\frac{k}{M}} = \frac{1}{2\pi}\sqrt{\frac{\gamma P_0A^2}{V_0M}}$$

Hence, the correct answer is (C).

37. According to the problem, we are given that

$$a = -bv$$

The equation of motion of a damped harmonic oscillator is written as

$$F = -ky - b'v$$

The damping force is $F = -b'v$, where b' is the damping constant

$$\Rightarrow \quad \frac{d^2y}{dt} + \left(\frac{b'}{m}\right)\frac{dy}{dt} + \frac{k}{m}y = 0$$

$$\Rightarrow \quad \frac{d^2y}{dt} + b\frac{dy}{dt} + \frac{k}{m}y = 0 \qquad \left\{\therefore\ b = \frac{b'}{m}\right\}$$

Actually, we take damping constant as b with force, but here in the problem it is given to be b with acceleration, so, we have written the actual damping constant as b'.

For this case of a damped oscillator, the amplitude is not constant and varies with time as

$$A' = Ae^{-\left(\frac{b'}{2m}\right)t} = Ae^{-\left(\frac{b}{2}\right)t} \qquad \left\{\because\ b = \frac{b'}{m}\right\}$$

According to the problem, we have

$$A' = \frac{A}{e}$$

$$\Rightarrow \quad \frac{A}{e} = Ae^{-bt/2}$$

$$\Rightarrow \quad e^{bt/2} = e$$

$$\Rightarrow \quad \frac{bt}{2} = 1$$

$$\Rightarrow \quad t = \frac{2}{b}$$

Hence, the correct answer is (C).

38. Let $x_1 = A\sin(\omega t)$ and $x_2 = A\sin(\omega t + \phi)$

$$\Rightarrow \quad x_1 - x_2 = 2A\sin\left(\frac{\phi}{2}\right)\cos\left(\omega t + \frac{\phi}{2}\right)$$

Given that, $2A\sin\left(\frac{\phi}{2}\right) = A$

$$\Rightarrow \quad \frac{\phi}{2} = \frac{\pi}{6}$$

$$\Rightarrow \quad \phi = \frac{\pi}{3}$$

Hence, the correct answer is (B).

39. Since, $T_1 = 2\pi\sqrt{\frac{M}{k}}$ $\quad \ldots(1)$

When a mass m is placed on mass M, the new system is of mass $(M+m)$ attached to the spring. So, new time period of oscillation is

$$T_2 = 2\pi\sqrt{\frac{(m+M)}{k}} \quad \ldots(2)$$

If v_1 is the velocity of mass M passing through mean position and v_2 is the velocity of mass $(m+M)$ passing through

mean position, then using law of conservation of linear momentum, we get

$$Mv_1 = (m+M)v_2$$

where $v_1 = A_1\omega_1$ and $v_2 = A_2\omega_2$

$$\Rightarrow \quad M(A_1\omega_1) = (m+M)(A_2\omega_2)$$

$$\Rightarrow \quad \frac{A_1}{A_2} = \frac{(m+M)}{M}\frac{\omega_2}{\omega_1} = \left(\frac{m+M}{M}\right)\times\frac{T_1}{T_2}$$

$$\Rightarrow \quad \frac{A_1}{A_2} = \sqrt{\frac{m+M}{M}}$$

Hence, the correct answer is (D).

40. For a particle executing simple harmonic motion, we have

Acceleration, $a = -\omega^2 x$ where ω is a constant $= \frac{2\pi}{T}$

$$\Rightarrow \quad a = -\frac{4\pi^2}{T^2}x$$

$$\Rightarrow \quad \frac{aT}{x} = -\frac{4\pi^2}{T}$$

Since, the period of oscillation T is a constant, so

$$\frac{aT}{x} = \text{constant}$$

Hence, the correct answer is (B).

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Single Correct Choice Type Problems

1. For the block, $T = \frac{2\pi}{\omega} = 6$ s

For block, the position in 1 s will be

$$x = 0.2\cos\omega t$$

At $t = 1$ s, $x = 0.1$ m i.e., at $t = 1$ s, block will be at a distance 4.9 m + 0.1 m = 5 m from wall

So, range of pebble is also $R = 5$ m

$$\Rightarrow \quad R = (v\cos 45°)(1\text{ s}) = 5\text{ m}$$

$$\Rightarrow \quad v = 5\sqrt{2} = \sqrt{50}\text{ ms}^{-1}$$

Hence, the correct answer is (A).

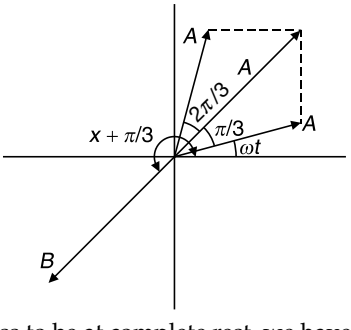
2. The frequency or time period of SHM depends on variable force. It does not depend on constant external force. Constant external force can only change the mean position. In the given problem, the mean position is at natural length of spring in the absence of electric field, whereas in the presence of electric field mean position will be obtained after a compression of x_0, where x_0 is given by

$$Kx_0 = QE$$

$$\Rightarrow \quad x_0 = \frac{QE}{K}$$

Hence, the correct answer is (A).

3. The situation is shown in the phasor diagram drawn.



For the mass to be at complete rest, we have

$$B = A \text{ and } \phi = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$$

Hence, the correct answer is (B).

4. From graph $x = 1\sin\left(\frac{2\pi}{8}t\right)$

Since $a = -\omega^2 A\sin(\omega t)$

$$\text{At } t = \frac{4}{3}\text{ s}, \quad a = -\left(\frac{\pi^2}{16}\right)(1)\left(\sin\frac{2\pi}{8}\right)\left(\frac{4}{3}\right)$$

$$\Rightarrow \quad a = -\frac{\sqrt{3}}{32}\pi^2\text{ cms}^{-2}$$

Hence, the correct answer is (B).

5. Internal forces in the springs are same, so we have

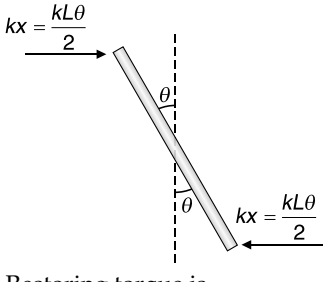
$$k_1x_1 = k_2x_2$$

Also, $x_1 + x_2 = A$

$$\Rightarrow \quad x_1 = \frac{k_2A}{k_1+k_2}$$

Hence, the correct answer is (B).

6.



Restoring torque is

$$\tau = -\left(k\frac{L}{2}\theta\right)\frac{L}{2}\times 2$$

$$\Rightarrow \quad \tau = -\frac{kL^2\theta}{2}$$

$$\Rightarrow \quad \alpha = \frac{\tau}{I} = \frac{-kL^2\theta/2}{ML^2/12}$$

$$\Rightarrow \quad \alpha = -\frac{6k\theta}{M} = -\omega^2\theta$$

$$\Rightarrow \quad \omega = \sqrt{\frac{6k}{M}} = 2\pi f$$

$$\Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\frac{6k}{M}}$$

Hence, the correct answer is (C).

7. Angular frequency of the system is

$$\omega = \sqrt{\frac{k}{m+m}} = \sqrt{\frac{k}{2m}}$$

Maximum acceleration of the system will be

$$\omega^2 A = \frac{kA}{2m}$$

This maximum acceleration to the lower block is provided by friction, so we have

$$f_{max} = ma_{max} = m\omega^2 A = m\left(\frac{kA}{2m}\right) = \frac{kA}{2}$$

Hence, the correct answer is (A).

8. Since, $y = Kt^2$

$$\Rightarrow \quad \frac{d^2y}{dt^2} = 2K$$

$$\Rightarrow \quad a_y = 2 \text{ ms}^{-2} \qquad \{\because K = 1 \text{ ms}^{-2}\}$$

Since, $T_1 = 2\pi\sqrt{\frac{\ell}{g}}$ and $T_2 = 2\pi\sqrt{\frac{\ell}{g+a_y}}$

$$\Rightarrow \quad \frac{T_1^2}{T_2^2} = \frac{g+a_y}{g} = \frac{10+2}{10} = \frac{6}{5}$$

Hence, the correct answer is (A).

9. Potential energy is minimum (in this case zero) at mean position $(x = 0)$ and maximum at extreme positions $(x = \pm A)$.

At time $t = 0$, $x = A$. Hence, PE should be maximum. Therefore, graph I is correct. Further in graph III, PE is minimum at $x = 0$. Hence, this is also correct.

Hence, the correct answer is (A).

10. In SHM, velocity of particle also oscillates simple harmonically. Speed is more near the mean position and less near the extreme positions. Therefore, the time taken for the particle to go from 0 to $\frac{A}{2}$ will be less than the time taken to go it from $\frac{A}{2}$ to A, or $T_1 < T_2$.

Conceptual Note(s)

From the equations of SHM we can show that

$$T_1 = T_{0\to A/2} = \frac{T}{12}$$

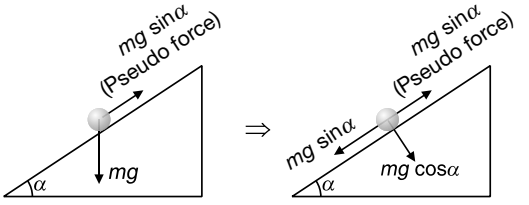
and $t_2 = T_{A/2\to A} = \frac{T}{6}$

So, $T_1 + T_2 = T_{0\to A} = \frac{T}{4}$

Hence, the correct answer is (B).

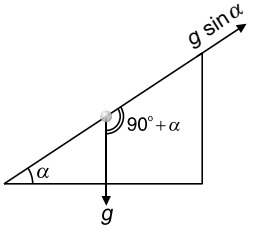
11. Free body diagram of bob of the pendulum with respect to the accelerating frame of reference is as shown in figure.

Net force on the bob is $F_{net} = mg\cos\alpha$



So, net acceleration of the bob is $g_{eff} = g\cos\alpha$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{L}{g_{eff}}} = 2\pi\sqrt{\frac{L}{g\cos\alpha}}$$



Conceptual Note(s)

Whenever point of suspension is accelerating take

$$T = 2\pi\sqrt{\frac{L}{g_{eff}}}$$

where, $\vec{g}_{eff} = \vec{g} - \vec{a}$

$\vec{a}$ = Acceleration of point of suspension

In this question $\vec{a} = g\sin\alpha$ (down the plane)

$$\Rightarrow \quad |\vec{g} - \vec{a}| = g_m$$

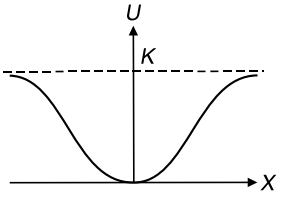
$$\Rightarrow \quad g_{eff} = g\sqrt{1+\sin^2\alpha + 2\sin\alpha\cos(90^\circ+\alpha)}$$

$$\Rightarrow \quad g_{eff} = g\cos\alpha$$

Hence, the correct answer is (B).

12. Since, $U(x) = k\left(1 - e^{-x^2}\right)$

It is an exponentially increasing graph of potential energy (U) with x^2. Therefore, U versus x graph will be as shown.



From the graph it is clear that at origin.

Potential energy U is minimum (therefore, kinetic energy will be maximum) and force acting on the particle is also zero because

$$F = \frac{-dU}{dx} = -(\text{slope of } U\text{-}x \text{ graph}) = 0$$

Therefore, origin is the stable equilibrium position. Hence, particle will oscillate simple harmonically about $x = 0$ for small displacements. Therefore, correct OPTION is (D).

(A), (B) and (C) OPTIONS are wrong due to following reasons:

(a) At equilibrium position $F = \frac{-dU}{dx} = 0$ i.e., slope U-x graph should be zero and from the graph we can see that slope is zero at $x = 0$ and $x = \pm\infty$.
Now among these equilibriums stable equilibrium position is that where U is minimum (Here $x = 0$).
Unstable equilibrium position is that where U is maximum (Here none).
Neutral equilibrium position is that where U is constant (Here $x = \pm\infty$)
Therefore, OPTION (A) is wrong.

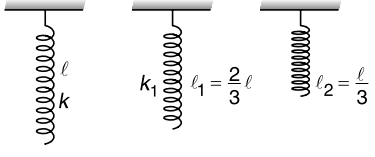
(b) For any finite non-zero value of x, force is directed towards the origin because origin is in stable equilibrium position. Therefore, OPTION (B) is incorrect.

(c) At origin, potential energy is minimum, hence kinetic energy will be maximum. Therefore, OPTION (C) is also wrong.

Hence, the correct answer is (D).

13. Since for a spring of force constant k and natural length ℓ, we have

$$k\ell = \text{constant}$$



Now, since $\ell_1 = 2\ell_2$

$\Rightarrow \quad \ell_1 = \frac{2}{3}\ell$

$\Rightarrow \quad k_1 = \frac{3}{2}k$

Hence, the correct answer is (B).

14. Given that, $U(x) = k|x|^3$

$\Rightarrow \quad [k] = \frac{[U]}{[x^3]} = \frac{[ML^2T^{-2}]}{[L^3]} = [ML^{-1}T^{-2}]$

Now, time period may depend on mass, amplitude and k, so

$$T \propto (\text{mass})^x (\text{amplitude})^y (k)^z$$

$\Rightarrow \quad [M^0 L^0 T] = [M]^x [L]^y [ML^{-1}T^{-2}]^z$

$\Rightarrow \quad [M^0 L^0 T] = [M^{x+z} L^{y-z} T^{-2z}]$

Applying the Principle of Homogeneity and equating the powers, we get

$$-2z = 1 \text{ and } y - z = 0 \text{ or } z = -\frac{1}{2}$$

$\Rightarrow \quad z = -\frac{1}{2}$ and $y = z = -\frac{1}{2}$

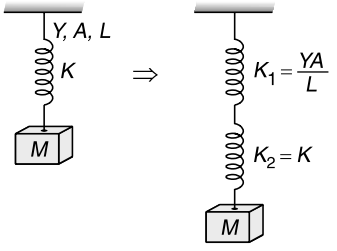
$\Rightarrow \quad T \propto (\text{amplitude})^{-1/2}$

$\Rightarrow \quad T \propto (a)^{-1/2}$

$\Rightarrow \quad T \propto \frac{1}{\sqrt{a}}$

Hence, the correct answer is (A).

15. Since, $K_{eq} = \frac{K_1 K_2}{K_1 + K_2} = \frac{L}{\frac{YA}{L} + K} = \frac{YAK}{YA + LK}$



$\Rightarrow \quad T = 2\pi\sqrt{\frac{m}{K_{eq}}} = 2\pi\sqrt{\frac{m(YA + LK)}{YAK}}$

Conceptual Note(s)

Equivalent force constant for a wire is given by $k = \frac{YA}{L}$. Since, in case of a wire, $F = \left(\frac{YA}{L}\right)x$ and in case of spring, $F = kx$. Comparing these two, we find the equivalent spring constant of the wire is $k = \frac{YA}{L}$.

Hence, the correct answer is (B).

16. Modulus of Rigidity, $\eta = \frac{F}{A\theta}$

where, $A = L^2$ and $\theta = \frac{x}{L}$

Therefore, restoring force, $F = -\eta A\theta = -\eta L x$

$\Rightarrow \quad$ Acceleration, $a = \frac{F}{M} = -\frac{\eta L}{M}x$

Since, $a \propto -x$, oscillations are simple harmonic in nature, time period of which is given by

$$T = 2\pi\sqrt{\left|\frac{\text{displacement}}{\text{acceleration}}\right|} = 2\pi\sqrt{\left|\frac{x}{a}\right|} = 2\pi\sqrt{\frac{M}{\eta L}}$$

Hence, the correct answer is (D).

17. When cylinder is displaced by an amount x from its mean position, spring force and upthrust both will increase. Hence,

$$\begin{pmatrix}\text{Net Restoring}\\ \text{Force}\end{pmatrix} = \begin{pmatrix}\text{Extra Spring}\\ \text{Force}\end{pmatrix} + \begin{pmatrix}\text{Extra}\\ \text{Upthrust}\end{pmatrix}$$

$\Rightarrow \quad F = -(kx + Ax\rho g)$

$\Rightarrow \quad a = -\left(\frac{k + \rho Ag}{M}\right)x$

$\Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\left|\frac{a}{x}\right|} = \frac{1}{2\pi}\sqrt{\frac{k + \rho Ag}{M}}$

Hence, the correct answer is (B).

18. Since, $(v_{max})_A = (v_{max})_B$

$\Rightarrow \quad A_1\omega_1 = A_2\omega_2$

CHAPTER 3

$\Rightarrow \quad A_1\left(\frac{2\pi}{T_1}\right) = A_2\left(\frac{2\pi}{T_2}\right)$

$\Rightarrow \quad \frac{A_1}{A_2} = \frac{T_1}{T_2} = \sqrt{\frac{K_2}{K_1}} \qquad \left\{\because T = 2\pi\sqrt{\frac{m}{k}}\right\}$

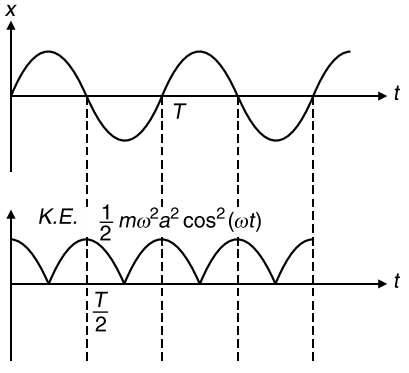
Hence, the correct answer is (D).

19. Let $x = a\sin(\omega t)$

$\Rightarrow \quad \dot{x} = v = a\omega\cos(\omega t)$

$\Rightarrow \quad \text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2a^2\cos^2(\omega t)$

$\Rightarrow \quad \text{PE} = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2a^2\sin^2(\omega t)$



So, x has a period T i.e., frequency f and kinetic energy has a period $\frac{T}{2}$ i.e., frequency $2f$

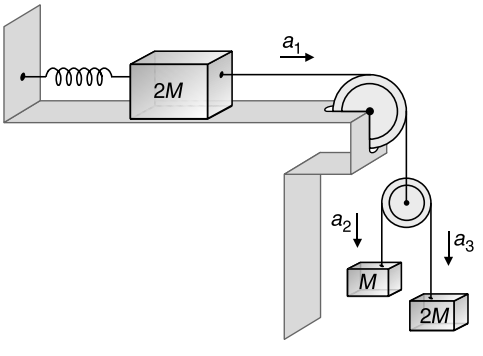
Hence, the correct answer is (C).

Multiple Correct Choice Type Problems

1. Using constraint relations, we get

$2a_1 = a_2 + a_3$

$\Rightarrow \quad a_1 - a_3 = a_2 - a_1$



Also $2mg - T = 2ma_3$ …(1)

$mg - T = ma_2$ …(2)

and $2T - kx = 2ma_1$ …(3)

Solving the above equations, we get

$T = \frac{4mg}{7} + \frac{2kx}{7}$ and $a_1 = \frac{4g}{7} - \frac{3kx}{14m}$

For $x = \frac{x_0}{4} = \frac{4mg}{3k}$, we get

$a_1 = \frac{4g}{7} - \frac{2g}{7} = \frac{2g}{7}$

For the oscillation of mass $2m$ about the mean position, we have

$a_1 = 0$

$\Rightarrow \quad x = \frac{8mg}{3k} = \text{Amplitude}$

$\Rightarrow \quad x_0 = 2A = \frac{16mg}{3k}$

Since $\omega = \sqrt{\frac{3k}{14m}}$

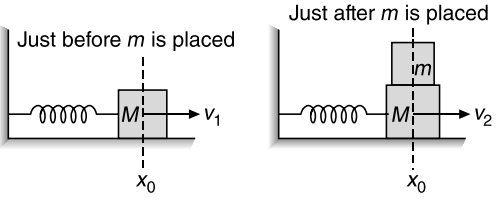
Also, at mean i.e., at $x = \frac{x_0}{2}$, we have

$v = v_{\max} = A\omega$

$\Rightarrow \quad v = \frac{x_0}{2}\sqrt{\frac{3k}{14m}} = \frac{8mg}{3k}\sqrt{\frac{3k}{14m}}$

Hence, the correct answer is (C).

2. **IN CASE-I**



$Mv_1 = (M+m)v_2$

$\Rightarrow \quad v_2 = \left(\frac{M}{M+m}\right)v_1$, where $v_2 = A_2\omega_2$ and $v_1 = A_1\omega_1$

i.e., velocity decreases at equilibrium position

$\Rightarrow \quad \sqrt{\frac{k}{M+m}}A_2 = \left(\frac{M}{M+m}\right)\sqrt{\frac{k}{M}}A_1$

$\Rightarrow \quad A_2 = \sqrt{\frac{k}{M+m}}A_1$

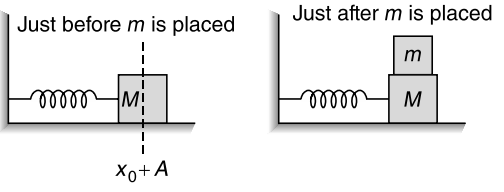
$\Rightarrow \quad E_2 = \frac{1}{2}(M+m)v_2^2$

$\Rightarrow \quad E_2 = \frac{1}{2}kA^2\left(\frac{M}{M+m}\right)$

IN CASE-II

$\omega_2 = \sqrt{\frac{k}{M+m}}$

No energy loss, so $A_1 = A_2$



$T' = 2\pi\sqrt{\frac{M+m}{k}}$, in both cases.

$v_2 = A\omega_2 = A\sqrt{\frac{k}{M+m}}$

Total energy decreases in first case whereas remain same in second case. Instantaneous speed at x_0 decreases in both cases (because ω decreases in both).
Hence, (A), (B) and (D) are correct.

3. **For First Oscillator**

$p = 0$ at $x = a$

So, a is the amplitude of oscillation A_1.
At $x = 0$ i.e., mean position, $p = b$

$$\Rightarrow \quad mv_{max} = b$$

$$\Rightarrow \quad v_{max} = \frac{b}{m}$$

$$\Rightarrow \quad E_1 = \frac{1}{2}mv_{max}^2 = \frac{m}{2}\left(\frac{b}{m}\right)^2 = \frac{b^2}{2m}$$

Also, $A_1\omega_1 = v_{max} = \frac{b}{m}$

$$\Rightarrow \quad \omega_1 = \frac{b}{ma} = \frac{1}{mn^2} \qquad \left\{\because A_1 = a,\ \frac{a}{b} = n^2\right\}$$

For Second Oscillator

$p = 0$ at $x = R$

$$\Rightarrow \quad A_2 = R$$

At $x = 0$ i.e., mean position, $p = R$

$$\Rightarrow \quad v_{max} = \frac{R}{m}$$

$$\Rightarrow \quad E_2 = \frac{1}{2}mv_{max}^2 = \frac{m}{2}\left(\frac{R}{m}\right)^2 = \frac{R^2}{2m}$$

Also, $A_2\omega_2 = \frac{R}{m}$

$$\Rightarrow \quad \omega_2 = \frac{R}{mR} = \frac{1}{m}$$

From above calculations, we see that

$$\frac{\omega_2}{\omega_1} = \frac{1/m}{1/mn^2} = n^2$$

$$\omega_1\omega_2 = \frac{1}{mn^2} \times \frac{1}{m} = \frac{1}{m^2n^2}$$

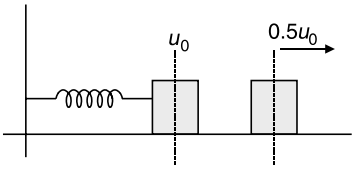
$$\frac{E_1}{\omega_1} = \frac{b^2/2m}{1/mn^2} = \frac{b^2n^2}{2} = \frac{a^2}{2n^2} = \frac{R^2}{2}$$

$$\frac{E_2}{\omega_2} = \frac{R^2/2m}{1/m} = \frac{R^2}{2}$$

$$\Rightarrow \quad \frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$$

Hence, (B) and (D) are correct.

4.



OPTION (A) is correct due to Law of Conservation of Energy
OPTION (B)

$$x = A\sin(\omega t)$$

$$\Rightarrow \quad \frac{dx}{dt} = A\omega\cos\omega t = \frac{u_0}{2} = u_0\cos\omega t$$

$$\Rightarrow \quad \cos(\omega t) = \frac{1}{2}$$

$$\Rightarrow \quad \omega t = \frac{\pi}{3}$$

$$\Rightarrow \quad t = \frac{\pi}{3\omega}$$

$$\Rightarrow \quad \Delta t = \frac{2\pi}{3\omega} = \frac{2\pi}{3}\sqrt{\frac{m}{k}}$$

So, this is incorrect
OPTION (C)
Maximum compression will occur at time

$$t = \frac{2\pi}{3}\sqrt{\frac{m}{k}} + \frac{\pi}{2}\sqrt{\frac{m}{k}}$$

$$\Rightarrow \quad t_1 = \frac{7\pi}{6}\sqrt{\frac{m}{k}}$$

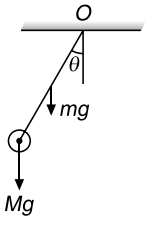
So, this is incorrect
OPTION (D)
Second time at equilibrium $t' = \frac{2\pi}{3}\sqrt{\frac{m}{k}} + \pi\sqrt{\frac{m}{k}}$

$$\Rightarrow \quad t' = \frac{5\pi}{3}\sqrt{\frac{m}{k}}$$

So, this is correct
Hence, (A) and (D) are correct.

5. Restoring torque about O is given by

$$\tau_A = \tau_B = \left(mg\frac{L}{2}\sin\theta + MgL\sin\theta\right)$$



In case A, moment of inertia will be more. Hence, angular acceleration $\left(\alpha = \frac{\tau}{I}\right)$ will be less. Therefore, angular frequency will be less.
Hence, (A) and (D) are correct.

6. For $A = -B$ and $C = 2B$, we get

$$x = B\cos 2\omega t + B\sin 2\omega t$$

$$\Rightarrow \quad x = \sqrt{2}B\sin\left(2\omega t + \frac{\pi}{4}\right)$$

This is equation of SHM of amplitude $\sqrt{2}B$.
If $A = B$ and $C = 2B$, then $x = B + B\sin 2\omega t$
This is also equation of SHM about the point $x = B$. Function oscillates between $x = 0$ and $x = 2B$ with amplitude B.
Hence, (B) and (D) are correct.

7. Applying the Principle of Superposition, we get

$$y = y_1 + y_2 + y_3$$

CHAPTER 3

$\Rightarrow \quad y = a\sin(\omega t) + a\sin(\omega t + 45°) + a\sin(\omega t + 90°)$

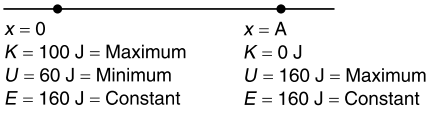
$\Rightarrow \quad y = a[\sin(\omega t) + \sin(\omega t + 90°)] + a\sin(\omega t + 45°)$

$\Rightarrow \quad y = 2a\sin(\omega t + 45°)\cos(45°) + a\sin(\omega t + 45°)$

Hence, (A) and (C) are correct.

8. $K_{max} = \frac{1}{2}(m\omega^2)A^2 = \frac{1}{2}kA^2 = \frac{1}{2}\times 2\times 10^6 \times (10^{-2}) = 100$ J

This is basically the energy of oscillation of the particle. K, U and E at mean position $(x = 0)$ and extreme position $(x = \pm A)$ are shown in Figure.



Hence, (B) and (C) are correct.

Linked Comprehension Type Questions

1. For motion to be periodic, it must reverse its path i.e., KE should become zero for a finite value of x.

Now $K + U = E$

$\Rightarrow \quad K = E - U$

$U_{max} = V_0$

$\Rightarrow \quad K_{min} = E - V_0$

If $K_{min} > 0$, particle will escape

So, $E - V_0 < 0$

$\Rightarrow \quad E < V_0$

Also $E = K + U > 0$

Hence, the correct answer is (C).

2. Since, $V = \alpha x^4$

$\Rightarrow \quad [\alpha] = ML^{-2}T^{-2}$

By methods of dimensional analysis, we have

$$\left[\frac{1}{A}\sqrt{\frac{m}{\alpha}}\right] = M^0L^0T$$

Hence, the correct answer is (B).

3. For $X > X_0$, potential energy is constant.

Since, $F = -\frac{dU}{dx}$, so $F = 0$

Hence, the correct answer is (D).

Matrix Match Type Questions

1. A → (p, s); B → (q, r, s); C → (s); D → (q)

For A

Potential energy of a simple pendulum varies curvilinearly with displacement and is minimum at a certain value.

For B

q and r correspond to zero acceleration, while s corresponds to accelerated motion p is not possible as velocity is negative corresponding to initial portion of the graph.

For C

$$R = \frac{u^2\sin(2\theta)}{g}$$

$\Rightarrow \quad R \propto u^2$

For D

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$\Rightarrow \quad T^2 \propto \ell$

2. A → (p); B → (q, r); C → (p); D → (q, r)

(A) Compare with the standard equation of SHM

$$v = \omega\sqrt{A^2 - x^2}$$

We see that the given motion is SHM with,

$\omega = C_1$ and $A^2 = C_2$

(B) The equation shows that the object does not change its direction and kinetic energy of the object keeps on decreasing.

(C) A pseudo force (with respect to elevator) will start acting on the object. Its means position is now changed and it starts SHM.

(D) The given velocity is greater than the escape velocity $v_e = \sqrt{\frac{2GM_e}{R_e}}$. Therefore, it keeps on moving towards infinity with decreasing speed.

Integer/Numerical Answer Type Questions

1. Since $k = \frac{YA}{L}$ and so,

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{YA}{lm}} = \sqrt{\frac{(n\times 10^9)(4.9\times 10^{-7})}{1\times 0.1}}$$

Substituting, $\omega = 140$ rads^{-1}, we get

$n = 4$

CHAPTER 4: MECHANICAL WAVES

Test Your Concepts-I
(Based On Wave Equation & Properties)

1. The amplitude of wave is, $A = 0.06$ m

Also, from diagram, $\frac{5}{2}\lambda = 0.2$ m

$\Rightarrow \quad \lambda = 0.08$ m

$\Rightarrow \quad f = \frac{v}{\lambda} = \frac{300}{0.08} = 3750$ Hz

$\Rightarrow \quad k = \frac{2\pi}{\lambda} = 78.5\ \text{m}^{-1}$

$\Rightarrow \quad \omega = 2\pi f = 23562\ \text{rads}^{-1}$

At $t = 0$, $x = 0$, $\frac{dy}{dx} =$ positive

and the given curve is a sine curve.

Hence, equation of wave travelling along positive x-direction has the form,

$$y(x,\ t) = A\sin(\omega t - kx)$$

Substituting the values, we get

$$y(x, t) = (0.06\ \text{m})\sin(23562t - 78.5x)$$

2. Since, $y = A\sin(\omega t - kx + \phi)$

At some given instant $y = 0$ at $x = 0$

$\Rightarrow \quad \omega t + \phi = 0$

Now, $-0.01\sqrt{3} = 0.02\sin(-0.1k)$

and $0.01\sqrt{3} = 0.02\sin(-0.8k)$

Satisfying these two conditions, we get

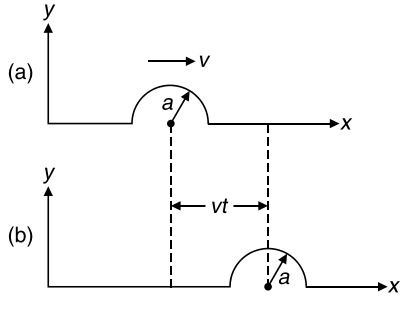
$$k = \frac{20\pi}{3}\ \text{m}^{-1},\ \omega = 2\pi f = 1000\pi\ \text{rads}^{-1}$$

$$v = \frac{\omega}{k} = 150\ \text{ms}^{-1}$$

3. Since, $y^2 = a^2 - x^2$ i.e., $x^2 + y^2 = a^2$

So, the pulse is semicircular in shape as shown in figure (a).

For the wave travelling in positive x-direction $y(x, t) = f(x - vt)$, so



$$y(x, t) = \begin{cases} \sqrt{a^2 - (x - vt)^2}, & \text{when} \quad |x - vt| \le a \\ 0, & \text{when} \quad |x - vt| \ge 0 \end{cases}$$

Above equation reduces to equation given in the problem for $t = 0$. The shape of pulse at a later time t is shown in Figure (b).

4. $\frac{\partial^2 y}{\partial t^2} = -\omega^2 a\sin(\omega t - kx)$ ...(1)

and $\frac{\partial^2 y}{\partial x^2} = -k^2 a\sin(\omega t - kx)$ ...(2)

From (1) and (2), we get

$$\frac{\partial^2 y}{\partial t^2} = \left(\frac{\omega^2}{k^2}\right)\frac{\partial^2 y}{\partial x^2}$$

$$\Rightarrow \quad \frac{\partial^2 y}{\partial t^2} = v^2\frac{\partial^2 y}{\partial x^2} \qquad \left\{\because\ v = \frac{\omega}{k}\right\}$$

The negative sign between ωt and kx implies that wave is travelling along positive x-direction.

5. We know that for a wave $v = f\lambda$

$$\Rightarrow \quad \lambda = \frac{v}{f} = \frac{360}{500} = 0.72\ \text{m}$$

Given $\Delta\phi = 60° = \frac{\pi}{180°} \times 60° = \frac{\pi}{3}$ rad

Since phase difference, $\Delta\phi = \frac{2\pi}{\lambda}\Delta x$

$$\Rightarrow \quad \Delta x = \frac{\lambda}{2\pi}\Delta\phi = \frac{0.72}{2\pi} \times \frac{\pi}{3} = 0.12\ \text{m}$$

6. Although all the four functions are written in the form $f(ax \pm bt)$, only (d) among the four functions is finite everywhere at all times. Hence $e^{-(x-vt)^2}$ represents a wave.

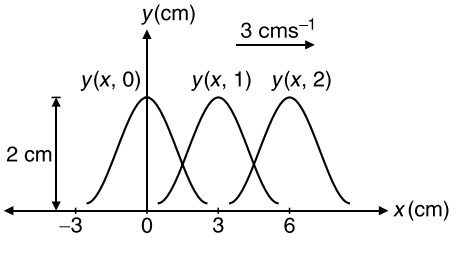
7. Since $y(x,\ t) = \frac{2}{(x - 3t)^2 + 1}$, so wave speed is $v = 3\ \text{cms}^{-1}$ and wave amplitude (the maximum value of y) is $A = 2$ cm. Since,

$$y(x, 0) = \frac{2}{x^2 + 1}$$

$$y(x, 1) = \frac{2}{(x - 3)^2 + 1}$$

and $y(x, 2) = \frac{2}{(x - 6)^2 + 1}$

The corresponding graph are shown in figure.



8. Since, $k_{\text{eff}} = 2k = 1\ \text{Nm}^{-1}$

$$\Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\frac{k_{\text{eff}}}{m}} = \frac{10}{2\pi}\ \text{s}^{-1}$$

$v = 0.1\ \text{ms}^{-1}$, $A = 0.02$ m

$\Rightarrow \quad \lambda = \dfrac{v}{f} = \dfrac{0.1}{\left(\dfrac{10}{2\pi}\right)} = \dfrac{2\pi}{100}$ m

Since, $y = A\cos\left(\dfrac{2\pi}{\lambda}\right)(vt - x)$

$\Rightarrow \quad y = 0.02\cos 100(0.1t - x)$

$\Rightarrow \quad y = 0.02\cos(10t - 100x)$ metre

Distance between two successive maxima is

$\lambda = \dfrac{2\pi}{100} = 0.0628$ m

9. Comparing given equation with standard equation $y = A\sin(kx - \omega t) = A\sin 2\pi\left(\dfrac{x}{\lambda} - \dfrac{t}{T}\right)$, we get, $A = 2$ cm, $f = \dfrac{1}{T} = \dfrac{1}{0.01} = 100$ Hz, $\lambda = 30$ cm and $v = f\lambda = 3000$ cms^{-1} $= 30$ ms^{-1}

10. Comparing this with $y = \dfrac{a}{b + (x + vt)^2}$, we get

$a = 10$, $b = 5$, $v = 2$ ms^{-1} along $-x$ direction. Also, the amplitude (at $x = 0$ and $t = 0$) is $y = \dfrac{a}{b} = \dfrac{10}{5} = 2$ units.

11. On comparing the given expression with

$y = f(x - vt)$

We get the velocity of the wave as

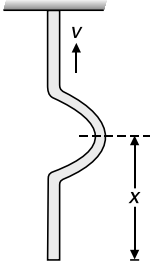
$v = 4$ ms^{-1}

Since these occurs negative sign between x and t in the given expression, the wave propagates **along the +ve *x*-axis**.

12. The negative sign between $3x$ and $4t$ implies that y_1 is travelling along positive x-direction and positive sign between them means y_2 is travelling along negative x-direction. They will cancel each other when $y_1 + y_2 = 0$. Substituting the values of y_1 and y_2 in above equation, we get $t = 0.75$ s and $x = 1$ m.

Test Your Concepts-II (Based On Transverse Wave in a String & Properties)

1. Since, $v = \dfrac{dx}{dt} = \sqrt{\dfrac{T}{\mu}}$



$\Rightarrow \quad T = g\int_0^x \mu dx = g\int_0^x \mu_0 x dx = \dfrac{g\mu_0 x^2}{2}$

$\Rightarrow \quad \dfrac{T}{\mu} = \dfrac{g\mu_0 x^2}{2\mu_0 x} = \dfrac{gx}{2}$

$\Rightarrow \quad \dfrac{dx}{dt} = \sqrt{\dfrac{gx}{2}}$

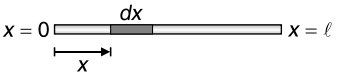
$\Rightarrow \quad \sqrt{\dfrac{g}{2}}\int_0^t dt = \int_0^{\ell_0} \dfrac{dx}{\sqrt{x}}$

$\Rightarrow \quad t = \sqrt{\dfrac{8\ell_0}{g}}$

2. Speed of transverse wave is given by

$v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{100}{\mu_0 + \alpha x}} = \dfrac{10}{\sqrt{\mu_0 + \alpha x}} = \dfrac{dx}{dt}$

$\Rightarrow \quad \int_0^l (\mu_0 + \alpha x)^{\frac{1}{2}} dx = \int_0^t 10 dt$



$\Rightarrow \quad \left.\dfrac{(\mu_0 + \alpha x)^{\frac{1}{2}+1}}{\left(\frac{1}{2}+1\right)\alpha}\right|_0^l = 10(t - 0)$

$\Rightarrow \quad \dfrac{2}{3\alpha}\left[(\mu_0 + \alpha l)^{\frac{3}{2}} - \mu_0^{\frac{3}{2}}\right] = 10t$

$\Rightarrow \quad t = \dfrac{1}{15\alpha}\left[(\mu_0 + \alpha l)^{\frac{3}{2}} - \mu_0^{\frac{3}{2}}\right]$

3. Tension is string AB is $T_{AB} = 6.4$ g $= 64$ N

So, speed of transverse waves in string AB is

$v_{AB} = \sqrt{\dfrac{T_{AB}}{\mu_{AB}}} = \sqrt{\dfrac{64}{10 \times 10^{-3}}}$

$\Rightarrow \quad v_{AB} = \sqrt{6400} = 80$ ms^{-1}

Tension in strong CD is $T_{CD} = 3.2$ g $= 32$ N

So, speed of transverse waves in string CD is

$v_{CD} = \sqrt{\dfrac{T_{CD}}{\mu_{CD}}} = \sqrt{\dfrac{32}{8 \times 10^{-3}}}$

$\Rightarrow \quad v_{CD} = \sqrt{4000} = 63.24$ ms^{-1}

4. Already Done in Theory.

5. Let the tension at point 1 and 2 be T_1 and T_2 respectively, then, $T_1 = 2g$, $T_2 = 6g$. If v_1 and v_2 are the speeds of wave at ends 1 and 2 respectively, then

$\dfrac{v_1}{v_2} = \dfrac{f\lambda_1}{f\lambda_2} = \sqrt{\dfrac{T_1/\mu}{T_2/\mu}} = \sqrt{\dfrac{T_1}{T_2}}$

$\Rightarrow \quad \dfrac{v_1}{v_2} = \sqrt{\dfrac{2g}{6g}} = \sqrt{\dfrac{1}{3}} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \quad \lambda_2 = \sqrt{3}\lambda_1 = 0.06\sqrt{3}$ m

6. The linear mass density is

$\mu = \dfrac{5 \times 10^{-3}}{50 \times 10^{-2}} = 1 \times 10^{-2}$ kgm^{-1}

Since, $v = \sqrt{\dfrac{T}{\mu}}$ i.e., $T = \mu v^2$

$\Rightarrow \quad T = (1 \times 10^{-2}) \times 6400 = 64$ N

Also, young's modulus is given by

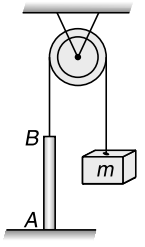
$$Y = \frac{T/A}{\Delta L/L}$$

$$\Rightarrow \quad \Delta L = \frac{TL}{AY} = \frac{64 \times 0.50}{1 \times 10^{-6} \times 16 \times 10^{11}} = 0.02 \text{ mm}$$

7. Tension in the rubber tube AB is

$$T = mg$$

$$\Rightarrow \quad T = (5)(9.8) = 49 \text{ N}$$



Mass per unit length of rubber tube is

$$\mu = \frac{0.9}{12} = 0.075 \text{ kgm}^{-1}$$

Speed of wave on the tube is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{49}{0.075}} = 25.56 \text{ ms}^{-1}$$

The required time is

$$t = \frac{AB}{v} = \frac{12}{25.56} = 0.47 \text{ s}$$

8. The velocity of the wave is $v_1 = \sqrt{\frac{T_1}{\mu}}$

On increasing the tension to $4T_1$, velocity v_2 is given by

$$v_2 = \sqrt{\frac{4\pi}{\mu}} = 2v_1$$

Also $\omega_1 = 2\pi v_1$ and $\omega_2 = 2\pi \frac{v_1}{4} = \frac{\omega_1}{4}$

Since, $P_1 = \frac{1}{2}\mu\omega_1^2 v_1 A^2$

$$\Rightarrow \quad P_2 = \frac{1}{2}\mu\omega_1^2 v_2 A^2 = \frac{1}{2}\mu\left(\frac{\omega_1}{4}\right)^2 (2v_1)A^2$$

$$\Rightarrow \quad P_2 = \frac{1}{8}\left(\frac{1}{2}\mu\omega_1^2 v_1 A^2\right) = \frac{P_1}{8}$$

9. Wave speed on a wire is $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T\ell}{m}}$

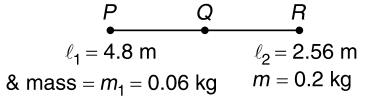
$$\Rightarrow \quad v = \sqrt{\frac{8 \times 0.64}{5 \times 10^{-3}}} = 32 \text{ ms}^{-1}$$

10. Given that, $v_1 = 2v_2$

$$\Rightarrow \quad \sqrt{\frac{T}{A\rho_1}} = 2\sqrt{\frac{T}{A\rho_2}} \qquad \left\{\because v = \sqrt{\frac{T}{A\rho}}\right\}$$

$$\Rightarrow \quad \frac{\rho_1}{\rho_2} = \frac{1}{4} = 0.25$$

11. The tension is maintained at 80 N and amplitude of incident wave is given to be 3.5 cm.



The mass per unit length of PQ and QR is

$$\mu_1 = \frac{0.06}{4.8} = \frac{1}{80} \text{ kgm}^{-1} \text{ and}$$

$$\mu_2 = \frac{0.2}{2.56} = \frac{1}{12.8} \text{ kgm}^{-1}$$

Speed of wave in PQ is

$$v_1 = \sqrt{\frac{T}{\mu_1}} = \sqrt{\frac{80}{1/80}} = 80 \text{ ms}^{-1}$$

Speed of wave in QR is

$$v_2 = \sqrt{\frac{T}{\mu_2}} = \sqrt{\frac{80}{1/12.8}} = 32 \text{ ms}^{-1}$$

(a) Time taken by the wave pulse to move from P to R is given by

$$t = \frac{l_1}{v_1} + \frac{l_2}{v_2} = \frac{4.8}{80} + \frac{2.56}{32} = 0.14 \text{ s}$$

(b) Since, $A_r = \left(\frac{v_2 - v_1}{v_2 + v_1}\right)A_i = \left(\frac{32 - 80}{32 + 80}\right)3.5$

$$\Rightarrow \quad A_r = -1.5 \text{ cm}$$

Also, $A_t = \left(\frac{2v_2}{v_2 + v_1}\right)A_i = \left(\frac{2 \times 32}{32 + 80}\right)3.5$

$$\Rightarrow \quad A_t = 2 \text{ cm}$$

12. Speed of a transverse wave on a string is $v = \sqrt{\frac{T}{\mu}}$ i.e., $v \propto \frac{1}{\sqrt{\mu}}$

Given that $\mu_2 > \mu_1$, $v_2 < v_1$

i.e., medium 2 is denser and medium 1 is rarer.

13. Tension in string is

$$T = kx = 160 \times 0.01 = 1.6 \text{ N}$$

So, wave speed is $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T\ell}{m}}$

$$\Rightarrow \quad v = \sqrt{\frac{1.6 \times 0.4}{0.01}} = 8 \text{ ms}^{-1}$$

Time taken by pulse is $t = \frac{\ell}{v} = \frac{0.4}{8} = 0.05 \text{ s}$

Test Your Concepts-III (Based On Sound Waves & Properties)

1. (a) At a distance r from a point source of power P, the intensity of the sound is

$$I = \frac{P}{4\pi r^2} = \frac{0.8}{(4\pi)(1.5)^2}$$

$$\Rightarrow \quad I = 2.83 \times 10^{-2} \text{ Wm}^{-2} \qquad \ldots(1)$$

Further, the intensity of sound in terms of $(\Delta P)_m$, ρ and v is given by

$$I = \frac{(\Delta P)_m^2}{2\rho v} \qquad \ldots(2)$$

From equations (1) and (2), we get

$$(\Delta P)_m = \sqrt{2 \times 2.83 \times 10^{-2} \times 1.29 \times 340}$$

$$\Rightarrow \quad (\Delta P)_m = 4.98 \text{ Nm}^{-2}$$

(b) Pressure oscillation amplitude $(\Delta P)_m$ and displacement oscillation amplitude A are related by the equation

$$(\Delta P)_m = BAk$$

Since, $v = \sqrt{\frac{B}{\rho}}$

Substituting $B = \rho v^2$, $k = \frac{\omega}{v}$ and $\omega = 2\pi f$, we get

$$(\Delta P)_m = 2\pi A\rho v f$$

$$\Rightarrow \quad A = \frac{(\Delta P)_m}{2\pi\rho v f} = \frac{4.98}{(2\pi)(1.29)(340)(600)}$$

$$\Rightarrow \quad A = 3\times10^{-6} \text{ m}$$

2. Since hydrogen is a diatomic gas, so $\gamma_{H_2} = \frac{7}{5}$

Speed of sound in hydrogen is $v_{H_2} = \sqrt{\frac{\gamma p}{\rho}}$

RMS speed of hydrogen molecules is

$$(v_{\text{rms}})_{H_2} = \sqrt{\frac{3P}{\rho}}$$

$$\Rightarrow \quad \frac{v_{H_2}}{(v_{\text{rms}})_{H_2}} = \sqrt{\frac{\gamma}{3}} = \sqrt{\frac{7/5}{3}} = \sqrt{\frac{7}{15}}$$

3. Since power is distributed uniformly in a hemisphere, intensity at a distance of 5 m from the source will be

$$I = \frac{P}{A} = \frac{P}{4\pi r^2/2} = \frac{10^{-3}}{2\pi(5)^2} = 6.37\ \mu\text{Wm}^{-2}$$

$$\Rightarrow \quad L = 10\log_{10}\left(\frac{I}{I_0}\right) = 10\log_{10}\left(\frac{6.37\times10^{-6}}{10^{-12}}\right)$$

$$\Rightarrow \quad L = 10(\log_{10}6.37 + 6\log_{10}10)$$

$$\Rightarrow \quad L = 10(0.80 + 6) = 68 \text{ dB}$$

If there are five dogs barking at the same time and at the same level, then $I_2 = 5I_1$. So

$$L_2 - L_1 = 10\log_{10}\left(\frac{I_2}{I_1}\right) = 10\log_{10}\left(\frac{5I_1}{I_1}\right)$$

$$\Rightarrow \quad L_2 = L_1 + 10\log_{10}5$$

$$\Rightarrow \quad L_2 = 68 + 10(0.7) = 75 \text{ dB}$$

4. Speed of sound wave

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{(2\times10^9)}{10^3}} = 1414 \text{ ms}^{-1}$$

Wavelength, $\lambda = \frac{v}{f} = 5.84$ m

5. Since, $\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{273+10}{273+27}} = \sqrt{\frac{283}{300}} = 0.97$

As frequency remains unchanged, so

$$\frac{v_2}{v_1} = \frac{\lambda_2}{\lambda_1} = 0.97 \qquad \{\because v = f\lambda\}$$

So, percentage change in wavelength is

$$\frac{\lambda_1 - \lambda_2}{\lambda_1}\times100 = \left(1 - \frac{\lambda_2}{\lambda_1}\right)\times100$$

So, percentage change is $(1 - 0.97)\times100 = 3\%$

6. Given that, $L = 10\log_{10}(I/I_0) = 60$

$$\Rightarrow \quad I/I_0 = 10^6$$

$$\Rightarrow \quad I = 10^{-12}\times10^6 = 1\ \mu\text{Wm}^{-2}$$

Power entering the room is $P = IA$

$$\Rightarrow \quad P = 1\times10^{-6}\times2 = 2\ \mu\text{W}$$

Energy collected in a day is $E = Pt$

$$\Rightarrow \quad E_{1\text{ day}} = 2\times10^{-6}\times86400 = 0.173 \text{ J}$$

7. $v = \sqrt{\frac{\gamma RT}{M}}$

Since, $v_{H_2} = v_{O_2}$

$$\Rightarrow \quad \sqrt{\frac{\gamma_{H_2}RT_{H_2}}{M_{H_2}}} = \sqrt{\frac{\gamma_{O_2}RT_{O_2}}{M_{O_2}}}$$

Further $\gamma_{H_2} = \gamma_{O_2}$ $\{\because$ both are diatomic$\}$

$$\Rightarrow \quad T_{H_2} = \left(\frac{M_{H_2}}{M_{O_2}}\right)(T_{O_2}) = \left(\frac{1}{16}\right)(100 + 273)$$

$$\Rightarrow \quad T_{H_2} = 23.31 \text{ K}$$

$$\Rightarrow \quad T_{H_2} \approx -249.7\ ^\circ\text{C}$$

8. Since, $v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{75\times10^9}{2.70\times10^3}} = 10^3\sqrt{\frac{75}{2.7}}$

$$\Rightarrow \quad v = 10^3\sqrt{\frac{75}{2.7}} = 5.27\times10^3 \text{ ms}^{-1}$$

9. Since, $I = \frac{(\Delta P)^2_{\max}}{2\rho v}$

$$\Rightarrow \quad (\Delta P)_{\max} = \sqrt{2\rho v\text{I}}$$

$$\Rightarrow \quad (\Delta P)_{\max} = \sqrt{2\times1.293\times340\times10^{-3}}$$

$$\Rightarrow \quad (\Delta P)_{\max} = 0.94 \text{ Nm}^{-2}$$

10. (a) Since $L = 10\log_{10}(I/I_0)$

$$\Rightarrow \quad 20 = 10\log_{10}\left(\frac{I}{10^{-12}}\right)$$

$$\Rightarrow \quad I = 10^{-10} \text{ Wm}^{-2}$$

Further, $v = \sqrt{\frac{B}{\rho}} = 200 \text{ ms}^{-1}$

Intensity of a sound wave is given by

$$I = \frac{(\Delta P_{\max})^2}{2\rho v}$$

$$\Rightarrow \quad (\Delta P)_{\max} = \sqrt{2I\rho v}$$

$$\Rightarrow \quad (\Delta P)_{\max} = \sqrt{2\times10^{-10}\times1\times200}$$

$\Rightarrow \quad (\Delta P)_{max} = 2\times10^{-4}\ \text{Nm}^{-2}$

(b) Since, $I = 2\pi^2 f^2 A^2 \rho v = \frac{1}{2}\omega^2 A^2 \rho v$

$\Rightarrow \quad A = \sqrt{\frac{2I}{\omega^2 \rho v}} = \sqrt{\frac{2\times10^{-10}}{(500)^2(1)(200)}}$

$\Rightarrow \quad A = 2\times10^{-9}\ \text{m}$

Further, $k = \frac{\omega}{v} = \frac{500}{200} = \frac{5}{2}\ \text{m}^{-1}$ $\quad \left\{\because v = \frac{\omega}{k}\right\}$

So, equation of the wave is

$$y = 2\times10^{-9}\sin\left(500t - \frac{5}{2}x\right)$$

11. Since, $v_{H_2} = \sqrt{\frac{\gamma P}{\rho_{H_2}}}$ at 0 °C and

$$\rho_{mix} = \frac{(2V)\rho_{H_2} + V\rho_{N_2}}{3V}$$

$\Rightarrow \quad \rho_{mix} = \frac{16\rho_{H_2}}{3}$ $\quad \left\{\because \rho_{N_2} = 14\rho_{H_2}\right\}$

$\Rightarrow \quad v_{mix} = \sqrt{\frac{\gamma P}{\rho_{mix}}}$ at 0 °C

$\Rightarrow \quad v_{mix} = \frac{1300\sqrt{3}}{4} = 325\sqrt{3}\ \text{ms}^{-1}$

So, velocity of sound in the mixture at 27 °C is given by

$$v = \left(\sqrt{\frac{300}{273}}\right)(325\sqrt{3}) \qquad \left\{\because v \propto \sqrt{T}\right\}$$

$\Rightarrow \quad v = 591\ \text{ms}^{-1}$

12. Intensity, $I = \frac{(\Delta P)_m^2}{2\rho v}$

where ΔP_m is the pressure amplitude

$\Rightarrow \quad I = \frac{(6\times10^{-5})^2}{2\times1.2\times344} = 4.4\times10^{-12}\ \text{Wm}^{-2}$

13. Since, $I = \frac{(\Delta P)_m^2}{2\rho v}$

$\Rightarrow \quad (\Delta P)_m = \sqrt{2I\rho v} = \sqrt{2(10^{-12})(1.3)(332)}$

$\Rightarrow \quad (\Delta P)_m = 2.94\times10^{-5}\ \text{Nm}^{-2}$

Also, $(\Delta P)_m = BAK = \rho v^2 A\left(\frac{\omega}{v}\right) = \rho v A\omega$

$\Rightarrow \quad A = \frac{(\Delta P)_m}{\rho v\omega} = \frac{2.94\times10^{-5}}{1.3\times332\times2p\times10^3}$

$\Rightarrow \quad A = 1.1\times10^{-11}\ \text{m}$

Test Your Concepts-IV (Based On Interference)

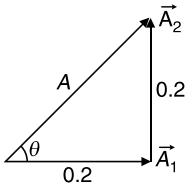
1. $y = y_1 + y_2$

$\Rightarrow \quad y = 0.2\sin(x-3t) + 0.2\sin\left(x-3t+\frac{\pi}{2}\right)$

$\Rightarrow \quad y = A\sin(x-3t+\theta)$

where, $A = \sqrt{(0.2)^2 + (0.2)^2} = 0.28$ m

and $\theta = \frac{\pi}{4}$



$\Rightarrow \quad y = 0.28\sin\left(x-3t+\frac{\pi}{4}\right)$

Since the amplitude of the resulting wave is 0.32 m and $A = 0.2$ m, so we get

$0.32 = 0.2\sqrt{1+1+2\cos\phi}$

$\Rightarrow \quad \phi = \pm1.29$ rad

2. Given, $\frac{A_1}{A_2} = \frac{3}{5}$

Since $I \propto A^2$, so $\sqrt{\frac{I_1}{I_2}} = \frac{3}{5}$

Intensity is maximum when

$\cos\phi = 1$ and $I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

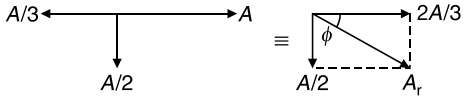
Intensity is minimum when

$\cos\phi = -1$ and $I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$

$\Rightarrow \quad \frac{I_{min}}{I_{max}} = \left(\frac{\sqrt{I_1} - \sqrt{I_2}}{\sqrt{I_1} + \sqrt{I_2}}\right)^2 = \left(\frac{\sqrt{I_1/I_2} - 1}{\sqrt{I_1/I_2} + 1}\right)^2$

$\Rightarrow \quad \frac{I_{min}}{I_{max}} = \left(\frac{\frac{3}{5} - 1}{\frac{3}{5} + 1}\right)^2 = \frac{4}{64} = \frac{1}{16}$

3. The phasor diagram for the situation discussed is drawn below



Resultant amplitude $A_r = \sqrt{\left(\frac{2A}{3}\right)^2 + \left(\frac{A}{2}\right)^2}$

$\Rightarrow \quad A_r = \frac{5}{6}A$

Also, $\tan\phi = -\frac{A/2}{2A/3} = -\frac{3}{4}$

$\Rightarrow \quad \phi = -\tan^{-1}\left(\frac{3}{4}\right)$

4. Path Difference = $ACP - ABP = \Delta x = 11.5$ cm

Since there is complete silence at P, so

$\Delta x = (2n-1)\frac{\lambda}{2}$, where $n = 1,\ 2,\ 3,\ ....$

$\Rightarrow \quad \frac{\lambda}{2} = 11.5$ cm

$\Rightarrow \quad \lambda = 23$ cm $= 0.23$ m

$\Rightarrow \quad f_{min} = \frac{v}{\lambda} = \frac{331.2}{0.23} = 1440$ Hz

5. $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\theta}$

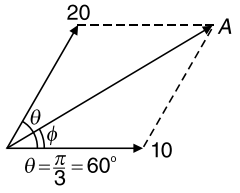
$\Rightarrow \quad A = \sqrt{(10)^2 + (20)^2 + (2)(10)(20)\cos(60°)}$

CHAPTER 4

$\Rightarrow \quad A = 26.46$ m

$$\tan\phi = \frac{20\sin(60°)}{10+20\cos(60°)} = 0.866$$

$\Rightarrow \quad \phi = 41° = 0.714$ rad



Therefore, phase of the resultant wave is

$(5x + 25t + 0.714)$ rad

6. Since $v = f\lambda$

$$\Rightarrow \quad \lambda = \frac{v}{f} = \frac{330}{220} = 1.5 \text{ m}$$

Further $\phi = \dfrac{2\pi}{\lambda}\Delta x$

$$\Rightarrow \quad \phi_1 = \left(\frac{2\pi}{\lambda}\right)(\Delta x_1) = \left(\frac{2\pi}{1.5}\right)(0.75) = \pi$$

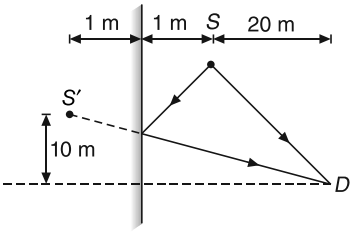
$$\Rightarrow \quad \phi_2 = \left(\frac{2\pi}{\lambda}\right)(\Delta x_2) = \left(\frac{2\pi}{1.5}\right)(3) = 4\pi$$

Since, they are out of phase at P, therefore destructive interference must be taking place. So, the resultant power at P is

$$P_1 - P_2 = (1.8\times10^{-3} - 1.2\times10^{-3}) \text{ watt}$$

$$\Rightarrow \quad P_1 - P_2 = 0.6\times10^{-3} \text{ watt}$$

7. Two sound waves reach point D. One reaches D directly from S and the other after reflection from the wall as shown in Figure.



The reflected wave appears to come from the virtual image S' of the source (remember that sound waves follows the ordinary laws of reflection and refraction). This image S' is distant 1 m from wall just like the source. The two path lengths, now are

$$SD = \sqrt{20^2 + 10^2} = 22.36 \text{ m}$$

and $S'D = \sqrt{22^2 + 10^2} = 24.17$ m

Path difference between two waves is

$$\Delta x = 24.17 - 22.36 = 1.81 \text{ m}$$

When path difference of 1.81 m equals $n\lambda$, a maxima is formed at D.

$\Rightarrow \quad 1.81 = n\lambda$ where $n = 0, 1, 2, \ldots\ldots$

$$\Rightarrow \quad 1.81 = \frac{n(340)}{f} \qquad \{\because v = f\lambda\}$$

$$\Rightarrow \quad f = n\left(\frac{340}{1.81}\right) = n(188) \text{ Hz}$$

$\Rightarrow \quad f = 188$ Hz, 376 Hz, 564 Hz, ……,

8. Path difference $\Delta x = S_1P - S_2P$

$$\Rightarrow \quad \Delta x = \sqrt{(2)^2 + (4)^2} - \sqrt{(1)^2 + (4)^2}$$

$\Rightarrow \quad \Delta x = 4.47 - 4.12$

$\Rightarrow \quad \Delta x = 0.35$ m

(a) Constructive interference occurs when

$$\Delta x = (2n)\frac{\lambda}{2} = \frac{nv}{f}$$

$$\Rightarrow \quad f = \frac{n(v)}{\Delta x}, \text{ where } n = 1,\ 2,\ 3, \ldots.$$

$$\Rightarrow \quad f = \frac{350}{0.35}, \frac{2\times350}{0.35}, \frac{3\times350}{0.35}, \ldots$$

$\Rightarrow \quad f = 1000$ Hz, 2000 Hz, 3000 Hz,….

(b) Destructive interference occurs when

$$\Rightarrow \quad \Delta x = (2n-1)\frac{\lambda}{2}, \text{ where } n = 1,\ 2,\ 3, \ldots.$$

$$\Rightarrow \quad \Delta x = (2n-1)\frac{v}{2f} \qquad \{\because v = f\lambda\}$$

$$\Rightarrow \quad f = \frac{(2n-1)v}{2\Delta x}$$

$$\Rightarrow \quad f = \frac{350}{2\times0.35}, \frac{3\times350}{2\times0.35}, \frac{5\times350}{2\times0.35}, \ldots.$$

$\Rightarrow \quad f = 500$ Hz, 1500 Hz, 2500 Hz,….

9. The path difference between the waves moving along the straight path and the semicircular path is

$$\Delta x = \pi r - 2r = (\pi - 2)r$$

For minima this path difference should be at least $\lambda/2$ i.e., 20 cm. Hence, we have, for the minimum value of r

$$(\pi - 2)r = 20 \text{ cm}$$

$$\Rightarrow \quad r = \frac{20}{\pi - 2} = 17.54 \text{ cm}$$

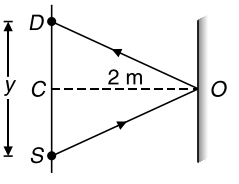
10. (a) $\lambda = \dfrac{v}{f} = \dfrac{360}{180} = 2$ m

A maximum will be observed at D, if

$$\Delta x = \sqrt{y^2 + 16} - y = \lambda = 2 \text{ m}$$

Solving this equation, we get

$y = 3$ m



(b) Similarly, for minima

$$\Delta x = \frac{3\lambda}{2} = 3 \text{ m}$$

Let the obstacle be shifted to the right by a distance z. Then

$$2\sqrt{\left(\frac{y}{2}\right)^2+(2+z)^2}-y=3$$

$$\Rightarrow \quad 2\sqrt{\left(\frac{3}{2}\right)^2+(2+z)^2}-3=3$$

$$\Rightarrow \quad z=0.6 \text{ m}$$

11. Let x be the distance travelled by the wave when it reaches the respective point.

For waves arriving at A from P and Q, phase difference is

$$\Delta\phi_A=(\phi_P-\phi_Q)+\frac{2\pi}{\lambda}(x_P-x_Q)$$

When the waves reach A, then Q is ahead of P in terms of path because wave emitted by Q reaches A before the wave emitted by P and also phase of P is ahead that of Q by 90°, so we have

$$\phi_P-\phi_Q=+\frac{\pi}{2} \text{ and } x_P-x_Q=-5 \text{ m}$$

$$\Rightarrow \quad \Delta\phi_A=+\frac{\pi}{2}+\frac{2\pi}{20}(-5)=0$$

Since, $I_R=I_1+I_2+2\sqrt{I_1I_2}\cos\Delta\phi$

$$\Rightarrow \quad I_A=I+I+2I\cos(0°)=4I$$

For waves arriving at C from P and Q, phase difference is

$$\Delta\phi_C=(\phi_P-\phi_Q)+\frac{2\pi}{\lambda}(x_P-x_Q)$$

When the waves reach C, then P is ahead of Q in terms of path, because wave emitted by P reaches C before the wave emitted by Q and also phase of P is ahead that of Q by 90°, so we have

$$\phi_P-\phi_Q=+\frac{\pi}{2} \text{ and } x_P-x_Q=+5 \text{ m}$$

$$\Rightarrow \quad \Delta\phi_C=+\frac{\pi}{2}+\frac{2\pi}{20}(+5)=\pi$$

$$\Rightarrow \quad I_C=I+I+2I\cos(\pi)=0$$

For waves arriving at B from P and Q, phase difference is

$$\Delta\phi_B=(\phi_P-\phi_Q)+\frac{2\pi}{\lambda}(x_P-x_Q)$$

When the waves reach B, then P and Q have same path and phase of P is ahead that of Q by 90°, so we have

$$\phi_P-\phi_Q=+\frac{\pi}{2} \text{ and } x_P-x_Q=0$$

$$\Rightarrow \quad \Delta\phi_B=+\frac{\pi}{2}+0=\frac{\pi}{2}$$

$$\Rightarrow \quad I_B=I+I+2I\cos(\pi/2)=2I$$

Hence, $I_A:I_B:I_C=4I:2I:0=2:1:0$

12. $\lambda=\frac{v}{f}=\frac{343}{21.5}\approx 16$ m

Path difference between two sound waves reaching at A is given by

$$\Delta x=S_1A-S_2A=(9-1)=8 \text{ m}$$

Since, we observe that

$$\Delta x=\frac{\lambda}{2}=(\text{Odd Multiple})\frac{\lambda}{2}$$

So, the minimum intensity is observed at A.

Since, $S_1=(-5,\ 0)$ and $S_2=(5,\ 0)$

For intensity to be minimum

$$S_1P-S_2P=(2n-1)\frac{\lambda}{2} \qquad n=1, 2, 3,\ldots$$

$$\Rightarrow \quad \sqrt{(x+5)^2+y^2}-\sqrt{(x-5)^2+y^2}=(16n-8) \quad n=1, 2, 3,\ldots$$

This is the desired relation between x and y when intensity is minimum.

Test Your Concepts-V (Based On Stationary Waves & Beats)

1. The given wave can be written as the sum of two component waves given by

$$y(x,t)=0.2\sin(0.5x-30t)+0.2\sin(0.5x+30t)$$

So, wave is made of two component waves

(i) $y_1(x,t)=0.2\sin(0.5x-30t)$, travelling along positive x-direction and

(ii) $y_2(x,t)=0.2\sin(0.5x+30t)$, travelling along negative x-direction.

Now, $\omega=30 \text{ rads}^{-1}$ and $k=0.5 \text{ cm}^{-1}$

So, frequency, $f=\frac{\omega}{2\pi}=\frac{15}{\pi}$ Hz

amplitude $A=0.2$ cm

and wave speed, $v=\frac{\omega}{k}=\frac{30}{0.5}=60 \text{ cms}^{-1}$

Particle velocity

$$v_P(x,\ t)=\frac{\partial y}{\partial t}=-12\sin(0.5x)\sin(30t)$$

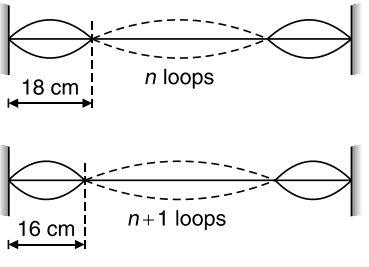
$$\Rightarrow \quad v_p\big|_{x=2.4 \text{ cm}, t=0.8 \text{ s}}=-12\sin(1.2)\sin(24)$$

$$\Rightarrow \quad v_P=10.12 \text{ cms}^{-1}$$

2. (a) Let ℓ be the length of the string. Then

$$18n=\ell \qquad \ldots(1)$$

$$16(n+1)=\ell \qquad \ldots(2)$$



From Equations (1) and (2), we get

$$n=8 \text{ and } \ell=144 \text{ cm}$$

Therefore, the minimum possible length of the string can be 144 cm.

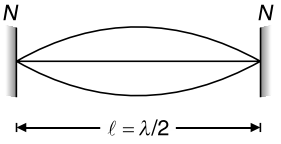
CHAPTER 4

(b) For fundamental frequency, $\ell = \frac{\lambda}{2}$

$\Rightarrow \quad \lambda = 2\ell = 288 \text{ cm} = 2.88 \text{ m}$

Speed of wave on the string

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{10}{4\times10^{-3}}} = 50 \text{ ms}^{-1}$$



So, fundamental frequency is given by

$$f = \frac{v}{\lambda} = \frac{50}{2.88} = 17.36 \text{ Hz} \approx 17 \text{ Hz}$$

3. (a) $A(x) = (0.5 \text{ cm})\sin\left[(1.57 \text{ cm}^{-1})x\right]$

At $x = 5.66$ cm, $A(x) = 2.56$ cm

This is also the maximum displacement at $x = 5.66$ cm.

So, $A_{max} = 2.56$ mm

(b) $\lambda = \frac{2\pi}{k} = \frac{2\pi}{1.57} = 4 \text{ cm}$

$$v = \frac{\omega}{k} = \frac{314}{1.57} = 200 \text{ cms}^{-1} = 2 \text{ ms}^{-1}$$

(c) $v_p = \frac{\partial y}{\partial t} = 157\sin(1.57x)\cos(314t)$

At, $x = 5.66$ cm and $t = 2$ sec, we get

$$v_p = 78.5 \text{ cms}^{-1}$$

(d) Nodes are the points where $A(x) = 0$

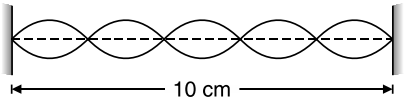
$\Rightarrow \quad x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots$

i.e., at $x = 0$, 2 cm, 4 cm,…, etc., we get NODES

At antinodes, $A(x)$ is a maximum

i.e., at $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots$, etc.

i.e., at $x = 1$ cm, 3 cm, 5 cm,…, etc., we get ANTINODES



So, number of loops is $p = \frac{\ell}{\lambda/2} = \frac{10}{2} = 5$

4. (a) $f = \frac{v}{2\ell} = \frac{1}{2\ell}\sqrt{\frac{Y}{\rho}}$

If S be the cross sectional area of the wire, then, $\rho = \frac{\mu}{S}$

$\{\because \mu = \rho S\}$

$\Rightarrow \quad f = \frac{1}{2\ell}\sqrt{\frac{YS}{\mu}} = 500\sqrt{2} \text{ Hz}$

(b) $T = YA\alpha\Delta\theta = (2\times10^{11})(10^{-6})(1.21\times10^{-5})(20)$

$\Rightarrow \quad T = 48.4 \text{ N}$

$\Rightarrow \quad f = \frac{v}{2\ell} = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} = 11 \text{ Hz}$

5. For a specific tension, the fundamental frequency of vibration $f \propto \frac{1}{L}$. So, for having fundamental frequencies in the ratio of 1 : 3 : 4, the vibrating lengths should be in the ratio

$L_1 : L_2 : L_3 = 1 : \frac{1}{3} : \frac{1}{4}$

$\Rightarrow \quad L_1 : L_2 : L_3 = 12 : 4 : 3$

Since $L_1 + L_2 + L_3 = 114$ cm, so

$114 = 12x + 4x + 3x = 19x$

$\Rightarrow \quad x = 6$

So, $L_1 = 12x = 72$ cm, $L_2 = 4x = 24$ cm, $L_3 = 3x = 18$ cm

6. Given that, $\mu = 0.05 \text{ gcm}^{-1} = 0.005 \text{ kgm}^{-1}$

So, $420 = n\left(\frac{v}{2\ell}\right)$ …(1)

and $490 = (n+1)\left(\frac{v}{2\ell}\right)$ …(2)

From Equations (1) and (2), we get $n = 6$

$\Rightarrow \quad 420 = 6\left(\frac{1}{2\ell}\sqrt{\frac{T}{\mu}}\right)$

$\Rightarrow \quad \ell = 6\left(\frac{1}{840}\sqrt{\frac{450}{0.005}}\right) = 2.142 \text{ m}$

7. Given that, $\frac{A_{max}}{A_{min}} = \frac{A_i + A_r}{A_i - A_r} = \frac{3}{2}$

$\Rightarrow \quad 2A_i + 2A_r = 3A_i - 3A_r$

$\Rightarrow \quad 5A_r = A_i$

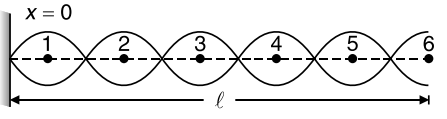
$\Rightarrow \quad \frac{A_r}{A_i} = \frac{1}{5}$

$\Rightarrow \quad \frac{I_r}{I_i} = \left(\frac{A_r}{A_i}\right)^2 = \left(\frac{1}{5}\right)^2 = \frac{1}{25}$

$\Rightarrow \quad I_r = 0.04I_i$

So, 4% of the incident energy is reflected or 96% energy passes across the obstacle.

8. (a)



From diagram, $\ell = 5\left(\frac{\lambda}{2}\right) + \left(\frac{\lambda}{4}\right) = \frac{11}{4}\lambda$

$\Rightarrow \quad \lambda = \frac{4\ell}{11} = \frac{4\times1}{11} = 0.36 \text{ m}$

(b) $v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{6.4\times10^{10}}{5\times10^{3}}} = 3578 \text{ ms}^{-1}$

$\Rightarrow \quad f = \frac{v}{\lambda} = \frac{3578}{0.36} = 9939 \text{ Hz}$

(c) Since, $A(x) = A\sin(kx)$

$\Rightarrow \quad A(x=0.5) = (4\times10^{-6})\sin\left(\frac{2\pi}{0.36}\right)\left(\frac{1}{2}\right)$

$\Rightarrow \quad A(x=0.5) = 2.57\times10^{-6}$ m

$\Rightarrow \quad y\big|_{\text{at } x=0.5 \text{ m}} = 2.57\times10^{-6}\sin(\omega t+\phi)$

where, $\omega = 2\pi f = 6.24\times10^{4}$ rads^{-1}

$\Rightarrow \quad y = 2.57\times10^{-6}\sin(62400t+\phi)$

9. (a) The equation of standing wave is

$$y = A\sin(kx-\omega t)+A\sin(kx+\omega t)$$

$\Rightarrow \quad y = 2A\sin(kx)\cos(\omega t)$

Particle velocity at location x is given by

$$v_P = \frac{\partial y}{\partial t} = -2A\omega\sin(kx)\cos(\omega t) \qquad \ldots(1)$$

The kinetic energy of an element dx is

$$dK = \frac{1}{2}(\mu dx)v_P^2 \qquad \ldots(2) \qquad \{\because\ dm = \mu dx\}$$

So, the kinetic energy contained between two nodes at any instant t is given by

$$K = \int_0^{\frac{\lambda}{2}} dK$$

Put $k = \frac{2\pi}{\lambda}$, $v = \frac{\omega}{k} = \sqrt{\frac{T}{\mu}}$ in (1) and then substitute (1) in (2), we get

$$K = \pi k A^2 T\sin^2(\omega t)$$

(b) $K_{\max} = \pi k A^2 T$ when $\sin^2(\omega t) = 1$

(c) Since, $\langle\sin^2\omega t\rangle_{\text{one cycle}} = \frac{1}{2}$

$\Rightarrow \quad \langle K\rangle_{\text{one cycle}} = \frac{\pi k A^2 T}{2}$

10. For first and second wave respectively $\lambda_1 = 204$ cm and $\lambda_2 = 208$ cm. Let velocity of sound in gas be v cms^{-1} then frequencies of first and second wave are

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{204} \text{ and } f_2 = \frac{v}{\lambda_2} = \frac{v}{208}$$

Beat frequency is $f_b = \frac{20}{6}$ Hz

$\Rightarrow \quad f_1 - f_2 = \frac{v}{204} - \frac{v}{208} = \frac{20}{6}$

$\Rightarrow \quad v = \frac{20}{6}\left(\frac{204\times208}{4}\right)$ cms^{-1} $= 35360$ cms^{-1}

$\Rightarrow \quad v = 353.6$ ms^{-1}

11. Fundamental frequency of an open pipe is

$$f_1 = \frac{v}{2\ell_0}$$

$\Rightarrow \quad \ell_0 = \frac{v}{2f_1} = \frac{330}{2\times300} = 0.55$ m $= 55$ cm

Since, $(f_{\text{closed}})_{\text{1st overtone}} = (f_{\text{open}})_{\text{1st overtone}}$

$\Rightarrow \quad 3\left(\frac{v}{4\ell_c}\right) = 2\left(\frac{v}{2\ell_0}\right)$

$\Rightarrow \quad \ell_c = \frac{3\ell_0}{4} = \frac{3}{4}(0.55) = 0.4125$ m

$\Rightarrow \quad \ell_c = 41.25$ cm

12. Fundamental frequency of closed pipe is

$$f_0 = \frac{v}{4\ell} = \frac{330}{4(1)} = 82.5 \text{ Hz}$$

At resonance, fundamental frequency of stretched wire equals the frequency of the air column, so we have

$\Rightarrow \quad \frac{1}{2l}\sqrt{\frac{T}{\mu}} = 82.5$

$\Rightarrow \quad T = \left(\frac{0.01}{0.3}\right)(2\times0.3\times82.5)^2$

$\Rightarrow \quad T = 81.675$ N

13. Let ℓ_1 and ℓ_2 be the lengths of closed and open pipes respectively. Fundamental frequency of closed organ pipe $f_1 = \frac{v}{4\ell}$, where $v = 330$ ms^{-1}. Given that $f_1 = 110$ Hz

$\Rightarrow \quad \frac{v}{4\ell_1} = 110$ Hz

$\Rightarrow \quad \ell_1 = \frac{v}{4\times110} = \frac{330}{4\times110}$ m $= 0.75$ m

So, the first overtone of a closed organ pipe will be given by

$$f_3 = 3f_1 = 3(110) \text{ Hz} = 330 \text{ Hz}$$

This produces a beat frequency of 2.2 Hz with first overtone of open organ pipe.

Therefore, first overtone frequency of open organ pipe is either

$(330+2.2)$ Hz $= 332.2$ Hz OR

$(330-2.2)$ Hz $= 327.8$ Hz

If it is 332.2 Hz, then $2\left(\frac{v}{2\ell_2}\right) = 332.2$ Hz

$\Rightarrow \quad \ell_2 = \frac{v}{332.2} = \frac{330}{332.2}$ m $= 0.99$ m

If it is 327.8 Hz, then $2\left(\frac{v}{2\ell_2}\right) = 327.8$ Hz

$\Rightarrow \quad \ell_2 = \frac{v}{327.8}$ m $= \frac{330}{327.8}$ m $= 1.0067$ m

So, the length of the closed organ pipe is $\ell_1 = 0.75$ m while length of open pipe is either $\ell_2 = 0.99$ m or 1.0067 m.

14. (a) Fundamental frequency when the pipe is open at both ends is

$$f_1 = \frac{v}{2\ell} = \frac{340}{2\times0.6} = 283.33 \text{ Hz}$$

(b) Let the hole be uncovered at a length ℓ from the mouthpiece, then the fundamental frequency will be $f_1 = \frac{v}{2\ell}$

CHAPTER 4

$$\Rightarrow \quad \ell = \frac{v}{2f_1} = \frac{340}{2\times 330}$$

$$\Rightarrow \quad \ell = 0.515 \text{ m}$$

$$\Rightarrow \quad \ell = 51.5 \text{ cm}$$

Conceptual Note(s)

Opening holes at the sides effectively shortens the length of the resonance column, thus increasing the frequency.

15. Number of beats $f_b = |f_1 - f_2|$

$$\Rightarrow \quad f_1 = f_2 + f_b \text{ or } f_1 = f_2 - f_b$$

$$\Rightarrow \quad f_1 = 364 + 3 \text{ or } f_1 = 364 - 3$$

$$\Rightarrow \quad f_1 = 367 \text{ Hz or } f_1 = 361 \text{ Hz}$$

Loading a fork with wax decreases its frequency. On loading the first fork, number of beats produced per second decreases, so

$$f_1 = 367 \text{ Hz}$$

16. Frequency of fundamental mode of closed pipe is $f_1 = \frac{v}{4\ell} = 200$ Hz

Since, decreasing the tension in string will decrease the beat frequency, so the first overtone frequency of the string should be 208 Hz (not 192 Hz).

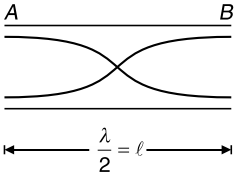
$$\Rightarrow \quad 208 = \frac{1}{\ell}\sqrt{\frac{T}{\mu}}$$

$$\Rightarrow \quad T = (208\ell)^2 \mu = (208\times 0.25)^2\left(\frac{2.5\times 10^{-3}}{0.25}\right)$$

$$\Rightarrow \quad T = 27.04 \text{ N}$$

17. Since $\frac{\lambda}{2} = \ell = 0.8$ m, so $\lambda = 1.6$ m

$$\Rightarrow \quad k = \frac{2\pi}{\lambda} = \frac{2\pi}{1.6} = 3.93 \text{ m}^{-1}$$



$$\Rightarrow \quad \omega = vk = (330)(3.93) = 1297 \text{ s}^{-1} \qquad \left\{\because v = \frac{\omega}{k}\right\}$$

$$\Rightarrow \quad y = A\cos(kx)\sin(\omega t)$$

$$\Rightarrow \quad y = A\cos(3.93x)\sin(1297t)$$

Test Your Concepts-VI (Based On Doppler's Effect)

1. Given $f_1 - f_2 = 3$

$$\Rightarrow \quad \left(\frac{v}{v - v_s}\right)f - \left(\frac{v}{v + v_s}\right)f = 3$$

$$\Rightarrow \quad \left[\frac{1}{\left(1 - \frac{v_s}{v}\right)} - \frac{1}{\left(1 + \frac{v_s}{v}\right)}\right]f = 3$$

$$\Rightarrow \quad \left[\left(1 - \frac{v_s}{v}\right)^{-1} - \left(1 + \frac{v_s}{v}\right)^{-1}\right]f = 3$$

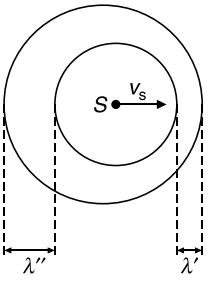
Since, difference in apparent frequencies is very small (3 Hz). So, we may conclude that speed of source $(v_s) \ll$ speed of sound (v). So, we can neglect higher terms of $\frac{v_s}{v}$.

$$\Rightarrow \quad \left[\left(1 + \frac{v_s}{v}\right) - \left(1 - \frac{v_s}{v}\right)\right]f = 3$$

$$\Rightarrow \quad \frac{2v_s f}{v} = 3$$

$$\Rightarrow \quad v_s = \frac{3v}{2f} = \frac{(3)(340)}{(2)(340)} = 1.5 \text{ ms}^{-1}$$

2. (a) Due to motion of the source, the wavelength is changed from λ to λ' (in the direction of v_s) and λ to λ'' (in the direction opposite to v_s), where



$$\lambda' = \frac{v - v_s}{f} = \frac{332 - 32}{1000} = 0.3 \text{ m}$$

and $\lambda'' = \frac{v + v_s}{f} = \frac{332 + 32}{1000} = 0.364$ m

(b) The number of waves arriving at the reflecting surface is the same as the number of waves received by an observer moving towards the source, i.e.,

$$f' = \left(\frac{v + v_0}{v - v_s}\right)f$$

$$\Rightarrow \quad f' = \left(\frac{332 + 64}{332 - 32}\right)\times 1000 = 1320 \text{ Hz}$$

OR $f' = \frac{v + v_0}{\lambda'} = \frac{332 + 64}{0.3} = 1320$ Hz

(c) Speed of a wave depends only on the characteristics of the medium. So, the speed of reflected wave is 332 ms^{-1}

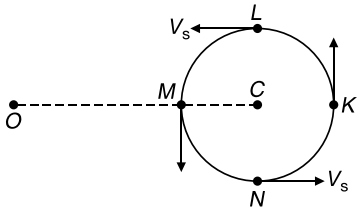
(d) Wavelength of the reflected wave is calculated in the similar manner as was calculated in part (a), i.e.,

$$\lambda_r = \frac{v - v_0}{f'} = \frac{332 - 64}{1320} \approx 0.2 \text{ m}$$

3. Apparent frequency will be minimum when the source is at N and moving away from the observer

$$f_{\min} = \left(\frac{v}{v + v_s}\right)f = \left(\frac{330}{330 + 30}\right)(540)$$

$$\Rightarrow \quad f_{\min} = 495 \text{ Hz}$$



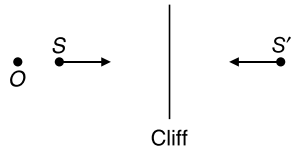
Frequency will be maximum when source is at L and approaching the observer

$$f_{max} = \left(\frac{v}{v - v_s}\right) f = \left(\frac{330}{330 - 30}\right)(540)$$

$\Rightarrow \quad f_{max} = 594$ Hz

Further when source is at M and K, angle between velocity of source and line joining source and observer is 90°, so $v_s \cos\theta = v_s \cos(90°) = 0$. Hence, there will be no change in the apparent frequency.

4. The situation is as shown in figure.



(a) Frequency of sound reaching directly to us (by S)

$$f_1 = \left(\frac{v}{v + v_s}\right) f = \left(\frac{330}{330 + 10}\right)(1000)$$

$\Rightarrow \quad f_1 = 970.6$ Hz

(b) Frequency of sound which is reflected off from the cliff (from S')

$$f_2 = \left(\frac{v}{v - v_s}\right) f = \left(\frac{330}{330 - 10}\right)(1000)$$

$\Rightarrow \quad f_2 = 1031.3$ Hz

(c) Beat frequency, $f_b = f_2 - f_1$

$\Rightarrow \quad f_b = 60.7$ Hz

5. (a) Initially the velocity of source is zero, so

$$\lambda_i = \frac{v}{f} = \frac{350}{1000} = 0.35 \text{ m}$$

(b) Instantaneous velocity of the source

$$v_s = at = 10t$$

$$\Rightarrow \quad f' = \left(\frac{v}{v - v_s}\right) f = \left(\frac{350}{350 - 10t}\right)(1000)$$

The average frequency over a time $t_0 = 1$ sec is given by

$$\langle f' \rangle = \frac{1}{t_0}\int_0^{t_0} f' dt = 1014.5 \text{ Hz}$$

6. Assuming that the airplane is approaching the observer with velocity v_a, then velocity of image of source from which the reflected wave is being produced is

$$v_s = 2v_a \qquad \ldots(1)$$

Just similar to the case of plane mirror, where if the mirror moves with a velocity v, then the image moves with a velocity $2v$. In this case,

$$f' = f\left(\frac{c}{c - v_s}\right) = f\left(1 - \frac{v_s}{c}\right)^{-1} \approx f\left(1 + \frac{v_s}{c}\right)$$

$$\Rightarrow \quad f' = f + f\left(\frac{v_s}{c}\right)$$

Since, $f' - f = 2.6 \times 10^3$ Hz, $n = 7.8 \times 10^9$ Hz and $c = 3 \times 10^8$ ms^{-1}, so we get

$$2.6 \times 10^3 = 7.8 \times 10^9 \left(\frac{v_s}{3 \times 10^8}\right) = 26 v_s$$

$\Rightarrow \quad v_s = 100$ ms^{-1}

Using equation (1), we get

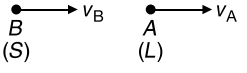
$$v_a = \frac{v_s}{2} = 25 \text{ ms}^{-1} = 180 \text{ kmh}^{-1}$$

7. At $t = 10$ sec, velocity of A is given by

$$v_A = u_A + a_A t$$

$$\Rightarrow \quad v_A = (2) + (2)(10)$$

$$\Rightarrow \quad v_A = 22 \text{ ms}^{-1}$$



Here B is the source and A is the listener. Hence

$$f' = f\left(\frac{v - v_A}{v - v_B}\right)$$

$$\Rightarrow \quad f = \left(\frac{v - v_B}{v - v_A}\right) f'$$

$$\Rightarrow \quad f = \left(\frac{330 - 2}{330 - 22}\right) \times 352$$

$$\Rightarrow \quad f = 374.8 \text{ Hz}$$

8. (a) $\lambda' = \left(\frac{v + w}{f}\right) = \left(\frac{v + w}{v}\right)\lambda$

$$\Rightarrow \quad \frac{\lambda' - \lambda}{\lambda} = \frac{w}{v}$$

So, percentage increase in wavelength is

$$\frac{w}{v} \times 100 = 10\%$$

(b) There will be no change in the frequency.

(c) $f' = \left(\frac{v + v_0}{v}\right) f$

$$\Rightarrow \quad v_0 = 0.1v$$

$$\Rightarrow \quad f' = 1.1f$$

So, the percentage increase in frequency is 10%

9. Since, $f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} = \frac{1}{2 \times 0.1}\sqrt{\frac{64}{\left(\frac{10^{-3}}{0.1}\right)}} = 400$ Hz

CHAPTER 4

Also, $f' = \left(\frac{v}{v+v_s}\right) f$ where $v = 300$ ms^{-1}

$\Rightarrow \quad f' < f$

Since, $f - f' = 1$ Hz

$\Rightarrow \quad 400 - \left(\frac{300}{300+v_s}\right) 400 = 1$

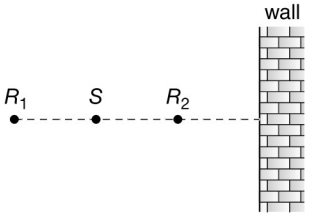
Solving this we get

$v_s = 0.752$ ms^{-1}

10. $f_b = \left(\frac{v}{v-v_s}\right) f - \left(\frac{v}{v+v_s}\right) f$

$\Rightarrow \quad f_b = \left(\frac{340}{340-0.17} - \frac{340}{340+0.17}\right) 1000$ Hz

$\Rightarrow \quad f_b = 1$ Hz



There are two cases possible.

Case 1: When S moves towards wall.

Case 2: When S moves away from the wall.

But in both the cases only R_1 will register the beats.

11. Since $v = \left(\frac{\Delta\lambda}{\lambda}\right) c$

$\Rightarrow \quad v = \left(\frac{0.5}{100}\right)(3\times10^8) = 1.5\times10^6$ ms^{-1}

12. Maximum value of the listener (v_L) or the source (v_s) is given by

$v_L = v_s = A\omega = 4\pi$ ms^{-1}

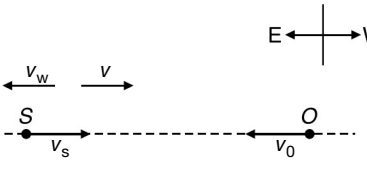
So, $f_{\min} = f_{\text{recede}} = \left(\frac{340-4\pi}{340+4\pi}\right)(300)$

$f_{\min} = 278.62$ Hz

and $f_{\max} = f_{\text{approach}} = \left(\frac{340+4\pi}{340-4\pi}\right)(300)$

$f_{\max} = 323$ Hz

13. Normally, we take east direction pointing to the right and west to the left. However, to be able to determine the sign of different velocities in the most general formula for Doppler's effect, we shall take east on the left, so that the source (i.e., the engine) is on the left of the observer. The directions of all the velocities are marked in the Figure.



The most general formulae to determine apparent frequency is

$$f' = f\left(\frac{v+v_w-v_0}{v+v_w-v_s}\right)$$

Here, $f = 1200$ Hz, $v = 330$ ms^{-1},

$v_0 = 72$ kmh$^{-1} = 20$ ms^{-1}

$\Rightarrow \quad f = 1200\left(\frac{330+(-10)-(-15)}{330+(-10)-20}\right)$

$\Rightarrow \quad f = 1200\left(\frac{330-10+15}{330-10-20}\right)$

$\Rightarrow \quad f = \frac{1200\times335}{300} = 1340$ Hz

14. Let the speed of the plane (source) be v_s. The maximum frequency is observed by the observer when v_s is along SO. The observer receives maximum frequency when the plane is nearest to him. That is as soon as the wave pulse reaches from S to O with speed v the plane reaches from S to S' with speed v_s.

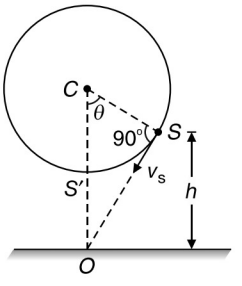
Hence, $t = \frac{SO}{v} = \frac{SS'}{v_s}$

$\Rightarrow \quad v_s = \left(\frac{SS'}{SO}\right) v$

$\Rightarrow \quad v_s = \frac{R\theta}{\sqrt{h^2-R^2}} v$

where $\cos\theta = \frac{R}{h}$ i.e., $\theta = \cos^{-1}\left(\frac{R}{h}\right)$

$\Rightarrow \quad v_s = \frac{Rv\cos^{-1}(R/h)}{\sqrt{h^2-R^2}}$



Single Correct Choice Type Questions

1. At $t = 0$, $x = 5$ cm, initial phase of insect

$\phi_1 = (20\pi)(5) - (50\pi)(0) = 100\pi$

After 5 s insect will most a distance $a = vt = 25$ cm
So, it will be at $x = 30$ cm
Phase at this instant is

$\phi_i = (20\pi)(30) - (50\pi)(5) = 350\pi$

$\Rightarrow \quad \Delta\phi = \phi_f - \phi_i = 250\pi$

Hence, the correct answer is (B).

2. $f - 5 \propto \sqrt{100}$ $\quad \{\because f \propto \sqrt{T}\}$

$f + 5 \propto \sqrt{121}$

$\Rightarrow \quad \frac{f+5}{f-5} = \frac{11}{10}$

$\Rightarrow \quad 10f + 50 = 11f - 55$

$\Rightarrow \quad f = 105$ Hz

Hence, the correct answer is (A).

3. $\nu_A = \dfrac{v}{4\ell} = \dfrac{v}{0\cdot 60}$

$\nu_B = \dfrac{v}{2\ell} = \dfrac{v}{0\cdot 61}$

Since, $\nu_A - \nu_B = 5$

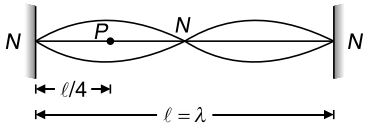
$\Rightarrow \quad \dfrac{v}{0\cdot 60} - \dfrac{v}{0\cdot 61} = 5$

$\Rightarrow \quad v = 183 \text{ ms}^{-1}$

$\Rightarrow \quad \nu_A = \dfrac{183}{0\cdot 6} = 305 \text{ Hz and } \nu_B = 300 \text{ Hz}$

Hence, the correct answer is (C).

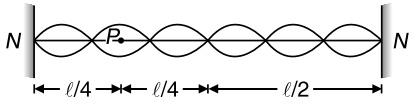
4. In the first case



Point P is an antinode i.e., the string is vibrating in its second harmonic. Let f_0 be the fundamental frequency. Then $2f_0 = 100$ Hz

$\Rightarrow \quad f_0 = 50$ Hz

Now P is an antinode (at length $\ell/4$ from one end) so centre should be a node. So, next higher frequency will be sixth harmonic or $6f_0$ which is equal to 300 Hz as shown below:



Hence, the correct answer is (D).

5. Frequency $f \propto \sqrt{mg}$, so $f \propto \sqrt{g}$

In water $f_w = 0.8\, f_{\text{air}}$

$\Rightarrow \quad \dfrac{g'}{g} = (0.8)^2 = 0.64$

$\Rightarrow \quad 1 - \dfrac{\rho_w}{\rho_m} = 0.64$

$\Rightarrow \quad \dfrac{\rho_w}{\rho_m} = 0.36 \qquad \ldots(1)$

where, ρ_w is relative density of water i.e. 1 and ρ_m is relative density of mass

In liquid, $\dfrac{g'}{g} = (0.6)^2 = 0.36$

$\Rightarrow \quad 1 - \dfrac{\rho_L}{\rho_m} = 0.36$

$\Rightarrow \quad \dfrac{\rho_L}{\rho_m} = 0.64 \qquad \ldots(2)$

where, ρ_L is relative density of liquid.

From equations (1) and (2), we get

$\dfrac{\rho_L}{\rho_w} = \dfrac{0.64}{0.36}$

$\Rightarrow \quad \rho_L = 1.77 \qquad \{\because \rho_w = 1\}$

Hence, the correct answer is (B).

6. For pipe P_1, $3f_c = 3\left(\dfrac{v}{4\ell_c}\right)$

For pipe P_2, $4f_0 = 4\left(\dfrac{v}{2\ell_0}\right)$

where, ℓ_c and ℓ_0 are lengths of closed and open organ pipe respectively. Since both are in resonance, so

$3\left(\dfrac{v}{4\ell_c}\right) = 4\left(\dfrac{v}{2\ell_0}\right)$

$\Rightarrow \quad \dfrac{\ell_c}{\ell_0} = \dfrac{3}{8}$

Hence, the correct answer is (B).

7. Since frequency remains constant, so $v \propto \lambda$

$\dfrac{\lambda_t}{\lambda_0} = \dfrac{v_t}{v_0}$

$\Rightarrow \quad \lambda_t = \sqrt{\dfrac{t+273}{273}}\lambda_0 = \left(1 + \dfrac{t}{273}\right)^{\frac{1}{2}}\lambda_0$

For $-1 < x < 1$, we have $(1+x)^n \approx 1 + nx$

$\Rightarrow \quad \lambda_t \approx \left(1 + \dfrac{t}{546}\right)\lambda_0 = \left(1 + \dfrac{25}{546}\right)110 = 115 \text{ cm}$

Hence, the correct answer is (B).

8. Since, $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{35}{3.5\times 10^3}} = 100 \text{ ms}^{-1}$, so $\lambda = \dfrac{v}{f} = 1 \text{ m}$

$\Rightarrow \quad k = \dfrac{2\pi}{\lambda} = 2\pi \text{ radm}^{-1}$

$\Rightarrow \quad \omega = vk = 200\pi \text{ rads}^{-1}$

At $x = 0$, $v_p = \text{maximum} = A\omega$

Since, $A\omega = (\text{Wave Velocity})(\text{Slope})$

$\Rightarrow \quad A = \dfrac{\text{slope} \times \text{wave velocity}}{\omega} = \dfrac{(\pi/20)\times(100)}{200\pi}$

$\Rightarrow \quad A = 0.025 \text{ m}$

Hence, the correct answer is (D).

9. Since, $f = f_0\left(\dfrac{v + v_0}{v}\right)$

$\Rightarrow \quad f = f_0\left(\dfrac{v + at}{v}\right) = f_0 + \left(\dfrac{f_0 a}{v}\right)t$

This is an equation of a straight line with positive intercept f_0 and positive slope $\dfrac{f_0 a}{v}$, where v is the speed of sound in air.

Hence, the correct answer is (A).

10. $y(x,t) = \dfrac{a}{(x \pm vt)^2 + b}$ is another form of progressive wave equation propagating with a speed v. Negative sign to be taken for propagation along $+x$-axis and positive sign to be taken for propagation along $-x$-axis.

Hence, the correct answer is (D).

CHAPTER 4

11. Let, Δx be the path difference, then

$$\Delta x = L_2P - L_1P$$

$$\Rightarrow \quad \Delta x = \sqrt{40^2 + 9^2} - 40 = 41 - 40 = 1 \text{ m}$$

For first maximum $\Delta x = (2n)\frac{\lambda}{2}$, where $n = 1$

$$\Rightarrow \quad 1 = 2(1)\left(\frac{\lambda}{2}\right)$$

$$\Rightarrow \quad \lambda = 1 \text{ m}$$

$$\Rightarrow \quad f = \frac{v}{\lambda} = 330 \text{ Hz}$$

Hence, the correct answer is (B).

12. Since, $\lambda = \frac{330}{500}$ m

So, first resonance length is

$$\ell_1 = \frac{\lambda}{4} = \frac{33}{200} \text{ m}$$

Second resonance length is

$$\ell_2 = \frac{3\lambda}{4} = \frac{66}{200} \text{ m}$$

Third resonance length is

$$\ell_3 = \frac{5\lambda}{4} = \frac{33}{40} \text{ m}$$

Fourth resonance length is

$$\ell_4 = \frac{7\lambda}{4} = \frac{231}{200} \text{ m}$$

which is greater than 1 m. Hence only three resonances are obtained.

Hence, the correct answer is (D).

13. Since $v = \sqrt{\frac{T}{\mu}}$, where T is tension in string, μ is mass per unit length i.e., $\mu = A\rho$, where A is the area of cross section of the wire.

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{Al\rho/l}} = \sqrt{\frac{T}{A\rho}}$$

$$\Rightarrow \quad \frac{v_1}{v_2} = \sqrt{\frac{T_1A_2}{T_2A_1}} = \sqrt{4} = 2$$

Hence, the correct answer is (B).

14. Since, $f = \left(\frac{v + v_0}{v}\right)f_0 = f_0 + \frac{f_0v_0}{v}$, where $v_0 = gt$

$$\Rightarrow \quad f = f_0 + \left(\frac{f_0g}{v}\right)t$$

i.e., f-t graph is a straight line of slope $\frac{f_0g}{v}$

$$\Rightarrow \quad \frac{f_0g}{v} = \text{slope}$$

$$\Rightarrow \quad v = \frac{f_0g}{\text{slope}} = \frac{(10^3)(10)}{10^3/30} = 300 \text{ ms}^{-1}$$

Hence, the correct answer is (C).

15. According to Snell's Law of Refraction

$$\frac{v_1}{v_2} = \frac{\sin\phi}{\sin\phi}$$

$$\Rightarrow \quad \frac{v_1}{v_2} = \frac{BD/AD}{AC/AD} = \frac{BD}{AC}$$

Hence, the correct answer is (C).

16. Since, $f_0 - f_c = 2$

$$\Rightarrow \quad \frac{v}{2\ell} - \frac{v}{4\ell} = \frac{v}{4l} = 2$$

$$\Rightarrow \quad \frac{v}{\ell} = 8 \qquad \ldots(1)$$

When length of open organ pipe is halved and that of closed organ pipe is doubled, beat frequency will be

$$f_0' - f_c' = \frac{v}{\ell} - \frac{v}{8\ell} = \frac{7}{8}\frac{v}{\ell}$$

Substituting $\frac{v}{\ell} = 8$ from equation (1), we get

$$f_b = 7$$

Hence, the correct answer is (B).

17. For Listener in the front

$$f' = f\left(\frac{v}{v - v_s}\right)$$

$$\Rightarrow \quad \lambda' = \frac{v}{f'} = \left(\frac{v - v_s}{f}\right) = \frac{345 - 30}{500} = 0.63 \text{ m}$$

For Listener at the back

$$f' = f\left(\frac{v}{v + v_s}\right)$$

$$\Rightarrow \quad \lambda' = \frac{v}{f'} = \frac{v + v_s}{f} = \frac{345 + 30}{500} = 0.75 \text{ m}$$

Hence, the correct answer is (B).

18. In front of the locomotive, effective value of velocity of sound is $v' = v + v_w$

So, $f' = f\left[\frac{(v + v_w) - 0}{(v + v_w) - v_s}\right]$

$$\Rightarrow \quad \lambda' = \frac{(345 + 10) - 30}{500} \qquad \left\{\because \lambda' = \frac{v'}{f'} = \frac{v + v_w}{f'}\right\}$$

$$\Rightarrow \quad \lambda' = \frac{325}{500} = 0.65 \text{ m}$$

At the back of the train the effective value of velocity of sound is $v' = v - v_w$

$$\Rightarrow \quad f' = f\left[\frac{v - v_w}{(v - v_w) + v_s}\right]$$

$$\Rightarrow \quad \lambda' = \left[\frac{(345 - 10) + 30}{500}\right] \qquad \left\{\because \lambda' = \frac{v'}{f'} = \frac{v - v_w}{f'}\right\}$$

$$\Rightarrow \quad \lambda' = \frac{365}{500} = 0.73 \text{ m}$$

Hence, the correct answer is (A).

19. The given pulse is of the form

$$y = \frac{a}{b + (x - vt)^2} \qquad \ldots(1)$$

where v is the wave velocity
Given equation is

$$y = \frac{1}{1 + (x-1)^2} \text{ at } t = 2 \text{ s} \qquad \ldots(2)$$

Comparing equations (1) and (2), we get

$$vt = 1$$

$$\Rightarrow \quad v = \frac{1}{t} = \frac{1}{2} = 0.5 \text{ ms}^{-1}$$

Hence, the correct answer is (B).

20. Standard form of a sine wave is

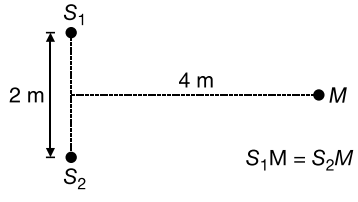
$$y = A\sin(\omega t - kx) \qquad \ldots(1)$$

Here, $\omega = \frac{2\pi}{T}$, $k = \frac{2\pi}{\lambda}$

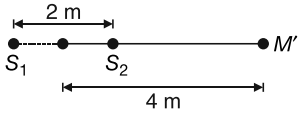
and $v = \frac{\omega}{k} = \frac{\lambda}{T}$

With these substitutions we can see that equation (1) can be re-written as OPTIONS (A), (C) and (D).
Hence, the correct answer is (B).

21. Path Difference $= S_1M' - S_2M'$



When the box is rotated



$$\Rightarrow \quad \Delta x = 5 - 3 = 2 \ m$$

For MAXIMA

Path Difference $= (\text{Even multiple})\frac{\lambda}{2}$

$$\Delta x = (2n)\frac{\lambda}{2}$$

For 5 maximum responses

$$\Rightarrow \quad 2 = 2(5)\frac{\lambda}{2} \qquad \left\{\because \Delta x = (2n)\frac{\lambda}{2}\right\}$$

$$\Rightarrow \quad \lambda = \frac{2}{5} = 0.4 \text{ m}$$

Hence, the correct answer is (B).

22. Since, $\omega_1 = 1000\pi$ and $\omega_2 = 1008\pi$

$$\Rightarrow \quad f_1 = 500 \text{ Hz and } f_2 = 504 \text{ Hz}$$

$$\Rightarrow \quad f_b = f_2 - f_1 = 4 \text{ Hz}$$

Hence, the correct answer is (A).

23. Since, $P = \frac{1}{2}\rho\omega^2 A^2 sv$ and $\frac{\lambda_1}{\lambda_2} = \frac{1}{2}$

$$\Rightarrow \quad \frac{f_1}{f_2} = \frac{\omega_1}{\omega_2} = \frac{\lambda_2}{\lambda_1} = \frac{2}{1} \qquad \left\{\because f \propto \frac{1}{\lambda}\right\}$$

Given that $P_1 = P_2$, so $\omega_1 A_1 = \omega_2 A_2$

$$\Rightarrow \quad \frac{A_1}{A_2} = \frac{\omega_2}{\omega_1} = \frac{1}{2}$$

Now pressure amplitude

$$P_0 = B_0 Ak$$

$$\Rightarrow \quad \frac{(P_0)_1}{(P_0)_2} = \left(\frac{A_1}{A_2}\right)\left(\frac{k_1}{k_2}\right) = \left(\frac{A_1}{A_2}\right)\left(\frac{\lambda_2}{\lambda_1}\right) = \left(\frac{1}{2}\right)\left(\frac{2}{1}\right) = 1$$

Hence, the correct answer is (A).

24. Since $f = \frac{100}{60}$ Hz, so $T = \frac{60}{100}$ s

$$\Rightarrow \quad d = \frac{vT}{2} = \frac{330(60/100)}{2} = 99 \text{ m}$$

Hence, the correct answer is (B).

25. At $t = 0$, $x = 0$ displacement $y = 0$. Therefore, OPTIONS (A) or (B) may be correct. Secondly slope at $x = 0$ at $t = 0$ is positive i.e., particle velocity is in negative y-direction because particle velocity is $v_p = -v(\partial y/\partial x)$. So, particle at $x = 0$ is travelling in negative y-direction.
Hence, the correct answer is (B).

26. $f' = f\left[\frac{v - (-v_s)}{v - v_s}\right] = f\left(\frac{v + v_s}{v - v_s}\right)$

$$\Rightarrow \quad f' = 1000\left(\frac{340 + 15}{340 - 15}\right) = 1092 \text{ Hz}$$

Hence, the correct answer is (C).

27. Initially frequency of vibrations of closed organ pipe is 10 kHz.

$$f = \frac{v}{4\ell} = \frac{\sqrt{\gamma RT/M}}{4\ell}$$

$$\Rightarrow \quad f \propto \sqrt{T}$$

$$\frac{f'}{f} = \sqrt{\frac{T'}{T}} = \left(1 + \frac{1}{300}\right)^{1/2} \approx 1 + \frac{1}{600}$$

$$\Rightarrow \quad f' = f\left(1 + \frac{1}{600}\right)$$

$$\Rightarrow \quad \Delta f = f' - f = \frac{f}{600}$$

$$\Rightarrow \quad \Delta f = \frac{10 \times 10^3}{600} = 16.67 \text{ Hz}$$

So, number of beats produced = 16.67 Hz.
Hence, the correct answer is (C).

28. According to Hook's Law, $F = T = k\Delta x$

$$\Rightarrow \quad v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{k(2l - l)}{M/3l}} = \sqrt{\frac{kl}{M/3l}}$$

CHAPTER 4

Also, $v' = \sqrt{\frac{k(4l)}{M/6l}} = 2\sqrt{2}v$

Hence, the correct answer is (D).

29. Power (which is directly proportional to intensity) is given by

$$P = \frac{1}{2}\mu\omega^2 A^2 v$$

where, μ is mass per unit length, A is amplitude, v is wave speed.

$$\Rightarrow \quad \frac{I_1}{I_2} = \frac{1}{36} = \frac{A_1^2\omega_1^2}{A_2^2\omega_2^2}$$

$$\Rightarrow \quad \frac{1}{36} = \frac{A_1^2}{A_2^2}\frac{1}{4}$$

$$\Rightarrow \quad \frac{A_1}{A_2} = \frac{1}{3}$$

Hence, the correct answer is (A).

30. For not hearing the echo the time interval between the beats of drum must be equal to time of echo.

$$\Rightarrow \quad t_1 = \frac{2d}{v} = \frac{60}{40} \qquad \ldots(1)$$

and $t_2 = \frac{2(d-90)}{v} = 1$

$$\Rightarrow \quad 2d - 180 = v \qquad \ldots(2)$$

From (1), we get

$$2d = \frac{3}{2}v.$$

Substituting in (2), we get

$$\Rightarrow \quad \frac{3}{2}v - 180 = v$$

$$\Rightarrow \quad 180 = \frac{v}{2}$$

$$\Rightarrow \quad v = 360 \text{ ms}^{-1}$$

$$\Rightarrow \quad \frac{2(d)}{360} = \frac{3}{2}$$

$$\Rightarrow \quad d = 270 \text{ m}$$

Hence, the correct answer is (D).

31. $y_1 = 4\sin[2\pi(200)t]$ and $y_2 = 3\sin[2\pi(202)t]$

Number of beats per second = 2 bps

$$\Rightarrow \quad \frac{I_{max}}{I_{min}} = \left(\frac{4+3}{4-3}\right)^2 = 49$$

Hence, the correct answer is (B).

32. $y_1 = a_1\sin\left(\omega t - \frac{2\pi x}{\lambda}\right)$

$$y_2 = a_2\sin\left(\omega t - \frac{2\pi x}{\lambda} + \phi + \frac{\pi}{2}\right) \qquad \{\because \sin(90+\theta) = \cos\theta\}$$

$$\Rightarrow \quad \Delta\phi = \phi + \frac{\pi}{2}$$

Since, $\Delta\phi = \frac{2\pi}{\lambda}\Delta x$

$$\Rightarrow \quad \Delta x = \frac{\lambda}{2\pi}\left(\phi + \frac{\pi}{2}\right)$$

Hence, the correct answer is (B).

33. Since, $f' = f\left(\frac{c-v}{c+v}\right)$

$$\Rightarrow \quad f' - f = \left(\frac{2v}{c-v}\right)f$$

where, c is velocity of waves emitted by radar i.e. c is velocity of light.

$$\Rightarrow \quad f' - f \cong \frac{2v}{c}f \qquad \{\because c - v \simeq c\}$$

$$\Rightarrow \quad 2\cdot 6\times 10^3 = \frac{2v}{3\times 10^8}\times 780\times 10^6$$

$$\Rightarrow \quad v = \frac{(3\times 10^8)(2.6\times 10^3)}{2\times 780\times 10^6} = 0.5\times 10^3 \text{ ms}^{-1} = 0.5 \text{ kms}^{-1}$$

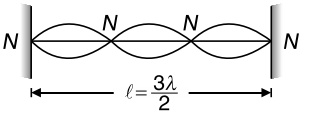
Hence, the correct answer is (B).

34. Since, $\ell = \frac{3\lambda}{2}$

$$\Rightarrow \quad \lambda = \frac{2\ell}{3} = \frac{(2)(0.6)}{3} \text{ m} = 0.4 \text{ m}$$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{80}{0.2}} = 20 \text{ ms}^{-1}$$

$$\Rightarrow \quad f = \frac{v}{\lambda} = \frac{20}{0.4} \text{ Hz} = 50 \text{ Hz}$$



Since, $v_{max} = (a_{max})\omega$

$$\Rightarrow \quad v_{max} = (0.5\times 10^{-2} \text{ m})(2\pi)(50) \text{ ms}^{-1} = \frac{\pi}{2} \text{ ms}^{-1}$$

$$\Rightarrow \quad v_{max} = 1.57 \text{ ms}^{-1}$$

Hence, the correct answer is (A).

35. Since, $f = \frac{c}{4\ell}$

$$\Rightarrow \quad \left|\frac{df}{dt}\right| = \frac{c}{4\ell^2}\left|\frac{d\ell}{dt}\right| = \frac{cv}{4\ell^2}$$

Hence, the correct answer is (B).

36. Since, $f = n\left(\frac{1}{2l}\sqrt{\frac{T}{\mu}}\right) = 4\left[\frac{1}{2(0.8)}\sqrt{\frac{0.4\times 9\cdot 8}{10^{-3}/0.8}}\right]$

$$\Rightarrow \quad f = \frac{40}{16}\sqrt{4\times 98\times 8} = \frac{40}{16}\times 4\times 14 = 160 \text{ Hz}$$

Hence, the correct answer is (C).

37. Since, $f = n\left(\frac{1}{2\ell}\sqrt{\frac{T}{\mu}}\right)$

$$\Rightarrow \quad Tn^2 = \text{constant}$$

where n is the number of loops

$$\Rightarrow \quad (0.4)(16) = (25)T$$

$\Rightarrow \quad T = (0\cdot 4)\left(\frac{4}{5}\right)^2$

Hence, the correct answer is (B).

38. Velocity of longitudinal wave $v_1 = \sqrt{\frac{Y}{\rho}}$ and velocity of transverse wave is $v_2 = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\rho S}}$

$$\frac{v_1}{v_2} = \sqrt{\frac{YS}{T}} = \sqrt{\frac{Y}{T/S}} = \sqrt{\frac{Y}{Y(\Delta l/l)}} = \sqrt{\frac{1}{1/n}} = \sqrt{n}$$

Since, $f \propto v$, so $\frac{f_1}{f_2} = \frac{v_1}{v_2} = \sqrt{n}$

In the above equations ρ is density of string, $S$is area of cross-section of string, Y is Young's modulus of elasticity, so $\frac{T}{S} = \text{Stress} = Y(\text{Strain}) = Y\left(\frac{\Delta l}{l}\right)$

Hence, the correct answer is (C).

39. At $x_1 = \frac{\pi}{3k}$ and $x_2 = \frac{3\pi}{2k}$

$\Rightarrow$ $\sin kx_1$ is not zero and $\sin kx_2$ is not zero.

Therefore, neither of x_1 or x_2 is a node

$\Rightarrow \quad \Delta x = x_2 - x_1 = \left(\frac{3}{2} - \frac{1}{3}\right)\frac{\pi}{k} = \frac{7\pi}{6k}$

Since $\frac{\pi}{k} < \Delta x < \frac{2\pi}{k}$, so $\frac{\lambda}{2} < \Delta x < \lambda$ $\left\{\because k = \frac{2\pi}{\lambda}\right\}$

Therefore $\phi_1 = \pi$ and $\phi_2 = k\Delta x = \frac{7\pi}{6}$

$\Rightarrow \quad \frac{\phi_1}{\phi_2} = \frac{6}{7}$

Hence, the correct answer is (D).

40. Since, $f \propto \sqrt{T}$

$\Rightarrow \quad \frac{f_{\text{air}}}{f_{\text{water}}} = \sqrt{\frac{w_{\text{air}}}{w_{\text{water}}}} = \sqrt{\frac{V\rho g}{V\rho g - V\rho_w g}}$

$\Rightarrow \quad \frac{f}{f/2} = 2 = \sqrt{\frac{\rho}{\rho - \rho_w}}$

$\Rightarrow \quad 4\rho - 4\rho_w = \rho$

$\Rightarrow \quad \rho = \frac{4}{3}\rho_w$ …(1)

Similarly, in second case

$$\frac{f}{f/3} = \sqrt{\frac{\rho}{\rho - \rho_L}}$$

$\Rightarrow \quad 3 = \sqrt{\frac{\frac{4}{3}\rho_w}{\frac{4}{3}\rho_w - \rho_L}} = \sqrt{\frac{4}{4 - 3\frac{\rho_L}{\rho_w}}}$

where, $\frac{\rho_L}{\rho_w}$ = specific gravity (say s)

$\Rightarrow \quad 9 = \frac{4}{4 - 3s}$

$\Rightarrow \quad 36 - 27s = 4$

$\Rightarrow \quad s = \frac{32}{27}$

Hence, the correct answer is (D).

41. From the given equation wave number

$$k = \frac{2\pi}{\lambda} = (10\pi)(0.01)$$

$\Rightarrow \quad k = 0.1\pi\, m^{-1}$

Since, phase difference $\Delta\phi = \left(\frac{2\pi}{\lambda}\right)\Delta x$

$\Rightarrow \quad \Delta\phi = (0.1\pi)(10) = \pi$

Hence, the correct answer is (B).

42. $\frac{v_{H_2}}{v_{CO_2}} = \sqrt{\frac{\gamma_{H_2} M_{CO_2}}{\gamma_{CO_2} M_{H_2}}} = \sqrt{\frac{(7/5)(44)}{(9/7)(2)}} = \sqrt{\frac{49}{45} \times 22}$

Hence, the correct answer is (A).

43. Since, 45 cm $= 5(9$ cm$)$ and 99 cm $= 11(9$ cm$)$

So, two other length between these two values are $7(9$ cm$)$ and $9(9$ cm$)$ i.e. 63 cm and 81 cm respectively. So, the fundamental length is 9 cm

$\Rightarrow \quad 9 = \frac{\lambda}{4}$ {for a closed organ pipe}

$\Rightarrow \quad \lambda = 36$ cm

Hence, the correct answer is (D).

44. Since, $f_{\text{approach}} = \left(\frac{v}{v - v_s}\right) f_0 = f_1$ and

$$f_{\text{recede}} = \left(\frac{v}{v + v_s}\right) f_0 = f_2 (< f_1)$$

Hence, the correct answer is (B).

45. On getting reflected from a rigid boundary the wave suffers an additional phase change of π (According to Stokes Law)

$\Rightarrow \quad y = 0.8A\sin(\omega t + kx + \pi)$

$\Rightarrow \quad y = -0.8A\sin(\omega t + kx)$

Hence, the correct answer is (B).

46. $y_r = -0.8A\sin(\omega t + kx)$

Hence, the correct answer is (C).

47. For change of pressure, velocity of sound remains the same

$\Rightarrow \quad \frac{v_{30}}{v_{15}} = \frac{v_{30}}{340} = \sqrt{\frac{303}{288}}$

$\Rightarrow \quad v_{30} = 340\sqrt{\frac{303}{288}}$

Hence, the correct answer is (A).

48. Since $\frac{I_2}{I_1} = 2$ and $L_2 - L_1 = 10\log_{10}\left(\frac{I_2}{I_1}\right)$

$\Rightarrow \quad L_2 - L_1 = 10\log_{10} 2 = 3$ dB $\{\because \log_e 2 \cong 0\cdot 3\}$

Hence, the correct answer is (C).

CHAPTER 4

49. Let equation of a plane progressive harmonic wave be

$$y = A\sin(\omega t - kx)$$

Wave speed is $v = \frac{\omega}{k}$, maximum particle speed is $v_p = A\omega$. Given that $v_p < v$

$$\Rightarrow \quad A\omega < \frac{\omega}{k}$$

$$\Rightarrow \quad A < \frac{1}{k}$$

$$\Rightarrow \quad A < \frac{\lambda}{2\pi}$$

Hence, the correct answer is (A).

50. Since, $\frac{f_{\text{approach}} - f_{\text{recede}}}{f} = 2\%$

$$\Rightarrow \quad \frac{2vv_s}{v^2 - v_s^2} = 0.02$$

$$v^2 >> v_s^2$$

$$\Rightarrow \quad \frac{2vv_s}{v^2} = 0.02$$

$$\Rightarrow \quad 2\frac{v_s}{v} = 0.02$$

$$\Rightarrow \quad 2v_s = (0.02)(300) = 3 \text{ ms}^{-1}$$

Hence, the correct answer is (D).

51. This problem is a Doppler effect analogy where $f = 20 \text{ min}^{-1}$, $v = 300 \text{ mmin}^{-1}$ and $v_s = 30 \text{ mmin}^{-1}$

Since, $f' = f\left(\frac{v}{v - v_s}\right)$

$$\Rightarrow \quad f' = (20)\left(\frac{300}{300 - 30}\right) = 22.22 \text{ min}^{-1}$$

Hence, the correct answer is (C).

52. Since, $f_0 = \frac{v}{2\ell}$ …(1)

Now beat frequency $f_b = f_1 - f_2$

$$\Rightarrow \quad f_b = \frac{v}{2\left(\frac{\ell}{2} - \Delta\ell\right)} - \frac{v}{2\left(\frac{\ell}{2} + \Delta\ell\right)}$$

$$\Rightarrow \quad f_b = \frac{v}{2}\left[\frac{1}{\frac{\ell}{2} - \Delta\ell} - \frac{1}{\frac{\ell}{2} + \Delta\ell}\right]$$

$$\Rightarrow \quad f_b = (f_0\ell)\left[\frac{2}{\ell - 2\Delta\ell} - \frac{2}{\ell + 2\Delta\ell}\right]$$

$$\Rightarrow \quad f_b = 2f_0\ell\left[\frac{\ell + 2\Delta\ell - \ell + 2\Delta\ell}{\ell^2 - 4(\Delta\ell)^2}\right]$$

$$\Rightarrow \quad f_b \approx 2f_0\ell\left(\frac{4\Delta\ell}{\ell^2}\right) \approx \frac{8f_0\Delta\ell}{\ell}$$

Hence, the correct answer is (A).

53. Let the third note of fixed frequency be f. Let f_1 and f_2 be the frequency of notes. Then

$$f_1 = \frac{v}{\lambda_1} = \frac{195v}{80} \text{ and } f_2 = \frac{v}{\lambda_2} = \frac{193v}{80}$$

Also, $f_1 - f = 5$ and $f - f_2 = 5$

$$\Rightarrow \quad f_1 - f_2 = 10$$

$$\Rightarrow \quad \frac{2v}{80} = 10$$

$$\Rightarrow \quad v = 400 \text{ ms}^{-1}$$

Hence, the correct answer is (D).

54. Fundamental frequency of open organ pipe is

$$f_0 = \frac{v}{2\ell} = \frac{330}{2 \times 0.3} = 550 \text{ Hz}$$

The given frequency

$$f = 1.1 \text{ kHz} = 1100 \text{ Hz} = 2f_0$$

Therefore, the given frequency corresponds to 2nd harmonic.

Hence, the correct answer is (A).

55. Since $f \propto \sqrt{T}$, so $\frac{\Delta f}{f} = \frac{1}{2}\frac{\Delta T}{T} = \frac{1}{2}\left(\frac{1}{100}\right)$

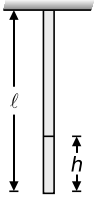
Given that $\Delta f = \frac{3}{2} = 1 \cdot 5$

$$\Rightarrow \quad \frac{1 \cdot 5}{f} = \frac{1}{200}$$

$$\Rightarrow \quad f = 300 \text{ Hz}$$

Hence, the correct answer is (C).

56. Let ℓ be the length of rope. Then tension in the string at height h will be



$$T = \frac{m}{\ell}hg$$

Since, $v = \sqrt{\frac{T}{\mu}}$

where, μ = mass per unit length $= \frac{m}{\ell}$

$$\Rightarrow \quad v = \sqrt{gh}$$

$$\Rightarrow \quad v^2 = gh$$

i.e., v versus h graph is a parabola.

Hence, the correct answer is (C).

57. Since, $\frac{f'}{f} = \sqrt{\frac{T + \Delta T}{T}} \approx 1 + \frac{\Delta T}{2T}$

$$\Rightarrow \quad \frac{\Delta T}{T} = 2\frac{\Delta f}{f} = 2\left(\frac{5}{500}\right) = \frac{2}{100}$$

So, %age change is 2%

Hence, the correct answer is (C).

58. Since $f = \frac{1}{2\ell}\sqrt{\frac{T}{m}}$ and $2f = \frac{1}{2\ell}\sqrt{\frac{T + W}{m}}$

$$\Rightarrow \quad 4 = \frac{T + W}{T} = \frac{m_0 + m}{m}$$

$$\Rightarrow \quad m = 3m_0 = 12 \text{ kg}$$

Hence, the correct answer is (C).

59. A monosyllabic sound reaches listener after reflection in $\frac{1}{5}$ s, so distance between source and listener is

$$d = v\left(\frac{t}{2}\right) = 330\left(\frac{1}{10}\right) = 33 \text{ m}$$

Hence, the correct answer is (B).

60. Let n_1 and n_2 be the number of loops in the wires. Also, they vibrate with same frequency, so

$$\frac{n_1}{2\ell}\sqrt{\frac{T}{\pi r^2\rho}} = \frac{n_2}{2\ell}\sqrt{\frac{T}{\pi(2r)^2\rho}}$$

$$\Rightarrow \quad \frac{n_1}{n_2} = \frac{1}{2}$$

Hence, the correct answer is (C).

61. Since, $\frac{\Delta\lambda}{\lambda} = +\frac{v_s}{c}$

$$\Rightarrow \quad \Delta\lambda = \frac{\lambda v_s}{c}$$

When a star recedes the λ increases (as f decreases)

Hence, the correct answer is (D).

62. $f' = f\left(\frac{v-0}{v-v_s}\right) = 600\left(\frac{330}{300}\right) = 660$ Hz

Hence, the correct answer is (B).

63. $R^2 = a^2 + a^2 + 2a^2\cos\phi$

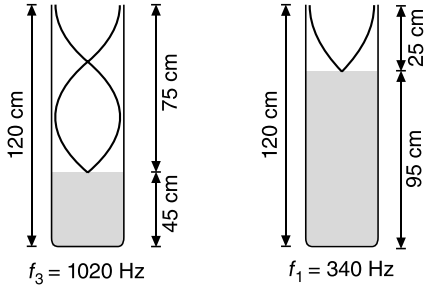
$$\Rightarrow \quad a^2 = a^2 + a^2 + 2a^2\cos\phi$$

$$\Rightarrow \quad \cos\phi = -\frac{1}{2}$$

$$\Rightarrow \quad \phi = \frac{2\pi}{3}$$

Hence, the correct answer is (D).

64. $\ell_0 = \frac{v}{4f_0} = \frac{340}{4(340)} = 25$ cm



Fundamental length = 25 cm

$$\left(\begin{array}{c}\text{Length of organ pipe/air}\\ \text{column for third harmonic}\end{array}\right) = \ell_3 = 75 \text{ cm}$$

(As organ pipe is closed, so even harmonics are absent)

$$\left(\begin{array}{c}\text{Length of organ pipe/air}\\ \text{column for fifth harmonic}\end{array}\right) = \ell_5 = 125 \text{ cm}$$

(as $\ell_5 > 120$ cm so this value is not permissible)

As water is being poured in the organ pipe it starts rising and gives the resonance firstly for 75 cm. Hence length of water column = 120 – 75 = 45 cm.

This is due to the fact that while pouring water f_3 comes first and f_1 comes later.

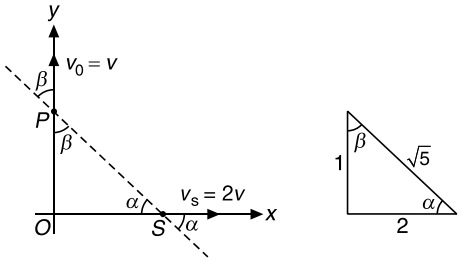
Hence, the correct answer is (B).

65. Let speed of observer be v along y-axis and speed of source is $2v$ along the x-axis.

$$OS = 2(OP) \qquad \{\because v_s = 2v_0\}$$

$$\Rightarrow \quad \cos\alpha = \frac{2}{\sqrt{5}} \text{ and } \cos\beta = \frac{1}{\sqrt{5}}$$

Now, $f = f_0\left(\frac{V - v_0\cos\beta}{V + v_s\cos\alpha}\right)$



$$\Rightarrow \quad f = f_0\left(\frac{V - \frac{v}{\sqrt{5}}}{V + \frac{4v}{\sqrt{5}}}\right) \qquad \ldots(1)$$

where, V is the speed of sound

From equation (1) we can see that f is constant but less than f_0.

Hence, the correct answer is (B).

66. $\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}}$ and $\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$

$$\Rightarrow \quad \frac{V_1}{V_2} = \frac{v_1}{v_2}$$

Hence, the correct answer is (B).

67. Since, $(\Delta P)_m = 2\pi f\rho vA$

$$\Rightarrow \quad A = \frac{(\Delta P)_m}{2\pi f\rho v}$$

$$\Rightarrow \quad A = \frac{30}{2\pi\times 10^3\times 300\times 1.5} = \frac{10^{-4}}{3\pi} \text{ m}$$

Hence, the correct answer is (D).

68. Since $y = 4\sin\left(\pi t + \frac{\pi x}{16}\right)$, where $\omega = \pi$, $k = \frac{2\pi}{\lambda} = \frac{\pi}{16}$

$$\Rightarrow \quad v = \frac{\omega}{k} = 16 \text{ cms}^{-1}$$

Also $\omega = \pi$, so $2\pi f = \pi$

$$\Rightarrow \quad f = 0.5 \text{ Hz and } \lambda = 32 \text{ cm}$$

Hence, the correct answer is (A).

69. $\Delta\phi = \left(\frac{2\pi}{\lambda}\right)\Delta x = \frac{2\pi}{32}(v\Delta t)$ $\{\because \Delta x = v\Delta t\}$

$$\Rightarrow \quad \Delta\phi = \frac{2\pi}{32}(16\times 0.4) = 0.4\pi$$

Hence, the correct answer is (B).

70. $\Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{32}(12) = \frac{3\pi}{4}$

Hence, the correct answer is (C).

71. Since $v_A - v_B = 2$ …(1)

and $v_B - v_A = 2$ …(2)

CHAPTER 4

On putting wax on prongs of B, frequency of B decreases and number of bps decreases to (1) bps. This condition is satisfied by equation (2). Hence

$$\nu_B - 100 = 2$$

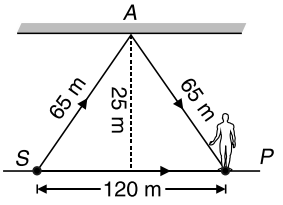
$$\Rightarrow \quad \nu_B = 102 \text{ Hz}$$

Hence, the correct answer is (D).

72. Path Difference $\Delta x = (SA + AP) - SP$

$$\Rightarrow \quad \Delta x = (65 + 65) - 120$$

$$\Rightarrow \quad \Delta x = 10 \text{ m}$$



So, net path difference between waves arriving at the person is $\Delta x = 10$ m

For maxima, i.e. constructive interference, we have

$$\Delta x = (2n)\frac{\lambda}{2};\ n = 0, 1, 2, ...$$

$$\Rightarrow \quad 10 = (2n)\frac{\lambda}{2};\ n = 0, 1, 2, ...$$

$$\Rightarrow \quad 10 = n\lambda;\ n = 0, 1, 2, ...$$

$$\Rightarrow \quad \lambda = \frac{10}{n};\ n = 0, 1, 2, ...$$

$$\Rightarrow \quad \lambda = 10, 5, \frac{10}{3}, ...$$

Hence, the correct answer is (B).

73. When the source approaches the observer the apparent frequency f' is increased and since, $I \propto f^2$ intensity of sound wave get increased.

Hence, the correct answer is (A).

74. Since $f' = f\left(\frac{v - v_\ell}{v - v_s}\right)$, so $\frac{f}{f'} = \left(\frac{v - v_s}{v - v_\ell}\right)$

Hence, the correct answer is (B).

75. Let ℓ be the length of the pipes and v the speed of sound. Then frequency of open organ pipe of nth overtone is

$$f_1 = (n+1)\frac{v}{2\ell}$$

and frequency of closed organ pipe of nth overtone is

$$f_2 = (2n+1)\frac{v}{4\ell}$$

The desired ratio is $\frac{f_1}{f_2} = \frac{2(n+1)}{2n+1}$

Hence, the correct answer is (B).

76. Since, $f \propto \frac{\sqrt{T/\mu}}{\ell} = \frac{\sqrt{T/\rho S}}{\ell}$

where $\mu = \rho S$ and S is cross-sectional area of wire.

$$\Rightarrow \quad f \propto \frac{\sqrt{T/\rho r^2}}{\ell}$$

$$\Rightarrow \quad f \propto \frac{\sqrt{T/\rho}}{r\ell}$$

$$\Rightarrow \quad \frac{f_1}{f_2} = \sqrt{\frac{T_1}{T_2}}\sqrt{\frac{\rho_2}{\rho_1}}\frac{r_2\ell_2}{r_1\ell_1} = (\sqrt{2})\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{4}\right)$$

$$\Rightarrow \quad f_2 = 4f_1$$

Hence, the correct answer is (C).

77. Velocity of wave pulse at distance x from the bottom

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{(\rho x s)g}{(\rho s)}} = \sqrt{xg}$$

As velocity is independent of ρ, time taken by both are same.

Hence, the correct answer is (C).

78. Beat frequency $f_b = f_1 - f_2$

$$\Rightarrow \quad f_b = \frac{v}{2\ell} - \frac{v}{2(\ell + x)}$$

$$\Rightarrow \quad f_b = \frac{v}{2\ell}\left[1 - \left(1 + \frac{x}{\ell}\right)^{-1}\right]$$

$$\Rightarrow \quad f_b = \frac{v}{2\ell}\left[1 - 1 + \frac{x}{\ell}\right] = \frac{vx}{2\ell^2}$$

Hence, the correct answer is (C).

79. Let density of Hydrogen be ρ, then that of Oxygen is 16ρ.

$$\rho_{\text{mixture}} = \frac{(4V)\rho + V(16\rho)}{4V + V}$$

$$\Rightarrow \quad \rho_{\text{mixture}} = 4\rho$$

Given that, $v_{H_2} = 1270 = \sqrt{\frac{\gamma p}{\rho}}$

So, $v_{\text{mixure}} = \sqrt{\frac{\gamma p}{4\rho}} = \frac{1}{2}(1270) = 635 \text{ ms}^{-1}$

Hence, the correct answer is (B).

80. Since $(v_p)_{\max} = A\omega$ and wave velocity, $v = \frac{\omega}{k}$

For both to be equal, $\lambda = 2\pi A$

Hence, the correct answer is (B).

81. Since, $f = f_0\left(\frac{v}{v-u}\right)\left(\frac{v+u}{v}\right) = 8\left(\frac{320+10}{320-10}\right) = 8\left(\frac{33}{21}\right)$

$$\Rightarrow \quad f = 8.5 \text{ kHz}$$

Hence, the correct answer is (A).

82. Since $n = 2\left(\frac{v}{2\ell}\right)$, so $n' = 3\left(\frac{v}{4\ell}\right) = \frac{3n}{4}$

Hence, the correct answer is (C).

83. $f = 1000$ Hz

Distance between two consecutive nodes is $\frac{\lambda}{2}$ so, distance between six nodes is $(6-1)\frac{\lambda}{2} = \frac{5\lambda}{2}$.

N N N N N N

$$\Rightarrow \quad \frac{5\lambda}{2} = 82 \cdot 5$$

$\Rightarrow \quad \lambda = 33$ cm

Since, $v = f\lambda$

$\Rightarrow \quad v = \left(\frac{33}{100}\right)(1000) = 330 \text{ ms}^{-1}$

Hence, the correct answer is (B).

84. $I \propto A^2 \propto \frac{1}{r^2}$ (Inverse Square Law)

$\Rightarrow \quad A \propto \frac{1}{r}$

$\Rightarrow \quad A' = \frac{A}{2}$

Hence, the correct answer is (C).

85. $n = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$

$n = \frac{1}{2\ell}\sqrt{\frac{T}{\pi D^2\rho/4}}$

$\Rightarrow \quad n = \frac{1}{\ell D}\sqrt{\frac{T}{\pi\rho}}$ $\left\{\because \mu = \frac{\pi(D/2)^2\ell\rho}{\ell}\right\}$

$\Rightarrow \quad n' = \frac{\sqrt{2}}{4}n = \frac{n}{2\sqrt{2}}$

Hence, the correct answer is (D).

86. $f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} = \frac{1}{2\ell}\sqrt{\frac{YA\Delta\ell}{A\rho}} = 35 \text{ Hz}$ $\left\{\because Y = \frac{T/A}{\Delta\ell/\ell}\right\}$

Hence, the correct answer is (A).

87. Since, $f = \frac{1}{2\ell D}\sqrt{\frac{T}{\pi\rho}}$

So $fD =$ constant and hence f becomes $\frac{f}{3}$

Hence, the correct answer is (B).

88. Since, $f_b = \frac{v}{2\ell} - \frac{v}{2(\ell+\Delta\ell)}$

$\Rightarrow \quad f_b = \frac{v}{2\ell}\left[1 - \left(1 + \frac{\Delta\ell}{\ell}\right)^{-1}\right]$

Since $\frac{\Delta\ell}{\ell} << 1$, so from Binomial Theorem, we get

$$f_b \approx \frac{v}{2\ell}\left[1 - \left(1 - \frac{\Delta\ell}{\ell}\right)\right] \approx \frac{v}{2\ell}\left(\frac{\Delta\ell}{\ell}\right) = \frac{v\Delta\ell}{2\ell^2}$$

Hence, the correct answer is (D).

89. Let v be the speed of sound and u the speed of train.

Then $v_s = v_0 = u$

and $f' = f\left(\frac{v + v_0\cos\theta}{v + v_s\cos\theta}\right)$

$\Rightarrow \quad f' = f\left(\frac{v + u\cos\theta}{v + u\cos\theta}\right) = f$

Hence, the correct answer is (C).

90. All of the options except (D) form equation of a progressive wave.

Hence, the correct answer is (D).

91. Since $v = \sqrt{\frac{T}{\mu}}$, where $\mu = \rho S$

$\Rightarrow \quad v = \sqrt{\frac{T}{\rho S}} = \sqrt{\frac{1000}{100 \times 10 \times 10^{-6}}} = 10^3 \text{ ms}^{-1}$

Hence, the correct answer is (B).

92. Since $L_2 - L_1 = 10\log_{10}\left(\frac{I_2}{I_1}\right)$

$\Rightarrow \quad 20 - 40 = 10\log_{10}\left(\frac{r_1^2}{r_2^2}\right) = 10\log_{10}\left(\frac{1}{r_2^2}\right)$

$\Rightarrow \quad -2 = \log_{10}\left(\frac{1}{r_2^2}\right) = -2\log_{10}(r_2)$

$\Rightarrow \quad r_2 = 10$ m

Hence, the correct answer is (C).

93. Since two coherent waves have different amplitude,

so $I_{\min} = (a_1 - a_2)^2 \neq 0$

Hence, the correct answer is (C).

94. $\frac{a_1}{a_2} = \frac{1}{3}$

$I_{\max} = (a_1 + a_2)^2$

$\Rightarrow \quad I_{\max} = (a_1 + 3a_1)^2 = 16a_1^2 = 16I$

Hence, the correct answer is (A).

95. $I_{\min} = (a_1 - a_2)^2$

$\Rightarrow \quad I_{\min} = (a_1 - 3a_1)^2 = 4a_1^2 = 4I$

Hence, the correct answer is (C).

96. $f_{\text{approach}} = 1000 = \left(\frac{v - 0}{v - v_s}\right)f = \frac{350}{300}f$

$\Rightarrow \quad f_{\text{recede}} = \left(\frac{v - 0}{v + v_s}\right)f = \frac{350}{400} \times \frac{300}{350} \times 1000$

$\Rightarrow \quad f_{\text{recede}} = 750$ Hz

Hence, the correct answer is (A).

97. Intensity $I = \frac{\text{Power}}{4\pi r^2} = 2\pi^2 f^2 A^2 \rho v$

$\Rightarrow \quad A = \frac{1}{2\pi fr}\sqrt{\frac{\text{Power}}{2\pi\rho v}}$

$\Rightarrow \quad A = \frac{1}{2\pi(1000)(10)}\sqrt{\frac{10}{2\pi(1.29)(340)}}$ m ≈ 0.96 μm

Hence, the correct answer is (D).

98. Since, $\nu_A - \nu_B = 5$ …(1)

or $\nu_B - \nu_A = 5$ …(2)

On filing A, the number of bps increases which is satisfied by (1). So

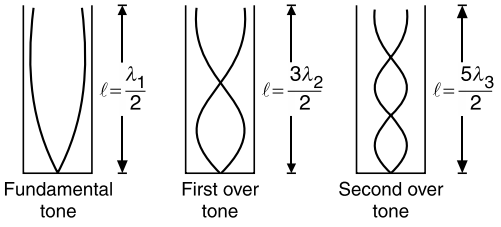
$\nu_A - 512 = 5$

$\Rightarrow \quad \nu_A = 517$ Hz

Hence, the correct answer is (C).

CHAPTER 4

99.



Since, $\ell = \frac{\lambda_1}{2}$

$\Rightarrow \quad \lambda_1 = 2\ell$

Similarly, $\ell = \frac{3\lambda_2}{2}$

$\Rightarrow \quad \lambda_2 = \frac{2\ell}{3}$

Also, $\ell = \frac{5\lambda_2}{2}$

$\Rightarrow \quad \lambda_3 = \frac{2\ell}{5}$

$\Rightarrow \quad \lambda_1 : \lambda_2 : \lambda_3 = 2\ell : \frac{2\ell}{3} : \frac{2\ell}{5} = 1 : \frac{1}{3} : \frac{1}{5}$

Hence, the correct answer is (B).

100. $f \propto \sqrt{T}$

$$f + \frac{f}{2} \propto \sqrt{T'}$$

$\Rightarrow \quad \frac{T'}{T} = \frac{9}{4}$

$\Rightarrow \quad \frac{T' - T}{T} = \left(\frac{9}{4} - 1\right) \times 100 = 125\%$

Hence, the correct answer is (C).

101. Speed of sound in a gas $v_1 = \sqrt{\frac{\gamma RT}{M}}$

while r.m.s. speed of gas molecules is

$$v_2 = \sqrt{\frac{3RT}{M}}$$

Since, γ is always less than 3

$$v_1 < v_2$$

Hence, the correct answer is (B).

102. $f\ell = \text{constant}$

$(256)\ell = (512)(0 \cdot 5)$

$\Rightarrow \quad \ell = 1 \text{ m}$

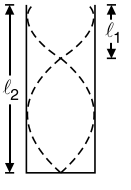
Hence, the correct answer is (C).

103. An open pipe and a closed pipe of equal length cannot have the same frequency at any harmonic.

Hence, the correct answer is (D).

104. $\ell_1 + x = \frac{\lambda}{4}$

$$\ell_2 + x = \frac{3\lambda}{4f}$$



$\Rightarrow \quad x = \frac{\ell_2 - 3\ell_1}{2}$

Hence, the correct answer is (C).

105. Since, $f = \frac{v}{\lambda}$

Given, $f_1 - f_2 = 6$

$\Rightarrow \quad \frac{v}{1} - \frac{v}{1.02} = 6$

Solving this we get, $v = 306 \text{ ms}^{-1}$

Hence, the correct answer is (B).

106. $\frac{I_1}{I_2} = \frac{16}{1} = \frac{a_1^2}{a_2^2}$

$\Rightarrow \quad \frac{a_1}{a_2} = 4$

$$\frac{I_{\max}}{I_{\min}} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2 = \left(\frac{4+1}{4-1}\right)^2 = \frac{25}{9}$$

Hence, the correct answer is (D).

107. $v \propto \sqrt{T}$ and $T \propto \Delta\ell$

$\Rightarrow \quad \frac{v_2}{v_1} = \sqrt{\frac{(\Delta\ell)_2}{(\Delta\ell)_1}}$

$\Rightarrow \quad \frac{v_2}{v_1} = \sqrt{\frac{(L/10)}{(L/20)}} = \sqrt{2}$

$\Rightarrow \quad v_2 = \sqrt{2}v$

Hence, the correct answer is (C).

108. Let a = amplitude due to s_1 and s_2 individually

Loudness due to $s_1 = I_1 = ka^2$ $\{k = \text{constant}\}$

Loudness due to $s_1 + s_2 = I = k(2a)^2 = 4I_1$

$\Rightarrow \quad n = 10\log_{10}\left(\frac{4I_1}{I_1}\right)$

$\Rightarrow \quad n = 10\log_{10}(4) = 10 \times 0.6 = 6$

Hence, the correct answer is (D).

109. Since, $\frac{\Delta f}{f} = 2.5\%$

$\Rightarrow \quad \frac{f' - f}{f} = \frac{2.5}{100} = \frac{1}{40}$

Since $f' = f\left(\frac{v}{v - v_s}\right)$

$\Rightarrow \quad \frac{f\left(\frac{v}{v - v_s}\right) - f}{f} = \frac{1}{40}$

$\Rightarrow \quad \frac{v - v + v_s}{v - v_s} = \frac{1}{40}$

$\Rightarrow \quad \frac{v_s}{320 - v_s} = \frac{1}{40}$

$\Rightarrow \quad 40v_s = 320 - v_s$

$\Rightarrow \quad 41v_s = 320$

$\Rightarrow \quad v_s = \dfrac{320}{41} = 7.8 \text{ ms}^{-1}$

Hence, the correct answer is (C).

110. $f = \dfrac{1}{2\ell}\sqrt{\dfrac{T}{\mu}}$

T becomes $4T$, so f becomes $2f$ i.e. all harmonics and overtones are multiplied by a factor of 2.
Hence, the correct answer is (A).

111. Let $y_1 = A\sin(\omega t)$, then

$\Rightarrow \quad y_2 = A\sin\left(\omega t + \dfrac{\pi}{2}\right)$

$\Rightarrow \quad R^2 = A^2 + A^2 + 2A^2\cos\left(\dfrac{\pi}{2}\right)$

$\Rightarrow \quad R = \sqrt{2}A$

However, both will have the same frequency on superimposing.
Hence, the correct answer is (D).

112. Path difference

$\Delta x = s_2D - s_1D = 5 - 4 = 1 \text{ m}$

The corresponding phase difference will be

$\phi = \left(\dfrac{2\pi}{\lambda}\right)\Delta x = \left(\dfrac{2\pi}{4}\right)(1) = \dfrac{\pi}{2}$

Using $I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi$, we get

$I = I_0 + I_0 + 2\sqrt{I_0I_0}\cos\left(\dfrac{\pi}{2}\right) = 2I_0$

Hence, the correct answer is (C).

113. Since it is being plucked at a distance of $\dfrac{\ell}{4}$. So, we must get two 100 ps. Hence

$f = 2f_1 = 2\left(\dfrac{1}{2\ell}\sqrt{\dfrac{T}{m}}\right)$

$\Rightarrow \quad f = \sqrt{\dfrac{20}{\dfrac{0\cdot5}{1000}}}$

$\Rightarrow \quad f = 200 \text{ Hz}$

Hence, the correct answer is (B).

114. Since $v_0 = \sqrt{\dfrac{\gamma RT}{M}}$, $v = \sqrt{\dfrac{3RT}{M}}$

$\Rightarrow \quad v_0 = v\sqrt{\dfrac{\gamma}{3}}$

Hence, the correct answer is (C).

115. Since, $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{200} = 0.01\ \pi\text{cm}^{-1}$ and $v = \dfrac{\omega}{k}$

$\Rightarrow \quad \omega = vk = (100 \text{ cms}^{-1})(0.01\ \pi \text{ cm}^{-1}) = \pi\text{s}^{-1}$

$\Rightarrow \quad y = 10\sin(0.01\ \pi x - \pi t)$ is the desired equation because the particle at $x = 0$ is moving downwards at $t = 0$. If we substitute $x = 0$ in this equation we get $y = -10\sin(\pi t)$, which is an equation of a SHM moving along negative of direction at $t = 0$.
Hence, the correct answer is (A).

116. Pressure variation has no effect on the speed of sound
Hence, the correct answer is (D).

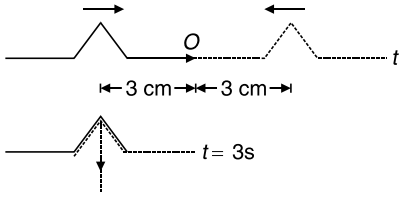
117. The maximum particle velocity is $a\omega$, a is the amplitude

$\Rightarrow \quad v_{\max} = (0.1)2\pi(300)$

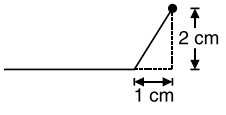
$\Rightarrow \quad v_{\max} = 60\pi \text{ cms}^{-1}$

Hence, the correct answer is (A).

118. The formation of the reflected pulse from a free support is similar to the overlap of two pulses of same nature travelling in opposite directions as shown below.

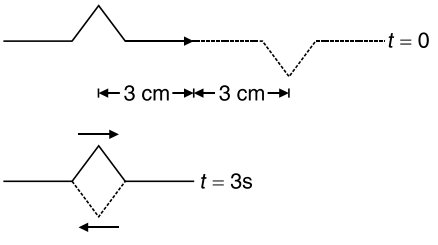


Therefore, net displacement of all points will become twice as shown below:



Hence, the correct answer is (D).

119. The formation of the reflected pulse from a fixed support is similar to the overlap of two inverted pulses travelling in opposite directions as shown below:



Hence, at $t = 3$ s, net displacement of all particles of the string will be zero i.e., string will be straight as shown below:

Hence, the correct answer is (A).

120. For 1st reading of oscillator

$f_A = (514 \pm 2) \text{ Hz}$

$\Rightarrow \quad f_A = 516 \text{ Hz or } 512 \text{ Hz}$

For 2nd reading of oscillator

$f_A = (510 \pm 6) \text{ Hz}$

$\Rightarrow \quad f_A = 516 \text{ Hz or } 504 \text{ Hz}$

A has a frequency of 516 Hz
Hence, the correct answer is (C).

CHAPTER 4

121. $\frac{I_1}{I_2}=\left(\frac{a_1}{a_2}\right)^2$ only when other properties like density of medium, speed of wave etc. remains the same $\left(I=\frac{1}{2}\rho v\omega^2a^2\right)$.

In incident and reflected waves medium is same. Therefore, other properties are same. Hence,

$$\frac{I_i}{I_r}=\left(\frac{a_i}{a_r}\right)^2$$

Hence, the correct answer is (A).

122. $v=v_0=\sqrt{\frac{T}{\mu}}$

Hence, the correct answer is (D).

123. $L_1=10\log\left(\frac{10}{I_0}\right)$ and $L_2=10\log\left(\frac{500}{I_0}\right)$

$\Rightarrow \quad L_2-L_1=10\log\left(\frac{500}{10}\right)=1.7$ dB

Hence, the correct answer is (B).

124. Next Resonance length after the fundamental is $3\ell_1=48$ cm
Hence, the correct answer is (C).

125. The maximum difference between the frequencies is

$$f_b=(n+1)-(n-1)=2 \text{ Hz}$$

Hence, the correct answer is (C).

126. Since, $\frac{\Delta\lambda}{\lambda}=0.01\%$

$\Rightarrow \quad \frac{v}{c}=\frac{0.01}{100}$

$\Rightarrow \quad v=\frac{0.01}{100}\times3\times10^8$

$\Rightarrow \quad v=3\times10^4 \text{ ms}^{-1}$

$\Rightarrow \quad v=30 \text{ kms}^{-1}$

Hence, the correct answer is (C).

127. Since, $v\propto\sqrt{T}$

$\Rightarrow \quad \frac{2v}{v}=2=\sqrt{\frac{T_2}{T_1}}$

$\Rightarrow \quad T_2=4\ T_1=4(273)$

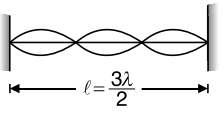
$\Rightarrow \quad T_2=1092 \text{ K}=819\ ^\circ\text{C}$

Hence, the correct answer is (C).

128. Wave number $k=\frac{2\pi}{\lambda}=0.6 \text{ cm}^{-1}$

$\Rightarrow \quad \frac{\lambda}{2}=\frac{\pi}{0.6}$ cm

$\Rightarrow \quad \ell=\frac{3\lambda}{2}=3\left(\frac{\pi}{0.6}\right)$ cm $=15.7$ cm

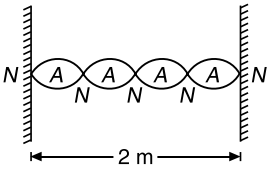


Hence, the correct answer is (D).

129. Since it vibrates in 4 segments, so we must get 5 Nodes and 4 Antinodes.

$\Rightarrow \quad 2=2\lambda$

$\Rightarrow \quad \lambda=1$ m



Distance between two consecutive nodes is $\frac{\lambda}{2}$

Hence, the correct answer is (A).

130. Two syllables in an echo means that time is $2\left(\frac{1}{5}\right)$ s

$\Rightarrow \quad d=v\left(\frac{t}{2}\right)=\left(\frac{330}{2}\right)\left(\frac{2}{5}\right)=66$ m

Hence, the correct answer is (C).

131. $y=ae^{x\pm vt}$ is one more form of progressive wave equation propagating with speed v.
Hence, the correct answer is (B).

132. $\Delta\phi=\left(\frac{2\pi}{\lambda}\right)\Delta x$

$\Rightarrow \quad \frac{\pi}{3}=\frac{2\pi}{(360/500)}\Delta x \qquad \left\{\because \lambda=\frac{v}{f}=\frac{360}{500}\right\}$

$\Rightarrow \quad \Delta x=\frac{120}{1000}$ m $=12$ cm

Hence, the correct answer is (B).

133. Third overtone is the fourth harmonic i.e.

$$f_4=4f_1=400 \text{ Hz}$$

Hence, the correct answer is (C).

134. $f_1=\frac{v}{4\ell}=\frac{340}{0\cdot80}$

$\Rightarrow \quad f_1=425$ Hz

Since in a closed organ pipe only odd harmonics are present. So, possible vibrations are $f_1,\ 3f_1,\ 5f_1$,.....

$\Rightarrow \quad 425,\ 1275,\ 2125,\ .....$

Hence, the correct answer is (B).

135. If the last fork gives octave of the first, then frequency of last must be 2 times frequency of first. So, the series becomes

$$v,\ v+4,\ v+8,.....2v$$

This is an Arithmetic Progression with common difference 4.

$\Rightarrow \quad 2v=v+(65-1)(4)$

$\Rightarrow \quad v=256$ Hz

Hence, the correct answer is (B).

136. Since $n=\frac{v}{2\ell}$, so $n'=\frac{v}{4\ell}=\frac{n}{2}$

Hence, the correct answer is (A).

137. Since $v=\sqrt{\frac{\gamma p}{\rho}}$, so $\frac{v_{\text{monatomic}}}{v_{\text{triatomc}}}=\sqrt{\frac{\gamma_{\text{monatomic}}}{\gamma_{\text{triatomic}}}}=\sqrt{\frac{5/3}{4/3}}$

$\Rightarrow \quad \frac{v_{\text{monatomic}}}{v_{\text{triatomc}}} = \sqrt{\frac{5}{4}} = 1.12$

Hence, the correct answer is (A).

138. $\omega = 50\pi$ and $k = 10\pi$

$\text{Speed} = \frac{\omega}{k} = \frac{50\pi}{10\pi} = 5 \text{ ms}^{-1}$

Hence, the correct answer is (C).

139. $350 = \frac{v}{2\ell}$

$\Rightarrow \quad 350 = \frac{350}{2\ell}$

$\Rightarrow \quad \ell = 0 \cdot 5 \text{ m}$

Hence, the correct answer is (B).

140. Since $f \propto \frac{1}{\ell}$, so $f + 5 \propto \frac{1}{1}$ and $f - 5 \propto \frac{1}{1 \cdot 05}$

$\Rightarrow \quad \frac{f+5}{f-5} = \frac{105}{100}$

$\Rightarrow \quad 100f + 500 = 105f - 525$

$\Rightarrow \quad 5f = 1025$

$\Rightarrow \quad f = 205 \text{ Hz}$

Hence, the correct answer is (D).

141. $f' = \left[\frac{v - \left(-\frac{v}{10}\right)}{v - \frac{v}{10}}\right] f = \frac{11}{9} f = 1.22 f$

Hence, the correct answer is (B).

142. Second overtone is the third harmonic i.e., 3 loops and hence 4 nodes and 3 antinodes.

Hence, the correct answer is (C).

143. $I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

$\Rightarrow \quad I_R = I + 4I + 0$

$\Rightarrow \quad I_R = 5I$

Hence, the correct answer is (B).

144. Fundamental frequency of a closed pipe is given by

$$f_0 = \frac{v}{4\ell}$$

Here, v = speed of sound in air

Length ℓ of air column will first decrease and then becomes constant (when rate of inflow = rate of outflow). Therefore, f_0 will first increase and then become constant.

Hence, the correct answer is (B).

145. Since, $L = 10\log_{10}\left(\frac{I}{I_0}\right)$

$\Rightarrow \quad L = 10\log_{10}\left(\frac{2\times10^{-8}}{10^{-12}}\right) = 10\log_{10}(2\times10^4)$

$\Rightarrow \quad L = 10\left[\log_{10}2 + \log_{10}(10^4)\right]$

$\Rightarrow \quad L = 10(4.3) = 43 \text{ dB}$

Hence, the correct answer is (C).

146. Since $f \propto \sqrt{T}$, so $2f \propto \sqrt{T'}$

$\Rightarrow \quad \frac{T'}{T} = 4$

$\Rightarrow \quad \frac{T' - T}{T} = \frac{3}{4}$

Hence, the correct answer is (D).

147. Apparent frequency for reflector (will act as an observer) would be

$$f_1 = \left(\frac{v+u}{v}\right) f$$

where f = actual frequency of source.

After being reflected the apparent frequency will further change and the reflector will now behave as a source. The apparent frequency will not become

$$f_2 = \left(\frac{v}{v-u}\right) f_1$$

Substituting value of f_1, we get

$$f_2 = \left(\frac{v+u}{v-u}\right) f$$

Hence, the correct answer is (C).

148. $\left(f_{\text{approach}}\right)_A = 5.5 \text{ kHz} = \left(\frac{v + v_A}{v}\right)5$ ...(1)

$\left(f_{\text{approach}}\right)_B = 6 \text{ kHz} = \left(\frac{v + v_B}{v}\right)5$ ...(2)

where v is the velocity of sound

$\Rightarrow \quad 5.5 = \left(1 + \frac{v_A}{v}\right)5$

$\Rightarrow \quad \frac{v_A}{v} = 0.1$ ...(3)

Similarly, $6 = \left(1 + \frac{v_B}{v}\right)5$

$\Rightarrow \quad \frac{v_B}{v} = 0.2$ ...(4)

$\Rightarrow \quad \frac{v_B}{v_A} = 2$

Hence, the correct answer is (B).

149. Time interval between compressional maximum and rarefactional maximum is $\frac{T}{2}$

$\Rightarrow \quad t = \frac{T}{2} = \frac{1}{2f} = \frac{1}{1000} \text{ s}$

Hence, the correct answer is (C).

150. $\text{Separation} = \frac{\lambda}{2} = \frac{v}{2f} = \frac{350}{1000} = 0.35 \text{ m}$

Hence, the correct answer is (C).

151. Since, $I \propto A^2\omega^2$

$\Rightarrow \quad \frac{I_1}{I_2} = \left(\frac{1}{4}\right)(16)$

CHAPTER 4

$\Rightarrow \quad \frac{I_1}{I_2} = 4$

$\Rightarrow \quad \frac{I_2}{I_1} = \frac{1}{4}$

Hence, the correct answer is (C).

152. Distance between two adjacent antinodes is $\frac{\lambda}{2}$ or $\frac{\pi}{k}$, volume of string between two adjacent antinodes $V = \frac{\pi}{k}a$

$E = (V)(u_1 + u_2)$

Here, u is the energy density (energy per unit volume) which is equal to $\frac{1}{2}\rho A^2\omega^2$

$\Rightarrow \quad E = \left(\frac{\pi}{k}\right)(s)\left[\frac{1}{2}\rho a^2\omega^2 + \frac{1}{2}\rho(2a)^2\omega^2\right] = \frac{5\pi s\rho a^2\omega^2}{2k}$

Hence, the correct answer is (C).

153. $L_2 - L_1 = 10\log_{10}\left(\frac{I_2}{I_1}\right) = 10\log_{10}(10^5) = 50 \text{ dB}$

Hence, the correct answer is (D).

154. Since $\frac{I_1}{I_2} = \frac{4}{1}$, so $a_1 = 2a_2$

$\Rightarrow \quad \frac{I_{\max}}{I_{\min}} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2 = \left(\frac{2+1}{2-1}\right)^2 = 9$

So, $L_1 - L_2 = 10\log_{10}\left(\frac{I_{\max}}{I_{\min}}\right) = 10\log_{10}(9)$

$\Rightarrow \quad L_1 - L_2 = 20\log_{10}(3)$

Hence, the correct answer is (B).

155. Since, $L = 10\log_{10}\left(\frac{I}{I_0}\right)$, where $I_0 = 10^{-12}$ Wm^{-2}

$\Rightarrow \quad 70 = 10\log_{10}\left(\frac{I}{I_0}\right)$

$\Rightarrow \quad \frac{I}{I_0} = 10^7$

$\Rightarrow \quad I = 10^{-5}$ Wm^{-2}

Hence, the correct answer is (A).

156. $f_1 = 900\left(\frac{300}{300 + v_1}\right) \approx 900\left(1 + \frac{v_1}{300}\right)^{-1}$

$\Rightarrow \quad f_1 = 900 - 3v_1$

Similarly, $f_2 = 900\left(\frac{300}{300 + v_2}\right) = 900 - 3v_2$

Given, $f_2 - f_1 = 6$

$\Rightarrow \quad 3(v_1 - v_2) = 6$

$\Rightarrow \quad v_1 - v_2 = 2$ ms^{-1}

Hence, the correct answer is (B).

157. $f_{\text{approach}} = \left(\frac{v - 0}{v - v_s}\right)f = \frac{340}{320}f$

Also, $f_{\text{recede}} = \left(\frac{v - 0}{v + v_s}\right)f = \left(\frac{340}{360}\right)f$

$\Rightarrow \quad \frac{f_{\text{approach}}}{f_{\text{recede}}} = \frac{360}{320} = \frac{9}{8}$

Hence, the correct answer is (A).

158. $P = \frac{1}{2}\mu\omega^2A^2v$ where $v = \sqrt{\frac{T}{\mu}}$

Hence, the correct answer is (C).

159. When the source approaches the observer

$f_1 = f\left(\frac{v}{v - v_s}\right) = f\left(1 - \frac{v_s}{v}\right)^{-1} \approx f\left(1 + \frac{v_s}{v}\right)$

$\Rightarrow \quad \left(\frac{f_1 - f}{f}\right) \times 100 = \frac{v_s}{v} \times 100 = 10 \qquad \ldots(1)$

In the second case when the source recedes the observer.

$f_2 = f\left(\frac{v}{v + v_s}\right) = f\left(1 + \frac{v_s}{v}\right)^{-1} \approx f\left(1 - \frac{v_s}{v}\right)$

$\Rightarrow \quad \left(\frac{f_2 - f}{f}\right) \times 100 = -\frac{v_s}{v} \times 100 = -10 \qquad \{\text{from (1)}\}$

In the first case observed frequency increase by 10% while in the second case observed frequency decrease by 10%.

Hence, the correct answer is (D).

160. Since on decreasing the temperature, velocity of sound also decreases $(v \propto \sqrt{T})$ and hence frequency of air column also decreases. Since beat frequency also decreases hence it is obvious that the frequency of air column must be higher than that of fork. Let f be the frequency of fork, then

$\frac{f + 4}{f + 1} = \frac{v_{51}}{v_{16}} = \sqrt{\frac{273 + 51}{273 + 16}} = \frac{18}{17}$

$\Rightarrow \quad f = 50$ Hz

Hence, the correct answer is (A).

161. $\Rightarrow \quad f_1 = \left(\frac{340}{340 - 34}\right)f = \frac{10}{9}f$

and $f_2 = \left(\frac{340}{340 - 17}\right)f = \frac{20}{19}f$

$\Rightarrow \quad \frac{f_1}{f_2} = \frac{10/9}{20/19} = \frac{19}{18}$

Hence, the correct answer is (D).

162. Since there is no relative motion between the source and listener, so apparent frequency equals original frequency.

Hence, the correct answer is (A).

163. Since $f_b = \frac{1}{t_b} = 2$ Hz, so $t_b = 0.5$ s

Hence, the correct answer is (A).

164. Since, $90 - 40 = 10\log_{10}\left(\frac{I_{90}}{I_{40}}\right)$

$\Rightarrow \quad \frac{I_{90}}{I_{40}} = 10^5$

Hence, the correct answer is (D).

165. Since $fD = \text{constant}$

$\Rightarrow \quad f_1D_1 = f_2D_2$

$\Rightarrow \quad fD = (4f)D'$

$\Rightarrow \quad D' = \frac{1}{4}D$

Hence, the correct answer is (B).

166. Given, $3\left(\frac{v}{4\ell_c}\right) = 2\left(\frac{v}{2\ell_0}\right)$

$\Rightarrow \quad \frac{\ell_c}{\ell_0} = \frac{3}{4}$

Now, $n\left(\frac{v}{4\ell_c}\right) = m\left(\frac{v}{2\ell_0}\right)$

$\Rightarrow \quad \frac{n}{m} = 2\left(\frac{\ell_v}{\ell_0}\right) = \frac{6}{4} = \frac{3}{2}$

$\Rightarrow \quad \frac{n}{m} = \frac{3}{2} = \frac{9}{6}$

So, $n = 9$ and $m = 6$.

Hence, the correct answer is (C).

167. Given that $\frac{\Delta f}{f} = 25\% = \frac{1}{4}$

$\Rightarrow \quad \frac{f' - f}{f} = \frac{1}{4}$

$\Rightarrow \quad 4f' - 4f = f$

$\Rightarrow \quad 4f' = 5f$

$\Rightarrow \quad \frac{f'}{f} = \frac{5}{4}$

$\Rightarrow \quad \frac{v}{v - v_s} = \frac{5}{4}$

$\Rightarrow \quad 4v = 5v - 5v_s$

$\Rightarrow \quad v_s = \frac{v}{5}$

Hence, the correct answer is (A).

168. For open pipe, the fundamental frequency is $f = \frac{v}{2l}$.

When dipped half in water, it becomes a closed organ pipe of length $l/2$, so fundamental frequency is

$$f' = \frac{v}{4(l/2)} = f$$

Hence, the correct answer is (B).

169. Fundamental tone of a closed organ pipe is less than the fundamental tone of an open organ pipe of same length. Given that $f_o - f_c = 4$

When the length of open organ pipe is increased f_0 will decrease, because $f \propto \frac{1}{l}$, so $|f_o \sim f_c|$ or beat frequency may increase or decrease.

Hence, the correct answer is (D).

170. Length of string $\ell = (\text{number of loops})\left(\frac{\lambda}{2}\right)$

Since, length is constant, wavelength λ will become half when the number of loops become two times.

Further frequency $f = \frac{v}{\lambda} = \text{consant}$

So, v should also become $\frac{1}{2}$ times $\quad \{\because v \propto \sqrt{T}\}$

Therefore, speed v will become $\frac{1}{2}$ times when mass on the pan is reduced to $\frac{M}{4}$

Hence, the correct answer is (B).

171. Since, $425 : 595 : 765 \equiv 5 : 7 : 9$

So, pipe must be a closed organ pipe

Hence, $\frac{5v}{4\ell} = 425$

$\Rightarrow \quad \frac{5(340)}{4\ell} = 425$

$\Rightarrow \quad \ell = 1 \text{ m}$

Hence, the correct answer is (A).

172. $f_0 = \frac{v}{4\ell} = \frac{340}{4} = 85 \text{ Hz}$

Hence, the correct answer is (D).

173. $v = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\gamma\left(\frac{RT}{M}\right)}$

$\Rightarrow \quad \frac{v_{H_2}}{v_{O_2}} = \sqrt{\frac{M_{O_2}}{M_{H_2}}} = \sqrt{\frac{32}{2}}$

$\Rightarrow \quad \frac{v_{H_2}}{v_{O_2}} = 4$

Hence, the correct answer is (C).

174. Due to weight of the rope, the tension will increase along the rope from the lower end to the upper end. Hence, the pulse will travel with increasing speed as

$$v = \sqrt{\frac{T}{\mu}}$$

Hence, the correct answer is (B).

175. $v = f\lambda$

$\Rightarrow \quad 25 = f(100)$

$\Rightarrow \quad f = 0.25 \text{ Hz}$

$\Rightarrow \quad T = \frac{1}{f} = 4 \text{ s}$

Hence, the correct answer is (A).

176. $y = 2a\sin(\omega t)\cos(kx)$

$\Rightarrow \quad k = \frac{\pi}{15}$

$\Rightarrow \quad \frac{2\pi}{\lambda} = \frac{\pi}{15}$

$\Rightarrow \quad \lambda = 30 \text{ cm}$

CHAPTER 4

Distance between a node and antinode is $\frac{\lambda}{4}$.

So, $d = 7 \cdot 5$ cm

Hence, the correct answer is (A).

177. Since, the frequencies are in the ratio of $5:7:9$, so, it is a closed pipe

$$425 \text{ Hz} = 5\left(\frac{v}{4\ell}\right)$$

$$\Rightarrow \quad l = \frac{5v}{4\times 425} = \frac{5\times 340}{4\times 425} \text{ m} = 1 \text{ m}$$

Hence, the correct answer is (C).

178. Let a_i and a_r be the amplitudes of incident and reflected waves. Then

$$\frac{a_i + a_r}{a_i - a_r} = n \qquad \{\text{given}\}$$

$$\Rightarrow \quad \frac{a_r}{a_i} = \left(\frac{n-1}{n+1}\right)$$

So, fraction of energy reflected is

$$\frac{E_r}{E_i} = \left(\frac{a_r}{a_i}\right)^2 = \left(\frac{n-1}{n+1}\right)^2$$

Hence, the correct answer is (B).

179. $I_{\text{total}} = n^2 a^2 = n^2 I_0$

$$\Rightarrow \quad I_{\text{total}} = (10)^2 I_0 = 100 I_0$$

$$\Rightarrow \quad I_{av} = \langle I \rangle = \frac{100 I_0}{10} = 10 I_0$$

Hence, the correct answer is (C).

180. $f_1 = 50$ Hz

Second overtone in a closed organ pipe is the fifth harmonic (as even harmonics are absent), So

$$f_5 = 5 f_1 = 250 \text{ Hz}$$

Hence, the correct answer is (D).

181. $f' = f\left(\frac{v-0}{v-v_s}\right) = 90\frac{v}{\left(v-\frac{v}{10}\right)} = 90\times\frac{10}{9} = 100$ Hz

Hence, the correct answer is (A).

182. Since, $f_b = 6$ bps $= 6$ Hz

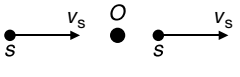
$$\Rightarrow \quad 6 = \frac{v}{50} - \frac{v}{50\cdot 4}$$

$$\Rightarrow \quad v = 378 \text{ ms}^{-1}$$

Hence, the correct answer is (C).

183. Since, $\Delta f = f_1 - f_2$

$$\Rightarrow \quad \Delta f = f\left(\frac{v}{v-v_s}\right) - f\left(\frac{v}{v+v_s}\right)$$



$$\Rightarrow \quad \Delta f = f\left[\left(1-\frac{v_s}{v}\right)^{-1} - \left(1+\frac{v_s}{v}\right)^{-1}\right]$$

$$\Rightarrow \quad \Delta f \approx f\left[\left(1+\frac{v_s}{v}\right) - \left(1-\frac{v_s}{v}\right)\right] = \frac{2 f v_s}{v}$$

Hence, the correct answer is (A).

Multiple Correct Choice Type Questions

1. $v = \sqrt{\frac{T}{\mu}}$

For equilibrium $Mg = mg\sin 30 = T$

$$\Rightarrow \quad M = \frac{m}{2}$$

$$\Rightarrow \quad 100 = \sqrt{\frac{Mg}{9.8\times 10^{-3}}} = \sqrt{\frac{M(9.8)}{9.8\times 10^{-3}}}$$

$$\Rightarrow \quad 100 = \sqrt{M(1000)}$$

$$\Rightarrow \quad M = 10 \text{ kg and } m = 20 \text{ kg}$$

Hence, (A) and (D) are correct.

2. $y = 10^{-4}\sin(60t + 2x)$

$A = 10^{-4}$ m, $\omega = 60$ rads^{-1}, $k = 2$ m^{-1}

Speed of wave $v = \frac{\omega}{k} = 30$ ms^{-1}

Frequency, $f = \frac{\omega}{2\pi} = \frac{30}{\pi}$ Hz. Wavelength, $\lambda = \frac{2\pi}{k} = \pi m$

Further, $60t$ and $2x$ are of same sign. Therefore, the wave should travel in negative x-direction.

Hence, (A), (B), (C) and (D) are correct.

3. $y = a\sin(\alpha t - \beta x)$

Compare with the standard equation of progressive wave i.e.

$$y = a\sin(\omega t - kx)$$

$$\Rightarrow \quad \omega = \alpha \text{ and } k = \beta$$

$$\Rightarrow \quad v = \frac{\omega}{k} = \frac{\alpha}{\beta} \qquad \{\text{OPTION } \textbf{(D)}\}$$

$$\Rightarrow \quad 2\pi f = \alpha$$

$$\Rightarrow \quad f = \frac{\alpha}{2\pi}$$

$$\Rightarrow \quad T = \frac{2\pi}{\alpha} \qquad \{\text{OPTION } \textbf{(B)}\}$$

Since, $k = \frac{2\pi}{\lambda} = \beta$

$$\Rightarrow \quad \lambda = \frac{2\pi}{\beta} \qquad \{\text{OPTION } \textbf{(C)}\}$$

Hence, (B), (C) and (D) are correct.

4. Let the progressive wave equation be

$$y = a\sin(\omega t - kx)$$

$$\Rightarrow \quad \frac{\partial y}{\partial t} = a\omega\cos(\omega t - kx) \qquad \ldots(1)$$

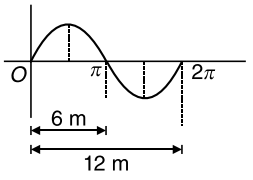
$$\Rightarrow \quad \frac{\partial^2 y}{\partial t^2} = -a\omega^2\sin(\omega t - kx) \qquad \ldots(2)$$

Also $\frac{\partial y}{\partial x} = -ak\cos(\omega t - kx)$ $\qquad \ldots(3)$

$$\Rightarrow \quad \frac{\partial^2 y}{\partial x^2} = ak^2 \sin(\omega t - kx) \qquad \ldots(4)$$

Hence, (A) and (B) are correct.

5. $\lambda = \dfrac{v}{f} = \dfrac{300}{25} = 12 \text{ m}$



Amplitude difference = zero = A

Phase Difference = $\pi = \phi$

Hence, (A) and (D) are correct.

6. Maximum speed of any point on the string is $v_{max} = a\omega$

$$\Rightarrow \quad v_{max} = a(2\pi f)$$

$$\Rightarrow \quad v_{max} = \frac{v}{10} = \frac{10}{10} = 1 \qquad \{\text{Given: } v = 10 \text{ ms}^{-1}\}$$

$$\Rightarrow \quad 2\pi a f = 1$$

$$\Rightarrow \quad f = \frac{1}{2\pi a}$$

Since, $a = 10^{-3}$ m

$$\Rightarrow \quad f = \frac{1}{2\pi \times 10^{-3}} = \frac{10^3}{2\pi} \text{ Hz}$$

Speed of wave, $v = f\lambda$

$$\Rightarrow \quad (10 \text{ ms}^{-1}) = \left(\frac{10^3}{2\pi} \text{ s}^{-1}\right)\lambda$$

$$\Rightarrow \quad \lambda = 2\pi \times 10^{-2} \text{ m}$$

Hence, (A) and (C) are correct.

7. Wavelength depends on length which is fixed. Therefore, wavelength does not change.

Further $v = \sqrt{\dfrac{T}{m}}$

$$\Rightarrow \quad v \propto T^{1/2}$$

$$\Rightarrow \quad (\%\text{ change in } v) = \frac{1}{2}(\%\text{ change in } T)$$

$$\Rightarrow \quad (\%\text{ change in } v) = \frac{1}{2}(2) = 1\%$$

i.e., speed and hence frequency will change by 1%

Change in frequency is 15 Hz which is 1% of 1500 Hz

Therefore, original frequency should be 1500 Hz.

Hence, (B), (C) and (D) are correct.

8. $T_1 > T_2$

$$\Rightarrow \quad v_1 > v_2$$

$$\Rightarrow \quad f_1 > f_2 \text{ and } f_1 - f_2 = 6 \text{ Hz}$$

Now, if T_1 is increased, f_1 will increase or $f_1 - f_2$ will increase. Therefore (D) OPTION is wrong.

If T_1 is decreased f_1 will decrease and it may be possible that now $f_2 - f_1$ become 6 Hz. Therefore (C) OPTION is correct.

Similarly, when T_2 is increased, f_2 will increase and again $f_2 - f_1$ may become equal to 6 Hz. Therefore (C) OPTION is correct.

Similarly, when T_2 is increased, f_2 will increase and again $f_2 - f_1$ may become equal to 6 Hz. So, (B) is also correct but (A) is wrong.

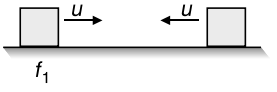
Hence, (B) and (C) are correct.

9. Conceptual

Hence, (B) and (C) are correct.

10. $f_2 = \left(\dfrac{V - w + u}{V - w - u}\right) f_1$

So, $f_2 > f_1$

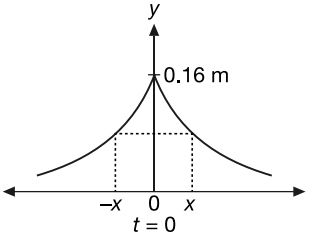


$$f_2 = \left(\frac{V + w + u}{V + w - u}\right) f_1$$

So, $f_2 > f_1$

Hence, (A) and (B) are correct.

11. The shape of pulse at $x = 0$ and $t = 0$ would be as shown, in Figure.



$$y(0,\ 0) = \frac{0.8}{5} = 0.16 \text{ m}$$

From the figure it is clear that $y_{max} = 0.16$ m

Pulse will be symmetric (Symmetry is checked about y_{max}) if at $t = 0$

$$y(x) = y(-x)$$

From the given equation

$$y(x) = \frac{0.8}{16x^2 + 5}$$

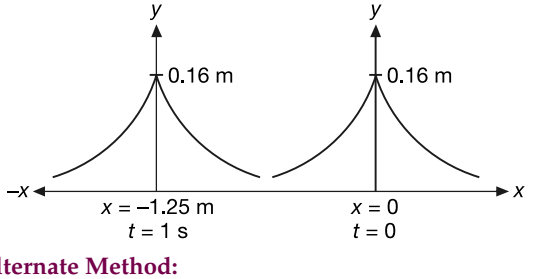
and $\quad y(-x) = \dfrac{0.8}{16x^2 + 5} \qquad \{\text{at } t = 0\}$

$$\Rightarrow \quad y(x) = y(-x)$$

Therefore, pulse is symmetric.

Speed of pulse

At $t = 1$ s and $x = -1.25$ m value of y is again 0.16 m, i.e., pulse has travelled a distance of 1.25 m in 1 s in negative x-direction or we can say that the speed of pulse is 1.25 ms^{-1} and it is travelling in negative x-direction. Therefore, it will travel a distance of 2.5 m in 2 s. The above statement can be better understood from Figure.



Alternate Method:

If equation of a wave pulse is

$$y = f(ax \pm bt)$$

CHAPTER 4

The speed of wave is $\frac{b}{a}$ in negative x-direction for $y = f(ax + bt)$ and positive x-direction for $y = f(ax - bt)$. Comparing this from given equation we can find that speed of wave is $\frac{5}{4} = 1.25\ \text{ms}^{-1}$ and it is travelling in negative x-direction.

Hence, (B), (C) and (D) are correct.

12. For closed organ pipe,

$$f = n\left(\frac{v}{4\ell}\right) \text{ where, } n = 1, 3, 5, ...$$

$$\Rightarrow \quad \ell = \frac{nv}{4f}$$

For $n = 1$, $\ell_1 = \frac{(1)(330)}{4 \times 264} \times 100\ \text{cm} = 31.25\ \text{cm}$

For $n = 3$, $\ell_3 = 3\ell_1 = 93.75\ \text{cm}$

For $n = 5$, $\ell_5 = 5\ell_1 = 156.25\ \text{cm}$

Hence, (A) and (C) are correct.

13. The fundamental frequency of oscillation is

$$f = \frac{1}{2\ell}\sqrt{\frac{T}{m}} = \frac{1}{2\ell}\sqrt{\frac{T}{\rho S}}$$

Here, m = mass per unit length of wire = ρS

where S = area of cross-section of wire

$$\frac{T}{S} = \text{stress} = Y(\text{strain}) = Y\left(\frac{\Delta\ell}{\ell}\right) = Y \propto t$$

$$\Rightarrow \quad f = \frac{1}{2\ell}\sqrt{\frac{Y\alpha t}{\rho}}$$

$$\Rightarrow \quad f \propto \frac{1}{\ell} \propto \sqrt{Y} \propto \sqrt{t} \text{ and } f \propto \sqrt{\frac{\alpha}{\rho}}$$

Hence, (A), (B), (C) and (D) are correct.

14. $\omega = 15\pi$, $k = 10\pi$

Speed of wave, $v = \frac{\omega}{k} = 1.5\ \text{ms}^{-1}$

Wavelength of wave $\lambda = \frac{2\pi}{k} = \frac{2\pi}{10\pi} = 0.2\ \text{m}$

$10\pi x$ and $15\pi t$ have the same sign. Therefore, wave is travelling in negative x-direction.

Hence, (B) and (C) are correct.

15. In plane progressive harmonic wave particles execute SHM and in SHM phase difference is

$\Delta\phi = \pi$ between displacement and acceleration

$\Delta\phi = \frac{\pi}{2}$ between displacement and velocity

and $\Delta\phi = \frac{\pi}{2}$ between velocity and acceleration

Hence, (B), (C) and (D) are correct.

16. For a plane wave intensity (energy crossing per unit area per unit time) is constant at all points.

But for a spherical wave, intensity at a distance r from a point source of power P (energy transmitted per unit time) is given by

$$I = \frac{P}{4\pi r^2}$$

$$\Rightarrow \quad I \propto \frac{1}{r^2}$$

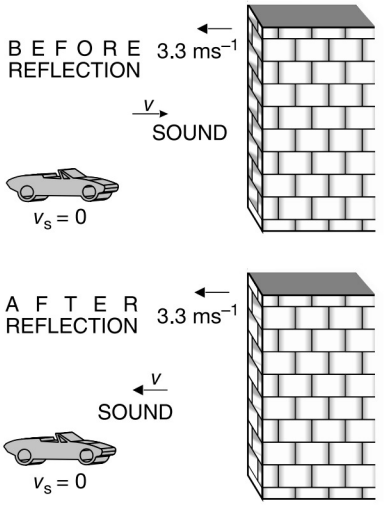
For a line source $I \propto \frac{1}{r}$ because $I = \frac{P}{\pi r \ell}$

Hence, (A), (C) and (D) are correct.

17. Conceptual

Hence, (B), (C) and (D) are correct.

18. $f' = f\left(\frac{v + v_{\text{wall}}}{v - v_{\text{wall}}}\right)$



$$\Rightarrow \quad f' = 1000\left(\frac{340 + 3.3}{340 - 3.3}\right) \simeq 1020\ \text{Hz}$$

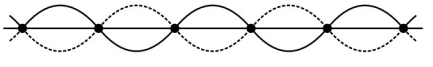
$$\Rightarrow \quad \frac{f' - f}{f} \times 100 = \frac{\Delta f}{f} \times 100 = 2\%$$

Hence, (A) and (C) are correct.

19. For fifth harmonic, number of loops formed is 5, so

No. of nodes = 6

$$\lambda = \frac{2\pi}{k} = \frac{2 \times 3.14}{62.8} = 0.1\ \text{m}$$



$$\text{Length} = \frac{5\lambda}{2} = 0.25\ \text{m}$$

The mid-point is antinode. Its maximum displacement = 0.01 m

$$\Rightarrow \quad f = \frac{v}{2\ell} = \frac{\omega}{k \times 2\ell} = 20\ \text{Hz}$$

Hence, (B) and (C) are correct.

20. Velocity of particle is given by $v_p = -v\left(\frac{dy}{dx}\right)$

Here, v is wave speed and $\frac{dy}{dx}$ the slope.

At point E slope is positive, therefore, v_p will be along negative x-direction. Similarly, slope at D is zero. Therefore, v_p at D will be zero.

Excess pressure $dP = -B \cdot \dfrac{dy}{dx}$

At C slope is negative. Therefore, dP is positive i.e., particles located near C are under compression.

At point D, slope is zero i.e., $dP = 0$.

Hence, (A), (B), (C) and (D) are correct.

21. Standing waves can be produced only when two similar type of waves (same frequency and speed, but amplitude may be different) travel in opposite directions.
Hence, (A), (B) and (C) are correct.

22. Any function $y = f(at \pm bx)$ represents a wave if it is finite everywhere and at all times. The function

$$y = \frac{1}{4t + 3x}$$

is not defined at $x = 0$ and $t = 0$
Hence, (A) and (B) are correct.

23. The wave travels from left to right. Therefore, points lying leftwards are always ahead in phase. Further

Particle velocity $= -(\text{wave speed})(\text{slope})$

Slope at A is positive, while at B is negative i.e., particle velocity at A is negative and at B is positive. Therefore, A is moving downwards while B is moving upwards.
Hence, (A) and (D) are correct.

24. Velocity of wave is $v = \dfrac{\omega}{k} = \dfrac{4}{5} = 0.8 \text{ ms}^{-1}$

The displacement of a particle of the string at t = 0 and $x = \dfrac{\pi}{30}$ m from the mean position is

$$y = 8\sin\left(\frac{5\pi}{30} - 0\right) = 8\sin\left(\frac{\pi}{6}\right) = 8\left(\frac{1}{2}\right) = 4 \text{ m}$$

Hence, (A) and (B) are correct.

25. From the given expression for y:

amplitude $A = 0.02$ m

angular frequency $\omega = 50\pi$ rads^{-1}

and wave number $k = 10\pi$ m^{-1}

Now wave speed $v = \dfrac{\omega}{k} = \dfrac{50\pi}{10\pi} = 5 \text{ ms}^{-1}$

Therefore, OPTION (D) is wrong

Displacement node occurs at $10\pi x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$ etc.

$\Rightarrow \quad x = \dfrac{1}{20}, \dfrac{3}{20}$

$\Rightarrow \quad x = 0.05$ m and 0.15 m

Displacement antinode occurs at

$10\pi x = 0, \pi, 2\pi, 3\pi$ etc.

$\Rightarrow \quad x = 0$, 0.1 m, 0.2 m and 0.3 m

Wavelength $\lambda = 2$ (distance between two consecutive nodes or antinodes)

$\Rightarrow \quad \lambda = 2(0.1) = 0.2$ m

Hence, (A), (B) and (C) are correct.

26. ABC has negative slope hence, represents compression, while CDE has positive slope hence, represents rarefaction.
Hence, (A) and (D) are correct.

27. The number of waves encountered by the moving plane per unit time is given by

$$n = \frac{\text{distance travelled}}{\text{wavelength}}$$

$$\Rightarrow \quad n = \frac{c + v}{\lambda} = \frac{c}{\lambda}\left(1 + \frac{v}{c}\right) = f\left(1 + \frac{v}{c}\right) \qquad \{\text{OPTION (A)}\}$$

The stationary observer meets the frequency f' of the incident wave and receives the reflected wave of frequency f'' emitted by the moving platform as

$$f'' = \frac{f'}{1 - \dfrac{v}{c}} = \frac{f\left(1 + \dfrac{v}{c}\right)}{1 - \dfrac{v}{c}} = \frac{f(c + v)}{(c - v)} \qquad \{\text{OPTION (C)}\}$$

$$\text{Wavelength, } \lambda'' = \frac{c}{f''} = \frac{c}{f}\left(\frac{c - v}{c + v}\right) \qquad \{\text{OPTION (B)}\}$$

Beat frequency, $f_b = f'' - f$

$$\Rightarrow \quad f_b = \frac{f\left(1 + \dfrac{v}{c}\right)}{\left(1 - \dfrac{v}{c}\right)} - f = f\left(\frac{1 + \dfrac{v}{c}}{1 - \dfrac{v}{c}} - 1\right)$$

$$\Rightarrow \quad f_b = \frac{\left(1 + \dfrac{v}{c}\right)f}{\left(1 - \dfrac{v}{c}\right)} = \frac{2vf}{c - v}$$

Hence, (A), (B) and (C) are correct.

28. Conceptual
Hence, (A), (B) and (D) are correct.

29. As $f_1 : f_2 : f_3 = 3 : 5 : 7$, string is fixed at one end. Its fundamental frequency is $f_0 = \dfrac{f_1}{3} = \dfrac{105}{3} = 35$ Hz.

Hence, (B) and (C) are correct.

30. Comparing the given equation with standard equation of stationary wave

$$y = 2a\sin(kx)\cos(\omega t)$$

we have $a = 2$ mm

$$k = \frac{2\pi}{\lambda} = 3.14$$

$$\Rightarrow \quad \frac{\lambda}{2} = 1 \text{ m}$$

So, the smallest possible length is $\dfrac{\lambda}{2}$ or 1 m.
Hence, (A) and (D) are correct.

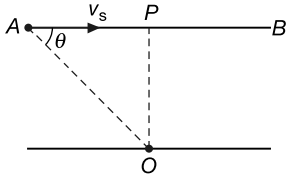
CHAPTER 4

31. $T = \left(\frac{M}{L}\right)xg = \begin{pmatrix}\text{weight of part of rope hanging}\\ \text{below the point under}\\ \text{consideration.}\end{pmatrix}$

Since $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{\left(\frac{M}{L}\right)xg}{\left(\frac{M}{L}\right)}} = \sqrt{xg}$

Hence, (B) and (D) are correct.

32. The graph shows the situation shown in figure. The observed frequency will initially be more than the natural frequency. When the source is at P, natural frequency i.e., 2000 Hz.



For region AP : $f = f_0\left(\frac{v}{v - v_s\cos\theta}\right)$

For PB : $f = f_0\left(\frac{v}{v + v_s\cos\theta}\right)$

Minimum value of f will be

$f_{\min} = f_0\left(\frac{v}{v + v_s}\right)$ when $\cos\theta = 1$

$\Rightarrow \quad 1800 = 2000\left(\frac{300}{300 + v_s}\right)$

Solving this we get,

$v_s = 33.33\ \text{ms}^{-1}$ and maximum value of f can be

$f_{\max} = f_0\left(\frac{v}{v - v_0}\right)$ when $\cos\theta = 1$

$\Rightarrow \quad f_{\max} = 2000\left(\frac{300}{300 - 33.33}\right) = 2250\ \text{Hz}$

Hence, (C) and (D) are correct.

33. For closed pipe, $f = n\left(\frac{v}{4\ell}\right) n = 1, 3, 5.....$

For $n = 1$, $f_1 = \frac{v}{4\ell} = \frac{320}{4\times 1} = 80\ \text{Hz}$

For $n = 3$, $f_3 = 3f_1 = 240\ \text{Hz}$

For $n = 5$, $f_5 = 5f_1 = 400\ \text{Hz}$

Hence, (A), (B) and (D) are correct.

34. Conceptual

Hence, (C) and (D) are correct.

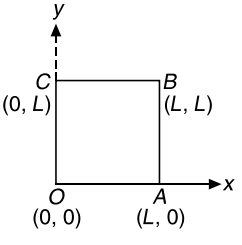
35. Since, the edges are clamped, displacement of the edges $u(x, y) = 0$ for

Line OA i.e., $y = 0, 0 \le x \le L$

AB i.e., $x = L, 0 \le y \le L$

BC i.e., $y = L\ 0 \le x \le L$

OC i.e., $x = 0, 0 \le y \le L$



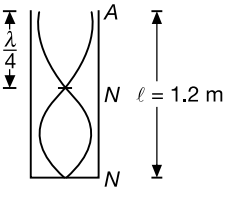
The above conditions are satisfied only in alternatives (B) and (C).

Note that $u(x, y) = 0$, for all four values e.g., in alternative (D), $u(x, y) = 0$ for $y = 0$, $y = L$ but it is not zero for $x = 0$ or $x = L$. Similarly, in OPTION (A) $u(x, y) = 0$ at $x = L$, $y = L$ but it is not zero for $x = 0$ or $y = 0$ while in OPTIONS (B) and (C), $u(x, y) = 0$ for $x = 0$, $y = 0$, $x = L$ and $y = L$.

Hence, (B) and (C) are correct.

36. $\ell = \frac{3\lambda}{4}$

$\Rightarrow \quad \frac{\lambda}{4} = \frac{\ell}{3} = 0.4\ \text{m}$



Pressure variation will be maximum at displacement nodes i.e., at 0.4 m from the open end and at closed end.

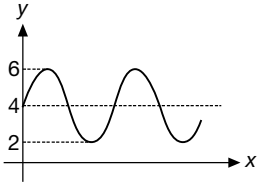
Hence, (B) and (C) are correct.

37. In case of sound wave, y can represent pressure and displacement, while in case of an electromagnetic wave it represents electric and magnetic fields.

NOTE: In general, y is any general physical quantity which is a made to oscillate at one place and these oscillations are propagated to other places also.

Hence, (A), (B), (C) and (D) are correct.

38. The shape of the equation is as follows:



From this we can see that amplitude of wave is 2 units.

Wave speed $v = \frac{\text{coefficient of } t}{\text{coefficient of } x}$

$\Rightarrow \quad v = \frac{6}{3} = 2\ \text{units}$

Hence, (B) and (C) are correct.

Reasoning Based Questions

1. In resonance tube, the vibrations are set-up in the air column which only depends on the length of the resonance column and velocity of sound. The liquid in the tube only reflects the waves which upon superposition produce stationary waves. Therefore, if oil of density higher than that of water is used, there will be no effect on the frequency of waves set-up.

Hence, the correct answer is (D).

2. It is known that all the stars are moving away from each other. Therefore, apparent frequency of light emitted from a star as received by an observer on the earth is less than its actual frequency. Since, wavelength is inversely proportional to the frequency, the apparent wavelength of the light from the star is more than the actual wavelength. In other

words, wavelength of light shifts towards longer end i.e., towards red end of the visible spectrum, this is red shift.
Hence, the correct answer is (A).

3. The gases possess only volume elasticity (gases do not possess shear elasticity). As such, only longitudinal waves can propagate in gases. On the other hand, the solids, possess both volume and shear elasticity and likewise both the longitudinal and transverse waves can propagate through the solids.
Hence, the correct answer is (B).

4. If a closed pipe of length L is in resonance with a tuning fork of frequency ν, then

$$\nu = \frac{v}{4L}$$

An open pipe of same length L produces vibrations of frequency $\frac{v}{2L}$. Obviously, it cannot be in resonance with the given tuning fork of frequency $\nu\left(=\frac{v}{4L}\right)$.

Hence, the correct answer is (C).

5. For an isothermal process, $PV = \text{constant}$.
Differentiating both sides, we get

$$PdV + VdP = 0$$

$$\Rightarrow \quad PdV = -VdP$$

$$\Rightarrow \quad B_{\text{isot}} = -\frac{dP}{\left(\frac{dV}{V}\right)} = P \qquad \ldots(1)$$

For an adiabatic process, $PV^{\gamma} = \text{constant}$
Differentiating both sides,

$$P\left(\gamma V^{\gamma-1}\right)dV + V^{\gamma}dP = 0$$

$$\Rightarrow \quad \gamma PdV + VdP = 0$$

$$\Rightarrow \quad B_{\text{ad}} = -\frac{dP}{\left(\frac{dV}{V}\right)} = \gamma P \qquad \ldots(2)$$

From equation (1) and (2)

$$B_{\text{ad}} = \gamma B_{\text{isot}}$$

As $\gamma > 1$, therefore $B_{\text{ad}} > B_{\text{isot}}$

Hence, the correct answer is (B).

6. Only transverse waves can be polarised. Sound waves (mechanical wave) cannot be polarised as they are longitudinal in nature whereas light waves can be polarised as they are transverse in nature.
Hence, the correct answer is (A).

7. According to Newton, speed of sound in gases,

$$v = \sqrt{\frac{K_{\text{iso}}}{\rho}} = \sqrt{\frac{P}{\rho}}$$

Laplace pointed out that since the wave propagates quickly in the medium and the changes taking place in the gases due to the propagation of sound cannot be isothermal but are adiabatic in nature, he corrected the Newton's formula accordingly, i.e.,

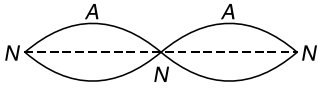
$$v = \sqrt{\frac{K_{\text{adia}}}{\rho}} = \sqrt{\frac{\gamma P}{\rho}}$$

Hence, the correct answer is (A).

8. For open organ pipe fundamental frequency is expressed as $n = \frac{v}{2\ell}$ here v is the velocity and ℓ is the length of organ pipe. Since, the velocity increases rapidly with the increase of temperature in comparison with the increase in its length. Therefore, it is clear that the frequency increases with the increase of temperature.
Hence, the correct answer is (A).

9. Intensity of sound at any point is energy flowing per second per unit area around that point. So, it will not change due to motion of listener.
The apparent wavelength changes with the motion of listener.
Hence, the correct answer is (B).

10. Stationary wave is represented as shown in figure.



From the figure we observe that at nodes the amplitude is zero and velocity of particle is also zero and at antinodes the amplitude is maximum. Hence the velocity of particle is also maximum and all particles cross mean position between two successive nodes.
Hence, the correct answer is (A).

11. The wood offers high damping to the sound waves. For making the bells the low damping is required and for low damping the metals are most suitable. Hence, the bells are made of metals.
Hence, the correct answer is (A).

12. Sound has greater speed in solid than in air. Hence, when ear is placed on the rails the sound of train coming from some distance is heard. Hence, Statement-1 is true and Statement-2 is false.
Hence, the correct answer is (C).

13. Sound coming from the different sources can be recognised by virtue of their quality which is characteristics of sound. That is why we recognise the voices of our friends.
Hence, the correct answer is (A).

14. The speed of sound in gaseous medium is given by

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\gamma\left(\frac{RT}{M}\right)} \qquad \ldots(1)$$

At constant temperature

$$PV = \text{constant} \qquad \ldots(2)$$

If V is the volume of one mole of gas, then density of gas $\rho = \frac{M}{V}$ or $V = \frac{M}{\rho}$

where M is the molecular weight of the gas.

CHAPTER 4

Therefore, Equation (2) becomes, $\frac{PM}{\rho} = \text{constant}$

$\Rightarrow \quad \frac{P}{\rho} = \text{constant as } M \text{ is a constant}$

$\Rightarrow \quad v = \text{constant}$

Thus, change in air pressure does not affect the speed of sound.

Statement-2 is clear from Equation (1).

Hence, the correct answer is (D).

15. In stationary wave, total energy associated with it is twice the energy of each of incidence and reflected wave.

Large amount of energy are stored equally in standing waves and is trapped within the waves. Hence, there is no transmission of energy through the waves.

Hence, the correct answer is (B).

16. When the source of sound (engines) moves towards the listener, the apparent frequency,

$$\nu_1 = \frac{v}{v - v_s}\nu$$

Or the other hand, if the source of sound moves away from the listener, the apparent frequency

$$\nu_2 = \frac{v}{v + v_s}\nu$$

If follow that $\nu_1 > \nu_2$. Since pitch of a sound depends on frequency, the whistle of the approaching engine is shriller that the receding engine.

Hence, the correct answer is (C).

17. Doppler's effect is observed readily in sound waves due to the larger wavelength. This effect is not followed with light due to shorter wavelength. The velocity of light is 3×10^8 ms^{-1} and 332 ms^{-1} velocity of sound.

Hence, the correct answer is (B).

18. At the point where a compression and a rarefaction meet, the displacement is minimum and it is called displacement node. At this point, the pressure difference is maximum i.e., at the same time, it is a pressure antinode. On the other hand, at the mid-point of a compression or a rarefaction, the displacement variation is maximum i.e., such a point is displacement antinode. However, such a point is pressure node, as pressure variation is minimum at such a point.

Hence, the correct answer is (B).

19. The resultant amplitude of two waves is given by

$$A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\theta}$$

Here, $a_1 = a_2 = A = a$

$$\frac{1}{2} = 1 + \cos\theta$$

$\Rightarrow \quad \cos\theta = -\frac{1}{2}$

$\Rightarrow \quad \theta = 120^o$

Statement-2 is incorrect as it is just opposite to the Principle of Superposition of waves.

Hence, the correct answer is (C).

20. Experimental value of velocity of sound in air is 332 ms^{-1}

Theoretical value of velocity of sound from Newton's formula is 280 ms^{-1}

Difference between the two values is $332 - 280 = 52$ ms^{-1}

$$\text{Percentage error} = \frac{332 - 280}{332} \times 100 = 16\%$$

Hence, the correct answer is (C).

Linked Comprehension Type Questions

1. $y = 4\sin\frac{\pi x}{15}\cos(96\pi t) = A_x\cos(96\pi t)$

Here, $A_x = 4\sin\frac{\pi x}{15}$

at $x = 5$ cm, $A_x = 4\sin\frac{5\pi}{15} = 4\sin\frac{\pi}{3} = 2\sqrt{3}$ cm

This is the amplitude or maximum displacement at $x = 5$ cm.

Hence, the correct answer is (C).

2. Nodes are located where $A_x = 0$

or $\quad \frac{\pi x}{15} = 0, \pi, 2\pi, ...$

or $\quad x = 0, 15$ cm, 30 cm etc.

Hence, the correct answer is (B).

3. Velocity of particle,

$$v_p = \left.\frac{\partial y}{\partial t}\right|_{x=\text{constant}} = -384\sin\left(\frac{\pi x}{15}\right)\cos(96\pi t)$$

At $\quad x = 7.5$ cm and $t = 0.25$ s

$$v_p = -384\pi\sin\left(\frac{\pi}{2}\right)\sin(24\pi) = 0$$

Hence, the correct answer is (A).

4. Amplitude of components waves is $A = \frac{4}{2} = 2$ cm

$\omega = 96\pi$ and $k = \frac{\pi}{15}$

Component waves are, $y_1 = 2\sin\left(\frac{\pi}{15}x - 96\pi t\right)$ and $y_2 = 2\sin\left(\frac{\pi}{15}x + 96\pi t\right)$

Hence, the correct answer is (D).

5. Since $f_{\text{app}} = \left(\frac{v}{v - v_s}\right)f_0$

$\Rightarrow \quad 2.2 \times 10^3 = \frac{300}{300 - 30}f_0$

$\Rightarrow \quad f_0 = \frac{2.2 \times 10^3 \times 270}{300}$ Hz

$\Rightarrow \quad f_0 = 1980$ Hz $= 1.98$ kHz

Hence, the correct answer is (B).

6. $f_{\text{app}} = 2.2 \times 10^3 = \frac{300}{300 - v_s}f_0$ …(1)

$f_{\text{recede}} = 1.8 \times 10^3 = \frac{300}{300 + v_s}f_0$ …(2)

Dividing equation (1) by (2), we get

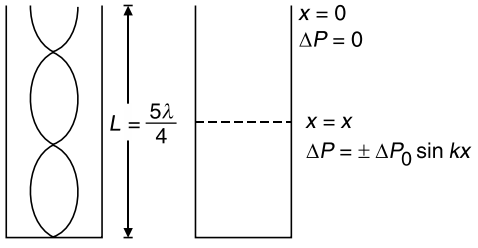
$$\frac{22}{18}=\frac{300-v_s}{300+v_s}$$

$$\Rightarrow \quad 40v_s=(22-18)300$$

$$\Rightarrow \quad v_s=\frac{1200}{40}\ \text{ms}^{-1}=30\ \text{ms}^{-1}$$

Hence, the correct answer is (D).

7. Frequency of second overtone of the closed pipe



$$\lambda_5=5\left(\frac{v}{4L}\right)=440$$

$$\Rightarrow \quad L=\frac{5v}{4\times 440}\ \text{m}$$

Substituting v = speed of sound in air = 330 ms^{-1}

$$L=\frac{5\times 330}{4\times 440}=\frac{15}{16}\ \text{m}$$

Hence, the correct answer is (B).

8. $\lambda=\dfrac{4L}{5}=\dfrac{4\left(\frac{15}{16}\right)}{5}=\dfrac{3}{4}\ \text{m}$

Hence, the correct answer is (D).

9. Open end is displacement antinode. Therefore, it would be a pressure node.

or at $x=0$, $\Delta P=0$

Pressure amplitude at $x=x$, can be written as

$$\Delta P=\pm\Delta P_o\sin kx$$

Where $k=\dfrac{2\pi}{\lambda}=\dfrac{2\pi}{3/4}=\dfrac{8\pi}{3}\ \text{m}^{-1}$

Therefore, pressure amplitude at $x=\dfrac{L}{2}=\dfrac{15/16}{2}$ m or $\left(\dfrac{15}{32}\right)$ m will be

$$\Delta P=\pm\Delta P_o\sin\left(\frac{8\pi}{3}\right)\left(\frac{15}{32}\right)=\pm\Delta P_o\sin\left(\frac{5\pi}{4}\right)$$

$$\Delta P=\pm\frac{\Delta P_o}{\sqrt{2}}$$

Hence, the correct answer is (C).

10. Open end is a pressure node i.e., $\Delta P=0$.

Hence, $P_{\max}=P_{\min}=$ Mean pressure (P_o)

Hence, the correct answer is (C).

11. Closed end is a displacement node or pressure antinode.

Therefore, $P'_{\max}=P_o+\Delta P_o$

and $P'_{\min}=P_o-\Delta P_o$

Hence, the correct answer is (B).

12. E is the mean position of oscillation of the particle

So, the kinetic energy is maximum at E

Hence, the correct answer is (B).

13. The stretching of string is maximum at E

So, the elastic potential energy is maximum at E.

Hence, the correct answer is (B).

14. B is moving up and D is moving down.

Hence, the correct answer is (C).

15. The wave reflected from denser medium will suffer an additional phase change of π. So

$$y_r=A\cos(\omega t-kx+\phi+\pi)$$

$$\Rightarrow \quad y_r=-A\cos(\omega t-kx+\phi)$$

Hence, the correct answer is (D).

16. Conceptual

Hence, the correct answer is (C).

17. Conceptual

Hence, the correct answer is (D).

18. **The correct answer is (B).**

19. **The correct answer is (A).**

Combined solution to 18 and 19

$$y_i+y_r=y_t \text{ (at point } P)$$

$$\Rightarrow \quad A_i+A_r=A_t \qquad \ldots(1)$$

Also, $\dfrac{\partial y_i}{\partial x}+\dfrac{\partial y_r}{\partial x}=\dfrac{\partial y_t}{\partial x}$ (at point P)

$$\Rightarrow \quad A_i+A_r=A_t\frac{k_t}{k_i} \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$A_r=\frac{(k_i-k_t)}{(k_i+k_t)}A_i \text{ and } A_r=\frac{2k_i}{(k_i+k_t)}A_i$$

Since, tension T is same in both the strings, so we have

$$v=\frac{\omega}{k}$$

$$\Rightarrow \quad \sqrt{\frac{T}{\mu}}=\frac{\omega}{k}$$

As T and ω are same in both the strings, so

$$k\propto\sqrt{\mu}$$

$$\Rightarrow \quad A_r=\frac{\left(\sqrt{\mu_1}-\sqrt{\mu_2}\right)}{\left(\sqrt{\mu_1}+\sqrt{\mu_2}\right)}A_i \text{ and } A_t=\frac{2\left(\sqrt{\mu_1}\right)}{\left(\sqrt{\mu_1}+\sqrt{\mu_2}\right)}A_i$$

20. Path difference between the waves reaching D is

$$\Delta x=\frac{3\pi R}{2}-\frac{\pi R}{2}=\frac{2\pi R}{2}=\pi R$$

Since, $I_R=I_1+I_2+2\sqrt{I_1I_2}\cos\phi$

CHAPTER 4

$\Rightarrow \quad I_R = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$ when $\phi = 2n\pi$, $n = 0, 1, 2, 3, \ldots$.

Since the wave generated is divided equally in two parts, so we have

$$I_1 = I_2 = \frac{I_0}{2}$$

$$\Rightarrow \quad I_R = \left(\sqrt{\frac{I_0}{2}} + \sqrt{\frac{I_0}{2}}\right)^2$$

$$\Rightarrow \quad I_R = 2I_0$$

Hence, the correct answer is (C).

21. Since $\Delta x = \pi R$

For maxima, we have

$\Delta\phi = (2n)\pi$, where $n = 0, 1, 2, 3, \ldots$.

$$\Rightarrow \quad \left(\frac{2\pi}{\lambda}\right)(\pi R) = (2n)\pi$$

$$\Rightarrow \quad \lambda_{\max} = \frac{\pi R}{n_{\min}} = \frac{\pi R}{1}$$

$$\Rightarrow \quad \lambda_{\max} = \pi R$$

Hence, the correct answer is (A).

22. Similarly, for minima, we have

$\Delta\phi = (2n+1)\pi$, where $n = 0, 1, 2, 3, \ldots$.

$$\Rightarrow \quad \left(\frac{2\pi}{\lambda}\right)(\pi R) = (2n+1)\pi$$

$$\Rightarrow \quad \lambda_{\max} = \frac{2\pi R}{2n+1}$$

$$\Rightarrow \quad \lambda_{\max} = 2\pi R$$

Hence, the correct answer is (B).

23. Since, $y = a\sin\left[\frac{2\pi}{\lambda}(vt - x)\right]$

From the graph, $a = 10$ mm $= 1$ cm

$\lambda = 8$ m, $v = 1$ ms^{-1}

$$\Rightarrow \quad y = 1\sin\left[\frac{2\pi}{8}(t - x)\right] \text{ cm}$$

Now $v_P = \frac{\partial y}{\partial t} = \frac{\pi}{4}\cos\left[\frac{\pi}{4}(t - x)\right]$

$$\Rightarrow \quad v_P(x = 4 \text{ m},\ t = 0 \text{ s}) = \frac{\pi}{4} \text{ cms}^{-1}$$

Hence, the correct answer is (B).

24. At the instant and location asked, the two waves will superimpose, so the net displacement is sum of displacement due to individual pulses.

$$y_1(x = 9 \text{ m},\ t = 8 \text{ s}) = \frac{1}{\sqrt{2}} \text{ cm and } y_2 = 1 \text{ cm}$$

$$\Rightarrow \quad y_{\text{net}} = y_1 + y_2 = \left(\frac{1}{\sqrt{2}} + 1\right) \text{ cm}$$

Hence, the correct answer is (A).

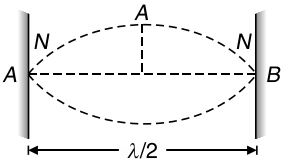
25. Net velocity is sum of velocity due to individual pulses.

Since, $v_1 = \frac{\pi}{4}\cos\left[\frac{\pi}{4}(t - x)\right]$

$$\Rightarrow \quad v_1(x = 9 \text{ m},\ t = 8 \text{ s}) = -\frac{\pi}{4\sqrt{2}}$$

Hence, the correct answer is (C).

26.



Hence, the correct answer is (A).

27. $\frac{\omega}{v} = k$

$$v = \sqrt{\frac{40 \times 2}{0.1}} = 20\sqrt{2}$$

$$\omega = \frac{\pi}{2}\left(20\sqrt{2}\right)$$

$$\Rightarrow \quad \omega \approx 44 \text{ rads}^{-1}$$

Hence, the correct answer is (A).

28. Conceptual

Hence, the correct answer is (A).

29. Velocity of particle undergoing wave motion is given by

$$\frac{\partial y}{\partial t} = -u\frac{\partial y}{\partial x}$$

At $t = 0$ and $x = 2$, we have

$$\frac{\partial y}{\partial t} = \frac{-1 \times 10 \times 10^{-3}}{4 \times 10^{-2}} = -\frac{1}{4} = -0.25 \text{ ms}^{-1}$$

Hence, the correct answer is (D).

30. Displacement and velocity of particle is vector sum of displacements and velocity due to individual pulses.

So, displacement at $x = 8$ cm and $t = 6$ sec is

$$y = 5 \text{ mm} - 5 \text{ mm} = 0$$

Hence, the correct answer is (D).

31. Velocity at $x = 8$ cm and $t = 6$ sec is

$$v = -0.25 + 0.125$$

$$v = -0.125 \text{ ms}^{-1}$$

Hence, the correct answer is (B).

32. Wavelength of incident wave $= \frac{2\pi}{a}$

Hence, the correct answer is (C).

33. Frequency of incident wave $= \frac{b}{2\pi}$

Hence, the correct answer is (B).

34. Intensity of reflected wave has become 0.64 times. But since $I \propto A^2$ amplitude of reflected wave will become 0.8 times. a and b will remain as it is. But direction of velocity of wave will become opposite. Further there will be a phase change

of π, as it is reflected by an obstacle (denser medium). Therefore, equation of reflected wave would be

$$y_r = 0.8A\cos[ax - bt + \pi] = -0.8A\cos(ax - bt)$$

Hence, the correct answer is (C).

35. The equation of resultant wave will be,

$$y = y_i + y_r = A\cos(ax + bt) - 0.8A\cos(ax - bt)$$

Particle velocity

$$v_p = \frac{\partial y}{\partial t} = -Ab\sin(ax + bt) - 0.8Ab\sin(ax - bt)$$

Maximum particle speed can be $1.8Ab$, where,

$$\sin(ax + bt) = \pm 1 \text{ and } \sin(ax - bt) = \pm 1$$

and minimum particle speed can be zero, where

$$\sin(ax + bt) \text{ and } \sin(ax - bt) \text{ both and zero}$$

Hence, the correct answer is (D).

36. Since, there is no relative motion along y-axis, so we have

$$f_{app} = f_0\left(\frac{v}{v + v_s}\right)$$

$$\Rightarrow \quad f_{app} = 960\left(\frac{310}{310 + 10}\right) = 930 \text{ Hz}$$

Hence, the correct answer is (B).

37. Throughout the apparent frequency will remain same.
Hence, the correct answer is (D).

38. Both the source and detector fall simultaneously on ground and relative velocity exists only along the horizontal.
Hence, the correct answer is (A).

39. Since, $(x - vt)$ is a pulse travelling in $+ve$ x-direction, so y_1 is along $+x$-axis.

Similarly, $(x + vt)$ is a pulse travelling in $-$ve x direction, so y_2 is along $-x$ axis.

Hence, the correct answer is (C).

40. Two waves cancel when

$$|y_1| = |y_2|$$

$$\Rightarrow \quad \frac{5}{(3x - 4t)^2 + 2} = \frac{+5}{(3x + 4t - 6)^2 + 2}$$

$$\Rightarrow \quad (3x - 4t)^2 + 2 = (3x + 4t - 6)^2 + 2$$

$$\Rightarrow \quad 3x - 4t = 3x + 4t - 6$$

$$\Rightarrow \quad 6 = 8t$$

$$\Rightarrow \quad t = \frac{6}{8} = \frac{3}{4} = 0.75 \text{ s}$$

Hence, the correct answer is (D).

41. At $t = 0$

$$y_1 = \frac{5}{(3x)^2 + 2} = \frac{5}{9x^2 + 2}$$

$$y_2 = \frac{-5}{(3x - 6)^2 + 2}$$

For, $|y_1| = |y_2|$, we have

$$\frac{1}{9x^2 + 2} = \frac{1}{9x^2 + 36 - 36x + 2}$$

$$\Rightarrow \quad 9x^2 + 2 = 9x^2 + 36 - 36x + 2$$

$$\Rightarrow \quad 36(x - 1) = 0$$

$$\Rightarrow \quad x = 1 \text{ m}$$

Hence, the correct answer is (D).

42. $v = \sqrt{\frac{B}{\rho}}$

$$\Rightarrow \quad \rho v^2 = B$$

$$\Rightarrow \quad \rho = \frac{B}{v^2}$$

But from the given equation, we have

$$\omega = 6000\pi \text{ and } k = 15\pi$$

$$\Rightarrow \quad v = \frac{\omega}{k} = 400 \text{ ms}^{-1}$$

$$\Rightarrow \quad \rho = \frac{1.6 \times 10^5}{(400)^2}$$

$$\Rightarrow \quad \rho = 1 \text{ kgm}^{-3}$$

Hence, the correct answer is (A).

43. $I_R = 2I = \frac{2P_0^2}{2\rho v} = \frac{2 \times (24\pi)^2}{2 \times 1 \times 400} = 1.44\pi^2$

Hence, the correct answer is (B).

44. For wave in string

$$I \propto A^2 v^2$$

$$\frac{I_1}{I_2} = \frac{A_1^2 v_1^2}{A_2^2 v_2^2} = (2)^2(2)^2 = 16$$

Hence, the correct answer is (C).

45. Assuming phase at $x = 0$ to be zero

Phase of point P on string 1 is $\frac{\pi}{2}$

Phase of point P' on string 2 is π

So, phase difference $= \pi - \frac{\pi}{2} = \frac{\pi}{2}$

Hence, the correct answer is (A).

46. Phase of particle at $x = 20$ cm on string $1 = 2\pi$
Phase of particle at $x = 20$ cm on string $2 = \pi$
So, phase difference $= 2\pi - \pi = \pi$
Hence, the correct answer is (D).

47. **The correct answer is (A).**

48. **The correct answer is (B).**

49. **The correct answer is (C).**

Combined solution to 47, 48 and 49

Since, $\mu_2 = 4\mu_1$ and $v = \sqrt{\frac{T}{\mu}}$

$$\Rightarrow \quad v_2 = \frac{v_1}{2}$$

CHAPTER 4

Also, $k = \dfrac{\omega}{v}$ $\left\{\because v = \dfrac{\omega}{k}\right\}$

$\Rightarrow \quad k_2 = 2k_1$

Also, $A_r = \left(\dfrac{v_2 - v_1}{v_2 + v_1}\right)A_i$

$A_t = \left(\dfrac{2v_2}{v_2 + v_1}\right)A_i$

Hence, the correct answer is (B).

50. $y(x = 0,\ t) = 8\sin(4t) = A\sin(\omega t)$

$y(x, t) = A\sin(\omega t - kx)$

$\omega = kv = \left(\dfrac{2\pi}{\lambda}\right)v$

Since, $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{1}{0.2}} = \sqrt{5}\ \text{ms}^{-1}$

$\Rightarrow \quad k = \dfrac{\omega}{v} = \dfrac{4}{\sqrt{5}}\ \text{m}^{-1}$

$\Rightarrow \quad y(x, t) = 8\sin\left(4t - \dfrac{4}{\sqrt{5}}x\right)$

Hence, the correct answer is (B).

51. Average power is given by

$P_{av} = \dfrac{1}{2}\mu v\omega^2 A^2$

$\Rightarrow \quad P_{av} = \dfrac{1}{2}(0.2)\sqrt{5}(4)^2(0.08)^2$

$\Rightarrow \quad P_{av} = 0.1 \times \sqrt{5} \times 16 \times 64 \times 10^{-2}$

$\Rightarrow \quad P_{av} = 0.02$ watt

Hence, the correct answer is (A).

52. Power transferred to bath is

$P' = 50\%$ of $P_{av} = \dfrac{0.02}{2} = 0.01$ W

Since, $dQ = mcdT$

$\Rightarrow \quad \dfrac{dQ}{dt} = mc\left(\dfrac{dT}{dt}\right) = mc\left(\dfrac{\Delta T}{\Delta t}\right) = 0.01$

$\Rightarrow \quad \Delta t = \dfrac{mc\Delta T}{0.01}$

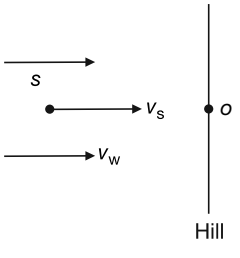
$\Rightarrow \quad \Delta t = \dfrac{(1)(1)(4200)(1)}{0.01}$

$\Rightarrow \quad \Delta t = 4.2 \times 10^5$ s

Hence, the correct answer is (A).

53. Given: $v_s = v_w = 40\ \text{kmh}^{-1}$ and

$v = 1200\ \text{kmh}^{-1}$ = Speed of sound



Frequency observed by observer

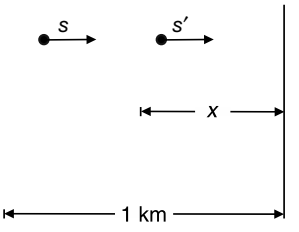
$f' = f\left(\dfrac{v + v_w}{v + v_w - v_s}\right)$

Substituting the values, we have

$f' = 580\left(\dfrac{1200 + 40}{1200 + 40 - 40}\right) = 599.33$ Hz

Hence, the correct answer is (B).

54. Let x be the distance of the source from the hill at which echo is heard of the sound which was produced when source was at a distance 1 km from the hill. Then, time taken by the source to reach from s to s' = time taken by the sound to reach from s to hill and then from hill to s'. Thus,



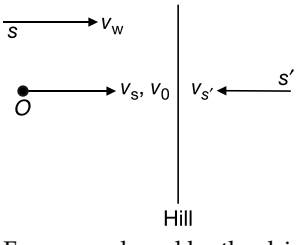
$\dfrac{1 - x}{40} = \dfrac{1}{1200 + 40} + \dfrac{x}{1200 - 40}$

Solving this equation, we get

$x = 0.935$ km

Hence, the correct answer is (A).

55.



Frequency heard by the driver of the reflected wave

$f'' = f\left[\dfrac{v - v_w + v_o}{v - v_w - v_{s'}}\right] = 580\left[\dfrac{1200 - 40 + 40}{1200 - 40 - 40}\right]$

$f'' = 621.43$ Hz

Hence, the correct answer is (D).

Matrix Match/Column Match Type Questions

1. A → (p, r); B → (q, s); C → (q, r); D → (p, s)

For the incident wave in (A), two cases arise

$y_i = A\sin(kx - \omega t)$

For reflection at flexible support,

$y_r = A\sin(kx + \omega t)$

$\Rightarrow \quad y = y_i + y_r = 2A\sin(kx)\cos(\omega t)$

$y_i = A\sin(kx - \omega t)$

For reflection at rigid support,

$y_r = -A\sin(kx + \omega t)$

$\Rightarrow \quad y = y_i + y_r = 2A\cos(kx)\sin(\omega t)$

For the incident wave in (B), two cases arise

$$y_i = A\cos(kx - \omega t)$$

For reflection at flexible support,

$$y_r = A\cos(kx + \omega t)$$

$$\Rightarrow \quad y = y_i + y_r = 2A\cos(kx)\cos(\omega t)$$

$$y_i = A\cos(kx - \omega t)$$

For reflection at rigid support,

$$y_r = -A\cos(kx + \omega t)$$

$$\Rightarrow \quad y = y_i + y_r = 2A\sin(kx)\sin(\omega t)$$

When, $x = 0$ is the rigid support then nodes are formed at $x = 0$ and at the nodes, $y = 0$.
This is satisfied by the equations

$$y = 2A\sin(kx)\cos(\omega t) \text{ and } y = 2A\sin(kx)\sin(\omega t)$$

When, $x = 0$ is the flexible support then antinodes are formed at $x = 0$ and at the antinodes, $y = \pm A$
This is satisfied by the equations

$$y = 2A\cos(kx)\sin(\omega t) \text{ and } y = 2A\cos(kx)\cos(\omega t)$$

2. A → (r); B → (p); C → (q); D → (s)

$$y = A\sin(\omega t - kx)$$

$$\Rightarrow \quad \omega = 10 \text{ rads}^{-1},\ k = 5 \text{ m}^{-1}$$

Since, $v = \dfrac{\omega}{k}$

$$\Rightarrow \quad v = 2 \text{ ms}^{-1}$$

Since, $k = 5$

$$\Rightarrow \quad \frac{2\pi}{\lambda} = 5$$

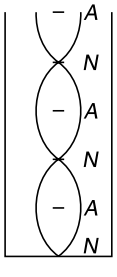
$$\Rightarrow \quad \lambda = 0.4\pi$$

$$v_P = \frac{\partial y}{\partial t} = A\omega\cos(\omega t - kx)$$

$$\Rightarrow \quad (v_P)_{\max} = A\omega = Avk$$

$$\Rightarrow \quad (v_P)_{\max} = (0.02)(5)(2) = 0.2 \text{ ms}^{-1}$$

3. A → (s); B → (p); C → (q); D → (r)
As discussed, third overtone frequency of closed pipe means seventh harmonic, which is 7 times the fundamental frequency. So, $x = 7$.
The second overtone, as shown, corresponds to fifth harmonic and has 3 Nodes and 3 Antinodes



Similarly, the third overtone has 4 Nodes and 4 Antinodes.

H	O
1	x
3	1
5	2
7	3
9	4
⋮	⋮
n	$\dfrac{n-1}{2}$
$(2n+1)$	n

4. A → (r); B → (p); C → (p); D → (p)
Frequency is the property of source. It remains unchanged.
In rarer medium, speed of wave is more.

$$\lambda = \frac{v}{f}$$

$$\Rightarrow \quad \lambda \propto v$$

$$A_t = \left(\frac{2v_2}{v_1 + v_2}\right)A_i$$

Since, $v_2 > v_1$

$$\Rightarrow \quad A_t > A_i$$

5. A → (r); B → (r); C → (s); D → (q)
OPTION (A)

At $t = \dfrac{T}{2}$, we have

$$y = A\cos kx\cos\left(\frac{2\pi}{T} \times \frac{T}{2}\right) = -A\cos kx$$

Hence plot is (r)
OPTION (B)

$$v_p = \frac{\partial y}{\partial t} = -A\omega\cos(kx)\sin(\omega t)$$

At $t = \dfrac{T}{4}$, $v_P = -A\omega\cos(kx)\sin\left(\dfrac{\pi}{2}\right) = -A\omega\cos(kx)$

Hence plot is (r)
OPTION (C)

$\Delta P = -B\dfrac{\partial y}{\partial x}$, where B is the Bulks Modulus. So,

$$\Delta P = +BkA\sin(kx)\cos(\omega t)$$

At $t = 0$, $\Delta P = +BkA\sin(kx)$
Hence plot is (s)
OPTION (D)
Pressure at any point is $P = P_0 \pm \Delta P$
Since, $\Delta P = (BAk)\sin(kx)$
Also, density $\rho = \dfrac{PM}{RT}$

$$\Rightarrow \quad \rho = \frac{M}{RT}(P_0 \pm \Delta P)$$

CHAPTER 4

$\Rightarrow \quad \rho = \rho_0 \pm \Delta\rho$, where $\Delta\rho = \dfrac{M\Delta P}{RT}$

$\Rightarrow \quad \rho = \rho_0 \pm \dfrac{M}{RT}\Delta P$

Now $\Delta P = BkA\sin(kx)\cos(\omega t)$

At $t = \dfrac{T}{2}$, we have

$\Delta P = -BAk\sin(kx)$

$\Rightarrow \quad \rho = \rho_0 \pm \dfrac{M}{RT}[-(BAk)\sin(kx)]$

$\Rightarrow \quad \rho = \rho_0 \pm \rho_0'[-\sin(kx)]$ where $\rho_0' = BAk$

Hence plot is (q)

6. A → (r); B → (p); C → (q); D → (r)

(A) $f' = f\left(\dfrac{v + v_0'}{v + v_s}\right) = f$

(B) $f' = f\left(\dfrac{v + v_0}{v - v_s}\right) > f$

(C) $f' = f\left(\dfrac{v - v_0}{v + v_f}\right) < f$

(D) $f' = f\left(\dfrac{v - v_0}{v - v_s}\right) = f$

7. A → (q); B → (p); C → (p); D → (r)

$u = \dfrac{1}{2}\rho\omega^2 A^2$

ω depends on source, which is same for both, so

$\dfrac{u_1}{u_2} = \dfrac{\rho_1}{\rho_2} \times \left(\dfrac{A_1}{A_2}\right)^2 = \left(\dfrac{1}{4}\right)\left(\dfrac{1}{2}\right)^2 = \dfrac{1}{16}$

$P = \dfrac{1}{2}\rho\omega^2 A^2 sv = usv$

$\Rightarrow \quad P \propto uv$

Since, $v \propto \dfrac{1}{\sqrt{\mu}}$

$\mu_2 = 4\mu_1$

$\Rightarrow \quad v_2 = \dfrac{v_1}{2}$

$\Rightarrow \quad \dfrac{v_1}{v_2} = 2$

Now, $\dfrac{P_1}{P_2} = \dfrac{u_1}{u_2} \times \dfrac{v_1}{v_2} = \dfrac{1}{16} \times 2 = \dfrac{1}{8}$

$I = \dfrac{P}{S}$

$\Rightarrow \quad I \propto P$

$\Rightarrow \quad \dfrac{I_1}{I_2} = \dfrac{P_1}{P_2} = \dfrac{1}{8}$

8. A → (q); B → (t); C → (s); D → (t)

$k = \dfrac{2\pi}{\lambda} = 2\pi a$

$\Rightarrow \quad \lambda = \dfrac{1}{a}$

$\omega = \dfrac{2\pi}{\lambda} = 2\pi b$

$\Rightarrow \quad T = \dfrac{1}{b}$ or $f = b$

$\Delta\phi = \dfrac{2\pi}{\lambda}\Delta x = (2\pi a)\left(\dfrac{1}{4a}\right) = \dfrac{\pi}{2}$

$\Delta\phi = \dfrac{2\pi}{T}\Delta t = (2\pi b)\left(\dfrac{1}{8b}\right) = \dfrac{\pi}{4}$

9. A → (r); B → (p, r); C → (q, r); D → (p, r, s)

Conceptual

10. A → (q); B → (r); C → (q); D → (s)

Since, $v = \sqrt{\dfrac{3RT}{M}}$

Change in pressure has no effect on speed of sound, where as

$v \propto \sqrt{T}$ $\qquad \left\{\because\ v = \sqrt{\dfrac{3RT}{M}}\right\}$

11. A → (s); B → (q); C → (r); D → (p)

$y = 2A\sin(kx)\cos(\omega t)$

$\Rightarrow \quad 2A = 0.06$

$\Rightarrow \quad A = 0.03$ m

At nodes, $y = 0$

$\Rightarrow \quad \sin(2\pi x) = 0$

$\Rightarrow \quad x = 0.5$ m

At antinodes, $y =$ maximum

$\Rightarrow \quad \sin(2\pi x) = 1$

$\Rightarrow \quad x = 0.25$ m

$|y| = \left|2A\sin\left(2\pi \times \dfrac{3}{4}\right)\right|$

$\Rightarrow \quad |y| = |y_{max}| = 0.06$ m

12. A → (p); B → (p); C → (t); D → (r)

For closed pipe, $100 = \dfrac{v}{4\ell_c}$

$\Rightarrow \quad \ell_c = \dfrac{v}{400} = \dfrac{330}{400} = 0.825$ m

For open pipe, $200 = \dfrac{v}{2\ell_0}$

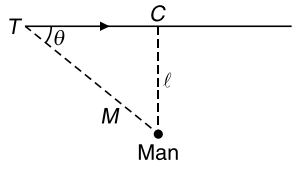
$\Rightarrow \quad \ell_0 = \dfrac{v}{400} = 0.825$ m

Different frequencies of closed pipes will be 100 Hz, 300 Hz, 500 Hz, etc., and different frequencies of open pipe are 200 Hz, 400 Hz, 600 Hz etc. i.e., none of them matches with each other.

So, the asked ratio is $\dfrac{600}{300} = 2$

Integer/Numerical Answer Type Questions

1. Let the velocity of truck at T when it blows the whistle be v_s. Then



$$600 = \left(\frac{v}{v - v_s\cos\theta}\right)500 \qquad \ldots(1)$$

During this time, speed of truck gets doubled, so

$$2v_s = v_s + at$$

$$\Rightarrow \quad v_s = at \qquad \ldots(2)$$

Now, $vt = AM = \dfrac{\ell}{\sin\theta}$ $\qquad \ldots(3)$

Also, $AB = \ell\cot\theta = v_s t + \dfrac{1}{2}at^2$ $\qquad \ldots(4)$

$$\Rightarrow \quad \frac{\ell\cos\theta}{\sin\theta} = at^2 + \frac{1}{2}at^2 = \frac{3}{2}at^2$$

$$\Rightarrow \quad \frac{\ell\cos\theta}{\sin\theta} = \frac{3}{2}a\left(\frac{\ell}{v\sin\theta}\right)^2$$

$$\Rightarrow \quad \frac{a\ell}{v} = \left(\frac{2}{3}\sin\theta\cos\theta\right)v$$

From Equation (1), we get

$$\frac{6}{5} = \frac{u}{v - at\cos\theta} = \frac{v}{v - \dfrac{a\ell}{v}\left(\dfrac{\cos\theta}{\sin\theta}\right)} \qquad \left\{\because\ t = \frac{\ell}{v\sin\theta}\right\}$$

$$\Rightarrow \quad \frac{6}{5} = \frac{v}{v - \left(\dfrac{2}{3}\sin\theta\cos\theta\right)\left(\dfrac{\cos\theta}{\sin\theta}\right)v}$$

$$\Rightarrow \quad \cos\theta = \frac{1}{2}$$

$$\Rightarrow \quad \theta = 60^\circ$$

2. Let the frequency of the tuning fork be f Hz. Let the frequency of the note from the open organ pipe at 20 °C and 30 °C be f_1 and f_2 respectively.

At 20 °C unison takes place. So,

$$f = f_1 \qquad \ldots(1)$$

At 30 °C, 5 beats per second are produced. So,

$$f_2 - f = 5 \qquad \ldots(2)$$

Since $f \propto \sqrt{T}$

$$\Rightarrow \quad \frac{f_1}{f_2} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{293}{303}} = 0.983 \qquad \ldots(3)$$

Solving above three equations (1), (2) and (3), we get

$$f = 296.2 \text{ Hz} \approx 296 \text{ Hz}$$

3. Frequency of sound reflected by the car is

$$f = \left(\frac{v + v_c}{v - v_c}\right)f_0$$

Since $v_c \ll v$

$$\Rightarrow \quad f = f_0\left(1 + \frac{v_c}{v}\right)\left(1 - \frac{v_c}{v}\right)^{-1} \approx \left(1 - \frac{2v_c}{v}\right)f_0$$

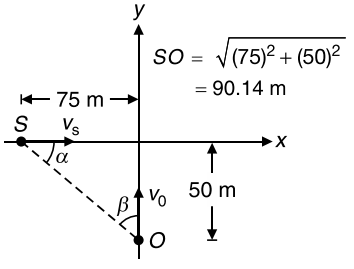
$$\Rightarrow \quad \Delta f = \frac{-2f_0}{v}\Delta v_c$$

$$\Rightarrow \quad \frac{1.2}{100}f_0 = \frac{-2f_0}{v}\Delta v_c$$

$$\text{Difference } |\Delta v_c| = \frac{1.2}{100} \times \frac{300}{2} \text{ ms}^{-1}$$

$$\Rightarrow \quad |\Delta v_c| = \frac{1.2}{100} \times \frac{330}{2} \times \frac{18}{5} \approx 7 \text{ kmh}^{-1}$$

4. After 5 seconds the observer will move a distance of 50 m while the source moves a distance of 25 m.



The apparent frequency is given by

$$f' = f\left(\frac{v + v_0\cos\beta}{v - v_s\cos\alpha}\right)$$

$$\Rightarrow \quad f' = (1000)\left(\frac{330 + 10 \times \dfrac{50}{90.14}}{330 - 5 \times \dfrac{75}{90.14}}\right)$$

$$\Rightarrow \quad f' = 1030 \text{ Hz}$$

5. Since, $y = A\sin\dfrac{2\pi}{\lambda}(vt - x)$

$$\Rightarrow \quad \frac{y}{A} = \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

In the first case, $\dfrac{y_1}{A} = \sin 2\pi\left(\dfrac{t}{T} - \dfrac{x_1}{\lambda}\right)$

where, $y_1 = +6$, $A = 8$, $x_1 = 10$ cm

$$\Rightarrow \quad \frac{6}{8} = \sin 2\pi\left(\frac{t}{T} - \frac{10}{\lambda}\right) \qquad \ldots(1)$$

Similarly, in the second case, we get

$$\frac{4}{8} = \sin 2\pi\left(\frac{t}{T} - \frac{25}{\lambda}\right) \qquad \ldots(2)$$

From Equation (1), we get

$$2\pi\left(\frac{t}{T} - \frac{10}{\lambda}\right) = \sin^{-1}\left(\frac{6}{8}\right) = 0.85 \text{ rad}$$

$$\Rightarrow \quad \frac{t}{T} - \frac{10}{\lambda} = 0.14 \qquad \ldots(3)$$

Similarly, from Equation (2), we get

$$2\pi\left(\frac{t}{T} - \frac{25}{\lambda}\right) = \sin^{-1}\left(\frac{4}{8}\right) = \frac{\pi}{6} \text{ rad}$$

$$\Rightarrow \quad \frac{t}{T} - \frac{25}{\lambda} = 0.08 \qquad \ldots(4)$$

CHAPTER 4

Subtracting Equation (4) from Equation (3), we get

$$\frac{15}{\lambda} = 0.06$$

$$\Rightarrow \quad \lambda = 250 \text{ cm}$$

6. When incident wave and reflected wave superimpose to produce stationary wave, the ratio of amplitudes at anti-node and at node is given by,

$$\frac{A_{\max}}{A_{\min}} = \frac{A_i + A_r}{A_i - A_r} = \frac{3}{2}$$

$$\Rightarrow \quad \frac{A_r}{A_i} = \frac{1}{5}$$

Since, $I \propto A^2$

$$\Rightarrow \quad \frac{I_r}{I_i} = \left(\frac{A_r}{A_i}\right)^2 = \frac{1}{25}$$

$$\Rightarrow \quad I_r = 0.04 I_i$$

Hence, 4% of the incident energy is reflected i.e., 96% energy passes across the obstacle.

7. If speed of astronaut is v, then apparent frequency of the echo received is

$$f' = f\left(\frac{c+v}{c-v}\right)$$

$$\Rightarrow \quad f' = f\left(1+\frac{v}{c}\right)\left(1-\frac{v}{c}\right)^{-1}$$

Since $v \ll c$, so we get

$$f' = f\left(1+\frac{2v}{c}\right)$$

$$\Rightarrow \quad f' - f = \frac{2fv}{c}$$

$$\Rightarrow \quad v = \frac{c|\Delta f|}{2f} = \frac{(3\times10^8)(10^3)}{2(5\times10^9)} = 30 \text{ ms}^{-1}$$

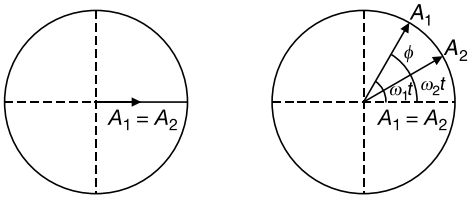
8. Since, $f \propto \frac{1}{\ell}$

$$\Rightarrow \quad \frac{\ell_2}{\ell_1} = \frac{f_1}{f_2}$$

$$\Rightarrow \quad \ell_2 = \left(\frac{f_1}{f_2}\right)\ell_1 = \left(\frac{124}{186}\right)(90) = 60 \text{ cm}$$

Thus, the string should be pressed at 60 cm from one end.

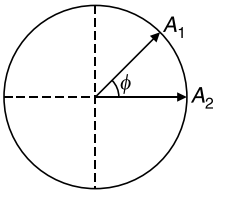
9. The problem can be represented in pictorial form as shown in figure.



This can also be seen as if A_2 is stationary and A_1 rotating with angular speed $(\omega_1 - \omega_2)$, so that phase difference between them at time t is

$$\phi = (\omega_1 - \omega_2)t$$

So, $T = \frac{2\pi}{(\omega_1 - \omega_2)} = \frac{1}{f_1 - f_2} = \frac{1}{10^3}$ sec



Now, it is given that the detector can detect signals only when resultant intensity $\geq 2I_0$ or resultant amplitude $\geq \sqrt{2}A_0$, because $I \propto A^2$. We can see that resultant amplitude is less than $\sqrt{2}A_0$ from $\phi = \frac{\pi}{2}$ to $\frac{3\pi}{2}$ i.e., for each half cycle. Hence, required time for which the detector remains idle is

$$t = \frac{T}{2} = \frac{(1/10^3)}{2} = 5\times10^{-4} \text{ s} = 500 \ \mu\text{s}$$

10. Difference in sound level is $L_2 - L_1$ given by

$$\Rightarrow \quad |\Delta L| = 10\log_{10}\left(\frac{I_2}{I_1}\right) = 10\log_{10}\left(\frac{r_1^2}{r_2^2}\right) \qquad \left\{\because I \propto \frac{1}{r^2}\right\}$$

Since $\frac{I_2}{I_1} = \left(\frac{r_1}{r_2}\right)^2 = \frac{1}{4}$ $\qquad \{\because \ r_2 = 2r_1\}$

$$\Rightarrow \quad |\Delta L| = \left|10\log_{10}\left(\frac{1}{4}\right)\right| = 6 \text{ dB}$$

11. (a) The given condition is possible only when

$$\sin(kx) = \cos(kx)$$

$$\Rightarrow \quad kx = \frac{\pi}{4}$$

Since, $A(x) = A\sin kx$

$$\Rightarrow \quad A = \frac{A(x)}{\sin(kx)} = \frac{3\sqrt{2}}{1/\sqrt{2}} = 6 \text{ mm}$$

(b) Since, $kx = \frac{\pi}{4}$

$$\Rightarrow \quad \left(\frac{2\pi}{\lambda}\right)\left(\frac{20}{2}\right) = \frac{\pi}{4}$$

$$\Rightarrow \quad \lambda = 80 \text{ cm}$$

$$\Rightarrow \quad \frac{\lambda}{2} = 40 \text{ cm}$$

So, number of loops is $p = \frac{\ell}{\lambda/2} = 6$

i.e., the string is vibrating in its fifth overtone mode.

12. Speed of a transverse wave on a wire is given by

$$v = \sqrt{\frac{T}{\mu}} \qquad \ldots(1)$$

Differentiating with respect to tension, we have

$$\frac{dv}{dT} = \frac{1}{2\sqrt{\mu T}} \qquad \ldots(2)$$

Dividing Equation (2) by Equation (1), we get

$$\frac{dv}{v}=\frac{1}{2}\frac{dT}{T}$$

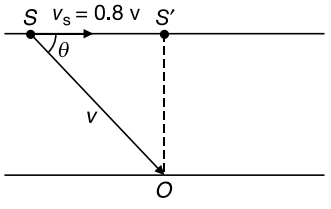
$$\Rightarrow \quad dT=(2T)\frac{dv}{v}$$

Substituting the values, we get

$$dT=\frac{(2)(50)(312-300)}{300}=4\text{ N}$$

So, tension must be increased by 4 N

13. (a) Assume that the pulse which is emitted when the source is at S, reaches the observer O in the same time in which the source reaches from S to S', then



$$\cos\theta=\frac{SS'}{SO}=\frac{v_s t}{vt}=\frac{v_s}{v}=0.8$$

Since, $f'=\left(\frac{v}{v-v_s\cos\theta}\right)f$

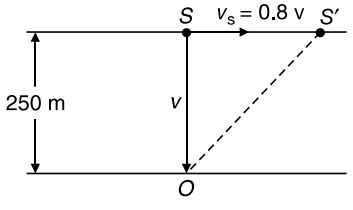
$$\Rightarrow \quad f'=\left(\frac{v}{v-(0.8v)(0.8)}\right)(900)$$

$$\Rightarrow \quad f'=\left(\frac{1}{1-0.64}\right)(900)=2500\text{ Hz}$$

(b) The observer will observe no change in the frequency when the source is at S as shown in figure. In the time when the wave pulse reaches from S to O, the source will reach from S to S'. Hence

$$t=\frac{SO}{v}=\frac{SS'}{v_s}$$

$$\Rightarrow \quad SS'=\left(\frac{v_s}{v}\right)SO=(0.8)(250)=200\text{ m}$$



Therefore, distance of observer from source at this instant is

$$S'O=\sqrt{(SO)^2+(SS')^2}$$

$$\Rightarrow \quad S'O=\sqrt{(250)^2+(200)^2}=320\text{ m}$$

14. (a) Intensity due to a point source varies with distance r from it as $I\propto\frac{1}{r^2}$. So, $\frac{I_2}{I_1}=\frac{r_1^2}{r_2^2}$

$$\Rightarrow \quad L_2-L_1=10\log_{10}\left(\frac{I_2}{I_1}\right)=10\log\left(\frac{r_1^2}{r_2^2}\right)$$

Since $L_1=30$ dB, $r_1=20$ m and $r_2=10$ m, so

$$L_2-30=10\log\left(\frac{20}{10}\right)^2=10\log_{10}(4)\approx 6$$

$$\Rightarrow \quad L_2=36\text{ dB}$$

(b) Since, $L_2-L_1=10\log_{10}\left(\frac{I_2}{I_1}\right)=10\log_{10}\left(\frac{r_1^2}{r_2^1}\right)$

Sound is not heard at a point where $L_2=0$

$$\Rightarrow \quad 30-0=10\log_{10}\left(\frac{r_1^2}{r_2^2}\right)$$

$$\Rightarrow \quad \left(\frac{r_2}{r_1}\right)^2=1000$$

$$\Rightarrow \quad \frac{r_2}{r_1}=31.62$$

$$\Rightarrow \quad r_2=(31.62)(20)\approx 632\text{ m}$$

15. $\omega=2\pi f=2\pi(125)=785\text{ rads}^{-1}$

$$v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{49\text{ N}}{0.0625\text{ kgm}^{-1}}}=28\text{ ms}^{-1}$$

$$\Rightarrow \quad P=\frac{1}{2}\mu\omega^2 vA^2$$

$$\Rightarrow \quad P=\frac{1}{2}\times 0.0625\times 785^2\times 28\times\left(9\times10^{-3}\right)^2$$

$$\Rightarrow \quad P=43.7\text{ W}$$

16. Since, $\lambda=\frac{v}{f}=\frac{\sqrt{T/\mu}}{f}$ $\qquad \left\{\because v=\sqrt{\frac{T}{\mu}}\right\}$

$$\Rightarrow \quad \lambda\propto\sqrt{T}$$

$$\Rightarrow \quad \frac{\lambda_{\text{Top}}}{\lambda_{\text{Bottom}}}=\sqrt{\frac{T_{\text{Top}}}{T_{\text{Bottom}}}}$$

$$\Rightarrow \quad \lambda_{\text{Top}}=(0.06)\sqrt{\frac{6+2}{2}}=0.12\text{ m}=12\text{ cm}$$

17. Since, $\frac{\lambda}{2}=(85-60)=25$ cm

$$\Rightarrow \quad \lambda=50\text{ cm}=0.5\text{ cm}$$

$$\Rightarrow \quad v=f\lambda=(500)(0.5)=350\text{ ms}^{-1}$$

18. Velocity of transverse wave in a string is $v=\sqrt{\frac{T}{\mu}}$ where, $T=mg=150$ N and

$$\mu=\frac{M}{L}=\frac{3\times10^{-2}}{2}=1.5\times10^{-2}\text{ kgm}^{-1}$$

$$\Rightarrow \quad v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{150}{1.5\times10^{-2}}}=100\text{ ms}^{-1}$$

CHAPTER 4

19. Since, $f=\frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$

$\Rightarrow \quad f\propto\frac{1}{\ell}$

$\Rightarrow \quad f_1:f_2:f_3=1:2:3$

$\Rightarrow \quad \ell_1:\ell_2:\ell_3=\frac{1}{1}:\frac{1}{2}:\frac{1}{3}=6:3:2$

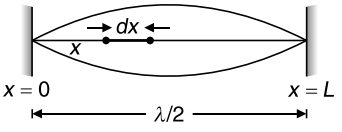
$\ell_1=(1.1)\left(\frac{6}{6+3+2}\right)=0.6\text{ m}=60\text{ cm}$

$\ell_2=(1.1)\left(\frac{3}{6+3+2}\right)=0.3\text{ m}=30\text{ cm}$

and $\ell_3=(1.1)\left(\frac{2}{6+3+2}\right)=0.2\text{ m}=20\text{ cm}$

Therefore, one bridge should be placed at 60 cm from one end and the other should be placed at 20 cm from the other end.

20. Mass of string is $m=\mu\ell$



Since, $\frac{\lambda}{2}=\ell$

$\Rightarrow \quad \lambda=2\ell$

$\Rightarrow \quad k=\frac{2\pi}{\lambda}=\frac{\pi}{\ell}$

Now, $A(x)=A\sin(kx)$

$\Rightarrow \quad dE=\frac{1}{2}(\mu dx)\omega^2A^2\sin^2kx$

$\Rightarrow \quad dE=\frac{1}{4}(\mu\omega^2A^2)(1-\cos(2kx))dx$

$\Rightarrow \quad E=\int_{x=0}^{x=\ell}dE=\frac{\mu\omega^2A^2}{4}\left(x-\frac{\sin 2kx}{2}\right)\Big|_0^\ell$

$\Rightarrow \quad E=\frac{\mu\ell\omega^2A^2}{4}=\frac{m\omega^2A^2}{4}$

21. Since, $v=\sqrt{\frac{T}{\mu}}v=40\text{ ms}^{-1}$

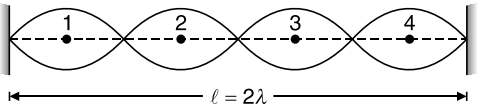
$\omega=2\pi f=377\text{ rads}^{-1}$

and $P=\frac{1}{2}\mu\omega^2A^2v=512\text{ watt}$

22. Four antinodes means four loops, so $\ell=2\lambda$

$\Rightarrow \quad \frac{\ell}{2}=\lambda$

$\Rightarrow \quad k=\frac{2\pi}{\lambda}=\frac{4\pi}{\ell}$



Since, $v=\frac{\omega}{k}$

$\Rightarrow \quad 400=\frac{(2\pi)(800)(\ell)}{(4\pi)}$

$\Rightarrow \quad \ell=1\text{ m}$

23. Since, strain $=\frac{\Delta\ell}{\ell}=\frac{0.1}{100}=10^{-3}$

So, tension in the wire is

$T=Y(\text{strain})(S)=10^{-3}YS$

Since, $f=\frac{v}{2\ell}=\frac{1}{2\ell}\sqrt{\frac{T}{\mu}}=\frac{1}{2\ell}\sqrt{\frac{T}{\rho S}}=\frac{1}{2\ell}\sqrt{\frac{10^{-3}Y}{\rho}}$

$\Rightarrow \quad f=\frac{1}{2\times1}\sqrt{\frac{(10^{-3})(20\times10^{10})}{8000}}=79\text{ Hz}$

24. Third overtone of closed organ pipe means seventh harmonic. Given that

$(f_7)_{\text{clsoed}}=(f_4)_{\text{open}}$

$\Rightarrow \quad 7\left(\frac{v}{4\ell_c}\right)=4\left(\frac{v}{2\ell_0}\right)$

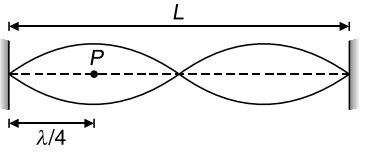
$\Rightarrow \quad \frac{\ell_c}{\ell_0}=\frac{7}{8}$

$\Rightarrow \quad \ell_0=\frac{8}{7}\ell_c=\frac{8}{7}(7\text{ cm})=8\text{ cm}$

25. (a) $v_{O_2}=\sqrt{\frac{B}{\rho}}=\sqrt{\frac{1.41\times10^5}{1.43}}=314\text{ ms}^{-1}$

(b) $v_{He}=\sqrt{\frac{B}{\rho}}=\sqrt{\frac{1.7\times10^5}{0.18}}=972\text{ ms}^{-1}$

26. Given that $\frac{L}{4}=\frac{\lambda}{4}$, so we have $L=\lambda$



In the next higher mode there will be total 6 loops and the desired frequency is $\left(\frac{6}{2}\right)(100)=300\text{ Hz}$

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1. Since, $f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$

For identical strings, l and μ will be same

$$f \propto \sqrt{T}$$

$$\Rightarrow \quad \frac{450}{300} = \sqrt{\frac{T_X}{T_Z}}$$

$$\Rightarrow \quad \frac{T_X}{T_Z} = \frac{9}{4} = 2.25$$

Hence, the correct answer is (C).

2. Given that, $\rho_{\text{wire}} = 9\times10^{-3}\ \text{kgcm}^{-3} = 9000\ \text{kgm}^{-3}$

$$\Delta l/l = 4.9\times10^{-4},\ Y = 9\times10^{10}\ \text{Nm}^{-2}$$

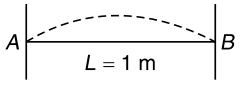
For lowest frequency, $L = 1\ \text{m} = \frac{\lambda}{2}$

$$\Rightarrow \quad \lambda = 2\ \text{m}$$

$$\Rightarrow \quad v = \sqrt{\frac{T}{\mu}} = f\lambda \qquad \ldots(1)$$

If A be the cross-sectional area of the wire, then

$$\mu = \frac{Al\rho}{l} = A\rho \text{ and } Y = \frac{T/A}{\Delta l/l}$$



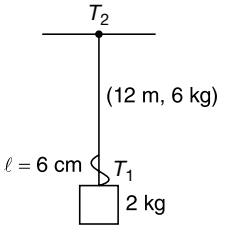
$$\Rightarrow \quad T = YA(\Delta l/l)$$

$$\Rightarrow \quad \sqrt{\frac{YA(\Delta l/l)}{A\rho}} = (2)f \qquad \{\text{from equation (1)}\}$$

$$\Rightarrow \quad f = \frac{1}{2}\sqrt{\frac{Y(\Delta l/l)}{\rho}} = \frac{1}{2}\sqrt{\frac{9\times10^{10}\times4.9\times10^{-4}}{9000}} = 35\ \text{Hz}$$

Hence, the correct answer is 35.

3. Since $v \propto \lambda$ i.e., $\sqrt{T/\mu} \propto \lambda$



Now, $T_2 = 8g$ and $T_1 = 2g$

$$\Rightarrow \quad \lambda_2 = \lambda_1\sqrt{\frac{T_2}{T_1}} = \lambda_1\sqrt{\frac{8g}{2g}} = (6)(2) = 12\ \text{cm}$$

Hence, the correct answer is (B).

4. The separation between two crests is

$$n_1\lambda = 5$$

and separation between crest and a trough is

$$(2n_2+1)\frac{\lambda}{2} = 1.5$$

where, n_1 and n_2 are positive integers.

$$\Rightarrow \quad \frac{1.5}{5} = \frac{(2n_2+1)}{2n_1}$$

$$\Rightarrow \quad n_1 = \frac{5}{3}(2n_2+1)$$

For n_1 and n_2 both to be integers, we must have

$$2n_2+1 = 3,\ 9,\ 15,\ 21,\ \ldots$$

because then only we get, $n_1 = 5,\ 15, 25, 35, \ldots$

So, $\lambda = \frac{5}{n_1} = \frac{3}{2n_2+1} = 1,\ \frac{1}{3},\ \frac{1}{5}, \ldots$

Hence, the correct answer is (D).

5. $\lambda = 2(l_2 - l_1) = 2(24.5-17) = 2(7.5) = 15\ \text{cm}$

Also, $v = f\lambda$

$$\Rightarrow \quad 330 = \lambda\times15\times10^{-2}$$

$$\Rightarrow \quad \lambda = \frac{330}{15}\times100 = \frac{1100\times100}{5} = 2200\ \text{Hz}$$

Hence, the correct answer is (C).

6. Since, $\Delta p = BAk = Bsk \qquad \{\because A = s\}$

$$\Rightarrow \quad \Delta p = (\rho v^2)s\left(\frac{\omega}{v}\right) \qquad \left\{\because v = \sqrt{\frac{B}{\rho}}\right\}$$

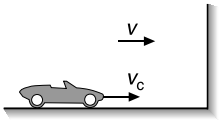
$$\Rightarrow \quad s = \frac{\Delta p}{\rho v\omega} \approx \frac{10}{1\times300\times1000}\ \text{m} = \frac{1}{30}\ \text{mm} = \frac{3}{100}\ \text{mm}$$

Hence, the correct answer is (B).

7. Frequency heard by wall is

$$f_1 = f\left(\frac{v}{v-v_c}\right) \qquad \ldots(1)$$

If f_2 be the frequency heard by driver after sound is reflected from the wall, then



$$f_2 = f_1\left(\frac{v+v_c}{v}\right) = f\left(\frac{v+v_c}{v-v_c}\right) \qquad \ldots(2)$$

From equations (1) and (2), we get

$$\frac{f_2}{f} = \frac{48}{44} = \frac{v+v_c}{v-v_c}$$

$$\Rightarrow \quad 12(v+v_c) = 11(v-v_c)$$

$$\Rightarrow \quad 23v_c = v$$

$$\Rightarrow \quad v_c = \frac{v}{23} = \frac{345}{23} = 15\ \text{ms}^{-1} = 54\ \text{kmh}^{-1}$$

Hence, the correct answer is (D).

8. Let x be the distance travelled by the wave when it reaches the respective point.
For waves arriving at A from P and Q, phase difference is

$$\Delta\phi_A = (\phi_P - \phi_Q) + \frac{2\pi}{\lambda}(x_P - x_Q)$$

CHAPTER 4

When the waves reach A, then Q is ahead of P in terms of path because wave emitted by Q reaches A before the wave emitted by P and also phase of P is ahead that of Q by 90°, so we have

$$\phi_P - \phi_Q = +\frac{\pi}{2} \text{ and } x_P - x_Q = -5 \text{ m}$$

$$\Rightarrow \quad \Delta\phi_A = +\frac{\pi}{2} + \frac{2\pi}{20}(-5) = 0$$

Since, $I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\Delta\phi$

$$\Rightarrow \quad I_A = I + I + 2I\cos(0°) = 4I$$

For waves arriving at C from P and Q, phase difference is

$$\Delta\phi_C = (\phi_P - \phi_Q) + \frac{2\pi}{\lambda}(x_P - x_Q)$$

When the waves reach C, then P is ahead of Q in terms of path, because wave emitted by P reaches C before the wave emitted by Q and also phase of P is ahead that of Q by 90°, so we have

$$\phi_P - \phi_Q = +\frac{\pi}{2} \text{ and } x_P - x_Q = +5 \text{ m}$$

$$\Rightarrow \quad \Delta\phi_C = +\frac{\pi}{2} + \frac{2\pi}{20}(+5) = \pi$$

$$\Rightarrow \quad I_C = I + I + 2I\cos(\pi) = 0$$

For waves arriving at B from P and Q, phase difference is

$$\Delta\phi_B = (\phi_P - \phi_Q) + \frac{2\pi}{\lambda}(x_P - x_Q)$$

When the waves reach B, then P and Q have same path and phase of P is ahead that of Q by 90°, so we have

$$\phi_P - \phi_Q = +\frac{\pi}{2} \text{ and } x_P - x_Q = 0$$

$$\Rightarrow \quad \Delta\phi_B = +\frac{\pi}{2} + 0 = \frac{\pi}{2}$$

$$\Rightarrow \quad I_B = I + I + 2I\cos(\pi/2) = 2I$$

Hence, $I_A : I_B : I_C = 4I : 2I : 0 = 2 : 1 : 0$

Hence, the correct answer is (D).

9. Since, $v' = v\left(\frac{v_{\text{sound}}}{v_{\text{sound}} - v\cos\theta}\right)$

Initially θ will be less, so $\cos\theta$ is more. Hence v' is more and then decreases.

Hence, the correct answer is (D).

10. Since, $v = \sqrt{\frac{T}{\mu}}$

$$\Rightarrow \quad 90 = \sqrt{\frac{YA(\Delta\ell/\ell)}{m/\ell}} = \sqrt{\frac{16\times10^{11}\times10^{-6}\times\Delta\ell}{6\times10^{-3}}}$$

$$\Rightarrow \quad \Delta\ell = \frac{8100\times3}{8}\times10^{-8} = 0.03 \text{ mm}$$

Hence, the correct answer is (C).

11. Since, $v_{\text{approach}} = \left(\frac{v}{v - v_s}\right)v_0$ and $v_{\text{recede}} = \left(\frac{v}{v + v_s}\right)v_0$

Given that beat frequency is $\Delta v = v_1 - v_2 = 2$ bps

$$\Rightarrow \quad \Delta v = v_{\text{app}} - v_{\text{rec}} = v_0\left(\frac{1}{c - v} - \frac{1}{c + v}\right)v$$

$$\Rightarrow \quad \Delta v = \left(\frac{2vv_s}{v^2 - v_s^2}\right)v_0$$

Since, $v_s \ll v$, so $\Delta v \approx 2\left(\frac{v_s}{v}\right)v_0 = 2$

$$\Rightarrow \quad 2\left(\frac{v_s}{350}\right)1400 = 2$$

$$\Rightarrow \quad v_s = \frac{1}{4} \text{ ms}^{-1}$$

Hence, the correct answer is (D).

12. Since, $v_s = \sqrt{\frac{\gamma P}{\rho}}$

$$\Rightarrow \quad \frac{v_{\text{gas}}}{v_{\text{air}}} = \sqrt{\frac{\rho_{\text{air}}}{\rho_{\text{gas}}}}$$

$$\Rightarrow \quad \frac{v_{\text{gas}}}{300} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad v_{\text{gas}} = \frac{300}{\sqrt{2}} = 150\sqrt{2} \text{ ms}^{-1}$$

$$\Rightarrow \quad f_2 - f_1 = \frac{v_{\text{gas}}}{2\ell} = \frac{150\sqrt{2}}{2(1)} = 75\sqrt{2} \text{ Hz} \approx 106 \text{ Hz}$$

Hence, the correct answer is 106.

13. Velocity of transverse wave $v \propto \sqrt{T}$

$$\Rightarrow \quad \frac{v'}{v} = \frac{v/2}{v} = \sqrt{\frac{T'}{T}}$$

$$\Rightarrow \quad T' = \frac{T}{4} = \frac{2.06\times10^4}{4} = 5.15\times10^3 \text{ N}$$

Hence, the correct answer is (B).

14. Let amplitude of each wave be A, then resultant wave equation is given by

$$x = A\sin\omega t + A\sin\left(\omega t - \frac{\pi}{4}\right) + A\sin\left(\omega t + \frac{\pi}{4}\right)$$

$$\Rightarrow \quad x = A\sin\omega t + \sqrt{2}A\sin\omega t = (\sqrt{2} + 1)A\sin\omega t$$

Resultant wave amplitude is $R = (\sqrt{2} + 1)A$

Since $I \propto A^2$, so $\frac{I}{I_0} = (\sqrt{2} + 1)^2$

$$\Rightarrow \quad I = 5.8I_0$$

Hence, the correct answer is (A).

15. Given that $\frac{nv}{2l} = 420$ …(1)

and $\frac{(n+1)v}{2l} = 490$ …(2)

From equation (1) and (2), we get

$$\frac{n}{n+1} = \frac{6}{7}$$

$$\Rightarrow \quad 7n = 6n + 6$$

$\Rightarrow \quad n = 6$

Substituting in (1), we get

$$\frac{v}{2l} = 70, \text{ where } v = \sqrt{\frac{T}{\mu}}$$

$$\Rightarrow \quad \ell = \frac{v}{140} = \frac{1}{140}\sqrt{\frac{540}{6\times10^{-3}}} = \frac{1}{140}\sqrt{90\times10^{3}}$$

$$\Rightarrow \quad \ell = \frac{300}{140} = 2.142 \text{ m}$$

Hence, the correct answer is (D).

16. Since $f = n\left(\frac{1}{2\ell}\sqrt{\frac{T}{\mu}}\right)$, where $\mu = \rho A = \pi r^2\rho$

$$\Rightarrow \quad f \propto \frac{n}{r}$$

Since frequency is same so, we have

$$\frac{p}{r_A} = \frac{q}{r_B}$$

$$\Rightarrow \quad \frac{p}{q} = \frac{r_A}{r_B} = \frac{1}{2}$$

Hence, the correct answer is (B).

17. $v = \frac{\omega}{k} = \frac{1000}{3}$ ms^{-1}

Since $v \propto \sqrt{T}$

$$\Rightarrow \quad \frac{dv}{v} = \frac{1}{2}\frac{dT}{T}$$

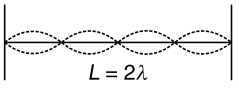
$$\Rightarrow \quad \frac{8\times3}{3\times1000} = \frac{1}{2}\times\frac{dT}{273}$$

$$\Rightarrow \quad dT = \frac{273\times2\times8}{1000} = 4.36 \text{ °C}$$

Hence, the correct answer is (A).

18. Since $k = 0.157 = \frac{2\pi}{\lambda}$

$$\Rightarrow \quad \lambda = \frac{2\pi}{0.157} = 40 \text{ m}$$

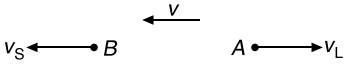


In fourth harmonic show in Figure, we have

$$L = \frac{4\lambda}{2} = 2\lambda = 80 \text{ m}$$

Hence, the correct answer is (D).

19.



Since, $f' = f\left(\frac{v - v_L}{v - v_S}\right) = f\left(\frac{v-20}{v-(-20)}\right)$

$$\Rightarrow \quad 2000 = \frac{320}{360}f$$

$$\Rightarrow \quad f = \frac{2000\times9}{8} = 2250 \text{ Hz}$$

Hence, the correct answer is (D).

20. For third harmonic, we have

$$\frac{3\lambda}{2} = L = 2$$

$$\Rightarrow \quad \lambda = \frac{4}{3} \text{ m}$$

$$\Rightarrow \quad v = f\lambda = \frac{4}{3}\times240 = 320 \text{ ms}^{-1}$$

Also, for third harmonic $f_3 = nf_1$

$$\Rightarrow \quad f_1 = \frac{f_3}{3} = \frac{240}{3} = 80 \text{ Hz}$$

Hence, the correct answer is (B).

21. Frequency of source is $f = 500$ Hz

When observer is moving away from the source with speed v_1, then apparent frequency is

$$f_{\text{recede}} = f_1 = 480 = f\left(\frac{v - v_1}{v}\right) \qquad \ldots(1)$$

$$\Rightarrow \quad 480 = 500\left(\frac{300 - v_1}{300}\right)$$

$$\Rightarrow \quad v_1 = 12 \text{ ms}^{-1}$$

When observer is moving towards the source with speed v_2, then apparent frequency is

$$f_{\text{approach}} = f_2 = 530 = f\left(\frac{v + v_2}{v}\right) \qquad \ldots(2)$$

$$\Rightarrow \quad 530 = 500\left(\frac{300 + v_2}{300}\right)$$

$$\Rightarrow \quad v_2 = 18 \text{ ms}^{-1}$$

Hence, the correct answer is (D).

22. Beat frequency $f_b = |f_1 - f_2| = 11 - 9 = 2$ Hz

Hence, beat time i.e., time interval between two consecutive maxima or minima is

$$t_b = \frac{1}{f_b} = \frac{1}{2} \text{ s}$$

Hence, the correct answer is (A).

23. Since, $f_{\text{approach}} = 1000 = f\left(\frac{v-0}{v-v_S}\right) = f\left(\frac{350}{350-50}\right)$

$$\Rightarrow \quad 1000 = f\left(\frac{350-0}{350-50}\right)$$

$$\Rightarrow \quad f = 1000\left(\frac{300}{350}\right) \text{ Hz}$$

On receding from the listener, we have

$$f_{\text{recede}} = f\left(\frac{v-0}{v+v_S}\right) = f\left(\frac{350}{350+50}\right) = f\left(\frac{350}{400}\right)$$

$$\Rightarrow \quad f_{\text{recede}} = 1000\left(\frac{300}{350}\right)\left(\frac{350}{400}\right) = 750 \text{ Hz}$$

Hence, the correct answer is (D).

24. Let v_1 be the frequency received by A, then

$$v_1 = v_0\left(\frac{1500-5}{1500-7.5}\right) = v_0\left(\frac{1495}{1492.5}\right)$$

Frequency received by B is

$$v_2 = v_1\left(\frac{v+v_L}{v+v_S}\right) = v_0\left(\frac{1495}{1492.5}\right)\left(\frac{1507.5}{1505}\right)$$

CHAPTER 4

$\Rightarrow \quad v_2 = v = 502$ Hz

Hence, the correct answer is (D).

25. Since, $y = A\sin(kx - \omega t + \phi)$

At $t = 0$ and $x = 0$, we observe that $y = 0$. So, particle is at mean position and will proceed in positive y direction.
Hence, the correct answer is (A).

26. Since, $120 = 10\log_{10}\left(\frac{I}{10^{-12}}\right)$

$\Rightarrow \quad \frac{I}{10^{-12}} = 10^{12}$

$\Rightarrow \quad I = 1\ \text{Wm}^{-2}$

$\Rightarrow \quad \frac{2}{4\pi r^2} = 1$

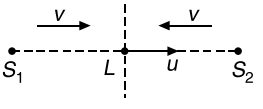
$\Rightarrow \quad r = \sqrt{\frac{2}{4\pi}}\ \text{m} = 0.399\ \text{m} = 40\ \text{cm}$

Hence, the correct answer is (B).

27. From Figure, we get

$f_1 = \left(\frac{v-u}{v}\right)f$ and $f_2 = \left(\frac{v+u}{v}\right)f$

$\Rightarrow \quad f_2 - f_1 = \left(\frac{2u}{v}\right)f$



$\Rightarrow \quad 10 = \left(\frac{2u}{330}\right)660$

$\Rightarrow \quad u = 2.5\ \text{ms}^{-1}$

Hence, the correct answer is (C).

28. Since, $\ell_1 = 30$ cm, $\ell_2 = 70$ cm

$\Rightarrow \quad \frac{\lambda}{2} = (\ell_2 - \ell_1) = 40$ cm

$\Rightarrow \quad \lambda = 80$ cm

$\Rightarrow \quad v = f\lambda = (480)(0.8) = 384\ \text{ms}^{-1}$

Hence, the correct answer is (B).

29. Since, $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{mg}{\mu}}$

Also, $v' = \sqrt{\frac{m\sqrt{g^2 + a^2}}{\mu}}$

$\Rightarrow \quad \frac{v'}{v} = \sqrt{\frac{\sqrt{g^2 + a^2}}{g}}$

$\Rightarrow \quad a \approx 1.83\ \text{ms}^{-2} \approx \frac{g}{5}$

Hence, the correct answer is (B).

30. For second harmonic, we have

$\lambda = L = 0.50$ m

$\Rightarrow \quad f = \frac{v}{\lambda} = 330 \times 2 = 660$ Hz

From Doppler's effect of sound, we get

$f' = f\left(\frac{330 + \frac{50}{18}}{330}\right) \approx 666$ Hz

Hence, the correct answer is (D).

31. Since, $f_1 = f\left(\frac{340}{340 - 34}\right)$ and $f_2 = f\left(\frac{340}{340 - 17}\right)$

$\Rightarrow \quad \frac{f_1}{f_2} = \frac{340 - 17}{340 - 34} = \frac{19}{18}$

Hence, the correct answer is (D).

32. Since, $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{8 \times 1}{5 \times 10^{-3}}} = 40\ \text{ms}^{-1}$

$\Rightarrow \quad \lambda = \frac{v}{f} = \frac{40}{100} = 0.4$ m

Separation between successive nodes is $\frac{\lambda}{2}$

$\Rightarrow \quad \Delta x = 0.2\ \text{m} \simeq 20\ \text{cm}$

Hence, the correct answer is (D).

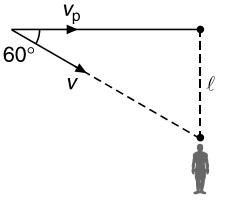
33. Since, $v = \frac{\omega}{k} = \frac{450}{9} = 50\ \text{ms}^{-1}$

$\Rightarrow \quad v = 50 = \sqrt{\frac{T}{\mu}}$

$\Rightarrow \quad T = 2500 \times 5 \times 10^{-3} = 12.5$ N

Hence, the correct answer is (D).

34. Since, $\frac{l\,\text{cosec}(60°)}{v} = \frac{l\cot(60°)}{v_p}$



$\Rightarrow \quad v_p = \frac{v}{2}$

Hence, the correct answer is (C).

35. Since, $y(x, t) = 10^{-3}\sin(50t + 2x)$

$\Rightarrow \quad v = \frac{\omega}{k} = \frac{50}{2} = 25\ \text{ms}^{-1}$

Also, wave is travelling along negative x direction.
Hence, the correct answer is (A).

36. Since, $\frac{\lambda_1}{4} = I_0 + 11$ and $\frac{\lambda_2}{4} = I_0 + 27$

$\Rightarrow \quad \frac{\lambda_2 - \lambda_1}{4} = 16$ cm

$\Rightarrow \quad v\left(\frac{1}{256} - \frac{1}{512}\right) = 0.64$ m

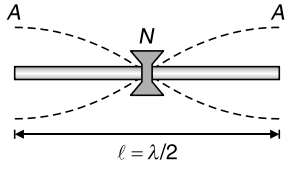
$\Rightarrow \quad v = (512)(0.64) = 328\ \text{ms}^{-1}$

Hence, the correct answer is (C).

37. Velocity of wave, $v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{9.27\times10^{10}}{2.7\times10^{3}}}$

$\Rightarrow \quad v = \sqrt{3.433\times10^{7}} = 10^{3}\sqrt{34.33} = 5.86\times10^{3}\ \text{ms}^{-1}$

Since rod is clamped at the middle, shape of fundamental wave is shown in Figure.



$\Rightarrow \quad \frac{\lambda}{2} = L = \frac{60}{100}\ \text{m}$

$\Rightarrow \quad \lambda = 1.2\ \text{m}$

So fundamental frequency is given by

$$f = \frac{v}{\lambda} = \frac{5.86\times10^{3}}{1.2} = 4.88\times10^{3}\ \text{Hz} = 5\ \text{kHz}$$

Hence, the correct answer is (A).

38. Frequency of organ pipe will either be 257 Hz or 255 Hz. If frequency of tuning fork is 257 Hz, then

$$257 = \frac{3v}{2\ell}$$

$\Rightarrow \quad \ell = \frac{3\times340}{2\times257}\ \text{m} = 1.98\ \text{m} = 198\ \text{cm}$

When frequency of tuning fork is 255 Hz, then

$$255 = \frac{3v}{2\ell}$$

$\Rightarrow \quad \ell = \frac{3\times340}{2\times255}\ \text{m} = 2\ \text{m} = 200\ \text{cm}$

So, length of pipe should be 200 cm

Hence, the correct answer is (C).

39. Frequency of a sonometer wire is $f = \frac{1}{LD}\sqrt{\frac{T}{\pi\rho}}$

$\Rightarrow \quad f \propto \frac{1}{L}$

Given that, $\frac{f_1}{f_2} = \frac{1}{0.95} = \frac{100}{95}$ …(1)

and $f_1 - f_2 = 10\ \text{Hz}$ …(2)

From equations (1) and (2), we get

$f_1 = 200\ \text{Hz}, f_2 = 190\ \text{Hz}$

So, frequency of tuning fork is

$f = f_1 - 5 = f_2 + 5 = 195\ \text{Hz}$

Hence, the correct answer is (C).

40. Frequency of sitar string B is either 420 Hz or 430 Hz. As tension in string B is increased, its frequency will increase. If the frequency is 430 Hz, then beat frequency will increase and if the frequency is 420 Hz, then beat frequency will decrease. Hence correct frequency is 420 Hz.

Hence, the correct answer is (C).

41. Given that $e = 1$ cm

For first resonance, $\frac{\lambda}{4} = \ell_1 + e = 11$ cm

For second resonance, $\frac{3\lambda}{4} = \ell_2 + e$

$\Rightarrow \quad \ell_2 = (3)(11) - 1 = 32\ \text{cm}$

Hence, the correct answer is (D).

42. The fundamental frequency in a stretched string is

$$f = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$$

where, n is number of antinodes (or number of loops), μ is the mass per unit length.

At the midpoint O of the two bridges, the node of stationary wave lies, hence length of two wires are equal, i.e., $L_1 = L_2 = L$. Also, frequency remains same for both wires, i.e., $f_1 = f_2$

$\Rightarrow \quad \frac{n_1}{2L}\sqrt{\frac{T}{\pi r^2\rho_1}} = \frac{n_2}{2L}\sqrt{\frac{T}{\pi r^2\rho_2}}$

$\Rightarrow \quad \frac{n_1}{\sqrt{\rho_1}} = \frac{n_2}{\sqrt{\rho_2}}$

$\Rightarrow \quad \frac{n_1}{n_2} = \sqrt{\frac{\rho_1}{\rho_2}} = \sqrt{\frac{\rho_1}{4\rho_1}} = \frac{1}{2} \qquad \{\because\ \rho_2 = 4\rho_1\}$

Hence, the correct answer is (B).

43. Given, $y(x, t) = 0.5\left(\frac{5\pi}{4}x\right)\sin\cos(200\pi t)$

Comparing, this equation with standard equation of standing wave i.e., $y(x, t) = 2a\sin kx\cos\omega t$, we get

$$k = \frac{5\pi}{4}\ \text{radm}^{-1} \text{ and } \omega = 200\pi\ \text{rads}^{-1}$$

So, speed of the travelling wave is

$$v = \frac{\omega}{k} = \frac{200\pi}{5\pi/4} = 160\ \text{ms}^{-1}$$

Hence, the correct answer is (B).

44. Velocity at point P is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{(\mu x)g}{\mu}} = \sqrt{gx}$$

$\Rightarrow \quad \frac{dx}{dt} = \sqrt{gx}$

$\Rightarrow \quad \int_0^{20}\frac{dx}{\sqrt{x}} = \int_0^{t}\sqrt{g}\,dt$

$\Rightarrow \quad t = 2\sqrt{2}\ \text{sec}$

Hence, the correct answer is (D).

45. Given that, $f_{\text{open}} = f = \frac{v}{2L}$

When pipe is half dipped in water, then

$$f' = f_{\text{closed}} = \frac{v}{4L'} = \frac{v}{4(L/2)} = f$$

Hence, the correct answer is (A).

CHAPTER 4

46. Frequency heard by the driver of second engine

$$f' = f\left(\frac{v+v_L}{v-v_S}\right)$$

$$\Rightarrow \quad f' = 540\left(\frac{330+30}{330-30}\right) = 540\left(\frac{360}{300}\right) = 648 \text{ Hz}$$

Hence, the correct answer is (D).

47. Given that $f_1 - f_2 = 5$ bps …(1)

Apparent frequency heard by the observer directly is

$$f_1 = f\left(\frac{v}{v-v_S}\right) = f\left(\frac{340}{340-5}\right) = f\left(\frac{340}{335}\right)$$

Apparent frequency heard by the observer after sound gets reflected from the wall is

$$f_2 = f\left(\frac{v}{v+v_S}\right) = f\left(\frac{340}{340+5}\right) = f\left(\frac{340}{345}\right)$$

Substituting in equation (1), we get

$$f\left(\frac{340}{335} - \frac{340}{345}\right) = 5$$

$$\Rightarrow \quad f = \frac{5\times 335\times 345}{340\times 10} = 169.96 \text{ Hz} \approx 170 \text{ Hz}$$

Hence, the correct answer is (D).

48. Since, $f_{\text{approach}} = f\left(\frac{v}{v-v_S}\right)$ and $f_{\text{recede}} = f\left(\frac{v}{v+v_s}\right)$

$$\Rightarrow \quad \Delta f = f_{\text{approach}} - f_{\text{recede}} = \left(\frac{2vv_S}{v^2 - v_S^2}\right) f \approx \left(\frac{2v_S}{v}\right) f$$

$$\Rightarrow \quad \frac{\Delta f}{f}\times 100\% = \left(\frac{2\times 20}{320}\right)\times 100\% = 12.5\%$$

Hence, the correct answer is (B).

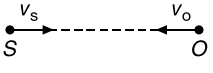
49. Apparent frequency heard by bat is

$$f' = f\left(\frac{v+v_{\text{bat}}}{v-v_{\text{bat}}}\right) = 8000\left(\frac{320+10}{320-10}\right)$$

$$\Rightarrow \quad f' = \frac{330}{310}\times 8000 = 8516 \text{ Hz}$$

Hence, the correct answer is (B).

50. For the given situation, frequency heard by observer is given by



$$f = f_0\left(\frac{v+v_0}{v-v_s}\right) = \frac{f_0 v}{v-v_s} + \frac{f_0 v_0}{v-v_s}$$

Comparing this equation with the equation of a straight line i.e., $y = mx + c$, we observe that slope of graph is $m = \frac{f_0}{v-v_s}$.

Hence, the correct answer is (A).

51. Since pipe is closed from one end, so it behaves like a closed organ pipe in which frequencies are given by

$$f = (2n-1)\left(\frac{v}{4L}\right), \text{ where } n = 1, 2, 3, 4, \ldots..$$

According to question, $f < 1250$ Hz

$$\Rightarrow \quad (2n-1)\left(\frac{v}{4L}\right) < 1250$$

$$\Rightarrow \quad (2n-1)\left(\frac{340}{4\times 0.85}\right) < 1250$$

$$\Rightarrow \quad (2n-1) < 12.5$$

Hence possible value of n are

$n = 1, 2, 3, 4, 5, 6$

So, number of possible natural frequencies that lie below 1250 Hz is 6.

Hence, the correct answer is (D).

52. Since, $f = \frac{1}{2L}\sqrt{\frac{T}{\mu}} = \frac{1}{2L}\sqrt{\frac{\text{Stress}}{\text{Density}}}$ …(1)

Also, $Y = \frac{\text{Stress}}{\text{Strain}}$ i.e., $\text{Stress} = Y(\text{Strain})$

Therefore, equation (1) becomes

$$f = \frac{1}{2L}\sqrt{\frac{Y(\text{Strain})}{\text{Density}}}$$

$$\Rightarrow \quad f = \frac{1}{2(1.5)}\sqrt{\frac{(2.2\times 10^{11})(1/100)}{7.7\times 10^3}}$$

$$\Rightarrow \quad f = \frac{1}{3}\sqrt{\frac{2}{7}\times 10^6} = \frac{1000}{3}\times\frac{\sqrt{2}}{\sqrt{7}} \approx 178.2 \text{ Hz}$$

Hence, the correct answer is (B).

53. Given, $y(x,t) = e^{-\left(ax^2+bt^2+2\sqrt{ab}xt\right)}$

$$\Rightarrow \quad y(x,t) = e^{-\left(\sqrt{a}x+\sqrt{b}t\right)^2} \quad \text{…(1)}$$

Comparing equation (1) with standard equation i.e.

$$y(x,t) = f(ax+bt)$$

Since there is positive sign between x and t terms, so wave travels along $-x$ direction such that wave speed is

$$v = \left|\frac{\text{Coefficient of } t}{\text{Coefficient of } x}\right| = \sqrt{\frac{b}{a}}$$

Hence, the correct answer is (B).

54. Wave velocity, $v = \frac{\omega}{k} = \frac{2\pi/0.04}{2\pi/0.5}$ ms^{-1}

Also $v = \sqrt{\frac{T}{\mu}}$, where T is the tension in the string and μ is the linear mass density given by $\mu = 0.04$ kgm^{-1}

$$\Rightarrow \quad v = \frac{\omega}{k} = \sqrt{\frac{T}{\mu}}$$

$$\Rightarrow \quad T = \frac{\mu\omega^2}{k^2} = \mu v^2$$

$$T = 0.04\left(\frac{2\pi/0.04}{2\pi/0.5}\right)^2 = 6.25 \text{ N}$$

Hence, the correct answer is (A).

55. The number of beats produced per second is

$$\Delta v = |(v+1) - (v-1)| = 2$$

Hence, the correct answer is (C).

56. Since source is at rest and observer is moving away from source, so

$$f' = f\left(\frac{v - v_L}{v - 0}\right) = f\left(\frac{v - v_L}{v}\right)$$

$$\Rightarrow \quad \left(\frac{f'}{f}\right)v = v - v_L$$

$$\Rightarrow \quad v\left(1 - \frac{f'}{f}\right) = v_L$$

$$\Rightarrow \quad 330(1 - 0.94) = v_L$$

$$\Rightarrow \quad v_L = 330 \times 0.06 = 19.80 \text{ ms}^{-1}$$

$$\Rightarrow \quad s = \frac{v_L^2 - 0^2}{2a} = \frac{(19.80)^2}{2 \times 2} = 98 \text{ m}$$

Hence, the correct answer is (B).

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Single Correct Choice Type Problems

1. Since, $\lambda = 4(L + e)$

$$\Rightarrow \quad L = \frac{\lambda}{4} - e$$

$$\Rightarrow \quad L = \frac{\lambda}{4} - 0.3d = 15.2 \text{ cm}$$

Hence, the correct answer is (B).

2. For hollow pipe, fundamental frequency is

$$f = \frac{v}{4\ell} = \frac{320}{4 \times 0.8}$$

For string in 2nd harmonic

$$f = \frac{1}{\ell}\sqrt{\frac{T}{\mu}} = \frac{1}{\ell}\sqrt{\frac{T\ell}{m}} = \frac{1}{0.5}\sqrt{\frac{50 \times 0.5}{m}}$$

Equating, we get

$$m = 0.01 \text{ kg} = 10 \text{ g}$$

Hence, the correct answer is (B).

3. $v = 10 \text{ cms}^{-1}$

$\lambda = 0.5$ m

$A = 10 \text{ cm} = 0.1$ m

From the figure, we observe that the equation of the wave must be

$$y = A\sin(kx - \omega t) \qquad \ldots(1)$$

where $k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.5} = 4\pi \text{ m}^{-1}$

Since, $v = \frac{\omega}{k}$

$$\Rightarrow \quad \omega = vk$$

$$\Rightarrow \quad \omega = \left(\frac{10}{100}\right)(4\pi) = 0.4\pi \text{ rads}^{-1}$$

So, equation (1) becomes

$$y = 0.1\sin(4\pi x - 0.4\pi t)\hat{j} \qquad \ldots(2)$$

Now $v_P = \frac{\partial y}{\partial t} = (0.1)(0.4\pi)\cos(4\pi x - 0.4\pi t)$ …(3)

Further, velocity at point P at displacement $y = 5$ cm is calculated first by substituting $y = \frac{5}{100}$ m in equation (2). So

$$0.05 = 0.1\sin(4\pi x - 0.4\pi t)$$

$$\Rightarrow \quad 4\pi x - 0.4\pi t = \frac{\pi}{6} \qquad \ldots(4)$$

$$\Rightarrow \quad v_P = (0.1)(0.4\pi)\cos\left(\frac{\pi}{6}\right)$$

$$\Rightarrow \quad v_P = (0.1)(0.4\pi)\frac{\sqrt{3}}{2} = \frac{4\pi\sqrt{3}}{200} = \frac{\pi\sqrt{3}}{50} \text{ ms}^{-1}$$

Hence, the correct answer is (C).

4. With increase in tension, frequency of vibrating string will increase. Since number of beats are decreasing. Therefore, frequency of vibrating string or third harmonic frequency of closed pipe should be less than the frequency of tuning fork by 4, so we have frequency of tuning fork (n) given by

$$n = f_3 + 4$$

$$\Rightarrow \quad n = 3\left(\frac{v}{4l}\right) + 4 = 3\left(\frac{340}{4 \times 0.75}\right) + 4 = 344 \text{ Hz}$$

Hence, the correct answer is (A).

5. Since, $f \propto v \propto \sqrt{T}$

Given that, $f_{AB} = 2f_{CD}$

$$\Rightarrow \quad T_{AB} = 4T_{CD} \qquad \ldots(1)$$

Further $\Sigma\tau_p = 0$

$$\Rightarrow \quad T_{AB}(x) = T_{CD}(l - x)$$

$$\Rightarrow \quad 4x = l - x \qquad \{\because T_{AB} = 4T_{CD}\}$$

$$\Rightarrow \quad x = \frac{l}{5}$$

Hence, the correct answer is (A).

6. Given, $\frac{\lambda}{2} = (63.2 - 30.7) \text{ cm}$

$$\Rightarrow \quad \lambda = 0.65 \text{ m}$$

So, the speed of sound observed is

$$v_0 = f\lambda = 512 \times 0.65 = 332.8 \text{ ms}^{-1}$$

Error in calculating velocity of sound is $\Delta v = 2.8 \text{ ms}^{-1}$

$$\Rightarrow \quad \Delta v = 280 \text{ cms}^{-1}$$

Hence, the correct answer is (D).

7. $f_1 = \frac{v}{\ell}$ {2nd harmonic of open pipe}

$f_2 = n\left(\frac{v}{4\ell}\right)$ {n^{th} harmonic of closed pipe}

Here, n is odd and $f_2 > f_1$

It is possible when $n = 5$ because with $n = 5$

CHAPTER 4

$$f_2 = \frac{5}{4}\left(\frac{v}{\ell}\right) = \frac{5}{4} f_1$$

Hence, the correct answer is (C).

8. The frequency is a characteristic of source. It is independent of the medium.

Hence, the correct answer is (D).

9. $f_c = f_0$ {both first overtone}

$$\Rightarrow \quad 3\left(\frac{v_c}{4L}\right) = 2\left(\frac{v_0}{2L_0}\right)$$

$$\Rightarrow \quad \ell_0 = \frac{4}{3}\left(\frac{v_0}{v_c}\right)L = \frac{4}{3}\sqrt{\frac{\rho_1}{\rho_2}}L \qquad \left\{\because v \propto \frac{1}{\sqrt{p}}\right\}$$

Hence, the correct answer is (C).

10. Let $\Delta\ell$ be the end correction.

Given that, fundamental tone for a length 0.1 m = first overtone for the length 0.35 m.

$$\frac{v}{4(0.1+\Delta\ell)} = \frac{3v}{4(0.35+\Delta\ell)}$$

Solving this equation, we get $\Delta\ell = 0.025 \text{ m} = 2.5 \text{ m}$

Hence, the correct answer is (B).

11. The motorcyclist observes no beats. So, the apparent frequency observed by him from the two source must be equal.

$$f_1 = f_2$$

$$\Rightarrow \quad 176\left(\frac{330-v}{330-22}\right) = 165\left(\frac{330+v}{330}\right)$$

Solving this equation, we get

$$v = 22 \text{ ms}^{-1}$$

Hence, the correct answer is (B).

12. $f = \frac{p}{2\ell}\sqrt{\frac{T}{\mu}}$

$$\Rightarrow \quad \frac{5}{2\ell}\sqrt{\frac{9g}{\mu}} = \frac{3}{2\ell}\sqrt{\frac{Mg}{\mu}}$$

$$\Rightarrow \quad 5(3) = 3\sqrt{M}$$

$$\Rightarrow \quad M = 25 \text{ kg}$$

Hence, the correct answer is (A).

13. Using the formula $f' = f\left(\frac{v+v_0}{v}\right)$

we get, $5.5 = 5\left(\frac{v+v_A}{v}\right)$ …(1)

and $6 = 5\left(\frac{v+v_B}{v}\right)$ …(2)

Here, v = speed of sound

v_A = speed of train A

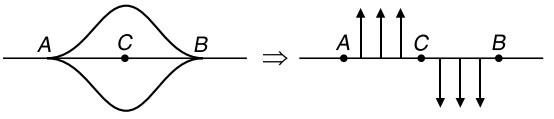
v_B = speed of train B

Solving equations (1) and (2), we get

$$\frac{v_B}{v_A} = 2$$

Hence, the correct answer is (B).

14. After two seconds both the pulse will move 4 cm towards each other. So, by their superposition, the resultant displacement at every point will be zero. Therefore, total energy will be purely in the form of kinetic. Half of the particles will be moving upwards and half downwards.



Hence, the correct answer is (B).

15. Energy $E \propto (\text{amplitude})^2 (\text{frequency})^2$

Amplitude (A) is same in both the cases, but frequency 2ω in the second case is two times the frequency (ω) in the first case.

$\Rightarrow \quad E_2 = 4E_1$

Hence, the correct answer is (C).

16. Fundamental frequency ν is given by

$$\nu = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} \qquad \{\text{with both the ends fixed}\}$$

So, fundamental frequency is

$$\nu \propto \frac{1}{\ell\sqrt{\mu}} \qquad \{\text{for same tension in both strings}\}$$

where μ is mass per unit length of wire

$\Rightarrow \quad \mu = \rho A = \rho(\pi r^2)$, where ρ is density of wire

$\Rightarrow \quad \sqrt{\mu} \propto r$

$$\Rightarrow \quad v \propto \frac{1}{r\ell}$$

$$\Rightarrow \quad \frac{v_1}{v_2} = \left(\frac{r_2}{r_1}\right)\left(\frac{\ell_2}{\ell_1}\right) = \left(\frac{r}{2r}\right)\left(\frac{2L}{L}\right) = 1$$

Hence, the correct answer is (D).

17. Since, $f_1 = f\left(\frac{v}{v-v_s}\right) = f\left(\frac{340}{340-34}\right) = f\left(\frac{340}{306}\right)$

and $f_2 = f\left(\frac{340}{340-17}\right) = f\left(\frac{340}{323}\right)$

$$\Rightarrow \quad \frac{f_1}{f_2} = \frac{323}{306} = \frac{19}{18}$$

Hence, the correct answer is (D).

18. Speed of sound in gases is given by

$$v = \sqrt{\frac{\gamma RT}{M}}$$

$$\Rightarrow \quad v \propto \frac{1}{\sqrt{M}}$$

$$\Rightarrow \quad \frac{v_1}{v_2} = \sqrt{\frac{m_2}{m_1}}$$

Hence, the correct answer is (B).

19. Speed of sound in an ideal gas is given by

$$v = \sqrt{\frac{\gamma RT}{M}}$$

$\Rightarrow \quad v \propto \sqrt{\frac{\gamma}{M}}$ {T is mass for both the gases}

$\Rightarrow \quad \frac{v_{N_2}}{v_{He}} = \sqrt{\frac{\gamma_{N_2}}{\gamma_{He}}\frac{M_{He}}{M_{N_2}}} = \sqrt{\frac{7/5}{5/3}\left(\frac{4}{28}\right)} = \frac{\sqrt{3}}{5}$

Because, $\gamma_{N_2} = r_{\text{diatomic}} = \frac{7}{5}$ and $\gamma_{He} = r_{\text{monatomic}} = \frac{5}{3}$

Hence, the correct answer is (C).

20. Mass per unit length of the string

$$m = \frac{10^{-2}}{0.4} = 2.5 \times 10^{-2} \text{ kgm}^{-1}$$

So, velocity of wave in the string $v = \sqrt{\frac{T}{m}}$

$\Rightarrow \quad v = \sqrt{\frac{1.6}{2.5 \times 10^{-2}}} = 8 \text{ ms}^{-1}$

For constructive interference between successive pulses

$$\Delta t_{\min} = \frac{2\ell}{v} = \frac{(2)(0.4)}{8} = 0.10 \text{ s}$$

Hence, the correct answer is (B).

21. This is an equation of a travelling wave in which particles of the medium are in SHM and maximum particle velocity in SHM is $A\omega$, where A is the amplitude and ω the angular velocity.

Hence, the correct answer is (A).

22. Source is moving towards the observer

$$f' = f\left(\frac{v}{v - v_s}\right)$$

$\Rightarrow \quad f' = 450\left(\frac{330}{330 - 33}\right)$

$\Rightarrow \quad f' = 500 \text{ Hz}$

Hence, the correct answer is (D).

23. From Hooke's Law

Tension in a string $(T) \propto$ extension (x)

and speed of sound in string $v = \sqrt{Tm}$

$\Rightarrow \quad v \propto \sqrt{T}$

Therefore, $v \propto \sqrt{x}$

x is increased to 1.5 times i.e., speed will increase by $\sqrt{1.5}$ times or 1.22 times. Therefore, speed of sound in new position will be 1.22 v.

Hence, the correct answer is (A).

24. Length of the organ pipe is same in both the cases. Fundamental frequency of open pipe is $f_1 = \frac{v}{2\ell}$ and frequency of third harmonic of closed pipe will be

$$f_2 = 3\left(\frac{v}{4\ell}\right)$$

Given that $f_2 = f_1 + 100$

$\Rightarrow \quad f_2 - f_1 = 100$

$\Rightarrow \quad \frac{3}{4}\left(\frac{v}{\ell}\right) - \left(\frac{1}{2}\right)\left(\frac{v}{\ell}\right) = 100$

$\Rightarrow \quad \frac{v}{4\ell} = 100 \text{ Hz}$

$\Rightarrow \quad \frac{v}{2\ell} = 200 \text{ Hz}$

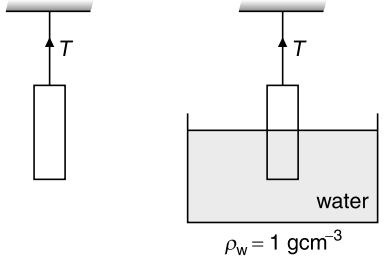
$\Rightarrow \quad f_1 = 200 \text{ Hz}$

Therefore, fundamental frequency of the open pipe is 200 Hz.

Hence, the correct answer is (A).

25. The diagrammatic representation of the given problem is shown in figure. The expression of fundamental frequency is

$$v = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$$



In air $T = mg = (V\rho)g$

$\Rightarrow \quad v = \frac{1}{2\ell}\sqrt{\frac{A\rho g}{\mu}}$ ...(1)

When the object is half immersed in water

$$T' = mg - \text{upthrust} = V_\rho g - \left(\frac{V}{2}\right)\rho_w g$$

$\Rightarrow \quad T' = \left(\frac{V}{2}\right)g(2\rho - \rho_w)$

The new fundamental frequency

$$v' = \frac{1}{2\ell} \times \sqrt{\frac{T}{m}} = \frac{1}{2\ell}\sqrt{\frac{(V_g/2)(2\rho - \rho_w)}{\mu}} \quad \ldots(2)$$

$\Rightarrow \quad \frac{v'}{v} = \left(\sqrt{\frac{2\rho - \rho_w}{2\rho}}\right)$

$\Rightarrow \quad v' = v\left(\frac{2\rho - \rho_w}{2\rho}\right)^{1/2} = 300\left(\frac{2\rho - 1}{2\rho}\right)^{1/2} \text{ Hz}$

Hence, the correct answer is (A).

26. The given equation can be written as

$$y = 2\left(2\cos^2\frac{t}{2}\right)\sin(1000t)$$

$\Rightarrow \quad y = 2(\cos t + 1)\sin(1000t)$

$\Rightarrow \quad y = 2\cos t \sin 1000t + 2\sin(1000t)$

$\Rightarrow \quad y = \sin(1001t) + \sin(999t) + 2\sin(1000t)$

i.e., the given expression is a result of superposition of three independent harmonic motions of angular frequencies 999, 1000 and 1001 rads^{-1}.

Hence, the correct answer is (B).

CHAPTER 4

27. Since $(f_1)_{\text{closed}} = (f_3)_{\text{open}}$

$$\Rightarrow \quad \frac{v}{4\ell_1} = 3\left(\frac{v}{2\ell_2}\right)$$

$$\Rightarrow \quad \frac{\ell_1}{\ell_2} = \frac{1}{6}$$

Hence, the correct answer is (C).

28. Since the point $x = 0$ is a node and reflection is taking place from point $x = 0$. This means that reflection must be taking place from the fixed end and hence the reflected ray must suffer an additional phase change of π or a path change of $\frac{\lambda}{2}$.

So, if $y_{\text{incident}} = a\cos(kx - \omega t)$

$$\Rightarrow \quad y_{\text{reflected}} = a\cos(-kx - \omega t + \pi)$$

$$\Rightarrow \quad y_{\text{reflected}} = -a\cos(\omega t + kx)$$

Hence, the correct answer is (B).

29. Only first equation is the equation of standing wave. The condition for a function of x and t to represent a wave is

$$\frac{\partial^2 y}{\partial x^2} = (\text{constant})\frac{\partial^2 y}{\partial t^2}$$

Only first equation satisfies the condition.
Hence, the correct answer is (A).

30. $f_c = \frac{v}{4\ell} = 512$ Hz

$$f_0 = \frac{v}{2\ell} = 2f_c = 1024 \text{ Hz}$$

Hence, the correct answer is (A).

31. $(v_P)_{\max} = a\omega = y_0\omega$

Since, $(v_P)_{\max} = 4v$

$$\Rightarrow \quad y_0\omega = 4\left(\frac{\omega}{k}\right)$$

$$\Rightarrow \quad y_0 = \frac{4}{k} = \frac{4}{(2\pi/\lambda)}$$

$$\Rightarrow \quad \lambda = \frac{\pi y_0}{2}$$

Hence, the correct answer is (B).

32. Initially the tube was open at both ends and then it is closed. Further

$$f_0 = \frac{v}{2\ell_0} \text{ and } f_c = \frac{v}{4\ell_c}$$

Since, tube is half dipped in water, $\ell_c = \frac{\ell_0}{2}$

$$\Rightarrow \quad f_c = \frac{v}{4\left(\frac{\ell_0}{2}\right)} = \frac{v}{2\ell_0} = f_0 = f$$

Hence, the correct answer is (C).

Multiple Correct Choice Type Questions

1. Let F be tension in the rope of linear mass density (ρ), then $v = \sqrt{\frac{F}{\mu}}$. Hence, speed at any position will be same for both pulses. Therefore, time taken by both pulses will be same.

Since, $\lambda = \frac{v}{f}$ and $v \propto \sqrt{F}$

$$\Rightarrow \quad \lambda \propto v \propto \sqrt{F}$$

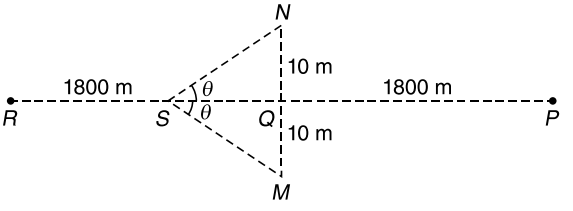
When pulse 1 reaches at A tension F decreases, so speed decreases so, λ decreases.

Conceptual Note(s)

If we refer velocity by magnitude only, then option (A, C, D) will be correct, else only (A, C) will be correct.

Hence, (A), (C) and (D) are correct.

2. Speed of car, $v_C = 60 \text{ kmh}^{-1} = \frac{500}{3} \text{ ms}^{-1}$



At a point S, between P and Q

$$v'_M = v_M\left(\frac{v + v_C\cos\theta}{v}\right) \text{ and } v'_N = v_N\left(\frac{v + v_C\cos\theta}{v}\right)$$

$$\Rightarrow \quad \Delta v = (v_N - v_M)\left(1 + \frac{v_C\cos\theta}{v}\right)$$

Similarly, between Q and R, we have

$$\Delta v = (v_N - v_M)\left(1 - \frac{v_C\cos\theta}{v}\right)$$

$$\Rightarrow \quad \frac{d(\Delta v)}{dt} = \pm(v_N - v_M)\frac{v_C}{v}\sin\theta\left(\frac{d\theta}{dt}\right)$$

At P and R, the car is very far from M and N, so $\theta \approx 0°$. Hence, slope of graph is zero.

At Q, we have $\theta = 90°$, so $\sin\theta$ is maximum. Also value of $\frac{d\theta}{dt}$ is maximum as $\left(\frac{d\theta}{dt}\right)_Q = \frac{v}{r_{\min}} = \frac{v}{10}$ constant

Hence, slope is maximum at Q.

At P, $v_P = \Delta v = (v_N - v_M)\left(1 + \frac{v_C}{v}\right)$ $\{\because \theta = 0°\}$

At R, $v_R = \Delta v = (v_N - v_M)\left(1 - \frac{v_C}{v}\right)$ $\{\because \theta = 0°\}$

At Q, $v_Q = \Delta v = v_N - v_M$ $\{\because \theta = 90°\}$

From these equations, we get

$$v_P + v_R = 2v_Q$$

Hence, (A), (B) and (C) are correct.

3. Minimum resonance length is $\ell_1 = \frac{\lambda}{4}$

$$\Rightarrow \quad \lambda = 4\ell_1$$

$$\Rightarrow \quad v = f\lambda = (244)(4\ell_1)$$

where $\ell_1 = (0.350 \pm 0.005)$ m

So, v lies between 336.7 ms^{-1} to 346.5 ms^{-1}

Now, $v = \sqrt{\dfrac{\gamma RT}{M \times 10^{-3}}}$, here M is molar mass of gas in gram

$\Rightarrow \quad v = \sqrt{100\gamma RT} \times \sqrt{\dfrac{10}{M}}$

For monoatomic gas i.e., for neon and argon, $\gamma = 1.67$

$\Rightarrow \quad v = 640 \times \sqrt{\dfrac{10}{M}}$

For diatomic gas i.e., for oxygen and nitrogen, $\gamma = 1.4$

$\Rightarrow \quad v = 590 \times \sqrt{\dfrac{10}{M}}$

$v_{Ne} = 640 \times \dfrac{7}{10} = 448$ ms^{-1}, $v_{Ar} = 640 \times \dfrac{17}{32} = 340$ ms^{-1}

$v_{O_2} = 590 \times \dfrac{9}{16} = 331.8$ ms^{-1}, $v_{N_2} = 590 \times \dfrac{3}{5} = 354$ ms^{-1}

Only possible answer is Argon.

Hence, the correct answer is (D).

4. There should be a node at $x = 0$ and antinode at $x = 3$ m.

Also, $v = \dfrac{\omega}{k} = 100$ ms^{-1}

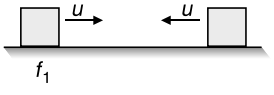
Hence $y = 0$ at $x = 0$ and $y = \pm A$ at $x = 3$ m

Only (A), (C) and (D) satisfy the condition.

Hence, (A), (C) and (D) are correct.

5. $f_2 = \left(\dfrac{V - w + u}{V - w - u}\right) f_1$

So, $f_2 > f_1$



$f_2 = \left(\dfrac{V + w + u}{V + w - u}\right) f_1$

So, $f_2 > f_1$

Hence, (A) and (B) are correct.

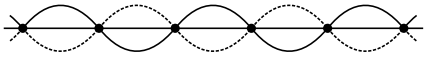
6. At open end phase of pressure wave changes by 180°. So, compression returns as rarefaction. At closed end there is no phase change. So, compression returns as compression returns as compression and rarefaction as rarefaction.

Hence, (A), (B) and (D) are correct.

7. For fifth harmonic, number of loops formed is 5, so

No. of nodes = 6

$\lambda = \dfrac{2\pi}{k} = \dfrac{2 \times 3.14}{62.8} = 0.1$ m



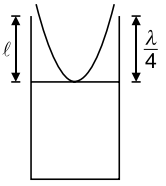
Length $\ell = \dfrac{5\lambda}{2} = 0.25$ m

The mid-point is antinode, so its maximum displacement is 0.01 m

$\Rightarrow \quad f = \dfrac{v}{2\ell} = \dfrac{\omega}{k(2\ell)} = 20$ Hz

Hence, (B) and (C) are correct.

8. Since $\ell < \dfrac{\lambda}{4}$



Further, larger the length of air column, weaker is the intensity.

Hence, (A) and (D) are correct.

9. Standing waves can be produced only when two similar type of waves (same frequency and speed, but amplitude may be different) travel in opposite directions.

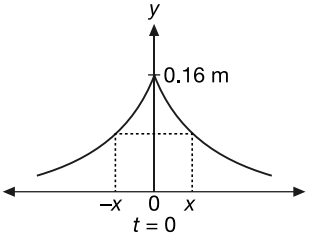
Hence, (A), (B) and (C) are correct.

10. In case of sound wave, y can represent pressure and displacement, while in case of an electromagnetic wave it represents electric and magnetic fields.

NOTE: In general, y is any general physical quantity which is a made to oscillate at one place and these oscillations are propagated to other places also.

Hence, (A), (B), (C) and (D) are correct.

11. The shape of pulse at $x = 0$ and $t = 0$ would be as shown, in Figure.



$y(0, 0) = \dfrac{0.8}{5} = 0.16$ m

From the figure it is clear that $y_{max} = 0.16$ m

Pulse will be symmetric (Symmetry is checked about y_{max}) if at $t = 0$

$y(x) = y(-x)$

From the given equation

$y(x) = \dfrac{0.8}{16x^2 + 5}$

and $y(-x) = \dfrac{0.8}{16x^2 + 5}$ {at $t = 0$}

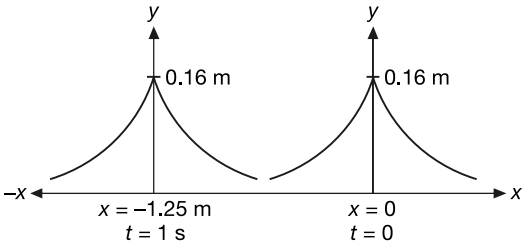
$\Rightarrow \quad y(x) = y(-x)$

Therefore, pulse is symmetric.

Speed of pulse

At $t = 1$ s and $x = -1.25$ m value of y is again 0.16 m, i.e., pulse has travelled a distance of 1.25 m in 1 s in negative x-direction or we can say that the speed of pulse is 1.25 ms^{-1} and it is travelling in negative x-direction. Therefore, it will travel a distance of 2.5 m in 2 s. The above statement can be better understood from Figure.

CHAPTER 4



Alternate Method:

If equation of a wave pulse is

$$y = f(ax \pm bt)$$

The speed of wave is $\frac{b}{a}$ in negative x-direction for $y = f(ax + bt)$ and positive x-direction for $y = f(ax - bt)$. Comparing this from given equation we can find that speed of wave is $\frac{5}{4} = 1.25\ \text{ms}^{-1}$ and it is travelling in negative x-direction.

Hence, (B), (C) and (D) are correct.

12. For a plane wave intensity (energy crossing per unit area per unit time) is constant at all points.
But for a spherical wave, intensity at a distance r from a point source of power P (energy transmitted per unit time) is given by

$$I = \frac{P}{4\pi r^2} \text{ i.e., } I \propto \frac{1}{r^2}$$

For a line source $I \propto \frac{1}{r}$, because $I = \frac{P}{\pi r \ell}$

Hence, (A), (C) and (D) are correct.

13. Maximum speed of any point on the string is $v_{max} = a\omega$

$$\Rightarrow \quad v_{max} = a(2\pi f)$$

$$\Rightarrow \quad v_{max} = \frac{v}{10} = \frac{10}{10} = 1 \qquad \{\text{Given: } v = 10\ \text{ms}^{-1}\}$$

$$\Rightarrow \quad 2\pi af = 1$$

$$\Rightarrow \quad f = \frac{1}{2\pi a}$$

Since, $a = 10^{-3}$ m

$$\Rightarrow \quad f = \frac{1}{2\pi \times 10^{-3}} = \frac{10^3}{2\pi}\ \text{Hz}$$

Speed of wave, $v = f\lambda$

$$\Rightarrow \quad (10\ \text{ms}^{-1}) = \left(\frac{10^3}{2\pi}\ \text{s}^{-1}\right)\lambda$$

$$\Rightarrow \quad \lambda = 2\pi \times 10^{-2}\ \text{m}$$

Hence, (A) and (C) are correct.

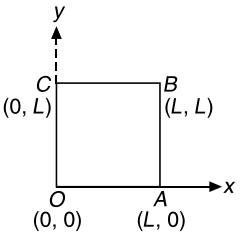
14. Since, the edges are clamped, displacement of the edges $u(x, y) = 0$ for

Line OA i.e., $y = 0, 0 \le x \le L$

AB i.e., $x = L, 0 \le y \le L$

BC i.e., $y = L\ 0 \le x \le L$

OC i.e., $x = 0, 0 \le y \le L$



The above conditions are satisfied only in alternatives (B) and (C).
Note that $u(x, y) = 0$, for all four values e.g., in alternative (D), $u(x, y) = 0$ for $y = 0$, $y = L$ but it is not zero for $x = 0$ or $x = L$.
Similarly, in OPTION (A) $u(x, y) = 0$ at $x = L$, $y = L$ but it is not zero for $x = 0$ or $y = 0$ while in OPTIONS (B) and (C), $u(x, y) = 0$ for $x = 0$, $y = 0$, $x = L$ and $y = L$.

Hence, (B) and (C) are correct.

15. $y = 0.02\cos(10\pi x)\cos\left(50\pi t + \frac{\pi}{2}\right)$

At node, amplitude = 0

$$\Rightarrow \quad \cos(10\pi x) = 0$$

$$\Rightarrow \quad 10\pi x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\Rightarrow \quad x = \frac{1}{20} = 0.05\ \text{m}, 0.15\ \text{m}.....$$

At antinode, amplitude is maximum

$$\Rightarrow \quad \cos(10\pi x) = \pm 1$$

$$\Rightarrow \quad x = 0, \pi, 2\pi...$$

$$\Rightarrow \quad x = 0, 0.1\ \text{m}, 0.2\ \text{m}.....$$

Now λ is twice the distance between two nodes or antinodes

$$\Rightarrow \quad \lambda = 2(0.1) = 0.2\ \text{m and } \frac{2\pi vt}{\lambda} = 50\pi t$$

$$\Rightarrow \quad v = 25\lambda = 25 \times 0.2 = 5\ \text{ms}^{-1}$$

Hence, (A), (B), (C) and (D) are correct.

16. The number of waves encountered by the moving plane per unit time is given by

$$n = \frac{\text{distance travelled}}{\text{wavelength}}$$

$$\Rightarrow \quad n = \frac{c+v}{\lambda} = \frac{c}{\lambda}\left(1 + \frac{v}{c}\right) = f\left(1 + \frac{v}{c}\right) \qquad \{\text{OPTION (A)}\}$$

The stationary observer meets the frequency f' of the incident wave and receives the reflected wave of frequency f'' emitted by the moving platform as

$$f'' = \frac{f'}{1 - \frac{v}{c}} = \frac{f\left(1 + \frac{v}{c}\right)}{1 - \frac{v}{c}} = \frac{f(c+v)}{(c-v)} \qquad \{\text{OPTION (C)}\}$$

$$\text{Wavelength, } \lambda'' = \frac{c}{f''} = \frac{c}{f}\left(\frac{c-v}{c+v}\right) \qquad \{\text{OPTION (B)}\}$$

Beat frequency, $f_b = f'' - f$

$$\Rightarrow \quad f_b = \frac{f\left(1 + \frac{v}{c}\right)}{\left(1 - \frac{v}{c}\right)} - f = f\left(\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}} - 1\right)$$

$$\Rightarrow \quad f_b = \frac{\left(1+\frac{v}{c}\right)f}{\left(1-\frac{v}{c}\right)} = \frac{2vf}{c-v}$$

Hence, (A), (B) and (C) are correct.

17. $T_1 > T_2$

$\Rightarrow \quad v_1 > v_2$

$\Rightarrow \quad f_1 > f_2$ and $f_1 - f_2 = 6$ Hz

Now, if T_1 is increased, f_1 will increase or $f_1 - f_2$ will increase. Therefore (D) OPTION is wrong.

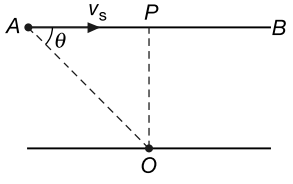
If T_1 is decreased f_1 will decrease and it may be possible that now $f_2 - f_1$ become 6 Hz. Therefore (C) OPTION is correct.

Similarly, when T_2 is increased, f_2 will increase and again $f_2 - f_1$ may become equal to 6 Hz. Therefore (C) OPTION is correct.

Similarly, when T_2 is increased, f_2 will increase and again $f_2 - f_1$ may become equal to 6 Hz. So, (B) is also correct but (A) is wrong.

Hence, (B) and (C) are correct.

18. The graph shows the situation shown in figure. The observed frequency will initially be more than the natural frequency. When the source is at P, natural frequency i.e., 2000 Hz.



For region AP : $f = f_0\left(\frac{v}{v - v_s\cos\theta}\right)$

For PB : $f = f_0\left(\frac{v}{v + v_s\cos\theta}\right)$

Minimum value of f will be

$$f_{\min} = f_0\left(\frac{v}{v+v_s}\right) \qquad \text{when } \cos\theta = 1$$

$$\Rightarrow \quad 1800 = 2000\left(\frac{300}{300+v_s}\right)$$

Solving this we get,

$v_s = 33.33$ ms^{-1} and maximum value of f can be

$$f_{\max} = f_0\left(\frac{v}{v-v_0}\right) \qquad \text{when } \cos\theta = 1$$

$$\Rightarrow \quad f_{\max} = 2000\left(\frac{300}{300-33.33}\right) = 2250 \text{ Hz}$$

Hence, (C) and (D) are correct.

19. For closed pipe, $f = n\left(\frac{v}{4\ell}\right) n = 1, 3, 5.....$

For $n = 1$, $f_1 = \frac{v}{4\ell} = \frac{320}{4\times 1} = 80$ Hz

For $n = 3$, $f_3 = 3f_1 = 240$ Hz

For $n = 5$, $f_5 = 5f_1 = 400$ Hz

Hence, (A), (B) and (D) are correct.

20. For closed organ pipe,

$$f = n\left(\frac{v}{4\ell}\right) \text{ where, } n = 1, 3, 5, ...$$

$$\Rightarrow \quad \ell = \frac{nv}{4f}$$

For $n = 1$, $\ell_1 = \frac{(1)(330)}{4\times 264}\times 100 \text{ cm} = 31.25$ cm

For $n = 3$, $\ell_3 = 3\ell_1 = 93.75$ cm

For $n = 5$, $\ell_5 = 5\ell_1 = 156.25$ cm

Hence, (A) and (C) are correct.

21. $y = 10^{-4}\sin(60t + 2x)$

$A = 10^{-4}$ m, $\omega = 60$ rads^{-1}, $k = 2$ m^{-1}

Speed of wave $v = \frac{\omega}{k} = 30$ ms^{-1}

Frequency, $f = \frac{\omega}{2\pi} = \frac{30}{\pi}$ Hz

Wavelength, $\lambda = \frac{2\pi}{k} = \pi m$

Further, $60t$ and $2x$ are of same sign. Therefore, the wave should travel in negative x-direction.

Hence, (A), (B), (C) and (D) are correct.

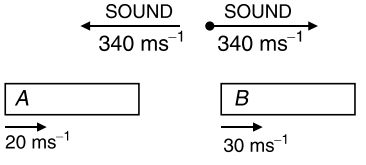
Linked Comprehension Type Questions

1. Velocity of sound w.r.t. passengers in train A is

$V_{SA} = 340 + 20 = 360$ ms^{-1}

Velocity of sound w.r.t. passengers in train B is

$V_{SB} = 340 - 30 = 310$ ms^{-1}



Hence, the correct answer is (B).

2. For the passengers in train A, there is no relative motion between source and observer, as both are moving with velocity 20 ms^{-1}. Therefore, there is no change in observed frequencies and correspondingly there is no change in their intensities.

Hence, the correct answer is (A).

3. For the passengers in train B, observer is receding with velocity 30 ms^{-1} and source is approaching with velocity 20 ms^{-1}.

$$\Rightarrow \quad f_1' = 800\left(\frac{340-30}{340-20}\right) = 775 \text{ Hz}$$

and $f_2' = 1120\left(\frac{340-30}{340-20}\right) = 1085$ Hz

CHAPTER 4

Spread of frequency $\Delta f' = f_2' - f_1' = 310$ Hz

Hence, the correct answer is (A).

4. In one second number of maximas is called the beat frequency. Hence,

$$f_b = f_1 - f_2$$

$$\Rightarrow \quad f_b = \frac{100\pi}{2\pi} - \frac{92\pi}{2\pi}$$

$$\Rightarrow \quad f_b = 4 \text{ Hz}$$

Hence, the correct answer is (A).

5. Speed of wave $v = \frac{\omega}{k}$

$$\Rightarrow \quad v = \frac{100\pi}{0.5\pi} = \frac{92\pi}{0.46\pi}$$

$$\Rightarrow \quad v = 200 \text{ ms}^{-1}$$

Hence, the correct answer is (A).

6. At $x = 0$, $y = y_1 + y_2$

$$x = 2A\cos(96\pi t)\cos(4\pi t)$$

Frequency of $\cos(96\pi t)$ function is 48 Hz and that of $\cos(4\pi t)$ function is 2 Hz.

In one second cos function becomes zero at $2f$ times, where f is the frequency. Therefore, first function will become zero at 96 times and the second at 4 times. But second will not overlap with first. Hence, net y will become zero 100 times in 1 s.

Hence, the correct answer is (C).

Matrix Match Type Questions

1. A → (p, r, s, q); B → (q, p, r, t); C → (q, s, r, p); D → (p, q, t, s)

(1) $f_0 = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$

(2) $f_1 = \frac{1}{2L_0}\sqrt{\frac{T_0}{2\mu}}$

(3) $f_2 = \frac{1}{2L_0}\sqrt{\frac{T_0}{3\mu}}$

(4) $f_3 = \frac{1}{2L_0}\sqrt{\frac{T_0}{4\mu}}$

For all highest fundamental is when length is L_0

2. A → (p, q, r, t); B → (p, q, t, u); C → (p, r, t, u); D → (t, q, r, u)

(1) $f_0 = \frac{1}{2L_0} \times \sqrt{\frac{T_1}{\mu}} = \frac{1}{2L}\sqrt{\frac{T_0}{\mu}}$

(2) $f_0 = \frac{3\times 2}{2\times 3L_0}\sqrt{\frac{T_2}{2\mu}}$

$$\Rightarrow \quad T_2 = \frac{T_0}{2}$$

(3) $f_0 = \frac{5\times 2}{5L_0}\sqrt{\frac{T_3}{3\mu}}$

$$\Rightarrow \quad T_3 = \frac{3T_0}{16}$$

(4) $f_0 = \frac{14\times 4}{2\times 7L_0}\sqrt{\frac{T_4}{4\mu}}$

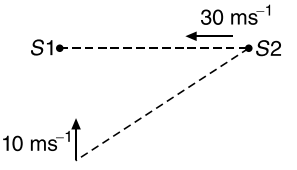
$$\Rightarrow \quad T_4 = \frac{T_0}{16}$$

3. A → (p, t); B → (p, s); C → (q, s); D → (q, r)

In organ pipes, longitudinal waves exist. In strings, transverse waves exist. Open end is an antinode, fixed end is a node. The least distance between node and antinode is $\frac{\lambda}{4}$ and between two nodes is $\frac{\lambda}{2}$.

Integer/Numerical Answer Type Questions

1. $v_{\text{sound}} = 330 \text{ ms}^{-1}$



$$f_1 = 120\left[\frac{330 + 10\sin 53^\circ}{330 - 30\cos 37^\circ}\right] \text{ Hz}$$

$$f_2 = 120\left[\frac{330 + 10}{330}\right] \text{ Hz}$$

$$\Delta f = 120 \times \left[\frac{336}{306} - \frac{34}{33}\right] = 8.13 \text{ Hz}$$

2. Frequency observed at car

If v is speed of sound and v_C is speed of car, then

$$f_1 = f_0\left(\frac{v + v_C}{v}\right)$$

Frequency of reflected sound as observed at the source

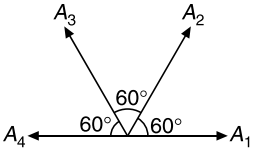
$$f_2 = f_1\left(\frac{v}{v - v_C}\right) = f_0\left(\frac{v + v_C}{v - v_C}\right)$$

Beat frequency $\Delta f = f_2 - f_0$

$$\Rightarrow \quad \Delta f = f_0\left(\frac{v + v_C}{v - v_C} - 1\right) = f_0\left(\frac{2v_C}{v - v_C}\right)$$

$$\Rightarrow \quad \Delta f = 492 \times \frac{2\times 2}{328} = 6 \text{ Hz}$$

3. Let individual amplitudes are A_0 each. Amplitudes can be added by vector method.



$$A_1 = A_2 = A_3 = A_4 = A_0$$

Resultant of A_1 and A_4 is zero. Resultant of A_2 and A_3 is

$$A = \sqrt{A_0^2 + A_0^2 + 2A_0A_0\cos 60^\circ} = \sqrt{3}A_0$$

This is also the net resultant.

Now, $I \propto A^2$

$\Rightarrow$ Net intensity will become $3I_0$

4. $A = \sqrt{4^2 + 3^2 + 2 \times 4 \times 3 \times \cos\left(\frac{\pi}{2}\right)} = 5$

5. $\ell = 20$ cm, $m = 1$ g, $T = 0.5$ N, $f = 100$ Hz

According to Melde's Formula, we have

$$f = \frac{n}{2\ell}\sqrt{\frac{T}{\mu}}$$

$$\Rightarrow \quad 100 = \frac{n}{2 \times 20 \times 10^{-2}}\sqrt{\frac{0.5}{10^{-3}/(20 \times 10^{-2})}}$$

$$\Rightarrow \quad 1 = \frac{n}{40}\sqrt{0.5 \times 20 \times 10} = \frac{n}{40} \times 10$$

$$\Rightarrow \quad n = 4$$

So, $l = \frac{n\lambda}{2}$

Separation between successive nodes on a string is

$$\frac{\lambda}{2} = \frac{20}{4} = 5 \text{ cm}$$